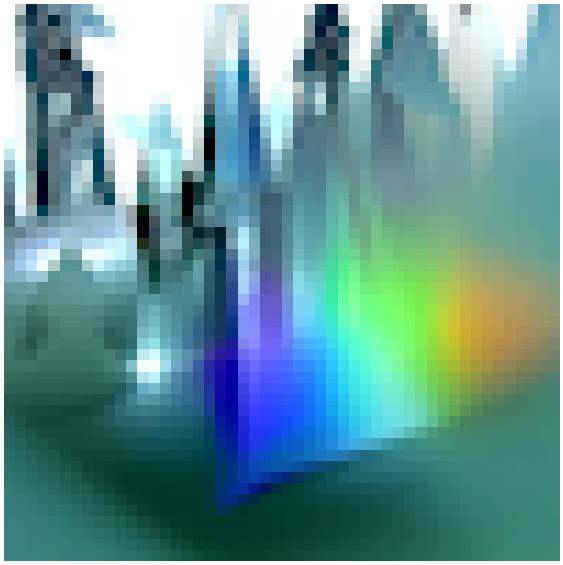
Noncommutative geometry: metric and spectral aspects



Report of Contributions

https://indico.global/e/10049

Contribution ID: **1** Type: **not specified**

Atiyah sequences of braided Lie algebras and their splittings

Wednesday 28 September 2022 09:00 (1 hour)

Presenter: LANDI, Giovanni

Session Classification: 28-morning

Contribution ID: **2** Type: **not specified**

Dual of the dual formula for the distance in noncommutative geometry

Wednesday 28 September 2022 10:00 (1 hour)

Connes's spectral distance is an extended metric on the state space of a C*-algebra, generalizing Kantorovich's dual formula of the Wasserstein distance of order 1 from optimal transport. It is expressed as a supremum. We present a dua formula - as an infimum - generalizing Beckmann's "dual of the dual" formulation of the Wasserstein distance.

Presenter: MARTINETTI, Pierre

Session Classification: 28-morning

Contribution ID: **3** Type: **not specified**

Spectral and metric aspects of the Dolbeault-Dirac spectral triple on quantum SO(5)/(SO(2)xSO(3)

Friday 30 September 2022 13:35 (25 minutes)

The Berstein-Gelfand-Gelfand resolution for irreducible quantum flag manifolds gives an algebraic description of the Dolbeault complex of (anti-)holomorphic k-forms by actions of quantum tangent space. Requiring equivariance and compatibility with the real form of the quantum enveloping algebra, there is an essentially unique hermitian metric on the (0,k)-forms given by the Haar state. Using equivariance, spectral computations can be reduced to determining the eigenvalues of the Laplace operator on 1-dimensional highest weight spaces.

Presenter: WAGNER, Elmar

Session Classification: 30-morning

Contribution ID: **4** Type: **not specified**

Around heat kernel

Friday 30 September 2022 10:00 (1 hour)

First, I present a few properties of heat kernel and its trace and then, revisit the way of computing the coefficients of the heat trace asymptotics for a differential operator acting on a fiber bundle over a Riemannian manifold in a way which avoids entering within the pseudodifferential theory.

Presenter: IOCHUM, Bruno

Session Classification: 30-morning

Contribution ID: **5** Type: **not specified**

Curvature for modules over spectral triples

Wednesday 28 September 2022 15:30 (1 hour)

In this talk we introduce the curvature of densely defined universal connections on Hilbert C*-modules relative to a spectral triple, obtaining a well-defined curvature operator. Algebraically, this curvature can be interpeted as the defect of the unbounded Kasparov product to commute with the operation of taking squares. The definition recovers the represented curvature of finitely generated projective modules as well as all the curvature data of a Riemannian submersion of compact manifolds, viewed as a KK-factorization.

Presenter: MESLAND, Bram

Session Classification: 28-afternoon

Contribution ID: **6** Type: **not specified**

The Gromov-Hausdorff distance in noncommutative geometry: convergence of quantum compact metric spaces and their modules

Wednesday 28 September 2022 16:30 (1 hour)

Connes' Spectral triples have emerged as the preferred tool to encode geometric information over possibly noncommutative C*-algebras. We present, in two lectures, a distance on the space of metric spectral triples, which then enables us to formally discuss ideas such as approximations or perturbations of spectral triples, and opens the possibility to study the geometry of spaces of spectral triples.

The foundation for our distance between spectral triples, called the spectral propinquity, is the Gromov-Hausdorff distance between compact metric spaces, for which we constructed an analogue on the class of quantum compact metric spaces, called the Gromov-Hausdorff propinquity. The propinquity can then be extended to certain modules over compact quantum metric spaces. This talk will present the constructions, and some examples of convergences, of the propinquity on compact quantum metric spaces and their modules.

Presenter: LATREMOLIERE, Frederic

Session Classification: 28-afternoon

Contribution ID: **7** Type: **not specified**

Spectral gaps for higher Laplacians and group cohomology

Wednesday 28 September 2022 11:20 (25 minutes)

I will focus on the cohomology of finitely presented groups. I plan to investigate two conditions concerning it: vanishing and reducibility (for all unitary representations). These conditions are related to Kazhdan's property (T): vanishing and reducibility coincide in degree one and are equivalent to this property. It is already known, due to the work of Dymara and Januszkiewicz, that this does not hold in higher cohomology. However, In 2020, Bader and Nowak gave a sufficient condition for vanishing and reducibility in two consecutive degrees. This condition translates these cohomological questions to the existence of a certain spectral gap of a Jacobian matrix, defined in the language of group rings. We were able to prove the existence of such a spectral gap for SL(3,Z), the special linear group of degree three. It was already known that this group does have property (T). The question of reducibility of the second cohomology remained opened, however. The existence of such spectral gap proves the reducibility. The key new tool we applied to prove this existence is the Fox calculus which can be used to compute group cohomology in lower degrees. I also comment on an application of the spectral gap for SL(3,Z) to expanders. This is the joint work with Marek Kaluba and Piotr Nowak (arxiv.org/abs/2207.02783).

Presenter: MIZERKA, Piotr

Session Classification: 28-morning

Contribution ID: **8** Type: **not specified**

Quantum Kaluza-Klein theory with M_2(C)

Friday 30 September 2022 12:45 (25 minutes)

We follow a quantum Kaluza-Klein formulation where we solve for the quantum Riemannian geometry on A = C∞(M) ⊗ M2(C) in terms of classical Riemannian geometry on M, the finite quantum geometry on M2(C) and gauge-field like cross term. We look at how scalar fields on the total space decompose into multiplets of fields in M differing in mass.

Presenter: LIU, Chengcheng

Session Classification: 30-morning

Contribution ID: **11** Type: **not specified**

Tolerance relations and quantization

Thursday 29 September 2022 10:00 (1 hour)

It is well known that "bad"quotient spaces can be studied by associating to them the groupoid C*-algebra of an equivalence relation. A similar procedure for relations that are reflexive and symmetric but fail to be transitive leads to a non associative algebra. I will discuss some of its properties based on a recent joint work with G. Landi and F. Lizzi.

Presenter: D'ANDREA, Francesco

Session Classification: 29-morning

Contribution ID: **12** Type: **not specified**

Noncommutative spaces at finite resolution

Thursday 29 September 2022 09:00 (1 hour)

We extend the traditional framework of noncommutative geometry in order to deal with two types of approximation of metric spaces. On the one hand, we consider spectral truncations of geometric spaces, while on the other hand, we consider metric spaces up to a finite resolution. In our approach the traditional role played by C*-algebras is taken over by so-called operator systems. We consider* C-envelopes and introduce a propagation number for operator systems, which we show to be an invariant under stable equivalence and use it to compare approximations of the same space. We illustrate our methods for concrete examples obtained by spectral truncations of the circle, and of metric spaces up to finite resolution.

Presenter: VAN SUIJLEKOM, Walter

Session Classification: 29-morning

Contribution ID: **13** Type: **not specified**

The Gromov-Hausdorff distance in noncommutative geometry: convergence of spectral triples

Thursday 29 September 2022 16:30 (1 hour)

We present a distance on the class of metric spectral triples. It thus becomes possible to formally discuss the idea of approximating a spectral triple with others: for instance, to approximate a spectral triple on the 2-torus by means of natural spectral triples on the finite dimensional so-called fuzzy tori.

A spectral triple induces an extended pseudo-metric on the state space of its underlying C*-algebra, which under reasonable conditions, gives rise to a quantum compact metric space. In addition, the Dirac operator of a spectral triple, as a self-adjoint operator, generates a 1-parameter unitary group. Putting these ideas together in a covariant version of the propinquity for modules, we obtain a well-defined, well-behaved metric on spectral triples. In addition to the convergence of spectral triples on fuzzy tori, or the approximations of the spectral triples on some fractals by means of spectral triples on graphs, we also prove that convergence for the spectral propinquity implies a natural convergence of the spectra of the Dirac operators, and under mild conditions, of the associated spectral actions.

Presenter: LATREMOLIERE, Frederic

Session Classification: 29-afternoon

Contribution ID: **14** Type: **not specified**

Spectral metric and Einstein tensors.

Thursday 29 September 2022 11:20 (1 hour)

We define bilinear functionals of vector fields and differential forms, the densities of which yield the metric and Einstein tensors on even-dimensional Riemannian manifolds. We generalise these concepts in non-commutative geometry and, in particular, we prove that for the conformally rescaled geometry of the noncommutative two-torus the Einstein functional vanishes.

Presenter: DABROWSKI, Ludwik

Session Classification: 29-morning

Contribution ID: **15** Type: **not specified**

Lorentzian spectral zeta functions

Thursday 29 September 2022 14:30 (1 hour)

The spectral theory of the Laplace–Beltrami operator on Riemannian manifolds is known to be intimately related to geometric invariants such as the Einstein-Hilbert action. These relationships have inspired many developments in physics including the Chamseddine–Connes action principle in the non-commutative geometry programme. However, a priori they do only apply to the case of Euclidean signature. The physical setting of Lorentzian manifolds has in fact remained problematic for very fundamental reasons. In this talk I will present results that demonstrate that there is a well-posed Lorentzian spectral theory nevertheless, and it is related to Lorentzian geometry in a way that resembles results known so far only in Euclidean signature. Namely, we consider perturbations of Minkowski space and more general spacetimes on which the d'Alembertian P is essentially self-adjoint. It is then possible to define functions of P, and we demonstrate (in a joint work with Nguyen Viet Dang, Sorbonne Université) that their Schwartz kernels have geometric content largely analogous to the Riemannian setting. In particular, we define a Lorentzian spectral zeta function and relate one of its poles to the Einstein–Hilbert action. If time permits, I will also sketch new advances on the closely related case of the square of the Lorentzian Dirac operator.

Presenter: WROCHNA, Michał

Session Classification: 29-afternoon

Contribution ID: **16** Type: **not specified**

Quantum metric structures on q-deformed spaces

Thursday 29 September 2022 15:30 (1 hour)

I will survey recent results concerning the quantum metric structures on the Podleś sphere and quantum SU(2). Along the way, I will provide a brief introduction to Rieffel's theory of compact quantum metric spaces and the notion of quantum Gromov-Hausdorff convergence. The talk is based on joint works with Konrad Aguilar, Thomas Gotfredsen and Jens Kaad.

Presenter: KYED, David

Session Classification: 29-afternoon

Contribution ID: **17** Type: **not specified**

Causality in noncommutative deformation spacetimes

Thursday 29 September 2022 12:20 (30 minutes)

A notion of causality specific to noncommutative geometry was introduced in 2013 by Franco and Eckstein, leading at the same time to a notion of Lorentzian metric. This notion has been widely explored with regards to almost-commutative space-times. In this talk, we present a more difficult exploration concerning "truly" noncommutative spacetimes, i.e. deformation spacetimes as Moyal spacetime and kappa-Minkowski spacetime. In both cases, a complete characterisation of the whole causal structure is still out of sight, but it is possible to highlight interesting allowed causal relations as well as constraints, which are some quantum analogues of classical causal relations and constraints but defined on non-local states.

Presenter: FRANCO, Nicolas

Session Classification: 29-morning

Contribution ID: **18** Type: **not specified**

TBA

TBA

Contribution ID: **19** Type: **not specified**

Geometrically realised spectral triples and dynamical mass

Friday 30 September 2022 09:00 (1 hour)

We apply the geometric realisation programme for spectral triples within quantum Riemannian geometry (QRG) to the algebra of 2x2 matrices with a Lorentzian metric to find an S^1 moduli of almost spectral triples where the Dirac operator is not hermitian but has natural hermitian and antihermitian parts (based on joint work with E. Lira-Torres). I will also explain how Kaluza Klein ideas look in QRG and lead to multiplets of fields on spacetime with a distribution of dynamically generated masses (based on joint work with J. Argota-Quiroz).

Presenter: MAJID, Shahn

Session Classification: 30-morning

Contribution ID: **20** Type: **not specified**

Localization and quantum observers in some Lie-algebra type noncommutative spacetimes.

Wednesday 28 September 2022 14:30 (1 hour)

I will discuss how the observers in a noncommutative spacetime need to be considered quantum objects as well, and discuss issues of localization of states for kappa-Minkowski spaces and some (angular type) variants of it.

Presenter: LIZZI, Fedele

Session Classification: 28-afternoon

Contribution ID: **21** Type: **not specified**

Sequences of Noncommutative Gauge Field Theories on AF-algebras

Friday 30 September 2022 11:00 (1 hour)

With Gaston Nieuviarts we have built and started to study a new framework to define sequences of Noncommutative Gauge Field Theories (NCGFT) on top of the defining sequence of an AF algebra. The main objective of this construction is to manage the way these NCGFT are related to each other along the sequence. A notion of "compatibility"is then necessary to handle this problem. In my talk, I will focus on the motivations for this work and on the formal aspects. Then I will illustrate some physical applications using derivation-based NCGFT. In his talk, Gaston Nieuviarts will focus on this construction for spectral triples.

Presenter: MASSON, Thierry

Session Classification: 30-morning

Contribution ID: **22** Type: **not specified**

Spectral triple based Noncommutative Gauge Field Theories on AF-algebras

Friday 30 September 2022 13:10 (25 minutes)

In the continuity of the presentation made by Thierry Masson, in which the general framework was introduced to define Noncommutative Gauge Field Theories (NCGFT) on top of the sequence of an AF algebra, I will present the part of our work that focuses on the study of these NCGFTs using spectral triples. In particular, I will insist on the
"compatibility" relations on the defining structure of (real, odd) spectral triples, and their impacts on the relations between Lagrangians.

Presenter: NIEUVIARTS, Gaston

Session Classification: 30-morning

Contribution ID: **23** Type: **not specified**

Differential nested pairs of quantum homogeneous spaces.

Friday 30 September 2022 12:20 (25 minutes)

TBA

Presenter: CAROTENUTO, Alessandro

Session Classification: 30-morning

Contribution ID: **24** Type: **not specified**

Spectral triple with real structure on fuzzy sphere

Wednesday 28 September 2022 11:45 (25 minutes)

Here we have illustrated the construction of a real structure on a fuzzy sphere Sˆ*_2 in its spin-1/2 representation. Considering the SU(2) covariant Dirac and chirality operator on Sˆ∗_2 given by U. C. Watamura et.al. [Commun. Math. Phys. 183, 365–382 (1997)], we have shown that the real structure is consistent with other spectral data for KO dimension-4 fulfilling the zero order condition, where we find it necessary to enlarge the symmetry group from SO(3) to the full orthogonal group O(3). However, the first order condition is violated, thus paving the way to construct a toy model for an SU(2) gauge theory to capture some features of physics beyond the standard model.

Presenter: CHAKRABORTY, Anwesha

Session Classification: 28-morning

Contribution ID: **25** Type: **not specified**

Hopf algebroids on quantum projective spaces

Wednesday 28 September 2022 12:10 (25 minutes)

We construct the Ehresmann-Schauenburg bialgebroid for a family of U(1)-quantum principal bundles over quantum projective spaces, showing that another antipode (related to K-theory on the base algebra) exists besides the "classical" flip. Moreover, we show how the theory of twists (generalized characters) applies in this situation.

Presenter: ZANCHETTIN, Jacopo

Session Classification: 28-morning