Pseudo-Nambu-Goldstone-Boson Dark Matter from Three Complex Scalars

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1. Introduction

a. The "What" and Why of Dark Matter

- From various astrophysical observations e.g., Galactic rotation curve (fig. 1), we hypothesize that some invisible matter exists.
- Nature: Electrically neutral, stable, and non-relativistic (cold).
- Out of many candidates, Weakly Interacting Massive Particle (WIMP) scenario is one of the most promising.

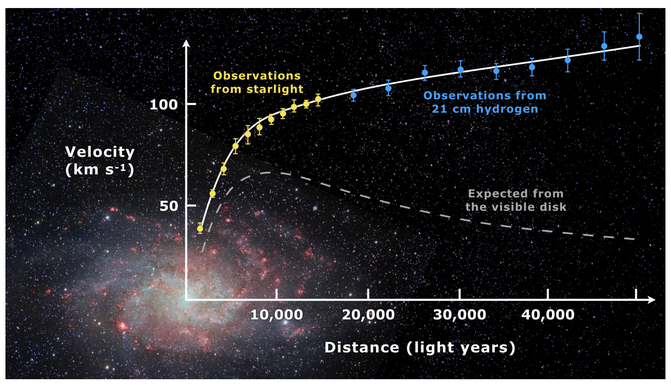


Fig. 1. Galactic rotation curve deviation [E. Corbelli et al.2000].

b. WIMP's role in DM search

- DM is produced thermally in chemical equilibrium.
- Freeze out abundance attained i.e.,

$$\Omega h^2 \simeq \frac{0.1\,\text{pb}}{\langle \sigma_{\text{ann.}} v \rangle} \simeq 0.12 \pm 0.001$$

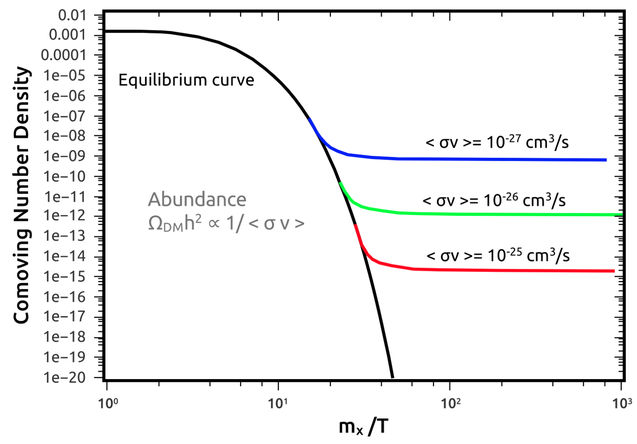
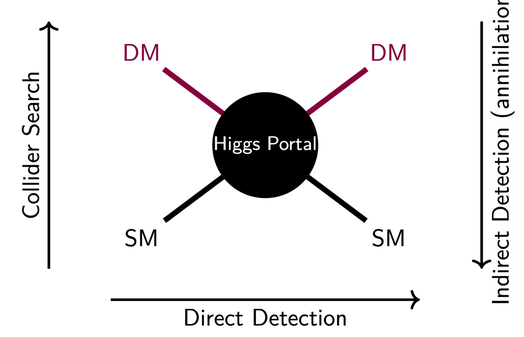


Fig. 2. Number density evolution. [Arcadi et al. 2018]



- Cross section boounds are getting stringent, e.g., fig. 3.

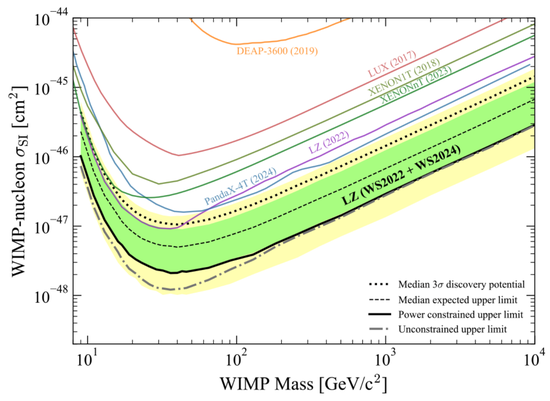


Fig. 3. DD constraint [LZ2024].

2. Background

c. Why pNGBs?

- Arises from spontaneous and soft breaking of global symmetries.
- Produces derivative interactions i.e, velocity suppressed.
- Naturally evades the stringent Direct Detection bound.

d. Original idea [Gross et al., 2017]

- One SM Singlet complex scalar S.
- Invariant under global $U(1)$ i.e, $S \to e^{i\alpha} S$.

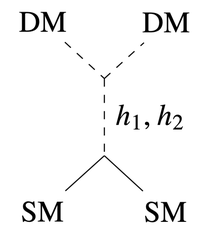
$$\mathcal{V}(S,\Phi) = -\frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_S}{2}|S|^4 \underbrace{- \mu_\Phi^2|\Phi|^2 + \frac{\lambda_\Phi}{2}|\Phi|^4}_{\text{SM Higgs}} + \underbrace{\lambda_{\Phi S}|\Phi|^2|S|^2}_{\text{Higgs portal}} \underbrace{- \frac{\mu_S'^2}{4}S^2 + \text{h.c.}}_{\text{soft } U(1) \text{ breaking } (\to \mathbb{Z}_2)} .$$

- After introducing VEVs: $S = (v_s + s + i\chi)/\sqrt{2}$, $\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}.$$

- Higgs bosons i.e., h_1, h_2 acts as mediator.

$$\mathcal{M} \propto \left(\frac{m_1^2}{t - m_1^2} - \frac{m_2^2}{t - m_2^2} \right) \simeq \frac{t\,(m_1^2 - m_2^2)}{m_1^2 m_2^2} \to 0$$



- Cons: Domain Wall Problem! $(\mathbb{Z}_2 \xrightarrow{\text{SSB}} \varnothing)$

3. Motivation

To provide a concrete theoretical framework for realizing the semi-annihilation-driven boosted DM (BDM) scenario investigated in [M. Aoki et al. 2023].

Fig. 4. BDM produced by semi-annihilation at Sun [T.Toma 2022].

4. The Model

- Three SM singlet complex scalars: S_j $(j = 1, 2, 3)$.
- Gauge $U(1)_V$ symmetry invariant: $S_j \to e^{i\theta_V(x)} S_j$.
- Softly-broken global $U(1)_A$ symmetry: $S_j \to e^{i\theta_A^j} S_j$.
- Discrete $S(3)$ permutation exchange symmetry: $S_1 \leftrightarrow S_2 \quad S_2 \leftrightarrow S_3 \quad S_3 \leftrightarrow S_1$.

- The soft-breaking part of our potential:

$$\mathcal{V}(S_j, \Phi) \supset \underbrace{-\frac{m_{12}^2}{3}\left(S_1^* S_2 + S_2^* S_3 + S_3^* S_1 + \text{h.c.} \right)}_{\text{soft breaking}}$$

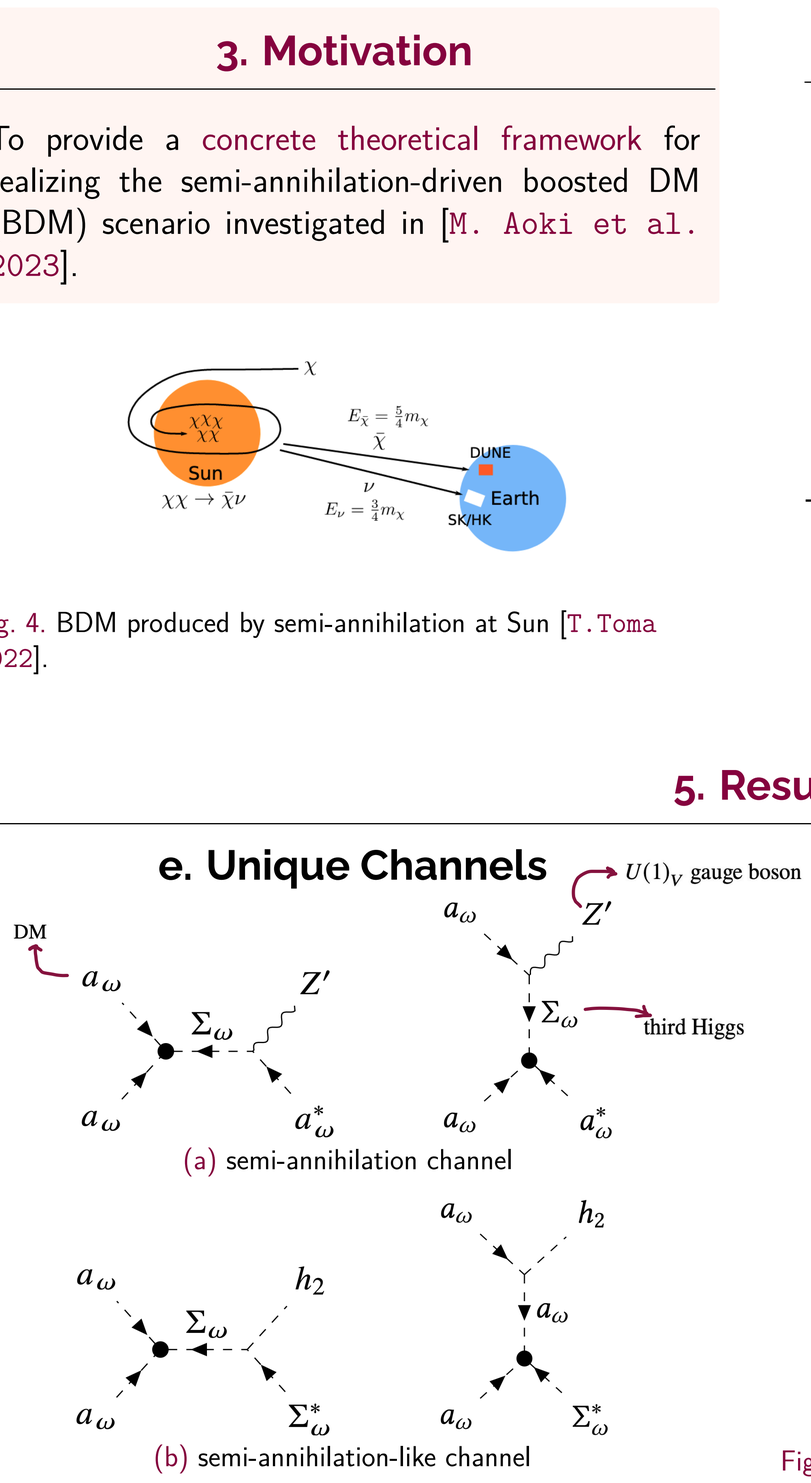
- In Higgs basis only one field gains VEV $\Sigma_1 = (v_s + s_1' + iz)/\sqrt{2}, \quad \omega = e^{i2\pi/3}$
- $(CP, \mathbb{Z}_3)$ charges for mass eigenbasis:
  $(h, s_1')_{2\times 2} : (+,1)_{2\times 2}, \quad z : (-,1),$
  $\Sigma_\omega : (+,\omega), \quad a_\omega : (-,\omega) \to$ Cx pNGB DM

The symmetry structure is as follows:

$$\overbrace{U(1)^{\text{global}} \times \underbrace{U(1)_A^{\text{global}}}_{\supset S(3)} \times U(1)_V^{\text{gauge}}}^{S_1, S_2, S_3} \xrightarrow[2\,\text{(p)NGBs}]{\text{soft break}} \overbrace{\mathbb{Z}_3 \times U(1)_V^{\text{gauge}}}^{\text{Higgs basis } (\Sigma_1, \Sigma_2, \Sigma_3)} \xrightarrow[1\,\text{(would-be)NGB}]{\text{SSB}} \underbrace{\mathbb{Z}_3\,\text{(DM stability)}}_{\text{mass eigenbasis } (h, s_1', z, \Sigma_\omega, a_\omega)} .$$

5. Results

e. Unique Channels



(a) semi-annihilation channel

(b) semi-annihilation-like channel

f. Relic abundance

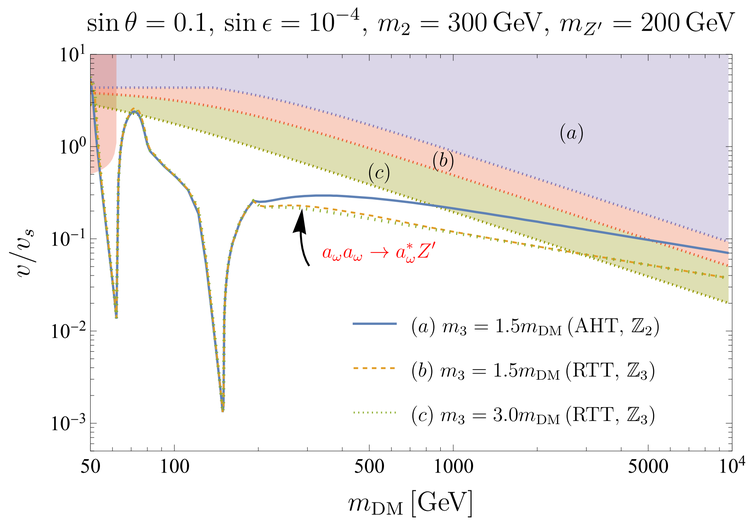


Fig. 6. Viable parameter space satisfying $\Omega_{\text{DM}}h^2 \approx 0.12$ and comparison with the AHT model [T. Abe et al. 2024].

6. Conclusion

- One complex pNGB DM stabilized by a residual $\mathbb{Z}_3$ symmetry.
- Relic density is set by both annihilation and semi-annihilation.
- Boosted DM flux is negligible in this minimal scenario, total cross section $\sigma_{\text{el}} \sim 10^{-54}\,\text{cm}^2$ well below the experimental sensitivity.
- Provides a minimal and consistent framework for semi-annihilation-driven pNGB BDM and opens the path for more predictive extensions.
- Future: We are working on a multi-component pNGB DM model to produce more boost.

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