A positive-definite formulation of tunneling action

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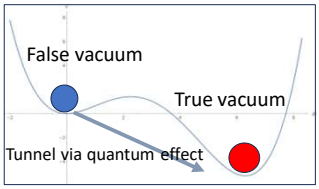
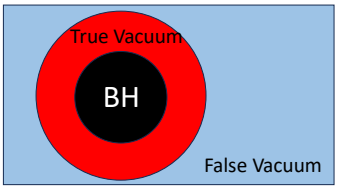
We propose a positive definite formulation of O(3)-symmetric tunneling by extending the Tunneling Potential method

1. Introduction





- Vacuum decay can occur depending on the structure of the Higgs sector
- It can be detected through the observation of the Gravitational waves

- Spherical objects (PBH, monopole, I-ball) enhance the vacuum decay via gravitational effects.

[Hiscook, 1996]

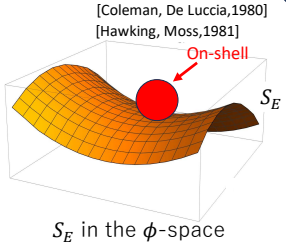
2. Decay rate and negative mode

Decay rate : $\Gamma \sim \exp(-S_E|_{on-shell})$

$$S_E = 2\pi^2 \int d\xi\, \rho(\xi)^3 \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + V(\phi)\right]$$

S_E has a decreasing direction (negative mode)

[Coleman, De Luccia,1980]
[Hawking, Moss,1981]

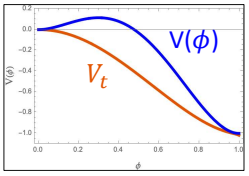


S_E in the ϕ-space

Difficult to extend from tunneling with O(4)-symmetry to O(3)-symmetry

3. Tunneling Potential method with O(4)

[Espinosa, 2019]



What is tunneling Potential method?

A positive-definite formulation of tunneling action with O(4)-symmetry

$$S_E = 2\pi^2 \int d\xi\, \rho(\xi)^3 \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + V(\phi)\right]$$

Change the dynamical variable & Canonical transformation

[Espinosa, Jinno, Konstandin 2022]

$$S_E = \frac{6\pi^2}{\kappa^2} \int d\phi\, \frac{(D+V_t')^2}{V_t^2 D} > 0 \ : \text{positive-definite}$$

$V_t(\phi)$: dynamical variable (tunneling potential)

No decreasing direction (negative mode)

We will extend this method from O(4) to O(3)-symmetry!

Our work: Positive definite formulation of O(3)-symmetric tunneling

4. O(3)-Tunneling Potential method

We formulate O(3)-symmetric tunneling analogous to O(4)-tunneling potential

O(3)-symmetric metric: $ds^2 = T^2\, dt^2 + R^2\, dr^2 + r^2 d\Omega_2^2$

$$S_E = 4\pi^2 \int dt \int dr\, r^3 \left[\frac{R}{2T}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{T}{2R}\left(\frac{\partial\phi}{\partial r}\right)^2 + TR\left\{V + \frac{M_p^2}{r^2}\left(\frac{\partial}{\partial r}\left(\frac{r}{R^2}\right) - 1\right)\right\}\right] - \text{(background)}$$

Change the dynamical variable

$\phi(t,r) \longrightarrow t(\phi,r)$

$$S_E = 4\pi^2 \int d\phi \int dr\, r^2 \left[\frac{R/T + T/R\, t'^2}{2\dot{t}} + \dot{t}TR\tilde{V} + \frac{2M_p^2 T\dot{R}t'}{R^2 r^2}\right] - \text{(background)}$$

Canonical transformation with the generating function being W=tP'

(t,p) ⟶ (Q,P)

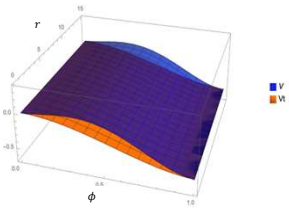
Positive definite tunneling action is

$$S_E = \int d\phi \int dr \sqrt{128\pi^2 \left[\frac{R}{T}\left(TR\tilde{V} - \left(V_t + \frac{r}{3}V_t'\right)\right) - \frac{R^2 r^2}{T^2 18}\left(\dot{V}_t + \frac{6M_p^2 T\dot{R}}{R^2 r^2}\right)\right]} - \text{(background)}$$

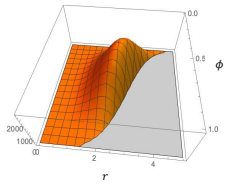
O(3)-symmetric version of the Tunneling Potential method!

5. Numerical illustration for consistency

The simplest example to perform a consistency check is tunneling in the fixed AdS spacetime



solution and potential



Action density

M_p^2	Positive-definite	Shooting method
10^4	39.90	39.87
10^3	40.90	40.87
10^2	53.90	53.89

We can reproduce the known result!

6. Future work

- On-going work: numerical analysis of tunneling in Schwarzschild spacetime
- To extend the positive-definite formulation to tunneling with lower symmetry (O(2)-symmetry or non-symmetric tunneling) and formulate the method to calculate the 1-loop determinant without gravity
- Positive-definite formulation for tunneling at finite temperature, which possesses O(3)-symmetry