Five Dimensional $SU(5) \times U(1)_{\rm PQ}$ Grand Unification

Norimi Yokozaki, Junhao Zhu

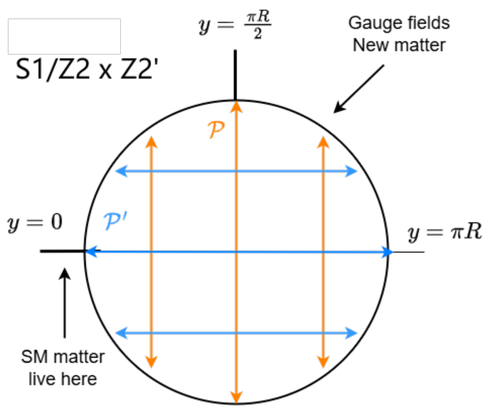
Yokozaki, N., Zhu, J. Eur. Phys. J. C 85, 591 (2025). arxiv:2412.15692

Summary : We investigate a GUT model that combines two famous mechanisms: GUT breaking via a 5D orbifold, and the QCD axion from a $U(1)$ 1-form field. Therefore, the axion decay constant and the GUT scale are unified. We introduce new bulk matter in an orbifolding-consistent way to achieve unification. The model is constrained by proton decay and the standard cosmological predictions. A trade-off between the unification scale and the reheating temperature leads to a proton decay signal and axion dark matter detectable by upcoming experiments.

Merits of Extra Dimensions

- Extra dimensions can exists if they are compactified
- Orbifolding provides an efficient way to break GUT symmetries
- A high quality axion can be obtained from a higher form field.

K-K Expansion and Symmetry Breaking



$$S \ni \int d^4x dy \tfrac{1}{2}(\partial_y \phi)^2 \to \int d^4x \textstyle\sum_n \tfrac{1}{2} M_n^2 \phi_n^2$$

$$M_n \sim n/R$$

$$\mathcal{P} : \phi(-y) = P\phi \qquad \mathcal{P}' : \phi(\pi R - y) = P'\phi$$

Parities forbid the appearance of zero mass modes

(+,+): yes, (-,-): no
(+,-):no (-,-): no

$$\begin{bmatrix} (+,+) & (+,-) \\ (+,-) & (+,+) \end{bmatrix}$$

$$\mathcal{P} : A_a T^a \to P A_a T^a P$$
$$\mathcal{P}' : A_a T^a \to P' A_a T^a P'$$
$$P' = \mathrm{diag}(-1,-1,-1,1,1)$$
$$P = \mathrm{diag}(1,1,1,1,1)$$
$$SU(5) \to SU(3) \times SU(2) \times U(1)$$

Kawamura Y 2001 , Progress of Theoretical Physics, 105(6), pp. 999-1006

A High Quality Axion

Axion as a Wilsonian operator

$$-\tfrac{1}{4g_P^2} C_{\mu\nu} C^{\mu\nu} + \tfrac{\kappa}{32\pi^2} \epsilon^{MNPQR} C_R \mathrm{Tr}(F_{MN} F_{PQ}), \quad \kappa \in Z$$

$$\tfrac{a}{f_a} = \tfrac{1}{2} \oint_{S_1} C \qquad f_a^{-1} = \kappa g_P L^{1/2} = \kappa g_{P,4D} L$$

1-form $U(1)$ symmetry: $C \to C + \Lambda, \ d\Lambda = 0$

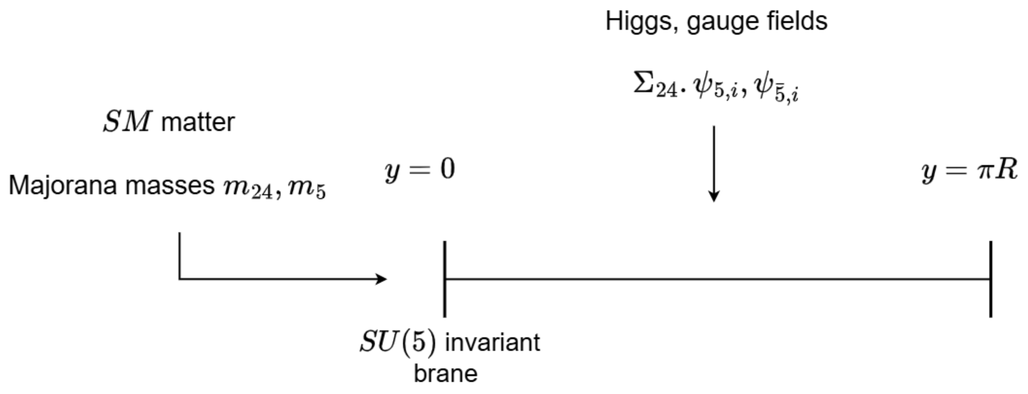
PQ symmetry: $\oint \Lambda = 2 f_a \theta, \ a \to a + \theta$

Non-locality ⟶ **Gravity effect expoentially suppressed**

see e.g. arxiv: 2408.10295

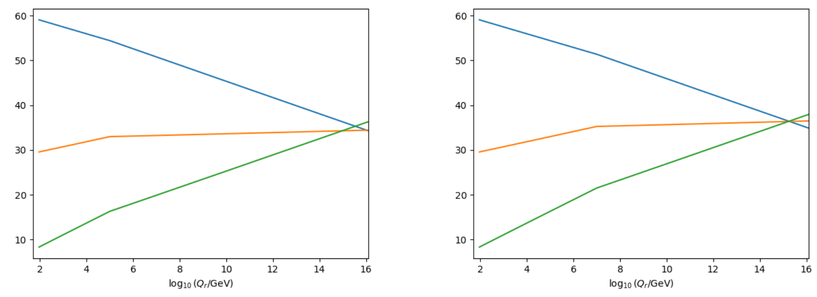
We can have C to be Z_2 odd, so that C_μ have parities $(-,-)$ and C_5 has $(+,+)$.

Gauge Unification with New Matter



Massless modes : $\Sigma_{24} \to \Sigma_8 + \Sigma_3 \qquad \psi_{5,i}, \psi_{\bar{5},i} \to \psi_{2,i}, \psi_{\bar{2},i}, \ i = 1,2$

The expected GUT scale is $\mathcal{O}(10^{15})$GeV



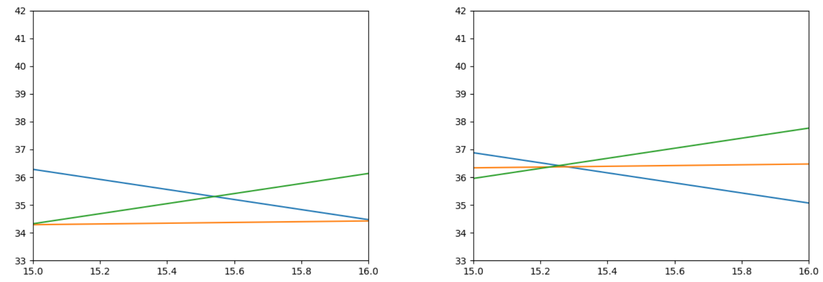


Figure 1: gauge running when $m_5 = m_{24}$, the left panel takes $m_{24} = 10^5\,\mathrm{GeV}$, and the right panel takes $m_{24} = 10^7\,\mathrm{GeV}$

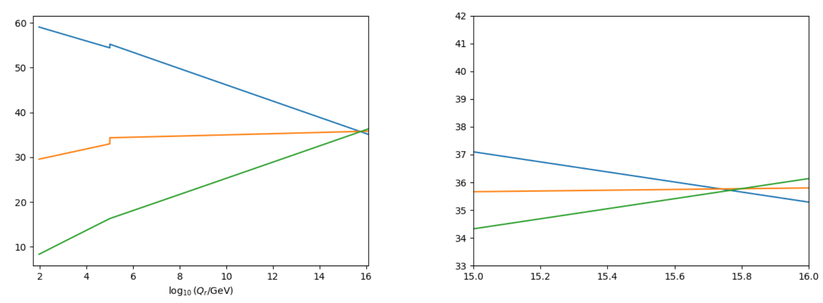


Figure 2: gauge running when $m_5 = 500 m_{24}$ and $m_{24} = 10^5\,\mathrm{GeV}$

Proton Decay Constraints

The color octet Σ_8 can decay to three-light quarks with a long lifetime:

$$\tau \sim 64\pi^3 M_*^4 m_8^{-5} \simeq 1.3 \times 10^{17}\, s \left(\frac{M_*}{10^{17}\,\mathrm{GeV}}\right)^4 \left(\frac{m_8}{10^6\,\mathrm{GeV}}\right)^{-5}$$

such long-lived colored particles would affect the predictions of Big Bang Nucleosynthesis and CMB spectrum. To avoid these constraints, we require $T_R < m_{\rm adj}$. The proton decay($p \to e^+\pi^0$) from the dimension 6 effective operator:

$$\mathcal{L}_{\rm eff} = \frac{g_{5,4D}^2}{M_X^2} \epsilon_{\alpha\beta\gamma} (A^{(1)} \overline{u^c}^\alpha P_R d^\beta \overline{e^c} P_L u^\gamma + A^{(2)} \overline{u^c}^\alpha P_L d^\beta \overline{e^c} P_R u^\gamma) + h.c.,$$

Given the proton decay constraint from Super-Kamiokande: $\Gamma > 2.4 \times 10^{34}$ years, the mass of the GUT gauge bosons should satisfy:

$$M_X \gtrsim 4.5 \times 10^{15}\,\mathrm{GeV}.$$

To achieve unification at this scale, the mass of the color octet should be smaller than 10^5-10^6GeV, so that the reheating temperature is bounded

$$T_R \leq 10^5\text{-}10^6\,\mathrm{GeV}$$

Resonant or non-thermal leptogenesis need to be considered to account for baryon asymmetry.

Axion Cosmology and Detection

With $f \sim 1/R$, the axion mass is estimated as:

$$m_a \approx 10^{-9}\mathrm{eV} \left(\frac{5 \times 10^{15}\,\mathrm{GeV}}{f_a}\right)$$

The energy density is approxiamted as:

$$\Omega_{\rm axion} h^2 \approx 0.2 \times 10^4 \theta_0^2 \left(\frac{f_a}{5 \times 10^{15}\,\mathrm{GeV}}\right)^{7/6}.$$

Assuming the axion explains all the dark matter, the initial value $\theta_0 = a(x)/f_a$ needs to be $\mathcal{O}(10^{-2})$ GeV. Inhomogenous contributions to the energy density of the axion may arise during inflation and affect the CMB spectrum. The observed CMB specturm is consistent without inhomogenous contributions, so the Hubble scale during inflation is constrainted as

$$H_I \lesssim 1.4 \times 10^9\,\mathrm{GeV} \left(\frac{\theta_0}{10^{-2}}\right) \left(\frac{f_a}{5 \times 10^{15}\,\mathrm{GeV}}\right).$$

The axion photon couling is estimated as

$$g_{a\gamma\gamma} \approx \frac{\alpha}{2\pi f_a}(8/3 - 1.92) \approx 1.7 \times 10^{-19}\,\mathrm{GeV}^{-1} \left(\frac{5 \times 10^{15}\,\mathrm{GeV}}{f_a}\right),$$

Provided that the QCD axion in our model accounts for the entire relic abundance of dark matter, this axion can be detected by future experiments.