Refining Two-Loop Corrections to Trilinear Higgs Couplings

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HELMHOLTZ

Introduction

Motivation

- Trilinear Higgs coupling(s) have not been measured yet

$\Rightarrow$ Potential portal into new physics

- Impact on di-Higgs production and early universe physics (electroweak phase transition)

Setup

- Follow up on [Braathen and Kanemura 2019, 2020]
- Calculate λ_{hhh} in the two-Higgs-doublet (2HDM) model in the alignment limit using the Higgs basis
- Leading two-loop results: gaugeless limit (i.e. $m_W, m_Z \to 0$) and neglect light Higgs mass ($m_h \to 0$)
- New: enforce alignment also at loop level

2HDM in the Higgs Basis

2HDM Lagrangian [Branco et al. 2012]

$$\Phi_{\mathsf{SM}} = \begin{pmatrix} G'^{+} \\ \frac{1}{\sqrt{2}}(v + \phi_{\mathsf{SM}} + iG'^{0}) \end{pmatrix}, \quad \Phi_{\mathsf{BSM}} = \begin{pmatrix} H'^{+} \\ \frac{1}{\sqrt{2}}(v_{\mathsf{BSM}} + \phi_{\mathsf{BSM}} + iA') \end{pmatrix},$$

$$\begin{aligned} V(\Phi_{\mathsf{SM}}, \Phi_{\mathsf{BSM}}) = {} & M_{11}^2 |\Phi_{\mathsf{SM}}|^2 + M_{22}^2 |\Phi_{\mathsf{BSM}}|^2 - M_{12}^2 (\Phi_{\mathsf{BSM}}^\dagger \Phi_{\mathsf{SM}} + \text{h.c.}) \\ & + \frac{\Lambda_1}{2} |\Phi_{\mathsf{SM}}|^4 + \frac{\Lambda_2}{2} |\Phi_{\mathsf{BSM}}|^4 + \Lambda_3 |\Phi_{\mathsf{SM}}|^2 |\Phi_{\mathsf{BSM}}|^2 + \Lambda_4 |\Phi_{\mathsf{BSM}}^\dagger \Phi_{\mathsf{SM}}|^2 \\ & + \left[\frac{\Lambda_5}{2} (\Phi_{\mathsf{BSM}}^\dagger \Phi_{\mathsf{SM}})^2 + (\Lambda_6 |\Phi_{\mathsf{SM}}|^2 + \Lambda_7 |\Phi_{\mathsf{BSM}}|^2) \Phi_{\mathsf{SM}}^\dagger \Phi_{\mathsf{BSM}} + \text{h.c.} \right], \end{aligned}$$

$$\mathcal{L}_{\mathsf{Yuk}} \supset -\bar{Q}_{\mathsf{L}} Y_{\mathsf{SM}}^t \tilde{\Phi}_{\mathsf{SM}} t_{\mathsf{R}} - \bar{Q}_{\mathsf{L}} Y_{\mathsf{BSM}}^t \tilde{\Phi}_{\mathsf{BSM}} t_{\mathsf{R}} \qquad (Q_{\mathsf{L}} = \begin{pmatrix} t_{\mathsf{L}} & b_{\mathsf{L}} \end{pmatrix}^{\mathsf{T}}, \; Y_{\mathsf{BSM}}^t = \zeta_t Y_{\mathsf{SM}}^t)$$

2HDM in the Higgs Basis

Rotations

$$\begin{pmatrix} h \\ H \end{pmatrix} = R_\alpha^\mathsf{T} \begin{pmatrix} \phi_\mathsf{SM} \\ \phi_\mathsf{BSM} \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A \end{pmatrix} = R_{\beta'}^\mathsf{T} \begin{pmatrix} G'^0 \\ A' \end{pmatrix}, \quad \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = R_{\beta'}^\mathsf{T} \begin{pmatrix} G'^+ \\ H'^+ \end{pmatrix}, \tan\beta' = \frac{v_\mathsf{BSM}}{v}$$

- Higgs basis: $\tan\beta' = 0$ ($v_\mathsf{BSM} = 0$)
- Alignment limit: $\alpha = 0$ ($\Lambda_6 = 0$)
- These are tree level conditions (not symmetry-protected), have to introduce $\delta\alpha$ ($\leftrightarrow \delta\Lambda_6$) and $\delta\beta'$ ($\leftrightarrow \delta v_\mathsf{BSM}$)

Input parameters

- v, m_h, m_H, $m_{H^\pm}$, m_A, M_{22}, Λ_2, Λ_7, $\alpha(=0)$, $\beta'(=0)$, ζ_t

Renormalization

Definitions

> Introduce WRC and mass counterterms for h, H (similarly for $H^{\pm}, A$):

$$\begin{pmatrix} h \\ H \end{pmatrix} \to \sqrt{1+\delta Z} \begin{pmatrix} h \\ H \end{pmatrix}, \delta Z = \begin{pmatrix} \delta Z_{hh} & \delta Z_{hH} \\ \delta Z_{Hh} & \delta Z_{HH} \end{pmatrix},$$

$$m^2_{h/H} \to m^2_{h/H} + m^2_{h/H} + \delta m^2_{h/H}$$

> Introduce counterterms for all input parameters

Tadpole Scheme

> Renormalize tadpoles in the parameter renormalized scheme (PRTS), renormalized tadpoles vanish, $\hat{T}_h = \hat{T}_H = 0$

⇒ Tadpoles cancel in the calculation

Calculation of $\lambda^{(2)}_{hhh}$

> Counterterms needed at one-loop order: δm^2_H, δm^2_A, $\delta m^2_{H^\pm}$,δM^2_{22}, δv, $\delta\alpha$, $\delta\beta'$, δZ

> Counterterms needed at two-loop order: δm^2_h

2HDM in the Higgs Basis

Mixing angle counterterms [Kanemura, Okada, et al. 2004; Kanemura, Kikuchi, and Yagyu 2015]

- Define $\delta\alpha$, $\delta\beta'$ via the off-diagonal WRC:

$$\delta\alpha^{(1)} = \frac{\delta Z_{hH}^{(1)} - \delta Z_{Hh}^{(1)}}{4}, \delta\beta' = \frac{\delta Z_{G^0A}^{(1)} - \delta Z_{AG^0}^{(1)}}{4}$$

Alternative Approach [Degrassi and Slavich 2023]

- Do not introduce mixing angles α, β' (also no δZ_{hH} , δZ_{Hh})
- Define counterterms $\delta\Lambda_6$ and δv_{BSM} by demanding that they cancel offdiagonal mass matrix contributions:

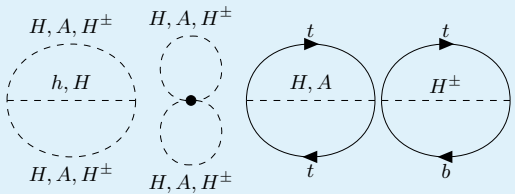
$$\delta v_{\text{BSM}}^{(1)} = -\frac{v}{m_A^2}\left(\Sigma_{G^0A}^{(1)}(0) - \delta T_{G^0A}^{(1)}\right) = 0$$

$$\delta\Lambda_6^{(1)} = \frac{1}{v^2}\left(\Sigma_{hH}^{(1)}(0) - \delta T_{hH}^{(1)} + \left(\frac{\delta v_{\text{BSM}}^{(1)}(2M_{22}^2 - m_H^2))}{v}\right)\right)$$

Two-Loop Corrections to λ_{hhh}

Effective Potential Approach

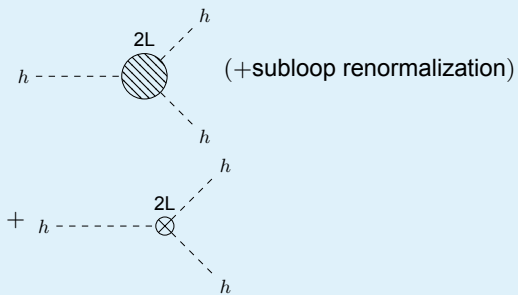
> Diagrams to consider:



> Include shifts of parameters to obtain OS result

Diagrammatic Approach

> $\lambda^{(2)}_{hhh} =$



> Leg corrections cancel due to OS counterterms

M_{22} Decoupling and Cross Checks

Definition of δM_{22} [Braathen and Kanemura 2019, 2020; Degrassi and Slavich 2023]

> Define δM^2_{22} by demanding:

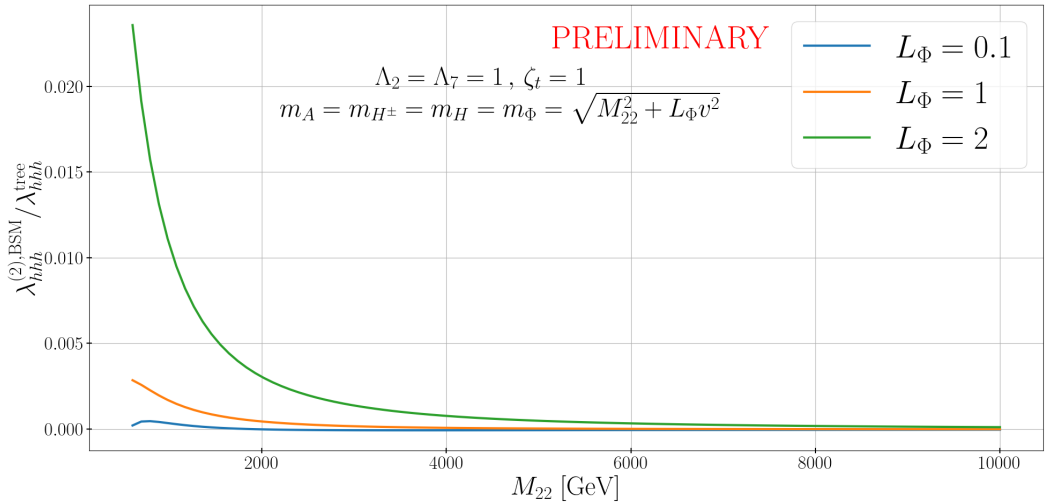
$$\lambda_{hhh}^{\text{2-loop,BSM}} \xrightarrow{M_{22}\to\infty} 0\,, \text{ where } m^2_\Phi \simeq M^2_{22} + \lambda_\Phi v^2 \ (\Phi = H, A, H^\pm)$$

$$\Rightarrow \ (\delta M^2_{22})^{\text{finite}} = \frac{3M^2_{22}}{16\pi^2}\left(\Lambda_2(1 - \log(M^2_{22}/Q^2)) + \frac{2\zeta_t^2 m_t^2}{v^2}(2 - \log(M^2_{22}/Q^2))\right)$$

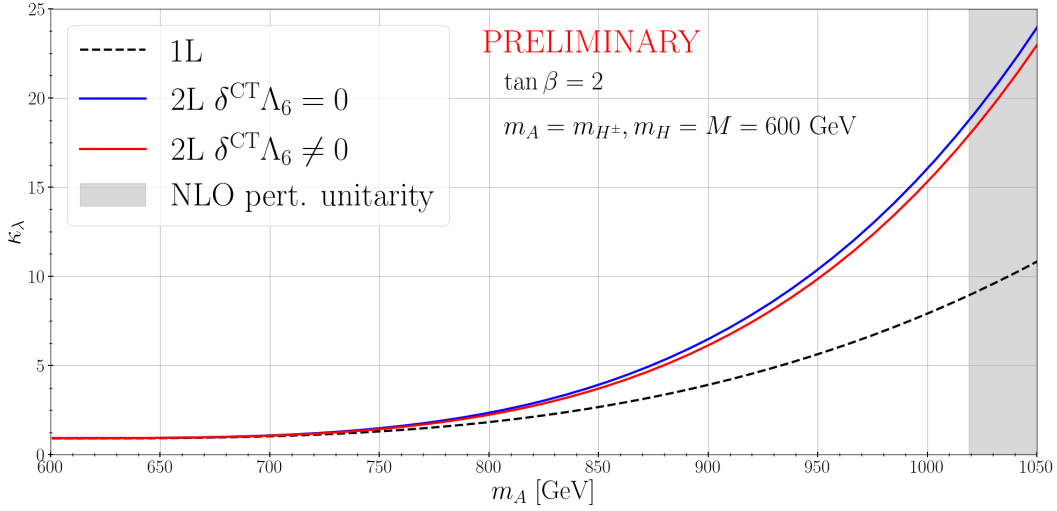
Cross Checks

> Comparison of results from diagramatic and effective potential approach
> Comparison of counterterms with RGE runnings from `SARAH` [Staub 2014]
> Cancellation of renormalization scale dependence, UV finiteness of final result

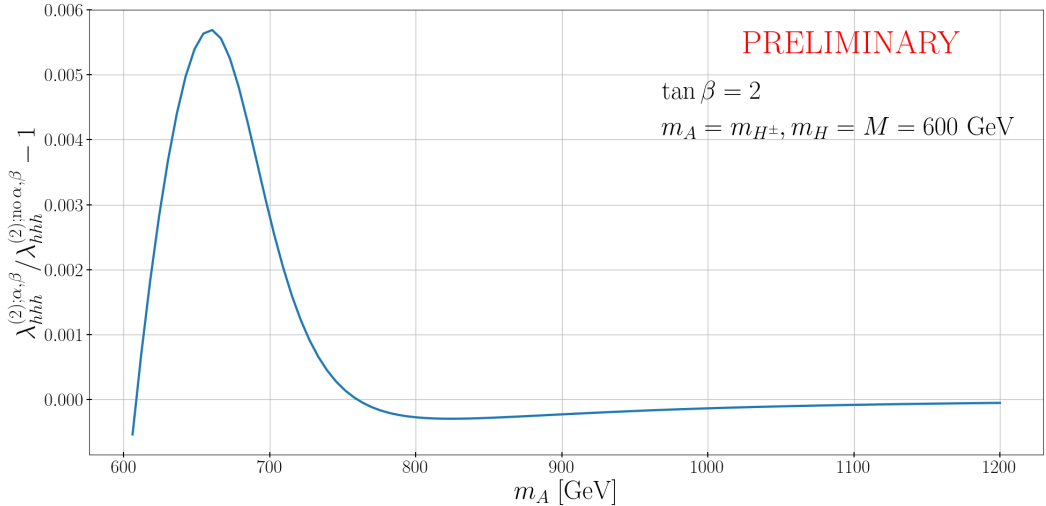
M_{22} Decoupling



Results


1L
2L $\delta^{\mathrm{CT}}\Lambda_6 = 0$
2L $\delta^{\mathrm{CT}}\Lambda_6 \neq 0$
NLO pert. unitarity
PRELIMINARY
$\tan\beta = 2$
$m_A = m_{H^\pm}, m_H = M = 600$ GeV
κ_λ
m_A [GeV]
0
5
10
15
20
25
600
650
700
750
800
850
900
950
1000
1050

Results Renormalization Difference



Conclusion

Summary

- Calculation of leading two-loop corrections to λ_{hhh} in the 2HDM in the alignment limit
- Alignment condition also enforced at loop order
- M_{22} counterterm defined to ensure apparent decoupling of two-loop corrections

Outlook

- Further comparison, check with viable benchmark points, impact on phenomenology?
- With completed renormalization setup, possible to calculate other quantities

Literature I

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Literature II

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2HDM in the Higgs Basis

Tadpole Equations

$$\left.\frac{\partial V_{\text{2HDM}}}{\partial \phi}\right|_{\{\phi\}=0} \stackrel{!}{=} T_\phi \quad (\phi = \phi_{\text{SM}}, \phi_{\text{BSM}}, G'^0, A'^0, G'^\pm, H'^\pm)$$

$$\Rightarrow M_{11}^2 = \frac{1}{2v^2}(2T_1 v - \Lambda_1 v^4 + v_{\text{BSM}}(-2T_2 - 2\Lambda_6 v^3 + 2M_{22}^2 v_{\text{BSM}} + 2\Lambda_7 v v_{\text{BSM}}^2 + \Lambda_2 v_{\text{BSM}}^3)$$

$$M_{12}^2 = \frac{1}{2v}(-2T_2 + \Lambda_6 v^3 + v_{\text{BSM}}(2M_{22}^2 + (\Lambda_3 + \Lambda_4 + \Lambda_5)v^2 + 3\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2)$$

$$T_1 = T_{\phi_{\text{SM}}}, \quad T_2 = T_{\phi_{\text{BSM}}}$$

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = R_\alpha \begin{pmatrix} T_h \\ T_H \end{pmatrix}, R_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$$

2HDM in the Higgs Basis

Mass Matrices

$$\mathcal{M}_{hH} = \begin{pmatrix} \frac{2\Lambda_1 v^4 + v_{\text{BSM}}(4\Lambda_6 v^3 + v_{\text{BSM}}(2M_{22}^2 + (\Lambda_3+\Lambda_4+\Lambda_5)v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2))}{2v^2} & \frac{2\Lambda_6 v^3 + v_{\text{BSM}}(-2M_{22}^2 + (\Lambda_3+\Lambda_4+\Lambda_5)v^2 - \Lambda_2 v_{\text{BSM}}^2)}{2v} \\ \frac{2\Lambda_6 v^3 + v_{\text{BSM}}(-2M_{22}^2 + (\Lambda_3+\Lambda_4+\Lambda_5)v^2 - \Lambda_2 v_{\text{BSM}}^2)}{2v} & \frac{1}{2}(2M_{22}^2 + (\Lambda_3+\Lambda_4+\Lambda_5)v^2 + 6\Lambda_7 v v_{\text{BSM}} + 3\Lambda_2 v_{\text{BSM}}^2) \end{pmatrix}$$

$$\mathcal{M}_{G^0 A} = \begin{pmatrix} \frac{1}{2v^2}(v_{\text{BSM}}^2(2M_{22}^2 + (\Lambda_3+\Lambda_4-\Lambda_5)v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2)) & -\frac{v_{\text{BSM}}}{2v}(2M_{22}^2 + (\Lambda_3+\Lambda_4-\Lambda_5)v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) \\ -\frac{v_{\text{BSM}}}{2v}(2M_{22}^2 + (\Lambda_3+\Lambda_4-\Lambda_5)v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) & \frac{1}{2}(2M_{22}^2 + (\Lambda_3+\Lambda_4-\Lambda_5)v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) \end{pmatrix}$$

$$\mathcal{M}_{G^\pm H^\pm} = \begin{pmatrix} \frac{1}{2}(v_{\text{BSM}}^2(2M_{22}^2 + \Lambda_3 v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) & -\frac{v_{\text{BSM}}}{2v}(2M_{22}^2 + \Lambda_3 v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) \\ -\frac{v_{\text{BSM}}}{2v}(2M_{22}^2 + \Lambda_3 v^2 + 2\Lambda_7 v v_{\text{BSM}} + \Lambda_2 v_{\text{BSM}}^2) & M_{22}^2 + \frac{1}{2}(\Lambda_3 v^2 + v_{\text{BSM}}(2\Lambda_7 v + \Lambda_2 v_{\text{BSM}})) \end{pmatrix}$$

$$T_{hH} = T_{G^0 A} = T_{G^\pm H^\pm} = \begin{pmatrix} \frac{T_1 v - T_2 v_{\text{BSM}}}{v^2} & \frac{T_2}{v} \\ \frac{T_2}{v} & 0 \end{pmatrix}$$

$$\mathcal{M}_{hH} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22} \end{pmatrix} \quad \Rightarrow \tan 2\alpha = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11} - \mathcal{M}_{22}}, \qquad \tan\beta' = \frac{v_{\text{BSM}}}{v}$$

2HDM in the Higgs Basis

Replacement Rules

$$\Lambda_1 = \frac{-1}{2v^4}(-(m_h^2 + m_H^2)v^2 + 2\Lambda_6 v^3 v_{\text{BSM}} + 4M_{22}^2 v_{\text{BSM}}^2 + 2\Lambda_7 v v_{\text{BSM}}^3 + 2\Lambda_2 v_{\text{BSM}}^4 + v\sqrt{S})$$

$$\Lambda_3 = \frac{-2M_{22}^2}{v^2} - \frac{v_{\text{BSM}}(2\Lambda_7 v + \Lambda_2 v_{\text{BSM}})}{v^2} + \frac{(2m_{H^\pm}^2)}{v^2 + v_{\text{BSM}}^2}$$

$$\Lambda_4 = \frac{1}{2v^2(v^2 + v_{\text{BSM}}^2)}((2m_A^2 + m_h^2 + m_H^2 - 4m_{H^\pm}^2)v^2 - 2\Lambda_6 v^3 v_{\text{BSM}} - 4\Lambda_7 v^3 v_{\text{BSM}} + 4M_{22}^2 v_{\text{BSM}}^2$$
$$- 2\Lambda_2 v^2 v_{\text{BSM}}^2 + 2\Lambda_7 v v_{\text{BSM}}^3 + 2\Lambda_2 v_{\text{BSM}}^4 + v\sqrt{S})$$

$$\Lambda_5 = \frac{1}{2v^2(v^2 + v_{\text{BSM}}^2)}((-2m_A^2 + m_h^2 + m_H^2)v^2 - 2\Lambda_6 v^3 v_{\text{BSM}} - 4\Lambda_7 v^3 v_{\text{BSM}} + 4M_{22}^2 v_{\text{BSM}}^2$$
$$- 2\Lambda_2 v^2 v_{\text{BSM}}^2 + 2\Lambda_7 v v_{\text{BSM}}^3 + 2\Lambda_2 v_{\text{BSM}}^4 + v\sqrt{S})$$

$$S \equiv (m_h^4 + m_H^4)v^2 - 4(\Lambda_6 v^3 - v_{\text{BSM}}(2M_{22}^2 + v_{\text{BSM}}(3\Lambda_7 v + 2\Lambda_2 v_{\text{BSM}})))^2 + 4m_H^2 v_{\text{BSM}}(-\Lambda_6 v^3$$
$$+ v_{\text{BSM}}(2M_{22}^2 + v_{\text{BSM}}(3\Lambda_7 v + 2\Lambda_2 v_{\text{BSM}}))) - 2m_h^2(2\Lambda_6 v^3 v_{\text{BSM}} + m_H^2(v^2 + 2v_{\text{BSM}}^2)$$
$$- 2v_{\text{BSM}}^2(2M_{22}^2 + v_{\text{BSM}}(3\Lambda_7 v + 2\Lambda_2 v_{\text{BSM}})))$$

2HDM in the Higgs Basis

Counterterms for Λ_6 and v_{BSM}

$$\delta v_{\text{BSM}}^{(1)} = \delta\beta'^{(1)} v$$

$$\delta\Lambda_6^{(1)} = \frac{\delta\beta'^{(1)}(M_{22}^2 - m_H^2)}{v^2} + \frac{\delta\alpha^{(1)}(m_h^2 - m_H^2)}{v^2}$$

2HDM in the Higgs Basis

Trilinear couplings

$$\lambda_{hhh} \to \left(\frac{3m_h^2}{v}\right) + \left(-\frac{3\delta v^{(1)} m_h^2}{v^2} + \frac{3(\delta m_h^2)^{(1)}}{v}\right) + \left(\frac{3(\delta v^{(1)})^2 m_h^2}{v^3} - \frac{3(\delta m_h^2)^{(1)}\delta v^{(1)}}{v^2} - \frac{3\delta v^{(2)} m_h^2}{v^2} + \frac{3(\delta m_h^2)^{(2)}}{v} - \frac{6(\delta\beta'^{(1)})^2 M_{22}^2}{v} + \frac{3\delta\alpha^{(1)}\delta\beta'^{(1)}(4M_{22}^2 - m_h^2)}{v} + \frac{3(\delta\alpha^{(1)})^2(-4M_{22}^2 + m_h^2)}{2v}\right)$$

$$\lambda_{hhH} \to \left(\frac{\delta\beta'^{(1)}(4M_{22}^2 - m_H^2)}{v} + \frac{\delta\alpha^{(1)}(-4M_{22}^2 + m_H^2)}{v}\right) + (\text{2-loop CT})$$

$$\lambda_{hHH} \to \frac{2(-M_{22}^2 + m_H^2)}{v} + (\text{1-loop CT}) + (\text{2-loop CT})$$

$$\lambda_{HHH} \to 3\Lambda_7 v + (\text{1-loop CT}) + (\text{2-loop CT})$$

2HDM in the Higgs Basis

Convert from the $\mathbb{Z}_2$ basis to the Higgs basis [Davidson and Haber 2005, Haber and Stål 2015, Bernon et al. 2015]

$$\Lambda_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2$$

$$\Lambda_2 = \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2$$

$$\Lambda_i = \frac{1}{4} s_{2\beta}^2 (\lambda_1 + \lambda_2 - 2\lambda_{345}) + \lambda_i \; (i = 3, 4, 5)$$

$$\Lambda_6 = -\frac{1}{2} s_{2\beta} (\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta})$$

$$\Lambda_7 = -\frac{1}{2} s_{2\beta} (\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta})$$

$$\Lambda_2 = \Lambda_1 + 2(\Lambda_6 + \Lambda_7) \cot 2\beta$$

$$\Lambda_3 + \Lambda_4 + \Lambda_5 = \Lambda_1 + 2\Lambda_6 \cot 2\beta - \frac{(\Lambda_6 - \Lambda_7)}{\cot 2\beta}$$

2HDM in the Higgs Basis

Convert from the $\mathbb{Z}_2$ basis to the Higgs basis [Davidson and Haber 2005; Haber and Stål 2015; Bernon et al. 2015]

$$M_{11}^2 = m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - m_{12}^2 s_{2\beta}$$
$$M_{12}^2 = m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + m_{12}^2 s_{2\beta}$$
$$M_{22}^2 = \frac{1}{2}(m_{22}^2 - m_{11}^2) s_{2\beta} - m_{12}^2 c_{2\beta}$$

2HDM in the Higgs Basis

Yukawa Couplings

$$Y^t_{\text{SM}} = \frac{m_t\sqrt{2}}{v}\frac{1}{1+\zeta_t\frac{v_{\text{BSM}}}{v}}\,,$$
$$Y^t_{\text{BSM}} = \frac{m_t\sqrt{2}}{v}\frac{\zeta_t}{1+\zeta_t\frac{v_{\text{BSM}}}{v}}$$

> Comparing with the $\mathbb{Z}_2$ symmetric basis:

$$\zeta_t = \cot\beta$$

Renormalization

SM VEV Renormalization

> Define v in the G_μ scheme:

$$
\begin{aligned}
\delta v^{(1)} &= \frac{(2\cos(\theta_W)^2-1)(\delta m_W^2)^{(1)}v}{2(\cos(\theta_W)^2-1)m_W^2} - \frac{\cos(\theta_W)^4(\delta m_Z^2)^{(1)}v}{2(\cos(\theta_W)^2-1)m_W^2} - \delta Z_e v \\
&= \frac{1}{16\pi^2 v}\left(-\frac{3m_t^2}{\epsilon} + \frac{m_A^2\log(m_{H^\pm}^2/m_A^2)m_{H^\pm}^2}{6m_A^2-6m_{H^\pm}^2} + \frac{m_H^2\log(m_{H^\pm}^2/m_H^2)m_{H^\pm}^2}{6m_H^2-6m_{H^\pm}^2}\right. \\
&+ \frac{2m_A^2\log(m_A^2/Q^2)m_{H^\pm}^2}{3m_A^2-3m_{H^\pm}^2} + \frac{2m_H^2\log(m_H^2/Q^2)m_{H^\pm}^2}{3m_H^2-3m_{H^\pm}^2} + \frac{1}{4}(-m_A^2-m_H^2-2(m_{H^\pm}^2+3m_t^2)) \\
&\left. - \frac{2(m_H^2m_{H^\pm}^4+m_A^2(m_{H^\pm}^4-2m_H^2m_{H^\pm}^2))\log(m_{H^\pm}^2/Q^2)}{3(m_A^2-m_{H^\pm}^2)(m_{H^\pm}^2-m_H^2)} + 3m_t^2\log(m_t^2/Q^2) + \mathcal{O}(\epsilon)\right)
\end{aligned}
$$

Renormalization

Mass counterterms

> Define masses on-shell (OS) (Σ: self-energy, δT: tadpole contribution):

$$(\delta m_h^2)^{(1)} = \Sigma_{hh}^{(1)}(m_h^2) - \delta T_{hh}^{(1)}\,, \qquad \delta Z_{hh}^{(1)} = -\left.\frac{\partial \Sigma_{hh}^{(1)}(p^2)}{\partial p^2}\right|_{p^2=m_h^2},$$

$$(\delta m_H^2)^{(1)} = \Sigma_{HH}^{(1)}(m_H^2) - \delta T_{HH}^{(1)}\,, \qquad \delta Z_{HH}^{(1)} = -\left.\frac{\partial \Sigma_{HH}^{(1)}(p^2)}{\partial p^2}\right|_{p^2=m_H^2},$$

$$\delta Z_{hH}^{(1)} = \frac{2}{m_h^2 - m_H^2}\left(\Sigma_{hH}^{(1)}(m_H^2) - \delta T_{hH}^{(1)}\right), \qquad \delta Z_{Hh}^{(1)} = \frac{2}{m_H^2 - m_h^2}\left(\Sigma_{hH}^{(1)}(m_h^2) - \delta T_{hH}^{(1)}\right)$$

$$(\delta m_h^2)^{(2)} = \Sigma_{hh}^{(2)}(m_h^2) - \delta T_{hh}^{(2)} - ((\delta m_h^2)^{(1)} + \delta T_{hh}^{(1)})\delta Z_{hh}^{(1)} - \delta T_{hH}^{(1)}\delta Z_{Hh}^{(1)} + \frac{1}{4}(\delta Z_{Hh}^{(1)})^2(m_h^2 - m_H^2)\,,$$

(similarly for A, $H^\pm$ and the Goldstone bosons G^0, $G^\pm$)