Evanescent operator contribution to radiative correction to QCD theta parameter at two-loop level in the BMHV scheme

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Is a CP-odd observable ($\bar{\theta}$) made of 4-dimensional rephasing invariants under DReg?

Introduction

Parity P and time-reversal T violating QCD

$$\mathcal{L}_{\not{P},\not{T}} = -\sum_{q=\text{all}} \text{Im}(m_q)\bar{q}i\gamma_5 q + \theta_G \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad m_q \equiv |m_q|e^{i\theta_q}$$

- rephasing-invariant: QCD θ angle $\bar{\theta} = \theta_G - \sum_q \theta_q$

due to the chiral anomaly

S. L. Adler, Phys. Rev. **177** (1969) 2426-38
J. S. Bell, R. Jackiw, Nuovo. Cim. A **60** (1969) 47-61
K. Fujikawa Phys. Lett. **42** (1979) 1195-1198

strong CP problem

neutron EDM exp.: $|d_n|_{\text{exp}} < 1.8 \times 10^{-26}\ e\,\text{cm}\ (90\%\ \text{CL})$

C. Abel, *et al.*, Phys. Rev. Lett. 124 (2020) 081803

naturalness?

$$|\bar{\theta}| \lesssim 1.2 \times 10^{-10} \quad (90\%\ \text{CL})$$

J. Liang, et al., *Phys.Rev.D* 108 (2023) 9, 094512

Accurate calculation of the QCD θ angle, including radiative corrections, is required for models other than the Pecchi-Quinn mechanism.

γ_5-problem

G. 't Hooft, M.J.G. Veltman, Nucl. Phys. B 1972, 44, 189–213
C. G. Bollini and J. J. Giambiagi, *Nuovo Cim.B* 12 (1972) 20-26
P. Breitenlohner, D. Maison, Commun. Math. Phys. 1977, 52, 11–38

The γ matrix algebra cannot be defined consistently in d-dimension. $d \equiv 4 - 2\epsilon$

- **BMHV scheme**
  - tensor

$$D\text{-dim.}: g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \qquad 4\text{-dim.}: \bar{g}^{\mu\nu} \qquad (D-4)\text{-dim.}: \hat{g}^{\mu\nu}$$

  - give up the anti-commutation property

$$\{\gamma_5, \overline{\gamma_\mu}\} = 0, \quad \boxed{[\gamma_5, \hat{\gamma_\mu}] = 0}$$

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\bar{\epsilon}_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

$$\epsilon_{\mu\nu\rho\sigma} = \bar{\epsilon}_{\mu\nu\rho\sigma}, \quad \hat{\epsilon}_{\mu\nu\rho\sigma} = 0$$

- **The calculation of $\bar{\theta}$ is suffered from the γ_5-problem.**
- **The γ_5-problem is caused by introduced evanescent operators, which vanish in a limit of $d \to 4$.**

QCD θ angle up to two-loop level

- setup: one-flavor QCD with a CP-odd Yukawa interaction

$$\mathcal{L} = |\partial_\mu\phi|^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{4}\text{Tr}[\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}] + \theta_G\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \bar{q}i\not{D}q - \bar{q}[\text{Re}(m_q) + \text{Im}(m_q)i\gamma_5]q - y_q\bar{q}P_Rq\phi + \text{h.c.}$$

CP-odd rephasing-invariant: $\boxed{\theta_G - \arg(m_q)}$, $y_q m_q^*$

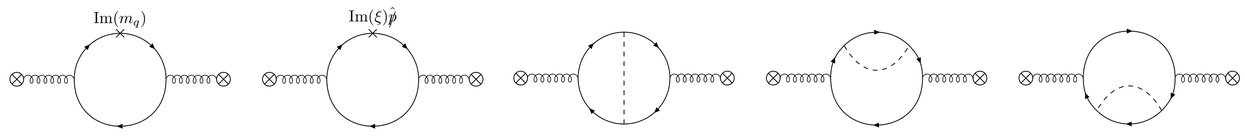
chiral anomaly

in d-dimension,

$$\mathcal{L} = |\bar{\partial}_\mu\phi|^2 + \boxed{|\hat{\partial}_\mu\phi|^2} - \frac{1}{2}m_\phi^2\phi^2 \boxed{- \frac{1}{4}\text{Tr}[\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}]} + \theta_G\frac{\alpha_s}{8\pi}\bar{G}^a_{\mu\nu}\tilde{\bar{G}}^{a\mu\nu} + \bar{q}i\bar{\not{D}}q + \boxed{(1+\xi)\bar{q}_L i\hat{\not{D}}q_R} - \bar{q}[\text{Re}(m_q) + \text{Im}(m_q)i\gamma_5]q - y_q\bar{q}P_Rq\phi + \text{h.c.}$$

CP-odd rephasing-invariant: $\boxed{\theta_G}$, $y_q m_q^\star$, $m_q(1+\xi)^\star$, $y_q(1+\xi)^\star$

- a result of the QCD θ angle



J. Hisano, et al., *JHEP* 03 (2023) 150

$$\begin{aligned}
&\theta_G(Q) + \delta_{\theta_G} + \delta\theta^{(1)} + \delta\theta^{(2)}_{\text{c.t.}} + \delta\theta^{(2)}_a + \delta\theta^{(2)}_b + \delta\theta^{(2)}_c \\
&= (\theta_G(Q) + \delta_{\theta_G}) + \left(-\frac{\text{Im}[m_q]}{\text{Re}[m_q]} - \text{Im}\left[\delta_{m_q}\right]\right) + (\text{Im}\left[\xi(Q)\right] + \text{Im}\left[\delta_\xi\right]) \\
&+ \frac{\text{Im}[y_q^2]}{16\pi^2}(\text{Re}[m_q])^2\left(I^{(2,2,1)} + 2I^{(3,1,1)}\right) - \frac{d-4}{2d}\frac{\text{Im}[y_q^2]}{16\pi^2}\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \frac{1}{\epsilon}\log Q^2(\text{Re}\left[m_q\right])^2 + O(\epsilon^0)\right] \\
&= \left(\theta_G(Q) + \text{Im}\left[\xi(Q)\right] + \frac{\text{Im}\left[y_q^2\right]}{16\pi^2}\frac{5}{8}\right) - \frac{\text{Im}[m_q]}{\text{Re}[m_q]} + \frac{\text{Im}[y_q^2]}{16\pi^2}(\text{Re}[m_q])^2\left(I^{(2,2,1)} + 2\bar{I}^{(3,1,1)}\right) + \left[\cancel{-\frac{\text{Im}\left[y_q^2\right]}{16\pi^2}\frac{1}{4\epsilon}} + \delta_{\theta_G}\right] + O(\epsilon^1) \\
&\qquad\qquad \| \\
&\qquad \theta_G^{(4-\text{dim})}
\end{aligned}$$

the rephasing-invariants of original $\mathscr{L}$!

Summary

- The result of $\bar{\theta}$ using DReg is made of the d-dimensional CP-odd rephasing-invariants, but it can be understood by 4-dimensional one.
  - **It is free from the γ_5-problem**. We can neglect the $(d-4)$-dimensional contributions at the first place.
- The result means θ_G is invariant under chiral transformation.
  - The chiral anomaly comes from the evanescent kinetic term(!?)