The kinetic gas universe
-
Lifting the Einstein Vlasov system to the tangent bundle

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University of Tartu

Joint work with N. Voicu and M. Hohmann

Virtual Conference of the Polish Society on Relativity 2020

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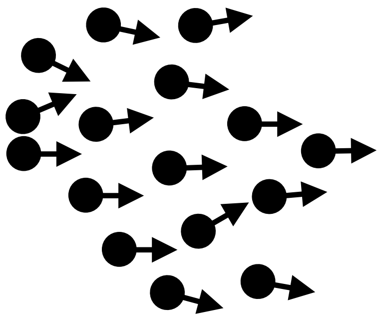
The gravitating kinetic gas

The kinetic gas

The kinetic gas



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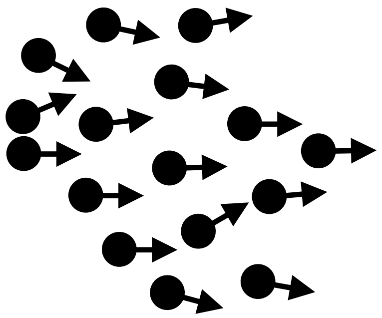

$$\phi(x, \dot{x})$$

$$\frac{\partial \phi}{\partial x^a}\dot{x}^a + \frac{\partial \phi}{\partial \dot{x}^a}\ddot{x}^a = C$$

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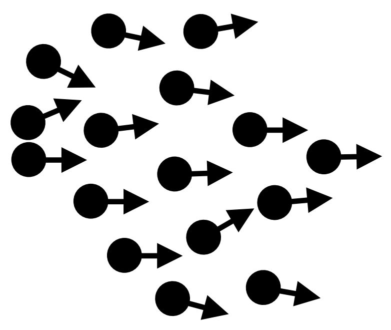

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Einstein Vlasov equations

$$R^{ab} - \frac{1}{2}g^{ab}R = \frac{8\pi G}{c^4}T^{ab}$$

$$T^{ab} = \int_{V_x} \frac{\dot{x}^a\dot{x}^b}{g_{cd}\dot{x}^c\dot{x}^d}\phi(x,\dot{x})\Sigma_x$$

[Andreasson 2011]

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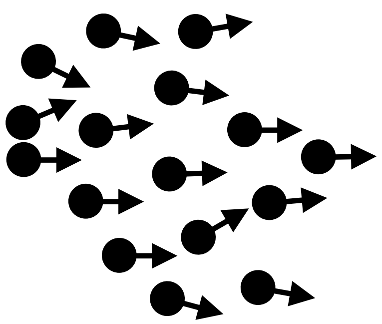

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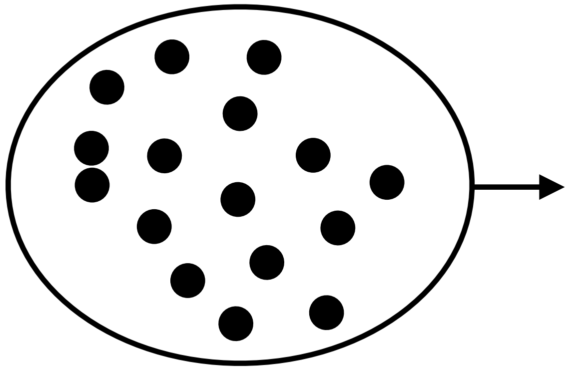
The fluid



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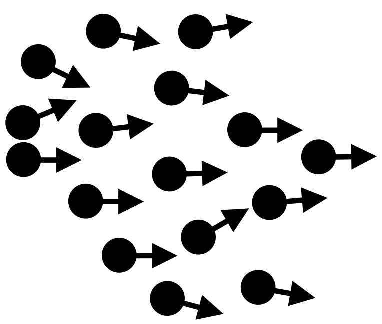

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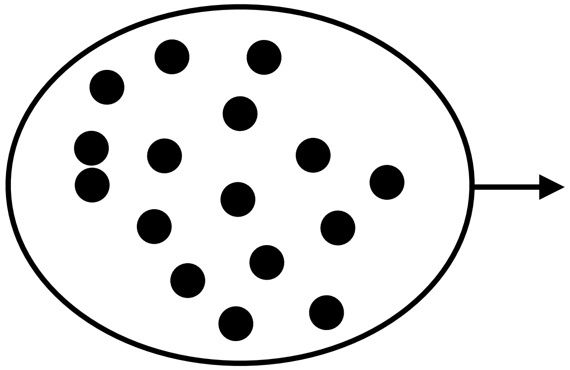

$$T^{ab} = \alpha(x)U^aU^b + \beta(x)g^{ab}$$

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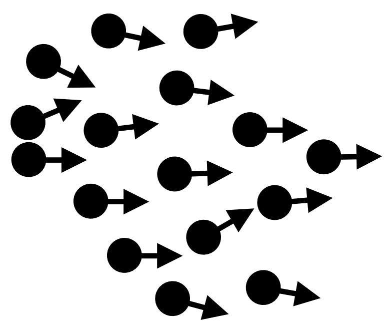

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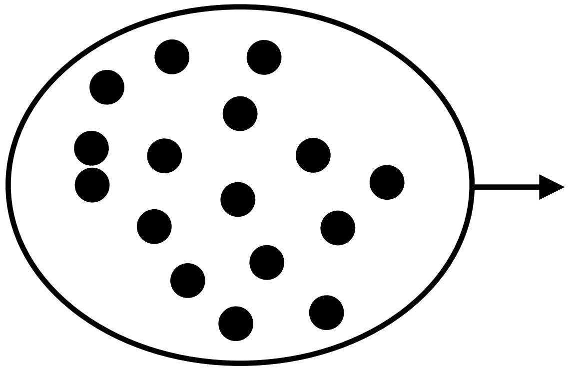

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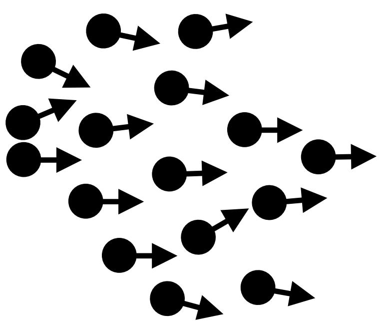

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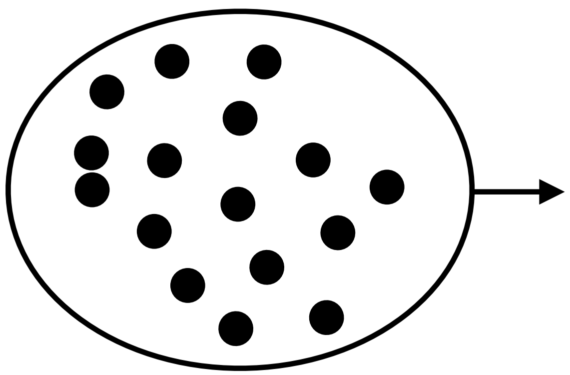

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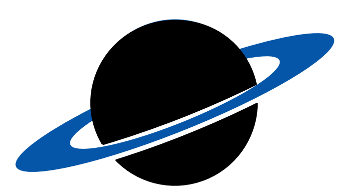
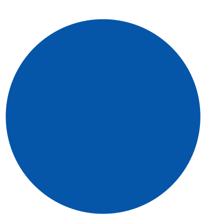
gas planets

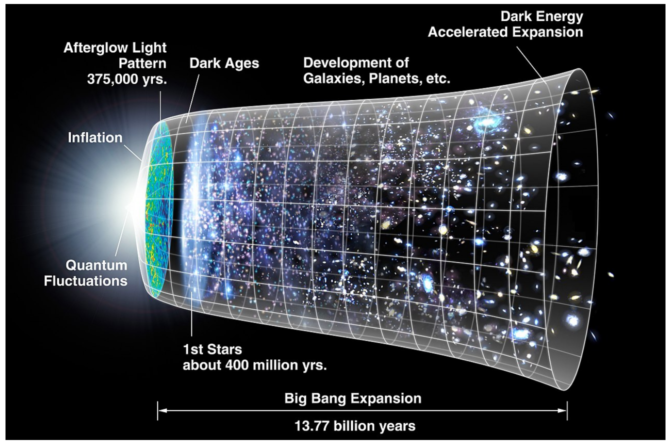
accretion discs



neutron stars



the universe



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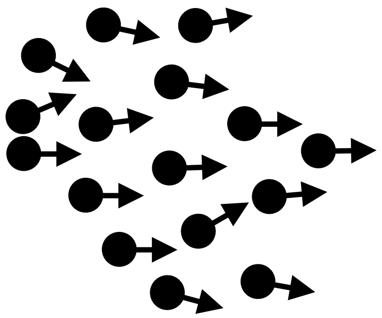

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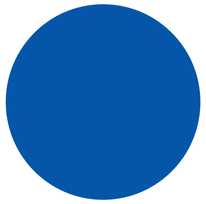
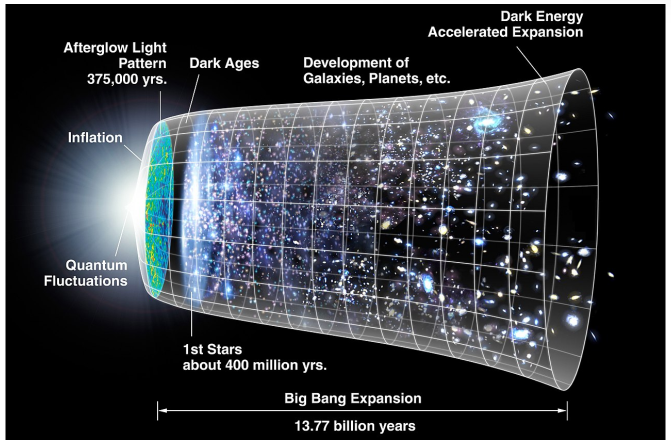
Information is lost through velocity averaging!

gas planets

accretion discs

neutron stars





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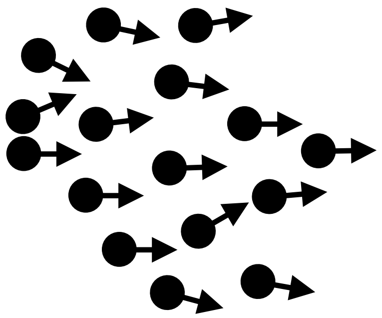

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Information is lost through velocity averaging!

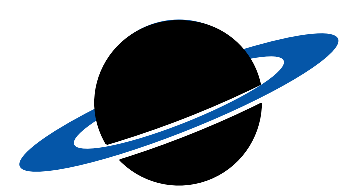
This talk: The gravitational field of kinetic gases without velocity averaging

$$G(x, \dot{x}) = \hat{\kappa}^2 \phi(x, \dot{x})$$

Taking the velocity distribution of the gas particles into account!

gas planets

accretion discs



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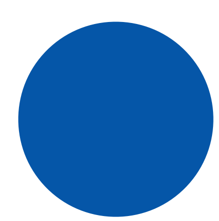
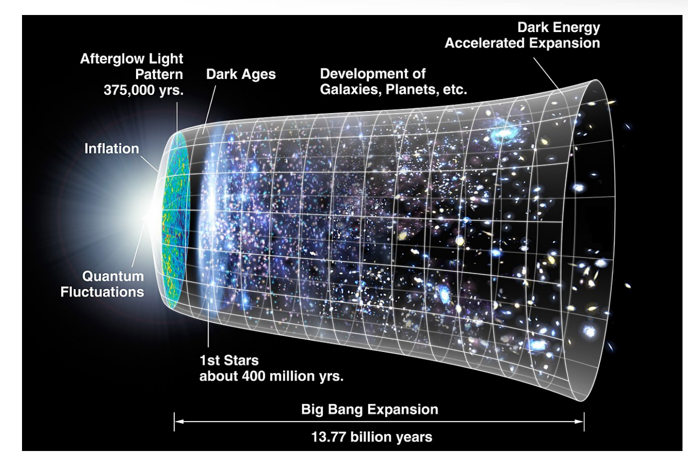
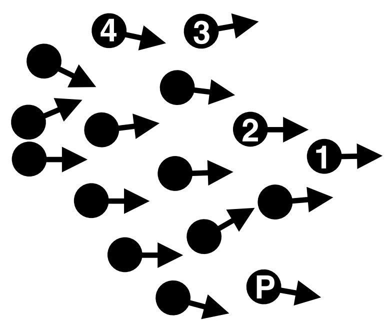




Point particles and Finsler geometry

The relativistic gas

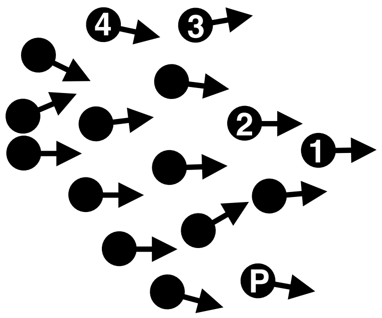


Individual gas particles

$$S[x_I] = m \int_{s_1}^{s_2} ds_I \sqrt{L(x_I, \dot{x}_I)}$$

$$= m \int_0^{t_I} d\tau_I = m t_I$$

The relativistic gas



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Finsler geometry

Geometry defined by $L(x, \dot{x})$

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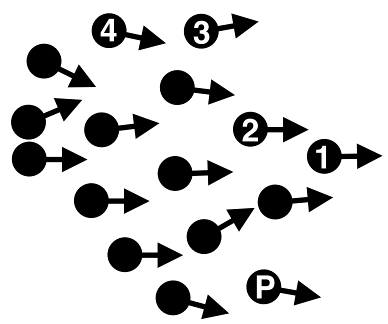

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Geometry defined by $L(x, \dot{x})$

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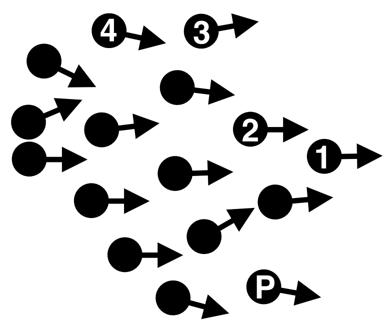
The relativistic gas



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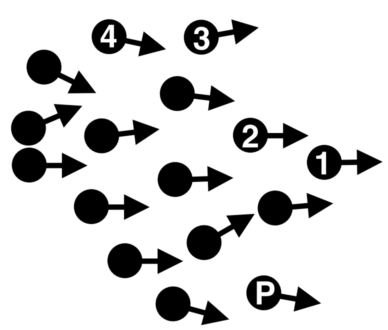

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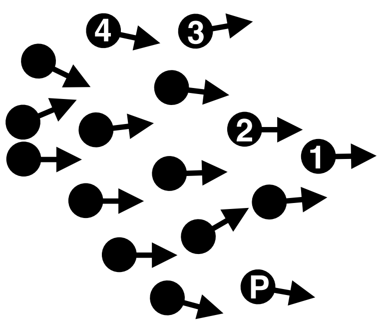

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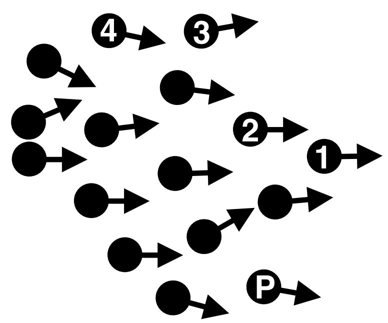

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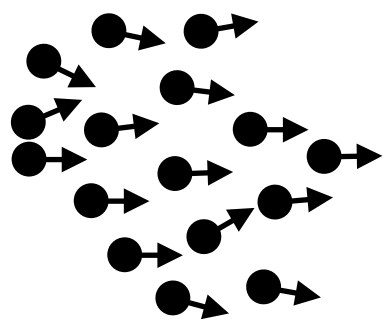
Kinetic gas



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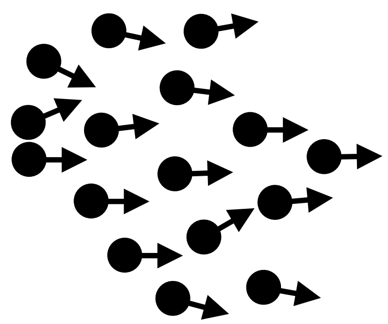

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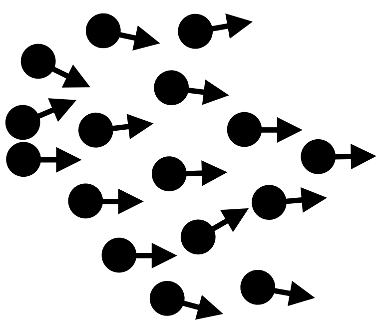

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The Gas action and Finsler geometry

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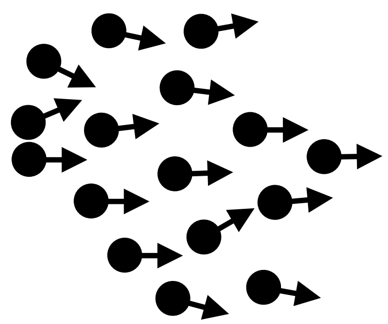

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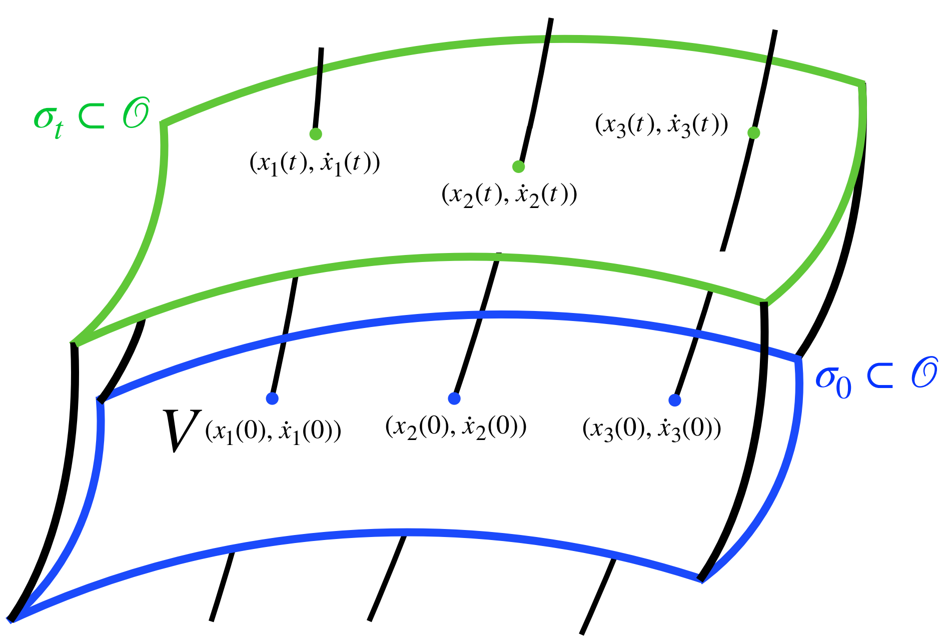
$$= mPt = m \int_0^t P ds$$



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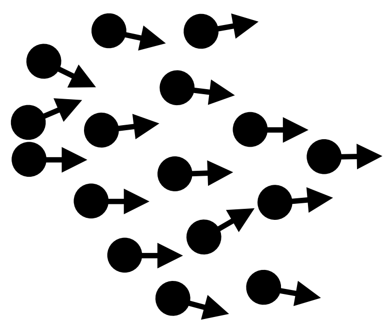
The relativistic gas

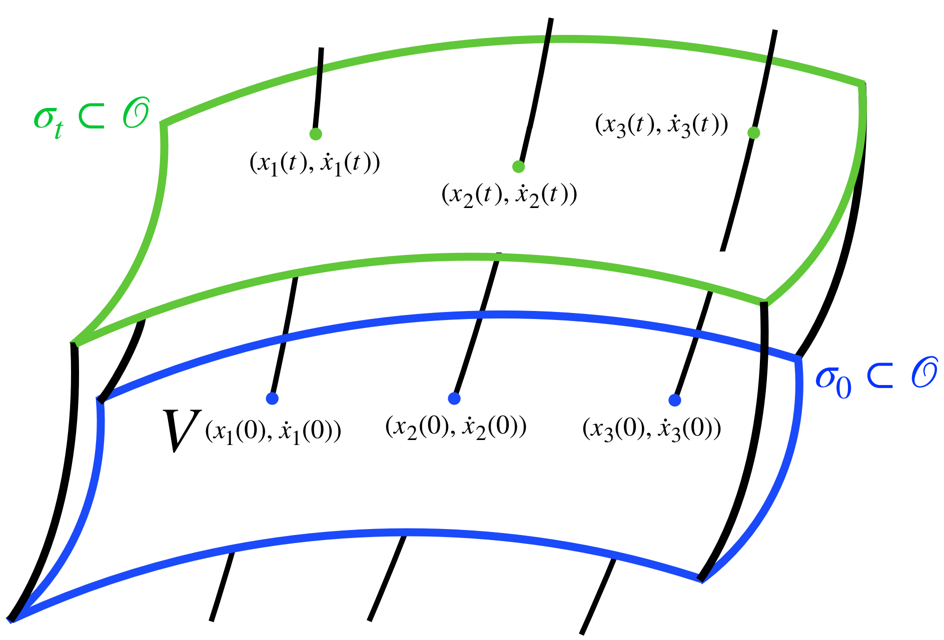


Kinetic gas

$$S_{gas} = m \sum_{i=1}^{P} \int_0^t ds_i \sqrt{L(x_I, \dot{x}_I)}$$

$$= mPt = m \int_0^t P ds$$

$$= m \int_V \phi \Sigma$$



Finsler geometry

Geometry defined by $L(x, \dot{x})$

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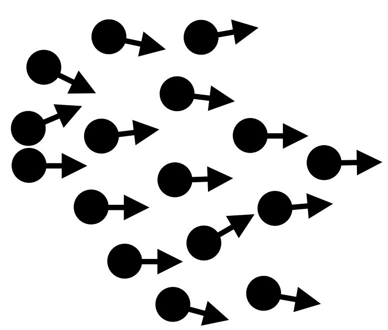
The Gas action and Finsler geometry

The relativistic gas



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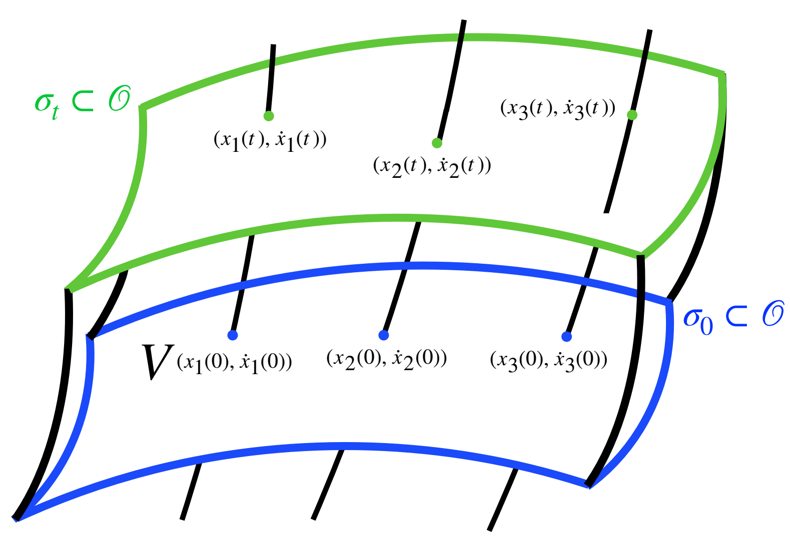
$$\Sigma \sim \det g^L \; d^4x \wedge d^3u$$



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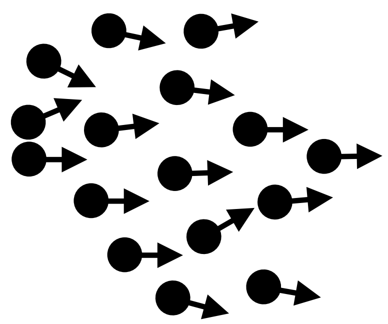

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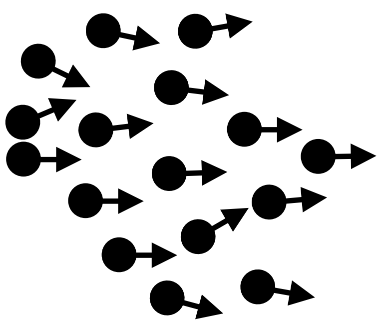
The relativistic gas



Kinetic gas action

$$S_{gas} = m\int_V \phi\Sigma$$

Covariance under coordinate changes

Finsler geometry

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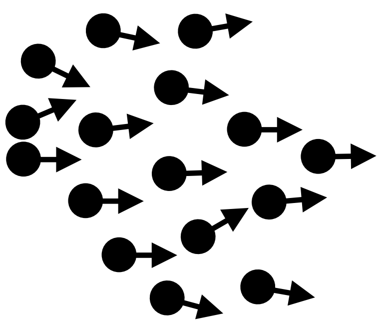
The relativistic gas

Kinetic gas action



$$S_{gas} = m \int_V \phi \Sigma$$

Covariance under coordinate changes

$$\delta_\xi S = m \int_V \xi^b \nabla_{\delta_a} \left(\frac{\phi \dot{x}^a \dot{x}_b}{L} \right) \Sigma + m \int_{\partial_V} J,$$

Finsler geometry

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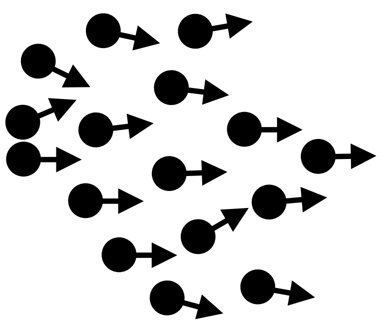
Energy - Momentum distribution tensor

The relativistic gas

Kinetic gas action



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Energy momentum distribution tensor

[Gotay, Marsden 1992]

Finsler geometry

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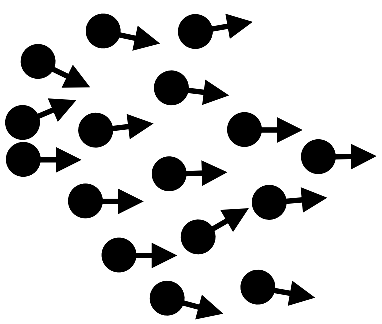
The relativistic gas

Kinetic gas action



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Covariance under coordinate changes

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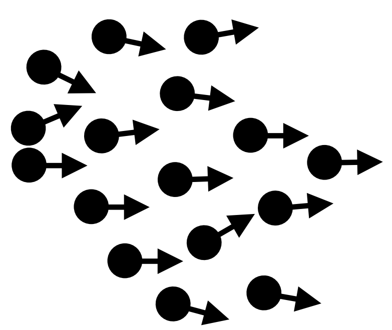
The relativistic gas

Kinetic gas action



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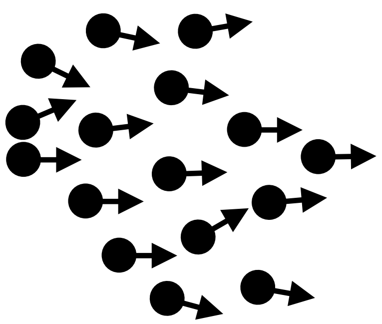
Energy - Momentum distribution tensor

The relativistic gas



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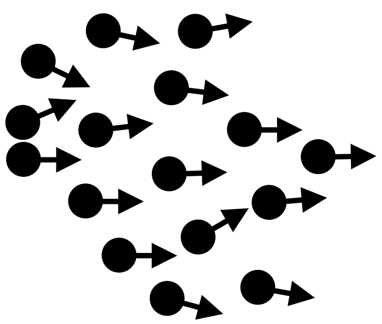

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Finsler geometry

Ge

- L

=

- g

sign$(g^L) = (+,-,-,-)$ on $\mathcal{T} \subset TM$

$$L = g_{ab}(x)\dot{x}^a\dot{x}^b$$

Lorentzian Geometry

- gas particle trajectories satisfy

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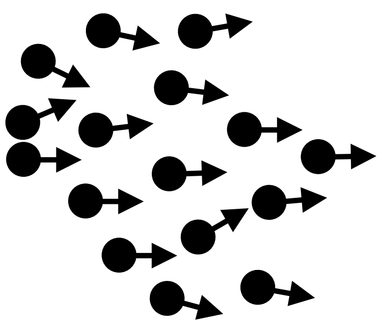
Energy - Momentum distribution tensor

The relativistic gas



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Energy - Momentum distribution tensor

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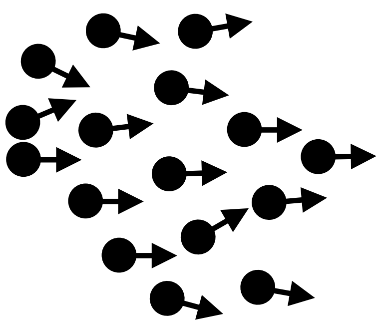

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Variational principle for the Einstein-Vlasov equations [Andersson, Korzynski 2019]

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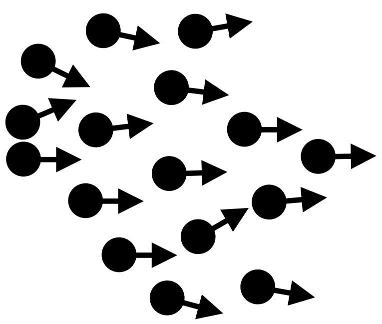

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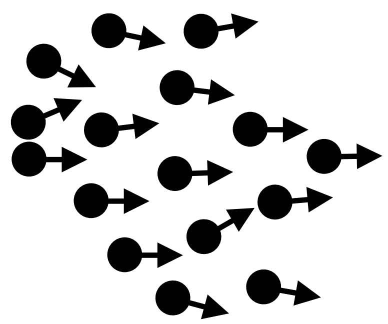
Finsler geometry

Geometry defined by $L(x,\dot{x})$

The relativistic gas



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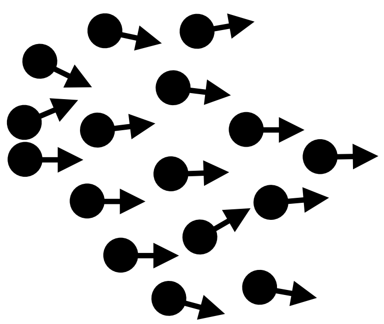
Coupling geometry and gas on the tangent bundle TM

The relativistic gas



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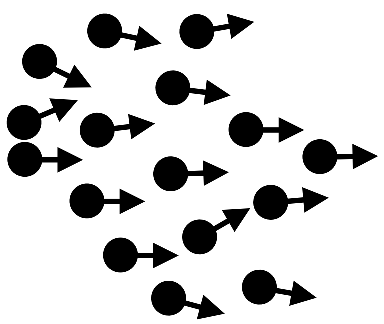
The gas-gravity action

$$S[L] = \tilde{\kappa} \int_V R_0 \, \Sigma + \int_V \phi \, \Sigma$$

The relativistic gas

Kinetic gas



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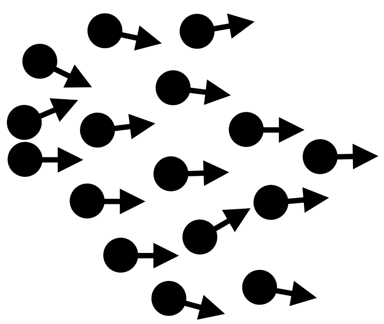
The 1PDF of a relativistic kinetic gas as source of gravity

$$\frac{1}{2} g^{Lab} \dot{\partial}_a \dot{\partial}_b R - 3\frac{R}{L} - g^{Lab}(\nabla_{\delta_a} P_b - P_a P_b + \dot{\partial}_a(\nabla P_b)) = -\frac{1}{2\tilde{\kappa}}\phi$$

Coupling geometry and gas on the tangent bundle TM

The relativistic gas

Kinetic gas



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Summary and Outlook

The gas-gravity action

$$S[L] = \tilde{\kappa} \int_V R_0 \, \Sigma + \int_V \phi \, \Sigma$$

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The gas-gravity action

$$S[L] = \tilde{\kappa} \int_V R_0 \, \Sigma + \int_V \phi \, \Sigma$$

The 1PDF of a relativistic kinetic gas as source of gravity

$$\underbrace{\frac{1}{2} g^{Lab} \dot{\partial}_a \dot{\partial}_b R - 3\frac{R}{L} - g^{Lab}(\nabla_{\delta_a} P_b - P_a P_b + \dot{\partial}_a(\nabla P_b))}_{G(x,\dot{x})} = -\frac{1}{2\tilde{\kappa}}\phi$$

The gas-gravity action

$$S[L] = \tilde{\kappa}\int_V R_0\, \Sigma + \int_V \phi\, \Sigma$$

The 1PDF of a relativistic kinetic gas as source of gravity

$$\underbrace{\frac{1}{2}g^{Lab}\dot{\partial}_a\dot{\partial}_b R - 3\frac{R}{L} - g^{Lab}(\nabla_{\delta_a}P_b - P_aP_b + \dot{\partial}_a(\nabla P_b))}_{G(x,\dot{x})} = -\frac{1}{2\tilde{\kappa}}\phi$$

The averaged equation and the Einstein Vlasov system

$$\underbrace{\int_{V_x}\frac{\dot{x}^a\dot{x}_b}{L}G(x,\dot{x})\Sigma_x}_{\mathfrak{G}^a{}_b(x)} = -\frac{1}{2\tilde{\kappa}}\int_{V_x}\frac{\dot{x}^a\dot{x}_b}{L}\phi\Sigma_x = T^a{}_b(x)$$

$$R^a{}_b - \frac{1}{2}\delta^a_b R = \frac{8\pi G}{c^4}\int_{V_x}\frac{\dot{x}^a\dot{x}_b}{g_{cd}(x)\dot{x}^c\dot{x}^d}\phi\Sigma_x$$

- Under which conditions is $\mathfrak{G}^a{}_b(x) \sim R^a{}_b - \frac{1}{2}\delta^a_b R$?

Summary and Outlook

The gas-gravity action

$$S[L] = \tilde{\kappa}\int_V R_0\, \Sigma + \int_V \phi\, \Sigma$$

The 1PDF of a relativistic kinetic gas as source of gravity

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Thank you for your attention

arxiv[1910.12152]: **Variational principle for the Einstein-Vlasov equations** - Andersson, Korzynski

arxiv[1910.14044]: **Relativistic kinetic gases as direct sources of gravity** - Hohmann, Pfeifer, Voicu

arxiv[2005.13561]: **The kinetic gas universe** - Hohmann, Pfeifer, Voicu

The gas-gravity action

$$S[L] = \tilde{\kappa}\int_V R_0\,\Sigma + \int_V \phi\,\Sigma$$

The 1PDF of a relativistic kinetic gas as source of gravity

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- Under which conditions is $\mathfrak{G}^a{}_b(x) \sim R^a{}_b - \frac{1}{2}\delta^a_b R$?

Homogeneous and Isotropic Ansatz

$$\Phi = \rho(x)e^{1-\frac{\dot{t}^2}{\dot{t}^2 - a(t)^2 w^2}}$$