Chiral Magnetohydrodynamics in Cosmology

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Plan of Talk

1. Motivation of PMF from Particle Cosmology

   Why is magnetic field interesting to cosmologist?

2. Inflationary Magnetogenesis

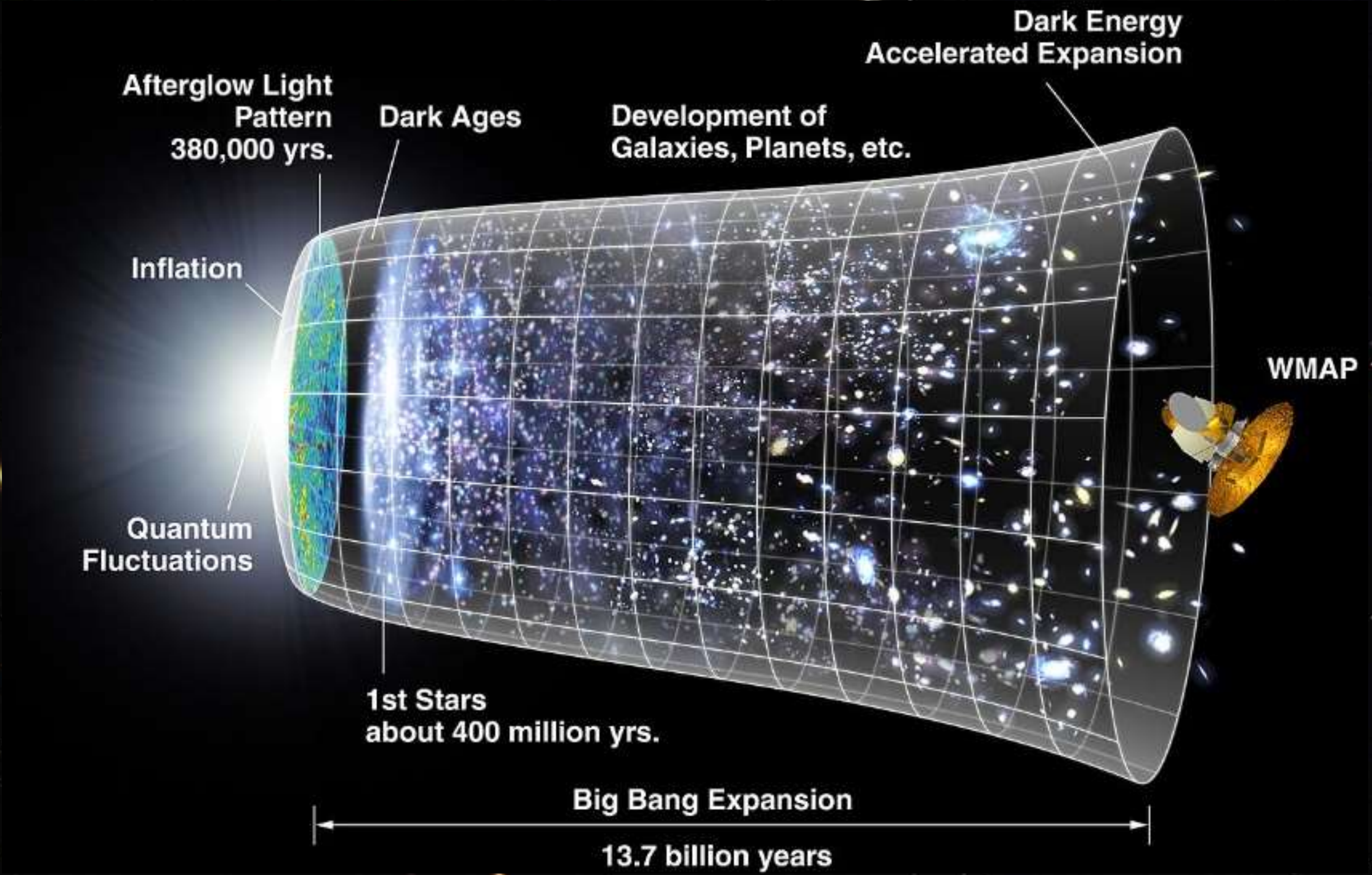
   How to set the initial condition of PMF?

3. Chiral MHD

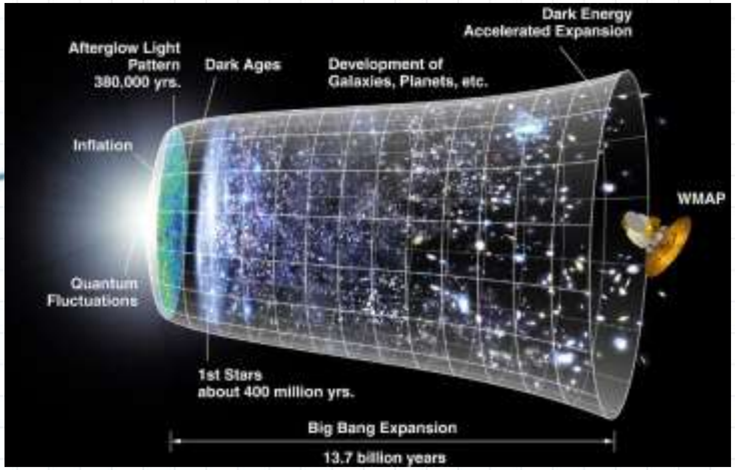
   How helical MF produces chiral chemical potential?

Why am I interested in PMF?

Primordial magnetic fields may be used as a probe of the early universe/new physics.


Dark Energy Accelerated Expansion
Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Inflation
WMAP
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years

Motivation



Inflation

- Inflation : $H \equiv \dot{a}/a \approx \mathrm{const} \longrightarrow a(t) \propto e^{Ht}$

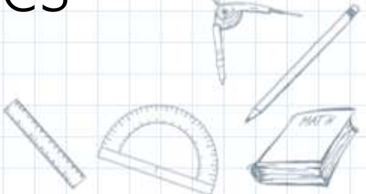
  Accelerating expansion era in primordial universe.

- It generates fluctuations seen in CMB/LSS

  Perturbations of all the light fields are produced.

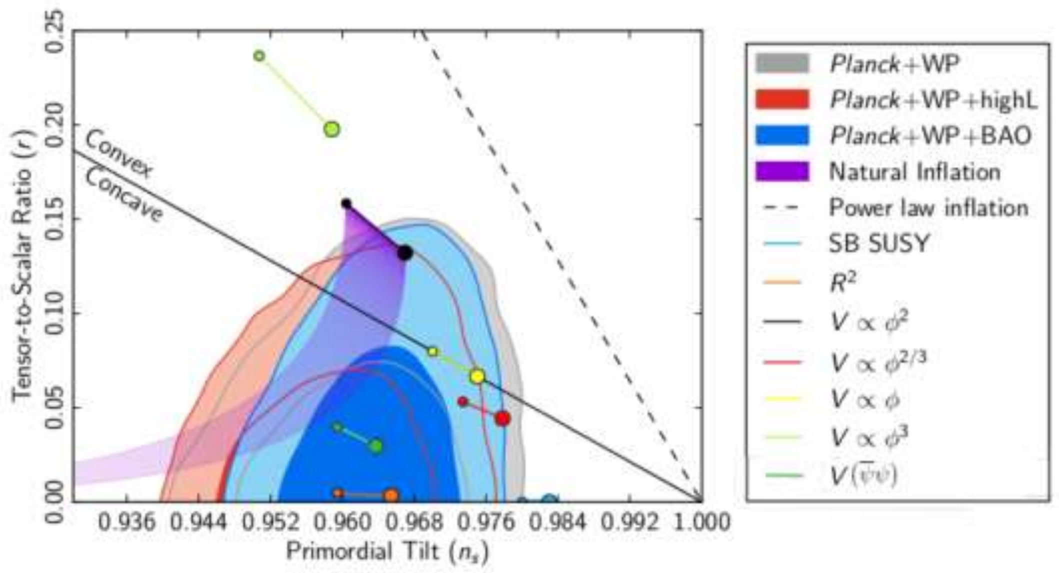
- Mechanism is unknown = New physics

  ρ_{inf} = energy scale of BSM physics.



What do we know?

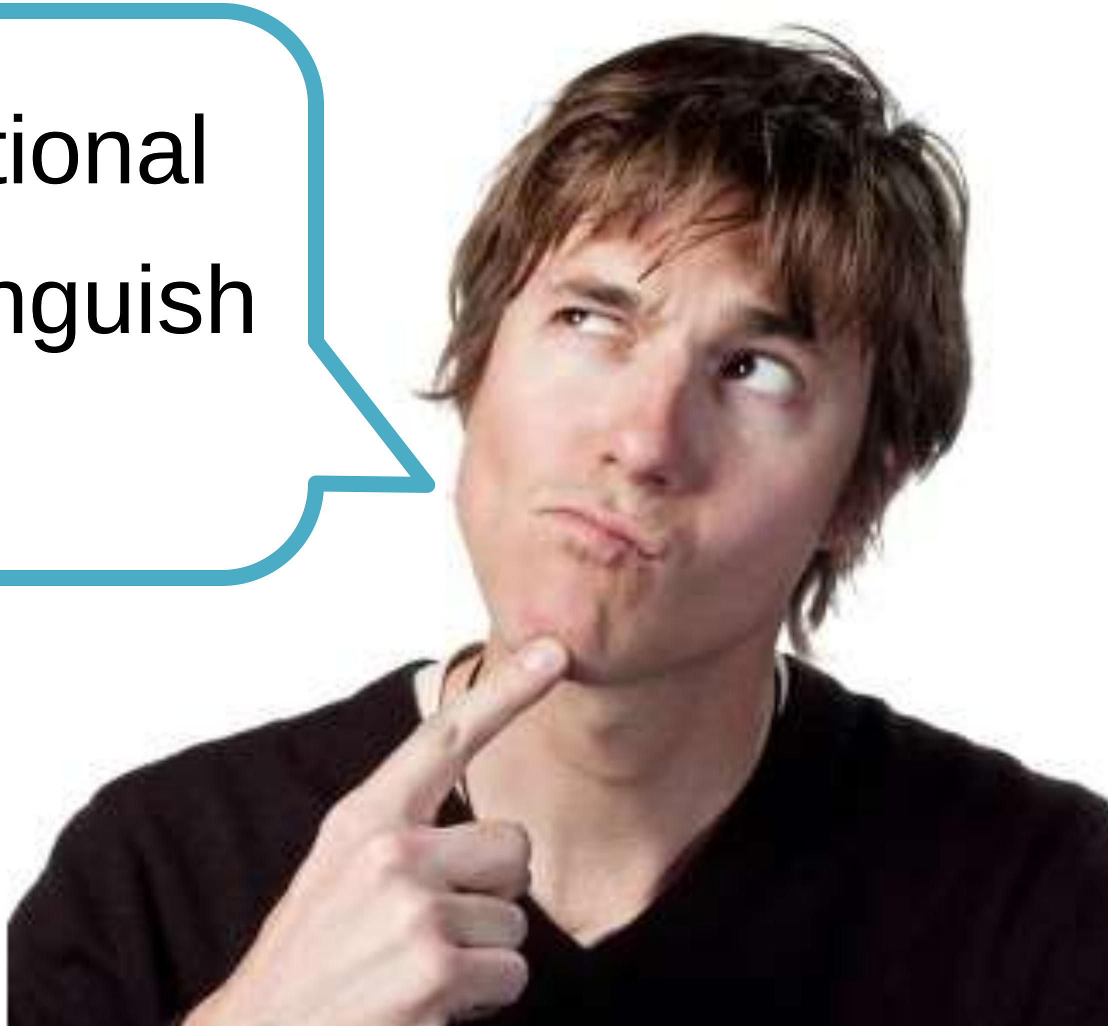
Very little from observations



2 parameters are not enough to reveal the beginning of the universe!

Motivation

Another observational signature to distinguish the models??



Primordial Magnetic Field

Electromagnetism (or QED) is generally **modified** on high energy scales

NB: Even within the standard model, QED is extended to electroweak for T>100GeV.

PMF can be an **unique probe** of the early universe/fundamental physics.

Unified Scenario

Primordial MF

Entire Universe

Unified Scenario

Primordial MF

Overdense Region

Amplification by Dynamo

Galactic MF

$$\mathbf{B}_{\boldsymbol{g}} \sim \mathbf{10^{-6}\ G}$$

Void Region

Dilutes by Cosmic Expansion

Void MF

Weak MF is expected

$$\mathbf{B}_{\boldsymbol{v}} \sim \mathbf{10^{-16}\ G}$$

Why am I interested in PMF?

Primordial magnetic fields may be used as a probe of the early universe/new physics.

↓

Need to link present MF to the early Universe

Study Generation & Evolution of MF

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   How helical MF produces chiral chemical potential?

Why inflationary magnetogenesis?

- Inflation = earliest epoch = highest energy scale

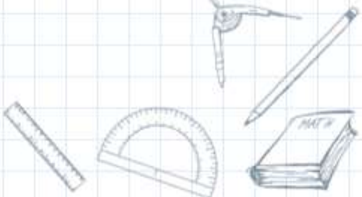
  ⇒ Biggest chance of modified electromagnetism

- Before the hot bigbang (reheating) = No plasma

  ⇒ Electric conductivity is zero. Easy to amplify MF.

- Seeds of the structures were produced during inflation

  ⇒ Similar mechanism for MF?



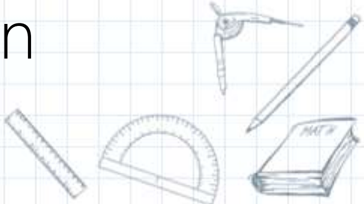
Basic idea of IM

- Initially zero MF. But quantum fluctuation is always there.

- Consider a model where EMF has an instability

- Quantum fluctuation is amplified

⇒ Significant EMFs are generated on small scale and then stretched by cosmic expansion



Model examples

●Kinetic Coupling [Ratra(1992)]

$$I^2(\phi)F_{\mu\nu}F^{\mu\nu}$$

●Axial Coupling [Garretson+(1992)]

$$+\frac{\phi}{M}F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

●Non-minimal Coupling
[Turner&Widrow(1988)]

$$+\xi R A_\mu A^\mu$$

●Higgs Coupling
[Finelli+(2001)]

$$+e^2\phi^2 A_\mu A^\mu$$

●Z boson projection
[Dimopoulos+(2001)]

$$A_\mu \simeq Z_\mu^{\mathrm{inf}} sin2\theta_w$$

etc…

Axial coupling model

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Axionic inflaton is coupled to EMF via Chern-Simon term

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - V(\phi)}_{\text{inflaton}} \underbrace{- \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Maxwell Theory}} \underbrace{- \frac{g}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}}_{\text{Coupling}}$$

Well motivated
by particle physics

Inflaton which has huge energy

is coupled to $F_{\mu\nu}\tilde{F}^{\mu\nu} = \boldsymbol{E}\cdot\boldsymbol{B}$,

⇨ Energy transfer to EMF is expected

EoM for vector potential

$'\equiv\partial_\eta$: Conformal time derivative

The Lagrangian leads to the following EoM:

F.T. $$A_i(t,\boldsymbol{x}) = \sum_{\lambda=\pm}\int\frac{d^3k}{(2\pi)^3}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\underline{\epsilon_i^\lambda\left(\hat{\boldsymbol{k}}\right)}A_\lambda(t,k)$$

circular polarization

EoM $$A_\pm'' + [k^2 \underline{\pm g\phi' k}]A_\pm = 0$$

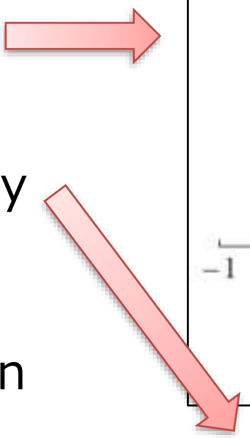
Modified part: $k < g\phi'$ unstable

Either one of two circular polarizations is produced.

Helical MF is generated!

[Fujita+(2015)]

Numerical Result 1



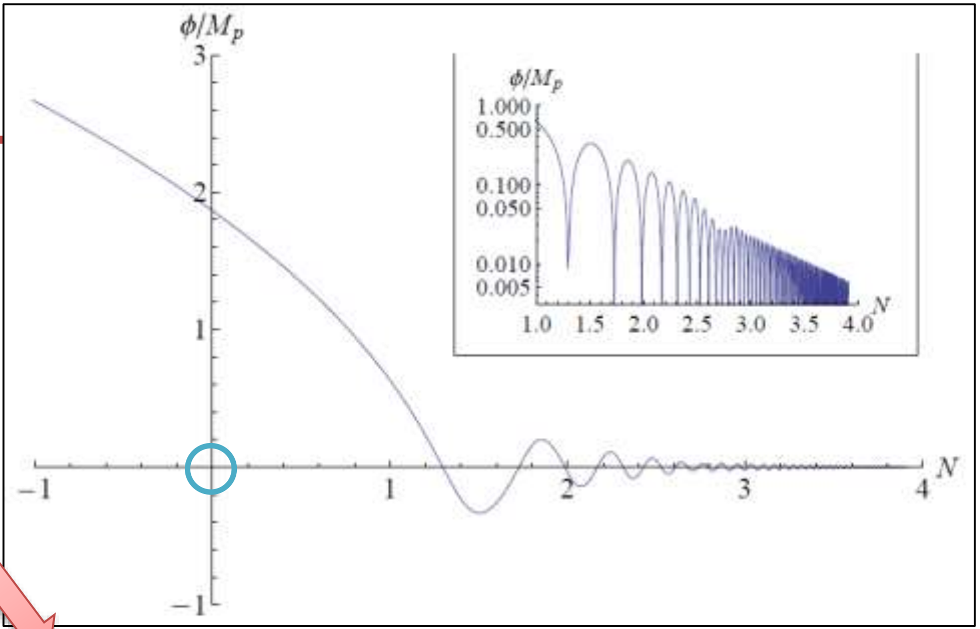
Behavior of inflaton $\phi(t)$

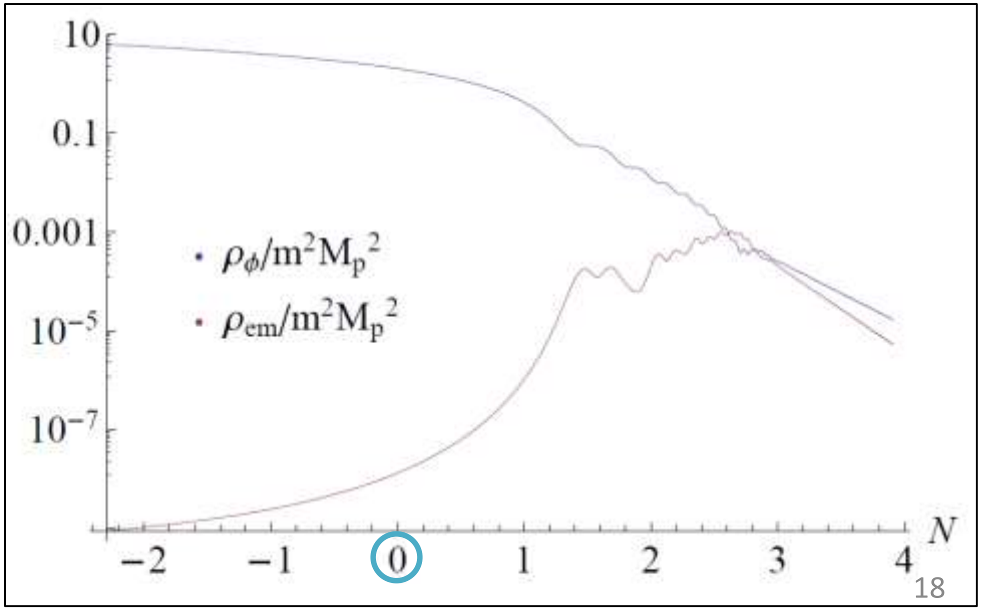
Evolution of energy density

$N = 0$ is the end of inflation

After inflation ends,
ϕ' increases and
EMF production gets efficient.

Back reaction is included.





Numerical Result 2

$$A''_{\pm} + [k^2 \pm g\phi' k]A_{\pm} = 0$$

Inflaton $\phi(t)$ shows a damped oscillation

First one A_- is dominant

Both pol. $A_{\pm}$ are produced

$$\sqrt{2k}A_- \sim \mathcal{O}(10^6) \qquad \sqrt{2k}A_+ \sim \mathcal{O}(10^2)$$

Maximally helical MF
is generated by inflation

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What happens after inflation?

- After inflation, reheating (hot big bang) occurs and charged particles (plasma) appear.

- Very high conductivity → $\boldsymbol{E}$ is erased

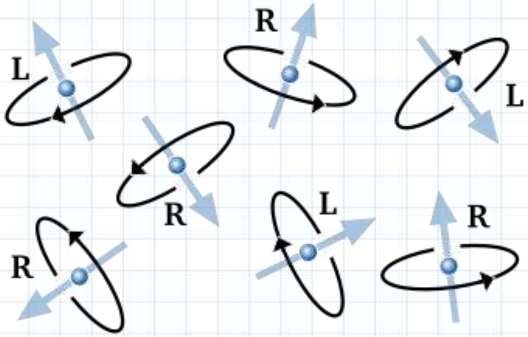
  Tight coupling btw plasma and $\boldsymbol{B}$ → MHD

- Temperature is very high, $T \gg m_e$.

  → Classical MHD should be extended to Chiral MHD.

Chiral Magnetic Effect

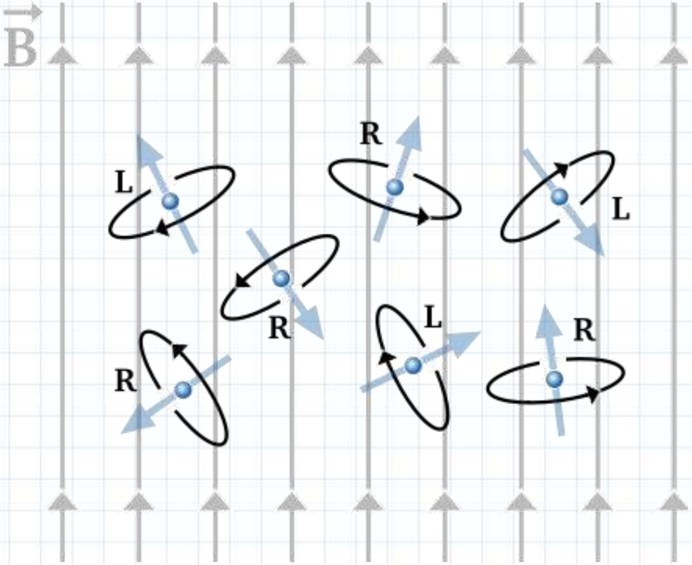
[*Vilenkin 1980*].



Consider there're some electrons which have left/right chirality

Chiral Magnetic Effect

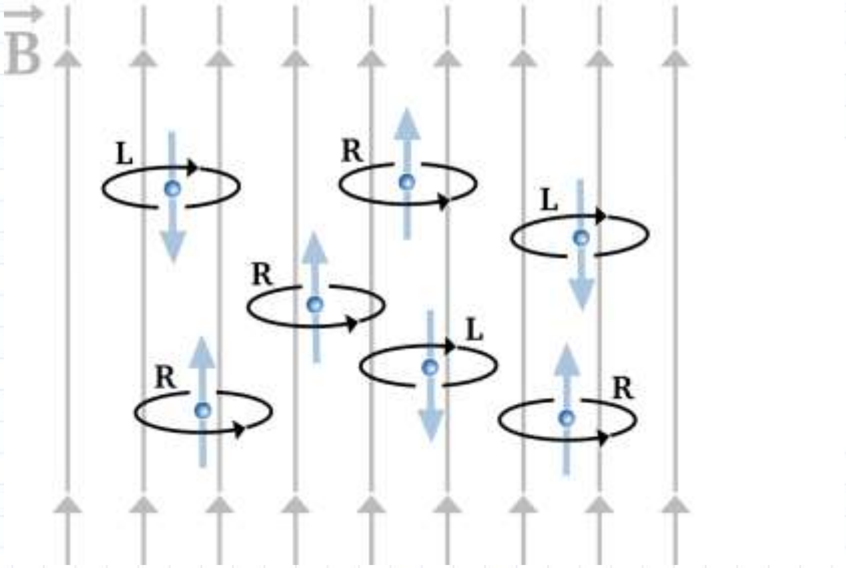
[*Vilenkin 1980*].



Apply magnetic field

Chiral Magnetic Effect

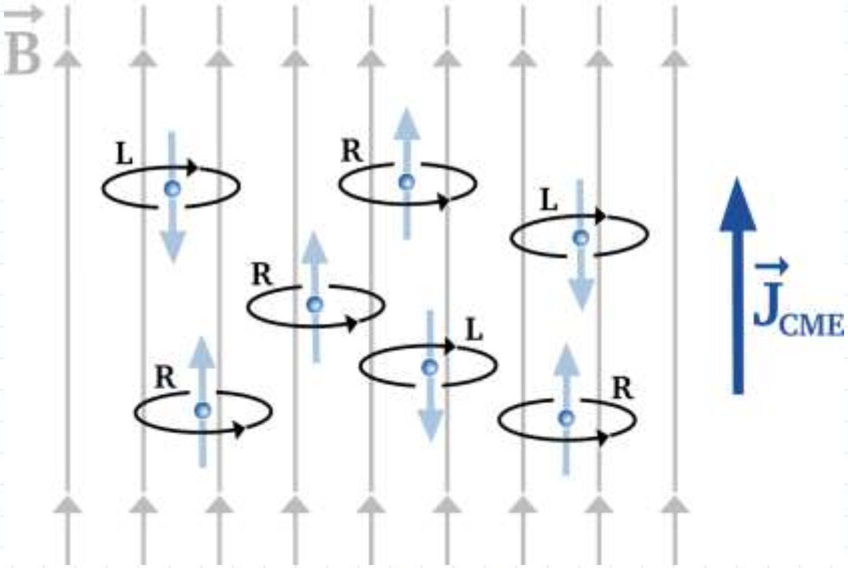
[*Vilenkin 1980*].



Spins are aligned to $\boldsymbol{B}$

Chiral Magnetic Effect

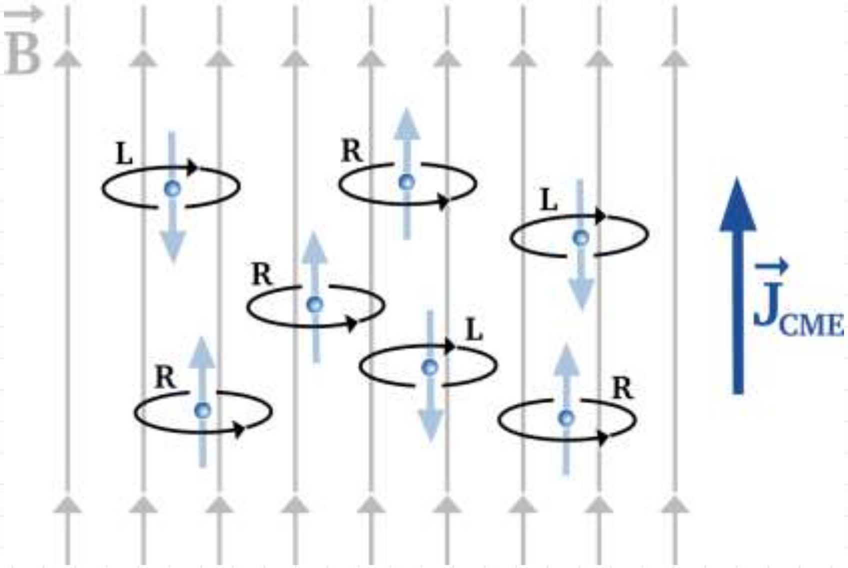
[*Vilenkin 1980*].



If the number of left and right-handed electrons are different, an current $J_{\mathbf{CME}}$ is induced.

Chiral Magnetic Effect

[*Vilenkin 1980*].



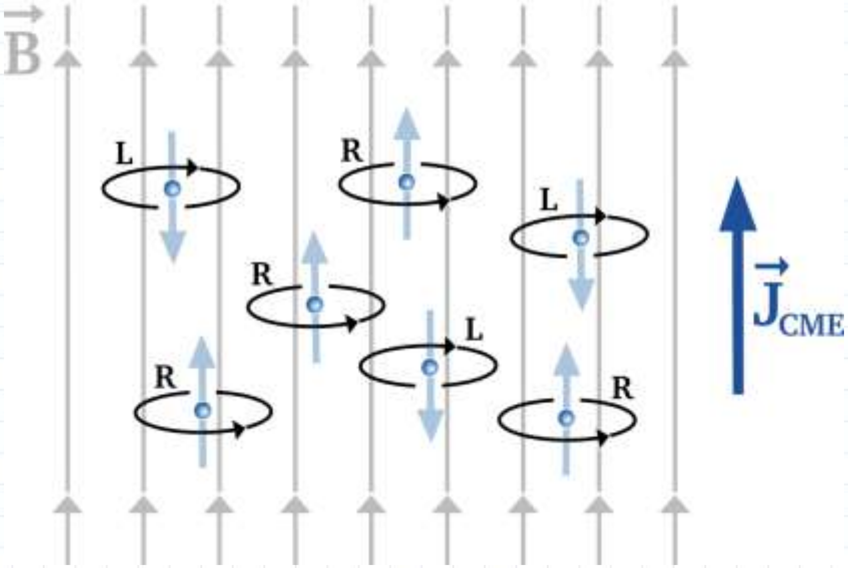
If the number of left and right-handed electrons are different, an current J_{CME} is induced.

For a chiral chemical potential $\mu_5 \equiv \mu_{\mathrm{L}} - \mu_{\mathrm{R}}$
an additional electric current occurs: $\mathbf{J}_{\mathrm{CME}} \propto \mu_5 \mathbf{B}$

Courtesy of Jenny Schober

Chiral Magnetic Effect

[*Vilenkin 1980*].



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Chiral induction eq.

$$\boxed{\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B} + \eta \mu_5 \boldsymbol{B} - \eta \nabla \times \boldsymbol{B})}$$

Courtesy of Jenny Schober

Where/When is chiral MHD important?

A chiral asymmetry can only survive at $k_{\rm B}T > 10 \ {\rm MeV}$ [*Boyarsky et al.* 2012]

Early Universe

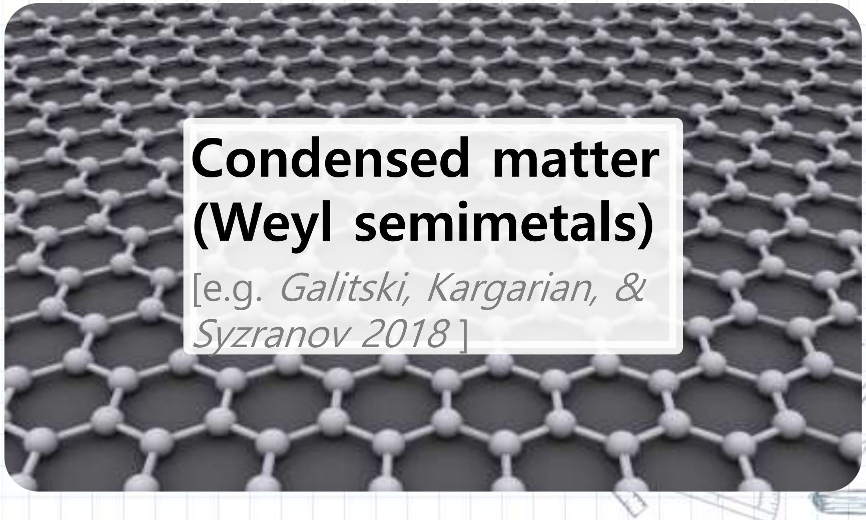
[e.g. *Joyce & Shaposhnikov 1997; Fröhlich & Pedrini 2000, Semikoz & Sokoloff 2004; Pavlovic et al 2017*]

Heavy-ion collisions

[e.g. *ALICE collaboration; 2013; Hirono, Hirano, & Kharzeev 2014*]

(Proto-)neutron stars

[e.g. *Dvornikov & Semikoz 2015; Grabowska et al. 2015; Sigl & Leite 2016; Yamamoto 2016*
]



Condensed matter (Weyl semimetals)

[e.g. *Galitski, Kargarian, & Syzranov 2018*]

Classical vs. chiral MHD

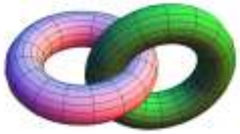
Classical MHD

Full set of evolution equations:

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times [\boldsymbol{U} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}] \\ \rho \frac{D\boldsymbol{U}}{Dt} &= (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p + \nabla \cdot (2\nu\rho S) \\ \frac{D\rho}{Dt} &= -\rho \nabla \cdot \boldsymbol{U} \end{aligned}$$

Conservation law (valid for $\eta \to 0$):

$$\frac{\partial}{\partial t}(\boldsymbol{A} \cdot \boldsymbol{B}) = 0$$



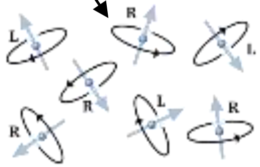
Chiral MHD

Full set of evolution equations:

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times [\boldsymbol{U} \times \boldsymbol{B} - \eta\, (\nabla \times \boldsymbol{B} \underline{- \mu_5 \boldsymbol{B}})] \\ \rho \frac{D\boldsymbol{U}}{Dt} &= (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} - \nabla p + \nabla \cdot (2\nu\rho S) \\ \frac{D\rho}{Dt} &= -\rho \nabla \cdot \boldsymbol{U} \\ \underline{\frac{D\mu_5}{Dt}} &\underline{= D_5\, \Delta\mu_5 + \lambda\eta \left[\boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) - \mu_5 \boldsymbol{B}^2\right]} \end{aligned}$$

Conservation law (valid for any η):

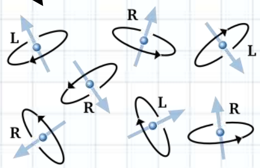
$$\frac{\partial}{\partial t}\left(\boldsymbol{A} \cdot \boldsymbol{B} + \frac{2\mu_5}{\lambda}\right) = 0$$



[Rogachevskii et al. 2017]

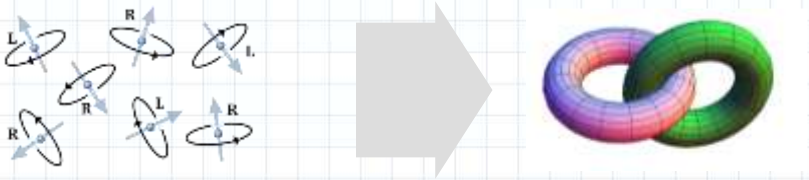
New phenomena due to new d.o.f.

$$\frac{\partial}{\partial t}\left(\langle \boldsymbol{A}\cdot\boldsymbol{B}\rangle + \frac{2\langle\mu_5\rangle}{\lambda}\right) = 0$$



Scenario 1

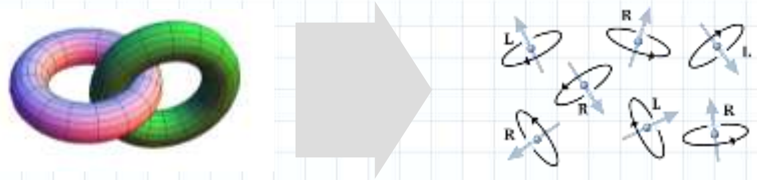
Start with initial chiral asymmetry and convert into magnetic helicity.



= **"chiral MHD dynamo"**

Scenario 2

Start with initial helical magnetic field and generate a chiral asymmetry.



= **What happens?**

[*Brandenburg et al. 2017*]

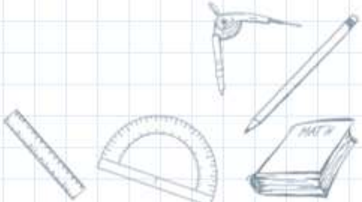
Analytic look at $\mu_5(t)$

$$\boxed{\frac{D\mu_5}{Dt} = \mathcal{D}_5\,\Delta\mu_5 + \lambda\,\eta\,\left[\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}) - \mu_5\boldsymbol{B}^2\right]}$$

Analytic look at $\mu_5(t)$

$$\boxed{\frac{D\mu_5}{Dt} = \mathcal{D}_5\,\cancel{\Delta\mu_5} + \lambda\,\eta\,\left[\underset{\substack{\rotatebox{90}{$\approx$} \\ k_p\boldsymbol{B}^2}}{\underline{\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})}} - \cancel{\mu_5\boldsymbol{B}^2}\right]}$$

- Initially $\mu_5 = 0$ $\Rightarrow$ $\mu_5 \simeq \left(\lambda\eta k_p B^2\right) t$



Analytic look at $\mu_5(t)$

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$$\wr$$

$$k_p\boldsymbol{B}^2$$

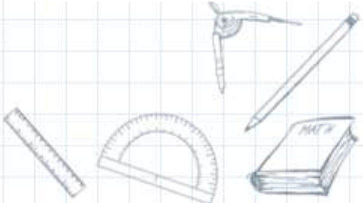
- Initially $\mu_5 = 0$ → $\mu_5 \simeq \left(\lambda\eta k_p B^2\right) t$
- But μ_5 can't exceed k_p → $\mu_5 \lesssim k_p(t) \searrow$

Analytic look at $\mu_5(t)$

$$\boxed{\frac{D\mu_5}{Dt} = \mathfrak{D}_5\,\Delta\mu_5 + \lambda\,\eta\,\Big[\underset{\substack{\wr \\ k_p\boldsymbol{B}^2}}{\underline{\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})}} - \mu_5\boldsymbol{B}^2\Big]}$$

- Initially $\mu_5 = 0$ → $\mu_5 \simeq \left(\lambda\eta k_p B^2\right) t$
- But μ_5 can't exceed k_p → $\mu_5 \lesssim k_p(t) \searrow$
- $\boldsymbol{A}\cdot\boldsymbol{B} + \frac{2}{\lambda}\mu_5 = \text{const.}$ → $\mu_5 \lesssim \lambda(\boldsymbol{A}\cdot\boldsymbol{B})_0 \simeq \lambda B_0^2/k_p$

Chiral MHD conservation law



Analytic look at $\mu_5(t)$

$$\boxed{\frac{D\mu_5}{Dt} = \mathcal{D}_5\,\Delta\mu_5 + \lambda\,\eta\,\Big[\underbrace{\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})}_{\simeq\, k_p\boldsymbol{B}^2} - \mu_5\boldsymbol{B}^2\Big]}$$

- Initially $\mu_5 = 0$ → $\mu_5 \simeq \left(\lambda\eta k_p B^2\right) t$
- But μ_5 can't exceed k_p → $\mu_5 \lesssim k_p(t)\ \searrow$
- $\boldsymbol{A}\cdot\boldsymbol{B} + \frac{2}{\lambda}\mu_5 = \text{const.}$ → $\mu_5 \lesssim \lambda(\boldsymbol{A}\cdot\boldsymbol{B})_0 \simeq \lambda B_0^2/k_0$

$$\boxed{\mu_5(t) \simeq \min\left[k_p(t), \lambda B_0^2/k_0\right]}$$

Analytic look at $B(t)$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \left[\boldsymbol{U} \times \boldsymbol{B} - \eta \left(\boldsymbol{\nabla} \times \boldsymbol{B} - \mu_5 \boldsymbol{B}\right)\right]$$

- For $\mu_{5,0} = 0$, $\mu_5 \boldsymbol{B}$ never overwhelms $\nabla \times \boldsymbol{B} \simeq k_p\, B$

Analytic look at $B(t)$

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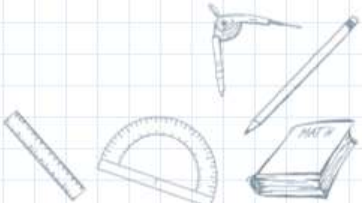
- For $\mu_{5,0} = 0$, $\mu_5 \boldsymbol{B}$ never overwhelms $\nabla \times \boldsymbol{B} \simeq k_p\, B$
- Magnetic Reynolds number $\mathrm{Re}_M \equiv u_{\mathrm{rms}}/k_p\eta$ characterizes the importance of the $\boldsymbol{U}$ term.



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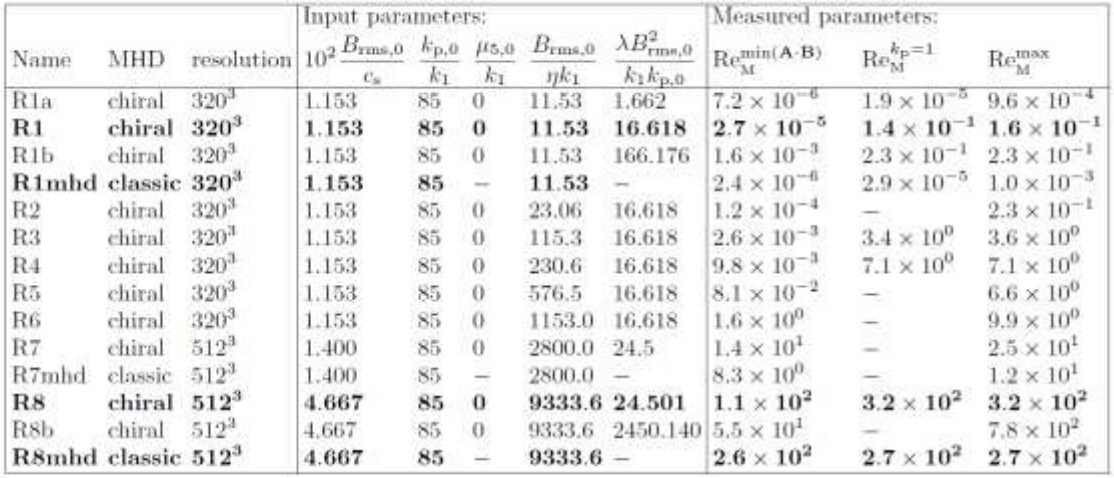
- For $\mu_{5,0} = 0$, $\mu_5 \boldsymbol{B}$ never overwhelms $\nabla \times \boldsymbol{B} \simeq k_p\, B$
- Magnetic Reynolds number $\mathbf{Re}_M \equiv u_{\mathbf{rms}}/k_p\eta$ characterizes the importance of the $\boldsymbol{U}$ term.
- Low $\mathbf{Re}_M$ → $\boldsymbol{U}$ is negligible (CME regime)

  High $\mathbf{Re}_M$ → Classical inverse cascade

Chiral MHD

DNS with varying Re_M

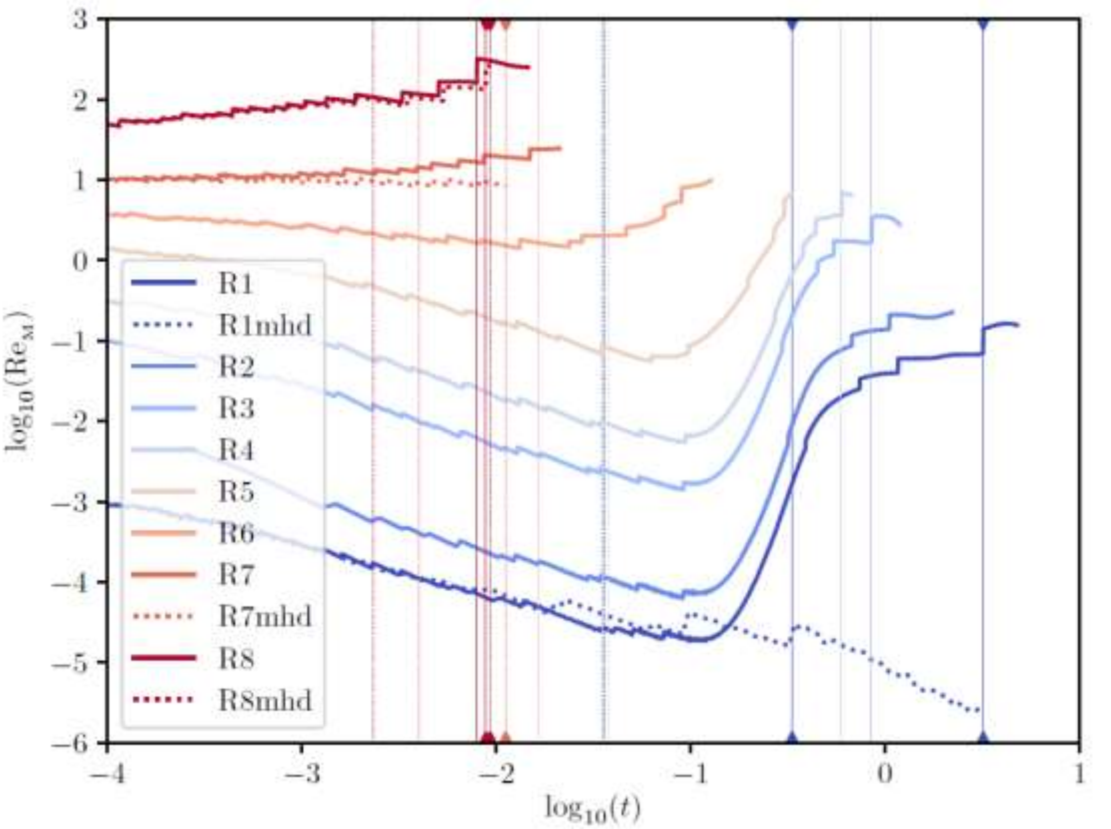


			Input parameters:					Measured parameters:		
Name	MHD	resolution	$10^2 \frac{B_{\mathrm{rms},0}}{c_s}$	$\frac{k_{\mathrm{p},0}}{k_1}$	$\frac{\mu_{5,0}}{k_1}$	$\frac{B_{\mathrm{rms},0}}{\eta k_1}$	$\frac{\lambda B^2_{\mathrm{rms},0}}{k_1 k_{\mathrm{p},0}}$	$\mathrm{Re}_{\mathrm{M}}^{\min(\mathbf{A}\cdot\mathbf{B})}$	$\mathrm{Re}_{\mathrm{M}}^{k_{\mathrm{p}}=1}$	$\mathrm{Re}_{\mathrm{M}}^{\max}$
R1a	chiral	320^3	1.153	85	0	11.53	1.662	7.2×10^{-6}	1.9×10^{-5}	9.6×10^{-4}
R1	**chiral**	$\mathbf{320^3}$	**1.153**	**85**	**0**	**11.53**	**16.618**	$\mathbf{2.7 \times 10^{-5}}$	$\mathbf{1.4 \times 10^{-1}}$	$\mathbf{1.6 \times 10^{-1}}$
R1b	chiral	320^3	1.153	85	0	11.53	166.176	1.6×10^{-3}	2.3×10^{-1}	2.3×10^{-1}
R1mhd	**classic**	$\mathbf{320^3}$	**1.153**	**85**	–	**11.53**	–	2.4×10^{-6}	2.9×10^{-5}	1.0×10^{-3}
R2	chiral	320^3	1.153	85	0	23.06	16.618	1.2×10^{-4}	–	2.3×10^{-1}
R3	chiral	320^3	1.153	85	0	115.3	16.618	2.6×10^{-3}	3.4×10^{0}	3.6×10^{0}
R4	chiral	320^3	1.153	85	0	230.6	16.618	9.8×10^{-3}	7.1×10^{0}	7.1×10^{0}
R5	chiral	320^3	1.153	85	0	576.5	16.618	8.1×10^{-2}	–	6.6×10^{0}
R6	chiral	320^3	1.153	85	0	1153.0	16.618	1.6×10^{0}	–	9.9×10^{0}
R7	chiral	512^3	1.400	85	0	2800.0	24.5	1.4×10^{1}	–	2.5×10^{1}
R7mhd	classic	512^3	1.400	85	–	2800.0	–	8.3×10^{0}	–	1.2×10^{1}
R8	**chiral**	$\mathbf{512^3}$	**4.667**	**85**	**0**	**9333.6**	**24.501**	$\mathbf{1.1 \times 10^{2}}$	$\mathbf{3.2 \times 10^{2}}$	$\mathbf{3.2 \times 10^{2}}$
R8b	chiral	512^3	4.667	85	0	9333.6	2450.140	5.5×10^{1}	–	7.8×10^{2}
R8mhd	**classic**	$\mathbf{512^3}$	**4.667**	**85**	–	**9333.6**	–	$\mathbf{2.6 \times 10^{2}}$	$\mathbf{2.7 \times 10^{2}}$	$\mathbf{2.7 \times 10^{2}}$

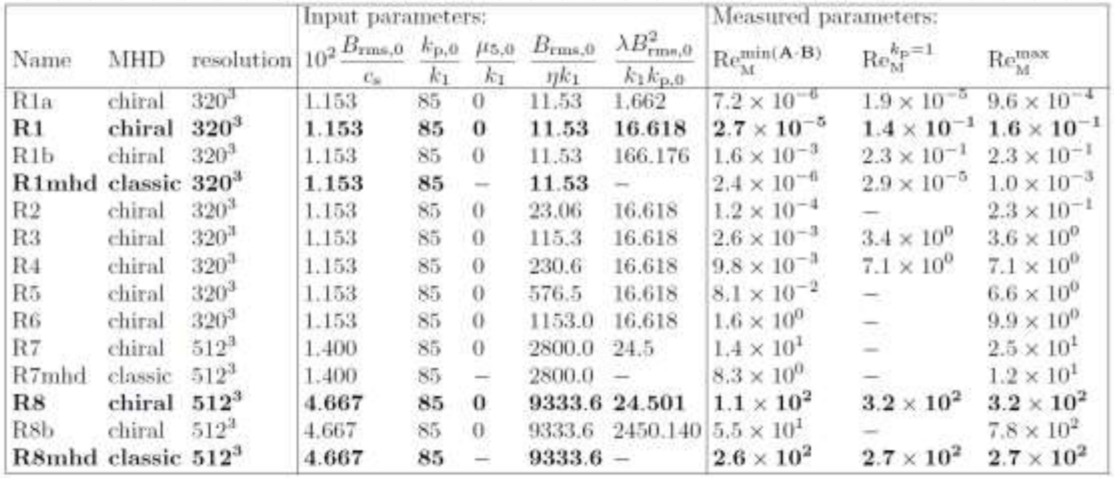
- Change η

⇒ different Re_M

- $\mathrm{Re}_M \lesseqgtr 1$ show distinct evolutions



Chiral MHD



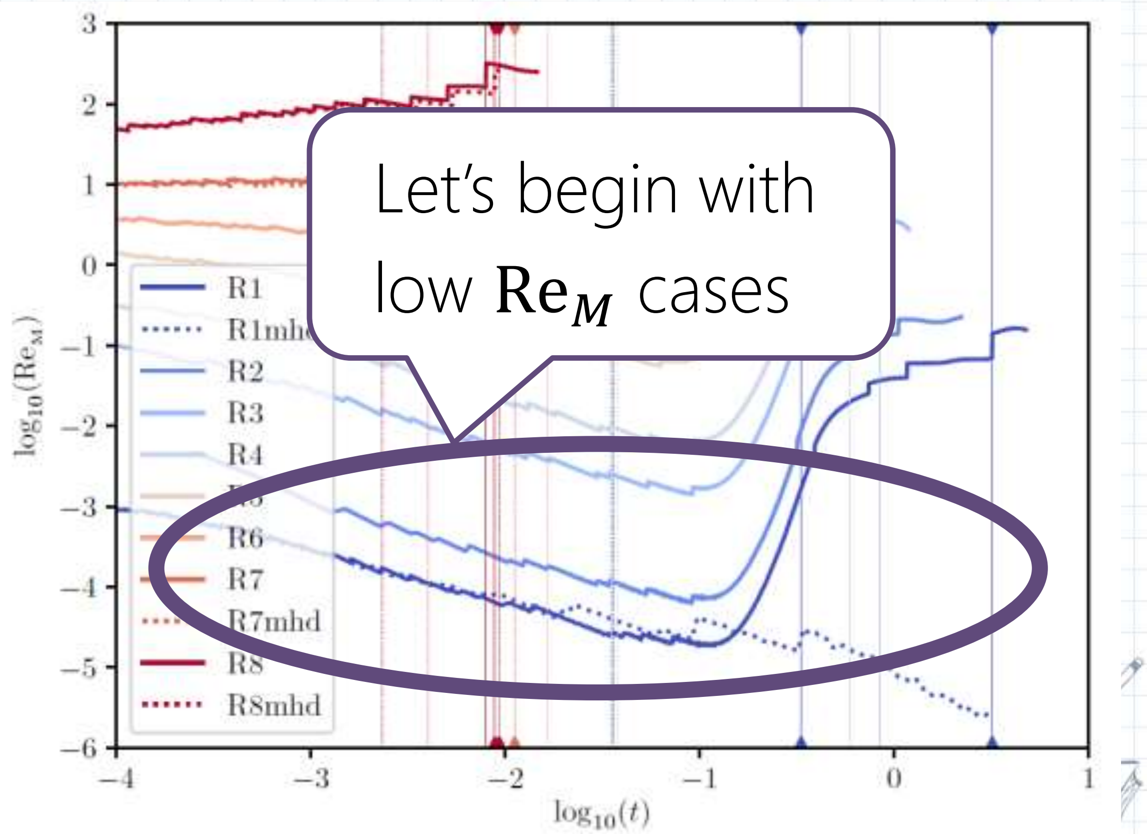
			Input parameters:					Measured parameters:		
Name	MHD	resolution	$10^2 \frac{B_{\mathrm{rms},0}}{c_s}$	$\frac{k_{\mathrm{p},0}}{k_1}$	$\frac{\mu_{5,0}}{k_1}$	$\frac{B_{\mathrm{rms},0}}{\eta k_1}$	$\frac{\lambda B^2_{\mathrm{rms},0}}{k_1 k_{\mathrm{p},0}}$	$\mathrm{Re}_\mathrm{M}^{\min(\mathbf{A}\cdot\mathbf{B})}$	$\mathrm{Re}_\mathrm{M}^{k_\mathrm{p}=1}$	$\mathrm{Re}_\mathrm{M}^{\max}$
R1a	chiral	320^3	1.153	85	0	11.53	1.662	7.2×10^{-6}	1.9×10^{-5}	9.6×10^{-4}
R1	**chiral**	$\mathbf{320^3}$	**1.153**	**85**	**0**	**11.53**	**16.618**	$\mathbf{2.7 \times 10^{-5}}$	$\mathbf{1.4 \times 10^{-1}}$	$\mathbf{1.6 \times 10^{-1}}$
R1b	chiral	320^3	1.153	85	0	11.53	166.176	1.6×10^{-3}	2.3×10^{-1}	2.3×10^{-1}
R1mhd	**classic**	$\mathbf{320^3}$	**1.153**	**85**	–	**11.53**	–	2.4×10^{-6}	2.9×10^{-5}	1.0×10^{-3}
R2	chiral	320^3	1.153	85	0	23.06	16.618	1.2×10^{-4}	–	2.3×10^{-1}
R3	chiral	320^3	1.153	85	0	115.3	16.618	2.6×10^{-3}	3.4×10^{0}	3.6×10^{0}
R4	chiral	320^3	1.153	85	0	230.6	16.618	9.8×10^{-3}	7.1×10^{0}	7.1×10^{0}
R5	chiral	320^3	1.153	85	0	576.5	16.618	8.1×10^{-2}	–	6.6×10^{0}
R6	chiral	320^3	1.153	85	0	1153.0	16.618	1.6×10^{0}	–	9.9×10^{0}
R7	chiral	512^3	1.400	85	0	2800.0	24.5	1.4×10^{1}	–	2.5×10^{1}
R7mhd	classic	512^3	1.400	85	–	2800.0	–	8.3×10^{0}	–	1.2×10^{1}
R8	**chiral**	$\mathbf{512^3}$	**4.667**	**85**	**0**	**9333.6**	**24.501**	$\mathbf{1.1 \times 10^{2}}$	$\mathbf{3.2 \times 10^{2}}$	$\mathbf{3.2 \times 10^{2}}$
R8b	chiral	512^3	4.667	85	0	9333.6	2450.140	5.5×10^{1}	–	7.8×10^{2}
R8mhd	**classic**	$\mathbf{512^3}$	**4.667**	**85**	–	**9333.6**	–	$\mathbf{2.6 \times 10^{2}}$	$\mathbf{2.7 \times 10^{2}}$	$\mathbf{2.7 \times 10^{2}}$

DNS with varying Re_M

- Change η

  ⇨ different $\mathbf{Re}_M$

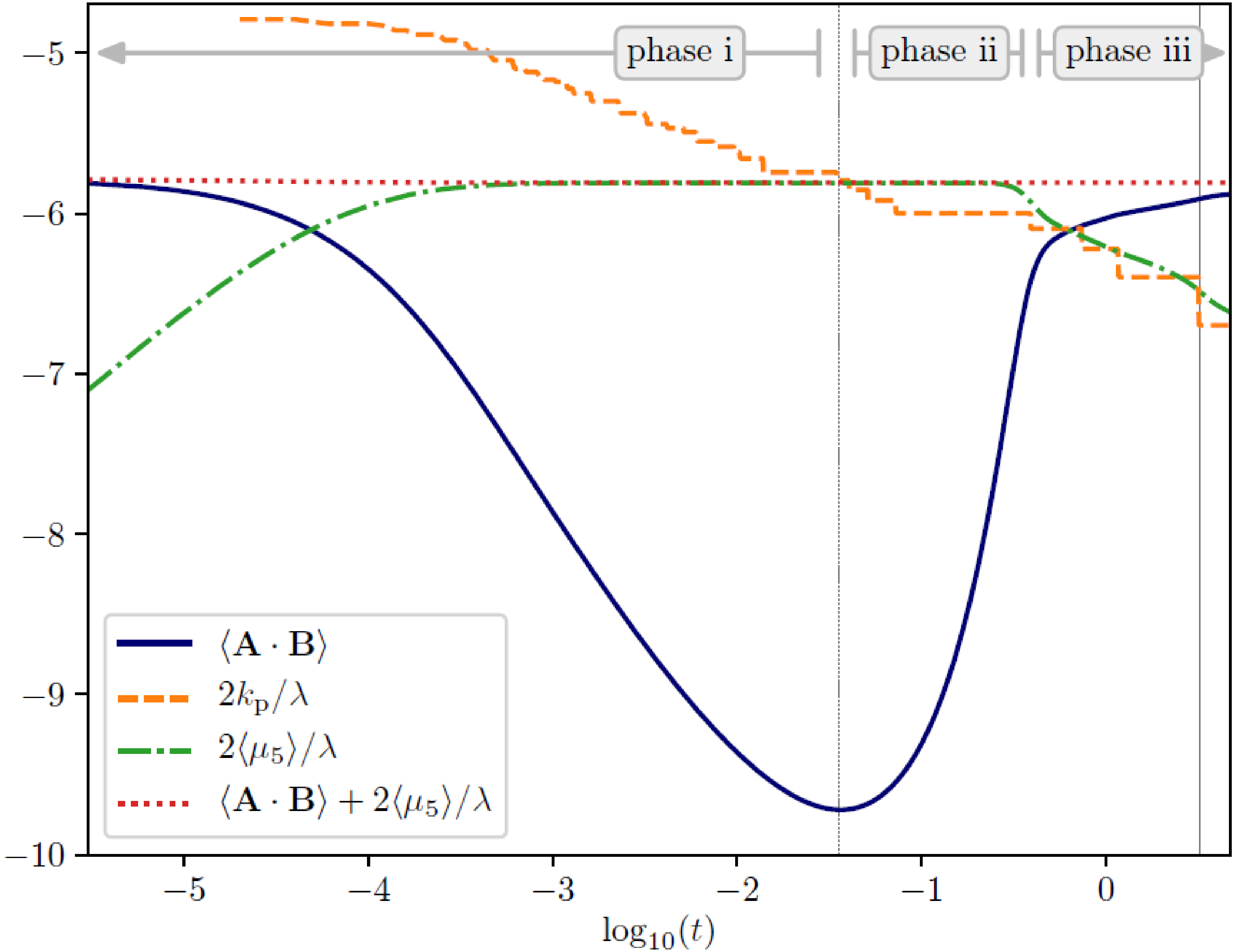
- $\mathbf{Re}_M \lesssim 1$ show distinct evolutions



Let's begin with low $\mathbf{Re}_M$ cases

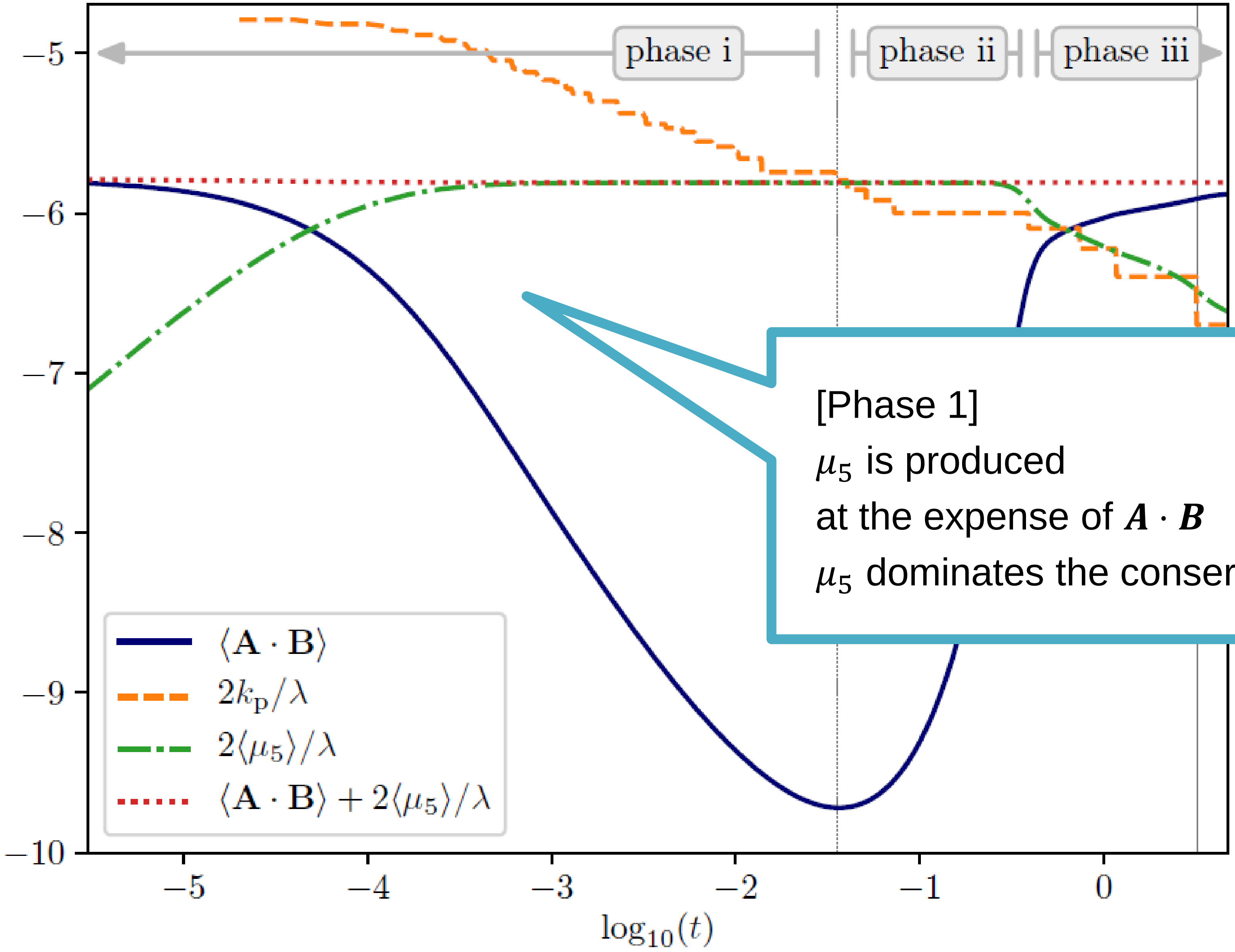
[Hirono et al. 2015]

Low Re_M case



[*Hirono et al. 2015*]

Low Re_M case



[Phase 1]

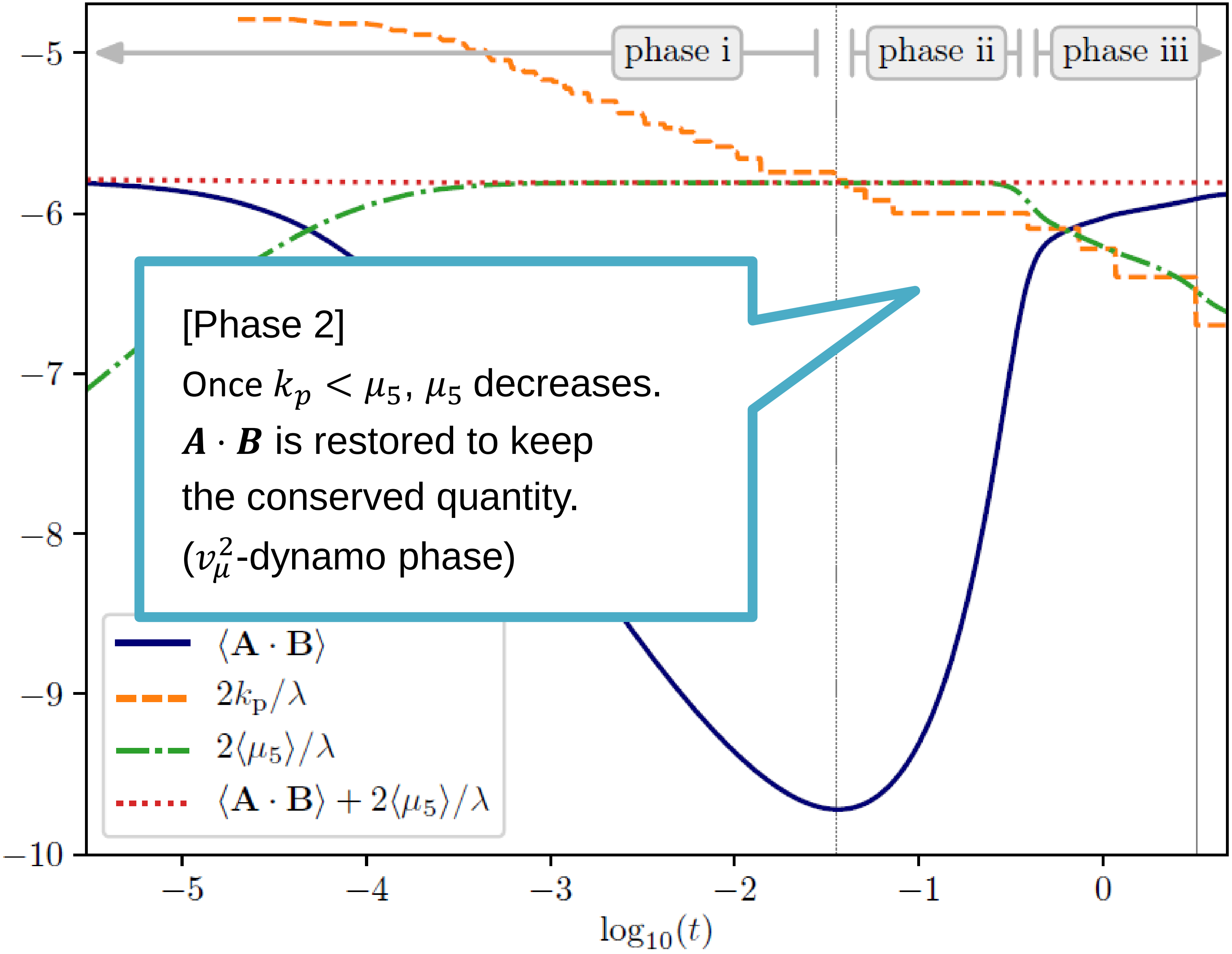
μ_5 is produced

at the expense of $\boldsymbol{A}\cdot\boldsymbol{B}$

μ_5 dominates the conserved #.

[Hirono et al. 2015]

Low Re_M case



[Phase 2]

Once $k_p < \mu_5$, μ_5 decreases.

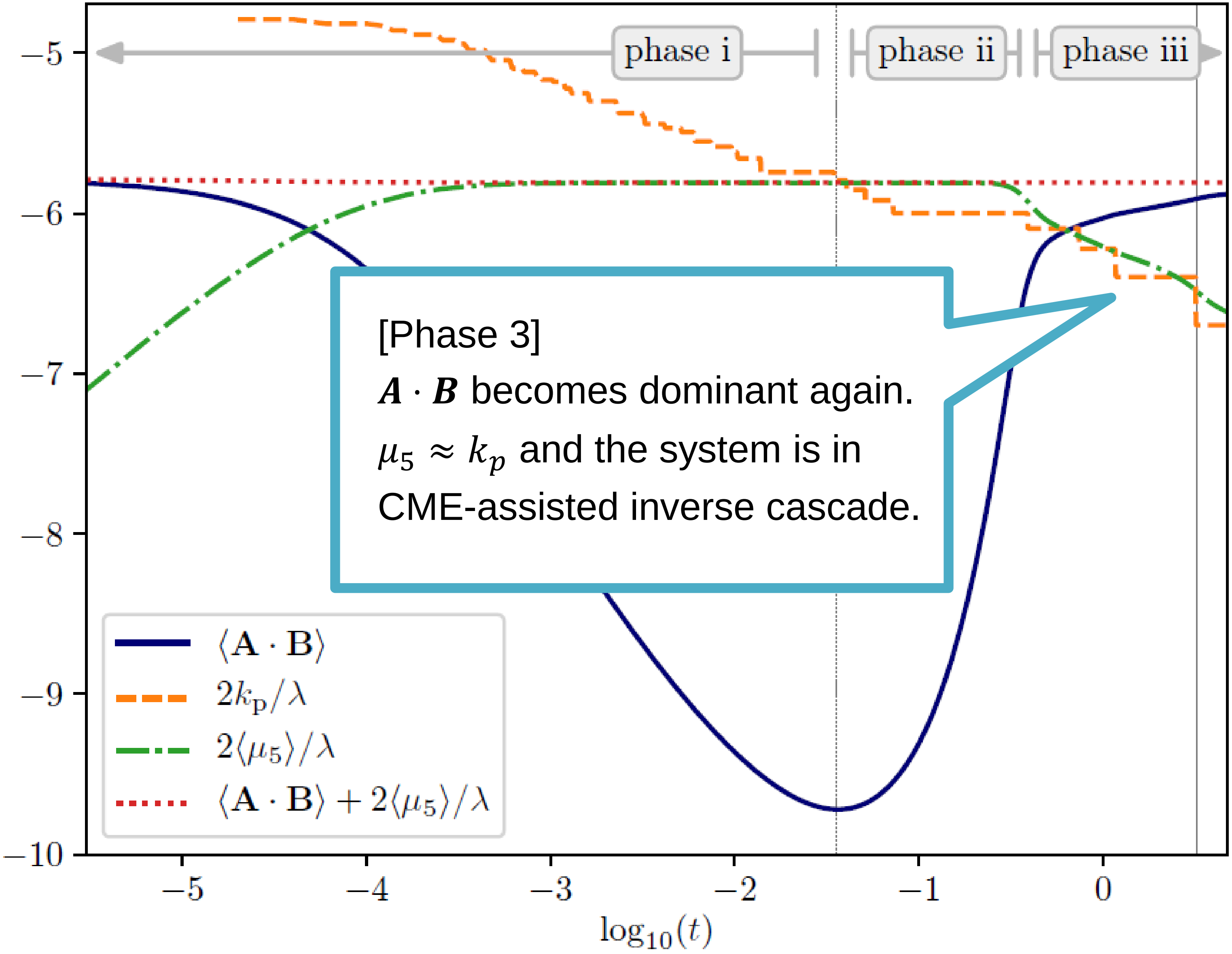
$\boldsymbol{A}\cdot\boldsymbol{B}$ is restored to keep

the conserved quantity.

(v_μ^2-dynamo phase)

[*Hirono et al. 2015*]

Low Re_M case



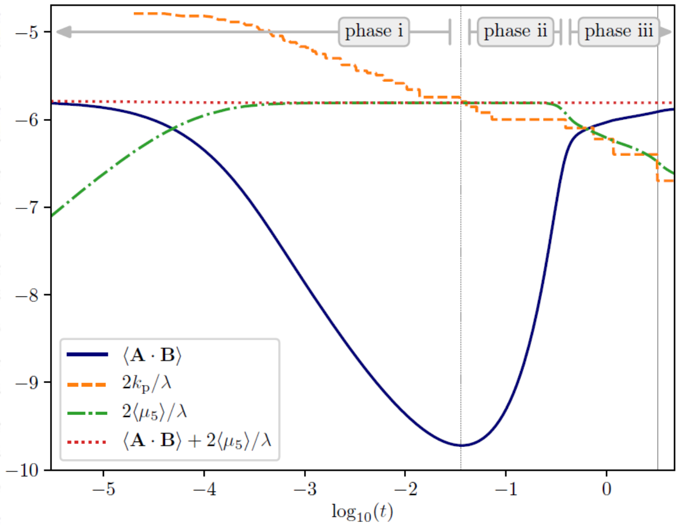
[Phase 3]

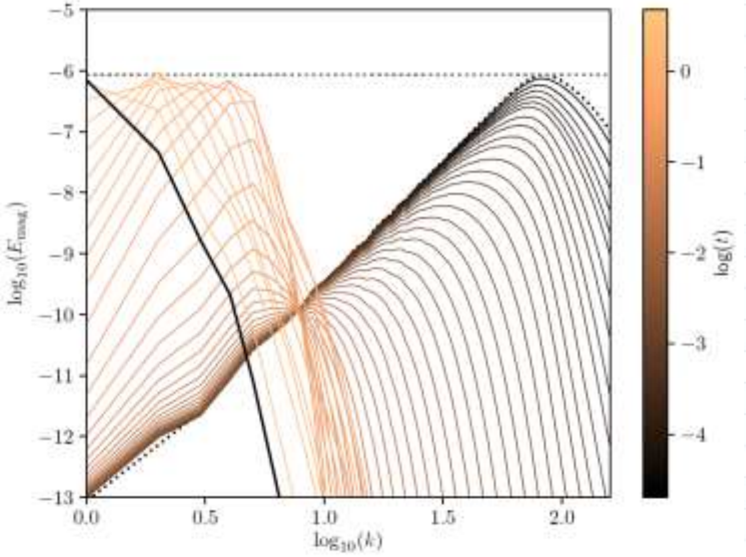
$\boldsymbol{A} \cdot \boldsymbol{B}$ becomes dominant again.

$\mu_5 \approx k_p$ and the system is in CME-assisted inverse cascade.

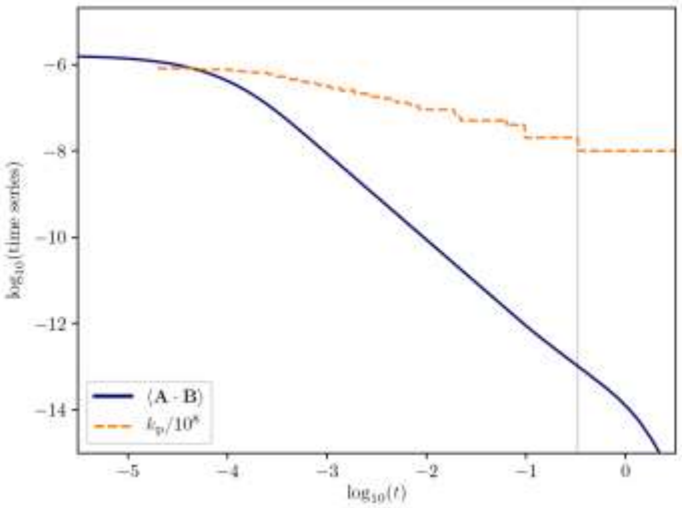
Low Re_M case

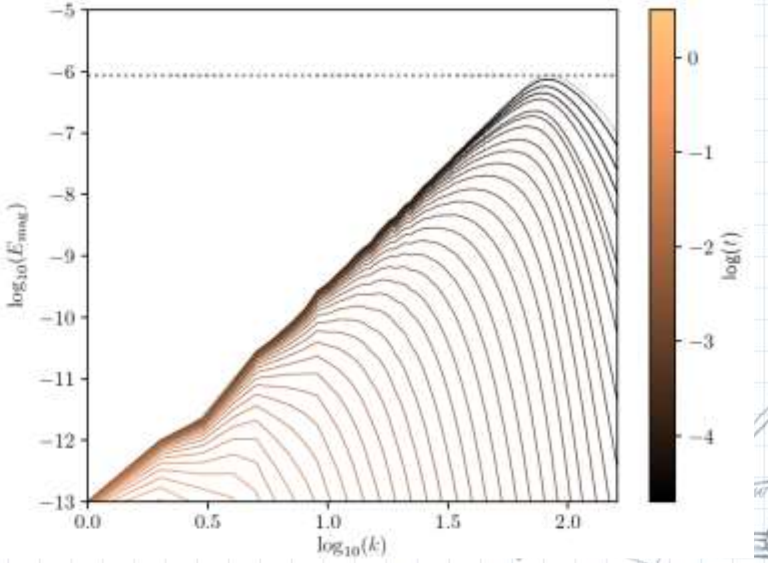
Chiral MHD





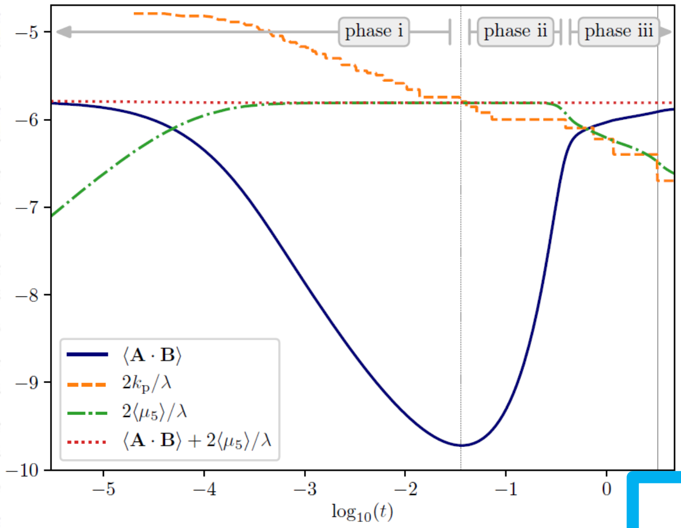
Classical MHD (no μ_5)

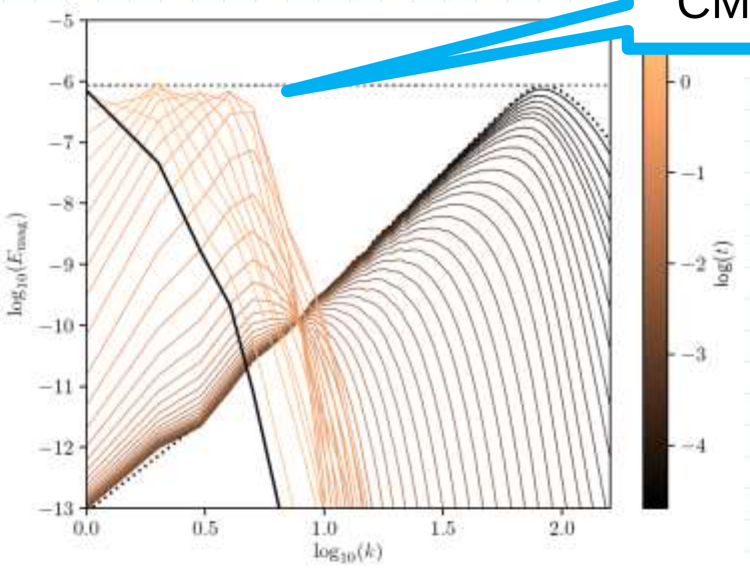




Low Re_M case

Chiral MHD





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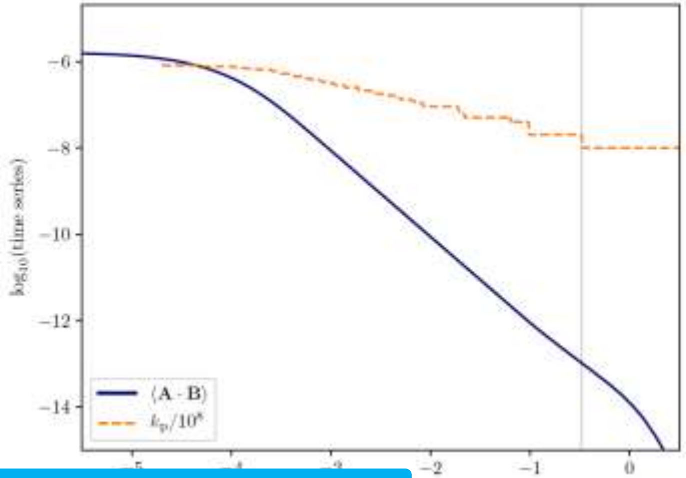
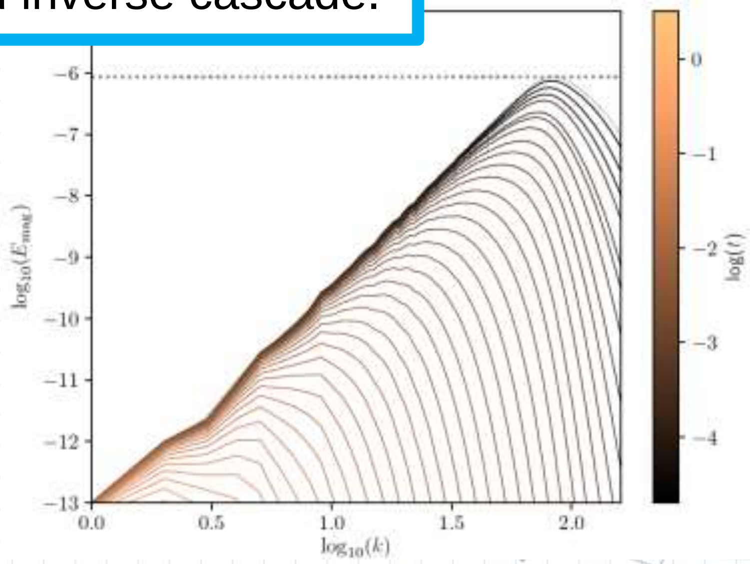




CME-assisted inverse cascade.

Analytic formula for μ_5

- We derive $\mu_5 \approx k_p$ in phase 3.

$$\mu_5(t) \simeq \frac{1}{2\sqrt{\eta t}} \frac{3 + n + \mathcal{C} + (3+n)\ln(t/t_{\mathcal{C}})}{\sqrt{\mathcal{C} + (3+n)\ln(t/t_{\mathcal{C}})}},$$

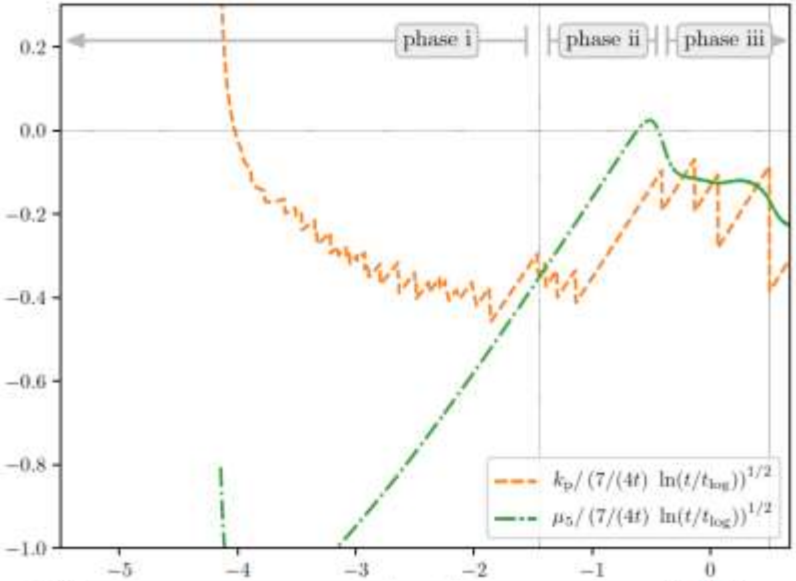
late-time limit → $\left[\frac{3+n}{4\eta t} \ln\left(\frac{t}{t_{\log}}\right)\right]^{\frac{1}{2}}$

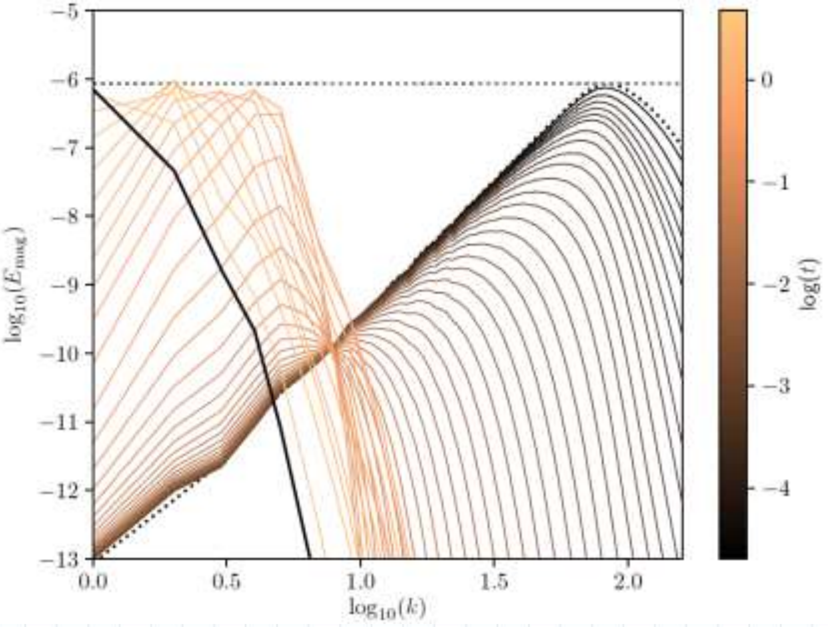
where $(\boldsymbol{A}\cdot\boldsymbol{B})_k \propto k^n$ is assumed

New scaling law $k_p \propto t^{-1/2}$
distinct form classical IC $t^{-2/3}$
with logarithmic correction
further slowing down IC.

$C = (3+n)\ln(t_C/t_{\log})$

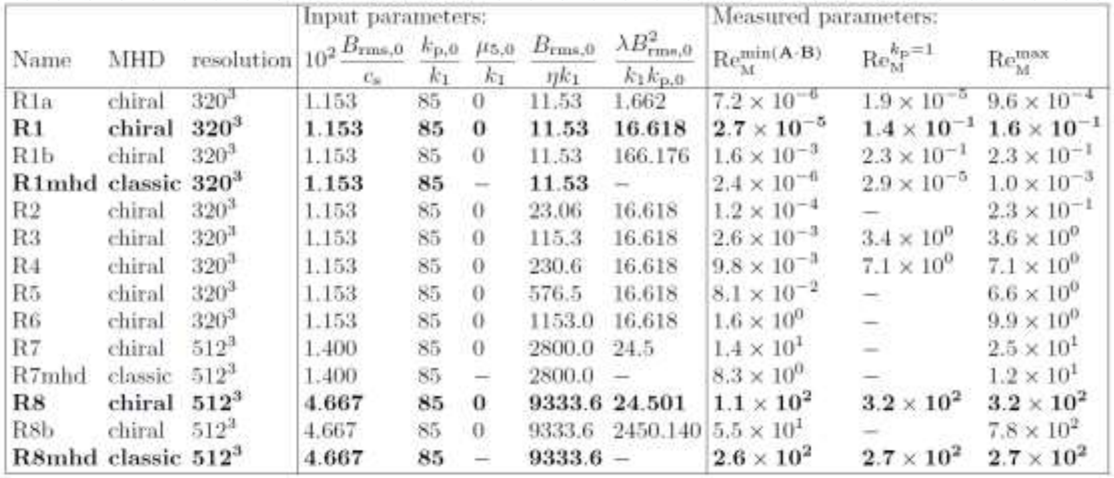
[*Hirono et al. 2015*]





Chiral MHD

DNS with varying Re_M



			Input parameters:					Measured parameters:		
Name	MHD	resolution	$10^2 \frac{B_{\mathrm{rms},0}}{c_s}$	$\frac{k_{\mathrm{p},0}}{k_1}$	$\frac{\mu_{5,0}}{k_1}$	$\frac{B_{\mathrm{rms},0}}{\eta k_1}$	$\frac{\lambda B^2_{\mathrm{rms},0}}{k_1 k_{\mathrm{p},0}}$	$\mathrm{Re}_\mathrm{M}^{\min(\mathbf{A}\cdot\mathbf{B})}$	$\mathrm{Re}_\mathrm{M}^{k_\mathrm{p}=1}$	$\mathrm{Re}_\mathrm{M}^{\max}$
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R1b	chiral	320^3	1.153	85	0	11.53	166.176	1.6×10^{-3}	2.3×10^{-1}	2.3×10^{-1}
R1mhd	**classic**	$\mathbf{320^3}$	**1.153**	**85**	–	**11.53**	–	$\mathbf{2.4 \times 10^{-6}}$	$\mathbf{2.9 \times 10^{-5}}$	$\mathbf{1.0 \times 10^{-3}}$
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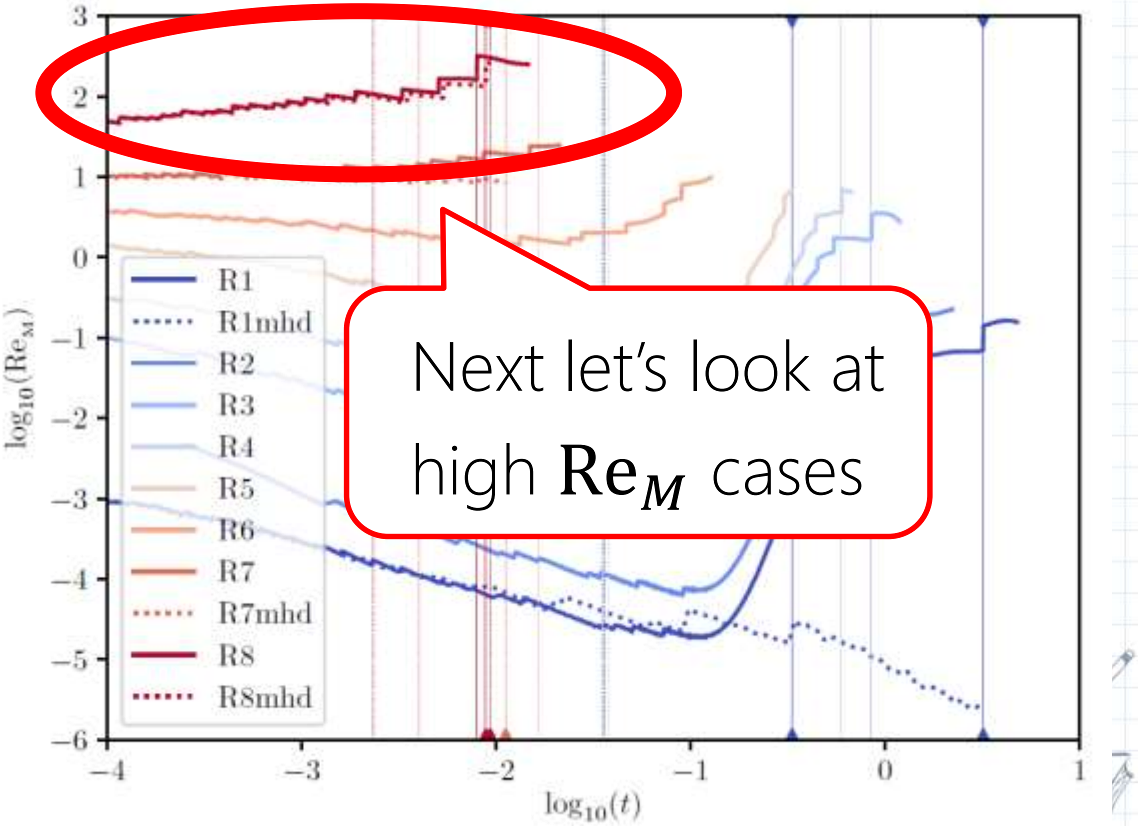
- Change η

  ➡ different Re_M

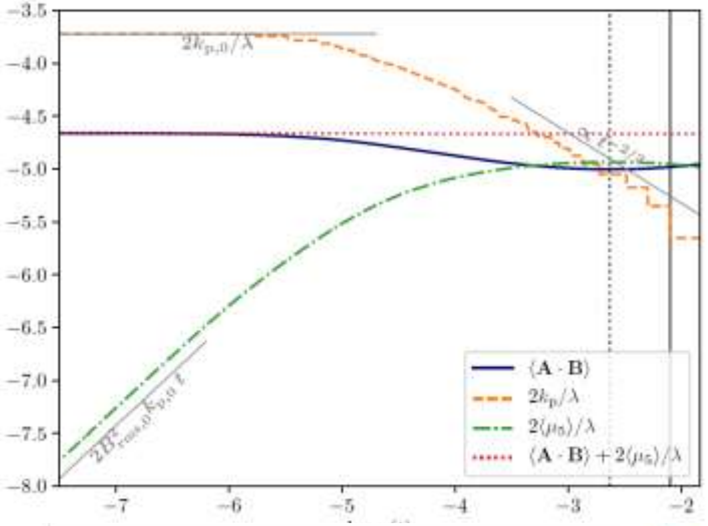
- $\mathrm{Re}_M \lesssim 1$ show distinct evolutions

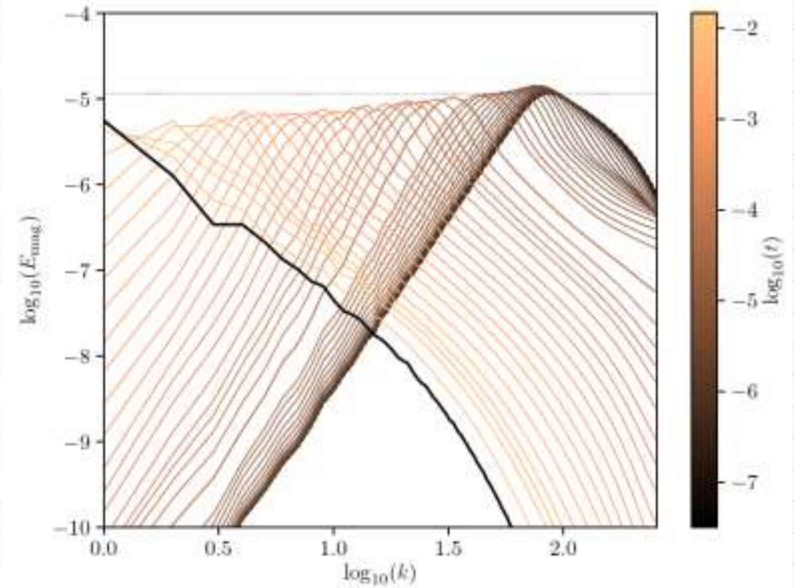


Next let's look at high Re_M cases

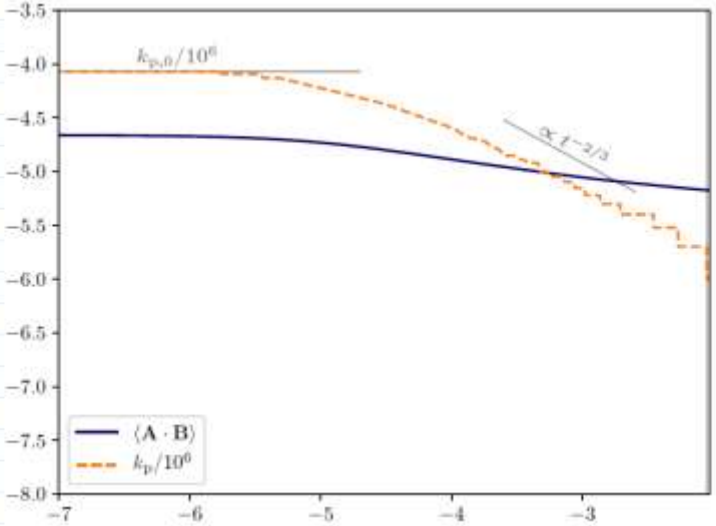
High Re_M case

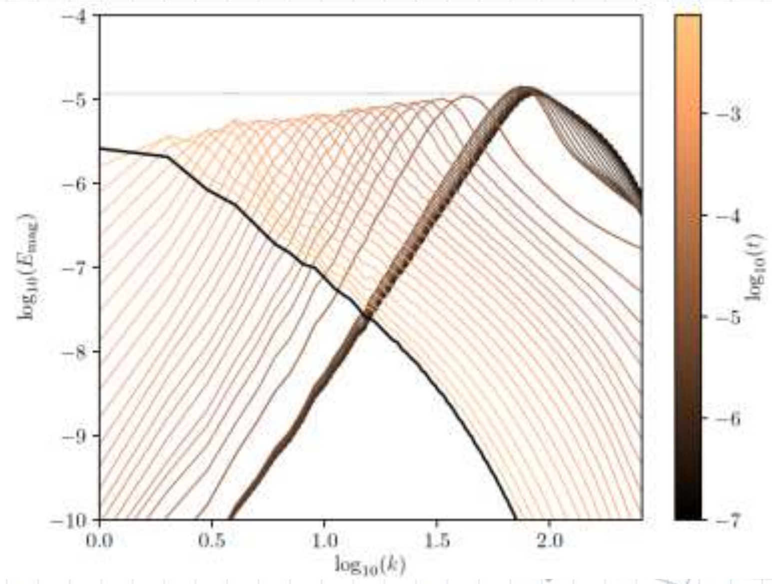
Chiral MHD





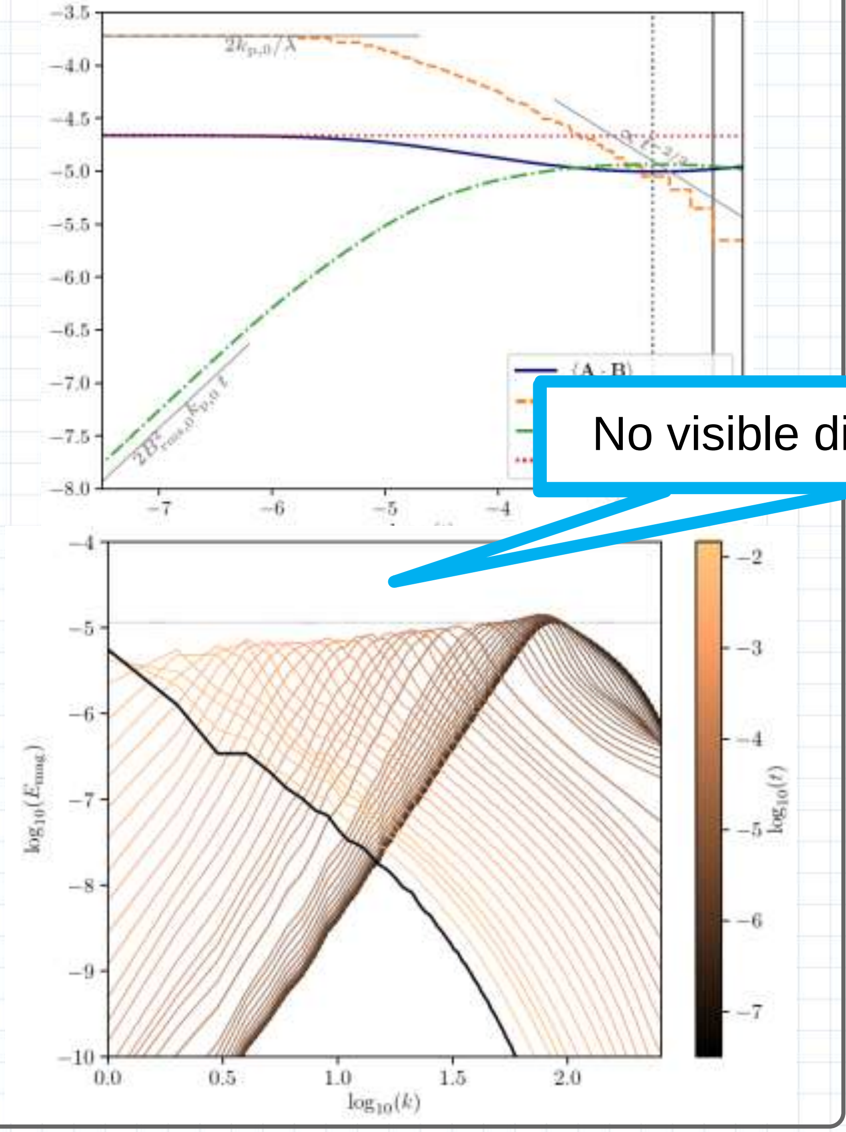
Classical MHD (no μ_5)



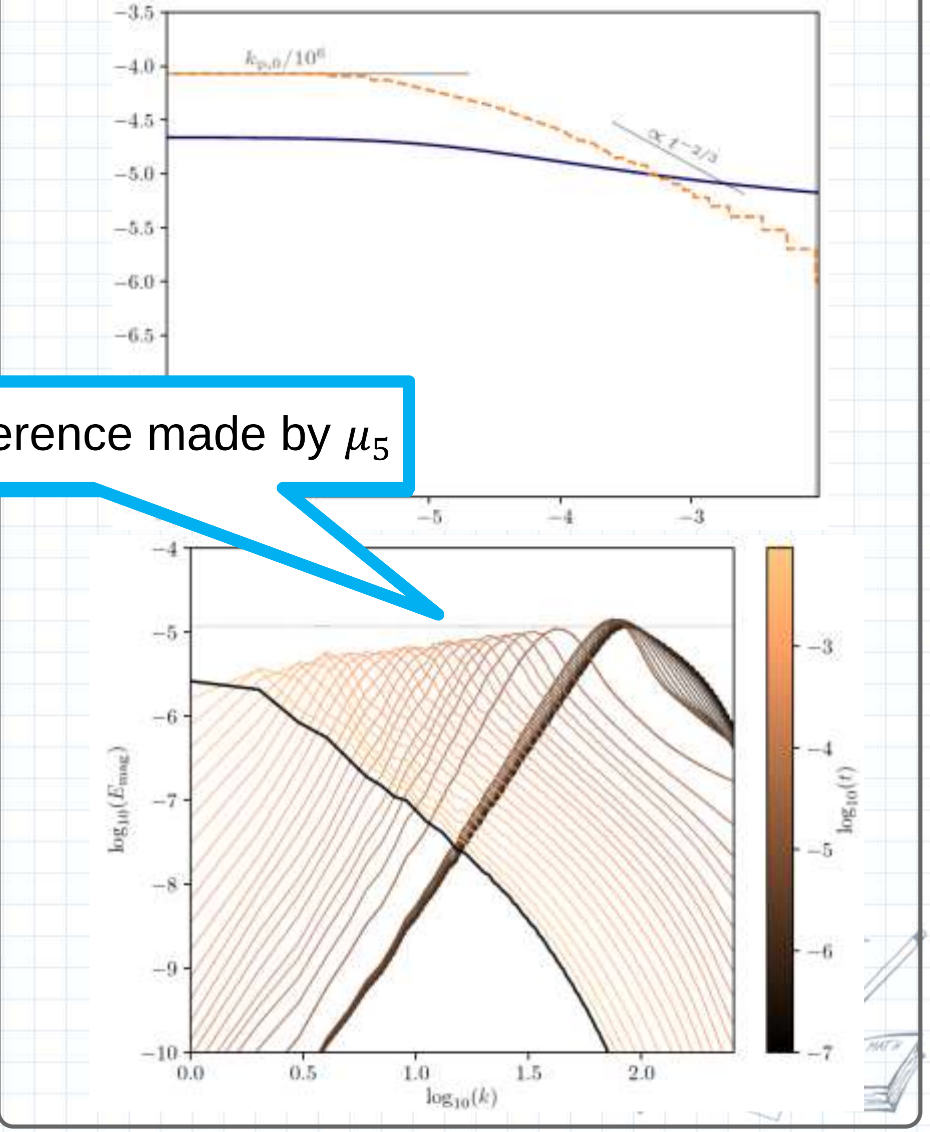


High Re_M case

Chiral MHD

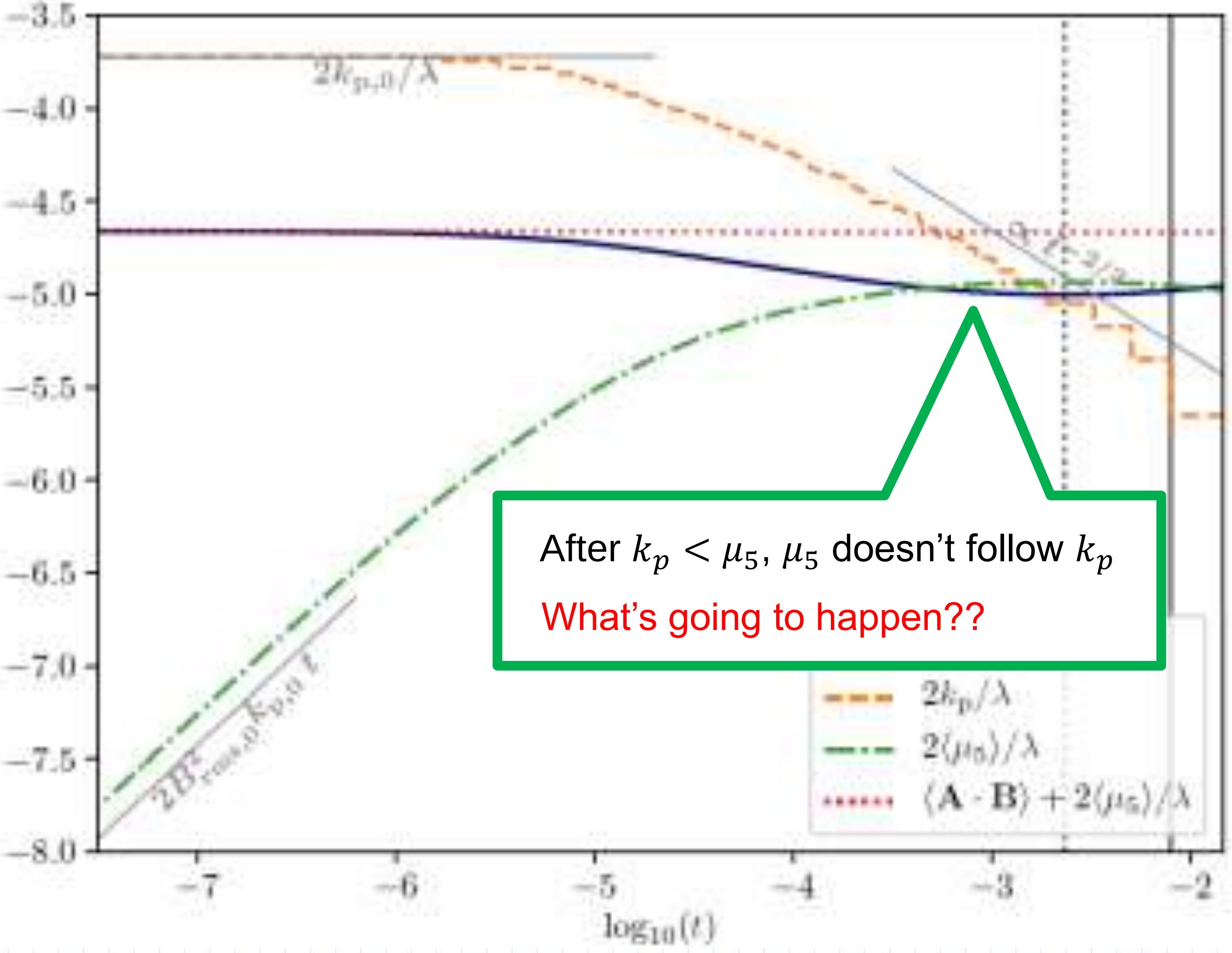


Classical MHD (no μ_5)



No visible difference made by μ_5

High Re_M case



After $k_p < \mu_5$, μ_5 doesn't follow k_p

What's going to happen??

Analytic formula for μ_5

$$\boxed{\frac{D\mu_5}{Dt} = \cancel{\mathcal{D}_5 \Delta\mu_5} + \lambda\eta\left[\underbrace{\boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})}_{\approx\, k_p \boldsymbol{B}^2} - \mu_5\boldsymbol{B}^2\right]}$$

Plug & Solve

- Simple model of classical inverse cascade

$$k_{\mathrm{p}}(t) = \begin{cases} k_{\mathrm{p},0} & (t \le t_{\mathrm{I}}) \\ k_{\mathrm{p},0}(t/t_{\mathrm{I}})^{-2/3} & (t_{\mathrm{I}} \le t) \end{cases} \qquad B(t) = \begin{cases} B_0 & (t \le t_{\mathrm{I}}) \\ B_0(t/t_{\mathrm{I}})^{-1/3} & (t_{\mathrm{I}} \le t) \end{cases}$$

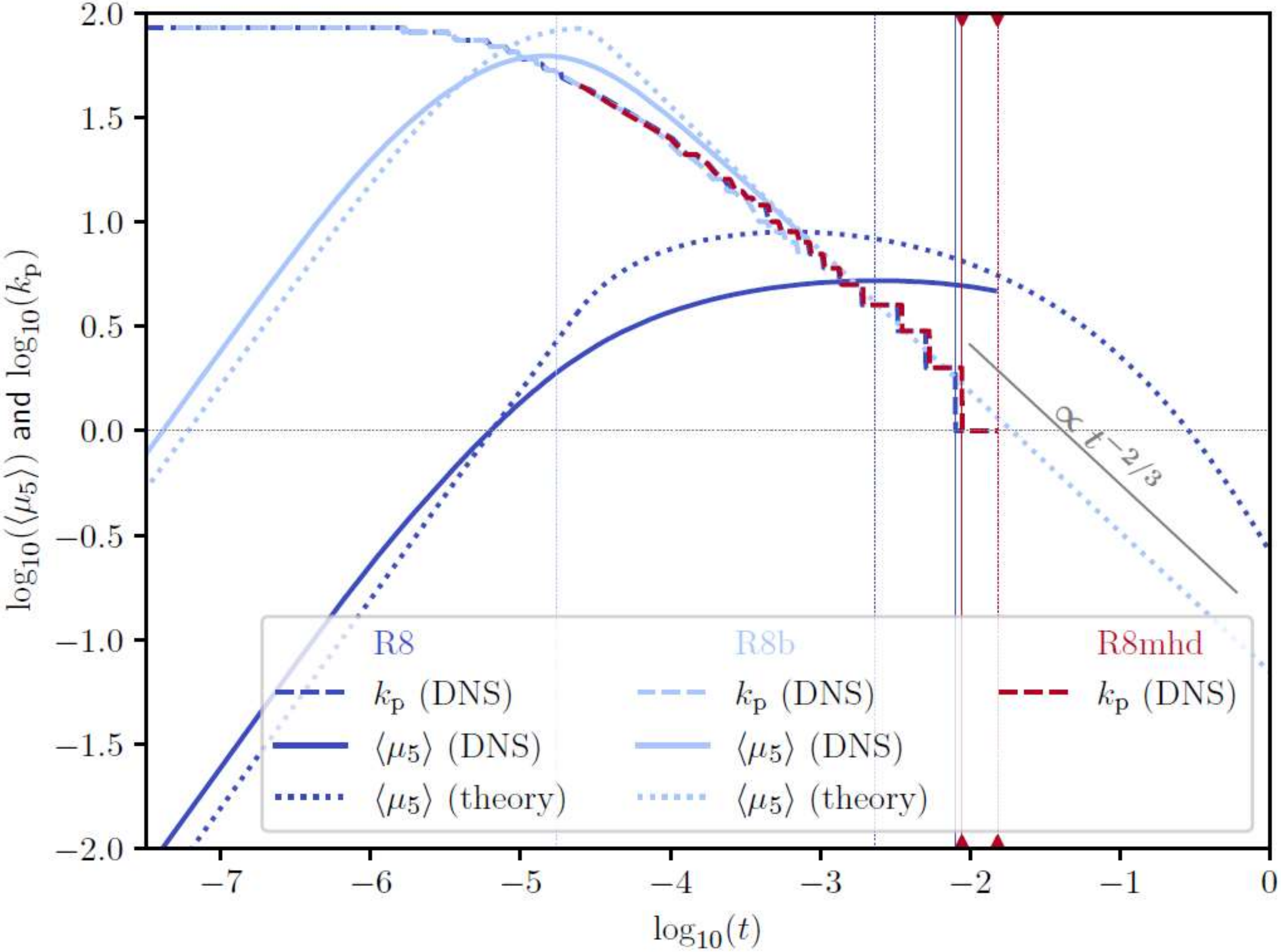
- Solution

$$\begin{aligned} \mu_5(t \ge t_{\mathrm{I}}) = k_{\mathrm{p},0}\Big[& -3\xi(t/t_{\mathrm{I}})^{-1/3} \\ & + e^{-3\xi(t/t_{\mathrm{I}})^{1/3}}\Big\{ \left(e^{\xi}(1+3\xi)-1\right)e^{2\xi} \\ & + 9\xi^2\left(\mathrm{Ei}(3\xi(t/t_{\mathrm{I}})^{1/3}) - \mathrm{Ei}(3\xi)\right)\Big\}\Big], \end{aligned}$$

$$\xrightarrow{\text{late-time limit}} \quad \boxed{\mu_5(t) \approx k_{\mathrm{p}}(t)}$$

$$\xi \equiv \lambda\eta B_0^2 t_{\mathrm{I}}$$

High Re_M case



Summary

- PMF may serve as an unique probe of the early universe and fundamental physics
- A model of inflationary magnetogenesis predicts maximally helical MF → its evolution?
- Studying Chiral MHD w/ initial helical MF, we found two distinct MF evolutions

$$\boxed{k_p \approx \mu_5 \propto \begin{cases} t^{-1/2}\log(t/t_{\log}) & (\mathrm{Re}_M \ll 1) \\ t^{-2/3} & (\mathrm{Re}_M \gg 1) \end{cases}, \qquad B \propto k_p^{1/2}}$$

Fin

Thank you so much!