Attempt at Constructing a Model of Grand Gauge-Higgs Unification with Family Unification

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Based on NM and Ryujiro Nago, PRD109, 115005 (2024)

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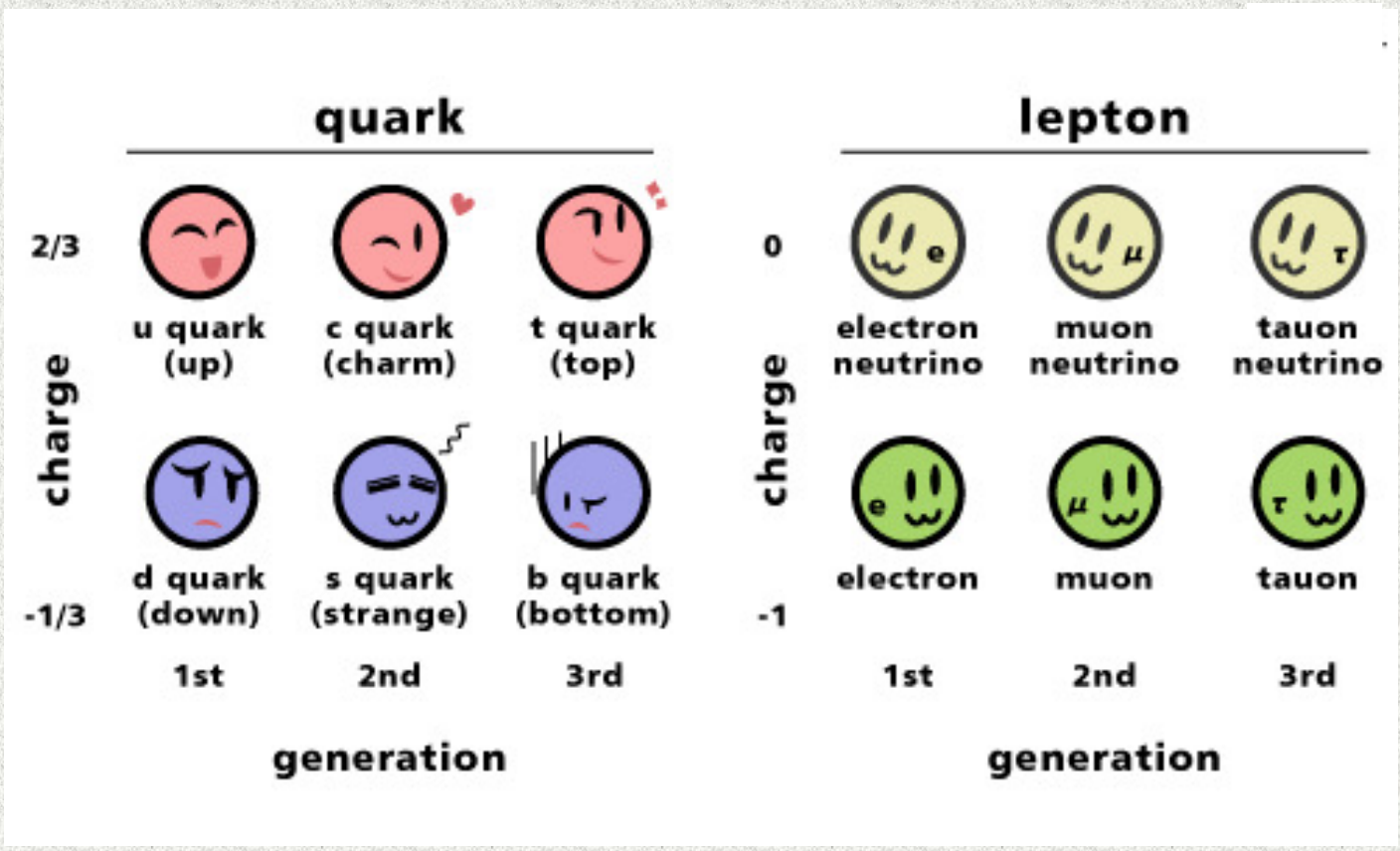
Introduction

Origin of 3 generations of quarks and leptons is one of the mysteries of the Standard Model



“Family unification”

Extending GUT gauge group,
3 generations are obtained from 1 generation
after symmetry breaking to SM

This program has a long history
and many models have been proposed so far

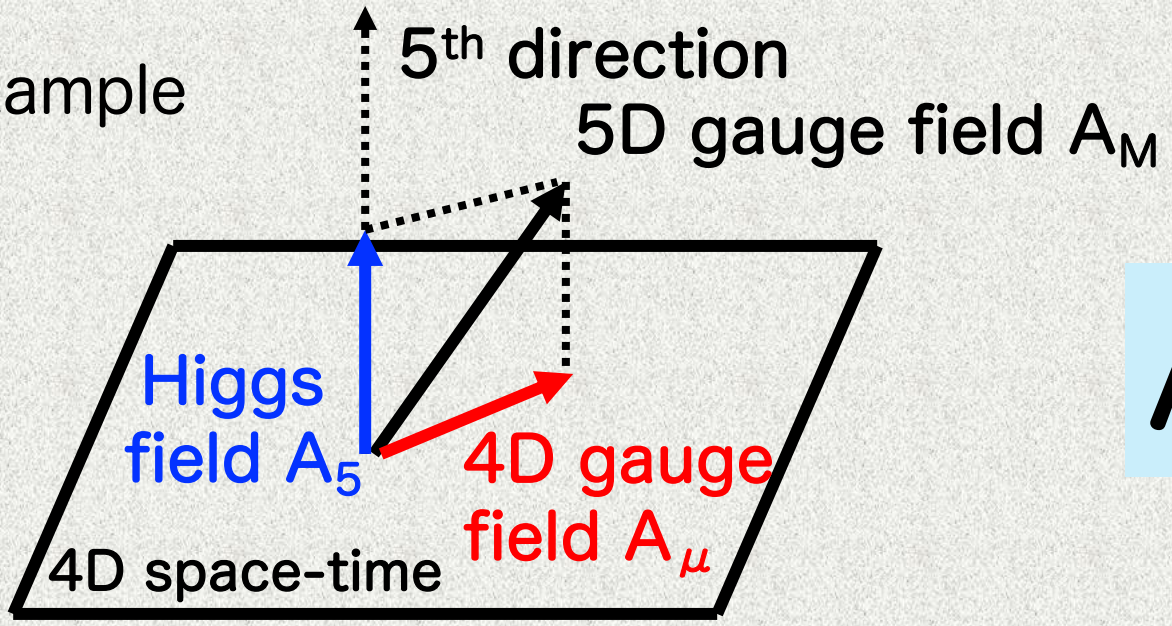
In this talk, we consider a possibility of

unification of 3 generations
in grand gauge-Higgs unification (GGHU)

Gauge-Higgs unification (GHU)

SM Higgs is regarded as an extra component of the higher dimensional gauge field

5D example



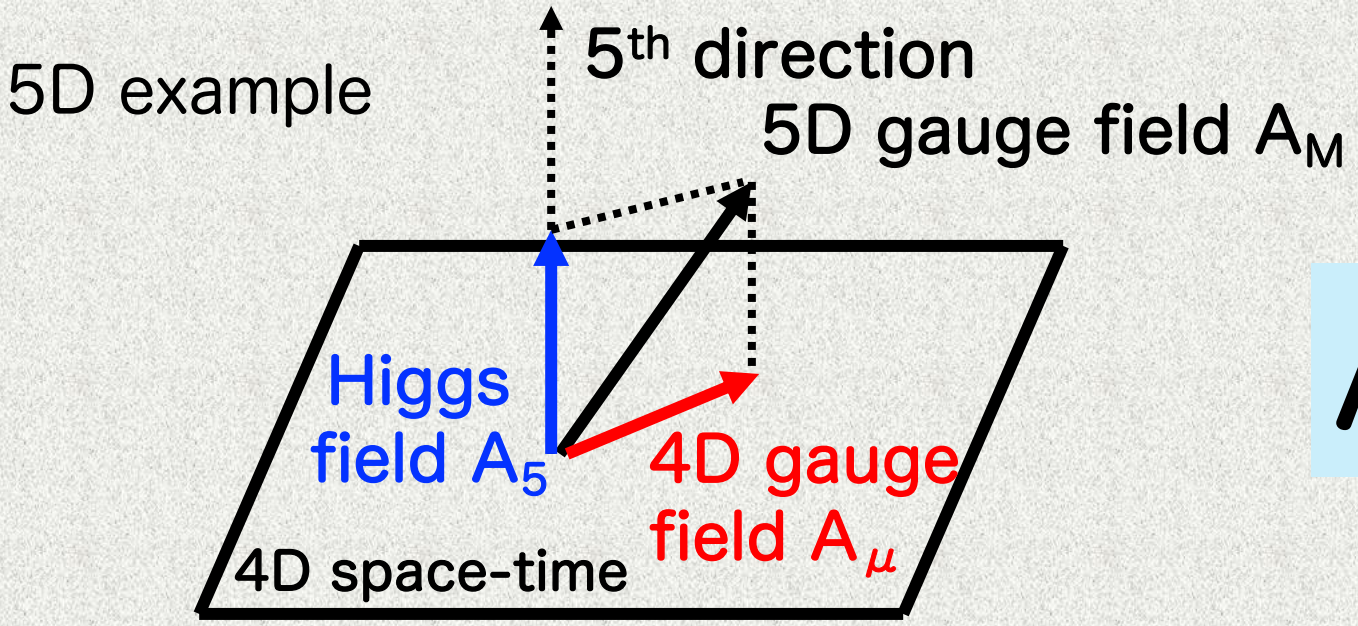
$$A_M = (A_\mu, A_5)$$

(M=0-3,5 μ=0-3)

4D gauge field A_μ and scalar field A_5 are unified into 5D gauge field A_M

Gauge-Higgs unification (GHU)

SM Higgs is regarded as an extra component of the higher dimensional gauge field



$$A_M = (A_\mu, A_5)$$

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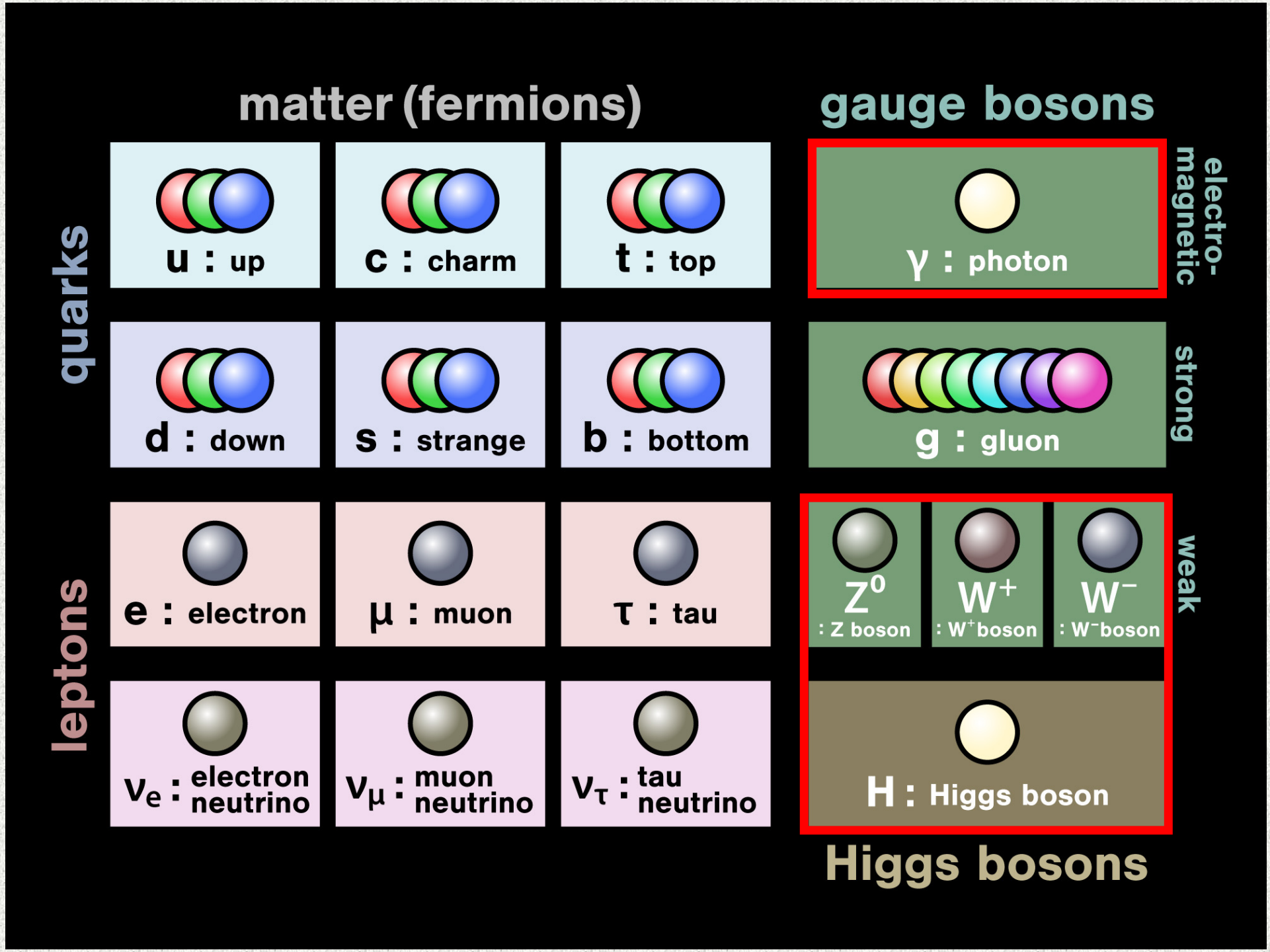
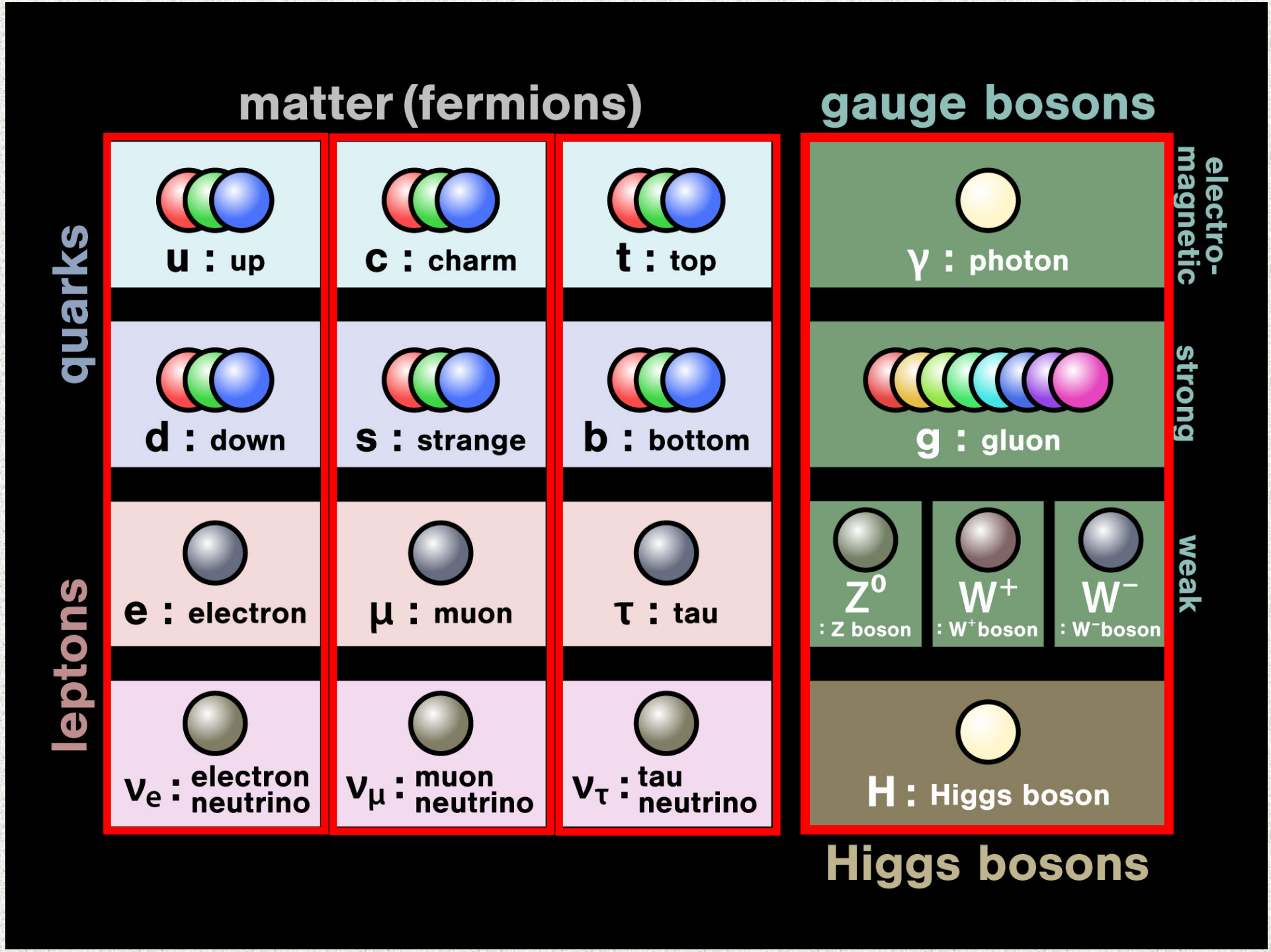
Higgs potential is finite
thanks to the higher dimensional gauge symmetry
→ solution to the hierarchy problem


matter (fermions)
gauge bosons
quarks
leptons
u : up
c : charm
t : top
d : down
s : strange
b : bottom
e : electron
μ : muon
τ : tau
νe : electron neutrino
νμ : muon neutrino
ντ : tau neutrino
γ : photon
electro-magnetic
g : gluon
strong
Z0 : Z boson
W+ : W+boson
W− : W−boson
weak
H : Higgs boson
Higgs bosons

GHU

GGHU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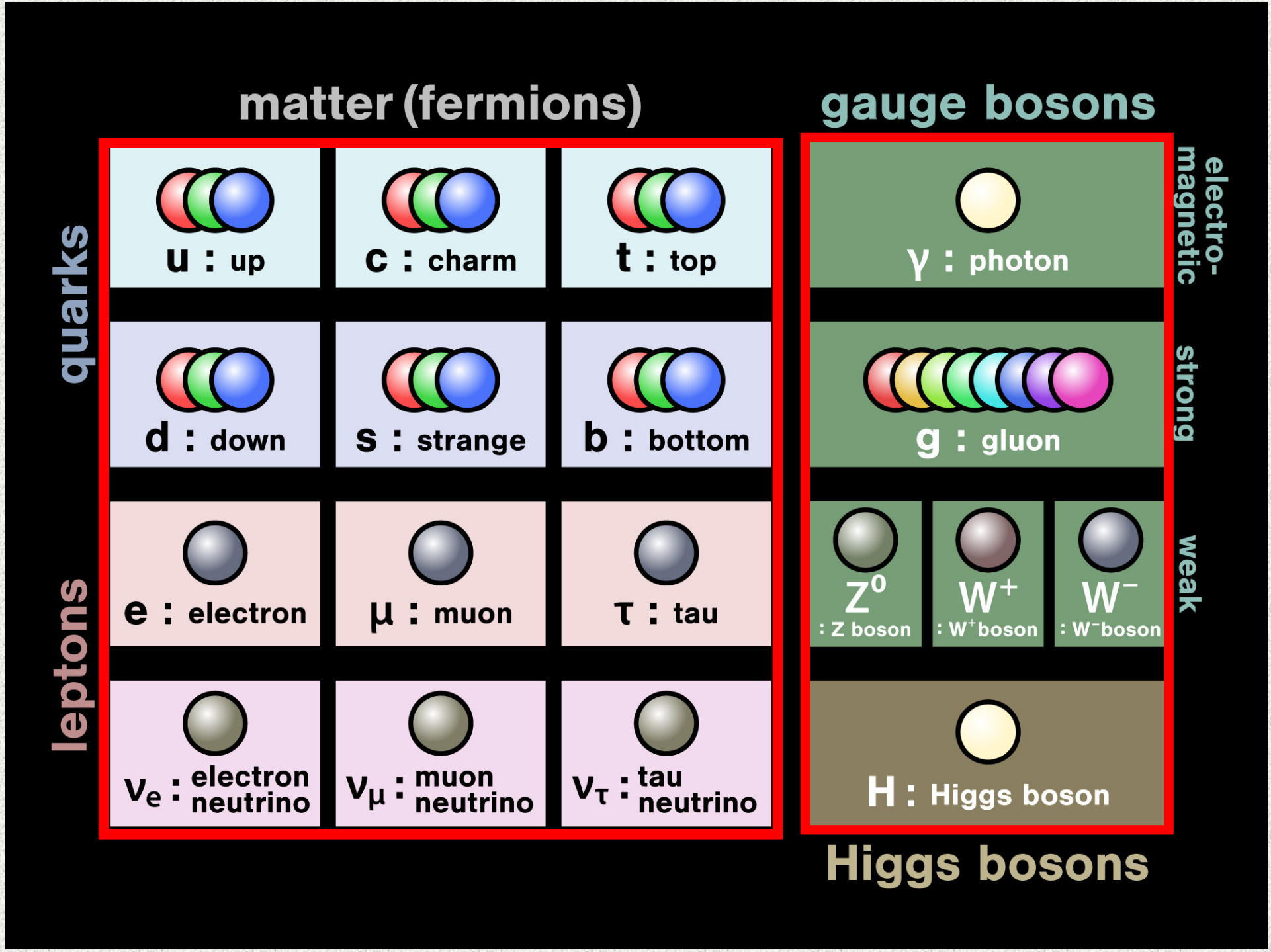
Family
Unified
GGHU



Georgi's idea

NPB156 (1979) 126

- 3 generations of SU(5) GUT 3 × [5*+10] from a larger group including SU(5)
- 10: 2-rank anti-symmetric rep.
  5*: 4–rank totally anti-symmetric rep.
- 3 generations can be obtained from anomaly free combinations of SU(N)(N>5) totally anti-symmetric reps

Georgi's idea

NPB156 (1979) 126

SU(11) with [4]+[8]+[9]+[10] reps (anomaly free)

[k]: k-rank totally anti-sym rep

$$[4](330\ \text{dim}) = \mathbf{5^*} + 6\times\mathbf{10^*} + 15\times\mathbf{10} + 20\times\mathbf{5} + 15\times\mathbf{1}$$
$$[8](165\ \text{dim}) = \mathbf{10} + 6\times\mathbf{10^*} + 15\times\mathbf{5^*} + 20\times\mathbf{1}$$
$$[9](55\ \text{dim}) = \mathbf{10^*} + 6\times\mathbf{5^*} + 15\times\mathbf{1}$$
$$[10](11\ \text{dim}) = \mathbf{5^*} + 6\times\mathbf{1}$$

Total = (1+15+6+1-20) × **5*** + (15+1-6-6-1) ×10

= 3 × [5*+10]!!

By applying Georgi's idea to GGHU, possibilities of family unification are studied here

Advantages of considering family unification in higher dimensions

- Anomaly cancellation constraints are relaxed

  → 4D chiral theory obtained from non-chiral theory in higher dimensions by compactification

- Symmetry breaking is easily realized by BCs in compactification

  → No need complicated Higgs potential

Constructing
6D theory with Family unification

Strategy of model building

2 step compactification

6D GGHU with family unification (one generation)

↓ orbifold compactification S^1/Z_2 in 6^{th} dimension

5D GGHU with 3 generations of quarks and leptons

↓ orbifold compactification S^1/Z_2 in 5^{th} dimension

4D SM with 3 generations of q & l

Strategy of model building

A candidate of 5D model with 3 generations
to be reproduced from 6D theory

5D SU(6) GGHU with [**20** + 2 × **6***]

- One generation of quarks & leptons just embedded
- No exotic fermions
- SM Higgs exists in A_5 Lim & Maru, PLB653 (2007) 320

But **20** in SU(6) is self-dual (**20**=**20***),
3 generations are impossible
→ simple extension to SU(7) GGHU with [**35** + 2 x **7***]

6D SU(N) theory

Orbifold S^1/Z_2 (R': radius of S^1)

Z_2 parity

$$P'_0 = \mathrm{diag}(\overbrace{+1,\ldots,+1,+1,\ldots,+1}^{N},-1,\ldots,-1) \quad \text{at } z=0,$$

$$P'_1 = \mathrm{diag}(\underbrace{+1,\ldots,+1}_{7},\underbrace{-1,\ldots,-1}_{p},\underbrace{+1,\ldots,+1}_{q}) \quad \text{at } z=\pi R'$$

N=7+p+q

z:
6th coordinate

Gauge field

$$A_M(x_\mu, y, z_{0,1} - z) = +P'_{0,1} A_M(x_\mu, y, z_{0,1} + z) P'^{\dagger}_{0,1}$$

$$A_6(x_\mu, y, z_{0,1} - z) = -P'_{0,1} A_6(x_\mu, y, z_{0,1} + z) P'^{\dagger}_{0,1}$$

(M=0,1,2,3,5
μ=0,1,2,3,
z_0=0, $z_1=\pi R'$)

gauge symmetry breaking

$$SU(N) \to SU(7) \times SU(p) \times SU(q) \times U(1)^2$$

Z_2 transformation of 6D Dirac fermion in SU(N) k-rank totally anti-symmetric representation $[k]_N$

$$[k]_N = (\boldsymbol{N} \times \cdots \times \boldsymbol{N})_a$$

$$(\boldsymbol{N} \times \cdots \times \boldsymbol{N})_a \to \eta_k (P_0' \boldsymbol{N} \times \cdots \times P_0' \boldsymbol{N})_a \quad \text{at } z = 0,$$
$$(\boldsymbol{N} \times \cdots \times \boldsymbol{N})_a \to \eta_k' (P_1' \boldsymbol{N} \times \cdots \times P_1' \boldsymbol{N})_a \quad \text{at } z = \pi R'$$

$\eta_k,\ \eta'_k = \pm 1$

Decomposition of $[k]_N$ into irreducible reps. of SU(7) x SU(p) x SU(q)

$$[k]_N = \sum_{l=0}^{k} \sum_{m=0}^{k-l} \sum_{n=0}^{k-l-m} ([l]_7, [m]_p, [n]_q)$$

Z_2 parity of ($[l]_7$, $[m]_p$, $[n]_q$) rep.

$$P_0' = \mathrm{diag}(\overbrace{+1,\ldots,+1}^{7},\overbrace{+1,\ldots,+1}^{p},\overbrace{-1,\ldots,-1}^{q})$$

$$P_1' = \mathrm{diag}(\underbrace{+1,\ldots,+1}_{7},\underbrace{-1,\ldots,-1}_{p},\underbrace{+1,\ldots,+1}_{q})$$

$$\mathcal{P}_0' = (-1)^n \eta_k = (-1)^{l+m}(-1)^k \eta_k,$$

$$\mathcal{P}_1' = (-1)^m \eta_k' = (-1)^{l+n}(-1)^k \eta_k',$$

(l+m+n=k is used)

Z_2 parity of 6D chiral fermion

$$\Psi_- : (-1)^{l+m}(-1)^k \eta_k, \quad (-1)^{l+n}(-1)^k \eta_k',$$

$$\Psi_+ : -(-1)^{l+m}(-1)^k \eta_k, \quad -(-1)^{l+n}(-1)^k \eta_k'.$$

(overall relative sign is due to 6D chirality

$(-1)^k \eta_k = +1$, $(-1)^k \eta'_k = +1$ are chosen below for simplicity)

Z_2 parities of 7, 21, 35 and their conjugates of SU(7)
± corresponds to 6D chirality

	$\mathcal{P}'_0$	$\mathcal{P}'_1$
$\mathbf{7}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\mathbf{21}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$
$\mathbf{35}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\overline{\mathbf{35}}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$
$\overline{\mathbf{21}}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\bar{\mathbf{7}}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$

$\Psi_+ : -(-1)^{l+m}, -(-1)^{l+n}$ $\Psi_- : (-1)^{l+m}, (-1)^{l+n}$

$3\times[(2\times7^*)+35^*]$

as massless fields possible

from a combination of SU(N) reps.??

↓

SU(14)!!

TABLE II. Numbers of massless fermions in $\bar{\mathbf{7}}_-$, $\mathbf{21}_-$, $\overline{\mathbf{35}}_-$ representations for various k-rank totally antisymmetric tensors of $SU(14)$.

k	$n^{(+,+)}_{\bar{\mathbf{7}}_-,k}$	$n^{(+,+)}_{\mathbf{21}_-,k}$	$n^{(+,+)}_{\overline{\mathbf{35}}_-,k}$
1	1	0	0
2	0	1	0
3	1	0	1
4	0	1	1
5	−5	1	1
6	1	−5	1
7	3	1	−5
8	1	3	−5
9	0	−5	3
10	−5	0	3
11	0	3	0
12	3	0	0
13	0	0	0

Solution

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Solution

$n_{7*}=1+1+1+3=6$, $n_{21}=1+1+1-5+3=0$, $n_{35*}=1+1+1=3$

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$3\times[(2\times 7^*)+35^*]$

Summary

We succeeded in constructing GGHU model with family unification

6D SU(14) gauge theory on $(S^1/Z_2)^2$ with fermions in [1]+[2]+[3]+[4]+[6]+[11]+[12] reps

Off course, there are many issues to be studied..

Future issues

- 3 generations from one fermion
- SO is hopeful → spinor rep. has large dimensions

  → recently done by NM and R.Nago 2503.12455
  6D SO(20) theory with one spinor rep.
- Models without exotic fermions are desired
- Compactification from 6D to 4D is interesting
- Realistic Yukawa couplings etc.

Thank you!!

TABLE I. Z_2 parity assignment of **7**, **21**, and **35** (and their conjugate) representations of $SU(7)$. $\pm$ are given by the corresponding Γ_7 eigenvalues.

	$\mathcal{P}'_0$	$\mathcal{P}'_1$
$\mathbf{7}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\mathbf{21}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$
$\mathbf{35}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\overline{\mathbf{35}}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$
$\overline{\mathbf{21}}_\pm$	$\pm(-1)^m$	$\pm(-1)^n$
$\bar{\mathbf{7}}_\pm$	$\mp(-1)^m$	$\mp(-1)^n$

$\Psi_+ : -(-1)^{l+m}, -(-1)^{l+n}$ $\Psi_- : (-1)^{l+m}, (-1)^{l+n}$

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z: 6th coordinate

N=7+p+q

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$$A_6(x_\mu, y, z_{0,1}-z) = - P'_{0,1} A_6(x_\mu, y, z_{0,1}+z) P'^{\dagger}_{0,1}$$

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gauge symmetry breaking

$$SU(N) \to SU(7) \times SU(p) \times SU(q) \times U(1)^2$$

Number of Massless modes (Z_2 even)

$$\mathbf{7}_{\pm} \quad \pm(-1)^m \quad \pm(-1)^n \qquad \mathbf{7}^*_{\pm} \quad \mp(-1)^m \quad \mp(-1)^n$$

→ $(m,n) = (\text{even, even}) \rightarrow \mathbf{7}_+, \mathbf{7}^*_-$ massless

$(m,n) = (\text{odd, odd}) \rightarrow \mathbf{7}_-, \mathbf{7}^*_+$ massless

$$n_{\bar{\mathbf{7}}_-,k}^{(+,+)} = \sum_{m,n=\text{even}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,1+m+n} + \delta_{k,6+m+n})$$

($\mathbf{7}_+$, $\mathbf{7}^*_-$)

$$- \sum_{m,n=\text{odd}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,1+m+n} + \delta_{k,6+m+n})$$

($\mathbf{7}_-$, $\mathbf{7}^*_+$)

${}_pC_m$: dim of [m] of SU(p)

・$n(R_+) = n((R^*_-)^c)$ ・$R^*_\pm$ and $R_\pm$ are massive

Number of massless mode(Z_2 even)

$$n^{(+,+)}_{\bar{7}_-,k} = \sum_{m,n=\text{even}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,1+m+n} + \delta_{k,6+m+n})$$

7_+ 7^*_-

$$- \sum_{m,n=\text{odd}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,1+m+n} + \delta_{k,6+m+n}),$$

7_- 7^*_+

$$n^{(+,+)}_{\mathbf{21}_-,k} = \sum_{m,n=\text{even}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,2+m+n} + \delta_{k,5+m+n})$$

21_- 21^*_+

$$- \sum_{m,n=\text{odd}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,2+m+n} + \delta_{k,5+m+n}),$$

21_+ 21^*_-

$$n^{(+,+)}_{\overline{\mathbf{35}}_-,k} = \sum_{m,n=\text{even}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,3+m+n} + \delta_{k,4+m+n})$$

35_+ 35^*_-

$$- \sum_{m,n=\text{odd}} {}_pC_m \cdot {}_qC_n \cdot (\delta_{k,3+m+n} + \delta_{k,4+m+n})$$

35_- 35^*_+

GGHU

物質を作る
第１世代
第２世代
第３世代
クォーク
レプトン
u
アップ
c
チャーム
t
トップ
d
ダウン
s
ストレンジ
b
ボトム
e
電子
μ
ミューオン
τ
タウオン
νe
e ニュートリノ
νμ
μ ニュートリノ
ντ
τ ニュートリノ
力を伝える（ゲージ粒子）
強い力
g
グルーオン
電磁気力
γ
光子
弱い力
W+
W−
Wボソン
Z
Z ボソン
質量を与える
H
ヒッグス粒子

世代統一されたSHU

物質を作る

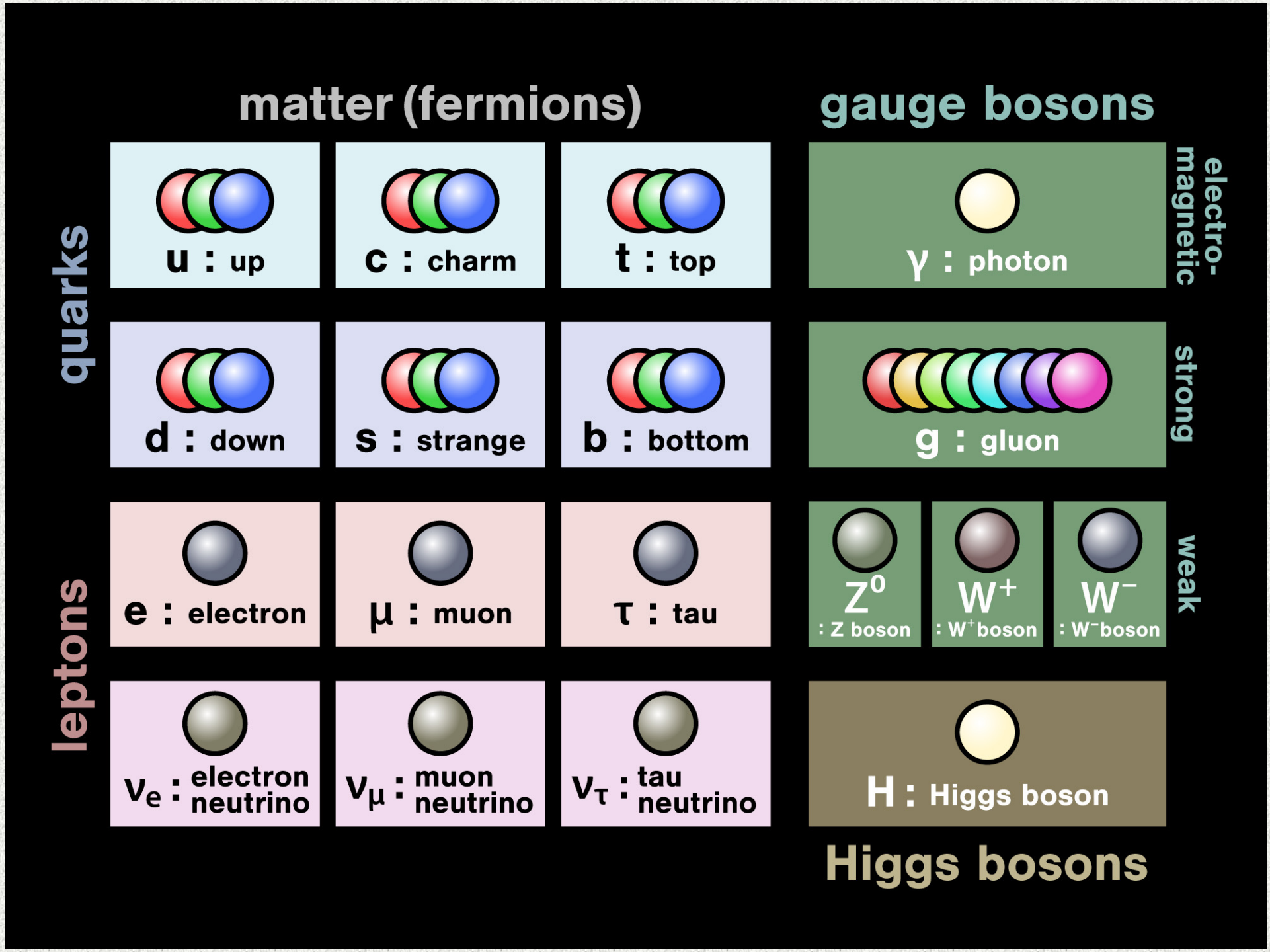
	第１世代	第２世代	第３世代
クォーク	u アップ	c チャーム	t トップ
クォーク	d ダウン	s ストレンジ	b ボトム
レプトン	e 電子	μ ミューオン	τ タウオン
レプトン	ν_e e ニュートリノ	ν_μ μ ニュートリノ	ν_τ τ ニュートリノ

力を伝える（ゲージ粒子）

強い力	g グルーオン	
電磁気力	γ 光子	
弱い力	W^+ W^- Wボソン	Z Zボソン

質量を与える

H ヒッグス粒子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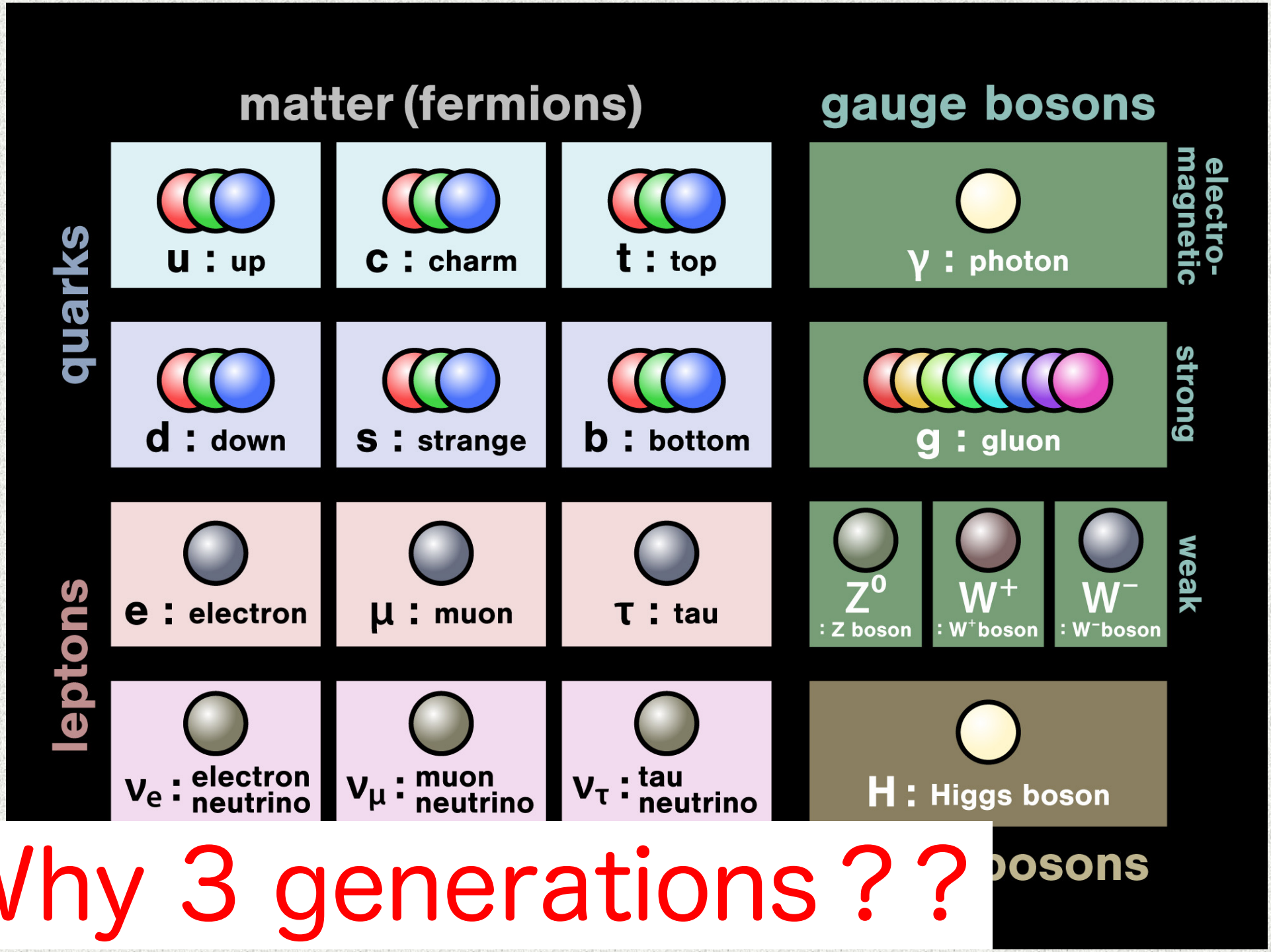
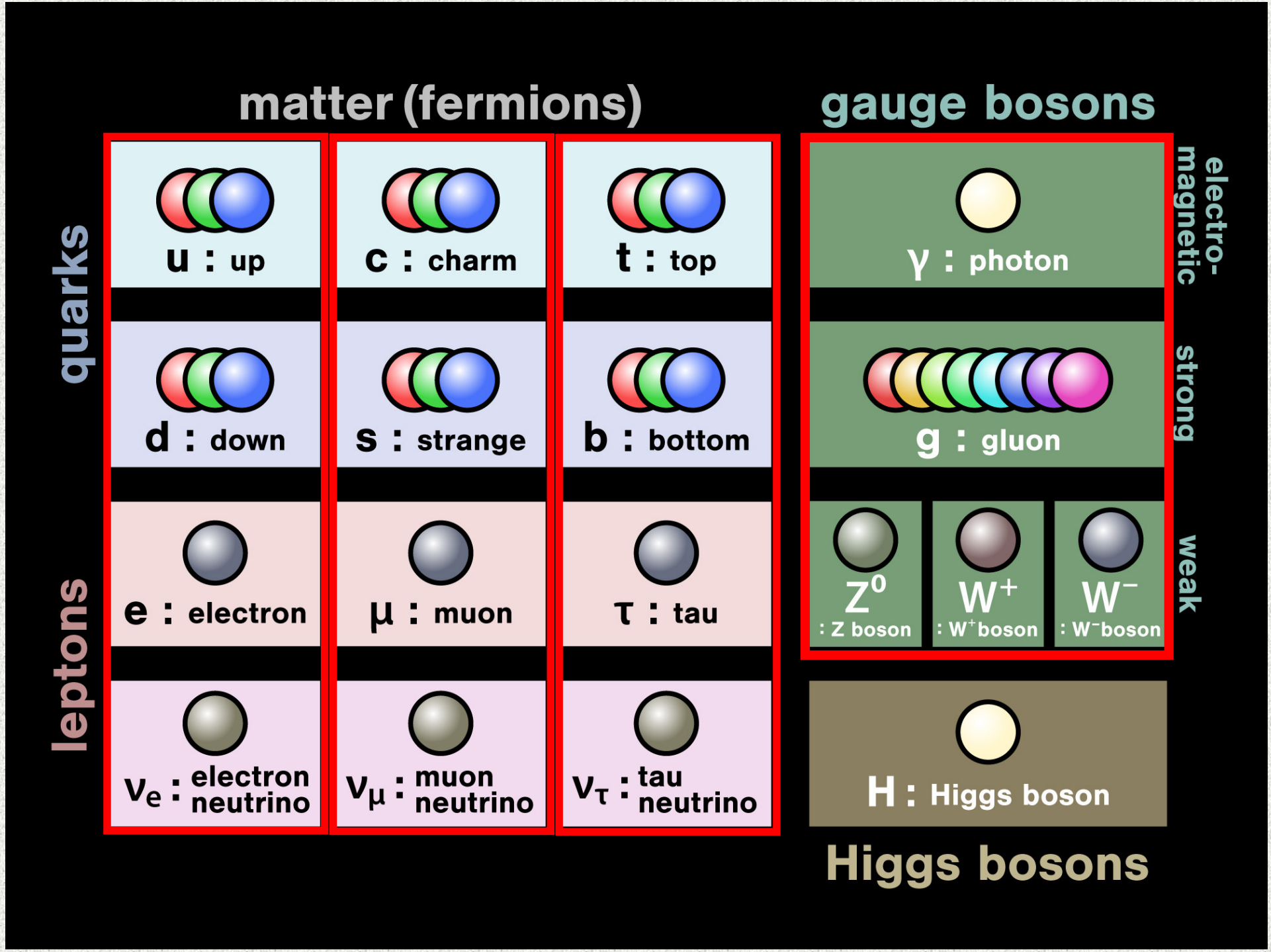


Why 3 generations ??

GUT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物質を作る

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クォーク	d ダウン	s ストレンジ	b ボトム
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レプトン	ν_e e ニュートリノ	ν_μ μ ニュートリノ	ν_τ τ ニュートリノ

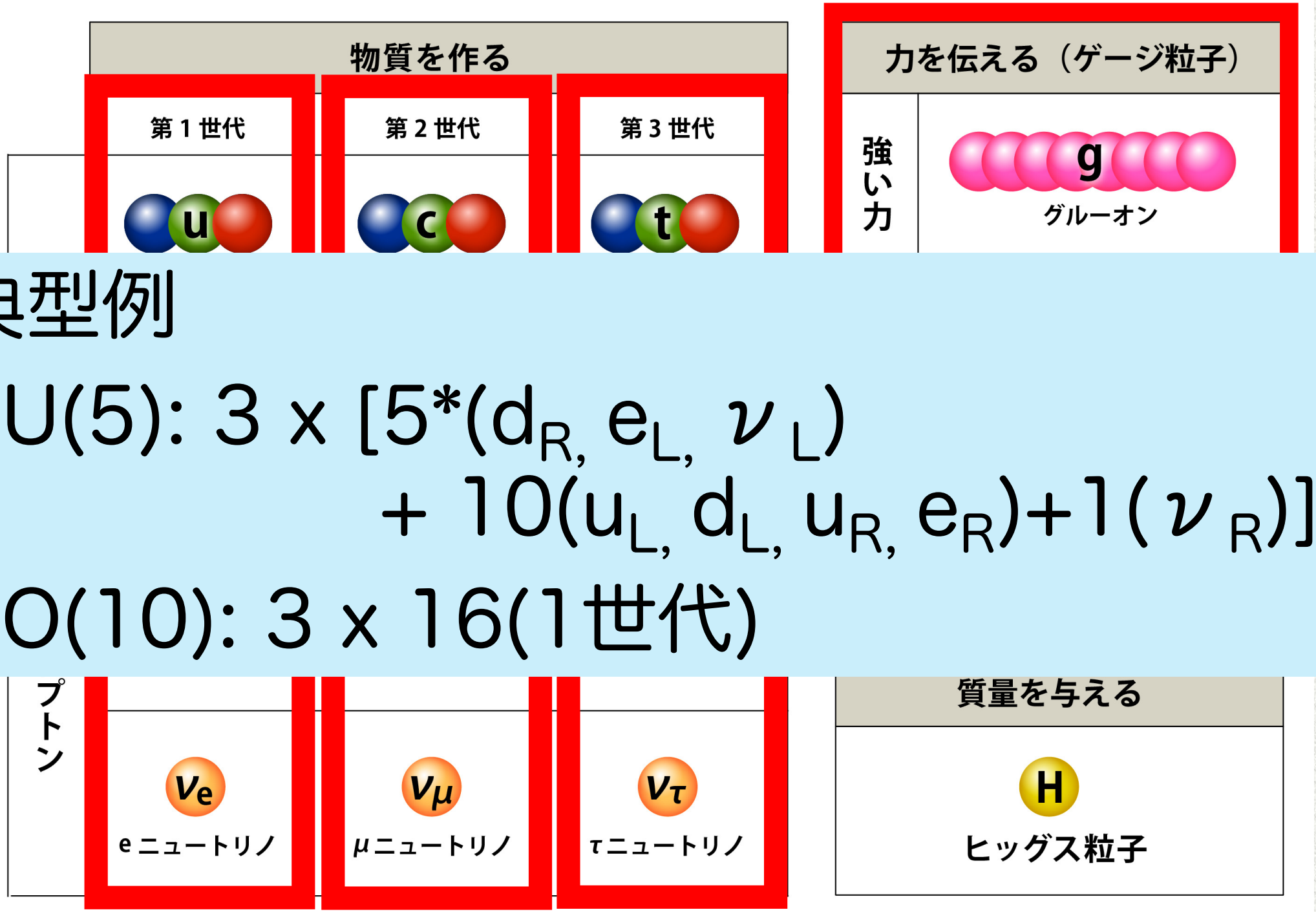
力を伝える（ゲージ粒子）

強い力	g グルーオン	
電磁気力	γ 光子	
弱い力	W^+ W^- Wボソン	Z Zボソン

質量を与える

H ヒッグス粒子

GUT



典型例

SU(5): 3 x [5*(d_R, e_L, ν_L)
+ 10(u_L, d_L, u_R, e_R)+1(ν_R)]

SO(10): 3 x 16(1世代)

	物質を作る		
	第 1 世代	第 2 世代	第 3 世代
クォーク	u アップ	c チャーム	t トップ
	d ダウン	s ストレンジ	b ボトム
レプトン	e 電子	μ ミューオン	τ タウオン
	ν_e e ニュートリノ	ν_μ μニュートリノ	ν_τ τニュートリノ

力を伝える（ゲージ粒子）	
強い力	g グルーオン
電磁気力	γ 光子
弱い力	W^+ W^- Wボソン　　Z Z ボソン
質量を与える	
	H ヒッグス粒子

Gauge-Higgs unification (GHU)

Family unification

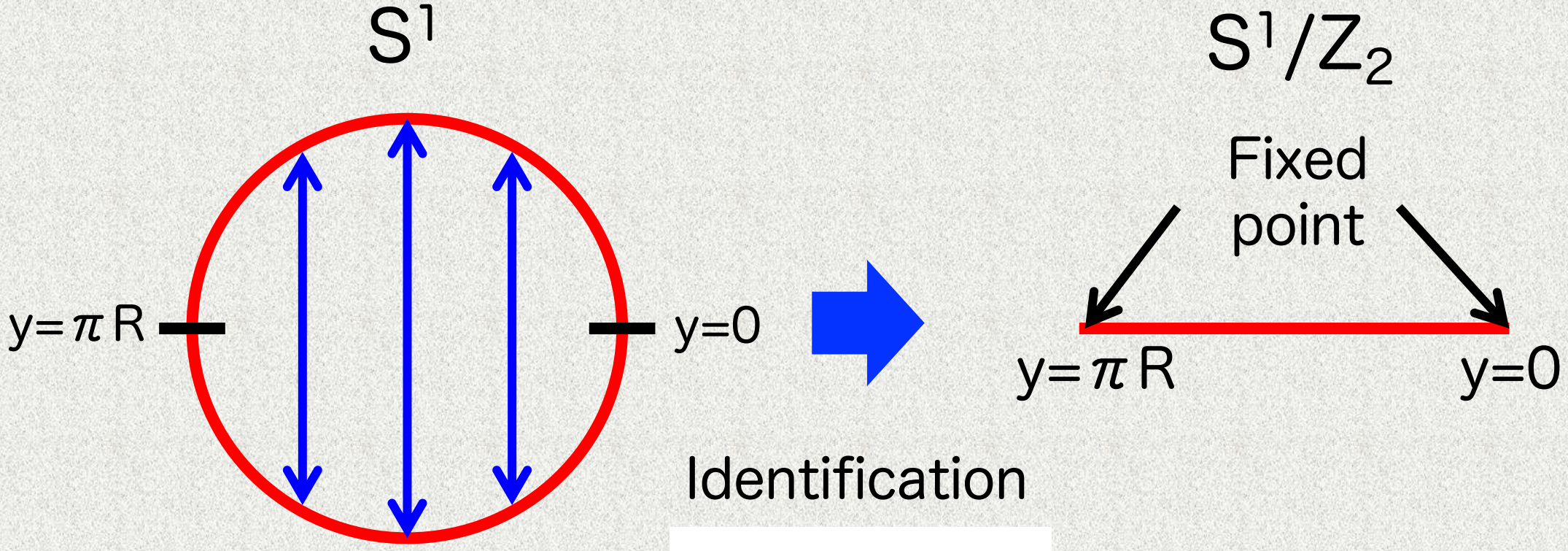
1. 1つのフェルミオンΨから、コンパクト化後に
   ３世代q・lのみがmasslessかつno massless exotic fermion
2. 複数の表現に属するフェルミオンΨ(R_1), Ψ(R_2), Ψ(R_3), …
   から、コンパクト化後に３世代q・lのみがmassless
   かつno massless exotic fermion
3. 1つのフェルミオンΨから、コンパクト化後に
   ３世代q・l + some massless exotic fermions
4. 複数の表現に属するフェルミオンΨ(R_1), Ψ(R_2), Ψ(R_3), ..
   からコンパクト化後に３世代q・lのみがmassless
   + some massless exotic fermions

Family unification (今回)

1. 1つのフェルミオンΨから、コンパクト化後に
   ３世代q・lのみがmasslessかつno massless exotic fermion
2. 複数の表現に属するフェルミオン$\Psi(R_1)$, $\Psi(R_2)$, $\Psi(R_3)$, …
   から、コンパクト化後に３世代q・lのみがmassless
   かつno massless exotic fermion
3. 1つのフェルミオンΨから、コンパクト化後に
   ３世代q・l + some massless exotic fermions
4. 複数の表現に属するフェルミオン$\Psi(R_1)$, $\Psi(R_2)$, $\Psi(R_3)$, ..
   からコンパクト化後に３世代q・lのみがmassless
   + some massless exotic fermions

5D theory of gauge-Higgs unification

5D SU(7) GGHU theory on S^1/Z_2



Identification

Z_2: $y \rightarrow -y$

- To obtain chiral fermions
- Symmetry Breaking

5D SU(7) GGHU theory on S^1/Z_2

Z_2 パリティ

$$P_0 = \mathrm{diag}(+1,+1,+1,+1,+1,-1,-1) \quad \text{at } y = 0,$$
$$P_1 = \mathrm{diag}(+1,+1,-1,-1,-1,-1,+1) \quad \text{at } y = \pi R,$$

yは5番目の座標
RはS¹の半径

ゲージ場のZ_2パリティ

$$A_\mu(x_\mu, y_{0,1}-y) = + P_{0,1}\, A_\mu(x_\mu, y_{0,1}+y)\, P^\dagger_{0,1}$$
$$A_5(x_\mu, y_{0,1}-y) = - P_{0,1}\, A_5(x_\mu, y_{0,1}+y)\, P^\dagger_{0,1}$$

(μ=0,1,2,3, $y_0 = 0$, $y_1 = \pi R$)

A_μのパリティは手で決める → A_5のパリティは$F_{\mu 5}$から決まる

モード展開

$$\Phi^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}}\phi_0(x) + \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty}\phi_n^{(+,+)}(x)\cos\left(\frac{n}{R}y\right)$$

$$\Phi^{(+,-)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi_n^{(+,-)}(x)\cos\left(\frac{n+1/2}{R}y\right),$$

$$\Phi^{(-,+)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi_n^{(-,+)}(x)\sin\left(\frac{n+1/2}{R}y\right),$$

$$\Phi^{(-,-)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=0}^{\infty}\phi_n^{(-,-)}(x)\sin\left(\frac{n}{R}y\right),$$

(+, +) parityだけ、massless mode (mass$^2 = -\partial_y^2$)

Z_2 Parity of gauge field

$$A_\mu(x_\mu, y_{0,1}-y) = + P_{0,1} A_\mu(x_\mu, y_{0,1}+y) P^\dagger_{0,1}$$
$$A_5(x_\mu, y_{0,1}-y) = - P_{0,1} A_5(x_\mu, y_{0,1}+y) P^\dagger_{0,1}$$

$$A_\mu = \left(\begin{array}{cc|ccc|cc}
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\
\hline
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\
\hline
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\
\hline
(-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) & (+,+)
\end{array}\right),$$

$$A_5 = \left(\begin{array}{cc|ccc|cc}
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\
(-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\
\hline
(-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\
(-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\
(-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\
\hline
(+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\
\hline
(+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-)
\end{array}\right),$$

ゲージ場のパリティ表示

$$A_\mu = \left(\begin{array}{cc|ccc|cc} (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\ (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\ \hline (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ \hline (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\ \hline (-,+) & (-,+) & (-,-) & (-,-) & (-,-) & (+,-) & (+,+) \end{array}\right),$$

$$\mathrm{SU}(7) \to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)^2$$

$$A_5 = \left(\begin{array}{cc|ccc|cc} (-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\ (-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\ \hline (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\ \hline (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \end{array}\right),$$

ゲージ場のパリティ表示

$$A_\mu = \left(\begin{array}{cc|ccc|cc} (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\ (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (-,-) & (-,+) \\ \hline (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (-,+) & (-,-) \\ \hline (-,-) & (-,- & & & & (+,+) & (+,-) \\ \hline (-,+) & (-,+ & & & & (+,-) & (+,+) \end{array}\right),$$

SM Higgs

$$A_5 = \left(\begin{array}{cc|ccc|cc} (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\ (-,-) & (-,-) & (-,+) & (-,+) & (-,+) & (+,+) & (+,-) \\ \hline (-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\ (-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\ (-,+) & (-,+) & (+,-) & (+,-) & (+,-) & (+,-) & (+,+) \\ \hline (+,+) & (+,+) & (+,-) & (+,-) & (+,-) & (- & \\ \hline (+,-) & (+,-) & (+,+) & (+,+) & (+,+) & (- & \end{array}\right),$$

Colored Higgs

フェルミオンの Z_2パリティ

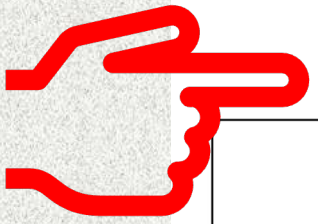
$\Psi(-y) = \eta R(P_0)\Gamma_5\Psi(y)$

$\Psi(\pi R-y) = \eta' R(P_1)\Gamma_5\Psi(\pi R+y)$ $(\eta, \eta' = \pm 1)$

$$
\overline{7} = \begin{cases} \overline{7}_L = \underbrace{(\overline{3},1)^{(+,-)}_{(\frac{1}{3},-1,-1)} \oplus l_L(1,2)^{(+,+)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(-,-)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(-,+)}_{(0,0,6)}}_{1} \\ \overline{7}_R = \underbrace{(\overline{3},1)^{(-,+)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(-,-)}_{(-\frac{1}{2},-1,-1)} \oplus \nu_R(1,1)^{(+,+)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(+,-)}_{(0,0,6)}}_{1} \end{cases}
$$

$$
\overline{7} = \begin{cases} \overline{7}_L = \underbrace{(\overline{3},1)^{(-,-)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(-,+)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(+,-)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{\chi_1(1,1)^{(+,+)}_{(0,0,6)}}_{1} \\ \overline{7}_R = \underbrace{\bar{d}_R(\overline{3},1)^{(+,+)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(+,-)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(-,+)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(-,-)}_{(0,0,6)}}_{1} \end{cases}
$$

$$
\overline{\mathbf{35}} = \begin{cases}
\overline{\mathbf{35}}_L = \underbrace{q_L(\mathbf{3},\mathbf{2})^{(+,+)}_{(\frac{1}{6},-3,-3)} \oplus (\bar{\mathbf{3}},\mathbf{1})^{(+,-)}_{(-\frac{2}{3},-3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(+,-)}_{(1,-3,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(-,+)}_{(-\frac{1}{6},3,-3)} \oplus (\mathbf{3},\mathbf{1})^{(-,-)}_{(\frac{2}{3},3.,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,-)}_{(-1,3,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(-,-)}_{(-\frac{1}{6},-2,4)} \oplus (\mathbf{3},\mathbf{1})^{(-,+)}_{(\frac{2}{3},-2.,4)} \oplus \chi_2(\bar{\mathbf{3}},\mathbf{1})^{(+,+)}_{(\frac{1}{3},4,4)} \oplus (\mathbf{1},\mathbf{2})^{(+,-)}_{(-\frac{1}{2},4,4)} \oplus (\mathbf{1},\mathbf{1})^{(-,+)}_{(-1,-2,4)}}_{\overline{\mathbf{15}}} \\
\overline{\mathbf{35}}_R = \underbrace{(\mathbf{3},\mathbf{2})^{(-,-)}_{(\frac{1}{6},-3,-3)} \oplus (\bar{\mathbf{3}},\mathbf{1})^{(-,+)}_{(-\frac{2}{3},-3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,+)}_{(1,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(+,-)}_{(-\frac{1}{6},3,-3)} \oplus u_R(\mathbf{3},\mathbf{1})^{(+,+)}_{(\frac{2}{3},3,-3)} \oplus e_R(\mathbf{1},\mathbf{1})^{(+,+)}_{(-1,3,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ \chi_3(\bar{\mathbf{3}},\mathbf{2})^{(+,+)}_{(-\frac{1}{6},-2,4)} \oplus (\mathbf{3},\mathbf{1})^{(+,-)}_{(\frac{2}{3},-2.,4)} \oplus (\bar{\mathbf{3}},\mathbf{1})^{(-,-)}_{(\frac{1}{3},4,4)} \oplus (\mathbf{1},\mathbf{2})^{(-,+)}_{(-\frac{1}{2},4,4)} \oplus (\mathbf{1},\mathbf{1})^{(+,-)}_{(-1,-2,4)}}_{\overline{\mathbf{15}}}
\end{cases},
$$



物質を作る

	第１世代	第２世代	第３世代
クォーク	u アップ	c チャーム	t トップ
	d	s	b
レプトン	e 電子	μ ミューオン	τ タウオン
	ν_e e ニュートリノ	ν_μ μ ニュートリノ	ν_τ τ ニュートリノ

力を伝える（ゲージ粒子）

強い力	g グルーオン
電磁気力	γ 光子
	Z ボソン

質量を与える

H ヒッグス粒子

なぜ3世代なのか？

3世代へのアプローチ

- 群論的アプローチ ← 本研究

  Georgi NPB156 (1979) 126, Wilczek and Zee PRD25 (1982) 553など多数

- Coset G/HにおけるNGフェルミオン

  $E_7/[SU(5)\times SU(3)\times U(1)] \rightarrow 3 \times [5^*+10]$

  Kugo and Yanagida PLB134 (1983) 313

- 余剰次元に磁場がかかった高次元理論

  余剰2次元磁場中のQMにおける基底状態の縮退数 → 世代数

- アノマリー相殺条件

  6次元$SU(2)_L$ globalアノマリー相殺 → 世代数 = 0 (mod 3)

  Dobrescu and Ponton PRL87 (2001) 031801

Yukawa coupling

ゲージ・ヒッグス統一模型では
ヒッグス場がゲージ場の余剰成分のため
湯川結合の構成は非自明

同一多重項に左巻きSU(2) 2重項、
右巻きSU(2) 1重項フェルミオンがあれば、
高次元のゲージ相互作用から湯川結合が生成

↓

一般に全ての湯川結合をゲージ相互作用で
再現するのは困難

Yukawa coupling

$$
\overline{7} = \begin{cases} \overline{7}_L = \underbrace{(\overline{3},1)^{(+,-)}_{(\frac{1}{3},-1,-1)} \oplus l_L(1,2)^{(+,+)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(-,-)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(-,+)}_{(0,0,6)}}_{1} \\ \overline{7}_R = \underbrace{(\overline{3},1)^{(-,+)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(-,-)}_{(-\frac{1}{2},-1,-1)} \oplus \nu_R(1,1)^{(+,+)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(+,-)}_{(0,0,6)}}_{1} \end{cases}
$$

$$
\overline{7} = \begin{cases} \overline{7}_L = \underbrace{(\overline{3},1)^{(-,-)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(-,+)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(+,-)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{\chi_1(1,1)^{(+,+)}_{(0,0,6)}}_{1} \\ \overline{7}_R = \underbrace{\bar{d}_R(\overline{3},1)^{(+,+)}_{(\frac{1}{3},-1,-1)} \oplus (1,2)^{(+,-)}_{(-\frac{1}{2},-1,-1)} \oplus (1,1)^{(-,+)}_{(0,5,-1)}}_{\overline{6}} \oplus \underbrace{(1,1)^{(-,-)}_{(0,0,6)}}_{1} \end{cases}
$$

$$
\overline{\mathbf{35}} = \begin{cases}
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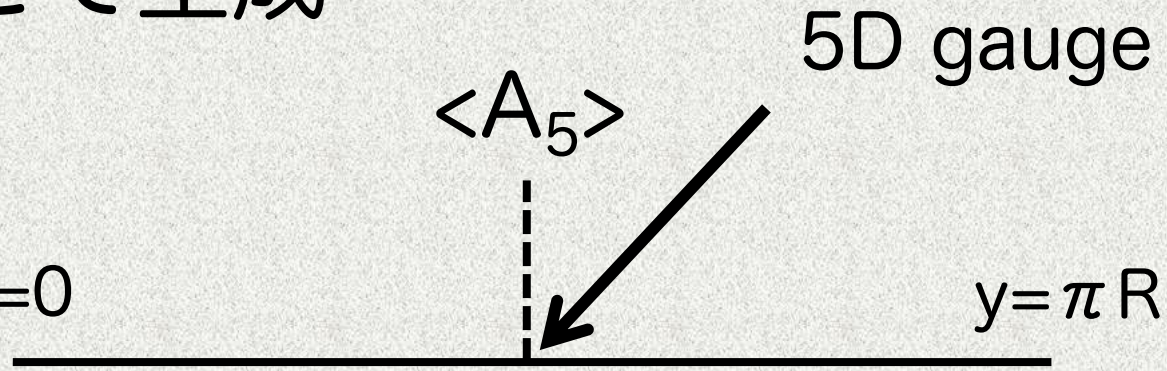
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(-,+)}_{(-\frac{1}{6},3,-3)} \oplus (\mathbf{3},\mathbf{1})^{(-,-)}_{(\frac{2}{3},3.,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,-)}_{(-1,3,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(-,-)}_{(-\frac{1}{6},-2,4)} \oplus (\mathbf{3},\mathbf{1})^{(-,+)}_{(\frac{2}{3},-2.,4)} \oplus \chi_2(\bar{\mathbf{3}},\mathbf{1})^{(+,+)}_{(\frac{1}{3},4,4)} \oplus (\mathbf{1},\mathbf{2})^{(+,-)}_{(-\frac{1}{2},4,4)} \oplus (\mathbf{1},\mathbf{1})^{(-,+)}_{(-1,-2,4)}}_{\overline{\mathbf{15}}} \\
\overline{\mathbf{35}}_R = \underbrace{(\mathbf{3},\mathbf{2})^{(-,-)}_{(\frac{1}{6},-3,-3)} \oplus (\bar{\mathbf{3}},\mathbf{1})^{(-,+)}_{(-\frac{2}{3},-3,-3)} \oplus (\mathbf{1},\mathbf{1})^{(-,+)}_{(1,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ (\bar{\mathbf{3}},\mathbf{2})^{(+,-)}_{(-\frac{1}{6},3,-3)} \oplus u_R(\mathbf{3},\mathbf{1})^{(+,+)}_{(\frac{2}{3},3,-3)} \oplus e_R(\mathbf{1},\mathbf{1})^{(+,+)}_{(-1,3,-3)}}_{\mathbf{20}} \\
\underbrace{\oplus\ \chi_3(\bar{\mathbf{3}},\mathbf{2})^{(+,+)}_{(-\frac{1}{6},-2,4)} \oplus (\mathbf{3},\mathbf{1})^{(+,-)}_{(\frac{2}{3},-2.,4)} \oplus (\bar{\mathbf{3}},\mathbf{1})^{(-,-)}_{(\frac{1}{3},4,4)} \oplus (\mathbf{1},\mathbf{2})^{(-,+)}_{(-\frac{1}{2},4,4)} \oplus (\mathbf{1},\mathbf{1})^{(+,-)}_{(-1,-2,4)}}_{\overline{\mathbf{15}}}
\end{cases},
$$

Yukawa coupling

- アップ型湯川とニュートリノ湯川は、
  5Dゲージ相互作用$\Psi^* A_5 \Psi$から生成
- 他の湯川については、SMフェルミオンを4Dブレーンに導入し、5D有質量フェルミオン場と結合させることで生成



$$\frac{\varepsilon_L}{\sqrt{\pi R}}\bar{\psi}^d Q_L \qquad \bar{\psi}^d A_5 \chi^s \qquad \frac{\varepsilon_R}{\sqrt{\pi R}}\bar{q}_R \chi^s$$

5Dフェルミオンの
Ψ^d: SU(2) 2重項
X^s: SU(2) 1重項

$$M_q \sim M_W \exp[-\pi MR]$$

Anomaly cancellation

In our model, all of the fermions are introduced as Dirac, not chiral → No gauge anomalies

As for the global gauge anomaly,
the 6th homotopy group of SU(14) $\Pi_6(SU(14)) = 0$
→ No global gauge anomaly