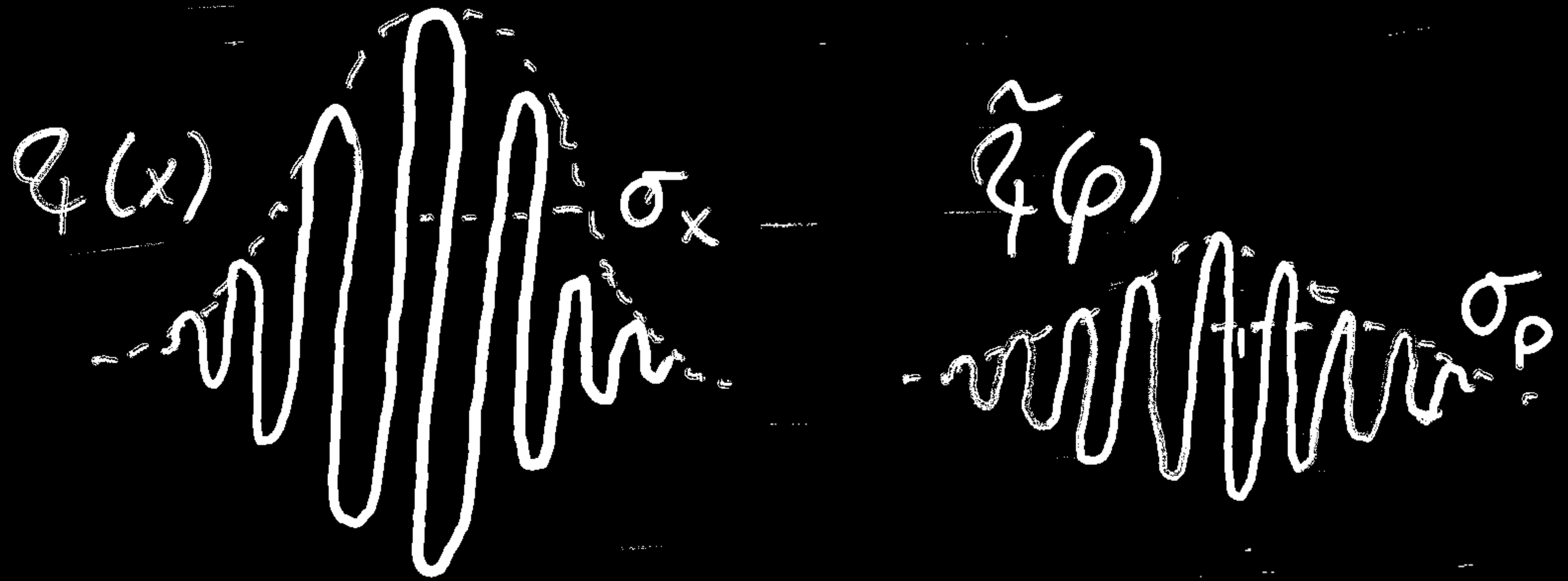
Asymptotic Extended Generalized Uncertainty Principle

Fabian Wagner

(fabian.wagner@usz.edu.pl)

based on: Mariusz P. Dąbrowski & FW, Eur. Phys. J. C (2019) 79: 716
Mariusz P. Dąbrowski & FW, Eur. Phys. J. C (2020) 80: 676

26/09/2020



$$\sigma_x \sigma_p \geq \hbar/2$$

How does space-time curvature affect the uncertainty principle?

What can we learn from this result?

Overview

1. What the adjectival hell?
    a. Generalized
    b. Extended
    c. Asymptotic
2. Method of derivation
3. Result
4. Conclusion

What the adjectival hell?

Generalised (GUP)

Generalised (GUP)

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

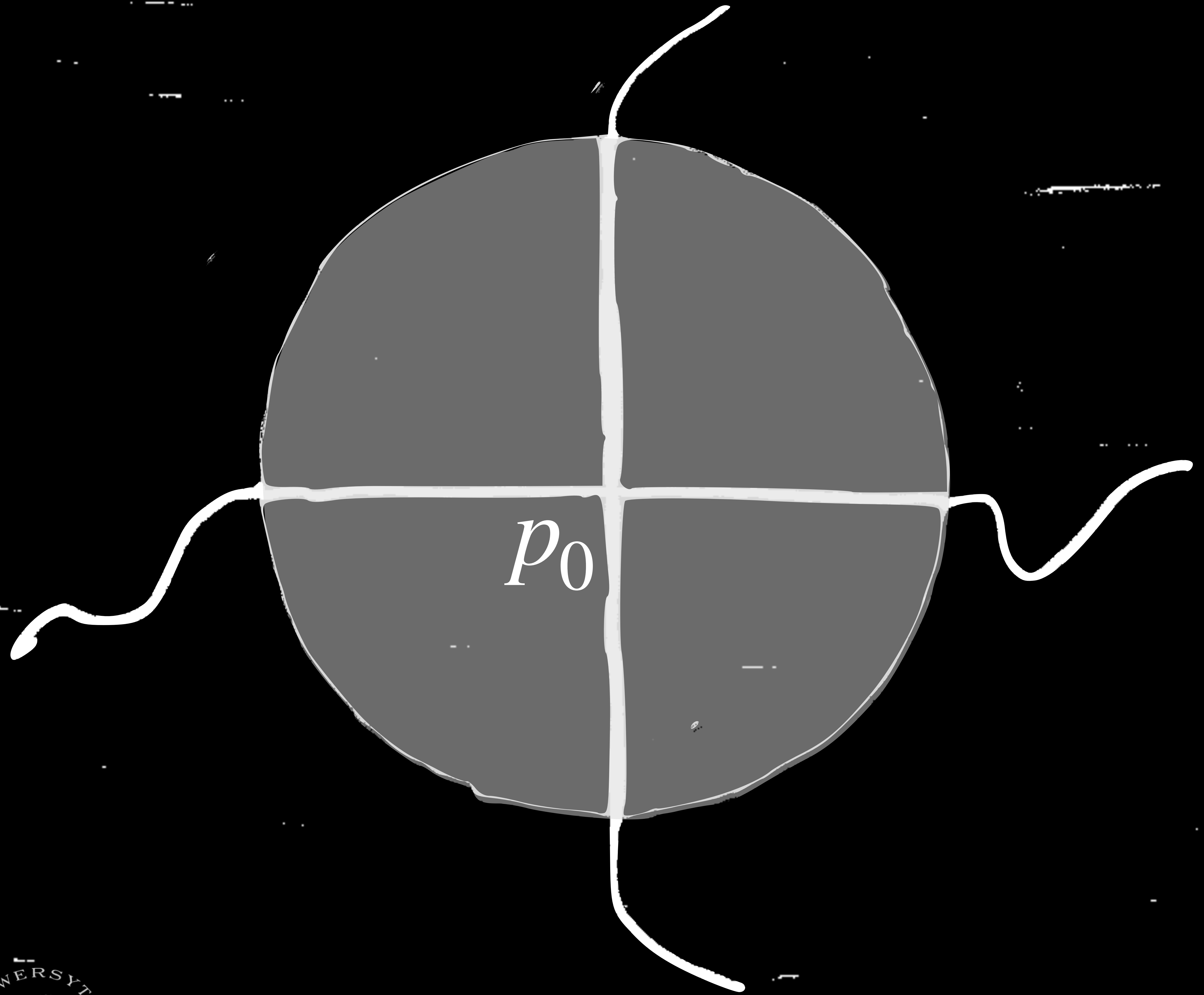
Generalised (GUP)

$$\sigma_x \sigma_p \gtrsim \frac{\hbar}{2}\left(1 + \alpha \frac{l_P^2 \sigma_p^2}{\hbar^2}\right)$$

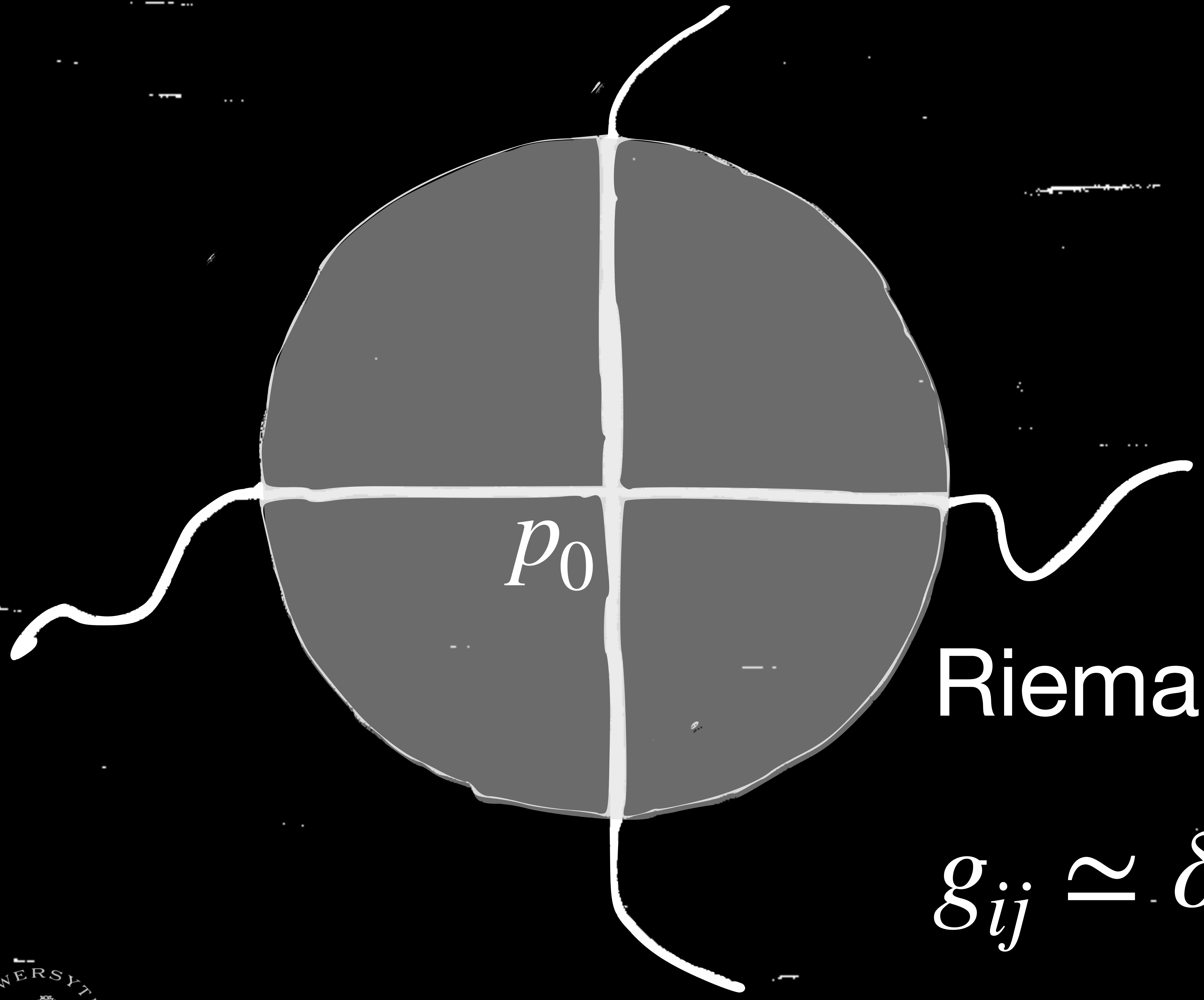
Extended (EUP)

$$\sigma_x \sigma_p \gtrsim \frac{\hbar}{2}\left(1 + \beta \frac{\sigma_x^2}{l_C^2}\right)$$

Asymptotic



Asymptotic

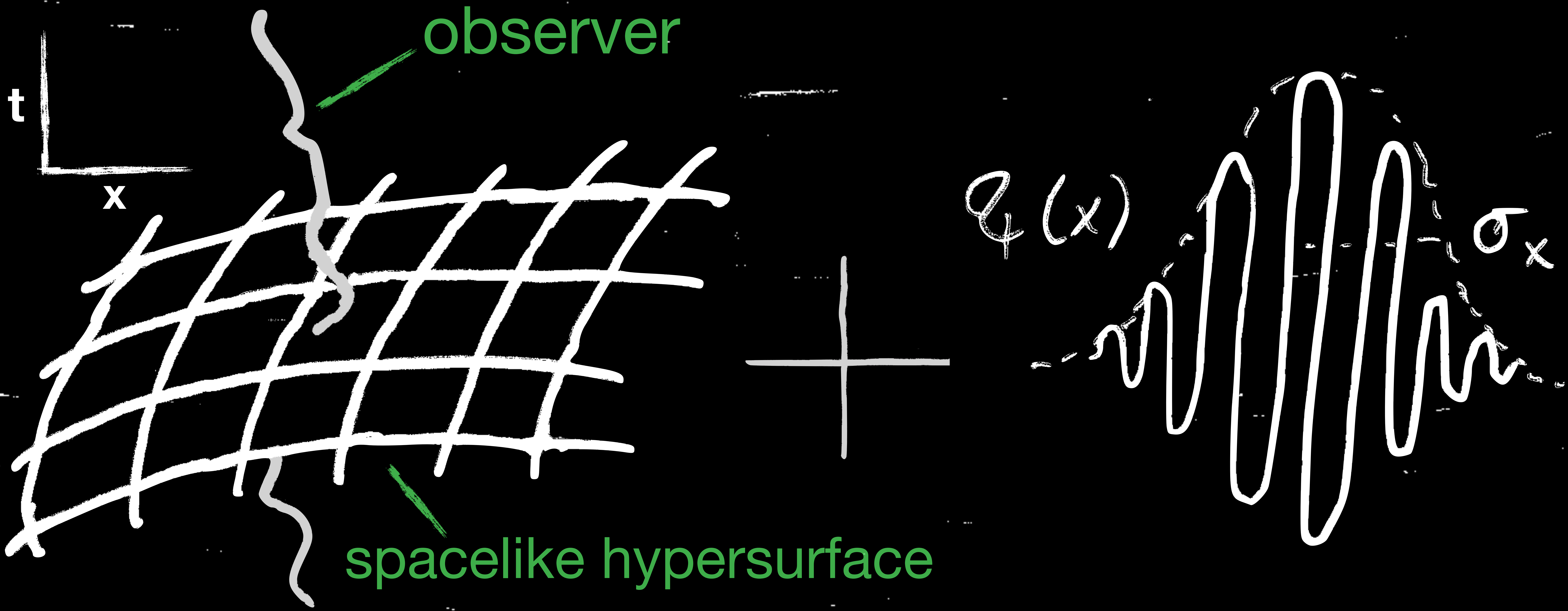


Riemann Normal Coordinates x^i

$$g_{ij} \simeq \delta_{ij} - \frac{1}{3} R_{ikjl}|_{p_0} x^k x^l$$

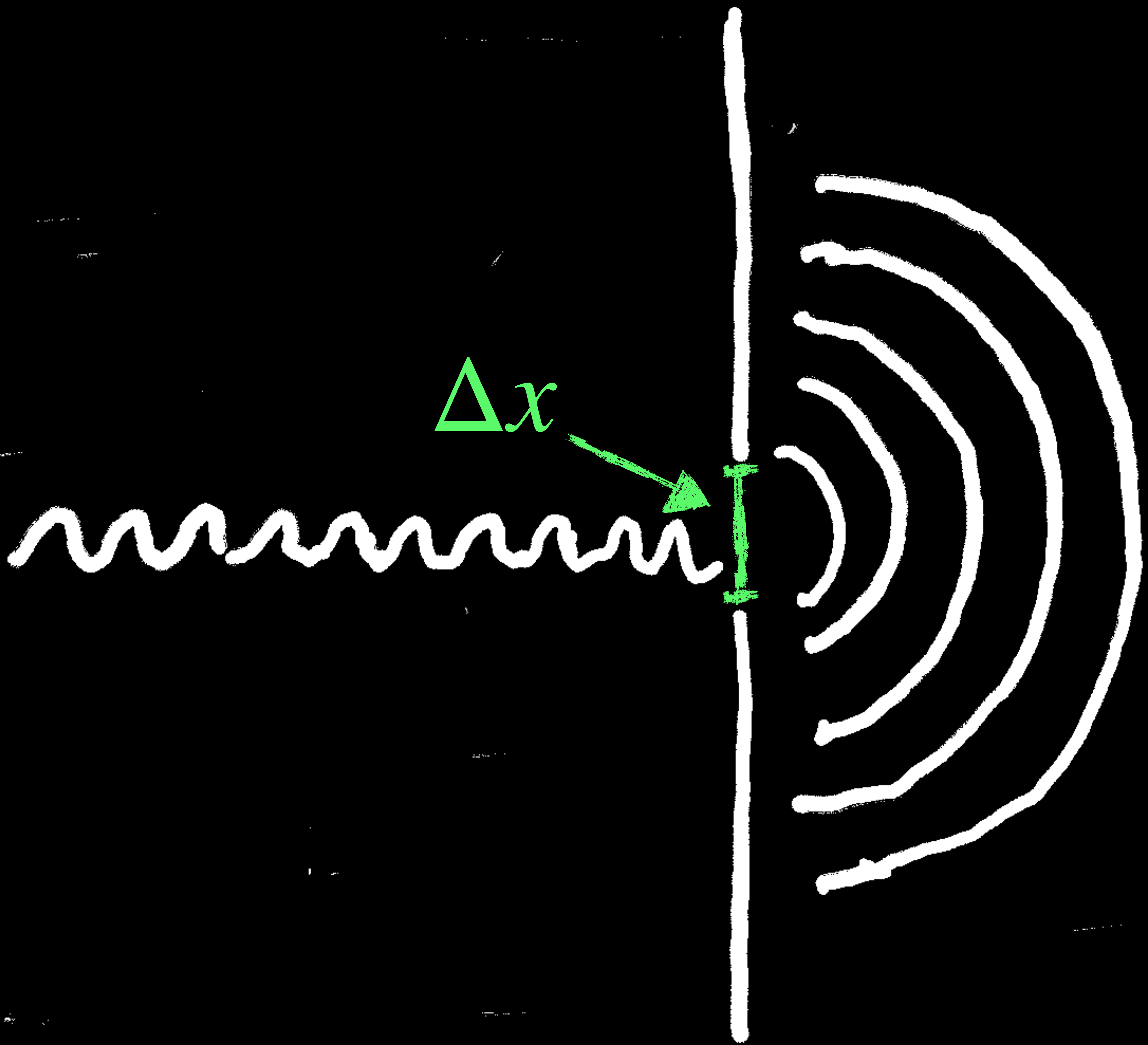
Method of derivation

QM on curved 3D manifolds



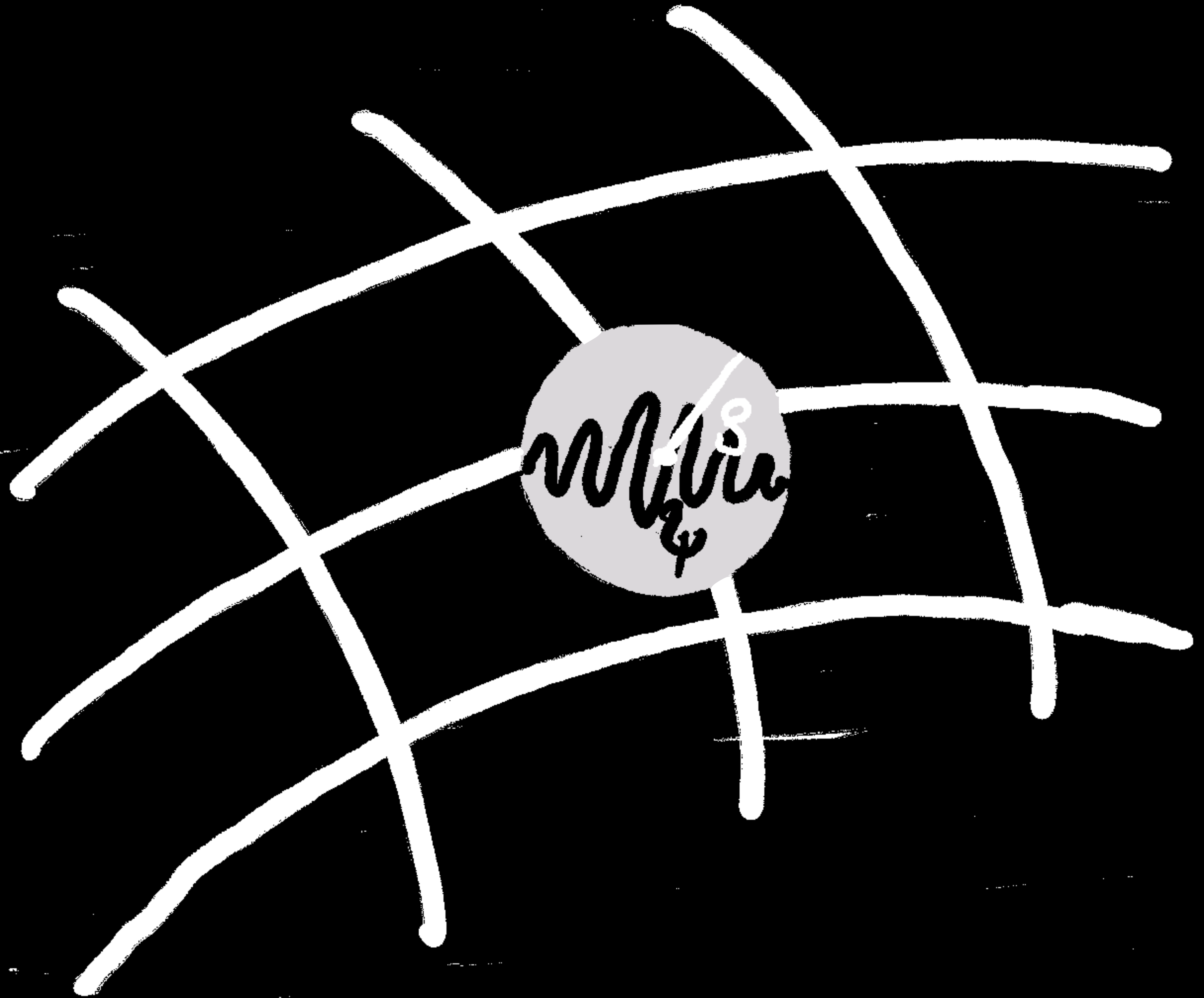
original idea: T. Schürmann, *Found.Phys.* 48 (2018) 716-725

Heisenberg's position uncertainty



Generalisation to curved 3D

Generalisation to curved 3D



Geodesic distance

$$\sigma(P, Q) = \int_P^Q \mathrm{d}s$$

Generalisation to curved 3D

Geodesic distance

$$\sigma(P, Q) = \int_P^Q \mathrm{d}s$$

Geodesic ball B_ρ

$$\sigma(p_0, x) \leq \rho$$

Momentum standard deviation

$$\sigma_p \rho \gtrsim \pi\hbar(1 + \ldots)$$

UNIWERSYTET SZCZECIŃSKI

Result

Asymptotic EGUP

$$\sigma_P \rho \gtrsim \pi \hbar \left(1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} - \frac{1}{12\pi^2} {}^{(3)}R \,\big|_{p_0} \rho^2 \right)$$

UNIWERSYTET SZCZECIŃSKI

Asymptotic EGUP

GUP

$$\sigma_P \rho \gtrsim \pi\hbar \left(1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} - \frac{1}{12\pi^2} {}^{(3)}R\,|_{p_0} \rho^2 \right)$$

US UNIWERSYTET SZCZECIŃSKI

Asymptotic EGUP

EUP

$$\sigma_P \rho \gtrsim \pi\hbar \left(1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} - \frac{1}{12\pi^2} {}^{(3)}R\,|_{p_0} \rho^2 \right)$$

Asymptotic EGUP

$$\sigma_P \rho \gtrsim \pi\hbar \left(1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} - \frac{1}{12\pi^2} {}^{(3)}R \big|_{p_0} \rho^2 \right)$$

l_C^{-2}

Asymptotic EGUP

$$\sigma_P \rho \gtrsim \pi\hbar \left(1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} - \frac{1}{12\pi^2} {}^{(3)}R \,|_{p_0} \rho^2 \right)$$

β

Conclusion

How does the curvature of space-time affect the uncertainty principle?

What can we learn from this result?

- Every kind of curved space leads to an EUP
  - even in Minkowski space-time (with appropriate slicing)
- Positive curvature decreases uncertainty ("quantumness")
  - Can vanish if manifold is closed
- Momentum space curvature = GUP
  - Point p_0 corresponds to $\overrightarrow{p} = 0$ (low energy)
- Approach well-suited for quantum mechanics in curved position or momentum space

UNIWERSYTET SZCZECIŃSKI

What can we learn from this result?

- Every kind of curved space leads to an EUP
    - even in Minkowski space-time (with appropriate slicing)
- Positive curvature decreases uncertainty ("quantumness")
    - Can vanish if manifold is closed
- Momentum space curvature = GUP
    - Point p_0 corresponds to $\overrightarrow{p} = 0$ (low energy)
- Approach well-suited for quantum mechanics in curved position or momentum space

What can we learn from this result?

- Every kind of curved space leads to an EUP
  - even in Minkowski space-time (with appropriate slicing)
- Positive curvature decreases uncertainty ("quantumness")
  - Can vanish if manifold is closed
- Momentum space curvature = GUP
  - Point p_0 corresponds to $\overrightarrow{p} = 0$ (low energy)
- Approach well-suited for quantum mechanics in curved position or momentum space

What can we learn from this result?

- Every kind of curved space leads to an EUP
  - even in Minkowski space-time (with appropriate slicing)
- Positive curvature decreases uncertainty ("quantumness")
  - Can vanish if manifold is closed
- Momentum space curvature = GUP
  - Point p_0 corresponds to $\overrightarrow{p} = 0$ (low energy)
- Approach well-suited for quantum mechanics in curved position or momentum space

What can we learn from this result?

- Every kind of curved space leads to an EUP
  - even in Minkowski space-time (with appropriate slicing)
- Positive curvature decreases uncertainty ("quantumness")
  - Can vanish if manifold is closed
- Momentum space curvature = GUP
  - Point p_0 corresponds to $\overrightarrow{p} = 0$ (low energy)
- Approach well-suited for quantum mechanics in curved position or momentum space

Can we generalize to curvature in both spaces simultaneously?


THE MORE UNCERTAINTY YOU CAN TOLERATE THE MORE CREATIVE YOU CAN BECOME.
the blue limb

Appendix

GEUP/ EGUP

$$\sigma_x \sigma_p \gtrsim \frac{\hbar}{2}\left(1 + \alpha \frac{l_P^2 \sigma_p^2}{\hbar^2} + \beta l_C^{-2} \sigma_x^2\right)$$

Momentum standard deviation

Dirichlet: $\psi|_{\partial B_\rho} = 0 \longrightarrow \langle \hat{p} \rangle = 0$

$$\sigma_p = \sqrt{\langle \hat{p}^2 \rangle}$$

$$\geq \sqrt{\lambda_1(\rho)}$$

UNIWERSYTET SZCZECIŃSKI

Schürmann's result

Constant curvature K

$$\sigma_p \rho \geq \pi \hbar \left(1 - \frac{K\rho^2}{2\pi^2} \right)$$

Eigenvalue problem

$$\left(\hbar^2\Delta + \lambda\right)\psi\big|_{B_\rho} = 0$$

$$\psi\big|_{\partial B_\rho} = 0$$

Perturbation theory

$$g_{ij} = g_{ij}^{(0)} + g_{ij}^{(2)} + \dots$$
$$\Delta = \Delta^{(0)} + \Delta^{(2)} + \dots$$

$$\psi = \psi^{(0)} + \psi^{(2)} + \dots$$
$$\lambda = \lambda^{(0)} + \lambda^{(2)} + \dots$$

UNIWERSYTET SZCZECIŃSKI

Perturbation theory

Complication:

$$\langle \psi | \phi \rangle = \int \sqrt{g}\, \mathrm{d}^3 x \,(\psi^* \phi)$$

$$= \langle \psi | \phi \rangle^{(0)} + \langle \psi | \phi \rangle^{(2)} + \ldots$$

UNIWERSYTET SZCZECIŃSKI

Deformed commutator (GUP)

In 1D: $[\hat{x}, \hat{P}] = i\hbar\left(1 + \frac{\alpha l_P^2}{\hbar^2}\hat{P}^2\right)$

$$\hat{P} \simeq \hat{p}\left(1 + \frac{\alpha l_P^2}{\hbar^2}\hat{p}^2\right)$$

In 3D: $\hat{P}_i \simeq \hat{p}_i\left(1 + \frac{\alpha l_P^2}{\hbar^2}\hat{p}^2\right)$

Asymptotic EGUP

$$\sigma_P \rho \gtrsim \pi\hbar \left[1 + \alpha \frac{l_P^2 \sigma_P^2}{\hbar^2} + \frac{\rho^2}{\pi^2} \left(\lambda_1^{(2)} + \ldots \right) \right]$$

$$\sigma_x \sigma_p \gtrsim \frac{\hbar}{2} \left(1 + \alpha \frac{l_P^2 \sigma_p^2}{\hbar^2} + \beta l_C^2 \sigma_x^2 \right)$$

UNIWERSYTET SZCZECIŃSKI