

Graph Representations and Circuit-Based Codes from GHZ States

GHZ states are fundamental in quantum information science, playing roles in communication, nonlocality tests, and quantum error correction. In this work, we explore two complementary aspects of GHZ states relevant for both theory and implementation.

First, we address the structure of non-symmetric GHZ states, defined as unequal superpositions of computational basis states, which frequently occur in qudit-based experiments due to decoherence and imperfect amplitude or phase control during state preparation. While symmetric GHZ states are known to be locally unitary (LU) equivalent to graph states, their non-symmetric counterparts lack a comparable stabilizer or graph-theoretic framework. We show that these non-symmetric GHZ states are LU-equivalent to two graphical constructions: (i) fully connected weighted hypergraph states with multi-qudit controlled-phase gates, and (ii) controlled-unitary (CU) star-shaped graphs. Although weighted hypergraph states typically do not admit a stabilizer description, we construct a full set of stabilizers using only a single ancilla qudit, independent of system size.

Second, we introduce a new method for constructing quantum error-correcting codes by embedding perfect (symmetric) GHZ states into brickwall circuit architectures. The building blocks of these circuits are GHZ states forming a $[[3, 1, 2]]_d$ code, which we use to generate larger codes through circuit composition. This approach allows us to recover quantum codes such as $[[6, 1, 3]]$, $[[9, 1, 4]]$, and $[[12, 1, 4]]$, while providing insight into how stabilizer structures evolve through circuit dynamics. The method offers a modular and scalable framework for circuit-based quantum error correction, potentially useful for fault-tolerant computing and entanglement-assisted protocols.

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