Mass spectrum and symmetry breaking of gauge fields in Yang-Mills theory with extra dimensions of S^2

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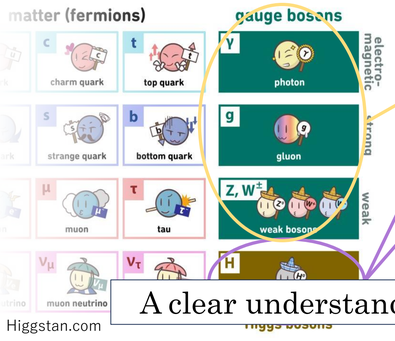
Introduction

1. Problems of the Standard Model

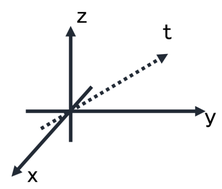


- Unification of the gauge group
- The origin of the Higgs boson
- Yukawa coupling constants

A clear understanding of **gauge symmetry breaking** is essential.

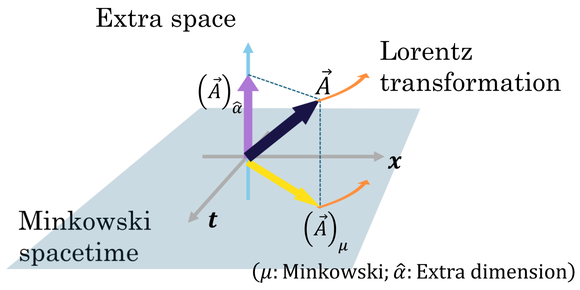
2. Higher-dimensional Yang-Mills theory with S^2

Minkowski spacetime × Extra-dimensional space (two-dimensional sphere S^2)



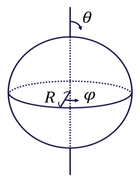
Gauge symmetry breaking by S^2 curvature.

3. Key features of the higher-dimensional theory (general)



- Extra-dimensional components appear as **Lorentz scalars** in Minkowski spacetime (left figure).
- Extra-dimensional momenta appear as **a mass** in Minkowski spacetime.

4. Key features of the extra-dimension S^2



$$g_{\hat{\alpha}\hat{\beta}} = \begin{pmatrix} -R^2 & 0 \\ 0 & -R^2 \sin^2\theta \end{pmatrix}$$

Euler-Lagrange equation of YM sector

$$\nabla_{\hat{\beta}} F^{\hat{\beta}\hat{\alpha}} - ig\left[A_{\hat{\beta}}, F^{\hat{\beta}\hat{\alpha}}\right] = 0$$

$A_\varphi \propto \cos\theta$ is a **non-trivial classical solution**.
We introduce a background field $\langle A_\varphi \rangle \propto \cos\theta$.

Gauge symmetry breaking with S^2

5. Set up

- Metric: $G_{MN} = \mathrm{diag}\left(1, -1, -1, -1, -R^2, -R^2\sin^2\theta\right)$
  → Coordinates: $X^M = (x^\mu, y^{\hat{\alpha}}) = (x^\mu, \theta, \varphi)$
- Gauge fields: $A_M(X) = \left(A_\mu(X),\ A_\theta(X),\ A_\varphi(X) + \mu\cos\theta\right)$

  Cartan-Weyl Basis:
  $$A_M = \sum_{i:\text{ all Cartans}} A_M^i H_i + \sum_{\alpha:\text{ all roots}} A_M^\alpha E_\alpha$$

  $\mu \equiv \mu^i H_i$ (constant) → The Background chosen along Cartan.
- Lagrangian:

$$S = \int d^4x\, d\theta\, d\varphi\, R^2 \sin\theta \left\{-\frac{1}{2}\mathrm{tr}[F_{MN}F^{MN}]\right\} \quad \left(F_{MN} = (\partial_M A_N - \partial_N A_M) - ig[A_M, A_N]\right)$$
$$= \int d^4x\, d\theta\, d\varphi\, R^2 \sin\theta \left\{-\frac{1}{2}\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] - \mathrm{tr}[F_{\mu\hat{\alpha}}F^{\mu\hat{\alpha}}] - \mathrm{tr}[F_{\theta\varphi}F^{\theta\varphi}]\right\}$$

6. Quadratic terms of four-dimensional gauge fields A_μ

$$\mathcal{L}^{\text{quadratic}}_{\text{gauge}} = \frac{1}{2}R^2\sin\theta\left[A^i_\mu\left(\Box + \frac{1}{R^2}\boldsymbol{L}^2\right)A^{\mu i} + A^{-\alpha}_\mu\left\{\Box + \frac{1}{R^2}\tilde{\boldsymbol{J}}^{(\alpha)2}\right\}A^{\mu\alpha}\right],$$

Kaluza-Klein expansion

$$\begin{cases} A^i_\mu(x,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{\sqrt{2}}{R}A^i_{\mu,lm}(x)\,Y_{lm}(\theta,\varphi), \\ A^\alpha_\mu(x,\theta,\varphi) = \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{R}A^\alpha_{\mu,jm}(x)\mathcal{Y}_{k_\alpha jm}(z,\varphi), \end{cases} \quad k_\alpha \equiv g\,(\mu\cdot\alpha)$$

$$S^{\text{quadratic}}_{\text{gauge}} = \int d^4x\left[\sum_{l=0}^{\infty}\sum_{m=-l}^{l}A^i_{\mu,lm}\left\{\Box + \frac{l(l+1)}{R^2}\right\}A^{\mu,i}_{lm} + \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}A^{-\alpha}_{\mu,jm}\left\{\Box + \frac{j(j+1)-k_\alpha^2}{R^2}\right\}A^{\mu,\alpha}_{jm}\right].$$

(Here, we omit the gauge-fixing term.)

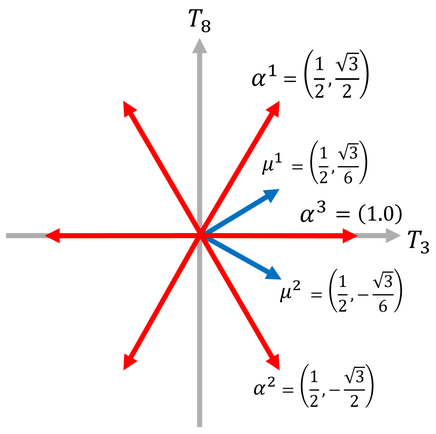
Kaluza-Klein masses

For Cartan components and root components with $k_\alpha = 0$,	For root components with $k_\alpha \neq 0$,
$\frac{l(l+1)}{R^2}$, which always have a massless mode, $l = 0$.	$\frac{j(j+1)-k_\alpha^2}{R^2}$, which never have massless modes.

A main result

When $k_\alpha = 0$, that is, **commuting with background fields,** the gauge symmetry in the α-direction is **unbroken** in four dimensions.

Application: gauge group = SU(3)



Ex. 1) when background $\mu = \mu^1 + \mu^2$,
$k_\alpha \neq 0$ (for all α) → $\mathrm{SU}(3) \to \mathrm{U}(1)\times\mathrm{U}(1)$.

Ex. 2) when background $\mu = \mu^1 - \mu^2$,
$k_{1,2} \neq 0$, $k_3 = 0$ → $\mathrm{SU}(3) \to \mathrm{SU}(2)\times\mathrm{U}(1)$.

T_3, T_8 : the Cartan generators of SU(3)
α^1, α^2 : the simple roots, $\alpha^3 \equiv \alpha^1 + \alpha^2$
μ^1, μ^2 : the fundamental weights $\left(\mu^i \cdot \alpha^j = \frac{1}{2}\delta^{ij}\right)$

7. Quadratic terms of extra-dimensional gauge fields A_θ, A_φ

Since A_θ, A_φ are interpreted as **scalar fields**, We should transform them to get a physical scalar boson and an unphysical **Nambu-Goldstone boson**.

$$A_\theta = -\frac{1}{\sin\theta}D_\varphi\phi + D_\theta\chi$$
$$A_\varphi = \sin\theta D_\theta\phi + D_\varphi\chi$$
$$\left(D_M \equiv \partial_M - ig\,[\langle A_M\rangle, \bullet\,]\right)$$

$$\mathcal{L}^{\text{quadratic}}_{\text{extra gauge}} = -\frac{1}{2}\sin\theta\Big\{\phi_i\Box\left(\boldsymbol{L}^2\phi^i\right) + \chi_i\Box\left(\boldsymbol{L}^2\chi^i\right) + \frac{1}{R^2}\left(\hat{\boldsymbol{L}}^2\phi_i\right)\left(\hat{\boldsymbol{L}}^2\phi^i\right) + \frac{\xi}{R^2}\left(\hat{\boldsymbol{L}}^2\chi_i\right)\left(\hat{\boldsymbol{L}}^2\chi^i\right)$$
$$+ \phi^{-\alpha}\Box\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\phi^\alpha\right) + \chi^{-\alpha}\Box\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\chi^\alpha\right) - 2ik_\alpha\phi^{-\alpha}\Box\chi^\alpha$$
$$+ \frac{1}{R^2}\left(\tilde{\boldsymbol{J}}^{(-\alpha)2}\phi^{-\alpha}\right)\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\phi^\alpha\right) - \frac{1}{R^2}k_\alpha^2\phi^{-\alpha}\phi^\alpha + \frac{\xi}{R^2}\left(\tilde{\boldsymbol{J}}^{(-\alpha)2}\chi^{-\alpha}\right)\left(\tilde{\boldsymbol{J}}^{(\alpha)2}\chi^\alpha\right)\Big\},$$

(ξ: gauge-fixing parameter)

Kaluza-Klein expansion

$$\begin{cases} \phi^i(\chi^i)(x,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{\sqrt{l(l+1)}}\phi^i_{lm}(\chi^i_{lm})(x)\,Y_{lm}(\theta,\varphi), \\ \phi^\alpha(\chi^\alpha)(x,\theta,\varphi) = \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\frac{\sqrt{2}}{\sqrt{j(j+1)-k_\alpha^2}}\phi^\alpha_{jm}(\chi^\alpha_{jm})(x)\mathcal{Y}_{k_\alpha jm}(z,\varphi), \end{cases}$$

$$S^{\text{quadratic}}_{\text{gauge}} = \int d^4x\left(\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\left[-\phi^i_{lm}\left\{\Box + \frac{l(l+1)}{R^2}\right\}\phi^i_{lm} - \chi^i_{lm}\left\{\Box + \xi\frac{l(l+1)}{R^2}\right\}\chi^i_{lm}\right]\right.$$
$$+ \sum_{j=|k_\alpha|}^{\infty}\sum_{m=-j}^{j}\left[-\phi^{-\alpha}_{1,jm}\left\{\Box + \frac{\left(j(j+1)-k_\alpha^2\right)^2 - k_\alpha^2}{R^2\left(j(j+1)-k_\alpha^2\right)}\right\}\phi^\alpha_{1,jm}\right.$$
$$\left.\left. - \chi^{-\alpha}_{jm}\left\{\Box + \xi\frac{j(j+1)-k_\alpha^2}{R^2}\right\}\chi^\alpha_{jm} + 2ik_\alpha\frac{1}{j(j+1)-k_\alpha^2}\phi^{-\alpha}_{jm}\Box\chi^\alpha_{jm}\right]\right).$$

(We treat these cross terms perturbatively)

ϕ as a physical scalar boson, χ as a Nambu-Goldstone boson.

★8. Details of SO(3) operators★

$$\boldsymbol{L}^2 \equiv -\frac{1}{\sin\theta}\partial_\theta\sin\theta\partial_\theta - \frac{1}{\sin^2\theta}\partial_\varphi^2$$
(the angular momentum operator)

$$\tilde{\boldsymbol{J}}^{(\alpha)2} \equiv \boldsymbol{L}^2 + 2\frac{\cos\theta}{\sin^2\theta}k_\alpha i\partial_\varphi + \frac{\cos^2\theta}{\sin^2\theta}k_\alpha^2$$

$\partial_\varphi \to \partial_\varphi - ik_\alpha\cos\theta$
Consider covariant derivative !

k_α is defined by
$g\,[\langle A_\varphi\rangle, E_\alpha] = g\,(\mu\cdot\alpha)\,E_\alpha \equiv k_\alpha\,E_\alpha$.

Eigenfunctions / Eigenvalues

$$\boldsymbol{L}^2 Y_{lm}(\theta,\varphi) = l(l+1)\,Y_{lm}(\theta,\varphi)$$
$$\begin{cases} l = 0, 1, 2, \cdots \\ m = -l, -l+1, \cdots, l \end{cases}$$

$$\left(\tilde{\boldsymbol{J}}^{(\alpha)2} + k_\alpha^2\right)\mathcal{Y}_{k_\alpha jm}(\theta,\varphi) = j(j+1)\,\mathcal{Y}_{k_\alpha jm}(\theta,\varphi)$$
$$\begin{cases} |k_\alpha| = 0, 1, 2, \cdots \\ j = |k_\alpha|, |k_\alpha|+1, |k_\alpha|+2, \cdots \\ m = -j, -j+1, \cdots, j \end{cases}$$

k_α is restricted by the condition requiring that the surface terms vanish.

Future work

- We obtained massless modes of ϕ. Then, we should consider effective mass term by an argument similar to Coleman-Weinberg potential.
- The couplings of gauge fields with scalars and fermions remain to be examined.