Schwinger model's parton distribution functions directly on the light-cone with tensor networks

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

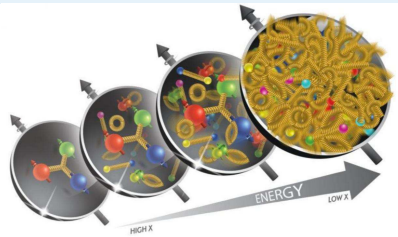
in collaboration with:

Mari Carmen Bañuls (MPQ Garching)
C.-J. David Lin (NYCU Hsinchu)
Manuel Schneider (NYCU Hsinchu)

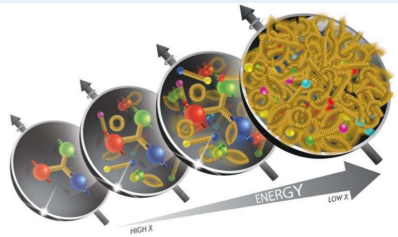
Based on:
M.C. Bañuls, KC, M. Schneider, C.-J.D. Lin
Parton Distribution Functions in the Schwinger model from Tensor Network States
arXiv:2504.07508

Motivation – nucleon structure

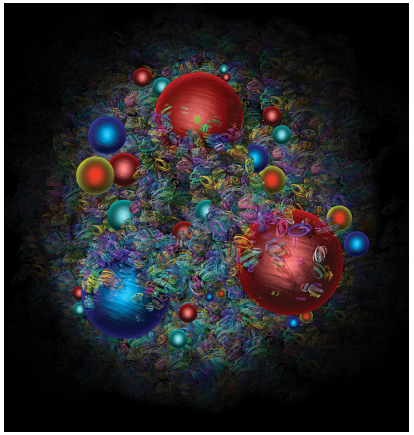
The nucleon is a very complicated system...
...and its structure is more complex
the closer we look!



Motivation – nucleon structure



The nucleon is a very complicated system...
...and its structure is more complex
the closer we look!

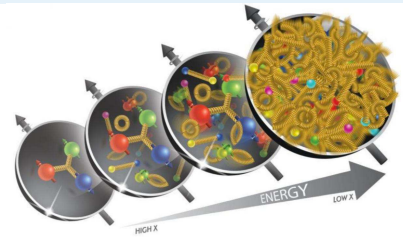


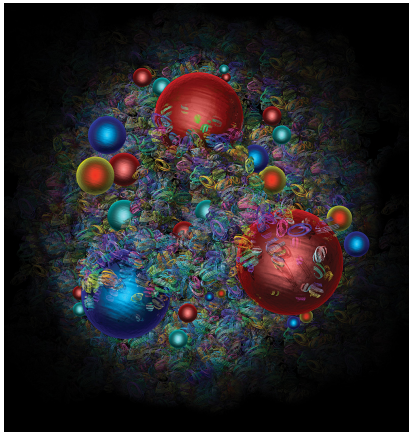
Different aspects:

- how the quarks and gluons move inside the nucleon
- 3D imaging of the nucleon – "hadron tomography"
- role of gluons and their emergent properties
- how is spin decomposed
- origin of nucleon mass
- ...

Motivation – nucleon structure

The nucleon is a very complicated system. . .
. . . and its structure is more complex
the closer we look!



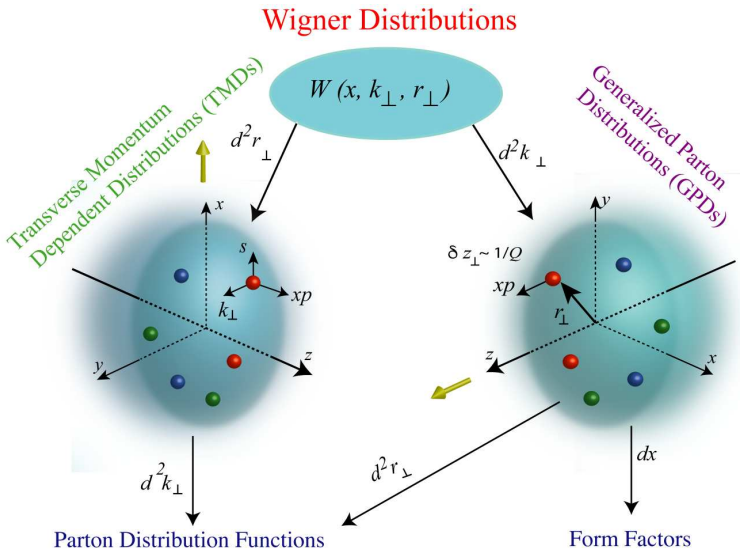


Different aspects:

- how the quarks and gluons move inside the nucleon
- 3D imaging of the nucleon – "hadron tomography"
- role of gluons and their emergent properties
- how is spin decomposed
- origin of nucleon mass
- . . .

Different functions characterizing the behavior of partons:

- 1D: form factors
- 1D: parton distribution functions (PDFs)
- 3D: generalized parton distributions (GPDs)
- 3D: transverse momentum dependent PDFs (TMDs)
- 5D: Wigner function

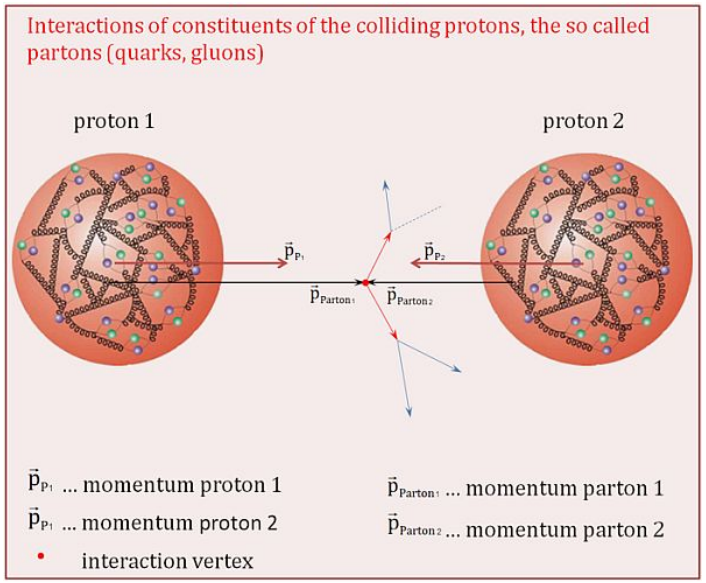


Parton distribution functions (PDFs)

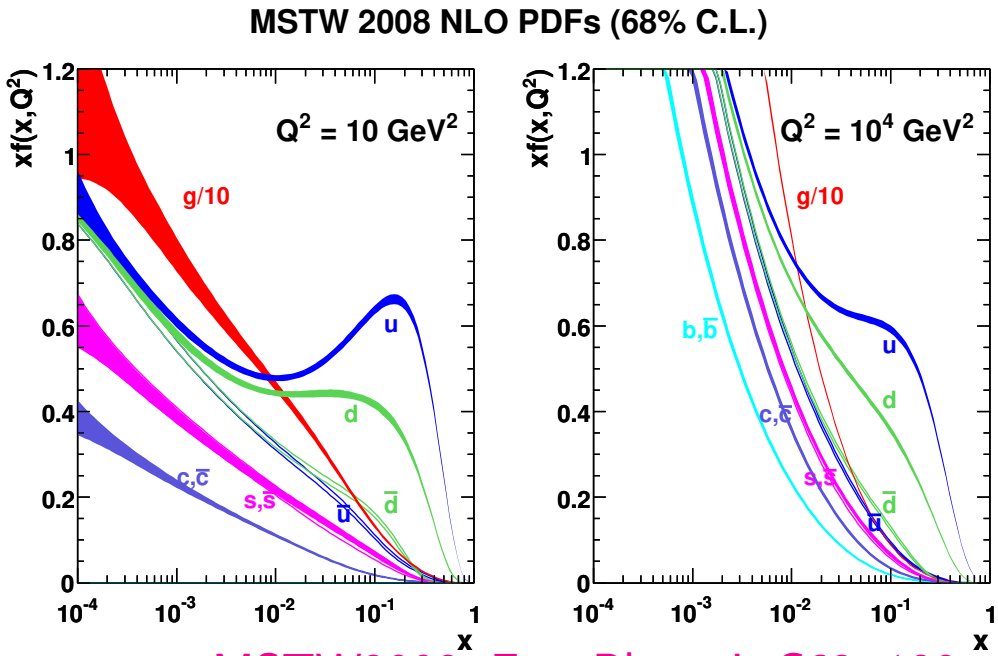
Introduction
Nucleon structure
PDFs
GPDs
LQCD
TNs for LGT
Schwinger model
Schwinger PDFs
Results
Summary

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.

$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$

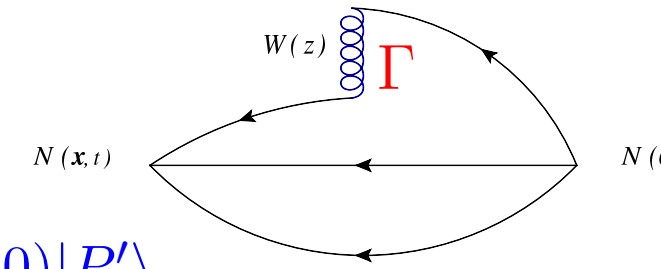


Source: LHC, CERN

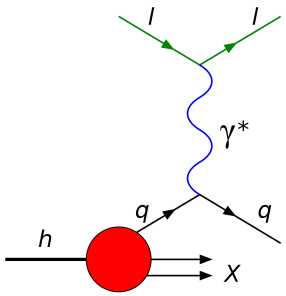


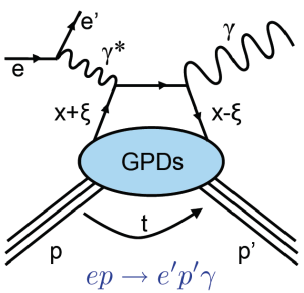
MSTW2008, Eur. Phys. J. C63, 189

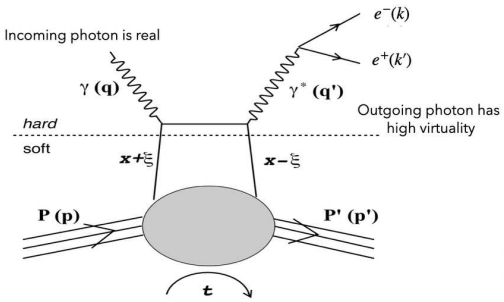
Generalized parton distributions (GPDs)

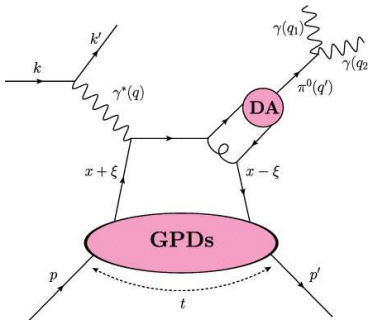


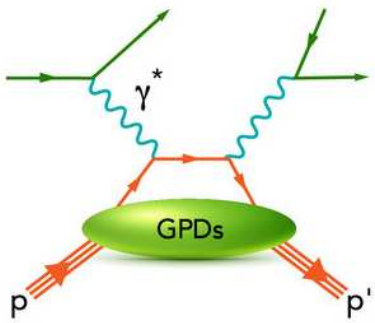
- Parton distribution functions (PDFs) – formal definition:
  $f(x,\mu) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle P|\overline{\psi}(\xi^-)\Gamma W(\xi^-,0)\psi(0)|P\rangle$
- Generalized parton distributions (GPDs):
  $F(x,\xi,t,\mu) = \frac{1}{2\pi}\int d\xi^- e^{-ixp^+\xi^-}\langle P''|\overline{\psi}(\xi^-)\Gamma W(\xi^-,0)\psi(0)|P'\rangle$
  The only difference: **momentum transfer**
  i.e. $P'' \neq P'$ $(P'' = P' + Q,\ t = -Q^2)$.
- Experimental access:
  - ⋆ PDFs – **Deep Inelastic Scattering (DIS)** – $ep \longrightarrow eX$
  - ⋆ GPDs – **Deeply Virtual Compton Scattering (DVCS)** – $ep \longrightarrow e'\gamma p'$
  - ⋆ GPDs – **Timelike Compton Scattering (TCS)** – $\gamma p \longrightarrow l\bar{l}p'$
  - ⋆ GPDs – **Deeply Virtual Meson Production (DVMP)** – $\gamma p \longrightarrow \pi^0 p' \longrightarrow \gamma\gamma' p'$
  - ⋆ GPDs – **Double Deeply Virtual Compton Scattering (DDVCS)** – $ep \longrightarrow e'p'l\bar{l}$











Partonic structure from Lattice QCD

Introduction
Nucleon structure
PDFs
GPDs
LQCD
TNs for LGT
Schwinger model
Schwinger PDFs
Results
Summary

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.

Partonic structure from Lattice QCD

Introduction
Nucleon structure
PDFs
GPDs
LQCD
TNs for LGT
Schwinger model
Schwinger PDFs
Results
Summary

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**
  (experiment) cross-section = perturbative-part $*$ partonic-distribution
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution

Introduction
Nucleon structure
PDFs
GPDs
LQCD
TNs for LGT
Schwinger model
Schwinger PDFs
Results
Summary

Partonic structure from Lattice QCD

- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**
  (experiment) cross-section = perturbative-part $*$ partonic-distribution
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution
- Which lattice observables one can use?
- Good "lattice cross sections" Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003
  - ⋆ computable on the lattice,
  - ⋆ having a well-defined continuum limit (renormalizable),
  - ⋆ perturbatively factorizable into PDFs.

Partonic structure from Lattice QCD

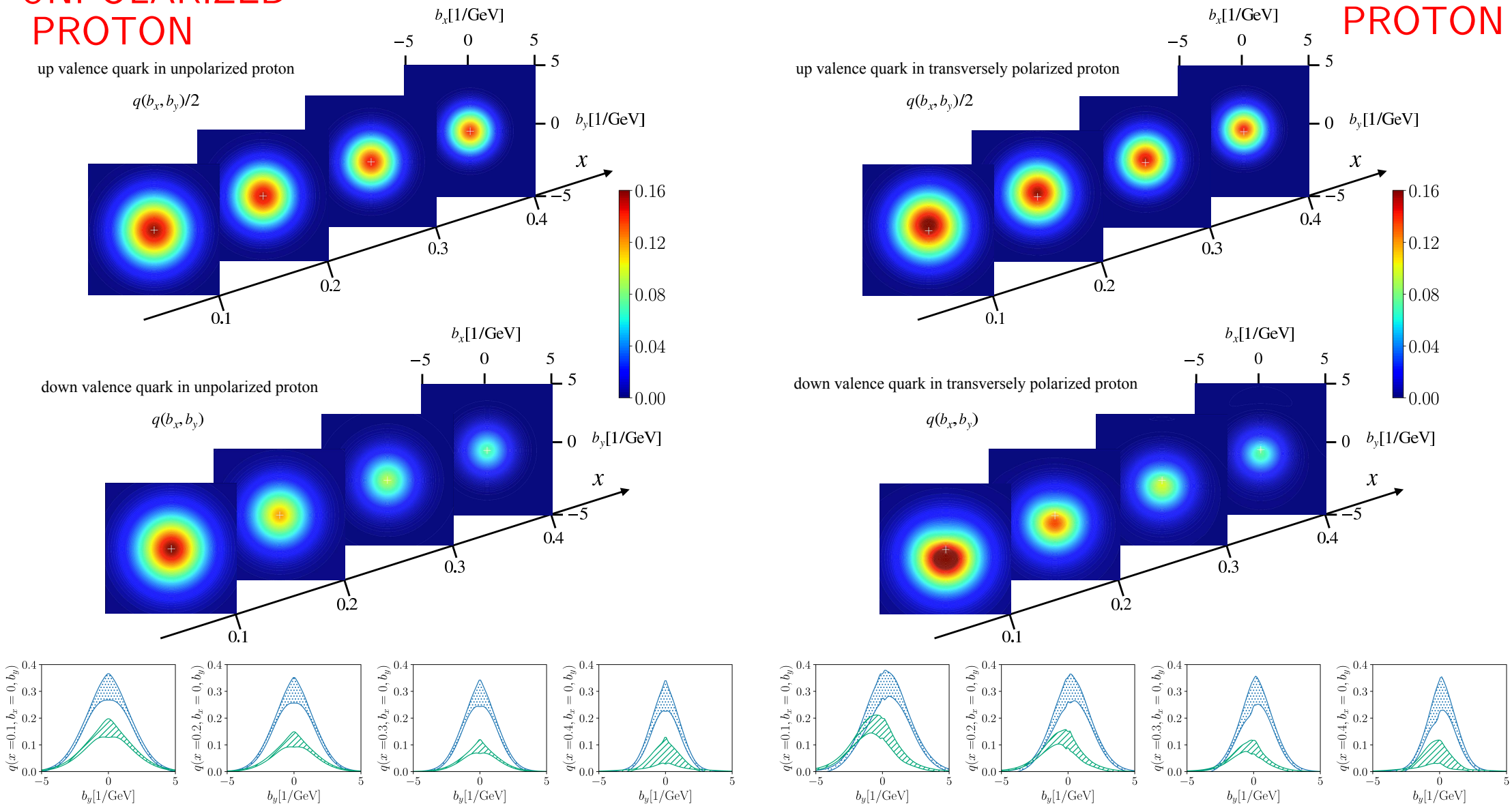
- Direct access to partonic distributions impossible in LQCD.
- Reason: **Minkowski** metric required, while LQCD works with **Euclidean**.
- Way out: similar as experimental access to these distributions – **factorization**
  (experiment) cross-section = perturbative-part $*$ partonic-distribution
  (lattice) lattice-observable = perturbative-part $*$ partonic-distribution
- Which lattice observables one can use?
- Good "lattice cross sections" Y.-Q. Ma, J.-W. Qiu, Phys. Rev. Lett. 120 (2018) 022003
  - ⋆ computable on the lattice,
  - ⋆ having a well-defined continuum limit (renormalizable),
  - ⋆ perturbatively factorizable into PDFs.
- Examples:
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

Nucleon tomography from GPDs

M. Chu, KC, M. Constantinou, P. Sznajder, J. Wagner, arXiv:2509.15931

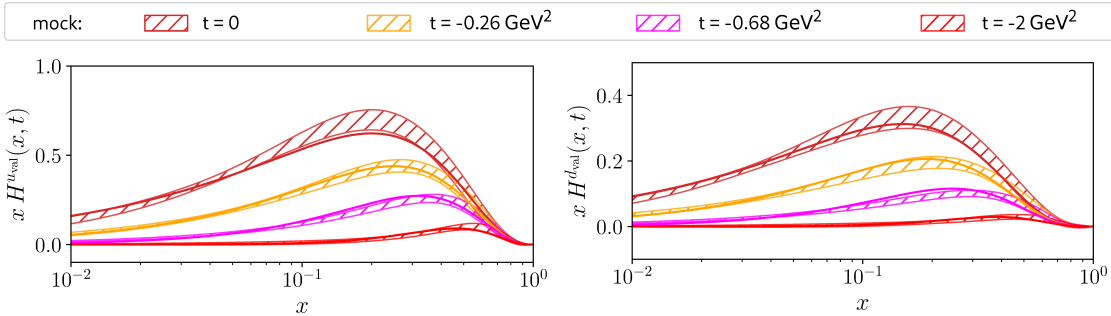
UNPOLARIZED PROTON

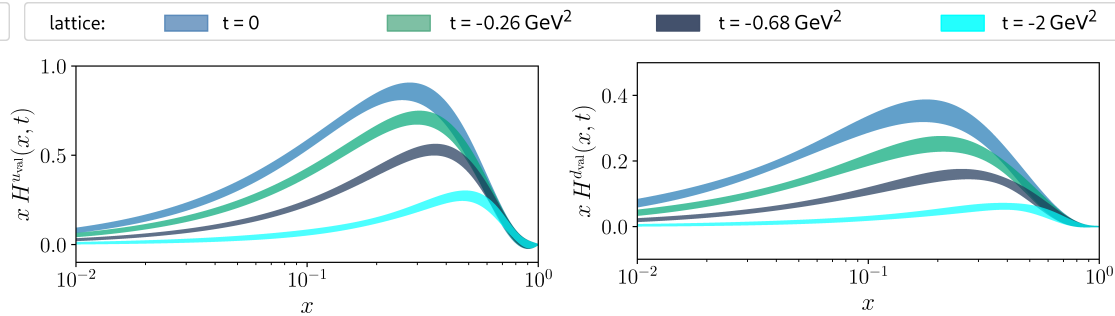
TRANSV. POLARIZED PROTON



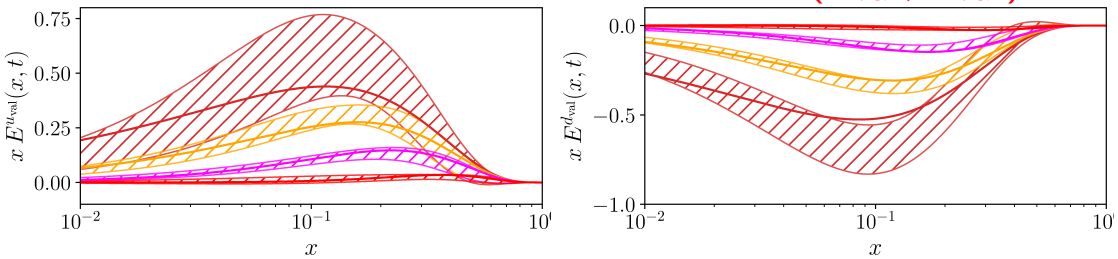
Results for GPDs

M. Chu, KC, M. Constantinou, P. Sznajder, J. Wagner, arXiv:2509.15931

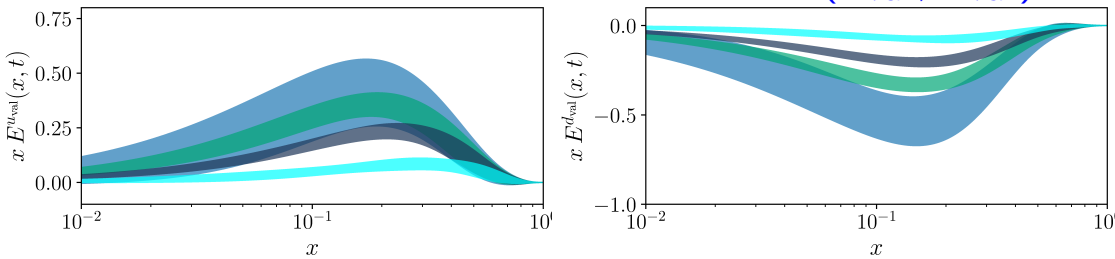




MOCK DATA TEST FOR H AND E ($u_{\rm val}$, $d_{\rm val}$)



LATTICE RESULTS FOR H AND E ($u_{\rm val}$, $d_{\rm val}$)



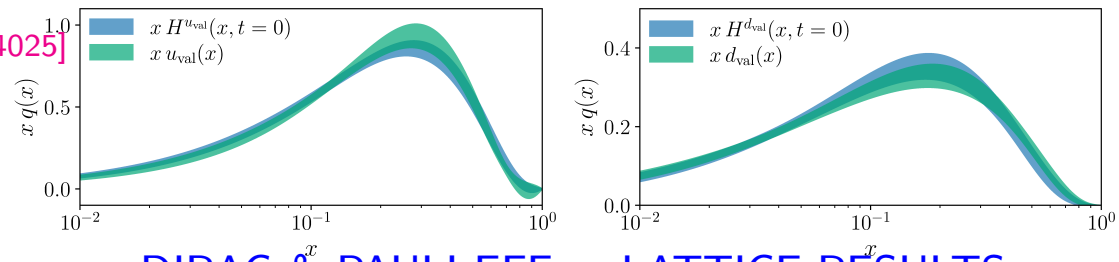
$$J^{u_{\rm val}} = 0.281(28),\ J^{d_{\rm val}} = -0.014(23)$$

cf. e.g. $J^{u_{\rm val}} = 0.195(10),\ J^{d_{\rm val}} = 0.017(5)$ [PRD110(2024)114025]

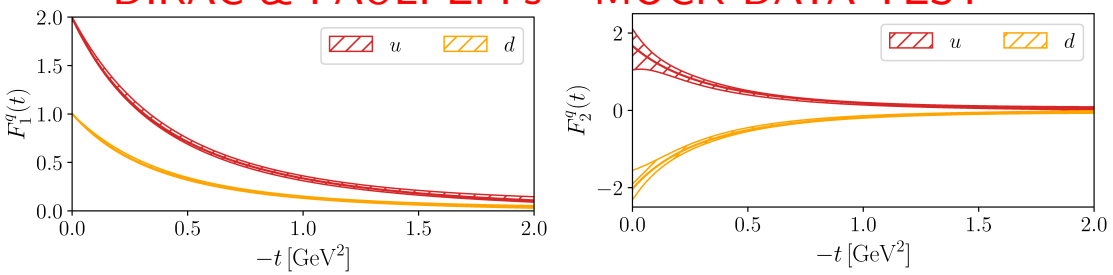
$$\kappa_p = (2/3)\kappa_u - (1/3)\kappa_d = 1.14(26)$$
$$\kappa_u = 1.03(43),\ \kappa_d = -1.37(39)$$

cf. experiment $\kappa_p^{\rm exp.} = 1.792847350(9)$ [Nature 509(2014)596]

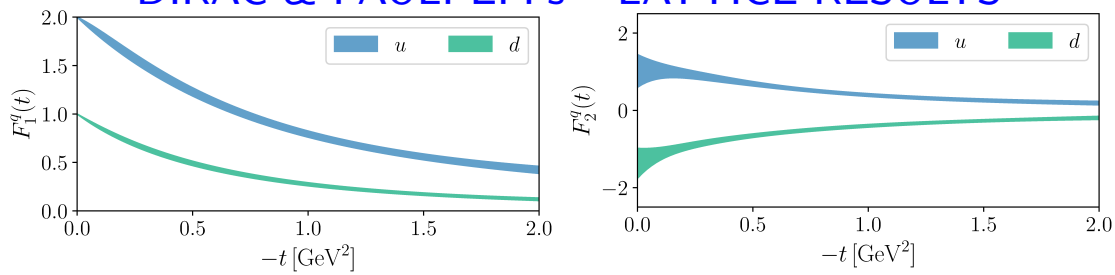
COMPARISON OF PDFs AND $t = 0$ GPDs



DIRAC & PAULI EFFs – MOCK DATA TEST



DIRAC & PAULI EFFs – LATTICE RESULTS



Motivation – TNs for gauge theories

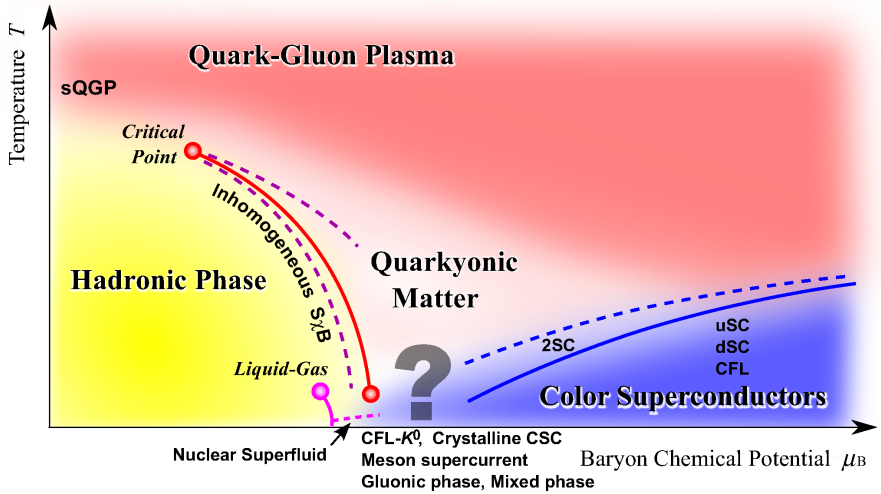
- The motivation for applying TN methods to LQCD comes mostly from the sign problem that appears for certain parameter regimes of MC simulations.

Motivation – TNs for gauge theories

- The motivation for applying TN methods to LQCD comes mostly from the sign problem that appears for certain parameter regimes of MC simulations.
- The (Euclidean) QCD path integral: $Z = \int D\bar{\psi} D\psi DU \; e^{-S_{gauge}[U] - S_{ferm}[\psi, \bar{\psi}, U]}$.
  static properties, zero baryon density: no sign problem! $\Rightarrow$ tremendous successes

Motivation – TNs for gauge theories

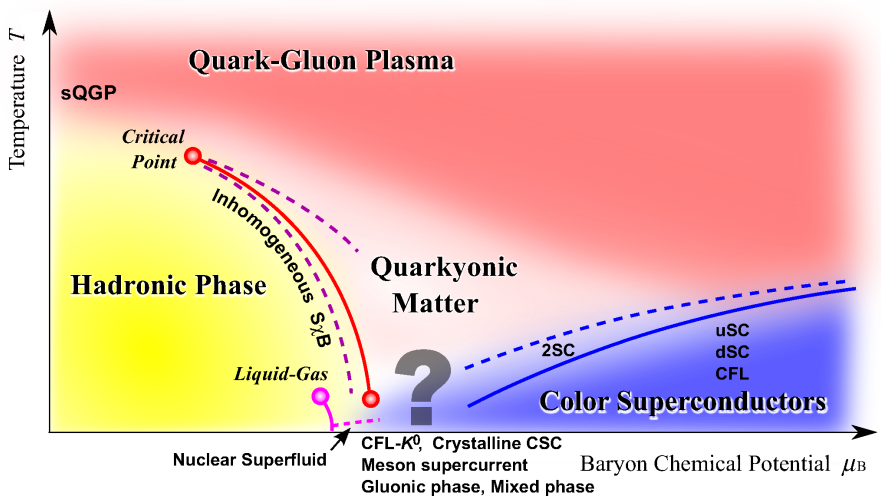
- The motivation for applying TN methods to LQCD comes mostly from the sign problem that appears for certain parameter regimes of MC simulations.
- The (Euclidean) QCD path integral: $Z = \int D\bar{\psi} D\psi DU \; e^{-S_{gauge}[U] - S_{ferm}[\psi,\bar{\psi},U]}$.
  static properties, zero baryon density: no sign problem! $\Rightarrow$ tremendous successes
  real-time dynamics: impossible in Euclidean spacetime...
  finite baryon density: mild or severe sign problem; at the same time huge physical significance of this regime



K. Fukushima, T. Hatsuda, Rep.Prog.Phys. 74(2011)14001

Motivation – TNs for gauge theories

- The motivation for applying TN methods to LQCD comes mostly from the sign problem that appears for certain parameter regimes of MC simulations.
- The (Euclidean) QCD path integral: $Z = \int D\bar{\psi} D\psi DU \; e^{-S_{gauge}[U] - S_{ferm}[\psi,\bar{\psi},U]}$.
  static properties, zero baryon density: no sign problem! $\Rightarrow$ tremendous successes
  real-time dynamics: impossible in Euclidean spacetime...
  finite baryon density: mild or severe sign problem; at the same time huge physical significance of this regime



K. Fukushima, T. Hatsuda, Rep.Prog.Phys. 74(2011)14001

- Ways to alleviate the (mild) sign problem: reweighting, Taylor expansion, analytic continuation from imaginary μ, ...
  – **basically hopeless...**
- Smarter ways:
  Lefschetz thimbles, complex Langevin, ...
  – **interesting progress and hope for good results for regimes with severe sign problem!**
- Still: **alternative approaches wanted!**
  Tensor Networks?

The Schwinger model

Our main toy model: the Schwinger model (QED in 1+1d)

J. S. Schwinger, Phys. Rev. **128** (1962) 2425

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\,\partial\!\!\!/ - g\,A\!\!\!/ - m)\psi$$

- Simplest gauge theory, but physics still surprisingly rich.
- In several aspects resembles much more complex theories (QCD):
  - ⋆ confinement,
  - ⋆ chiral symmetry breaking (via anomaly, not spontaneous),
  - ⋆ non-perturbative generation of mass gap.
- Massless case analytically solvable ⇒ precise checks possible.
- Standard toy model for testing lattice techniques.

Formulation of the model:

- staggered discretization,
- Jordan-Wigner transformation to spin-1/2 variables.

Multi-flavour Schwinger model as a spin model

HOPPING TERM

$$H = -xi^{N_f-1} \sum_{f=0}^{N_f-1} \sum_{n=0}^{N-2} \Big[\sigma^+_{N_f n+f} \sigma^z_{N_f n+f+1} \cdots \sigma^z_{N_f n+N_f+f-1} \sigma^-_{N_f n+N_f+f} +$$

HOPPING TERM H.c.

$$(-1)^{N_f-1} \sigma^-_{N_f n+f} \sigma^z_{N_f n+f+1} \cdots \sigma^z_{N_f n+N_f+f-1} \sigma^+_{N_f n+N_f+f} \Big] +$$

$$+ \sum_{k=0}^{N_f N-1} \alpha_1(k) 1 + \sum_{k=0}^{N_f N-1} \alpha(k) \sigma^z_k + \sum_{k=0}^{N_f N-1} \sum_{k'=k+1}^{N_f N-1} \beta'(k') \sigma^z_k \sigma^z_{k'},$$

INHOMOGENEOUS OFFSET

INHOMOGENEOUS MAGNETIC FIELD

GAUGE TERM

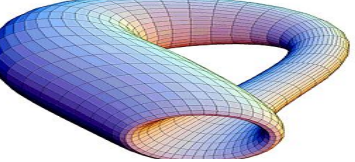
long-distance interaction!

with:

$$\alpha_1(k) = \frac{l_0^2}{N_f} \left(1 - \delta_{k/F,N-1}\right) + l_0 \left(1 - (k/N_f)\%2\right) + \frac{1}{2} M_{k\%N_f} + \frac{1}{8}(N + N_f - 1) + \xi,$$

$$\alpha(k) = l_0(N - k/N_f - 1) + \frac{\widetilde{\mu}_{k\%N_f}}{2} + \frac{1}{2} M_{k\%N_f} (-1)^{k/N_f} + \frac{N_f}{4} \left(N - k/N_f - (k/N_f)\%2\right),$$

$$\beta'(k) = \frac{1}{2} \left(N - k/N_f - 1 + 2\xi\right),$$



Our formulation

arXiv:2504.07508

Spin Hamiltonian:

$$H_{\text{spin}} = x \sum_n \left(\sigma_n^+ \sigma_{n+1}^- + \sigma_{n+1}^- \sigma_n^+\right) + \frac{\mu}{2} \sum_n \left[1 + (-1)^n \sigma_n^z\right] + \sum_n L_n^2,$$

with $\mu \equiv \frac{2}{ag^2} m_{\text{lat}} = \frac{2}{ag^2}\left(m - \frac{ag^2}{8}\right)$ and $x \equiv \frac{1}{a^2 g^2}$.

Physical states fulfill Gauss's law: $L_n - L_{n-1} = Q_n = \frac{1}{2}\left[(-1)^n + \sigma_n^z\right] + q_n$.

external (static) charge

This leads to $L_n = \frac{1}{2} \sum_{k \leq n} \left((-1)^k + \sigma_k^z + 2q_k\right)$

and the final form of the lattice spin Hamiltonian:

$$H = x \sum_{n=0}^{N-2} \left[\sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+\right] + \frac{\mu}{2} \sum_{n=0}^{N-1} \left[1 + (-1)^n \sigma_n^z\right] + \sum_{n=0}^{N-2} \left[\sum_{k=0}^{n} \left(\frac{1}{2}\left((-1)^k + \sigma_k^z\right) + q_k\right)\right]^2.$$

PDFs in the Schwinger model

Continuum PDF: arXiv:2504.07508

$$f(\xi) = \int_{-\infty}^{\infty} \frac{dz^-}{4\pi} e^{-i\xi P^+ z^-} \langle h|\bar{\psi}(z^-) W_{z^- \leftarrow 0} \gamma^+ \psi(0)|h\rangle_c.$$

Matrix element in the staggered discretization and the spin formulation:

$$\mathcal{M} = \gamma^+_{0,0}\mathcal{M}_{0,0} + \gamma^+_{0,1}\mathcal{M}_{0,1} + \gamma^+_{1,0}\mathcal{M}_{1,0} + \gamma^+_{1,1}\mathcal{M}_{1,1} = \mathcal{M}_{0,0} - \mathcal{M}_{0,1} - \mathcal{M}_{1,0} + \mathcal{M}_{1,1},$$

with

$$\mathcal{M}_{i,j}(\Delta z) = \langle h| e^{iH\frac{\Delta t}{2}\Delta z} \prod_{k<\Delta z+i} (i\sigma^z_k)\, \sigma^+_{\Delta z+i} e^{-iH_{\Delta z-1}\Delta t} \dots e^{-iH_3\Delta t} e^{-iH_1\Delta t} \prod_{k'<j} (-i\sigma^z_{k'})\, \sigma^-_j |h\rangle_c,$$

where:
subscript c – subtraction of vacuum contribution, $\langle h|\hat{O}|h\rangle_c = \langle h|\hat{O}|h\rangle - \langle 0|\hat{O}|0\rangle$,
$|h\rangle$ – first excited state (vector meson "at rest"), with mass $M = \langle h|H|h\rangle_c$,
H_k – inclusion of a single positive static charge at (odd) position k.

Lattice PDF: $f(\xi) = \frac{NM}{8\pi x} \sum_{\Delta z=0,2,4,\dots,N} e^{-i\xi\frac{M\Delta z}{2x}} \mathcal{M}(\Delta z)$.

PDFs in the Schwinger model

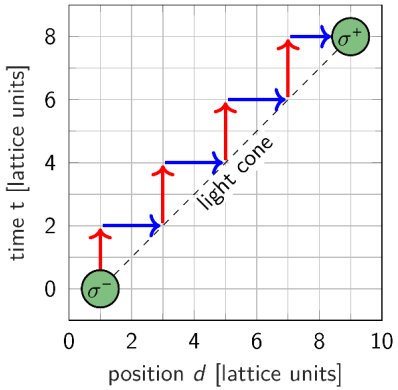
Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

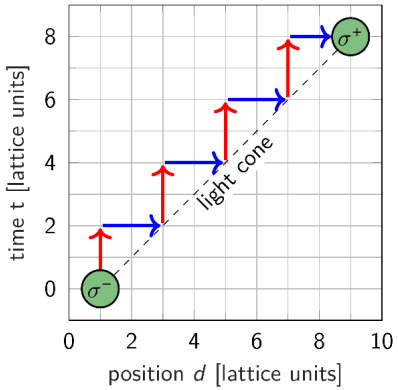
arXiv:2504.07508



PDFs in the Schwinger model

arXiv:2504.07508

Evaluate light-front correlations:
interchange small temporal and spatial steps



time evolution:

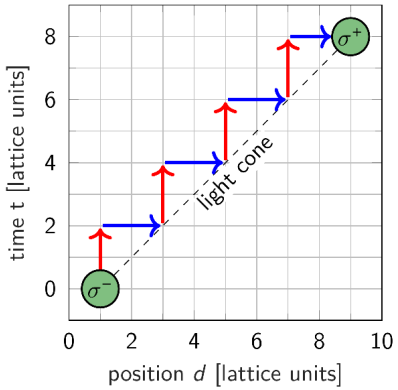
$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

PDFs in the Schwinger model

arXiv:2504.07508

Evaluate light-front correlations:
interchange small temporal and spatial steps



time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

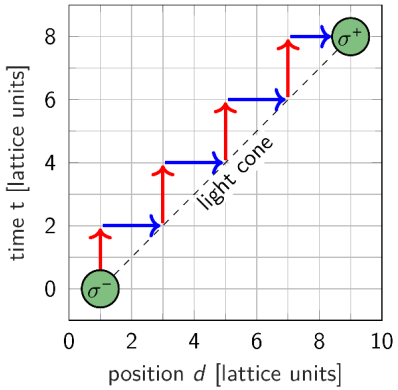
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

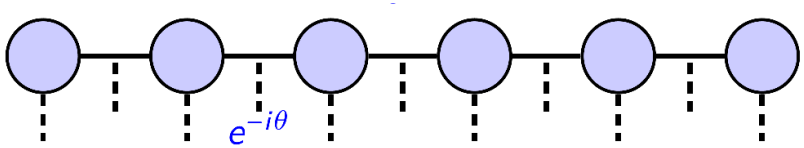
$$q_a \downarrow_{\rm 1unit} \longrightarrow q_b \uparrow_{\rm 1unit}$$

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

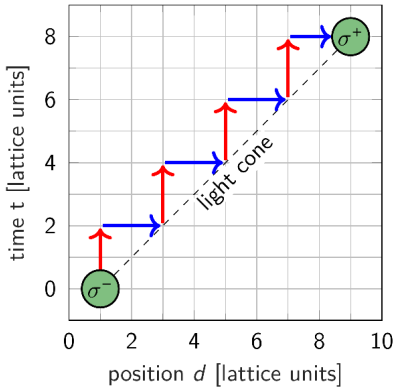
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

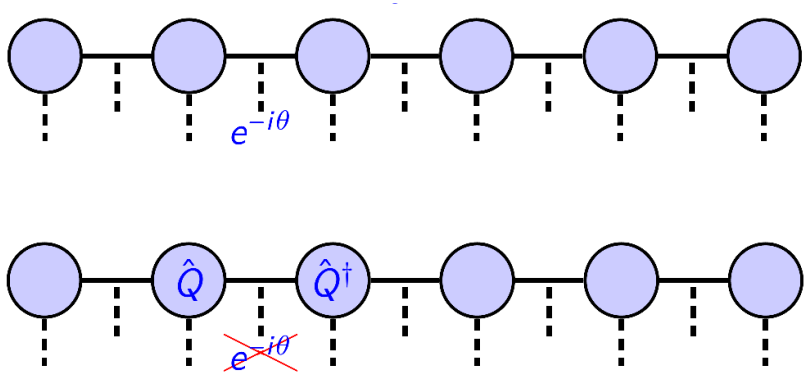
$$q_a \downarrow_{\rm 1unit} \longrightarrow q_b \uparrow_{\rm 1unit}$$

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

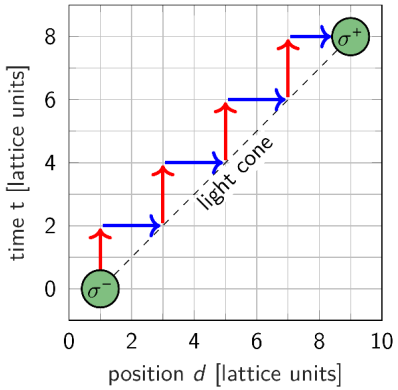
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

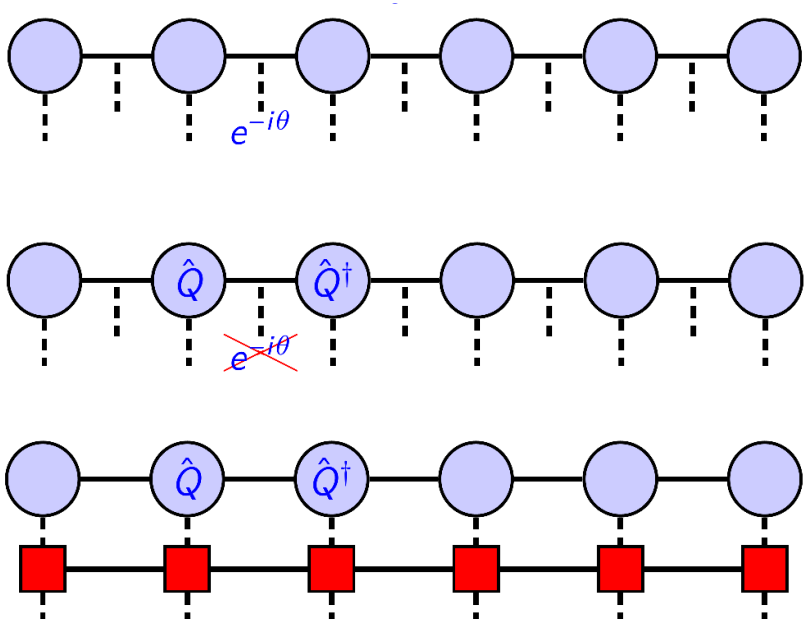
$$q_a \downarrow_{1\rm unit} \longrightarrow q_b \uparrow_{1\rm unit}$$

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

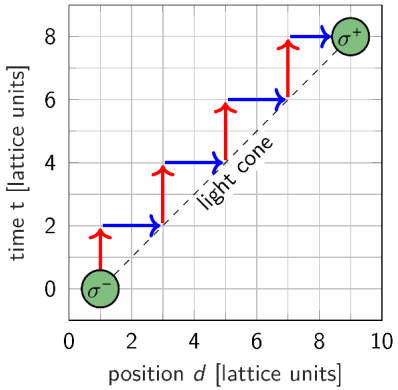
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

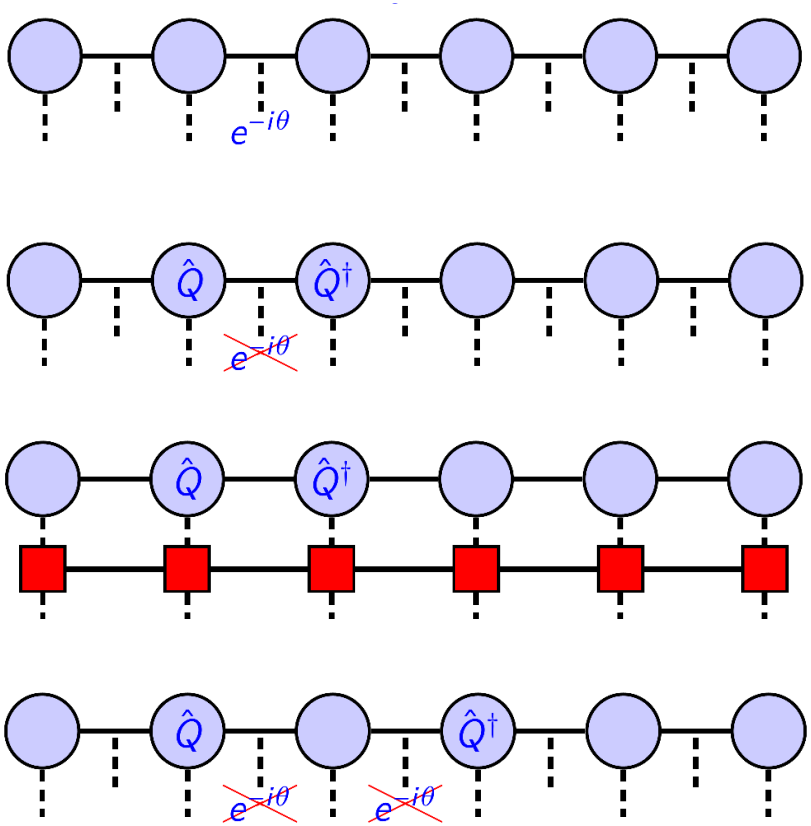
$$q_a \downarrow_{1\rm unit} \longrightarrow q_b \uparrow_{1\rm unit}$$

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

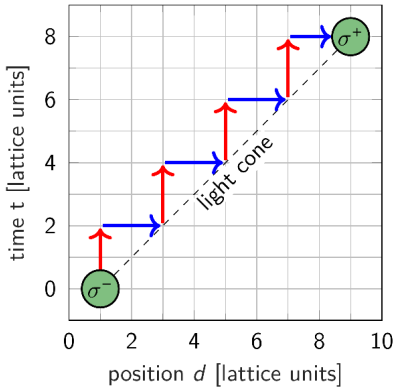
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

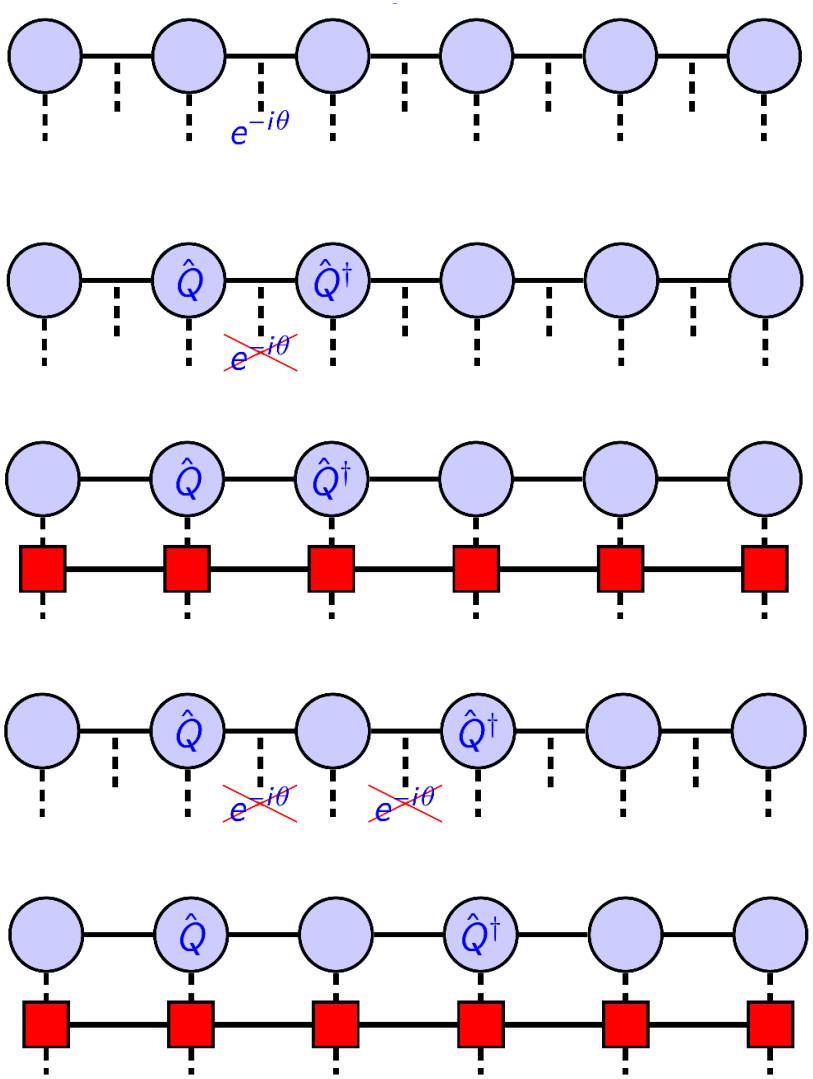
$$q_a \downarrow_{1\rm unit} \longrightarrow q_b \uparrow_{1\rm unit}$$

PDFs in the Schwinger model

Evaluate light-front correlations:
interchange small temporal and spatial steps

arXiv:2504.07508





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

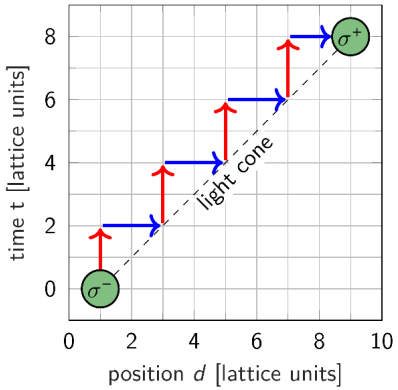
non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

$$q_a \downarrow_{\rm 1unit} \longrightarrow q_b \uparrow_{\rm 1unit}$$

PDFs in the Schwinger model

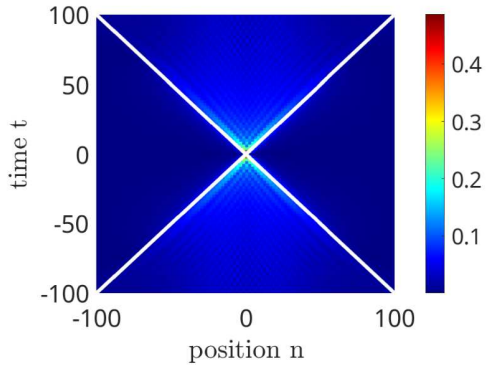
Evaluate light-front correlations:
interchange small temporal and spatial steps

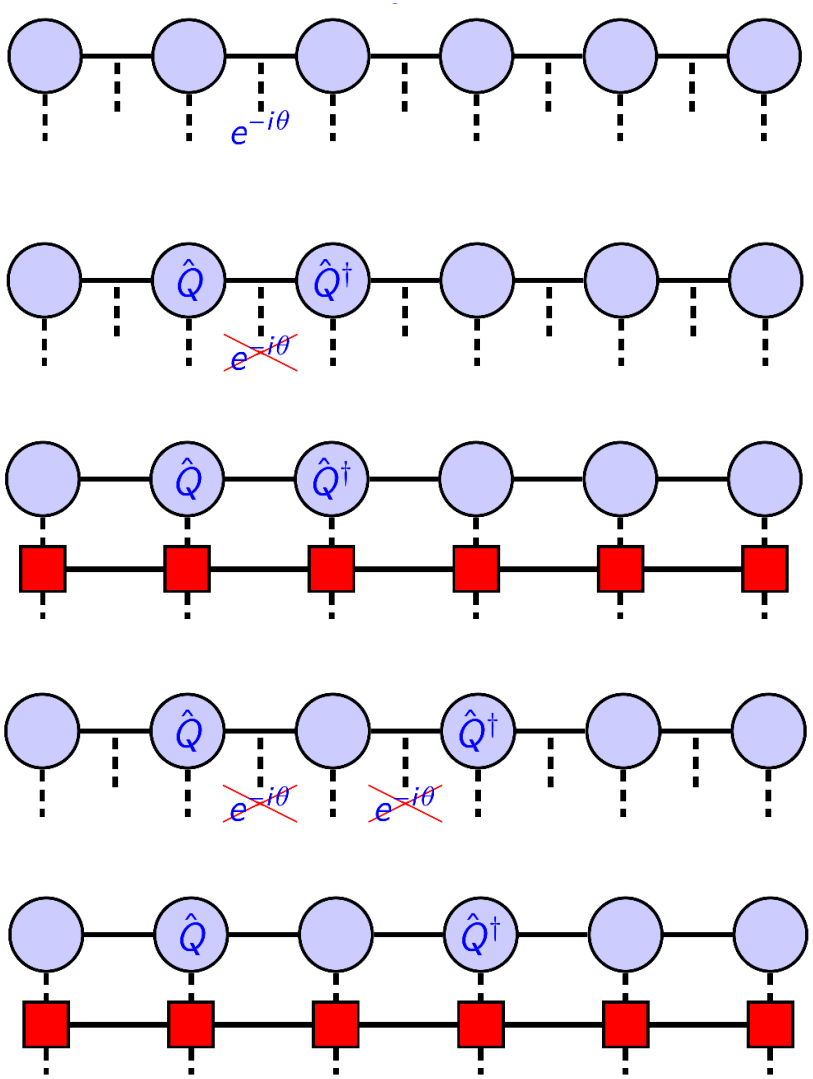
arXiv:2504.07508



Emerging light-cone structure

$\mathcal{M}_{0,0}(\Delta z)$ with 1 static charge at the origin





time evolution:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{\rm eo}} e^{-i\delta\tau H_{\rm oe}} e^{-i\delta\tau H_{\rm L}}\right)^{N_\tau}$$

The total evolution time τ divided into N_τ small steps of $\delta\tau$

space evolution:

non-trivial action on the gauge links
shifting the electric flux by one unit at a time
$\rightarrow$ like having a pair of charges around a link
$\Rightarrow$ string can be absorbed in the time evolution
by introducing a static charge q_k that ends
the Wilson line and moves along the light cone

$$q_a \downarrow_{1\rm unit} \longrightarrow q_b \uparrow_{1\rm unit}$$

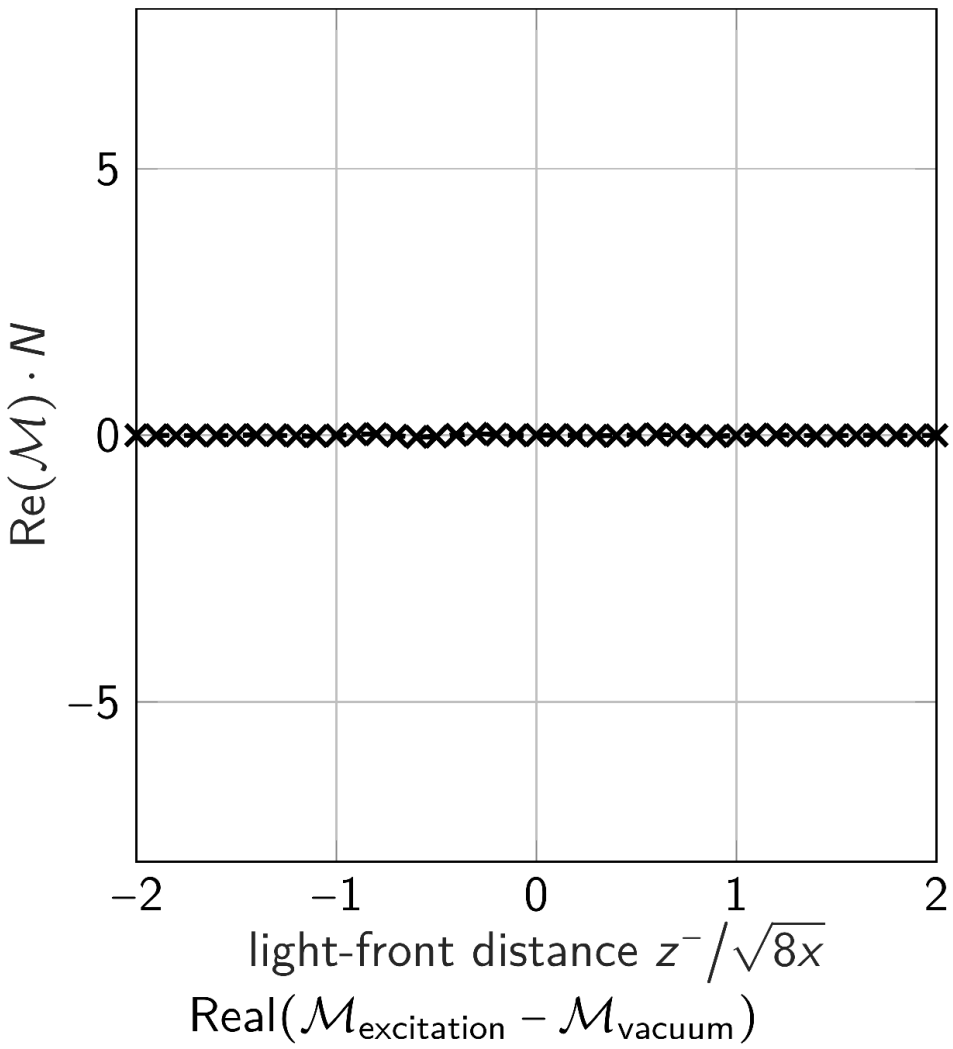
Introduction
Results
MEs
PDF
Summary

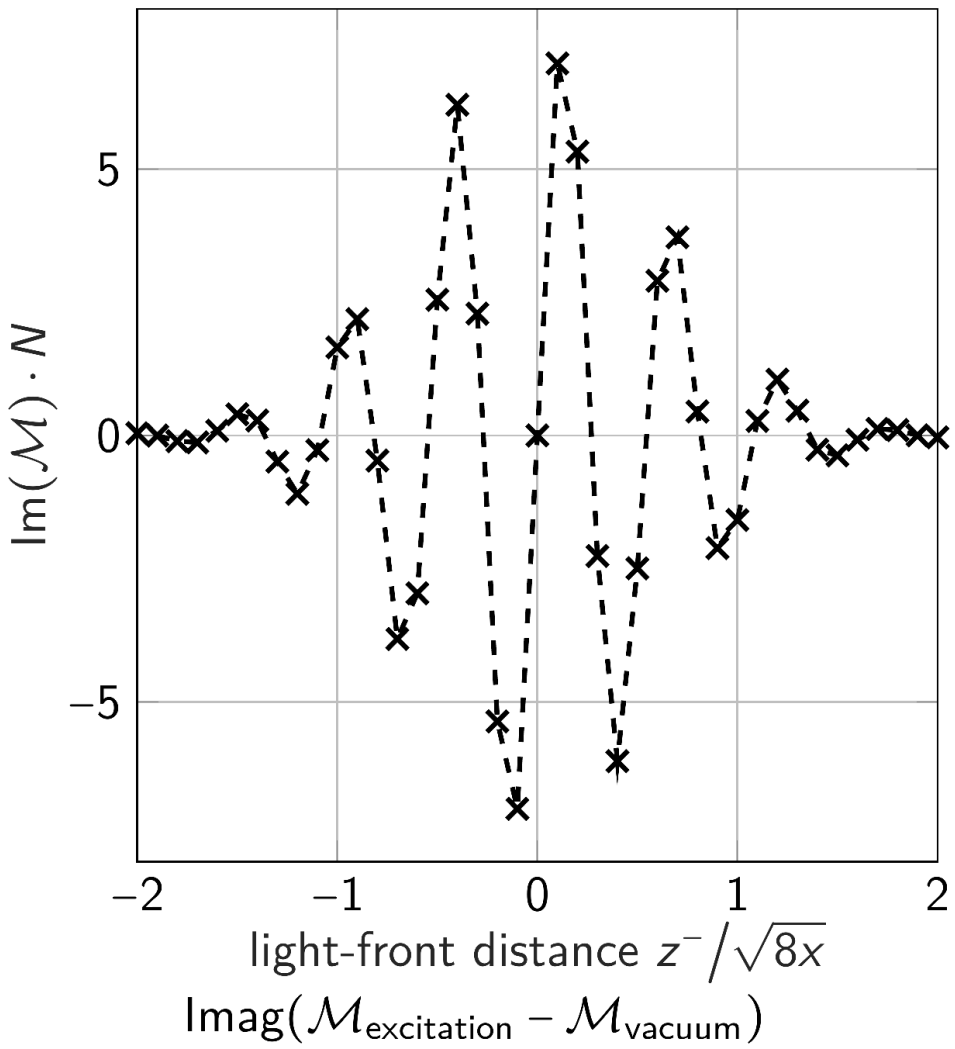
Results

Matrix elements

$\tilde{m} = 10, \quad x = 100, \quad D = 80, \quad N_\tau = 100, \quad \tilde{V} = 100$

in units of $m_V/g = 1/\sqrt{\pi}$ $\qquad$ $\tilde{V} = \tilde{m}N/\sqrt{x}$





arXiv:2504.07508

Systematic effects in our calculation

Introduction
Results
MEs
PDF
Summary

Investigate and quantify effects of:

Systematic effects in our calculation

Investigate and quantify effects of:
(stage of matrix elements)

- finite bond dimension $D = 40, 80, 120 \to D = 80$, error as max. deviation from $D \in \{40, 120\}$,

Systematic effects in our calculation

Investigate and quantify effects of:
(stage of matrix elements)

- finite bond dimension $D = 40, 80, 120 \to D = 80$, error as max. deviation from $D \in \{40, 120\}$,
- finite Trotter step $N_\tau = 10, 20, 50, 100, 200 \to N_\tau = 100$, error as max. deviation $|\mathcal{M}(N_\tau = 200) - \mathcal{M}(N_\tau = 100)|$, $|\mathcal{M}(N_\tau = \infty_{\mathrm{3fits}}) - \mathcal{M}(N_\tau = 100)|$,

Systematic effects in our calculation

Investigate and quantify effects of:
(stage of matrix elements)

- finite bond dimension $D = 40, 80, 120 \to D = 80$, error as max. deviation from $D \in \{40, 120\}$,
- finite Trotter step $N_\tau = 10, 20, 50, 100, 200 \to N_\tau = 100$, error as max. deviation $|\mathcal{M}(N_\tau = 200) - \mathcal{M}(N_\tau = 100)|$, $|\mathcal{M}(N_\tau = \infty_{\text{3fits}}) - \mathcal{M}(N_\tau = 100)|$,
- truncation of electric flux $L_{\text{cut}} = 2, 4, 8, 10, 12, 20 \to L_{\text{cut}} = 10$, error as max. deviation $|\mathcal{M}(L_{\text{cut}} > 10) - \mathcal{M}(L_{\text{cut}} = 10)|$, $|\mathcal{M}(L_{\text{cut}} = \infty_{\text{3fits}}) - \mathcal{M}(L_{\text{cut}} = 10)|$,

Systematic effects in our calculation

Investigate and quantify effects of:
(stage of matrix elements)

- finite bond dimension $D = 40, 80, 120 \to D = 80$, error as max. deviation from $D \in \{40, 120\}$,
- finite Trotter step $N_\tau = 10, 20, 50, 100, 200 \to N_\tau = 100$, error as max. deviation $|\mathcal{M}(N_\tau = 200) - \mathcal{M}(N_\tau = 100)|$, $|\mathcal{M}(N_\tau = \infty_{\text{3fits}}) - \mathcal{M}(N_\tau = 100)|$,
- truncation of electric flux $L_{\text{cut}} = 2, 4, 8, 10, 12, 20 \to L_{\text{cut}} = 10$, error as max. deviation $|\mathcal{M}(L_{\text{cut}} > 10) - \mathcal{M}(L_{\text{cut}} = 10)|$, $|\mathcal{M}(L_{\text{cut}} = \infty_{\text{3fits}}) - \mathcal{M}(L_{\text{cut}} = 10)|$,

(stage of PDFs – MEs Fourier-transformed)

- finite lattice spacing $x = 80, 100, 200, 300, 600, 1000 \to x = 100$, error as max. deviation $|f(x > 100) - f(x = 100)$, $|f(x = \infty_{\text{2fits}}) - f(x = 100)|$

Systematic effects in our calculation

Investigate and quantify effects of:
(stage of matrix elements)

- finite bond dimension $D = 40, 80, 120 \to D = 80$, error as max. deviation from $D \in \{40, 120\}$,
- finite Trotter step $N_\tau = 10, 20, 50, 100, 200 \to N_\tau = 100$, error as max. deviation $|\mathcal{M}(N_\tau = 200) - \mathcal{M}(N_\tau = 100)|$, $|\mathcal{M}(N_\tau = \infty_{3\text{fits}}) - \mathcal{M}(N_\tau = 100)|$,
- truncation of electric flux $L_{\text{cut}} = 2, 4, 8, 10, 12, 20 \to L_{\text{cut}} = 10$, error as max. deviation $|\mathcal{M}(L_{\text{cut}} > 10) - \mathcal{M}(L_{\text{cut}} = 10)|$, $|\mathcal{M}(L_{\text{cut}} = \infty_{3\text{fits}}) - \mathcal{M}(L_{\text{cut}} = 10)|$,

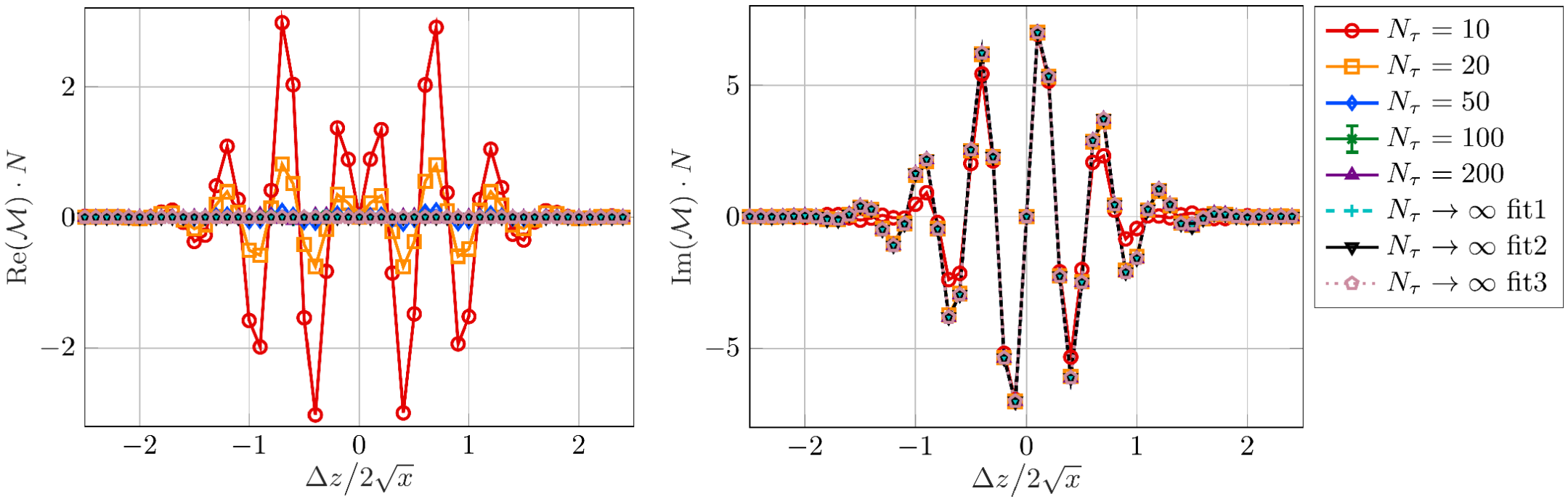
(stage of PDFs – MEs Fourier-transformed)

- finite lattice spacing $x = 80, 100, 200, 300, 600, 1000 \to x = 100$, error as max. deviation $|f(x > 100) - f(x = 100)$, $|f(x = \infty_{2\text{fits}}) - f(x = 100)|$ or $x = \infty$, central value as $(f_{\max} + f_{\min})/2$ from $x = 100$ and 2 extrapolations, error as $\pm(f_{\max} - f_{\min})/2$,
- finite volume $N = 80, 100, 120, 140, 160, 200 \to N = \infty$, central value as $(f_{\max} + f_{\min})/2$ from $N = 100$ and 3 extrapolations, error as $\pm(f_{\max} - f_{\min})/2$.

Matrix elements – Trotter step

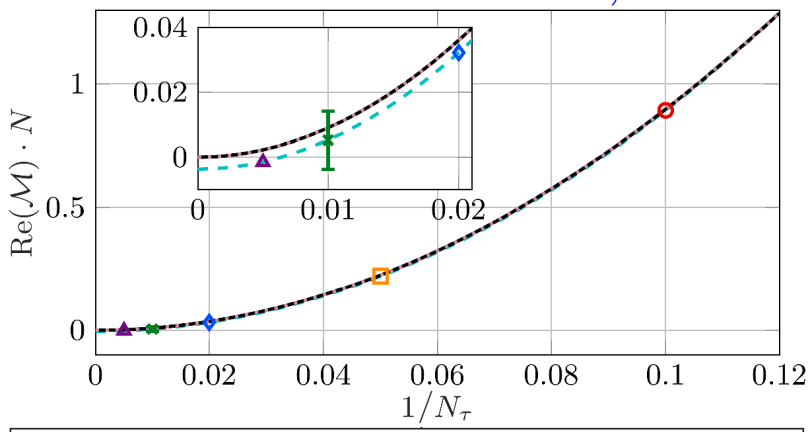
arXiv:2504.07508

$$\tilde{m} = 10, \quad x = 100, \quad D = 80, \quad \tilde{V} = 100$$

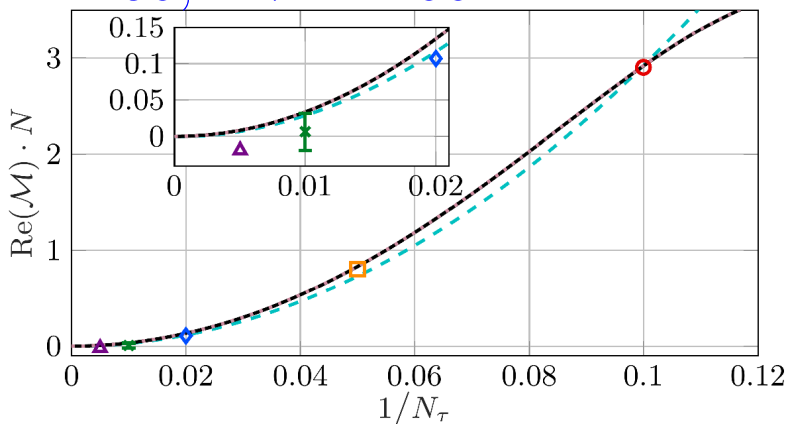


Matrix elements – Trotter step

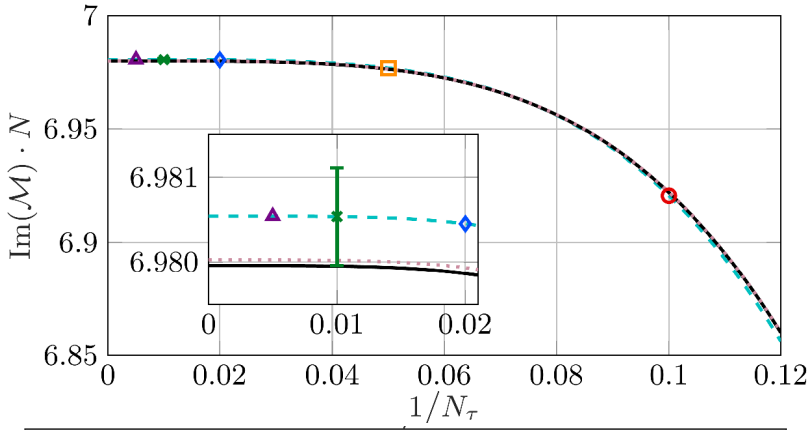
$\tilde{m} = 10, \quad x = 100, \quad D = 80, \quad \tilde{V} = 100$



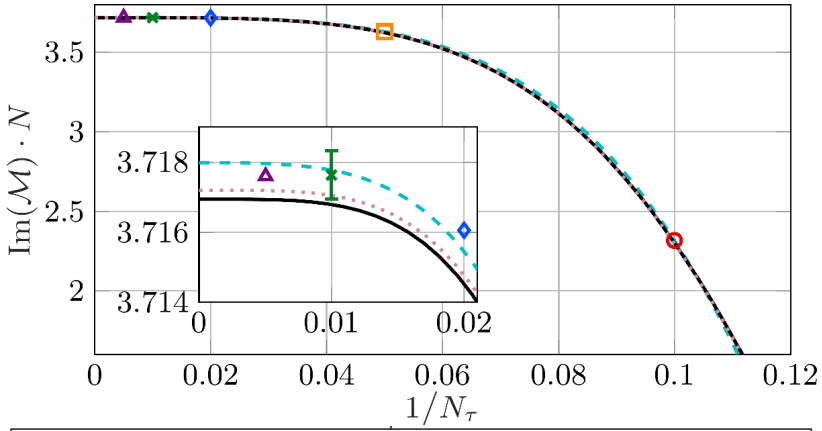
fit1: $-0.0037 + 89.6420 N_\tau^{-2}$
fit2: $Re\left(6.9800 exp\left(-6.4460 i N_\tau^{-2} \Delta z\right)\right)$
fit3: $Re\left(6.9800 exp\left(-6.4411 i N_\tau^{-2} \Delta z\right)\right)$



fit1: $-0.0005 + 292.1300 N_\tau^{-2}$
fit2: $Re\left(3.7169 exp\left(-6.4460 i N_\tau^{-2} \Delta z\right)\right)$
fit3: $Re\left(3.7172 exp\left(-6.4569 i N_\tau^{-2} \Delta z\right)\right)$



fit1: $6.9805 + 0.0179 N_\tau^{-2} - 600.9051 N_\tau^{-4}$
fit2: $Im\left(6.9800 exp\left(-6.4460 i N_\tau^{-2} \Delta z\right)\right)$
fit3: $Im\left(6.9800 exp\left(-6.4411 i N_\tau^{-2} \Delta z\right)\right)$

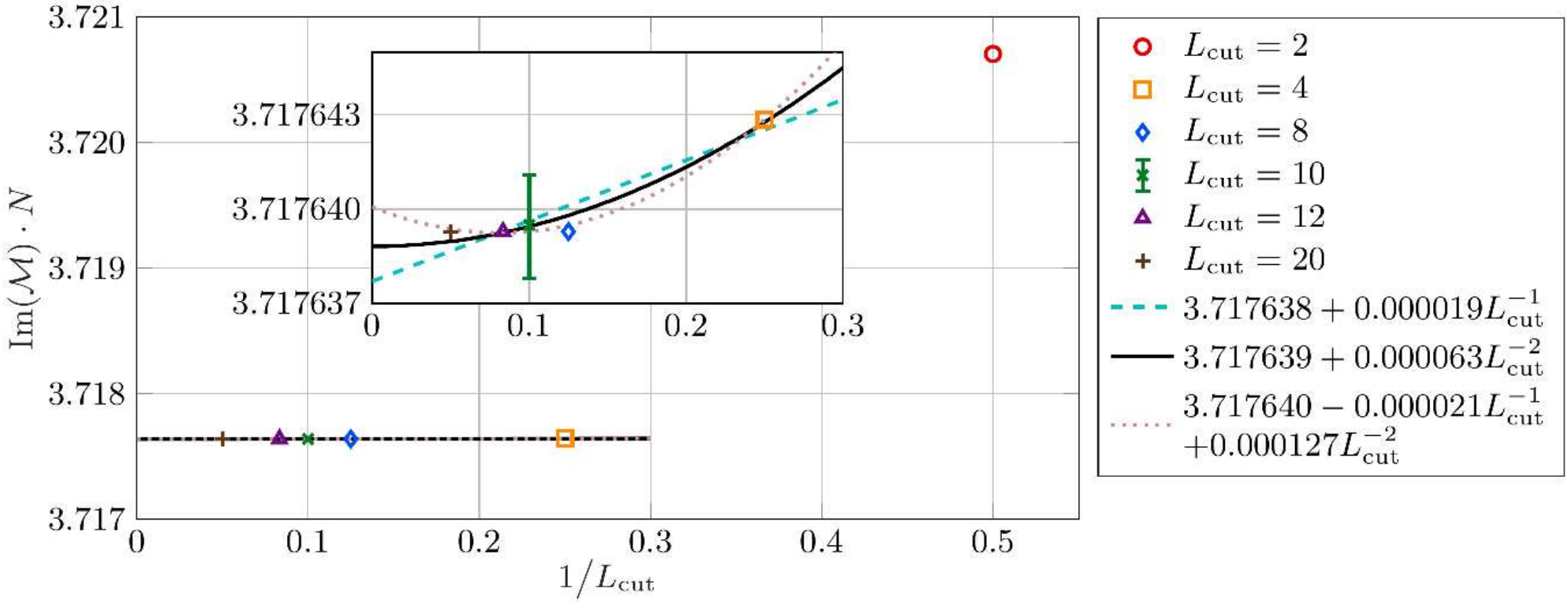


fit1: $3.7180 - 0.7655 N_\tau^{-2} - 13910.7396 N_\tau^{-4}$
fit2: $Im\left(3.7169 exp\left(-6.4460 i N_\tau^{-2} \Delta z\right)\right)$
fit3: $Im\left(3.7172 exp\left(-6.4569 i N_\tau^{-2} \Delta z\right)\right)$

Matrix elements – electric flux truncation

arXiv:2504.07508

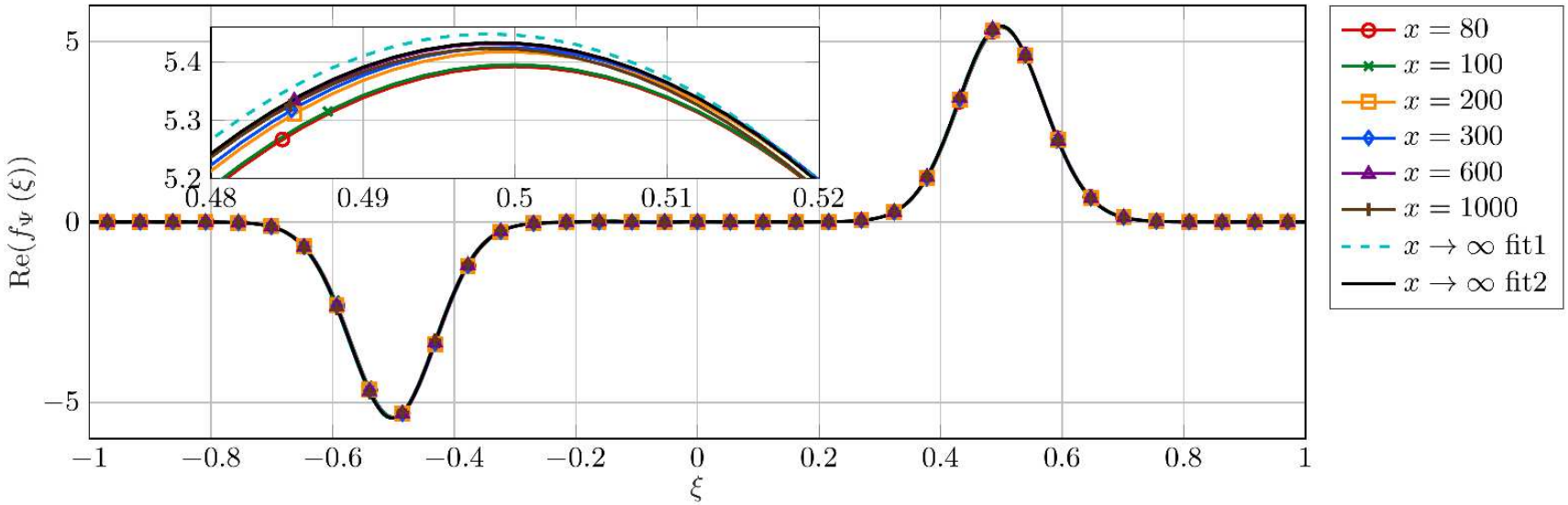
$$\tilde{m} = 10, \quad x = 100, \quad D = 80, \quad N_\tau = 100, \quad \tilde{V} = 100$$



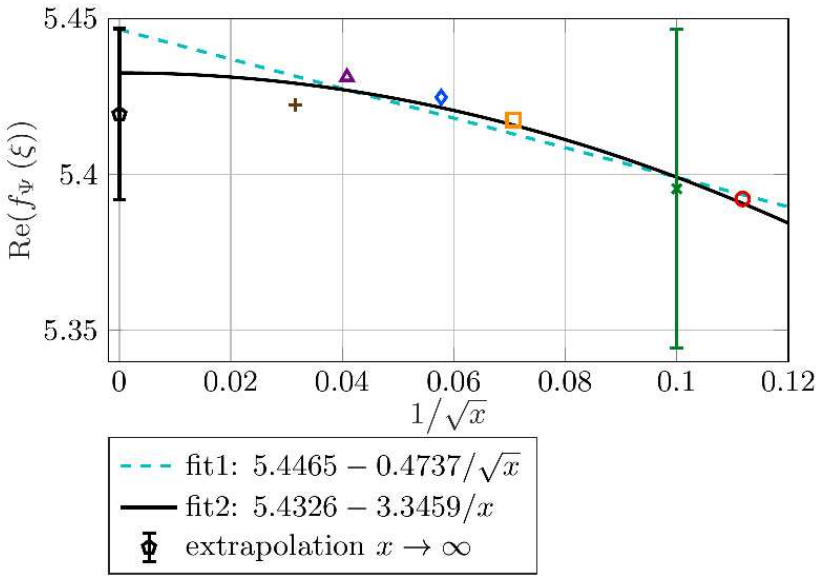
PDF – lattice spacing effects

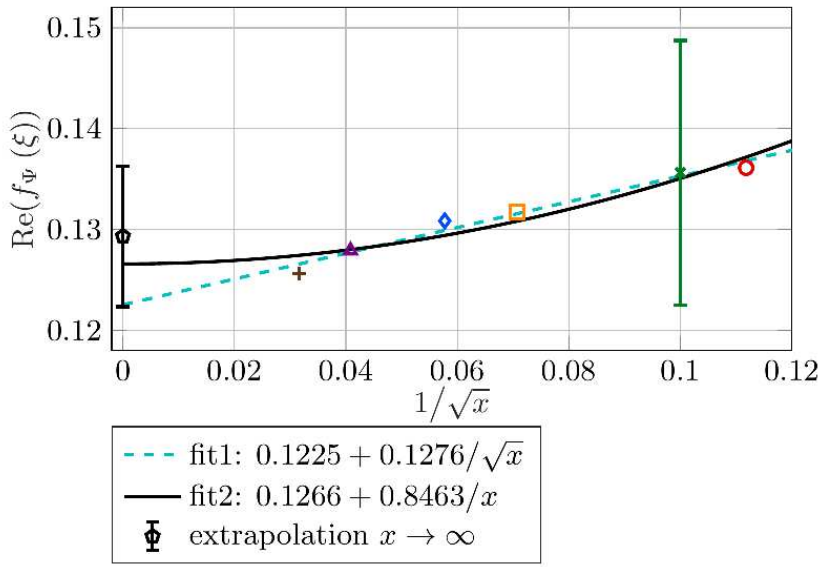
$\tilde{m} = 10, \quad D = 80, \quad N_\tau = 100, \quad \tilde{V} = 100$

arXiv:2504.07508



$\xi = 0.5$



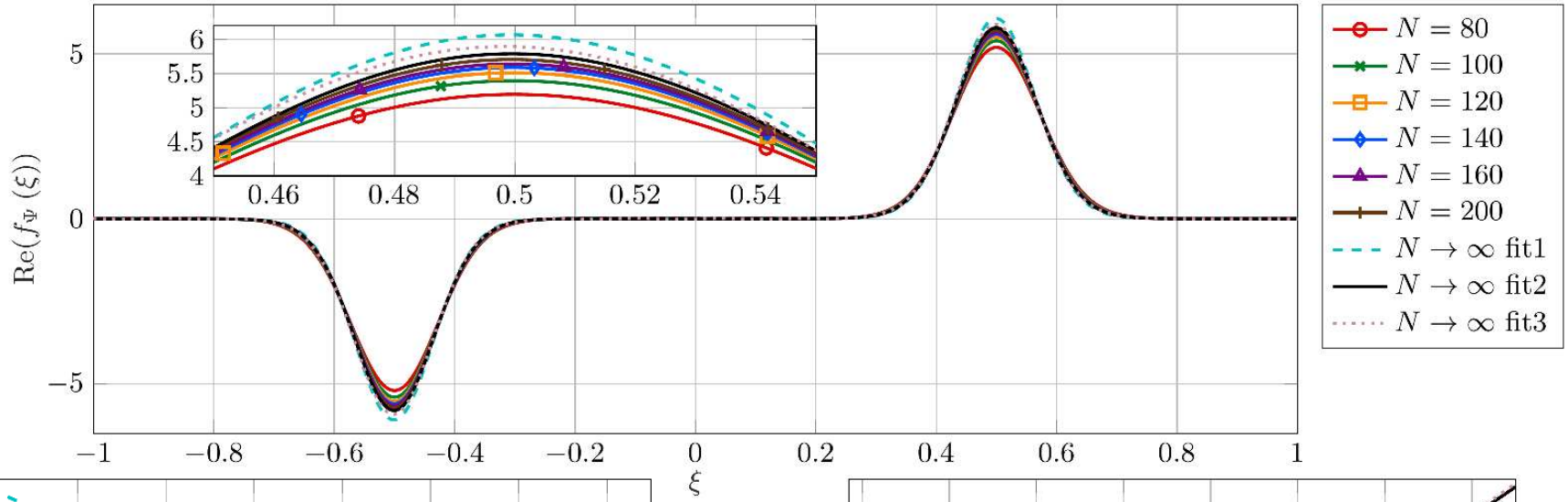


$\xi = 0.7$

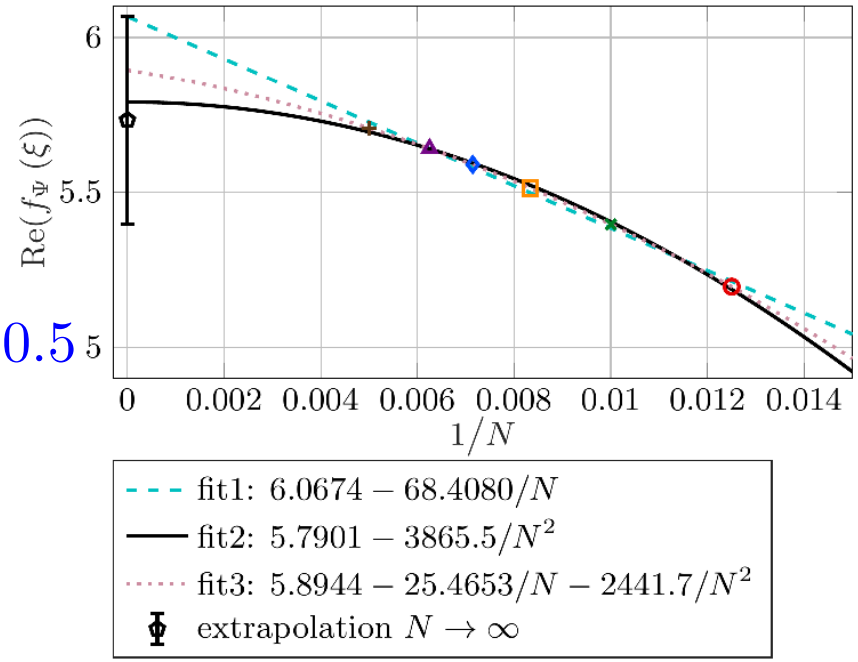
PDF – finite volume effects

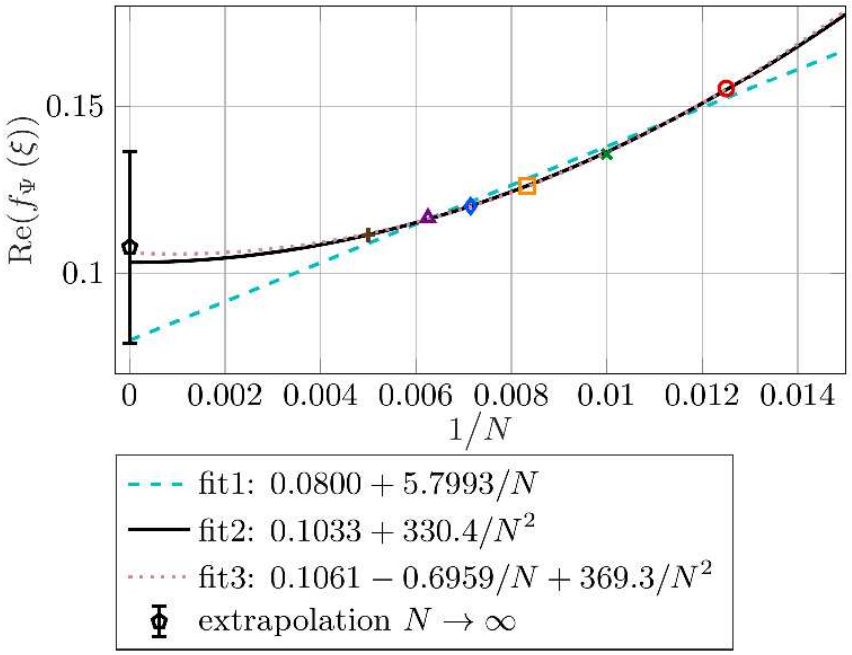
$\tilde{m} = 10, \quad D = 80, \quad N_\tau = 100, \quad x = 100$

arXiv:2504.07508



$\xi = 0.5$

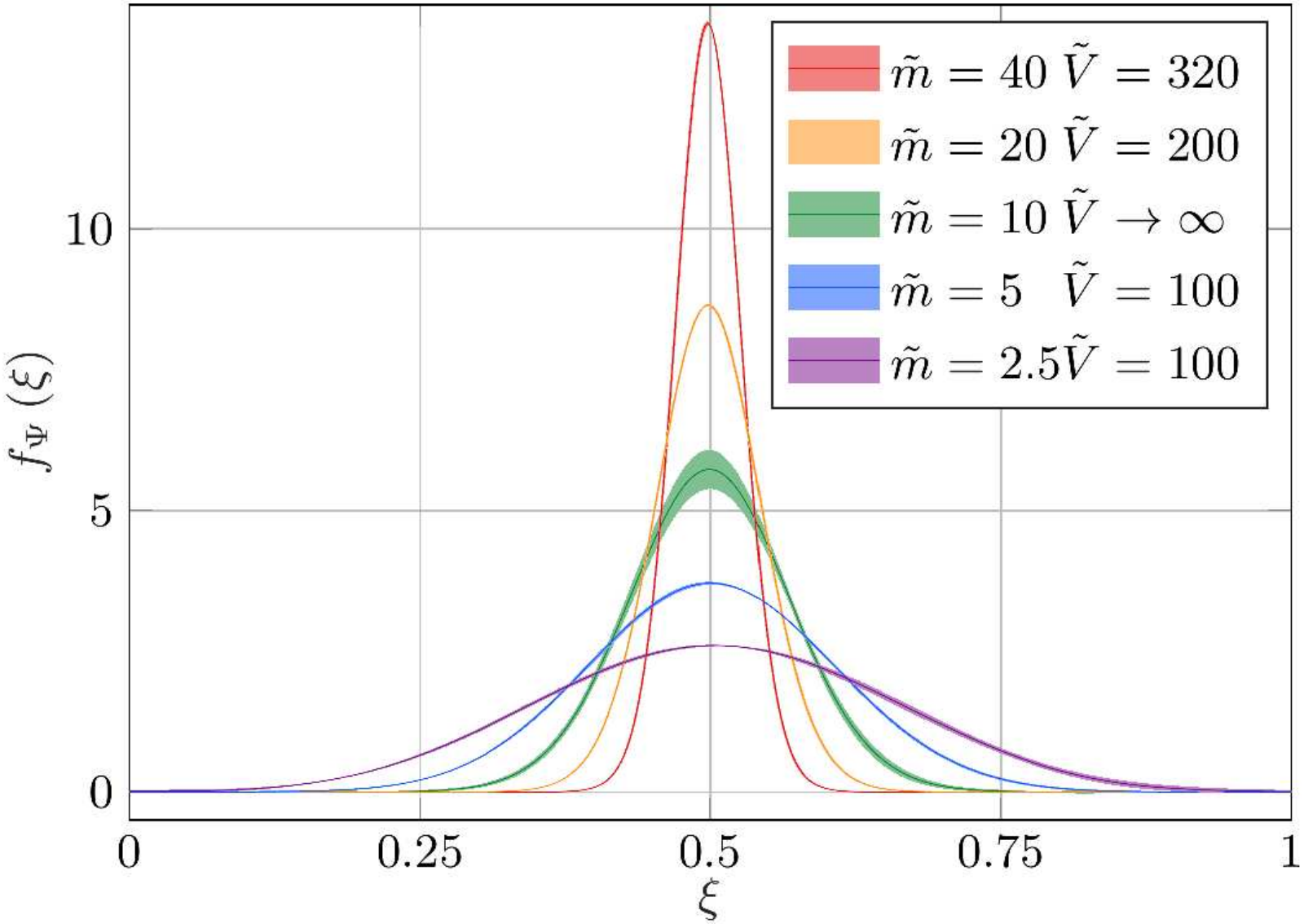




$\xi = 0.7$

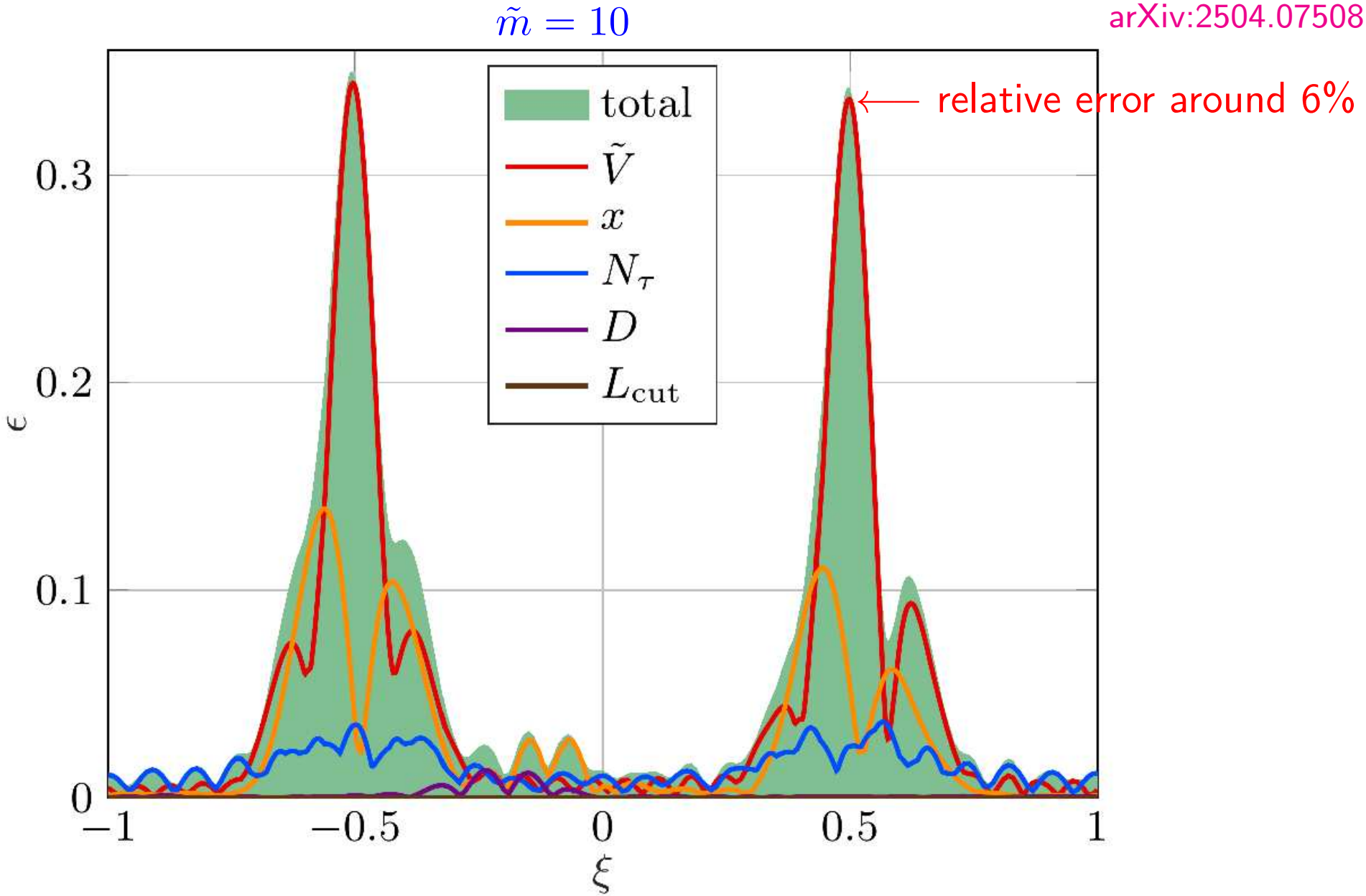
Final PDF for different masses

arXiv:2504.07508

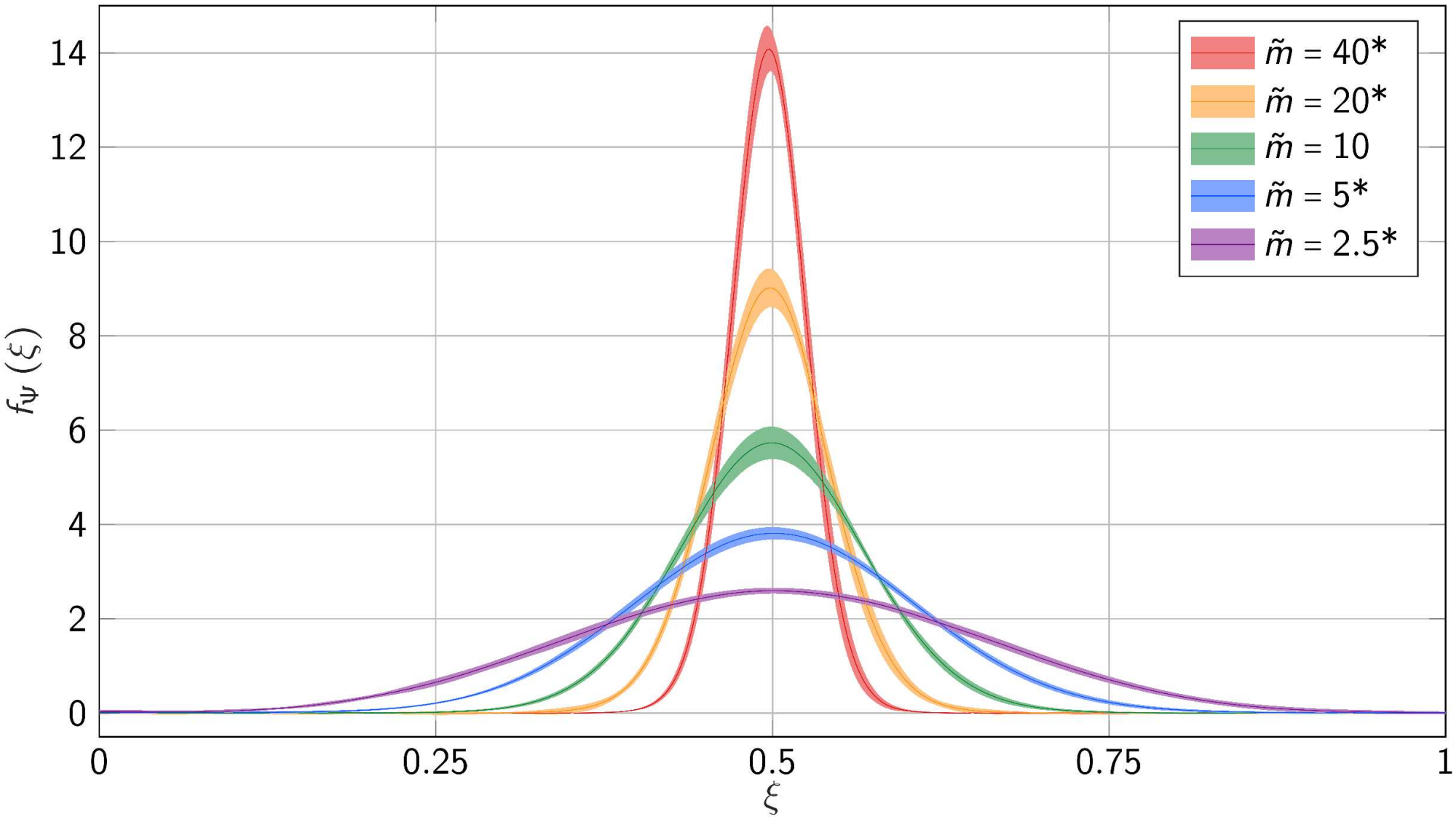


Systematic error budget for the final PDF

arXiv:2504.07508



Final PDF for different masses



Systematic errors for masses marked with a star preliminary

Summary

- PDFs accessible with indirect methods in standard LQCD, but desirable to have direct access to them!
- Such direct access possible with TNs.
- First calculation for the Schwinger model, focused on exploring the control over different systematic effects.
  arXiv:2504.07508
- We are exploring also the light-cone distribution amplitude.
- The biggest challenge obviously is to make it all practical in higher dimensions.

Thank you for your attention!