Physics Frontiers with Quantum Science and Technology
@University of Tokyo, 2022 Mar. 9-10

Extreme Universe of spacetime and matter from quantum information

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YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

It from Qubit
Simons Collaboration

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

① Introduction

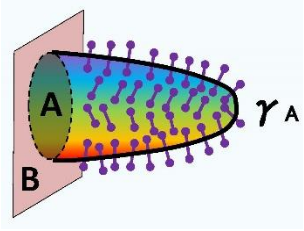
In science, "microscopes" are basic important devices.

Biology, Chemistry
Materials Physics → (Optical, electronic, ..) Microscopes → Cells, DNAs, Atoms,･･･

High Energy Physics → Accelerators → Elementary particles

Quantum Gravity
(String Theory) → Holography (AdS/CFT) → Qubits !
(Quantum Entanglement)

A thought experiment



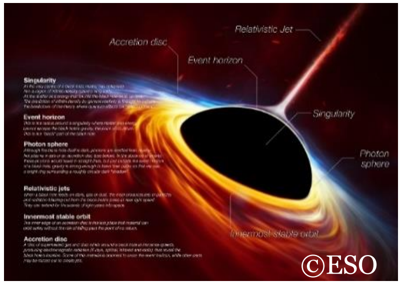
***Extreme Universe* emerges !**

What does holography magnify ?

Gravitational Spacetimes !

In particular, one of the most interesting and simplest targets for our thought experiments is

Black holes !



We can also magnify more general spacetimes

e.g. (Anti-)de Sitter spaces, big-bang Universe, etc.

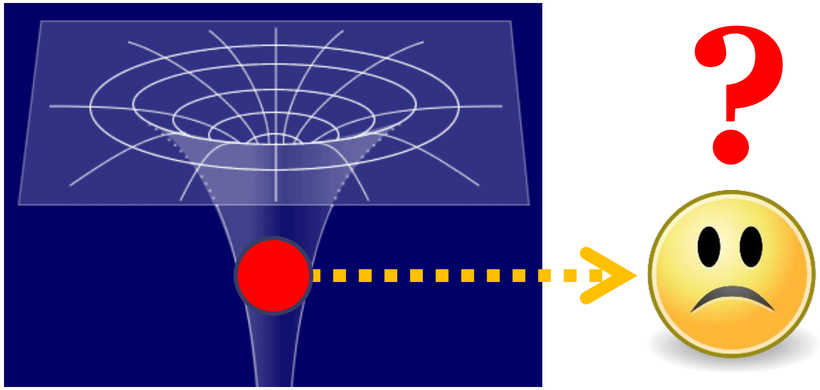
What is BH Entropy ?

After stars collapsed into a BH, outside observers cannot access the information inside the BH.

Stars

A lot of Information

BH



Hidden Information !

BH entropy !

Bekentein−Hawking Formula of BH Entropy [1972−1976]

Calculations in general relativity show that a BH has the following entropy:

➔Still mysterious !

$$\boxed{S_{BH} = \frac{k_B c^3}{\hbar} \times \frac{A_{BH}}{4G_N}}$$

➡ BH thermodynamics !

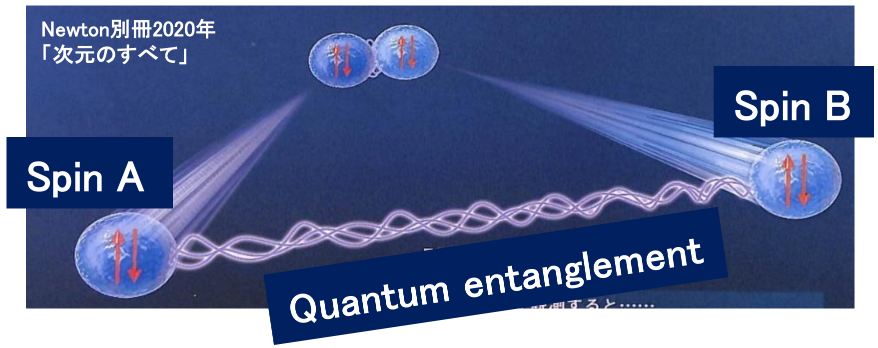
A_{BH}= Surface Area of Black hole ⇒ **Geometry**
G_N=Newton constant ⇒ **Gravity**
$\hbar$=Planck constant ⇒ **Quantum Mechanics**
k_B=Boltzmann const.⇒ **Stat. Mech. and Quant Info.**

} **Quantum Gravity!**

BH Entropy is proportional to the **area**, not to the volume !

What is Quantum Entanglement ?

Quantum Entanglement (QE) = Quantum correlation between two subsystems A and B



For pure states: Non-zero QE ⇔ $\left|\Psi\right\rangle_{AB} \neq \left|\Psi_1\right\rangle_A \otimes \left|\Psi_2\right\rangle_B$.

Not a Direct Product

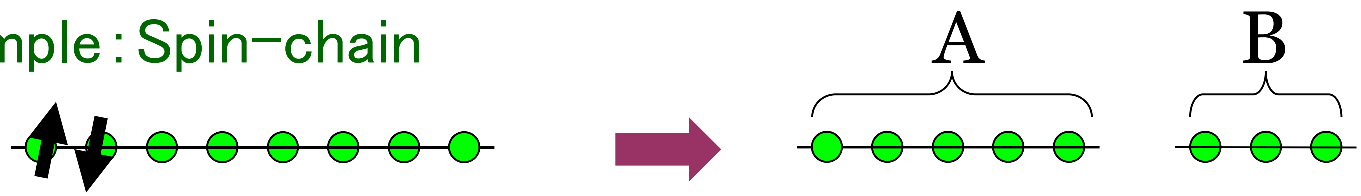
e.g. Bell state : $\left|\Psi_{Bell}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\uparrow\right\rangle_A \otimes \left|\downarrow\right\rangle_B + \left|\downarrow\right\rangle_A \otimes \left|\uparrow\right\rangle_B\right]$ ➡ Minimal Unit of Entanglement

Entanglement Entropy (EE)

An amount of QE is measured by **Entanglement Entropy (EE)**.

First we decompose the Hilbert space: $H_{tot} = H_A \otimes H_B$.

Example : Spin-chain



We introduce the reduced density matrix ρ_A

by tracing out B $\rho_A = \mathrm{Tr}_B\left[\left|\Psi_{tot}\right\rangle\left\langle\Psi_{tot}\right|\right]$.

The entanglement entropy (EE) S_A is defined by

$$\boxed{S_A = -\mathrm{Tr}[\rho_A \log \rho_A]} \propto$$

# of EPR Pairs
between A and B

Measurement of EE in Cond-mat Experiments

Example1: Ultracold bosonic atoms in optical lattices

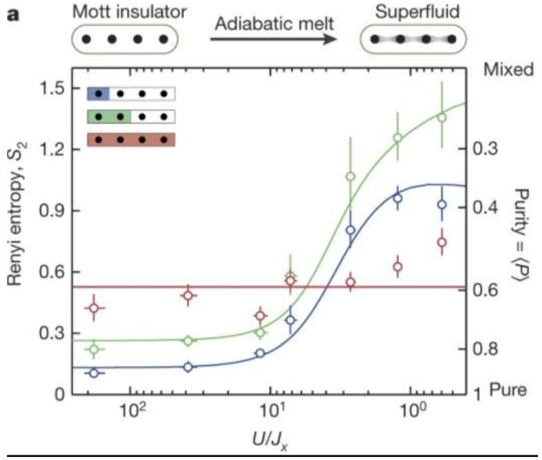
Published: 02 December 2015

Measuring entanglement entropy in a quantum many-body system

Rajibul Islam, Ruichao Ma, Philipp M. Preiss, M. Eric Tai, Alexander Lukin, Matthew Rispoli & Markus Greiner

Nature **528**, 77–83 (2015) | Cite this article

$$H = -J\sum_{\langle i,j\rangle} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i(n_i - 1) \qquad (4)$$



Example2: Trapped-ion quantum simulator

Science

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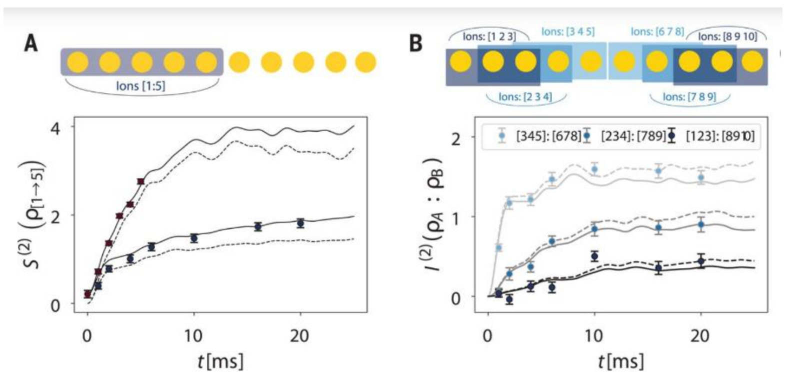
REPORT

Probing Rényi entanglement entropy via randomized measurements

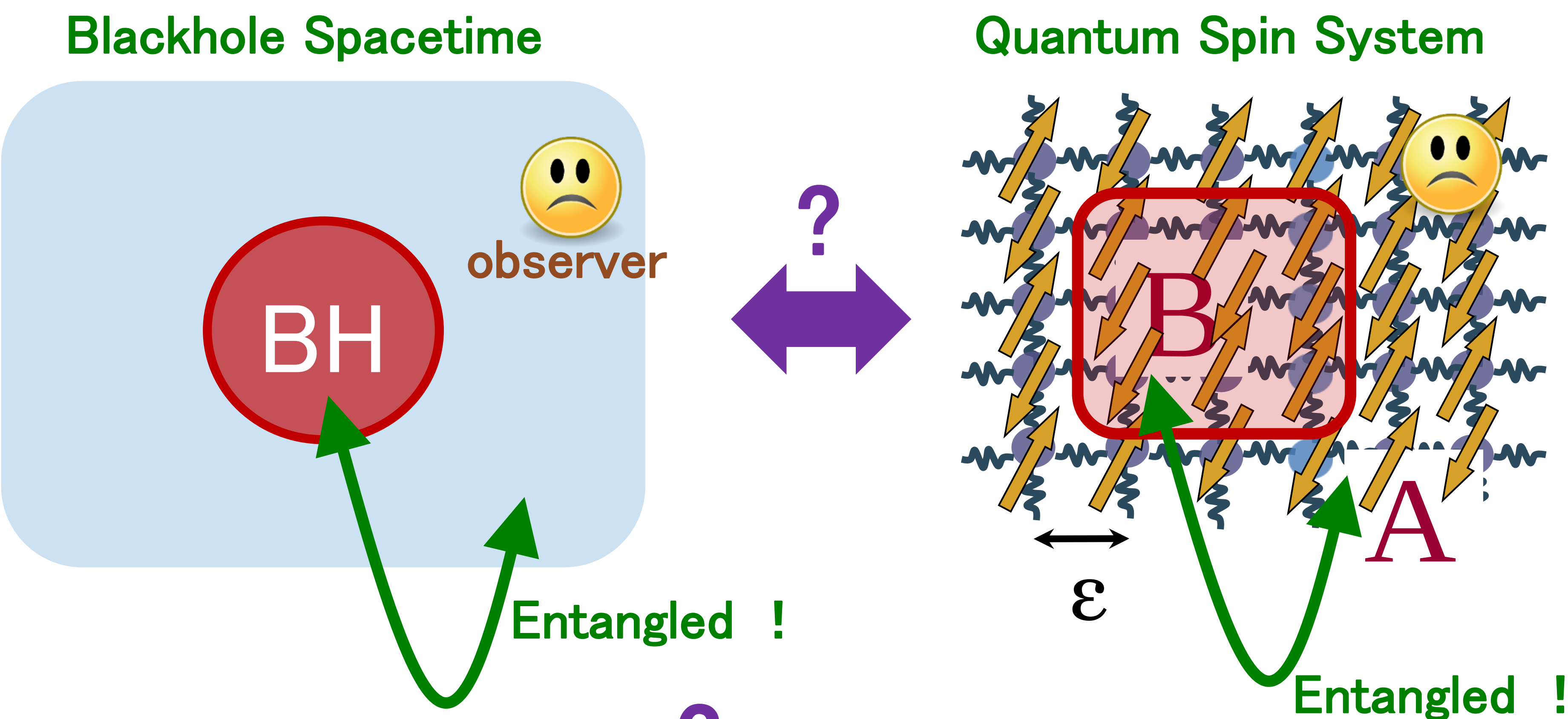
TIFF BRYDGES, ANDREAS ELBEN, PETAR JURCEVIC, BENOÎT VERMERSCH, CHRISTINE MAIER, BEN P. LANYON, PETER ZOLLER, RAINER BLATT, AND CHRISTIAN F. ROOS Authors Info & Affiliations

SCIENCE • 19 Apr 2019 • Vol 364, Issue 6437 • pp. 260-263 • DOI: 10.1126/science.aau4963

$$H_{\mathrm{XY}} = \hbar\sum_{i<j} J_{ij}\left(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+\right) + \hbar B\sum_j \sigma_j^z$$



Analogy between BH and Qubits



?

BH entropy S_{BH} ⟷ **Entanglement entropy S_A**

Spacetime — Matter

② Holography and Entanglement

BH Entropy Formula

$$\boxed{S_{BH} = \frac{A_{BH}}{4G_N}}$$

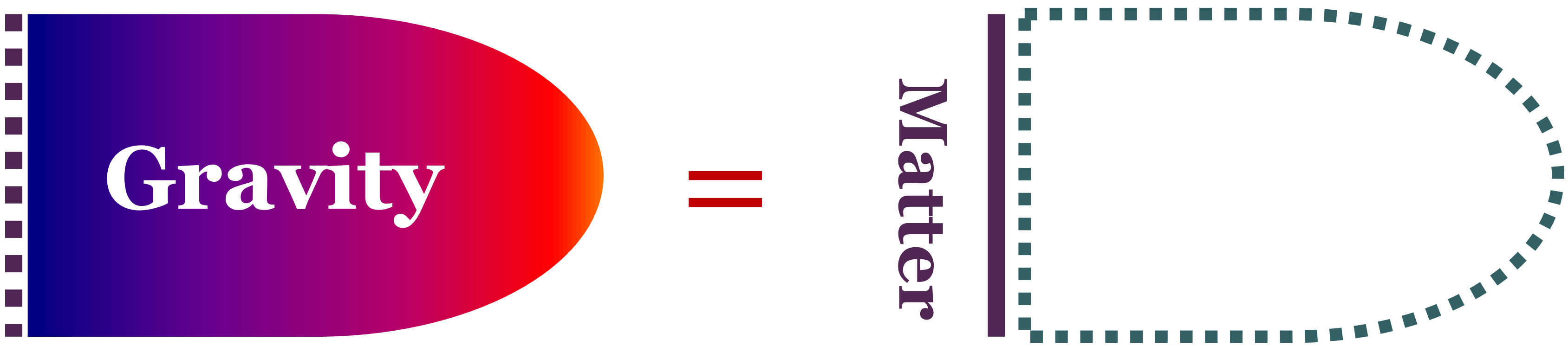
⇒ **Degrees of freedom in Gravity ∝ Area**

['t Hooft 1993, Susskind 1994]

Holography

Boundary of M

Gravity on M = Quantum Matter on ∂M



BH entropy(∝Area)= Thermal Entropy of Matter (∝Volume)

AdS/CFT Correspondence

[Maldacena 97]

AdS/CFT

Massless (gapless) theory

Gravity (String theory) on D+1 dim. AdS (anti de-Sitter space) **=** Conformal Field Theory (CFT) on D dim. Minkowski spacetime

Classical limit

Large N + Strong coupling

General relativity with $\Lambda < 0$

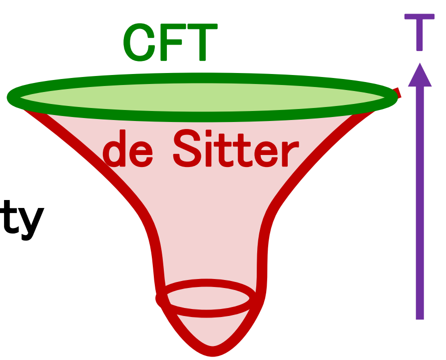
$$Z_{Gravity} = Z_{CFT}$$

Strongly interacting Quantum Field Theories

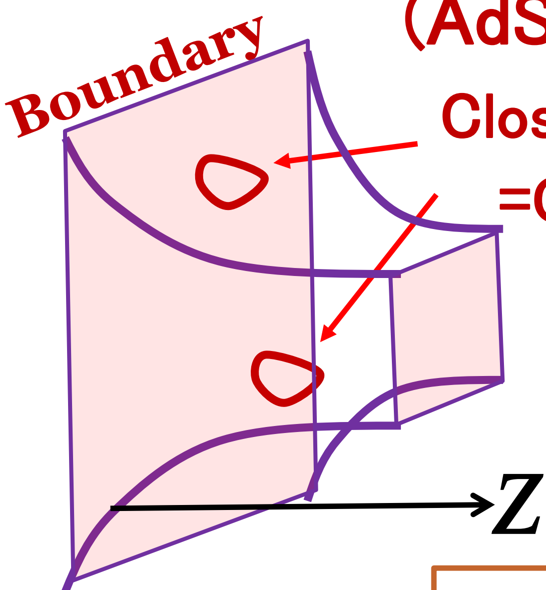
How about de Sitter space ($\Lambda > 0$) ? ➡ dS/CFT [Strominger 2001]

[dS/CFT] dS Gravity in D+1 dim. = D dim. (non-unitary) CFT

➡ We recently found the first example dual to Einstein gravity (for 3 dim. dS). [Hikida-Nishioka-Taki-TT, 2021]



Gravity in Anti de-Sitter space (AdS)

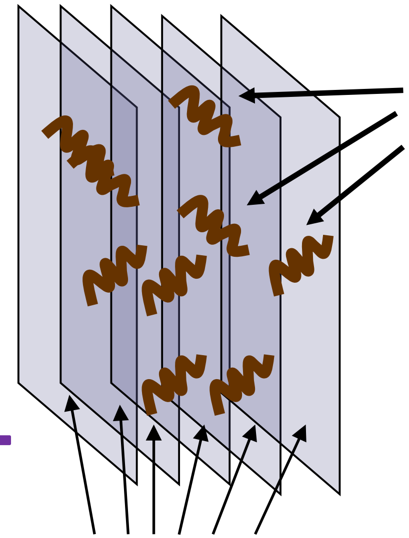


Closed Strings
=Gravitons

Equivalent

AdS/CFT

Conformal Field Theories (CFTs)



Open Strings
=Gluons in U(N)
gauge theories

N D-branes

AdS metric

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Thermodynamics of
Black holes (branes)

Equivalent

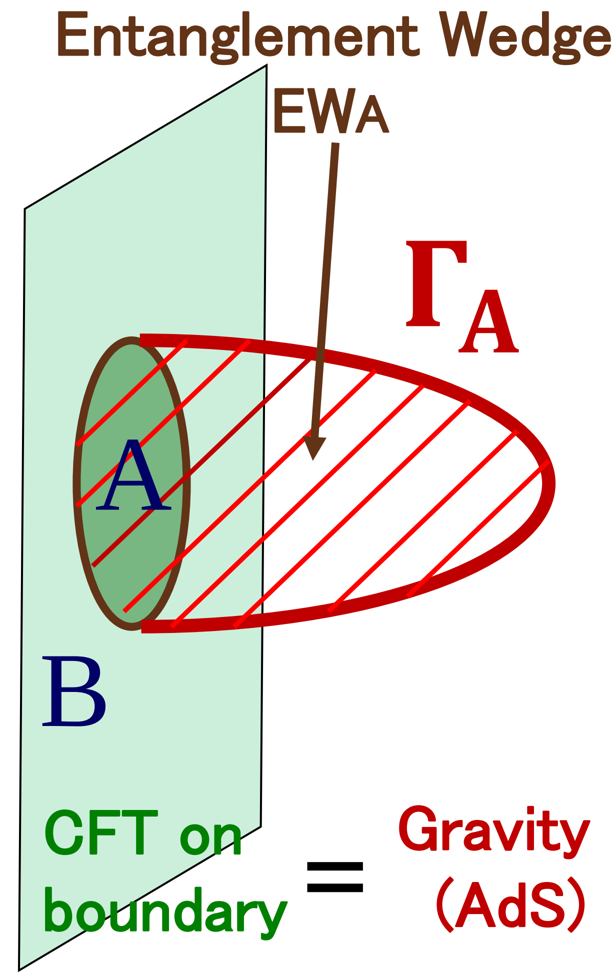
Thermodynamics
of various materials

Holographic Entanglement Entropy (HEE) [Ryu–TT 06, Hubeny–Rangamani–TT 07]

S_A can be computed from the minimal area surface Γ_A:

$$\boxed{S_A = \min_{\Gamma_A}\left[\frac{\mathrm{Area}(\Gamma_A)}{4G_N}\right]}$$

Note: $\partial\Gamma_A = \partial A$ and Γ_A is homolgous to A.

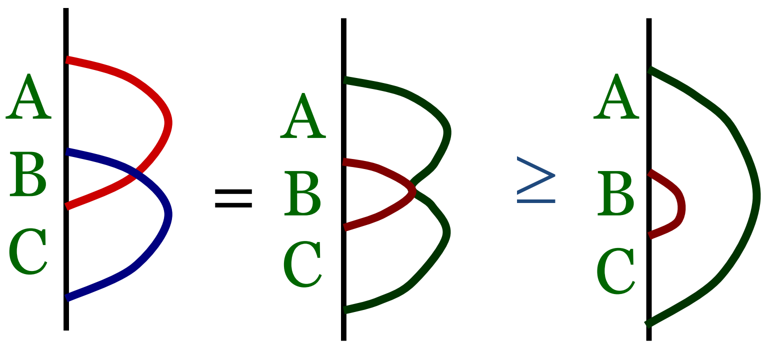


Bulk Reconstruction

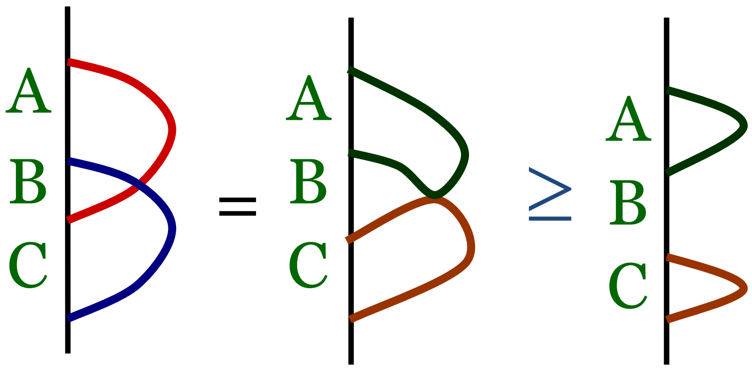
The information ρ_A in the region A is encoded in the entanglement wedge EW_A.

Algebraic properties in Quantum Information
⇔ **Geometric properties** in Gravity

<u>Holographic Proof of Strong Subadditivity</u> [Headrick–TT 07]



$$\Rightarrow \; S_{AB} + S_{BC} \geq S_{ABC} + S_{B}$$



$$\Rightarrow \; S_{AB} + S_{BC} \geq S_{A} + S_{C}$$

$$(\text{Note}: \quad AB \equiv A \cup B)$$

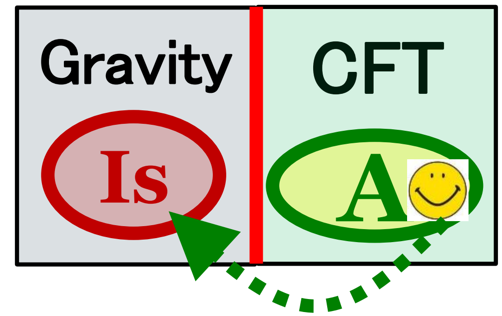
Black hole information problem and Island Formula

Recently, HEE was extended to CFTs which is coupled to gravity !

Island Formula:
[Penington 2019, Almheiri et.al. 2019]

$$\boxed{S_A = \mathrm{Min}\left[\frac{\mathrm{Area}(Is)}{4G_N} + S_{A\cup Is}\right]}$$

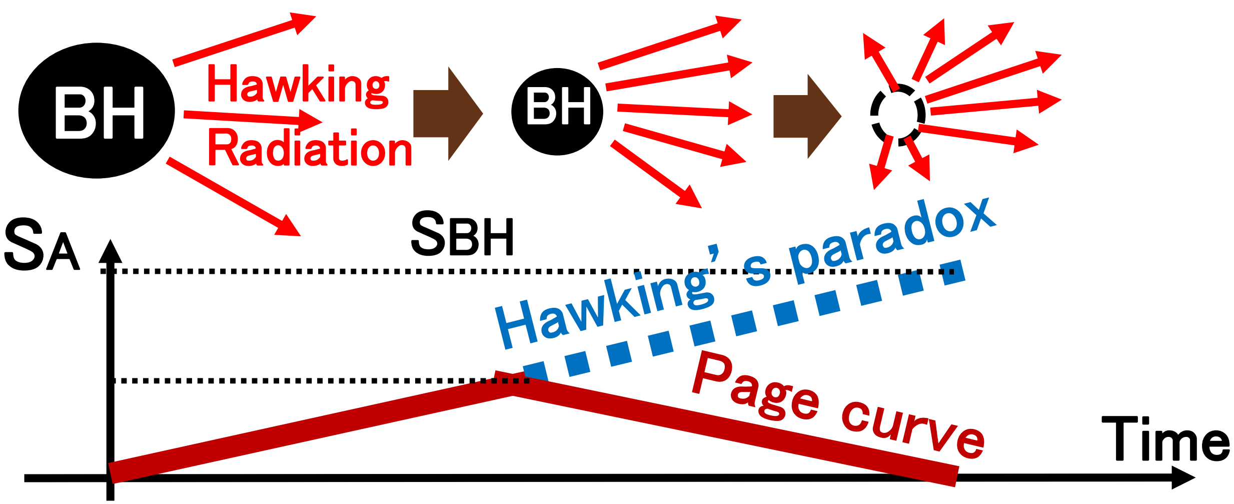
EE for Radiations　BH entropy　Bulk EE

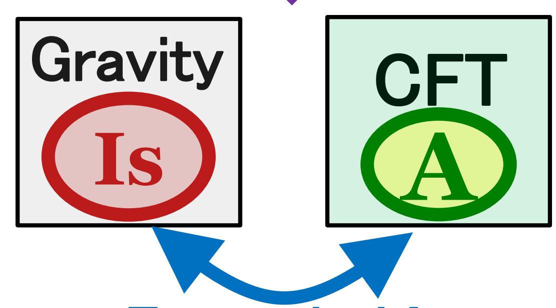


Accesible to BH interior

This explains the Page curve
→ Unitarity of BH evaporation !

Generalization



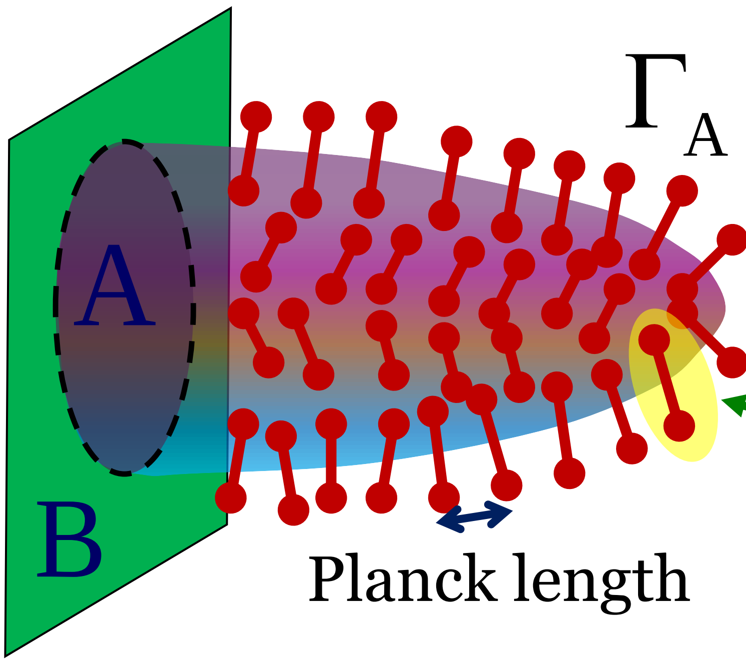


Entangled !

[Balasubramanian–Kar–Ugajin 2020–2021]

③ Extreme Universe from Quantum Entanglement

The HEE suggests that there is one qubit of entanglement for each Planck length area !



$$S_A = \frac{\mathrm{Area}(\Gamma_A)}{4 l_{pl}^{D-1}}$$

~10^{65} qubits per 1cm² !

Bell pair

= Planck scale mini Universe

Spacetime may emerge from entangled Qubits !

➔ Tensor Network (TN) realizes this idea !

Tensor network (TN) = Graphical description of quantum states

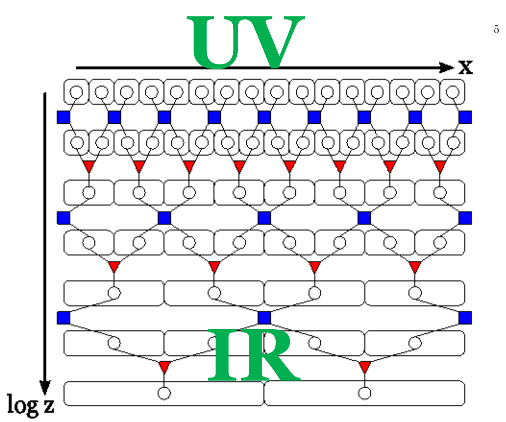
⇒ Network of quantum entanglement

[DMRG: White 92, CTM: Nishino-Okunishi 96, PEPS: Verstraete-Cirac 04, ….]

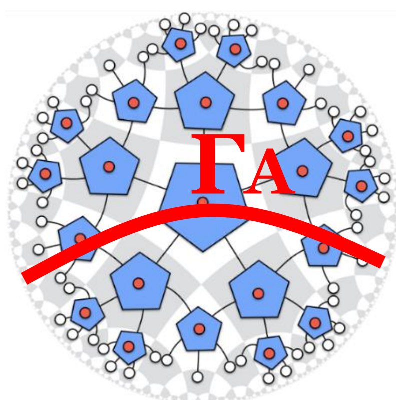
Conjecture: AdS geometry = TN

[Swingle 09,…
Pastawski-Yoshida-Harlow-Preskill 15,
….Caputa-TT et.al. 17-21,…]

MERA TN [Vidal 2005]

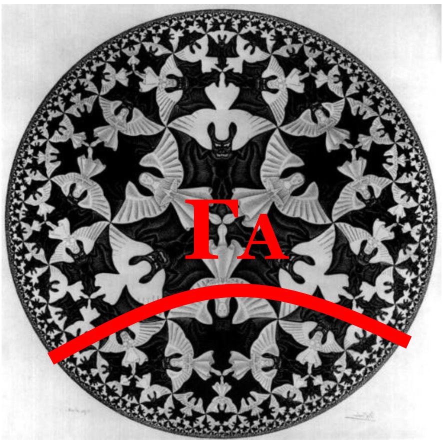


HaPPY TN



≈

AdS



Solve !

Emerge !

Quantum Matter
(e.g. spin systems)

Tensor Network
(≈ Quantum Circuit)

Universe
in Gravity

④ What is “Extreme Universe” ?

Conventional description in physics ➔ Space, Time and Matter as basic building blocks.

However, in the ***extreme situations***, due to strong quantum fluctuations, the degrees of freedom of space, time, and matter fluctuate enormously,

We cannot apply standard geometry due to quantum effects !

Quantum matter has too much degrees of freedom ！

We call such a difficult class of targets “Extreme Universe”.

Extreme Universe ＝ The three fundamental limits in nature:

①Limit of Space ②Limit of Time ③Limit of Matter

We think these are ultimate problems in physics.

Three problems of Extreme Universe

① Limit of Space

[Quantum Black hole (B)]

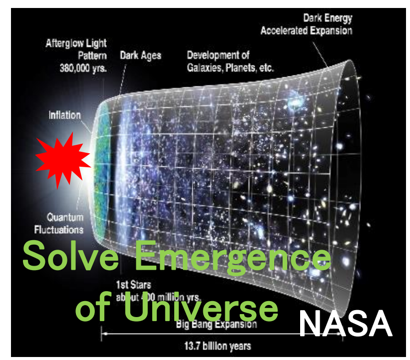
If a black hole evaporates, does the information inside disappear?

Quantum Gravity

② Limit of Time

[Quantum Cosmology (C)]

What is the ultimate law that explains the creation of the universe?

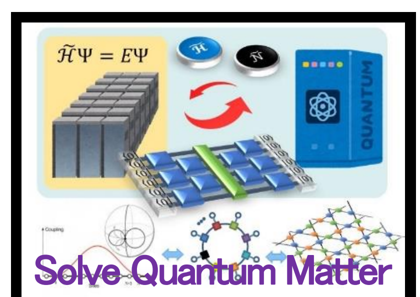


Holography

③ Limit of Matter

[Dynamics of Quantum Matter (D)]

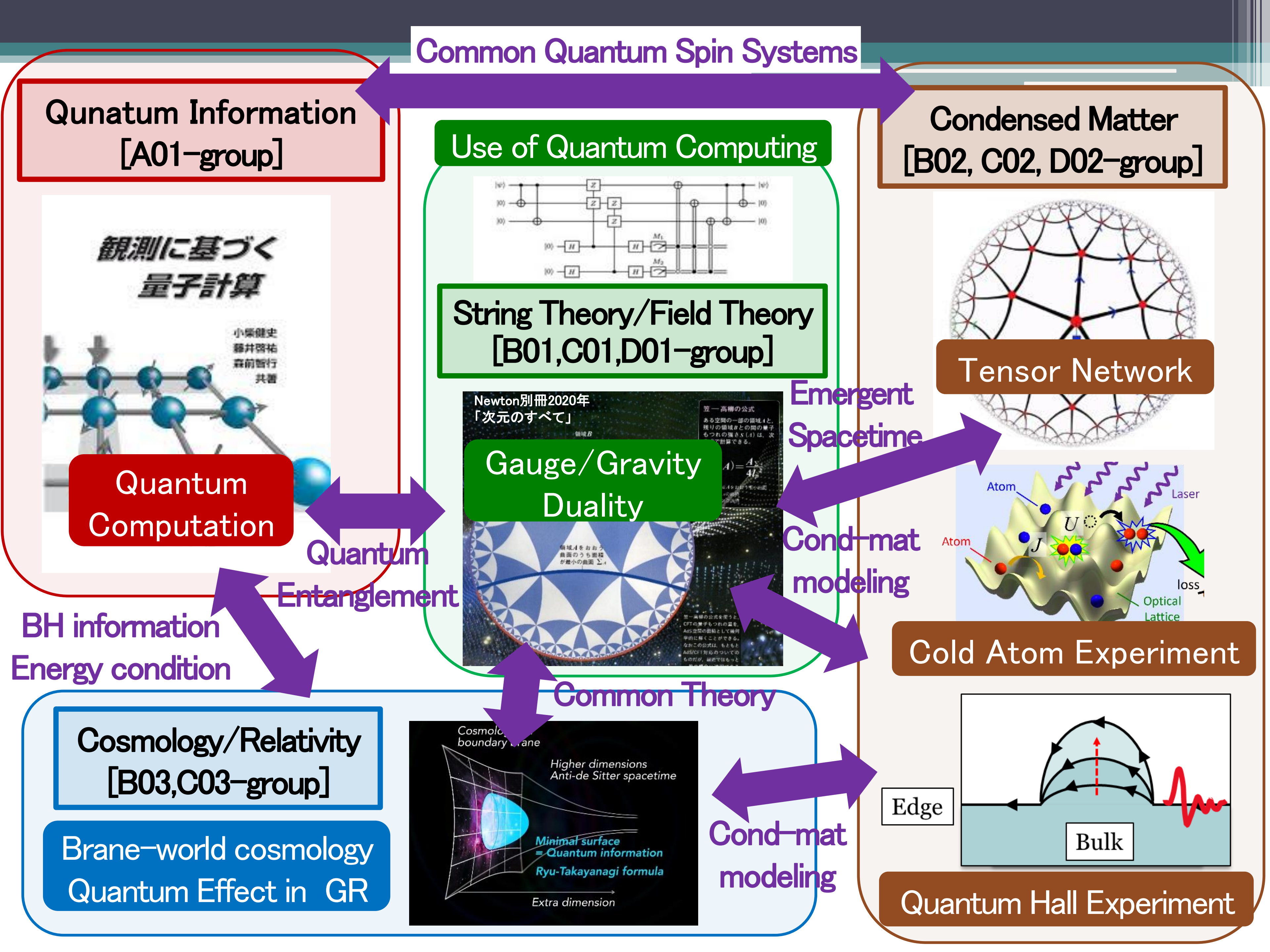
How can the dynamics of quantum matter be solved efficiently?



Quantum Chaos

Solve these problems in the light of quantum information

- Quantum Communication
- Quantum Error Correction
- Quantum Entanglement
- Computational Complexity
- Quantum Computing

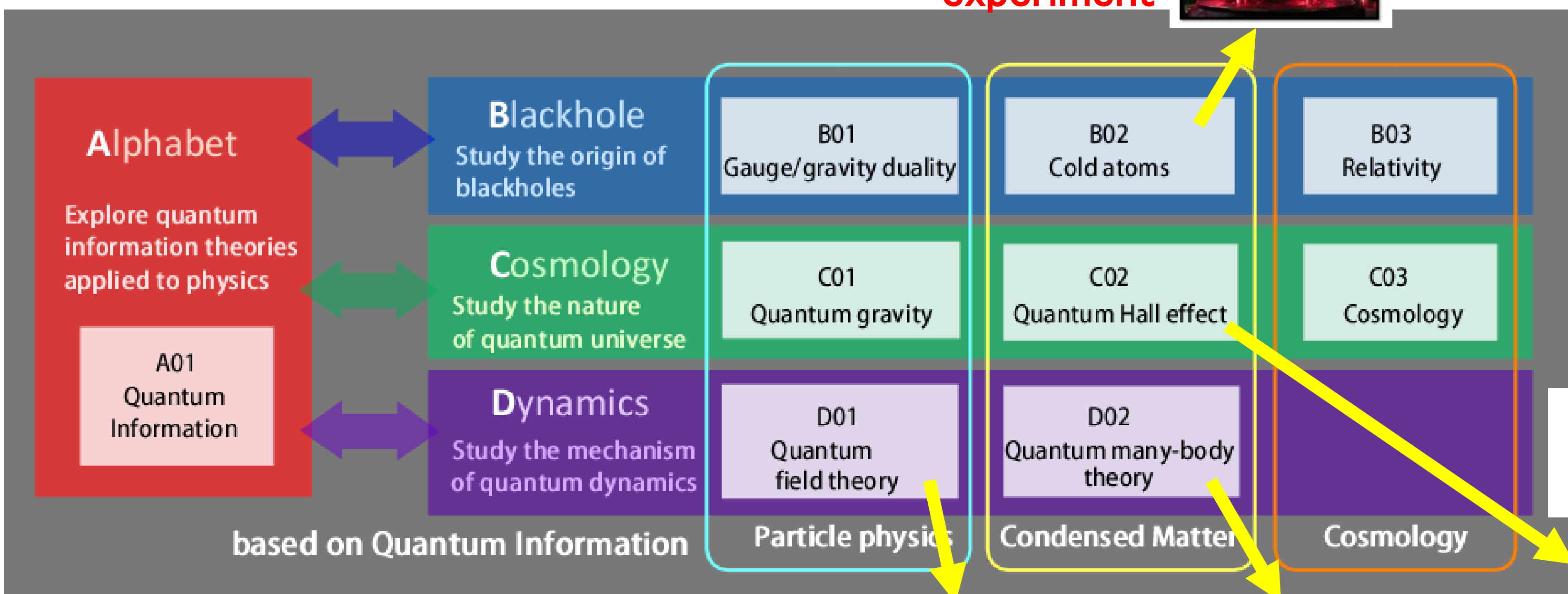

Common Quantum Spin Systems
Qunatum Information
[A01-group]
観測に基づく
量子計算
小柴健史
藤井啓祐
森前智行
共著
Quantum
Computation
Use of Quantum Computing
String Theory/Field Theory
[B01,C01,D01-group]
Newton別冊2020年
「次元のすべて」
Gauge/Gravity
Duality
Condensed Matter
[B02, C02, D02-group]
Tensor Network
Emergent
Spacetime
Atom
Laser
Atom
loss
Optical
Lattice
Cond-mat
modeling
Cold Atom Experiment
Quantum
Entanglement
BH information
Energy condition
Common Theory
Cosmology/Relativity
[B03,C03-group]
Brane-world cosmology
Quantum Effect in GR
Higher dimensions
Anti-de Sitter spacetime
Minimal surface
= Quantum information
Ryu-Takayanagi formula
Extra dimension
Cond-mat
modeling
Edge
Bulk
Quantum Hall Experiment

Extreme Universe Program

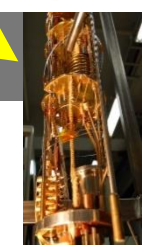
FY2021-2025 MEXT-KAKENHI-Grant-in-Aid for Transformative Research Areas (A)
“The Natural Laws of Extreme Universe--A New Paradigm for Spacetime and Matter from Quantum Information” https://www2.yukawa.kyoto-u.ac.jp/~extremeuniverse/en/

Planned Research

Cold atom experiment



Quantum Hall experiment



+ Publicly Offered Research (E01,E02,E03)

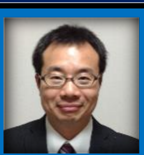
Quantum Computer

Super Computer

Management Group Members

Totally 42 members, including 9 PIs
See our webpage for details


A01PI Quantum Computation
Tomoyuki Morimae (YITP, Kyoto)


B01PI Quantum BH
Norihiro Iizuka (Osaka U.)


B02PI Cold Atom Theory
Masaki Tezuka (Kyoto U.)


B03PI BH in GR and Math
Akihiro Ishibashi (Kindai U.)


C01PI Holography and QI
Tadashi Takayanagi (YITP, Kyoto)


C02PI Quantum Hall Experiment
Go Yusa (Tohoku U.)


C03PI Cosmology in GR and Math
Testuya Shiromizu (Nagoya U.)


D01PI QFT and QI
Tatsuma Nishioka (YITP→Osaka U.)


D02PI Tensor Network
Koichi Okunishi (Niigata U.)


A01CoI Quantum Information
Yoshifumi Nakata (U. of Tokyo)


B02CoI Cold Atom Experiment
Shuta Nakajima (Kyoto U.)


C02CoI Relativistic Quantum Info.
Masahiro Hotta (Tohoku U.)


C03CoI Global Structure in GR
Keisuke Izumi (Nagoya U.)

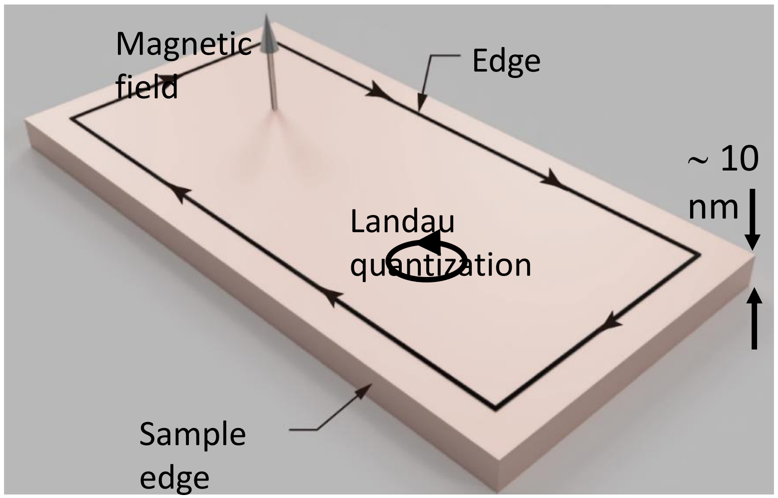

C03CoI General Cosmology Models
Tsutomu Kobayashi (Rikkyo U.)


D02CoI Tensor Network and QC
Hiroshi Ueda (Osaka U.)

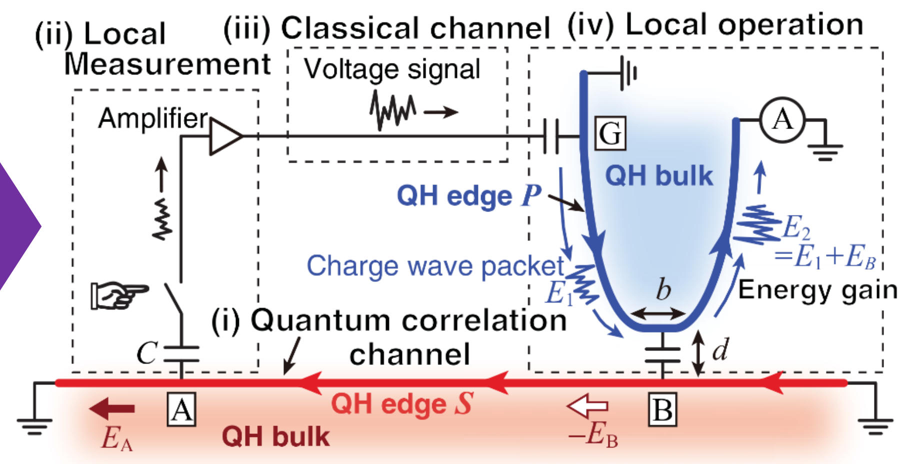

D02CoI Quantum Spin System Theory
Chisa Hotta (U. of Tokyo)

Quantum Hall Experiment in Yusa-group C02

Quantum Hall Effect

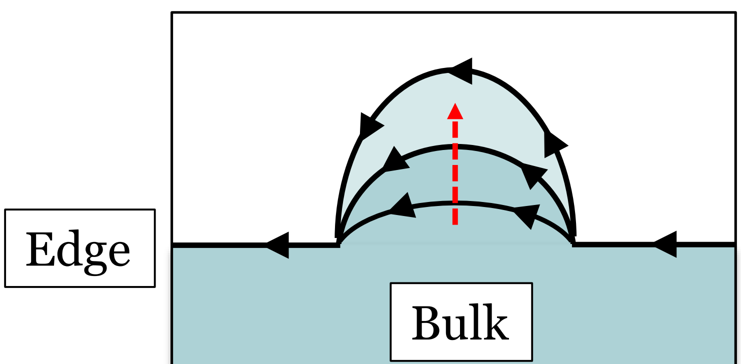


Quantum Energy Teleportation



Theory for Quantum Energy Teleportation in QHE
Hotta, Matsumoto, Yusa, PRA 2014.

Toy model of Expanding Universe

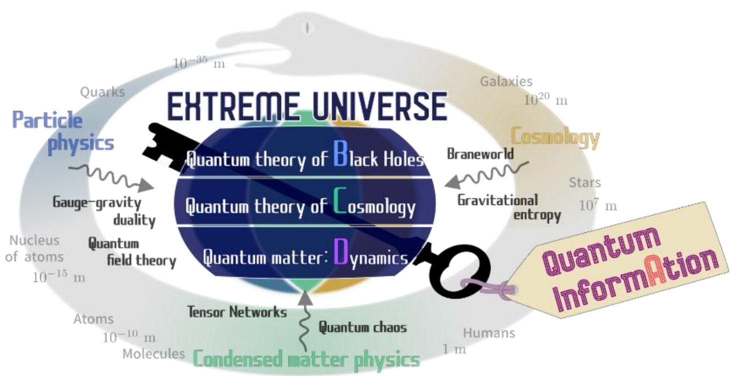


Theory for Expansing Universe in QHE:
Hotta-Nambu-Sugiyama-Yamamoto-Yusa,
arXiv:2202.03731

⑤ Conclusions

Starting from black holes, holography and quantum entanglement, we reviewed recent remarkable progresses on deep connections between quantum information, quantum matter and gravity.

These unsolved problems are boiled down to Extreme Universe !



Concept of ExU Collaboration

In spite of different scales, the laws of physics will be universal, all as a collection of qubits!

1st Annual meeting Mar 7–8

1st ExU School Mar 3–5

Monthly Colloquium

Grant-in-Aid for Transformative Research Areas A
Extreme Universe

Thank you very much !