Primordial Black Hole catalyze the First-Order Electroweak Phase Transition and its Parameter Space

Jiang Zhu

Preliminary work 250x.xxxx
In collaboration with Michael J. Ramsey-Musolf

李政道研究所
TSUNG-DAO LEE INSTITUTE

Introduction

FOPT in early universe is a probe to detect BSM

- ***The question:***
  How did the PBH affect the PT in the early universe

Uncertainty！ ！

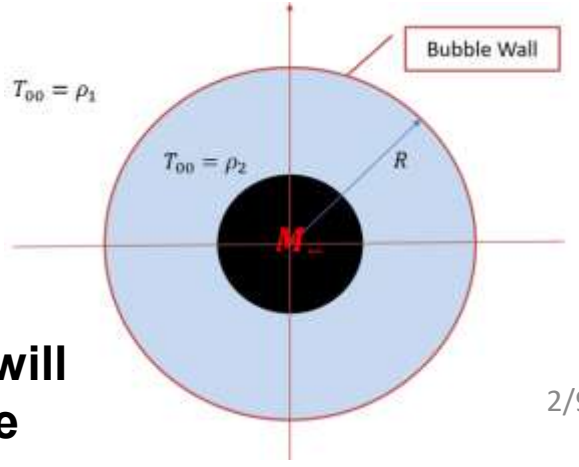
➢ **Phase Transition in Condense Matter**

Put a hot stone in superheated water

- Impurity will drametically affect the Phase transitions in Condense Matter systems

➢ **PBH as an impurity of Early Universe**

- The PBH as then impurity of the early universe will dramatically affect the phase transition and maybe change change **the viable parameter** **&** **Observation signal**.



Take Home Massage: The existence of the PBH will significantly increase the viable parameter space

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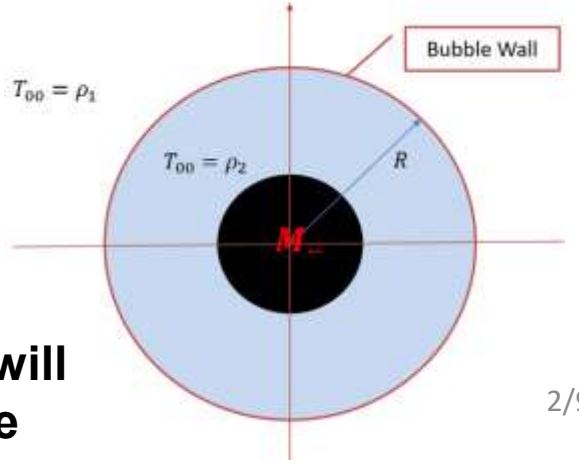


Take Home Massage: The existence of the PBH will significantly increase the viable parameter space

Nucleation in Black Hole Background

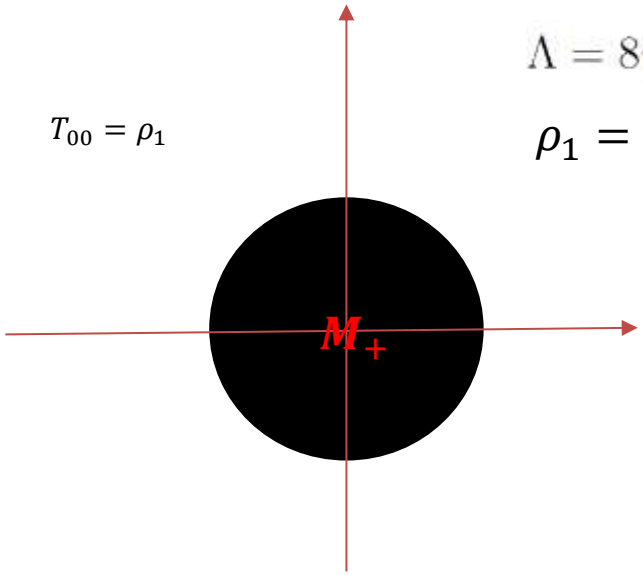
- **Nucleation for Gravity Tunneling**

Before bubble appeared

$$\Lambda = 8\pi G V_{eff}(\Phi)$$

$$\rho_1 = V(\phi_f)$$

$T_{00} = \rho_1$



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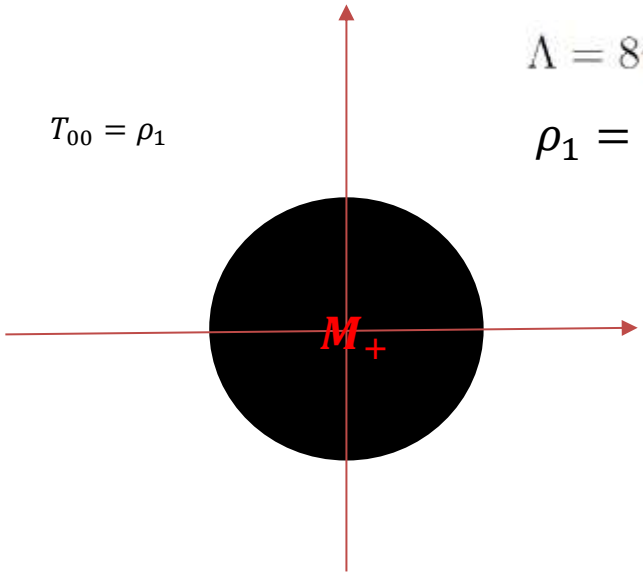

- PBH+Bubble (scalar field) impact on the metric tensor:

$$ds^2 = f(r)e^{2\delta(r)}d\tau^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,,$$

Metric effect of departure from Vacuum solution

Metric effect of Horizon, just like Schwarzschild BH $f(r) = 1 - \frac{2G\mu(r)}{r}$.

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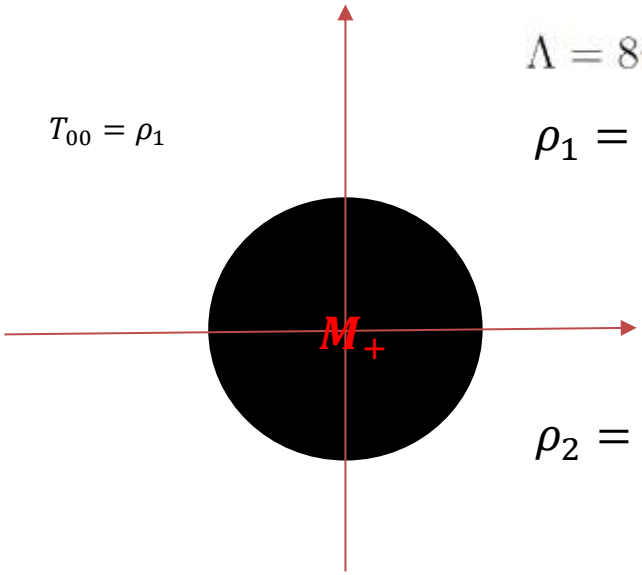
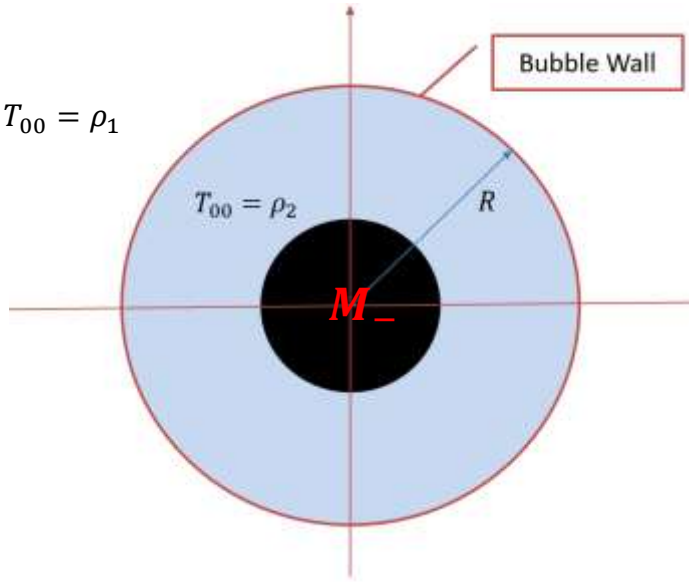


$\rho_2 = V(\phi_t) \approx 0$

After bubble appeared



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Nucleation in Black Hole Background

- **Nucleation for Gravity Tunneling vs Thermal Tunneling**
  - General tunneling Rate $\sim A\, e^{-B}$

$$\Gamma_{th} \sim \int_0^{E_{sp}} dE e^{-\frac{E}{T}} e^{-B(E)}$$

Boltzmann factor for thermal fluctuation with energy E_*

Probability for bubble tunneling from the QM

  - Thermal tunneling: $T = T_{plasma}$
  - PBH tunneling[1] : $T = TBH$ (Hawking temperature)

If, $T_{BH} \gg T_{pl}$, then the vacuum decay rate around the black hole will be significantly larger than the thermal tunneling rate

W. A. Hiscock. 1987, 1992; R. Gregory 2014, 2015, 2016, 2020, 2023; K. Mukaida 2017, 2019; B. K El-Menoufi 2020; A. Shkerin 2021, 2022; L. Heurtier 2023; Z-M Zeng 2024.

Application of PBH effect in BSM: ΣSM

- **Real Triplet Model Thermal History**

Model: ΣSM 2203.05889

$$V(H,\Sigma) = -\mu_H^2 H^\dagger H - \frac{1}{2}\mu_\Sigma^2 \mathrm{Tr}(\Sigma^2) + \lambda_H (H^\dagger H)^2 + \frac{1}{4} b_4 [\mathrm{Tr}(\Sigma^2)]^2 + \frac{1}{2} a_2 \mathrm{Tr}(\Sigma^2) H^\dagger H ,$$

Free parameter:

$$m_\Sigma \qquad a_2 \qquad b_4$$

[1] Andreas Ekstedt, Philipp Schicho, and Tuomas V. I. Tenkanen. DRalgo: A package for effec-tive field theory approach for thermal phase transitions. Comput. Phys. Commun., 288:108725,2023.

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To get the viable parameter space

- The finite temperature effective potential: 3DEFT[1]

⇩

- Solve the bounce equation& vacuum Decay: $\Gamma = Ae^{-B}$

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- Nucleation conditions: $B \sim 140$

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By short solve the bounce equation in curved spacetime + Einstein equation

Details in backup slides

⇩

Become different with PBH

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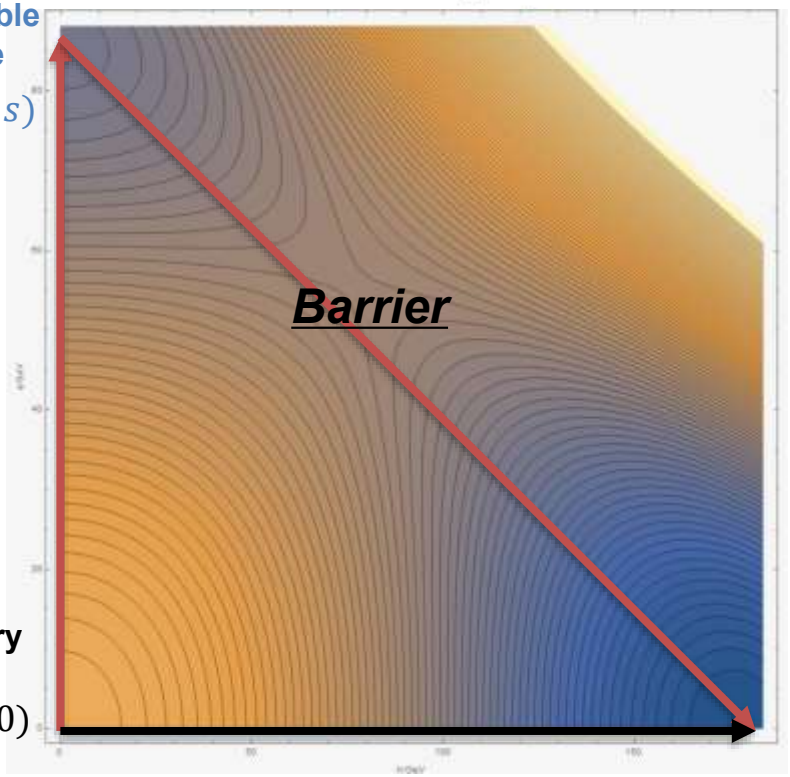

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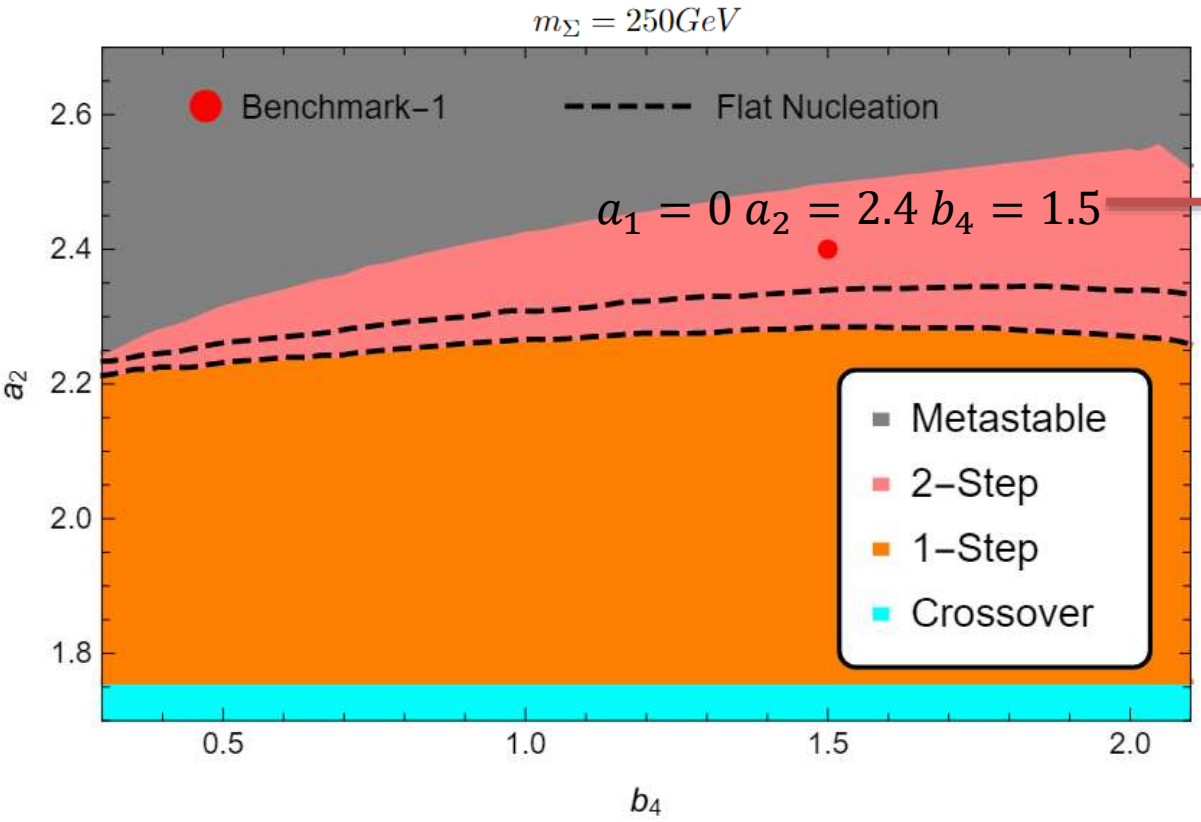
- **Phase Diagram**

Nucleation conditions $B \sim 140$



To study the PBH effect on this parameter regions

Application of PBH effect in BSM: ΣSM

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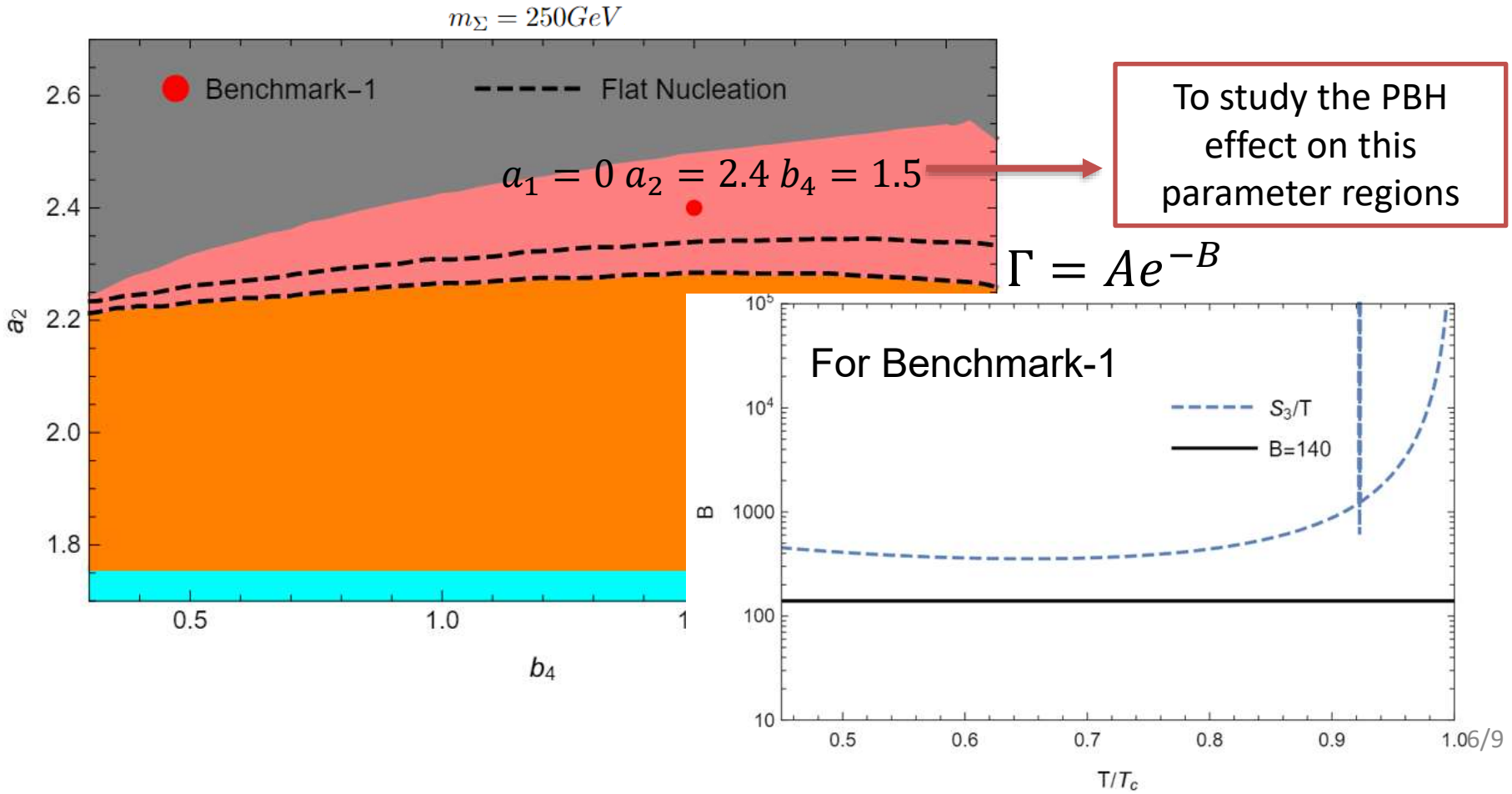
Nucleation conditions $B \sim 140$



Application of PBH effect in BSM: ΣSM

- **Numerical result of Bounce equation**

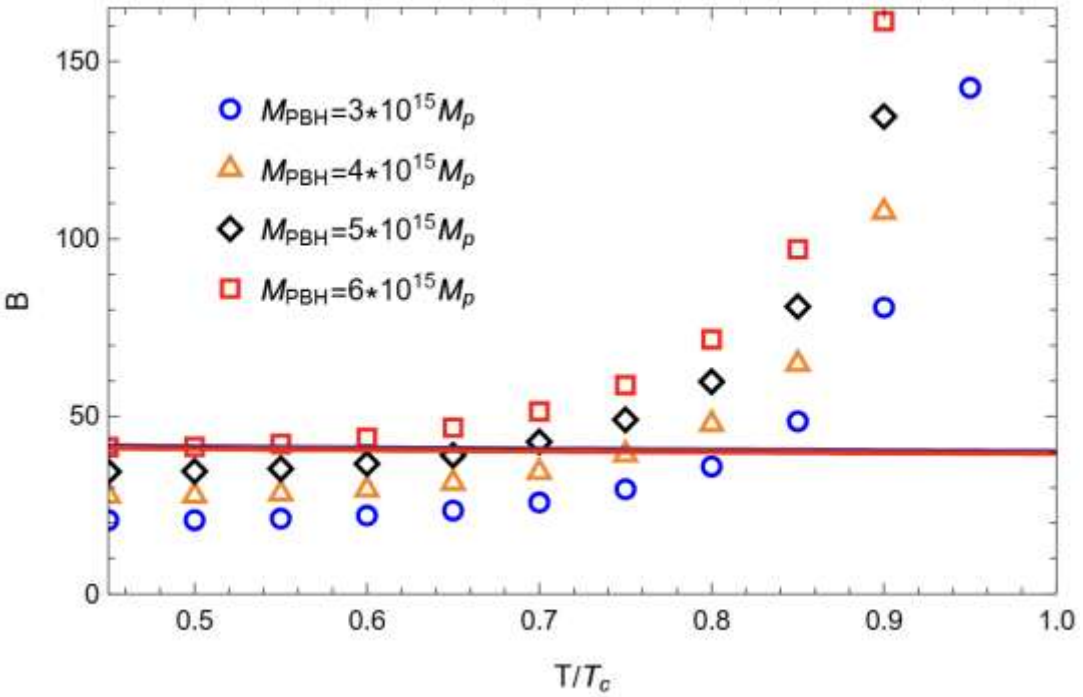
Number of the PBH per Hubble volume constraint by BBN[1]

$$\frac{n_{PBH}}{H^3} \sim 10^3 \quad \xrightarrow{\text{Minimum}} \quad \frac{n_{PBH}}{H^3} = 1$$

One bubble at 1 Hubble volume:

Nucleation conditions for PBH case is $B \sim \log\left(\frac{M_p^3}{T^2 M_{PBH}}\right)$

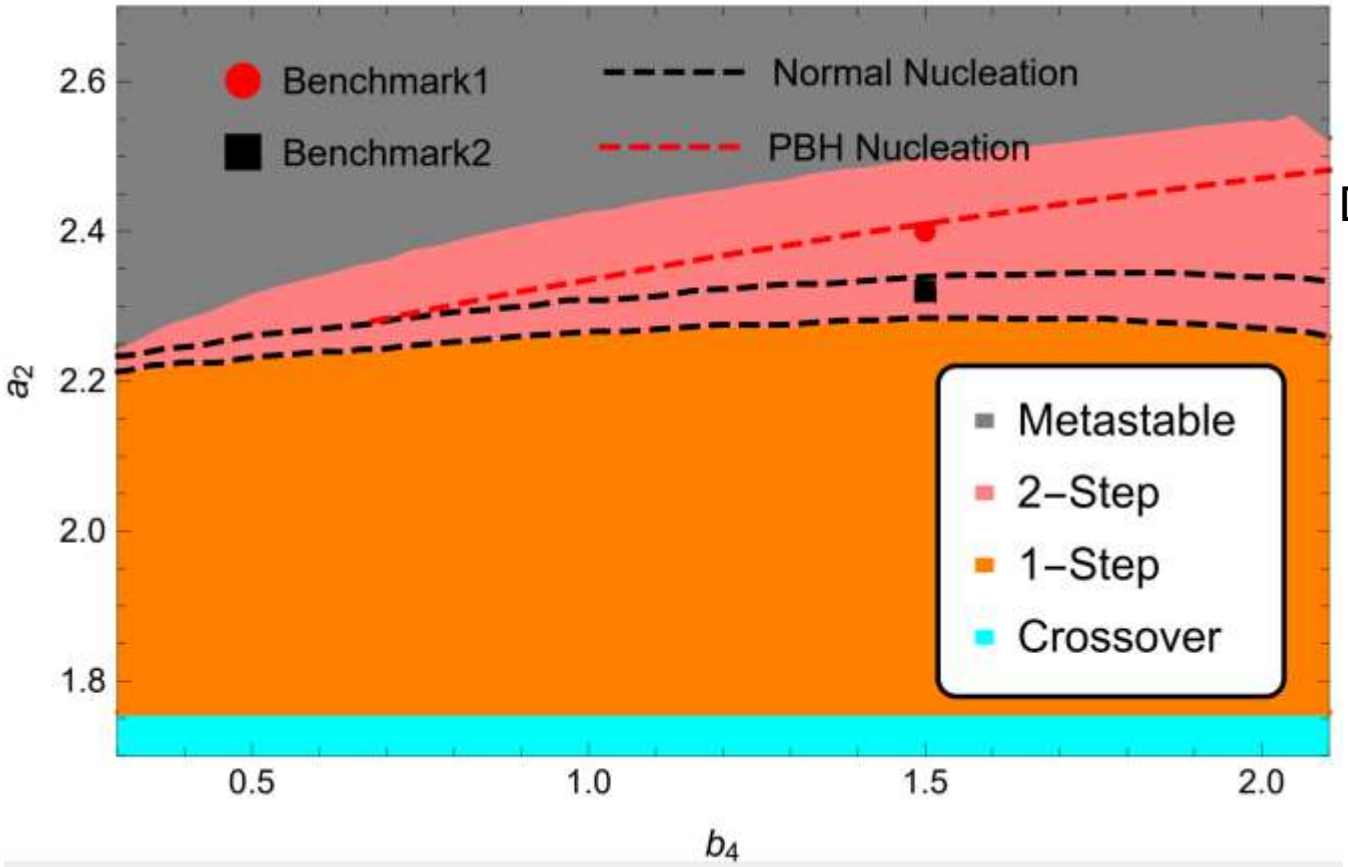
$$\Gamma = Ae^{-B}$$



Application of PBH effect in BSM: ΣSM

- **For Parameter Space**

$M_{PBH} = 5 \times 10^{15} M_{pl}$ $m_\Sigma = 250GeV$ $\frac{n_{PBH}}{H^3} = 1$



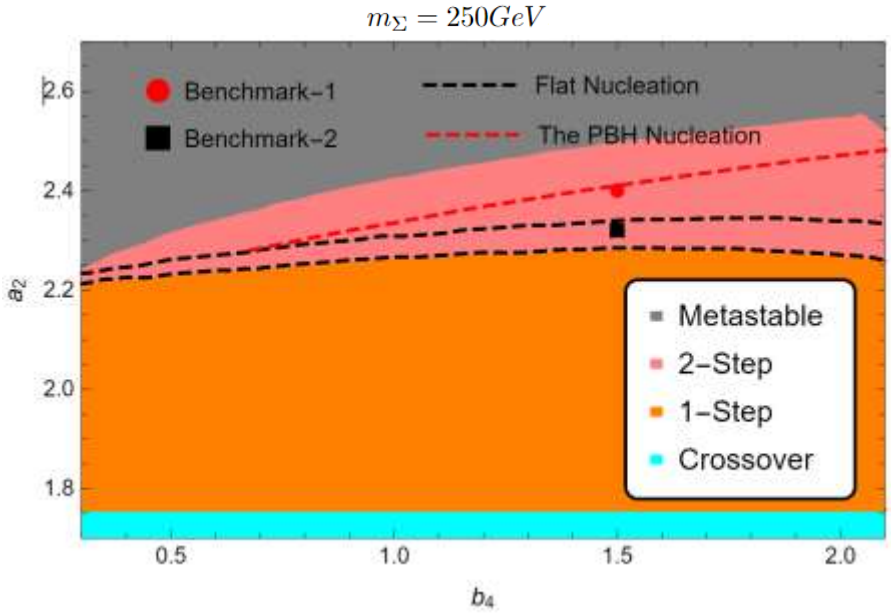
Doubled!

A little bit of PBH can significantly increase the viable parameter space!

Summary

- **Summary**

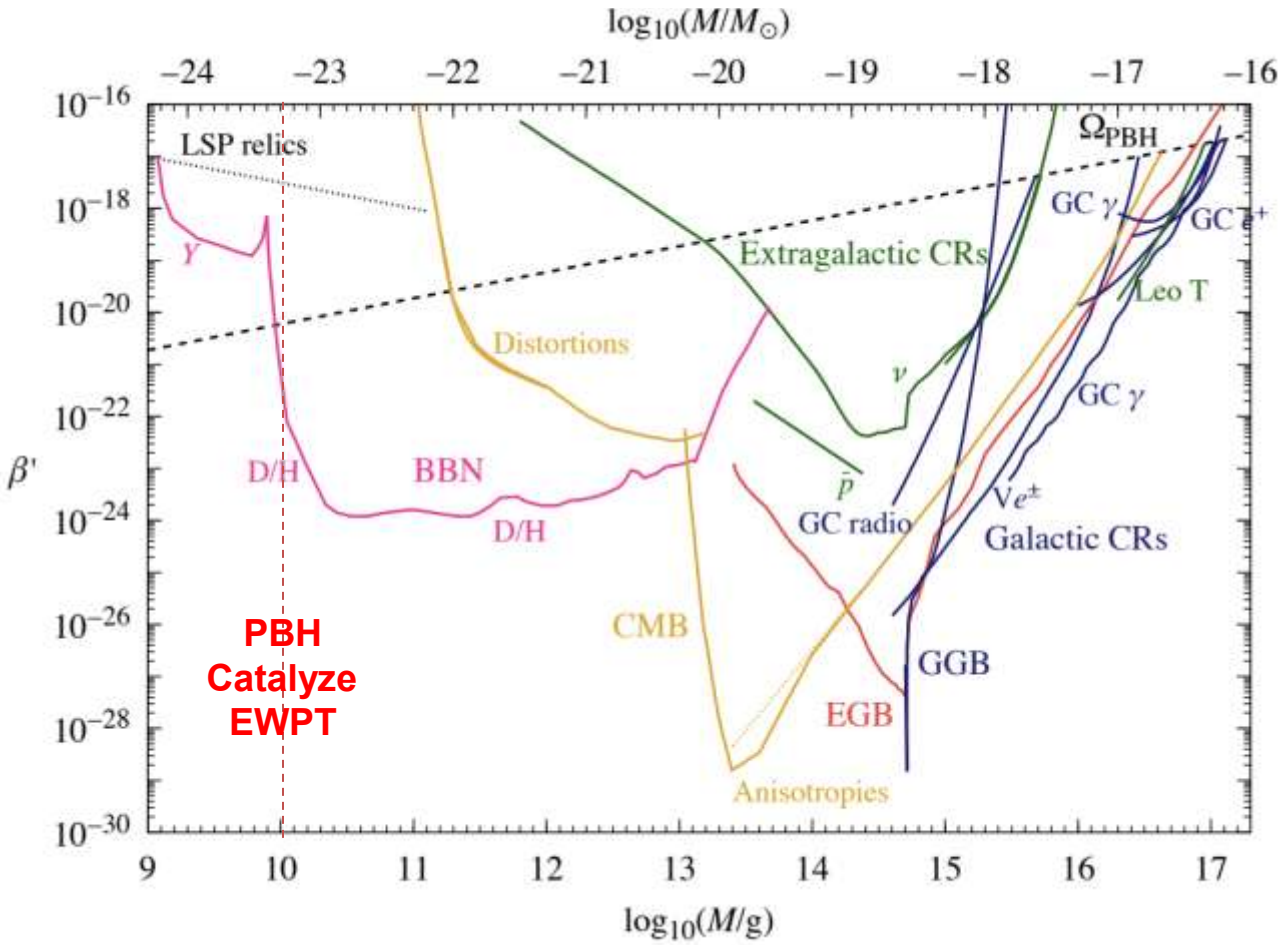
1. The PBH canb dramatically increase the phase transition in the early universe.
2. The PBH with $M_{PBH} \sim 10^{10} g$ will make the electroweak phase transition in BSM become easier to complete.
3. The existence of PBH will enlarge the corresponding parameter space for FOPT in BSM. A small PBH distribution may even double the parameter space



Backup: Constrain

- **Constrain on PBH**

$$Y_{\rm PBH} \equiv n_{\rm PBH}/s. \quad \beta' = 5.4\times10^{21}\left(\frac{\tau}{1\,{\rm s}}\right)^{1/2} Y_{\rm PBH}\,.$$



Nucleation in Black Hole Background

- **Nucleation for Thermal Tunneling in Flat Space Time vs Gravity Tunneling**

The tunneling rate is formulated by $\Gamma = Ae^{-B}$

	Flat spacetime	Curved spacetime
Action	$I[\Phi] = \int_0^\beta dt \int d^3\mathbf{x}\left[\frac{1}{2}(\nabla\Phi)^2 + V_{eff}(\Phi)\right]$	$I_{EH}[T] = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \left[\frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + V_{eff}(\Phi)\right]\right)$
Bounce Action	$B = I[\Phi_b] - I[\Phi_f]$	$B[T] = I_{EH}[T] - I_{EH}^{NB}[T]$
Scalar EoM	$\Phi_b'' + \frac{2}{r}\Phi_b' = \frac{\partial V}{\partial\Phi}$	$f\Phi'' + f'\Phi' + \frac{2}{r}f\Phi' + \delta' f\Phi' = \frac{\partial V_{eff}}{\partial\Phi}$
Gravity EoM	NONE	$\mu' = 4\pi r^2(\frac{1}{2}f\Phi'^2 + V_{eff}) \quad \delta' = 4\pi G r\Phi'^2$

Backup: General Spherical Metric

- **Gravity-Bounce equation**

Thermal nucleation action, the essential different is the metric

$$I_{\rm b}[T] = \beta \int dx^3 \sqrt{-h} \left(-\frac{R}{16\pi G} + \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right)$$

- The general spherical symmetric metric is

$$ds^2 = A(r) d\tau^2 + B(r) dr^2 + r^2 (d\theta^2 + \sin\theta d\phi^2)$$

The Einstein equation is $G_{\mu\nu} = -8\pi G T_{\mu\nu}$, if $T_{\mu\nu} = 0$, then $A(r)B(r) = 1$. If $T_{\mu\nu} \neq 0$, then normally $A(r)B(r) \neq 1$.

- For Vacuum solution $T_{\mu\nu} = 0$, the metric can be write as

$$ds^2 = f(r) d\tau^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin\theta d\phi^2)$$

$f(r) = 1$ is the flat space solution, $f(r) = 1 - \frac{2GM}{r}$ is the Schwarzschild solution

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- The general spherical symmetric metric is

$$ds^2 = A(r) d\tau^2 + B(r) dr^2 + r^2 (d\theta^2 + \sin\theta d\phi^2)$$

- However if the bubble is exist then $T^{\mu\nu} \neq 0$, we need a parameter to different $A\&B$ by $AB \neq 1$, we can parametrize this metric as

$$ds^2 = f(r) e^{2\delta(r)} d\tau^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)\,, \qquad f(r) = 1 - \frac{2G\mu(r)}{r}\,.$$

- For the system we want to solve

  Inside the bubble is the ***Schwarzschild*** metric, $\delta = 0$, $\mu = M_-$

  Outside the bubble is the ***Schwarzschild-deSitter*** metric, $\delta \neq 0$, $\mu = M_+ + \Lambda r^3$

Nucleation in Black Hole Background

- **Gravity-Bounce equation**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}, \qquad g^{\mu\nu}\nabla_\mu \partial_\nu \phi = \frac{\partial V}{\partial \phi}$$

General Spherical symmetric $O(3)$ metric

We can also write down the Gravity-Bounce equation(Einstein Equation+Bounce Equation)

$$B = S_{\text{bounce}} - S_{\text{E},0},$$

$O(3)$ K-G equation in the curved metric

$$f\phi'' + f'\phi' + \frac{2}{r} f\phi' + \delta' f\phi' - V_{,\phi} = 0$$

Einstein Equation at $tt\&rr$ component

$$\mu' = 4\pi r^2 \left(\frac{1}{2} f\phi'^2 + V \right)$$

$$\delta' = 4\pi G r \phi'^2,$$

Solve this equation by the Altered-shooting method ! !

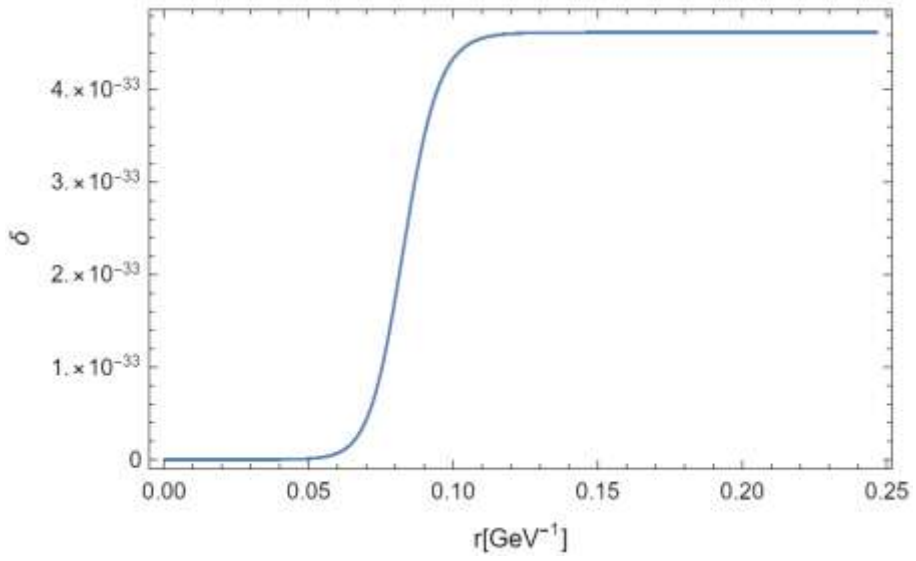
$$I_{EH}[T] = \int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G} + \left[\frac{1}{2} g^{ab}\partial_a \Phi \partial_b \Phi + V_{eff}(\Phi) \right] \right) \qquad B[T] = I_{EH}[T] - I_{EH}^{NB}[T]$$

PBH effect in BSM: ΣSM

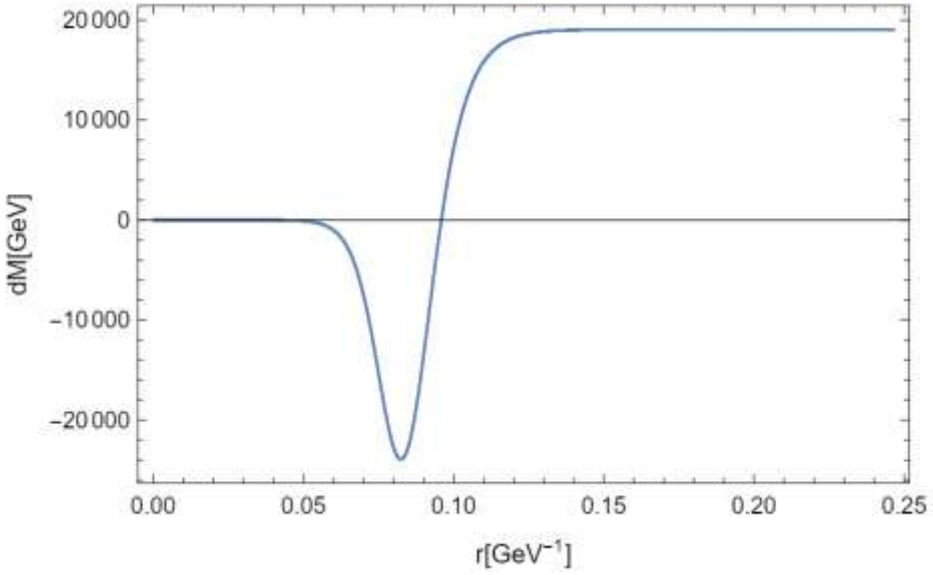
- **Numerical result of Gravity-Bounce equation**

$$ds^2 = f(r)e^{2\delta(r)}d\tau^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \qquad f(r) = 1 - \frac{2G\mu(r)}{r}\,.$$

$M_{PBH} \sim 5 \times 10^{15} M_p$



$$dM(r) \equiv \mu(r) - M_- - \Lambda(r)r^3/6G.$$



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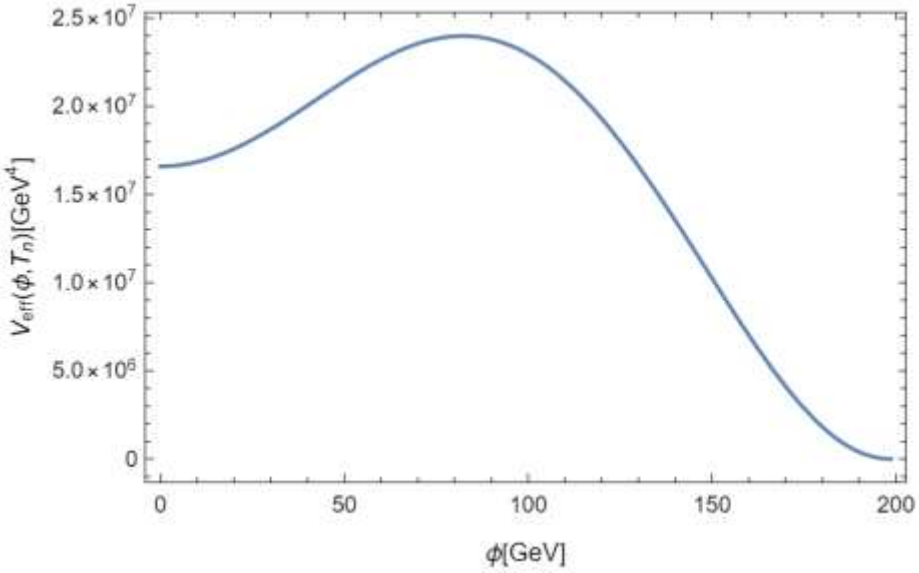
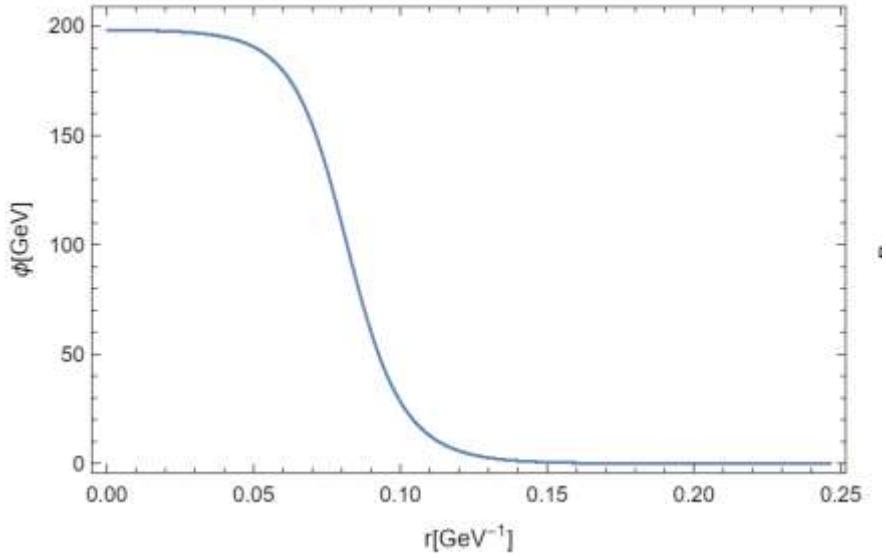
- **Numerical result of Bounce equation**

Benchmark-1 for $M_{PBH} \sim 5 \times 10^{15} M_p$





Tunneling action: $$I_{\rm b}[T] = \beta \int dx^3 \sqrt{-h} \left(-\frac{R}{16\pi G} + \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right)$$

$$B[T] = I_b[T] - I_b^{NBH}[T] \approx \frac{A_+}{4G} - \frac{A_-}{4G} \qquad A = 4\pi r^2, r = 2GM$$