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Low-scale flavon model with a Z_N flavor symmetry

Tetsutaro Higaki

(Keio U)

Keio University

CALAMVS GLADIO FORTIOR

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In collaboration with **Junichiro Kawamura** (Ohio state U, USA)

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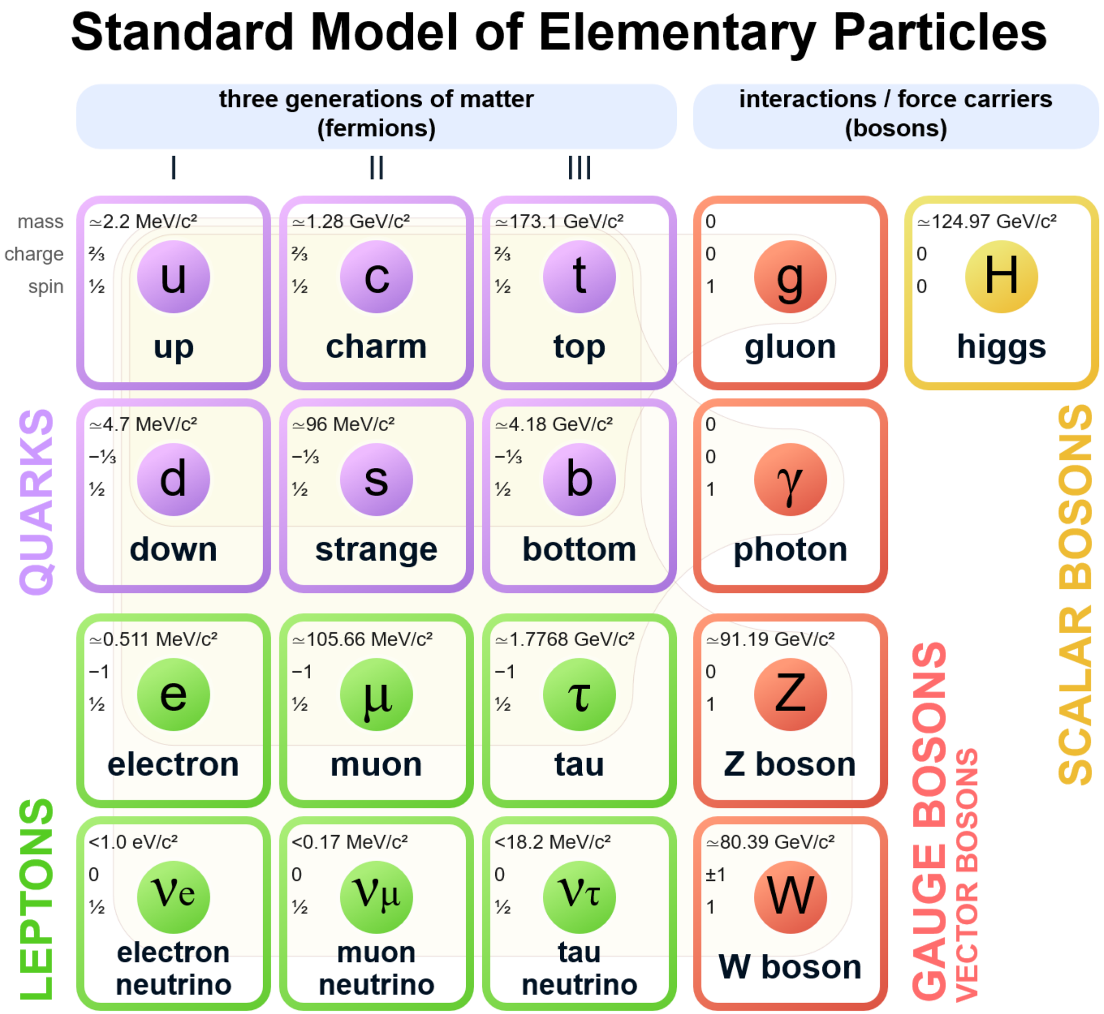
Froggatt-Nielsen (FN) mechanism with a Z_N flavor symmetry focused.

∵ $U(1)_{\mathrm{FN}}$ is anomalous typically.
Such models are less discussed.

Flavor mass hierarchy in the Standard Model



Why?

$$\frac{m_\tau}{m_e} \sim 3 \times 10^3$$

$$\frac{m_t}{m_u} \sim 3 \times 10^4$$

$$\frac{m^2_{\text{atomo}}}{m^2_{\text{solar}}} \sim 3 \times 10^{-2}$$

Wikipedia

Flavor mass hierarchy = Yukawa hierarchy

- Yukawa coupling y_{ij} gives mass of an elementary particle via spontaneous electroweak symmetry breaking:

$$\mathcal{L}_{\text{yukawa}} = y_{ij}\,\overline{U_i} Q_j H \quad \rightarrow \quad y_{ij}\langle H\rangle \overline{U_i} Q_j \equiv m_{ij} \overline{U_i} Q_j$$

$$\langle H\rangle \neq 0$$

$m_{ij} = y_{ij}\langle H\rangle$: mass of a particle.

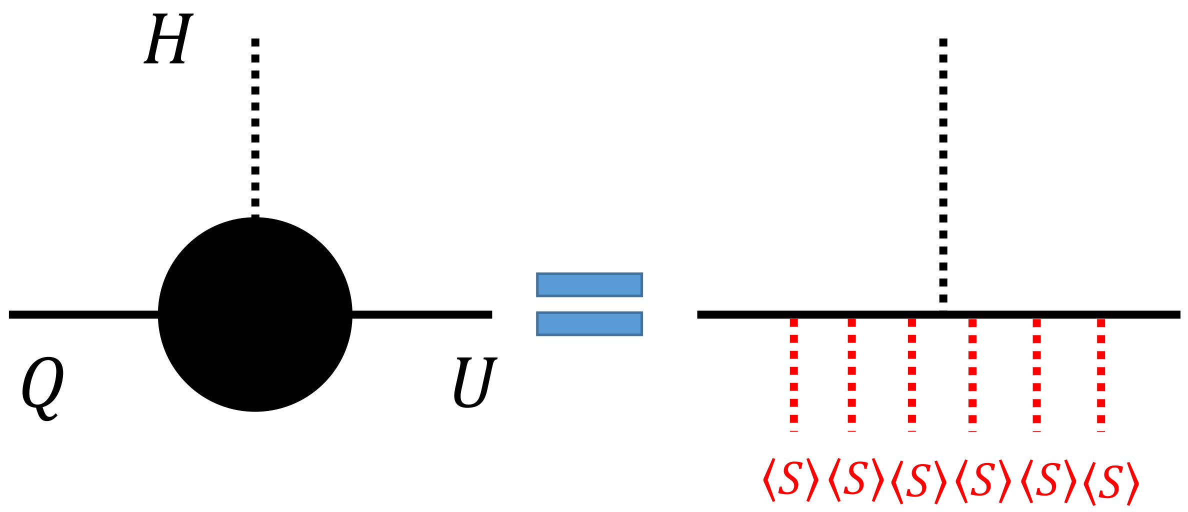
Hierarchy: $$\frac{m_j}{m_i} = \frac{y_j}{y_i} \ll 1$$

Hierarchal Yukawa obtained by the Froggatt-Nielsen (FN) mechanism

- FN mechanism can explain hierarchy with natural parameter choices

[Froggatt-Nielsen]

$$\mathcal{L}_{\text{yukawa}} = \left(\frac{S}{\Lambda}\right)^{n_{ij}} \overline{U_i} Q_j H, \qquad \Lambda\text{: cutoff scale,} \qquad n_{ij} > 0.$$



$$y_{ij} = \left(\frac{\langle S \rangle}{\Lambda}\right)^{n_{ij}}$$

The more S in y_{ij},

the smaller Yukawa for $\frac{\langle S \rangle}{\Lambda} < 1$.

Is FN mechanism controlled by $U(1)_{\mathrm{FN}}$?

- Yukawa coupling is invariant under a $U(1)_{\mathrm{FN}}$:

$$\mathcal{L}_{\mathrm{yukawa}} = \left(\frac{S}{\Lambda}\right)^{n_{ij}} \overline{U_i} Q_j H$$

$$U(1)_{\mathrm{FN}}:\ \overline{U_i} \to e^{\mathrm{i}\theta n_{Ui}} \overline{U_i},\quad Q_j \to e^{\mathrm{i}\theta n_{Qj}} Q_j,\quad H \to e^{\mathrm{i}\theta n_H} H,\quad S \to e^{\mathrm{i}\theta n_S} S;$$

$$n_{U_i} + n_{Qj} + n_H + n_{ij} \cdot n_S = 0.$$

- Chiral $U(1)_{\mathrm{FN}}$ can be anomalous $\rightarrow$ Z_N flavor symmetry!

  Cf. DW problem of QCD axion, discrete gauge symmetry in string model

[Sikivie], [Berasaluce-Gonzalez et al]: See backup.

SUSY FN mechanism with Z_4 instead of U(1)

- Model: MSSM with R-parity + singlet flavon S

$$W_{Z_N} = \frac{c_N}{4\ \Lambda} S^4 + \frac{c_m}{m\Lambda^{m-1}} S^m H_u H_d + W_{\text{fermion}}$$

$$W_{\text{Fermion}} = c^u_{ij} \left(\frac{S}{\Lambda}\right)^{\eta^u_{ij}} \overline{u}_{R_i} Q_{L_j} H_u + c^d_{ij} \left(\frac{S}{\Lambda}\right)^{\eta^d_{ij}} \overline{d}_{R_i} Q_{L_j} H_d$$
$$+ \ c^e_{ij} \left(\frac{S}{\Lambda}\right)^{\eta^e_{ij}} \overline{e}_{R_i} L_{L_j} H_d + c^n_{ij} \left(\frac{S}{\Lambda}\right)^{\eta^n_{ij}} \overline{N}_{R_i} L_{L_j} H_u + \frac{1}{2} M_{ij} \overline{N}_{R_i} \overline{N}_{R_j}$$

- Z_4 invariance modulo 4 with $S \to e^{i\pi/2} S,\quad \Phi_{\text{MSSM}} \to e^{i\pi n_\Phi/2} \Phi_{\text{MSSM}}$:

$$-\eta^u_{ij} \equiv n_{H_u} + n_{u_i} + n_{Q_j}, \qquad -\eta^d_{ij} \equiv n_{H_d} + n_{d_i} + n_{Q_j}$$
$$-\eta^e_{ij} \equiv n_{H_d} + n_{e_i} + n_{L_j}, \qquad -\eta^n_{ij} \equiv n_{H_u} + n_{n_i} + n_{L_j}$$

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Assumption: N_R has charge 2

- Yukawa coupling structure given by :

$$Y_{ij} = c_{ij}\,\epsilon^{\eta_{ij}}, \qquad \epsilon := \frac{\langle S \rangle}{\Lambda} \qquad c_{ij} = O(1).$$

1^{st} difference between Z_N and $U(1)$

- For Z_4 $\epsilon^{N-1} = \epsilon^3$ is the smallest Yukawa; model variety is limited

$$\epsilon^3 \sim \frac{m_u}{m_t} = 7.5 \times 10^{-6} \quad \rightarrow \quad \epsilon \sim 0.02.$$

$\because$ ϵ^N coupling exists $\leftrightarrow$ $O(1)$ coupling exists.

- Realistic flavor structure with $k = 0{,}1$ (d, e) & $l = 0{,}1{,}2{,}3$ (ν)

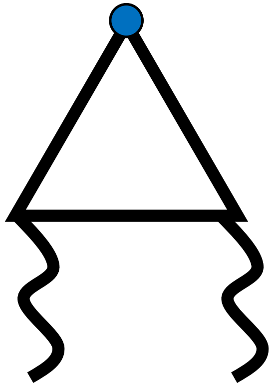
$$(m_u, m_c, m_t) \sim (\epsilon^3, \epsilon, 1), \quad (m_d, m_s, m_b) \sim \epsilon^k (\epsilon^2, \epsilon, 1), \quad (m_e, m_\mu, m_\tau) \sim \epsilon^k (\epsilon^2, 1, 1)$$

$$Y_u \sim \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^2 \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad Y_d \sim \epsilon^k \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad Y_e \sim \epsilon^k \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad Y_n \sim \epsilon^l \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}; \; V_{\rm CKM} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad V_{\rm PMNS} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

2nd difference between Z_N and $U(1)$

- Vanishing anomaly between Z_4 and the SM/gravity

Z_4 charge



$$\frac{2\pi}{4}\mathcal{A}F\tilde{F}$$

$$\downarrow$$

$$S = \frac{2\pi}{4}k\cdot\mathcal{A}$$

$SU(3)_C$	$SU(3)_C$
$SU(2)_L$	$SU(2)_L$
$U(1)_Y$	$U(1)_Y$
gravity	gravity

\+ Z_4 invariance for realistic hierarchy (modulo 4):

$$n_{H_u} + n_{H_d} \equiv 3(1+k), \quad n_{Q_3} + 3n_{L_3} \equiv 3+k, \quad n_{u_3} \equiv 3(n_{H_u} + n_{Q_3}),$$
$$n_{d_3} \equiv -k + 3(n_{H_d} + n_{Q_3}), \quad n_{e_3} \equiv 3(1 + n_{H_d} + n_{Q_3}), \quad 2 - \ell \equiv n_{H_u} + n_{L_3}.$$

$$m + n_{H_u} + n_{H_d} = m + 3(1+k) \equiv 0$$

$$k = 0 \leftrightarrow m = 1$$
$$k = 1 \leftrightarrow m = 2$$

24 choices

$$\frac{m_b}{m_t} \sim \frac{m_\tau}{m_t} \sim \epsilon^k \cot\beta \qquad \Longleftrightarrow \qquad W \ni \left(\frac{S}{\Lambda}\right)^m \Lambda H_u H_d.$$

Flavon-Higgs mixing

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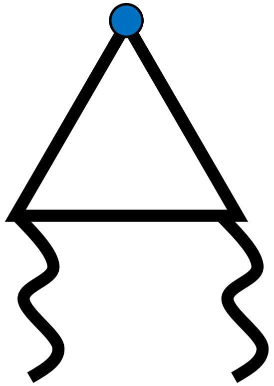

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$$m + n_{H_u} + n_{H_d} = m + 3(1+k) \equiv 0$$

Vacuum stability kills $k = 0$ $(m = 1)$.
(the potential minimal = the EW vacuum)

$$k = 1 \leftrightarrow m = 2$$

12 choices

See parameter space for k=1 (m=2) in backup.

$$\frac{m_b}{m_t} \sim \frac{m_\tau}{m_t} \sim \epsilon \cot\beta \quad \Longleftrightarrow \quad W \ni \frac{S^2}{\Lambda} H_u H_d.$$

Flavon-Higgs mixing

12 choices of charge assignments consistent with anomalies

k	n_{H_u}	n_{Q_3}	ℓ	m	$\tilde{\mathcal{A}}_Y$	$\mathcal{A}_{\text{gr}}$
1	0	0	2	2	1	0
1	0	1	1	2	3	1
1	0	2	0	2	1	0
1	0	3	3	2	3	1
1	1	0	1	2	1	1
1	1	1	0	2	3	0
1	1	2	3	2	1	1
1	1	3	2	2	3	0
1	2	0	0	2	1	0
1	2	1	3	2	3	1
1	2	2	2	2	1	0
1	2	3	1	2	3	1
1	3	0	3	2	1	1
1	3	1	2	2	3	0
1	3	2	1	2	1	1
1	3	3	0	2	3	0

If $U(1)_Y$ is embedded into $U(12n)$, no $Z_4 - U(1)_Y^2$ anomaly exists.

A numerical example for observables

$$(m_u, m_c, m_t) = (0.001288, 0.6268, 171.7), \quad (m_d, m_s, m_b) = (0.002751, 0.05432, 2.853),$$
$$(m_e, m_\mu, m_\tau) = (0.0004866, 0.1027, 1.746), \quad (\alpha_{\mathrm{CKM}}, \sin 2\beta_{\mathrm{CKM}}, \gamma_{\mathrm{CKM}}) = (1.518, 0.6950, 1.240),$$

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.974461 & 0.224529 & 0.00364284 \\ 0.224379 & 0.97359 & 0.0421456 \\ 0.00896391 & 0.0413421 & 0.999105 \end{pmatrix}$$

Neutrino & PMNS:

$$\Delta m^2_{12} = 7.37 \times 10^{-5}, \quad \Delta m^2_{23} = 2.56 \times 10^{-3},$$
$$\sin^2\theta_{12} = 0.297, \quad \sin^2\theta_{23} = 0.425, \quad \sin^2\theta_{13} = 0.0215.$$

Cutoff scale

$$\Lambda \sim 500\ \mathrm{TeV} \times \left(\frac{0.02}{\epsilon}\right)\left(\frac{v_s}{10\ \mathrm{TeV}}\right).$$

A numerical example for couplings/N_R masses

$$c^u = \begin{pmatrix} -2.23656 & -3.78792 & 5.07947 \cdot e^{-2.23037i} \\ -1.8029 & 1.51612 & -0.62796 \\ 2.43468 \cdot e^{0.019714i} & -2.11793 & 0.782311 \end{pmatrix}, \qquad c^e = \begin{pmatrix} -1.83414 & -4.06715 & -4.55088 \\ 0.814655 & -1.04839 & -1.16518 \\ -0.702312 & 1.27439 & 1.27222 \end{pmatrix}$$

$$c^d = \begin{pmatrix} 7.11034 & 4.75778 & 4.38956 \cdot e^{-1.64741i} \\ 6.74255 & -5.32201 & 3.39087 \\ 2.85434 \cdot e^{2.96002i} & -0.578767 & -2.59023 \end{pmatrix}, \qquad c^n = \begin{pmatrix} 3.63525 & -4.36595 & -4.00992 \\ -5.94856 & -2.38206 & 3.74011 \\ -2.19846 & -1.4343 & 0.589928 \end{pmatrix},$$

$$M = M_0 \begin{pmatrix} -6.07582 & 2.75669 & 4.32291 \\ 2.75669 & -4.43903 & 1.68412 \\ 4.32291 & 1.68412 & 5.09895 \end{pmatrix}$$

$\tan\beta = 5$. With $M_0 = 33.1474$ TeV and $\ell = 3$,

Suppressed flavon coupling to the SM fermion

- Coupling of flavon $S = \sigma + ia$ to the SM fermions f

$$\mathcal{L} \sim \hat{\lambda}^f S \bar{f}_R f_L$$

$$\hat{\lambda}^{u,S} \sim \rho_u \frac{v_u}{v_s} \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & 1 \end{pmatrix}, \quad \hat{\lambda}^{d,S} \sim \rho_d \frac{v_d}{v_s} \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon \end{pmatrix}, \quad \hat{\lambda}^{e,S} \sim \rho_e \frac{v_d}{v_s} \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^5 & \epsilon & \epsilon^5 \\ \epsilon^5 & \epsilon^5 & \epsilon \end{pmatrix}$$

suppressed by $\Gamma_{ij} = \frac{\langle H \rangle}{\langle S \rangle} \eta_{ij} Y_{ij}$ from $W = \left(\frac{S}{\Lambda}\right)^{\eta_{ij}} H \Phi_i \Phi_j$

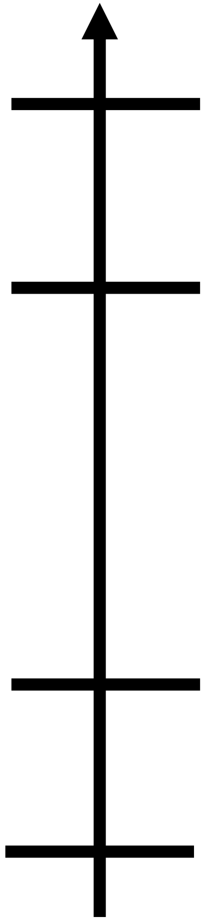
and alignment in diagonalizing fermion mass (off-diagonal element).

- Flavino: heavier than a/higgsino DM, and coupled to the MSSM via Γ_{ij}.

Energy scales in a model

Flavor :

$$\epsilon^3 \sim \frac{m_u}{m_t} \quad \rightarrow \quad \epsilon \sim 0.02.$$



$$\Lambda \sim \frac{\langle S \rangle}{\epsilon} \sim 1 \text{ PeV}$$

$$\langle S \rangle \sim 10 \text{ TeV} \sim m_{\text{SUSY}}$$

(Heavy SUSY assumed)

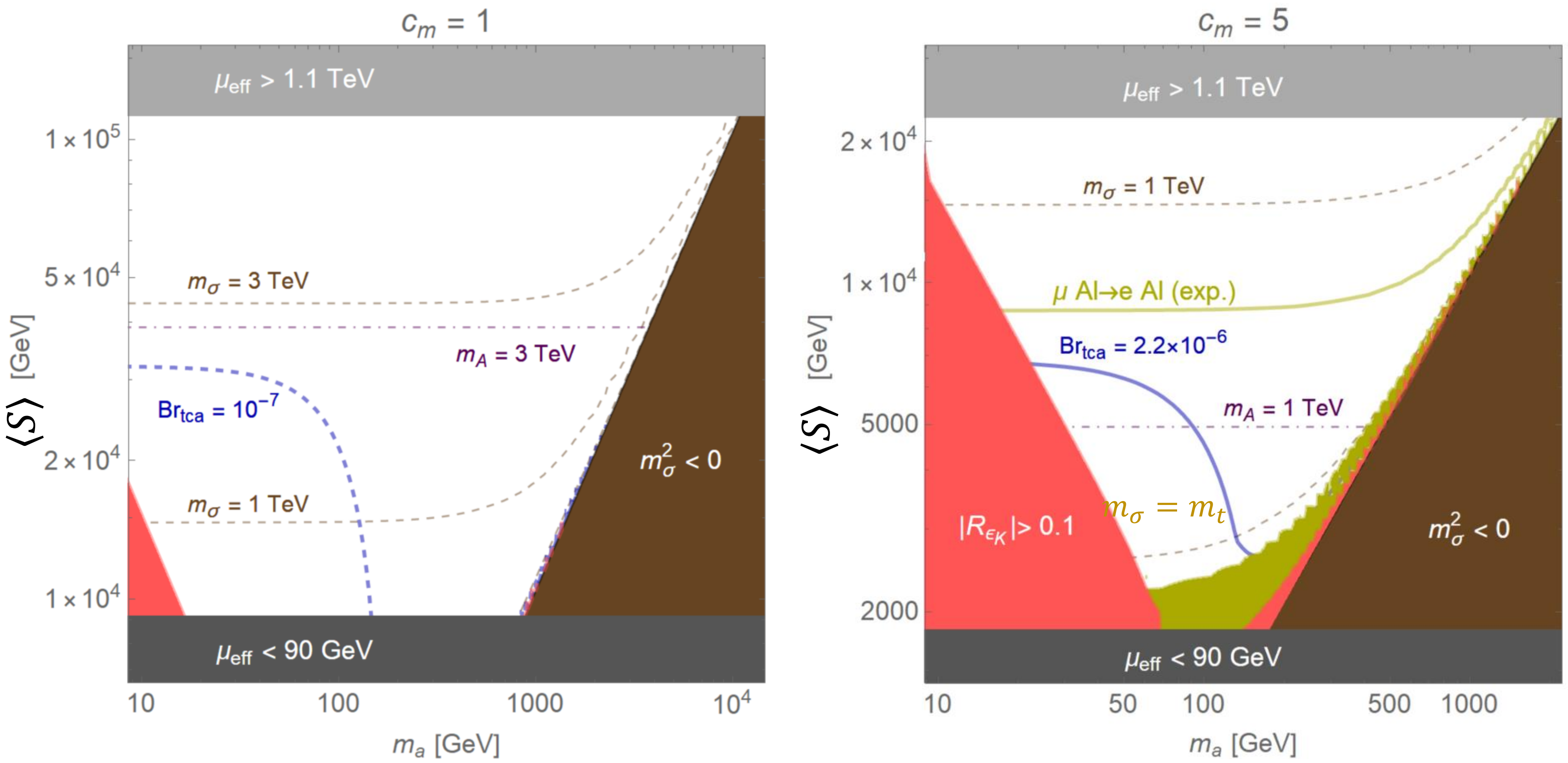
$$m_{\tilde{S}} \sim \epsilon \langle S \rangle$$

$$m_\sigma \sim \epsilon \langle S \rangle$$

$$\mu_{\text{higgsino}} \sim \epsilon \langle S \rangle \sim 1 \text{ TeV}$$ (LSP assumed)

$$m_a \sim 100 \text{ GeV}$$

Flavon constraint on model with Z_4 symmetry



- Colored region excluded by:

Red: $K - \overline{K}$

Yellow: $\mu \to e$ with Gold

Brown: tachyonic flavon σ

- (Right) testable:

1. $\mu \to e$ with Alminium: $6 \times 10^{-17} < \mathrm{Br} < 7 \times 10^{-13}$
2. $t \to ca$ at 100TeV collider

$W \sim c_m \frac{S^2}{\Lambda} H_u H_d$; larger c_m = larger S coupling to the fermions via scalar mixing.

Summary

- FN mechanism with Z_N flavor symmetry considered.

FN flavor symmetry can be discrete rather than continuous.

- A viable model constructed for Z_4 flavor symmetry.

(The Kähler potential can be included in the model. See backup or paper.)

Hierarchy bound: ϵ^3, discrete anomaly constraints, vacuum stability.

- Suppressed flavon coupling to the SM fermions.

Model consistent with current experiments & testable in future.

Back up

Ex: Discrete symmetry from $U(1)$ via Anomaly

- QCD axion with a global U(1)_PQ symmetry:

$$N\frac{a}{f}G_{\mu\nu}\tilde{G}^{\mu\nu} \quad \text{or} \quad V(a) \sim \cos\left(N\frac{a}{f}\right)$$

$$U(1)_{\text{PQ}} \rightarrow Z_N:\ \delta a = \frac{2\pi}{N}f.$$

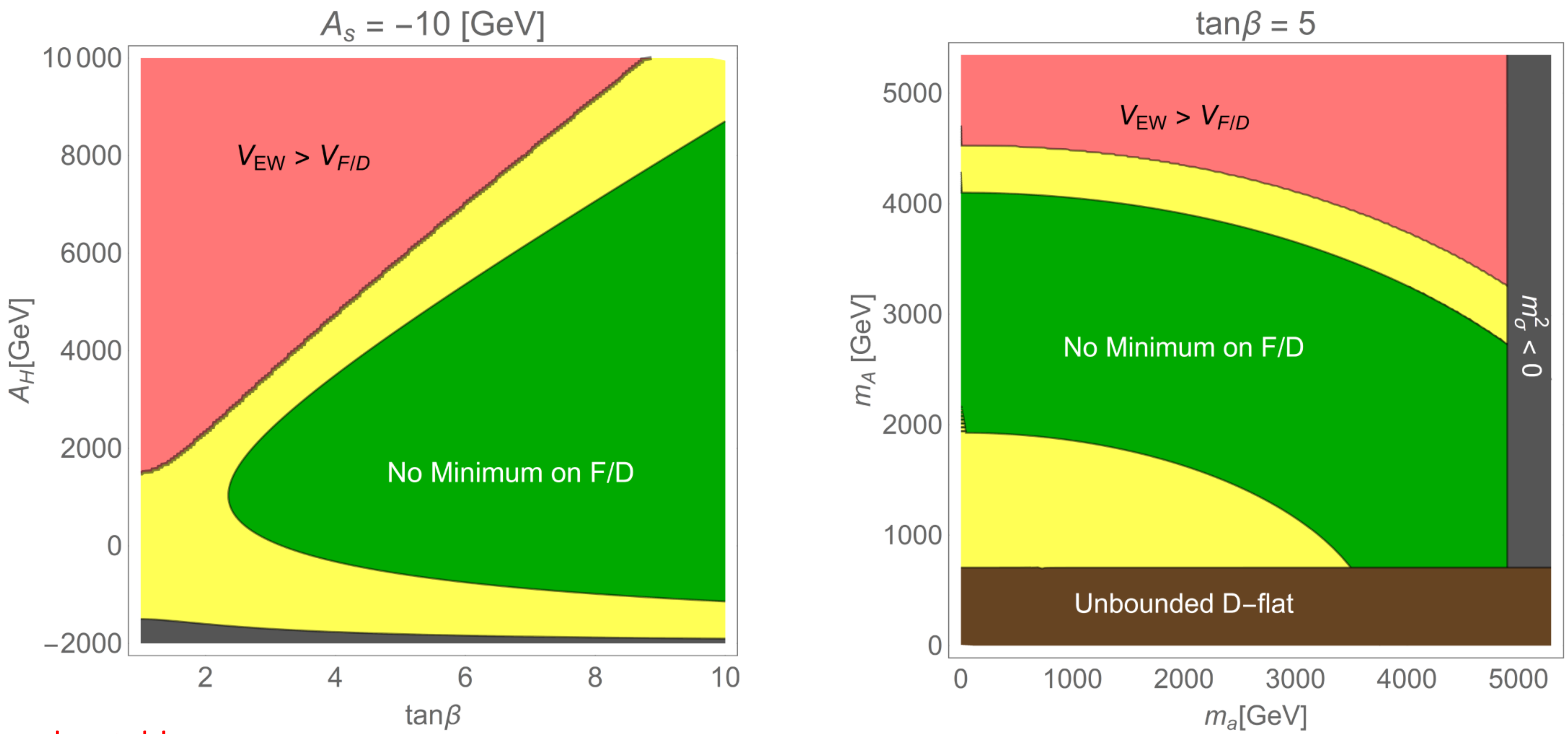
- In string theory, Z_N gauge symmetry from a U(1) gauge symmetry:

$$\boldsymbol{\mathcal{L}} = e^{-\mathrm{i}\phi}\boldsymbol{\mathcal{O}}_N + \phi F_{\mu\nu}\tilde{F}^{\mu\nu} + f^2\left(\partial_\mu\phi - NA_\mu\right)^2 + \text{chiral fermions}$$

$$U(1):\ \delta\phi = N\alpha(x),\ \ \delta\boldsymbol{\mathcal{O}}_N = \mathrm{i}N\alpha(x)\boldsymbol{\mathcal{O}}_N \rightarrow\ Z_N\text{: same with } \alpha(x) = \frac{2\pi}{N}.$$

ϕ: string theoretic axion, $\boldsymbol{\mathcal{O}}_N$: an operator with charge N

Vacuum stability for $m = 2$ $(k = 1)$



This case can be stable,
is different from m=1 (k=0).

$$V_{\text{soft}} := m_S^2 |S|^2 + m_{H_u}^2 |H_u^0|^2 + m_{H_d}^2 |H_d^0|^2 + \left(A_S \frac{S^N}{N\Lambda^{N-3}} - A_H \frac{S^m}{m\Lambda^{m-1}} H_u^0 H_d^0 + \text{h.c.} \right),$$

Possible Kähler potential corrections

$$\Delta_Q K = \left(\frac{a^i_j}{\Lambda} S Q^\dagger_i Q_j + \frac{\tilde{a}^i_j}{\Lambda^2} D^\alpha D_\alpha S \cdot Q^\dagger_i Q_j + \frac{b^i_j}{\Lambda^2} S^2 Q^\dagger_i Q_j + \frac{c^{ij}}{\Lambda^2} S^\dagger H_a Q_i Q_j + h.c.\right)$$
$$+ \frac{d^i_j}{\Lambda^2} S^\dagger S Q^\dagger_i Q_j + \frac{e_{ijkl}}{\Lambda^2} Q^\dagger_i Q_j Q^\dagger_k Q_l + \mathcal{O}\left(\Lambda^{-3}\right),$$

$$\int d^4\theta \; \frac{e_{Q_2 Q_1 \overline{d}_1 \overline{d}_2}}{\Lambda^3} S Q^\dagger_{L_2} Q_{L_1} \overline{d}^\dagger_{R_1} \overline{d}_{R_2} \supset \frac{\epsilon e_{Q_2 Q_1 \overline{d}_1 \overline{d}_2}}{2\Lambda^2} \; \overline{s}\gamma^\mu P_L d \cdot \overline{s}\gamma_\mu P_R d.$$

$$|\Delta\epsilon_K| = \frac{\kappa_\epsilon}{\sqrt{2}\Delta M_K} \frac{\epsilon \cdot \mathrm{Im}\left(e_{Q_2 Q_1 \overline{d}_1 \overline{d}_2}\right)}{2\Lambda^2} \left|\mathcal{O}^{\mathrm{LR}}_1\right|$$
$$\sim 10^{-2} \times \left(\frac{\epsilon}{0.02}\right)^3 \left(\frac{100\ \mathrm{TeV}}{v_s}\right)^2 \left(\frac{\mathrm{Im}\left(e_{Q_2 Q_1 \overline{d}_1 \overline{d}_2}\right)}{1.0}\right).$$