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CONNECTING SCIENCES

Dynamically induced Planck scale and inflation in the Palatini formulation

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Virtual conference of the Polish Society on Relativity 2020

based on arXiv:2006.09124
with
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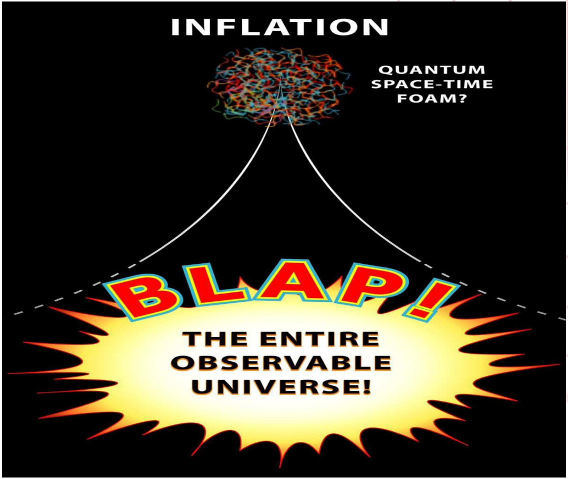
Euroopa Liit
Euroopa Sotsiaalfond
Eesti tuleviku heaks

Dark Universe

• Inflation •



- Solution for:
  - ◇ horizon problem
  - ◇ flatness problem
- Realizable with a constant Λ:

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left[-\frac{M_P^2}{2}R + \Lambda^4\right]$$

$$\Downarrow$$

FRW metric: $\delta s^2 = \delta t^2 - a(t)^2 \delta \mathbf{x}^2$

EoM $\rightarrow$ $a(t) \sim e^{\frac{\Lambda^2}{M_P} t}$

- PROBLEM: it would never stop!
- SOLVED: ϕ with a quasi-flat V
- assumptions on $\mathcal{L}$:
  GUT, SUGRA, string theory, ...
  $\rightarrow V(\phi) \rightarrow$ predictions

• Metric vs Palatini formulation •

The properties of spacetime are essentially described by two objects:

- the connection: $\Gamma^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport $\rightarrow$ curvature: $R^{\lambda}{}_{\mu\nu\sigma}(\Gamma, \partial\Gamma)$
- the metric tensor: $g_{\mu\nu} \rightarrow$ distance between two points

Einstein-Hilbert action: $\sqrt{-g^E}\mathcal{L}^E = \sqrt{-g^E}\left[-\frac{M_P}{2}R_E\right]$

Metric	**Palatini**
No torsion $\Rightarrow \Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$	No torsion $\Rightarrow \Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$
$\boxed{\nabla_\alpha g_{\mu\nu} = 0} \Rightarrow \Gamma =$ Levi-Civita $\Gamma = \bar{\Gamma}$	$\boxed{\Gamma\text{'s EoM}} \rightarrow \nabla_\alpha[\sqrt{-g}g_{\mu\nu}] = 0$
$\bar{\Gamma}^{\lambda}_{\alpha\beta} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\rho\alpha} - \partial_\rho g_{\alpha\beta}\right)$	$\Gamma^{\lambda}_{\alpha\beta} = \bar{\Gamma}^{\lambda}_{\alpha\beta}$

- However, with a non-minimally coupled scalar field, metric and Palatini formalism give different physical theories. (Bauer and Demir: 0803.2664)

• Metric vs Palatini formulation •

Jordan frame: $\sqrt{-g^J}\mathcal{L}^J = \sqrt{-g^J}\left[-\frac{1}{2}A(\phi)R(\Gamma) + \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$

Metric

$\nabla_\alpha g^J_{\mu\nu} = 0 \Rightarrow \Gamma = \text{Levi-Civita } \Gamma \quad = \bar{\Gamma}$

$\bar{\Gamma}^\lambda_{\alpha\beta} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\rho\alpha} - \partial_\rho g_{\alpha\beta}\right)$

Palatini

Γ's EoM: $\nabla_\alpha[A(\phi)\sqrt{-g^J}g^J_{\mu\nu}] = 0$

$\Gamma^\lambda_{\alpha\beta} = \bar{\Gamma}^\lambda_{\alpha\beta} + \delta^\lambda_\alpha\partial_\beta\omega + \delta^\lambda_\beta\partial_\alpha\omega - g_{\alpha\beta}\partial^\lambda\omega$

$\omega(\phi) = \ln\left[\sqrt{A(\phi)/M_P^2}\right]$, $\quad g^E_{\mu\nu} = \Omega(\phi)^2\, g_{\mu\nu}$, $\quad \Omega(\phi) = e^{\omega(\phi)}$

Einstein frame: $\sqrt{-g^E}\mathcal{L}^E = \sqrt{-g^E}\left[-\frac{M_P}{2}R_E + K(\phi)\frac{(\partial\phi)^2}{2} - \frac{V(\phi)}{\Omega(\phi)^4}\right]$

Metric

$$\Gamma_E = \bar{\Gamma}_E$$

$$K(\phi)_M = \frac{M_P^2}{A(\phi)} + \frac{3}{2}\frac{M_P^2}{A(\phi)^2}\left(\frac{\partial A}{\partial\phi}\right)^2$$

Palatini

$$\Gamma_E = \bar{\Gamma}_E$$

$$K(\phi)_P = \frac{M_P^2}{A(\phi)}$$

$\bullet$ ξ CW: Jordan Frame Lagrangian $\bullet$

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}A(\phi)R(\Gamma) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- Palatini Γ
- non-minimal: $A(\phi) = \xi\phi^2 \Rightarrow v = \frac{1}{\sqrt{\xi}}$ (we set $M_P = 1$)
- CW inflation: $V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4 + \Lambda^4$ with $\lambda(\phi)$ a running quartic coupling
- $V(v) = \frac{1}{4}\lambda(v)v^4 + \Lambda^4 = 0$ where v is the VEV of the inflaton
- Given $V(\phi)$ the min. eq. is $\phi\frac{\partial}{\partial\phi}\lambda(v) + 4\lambda(v) = \boxed{\beta(v) + 4\lambda(v) = 0}$ where $\beta(\mu) = \mu\frac{\partial}{\partial\mu}\lambda(\mu)$
- Therefore, 3 possible solutions:

  a) $\beta(v) = \lambda(v) = 0 \;\Rightarrow\; \lambda(\phi) \simeq \frac{\beta'(v)}{2}\ln^2(\phi/v) \;\rightarrow\;$ 2nd order CW

  b) $\beta(v) > 0,\; \lambda(v) < 0 \;\Rightarrow\; \lambda(\phi) \simeq \lambda(v) + \beta(v)\ln(\phi/v) \;\rightarrow\;$ 1st order CW

  c) ~~$\beta(v) < 0,\; \lambda(v) > 0$~~ $\rightarrow$ NO! local max.

From now on we omit "(v)" and restore it only when needed.

• ξ-1st order CW •

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- $V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4 + \Lambda^4$
- $\lambda(\phi) \simeq \lambda(v) + \beta(v)\ln(\phi/v)$

Using $V(v) = 0$: $$V(\phi) = \Lambda^4\left\{1 + \left[4\ln\left(\frac{\phi}{v}\right) - 1\right]\frac{\phi^4}{v^4}\right\}$$

Moving to the Einstein frame . . .

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\zeta}\partial_\nu\bar{\zeta} - \bar{U}(\bar{\zeta})\right]$$

- $\bar{U}(\bar{\zeta}) = \Lambda^4\left(4\,\frac{\bar{\zeta}}{v} + e^{-4\,\frac{\bar{\zeta}}{v}} - 1\right)$
- $\phi = e^{\bar{\zeta}/v}v$

• Slow-roll formalism •

Assuming slow-roll, the inflationary dynamics is described by the slow-roll parameters and the total number of e-folds. The slow-roll parameters are defined as

$$\epsilon_{\bar{U}} \equiv \frac{1}{2}\left(\frac{1}{\bar{U}}\frac{\mathrm{d}\bar{U}}{\mathrm{d}\bar{\zeta}}\right)^2 , \qquad \eta_{\bar{U}} \equiv \frac{1}{\bar{U}}\frac{\mathrm{d}^2\bar{U}}{\mathrm{d}\bar{\zeta}^2} ,$$

and the number of e-folds as

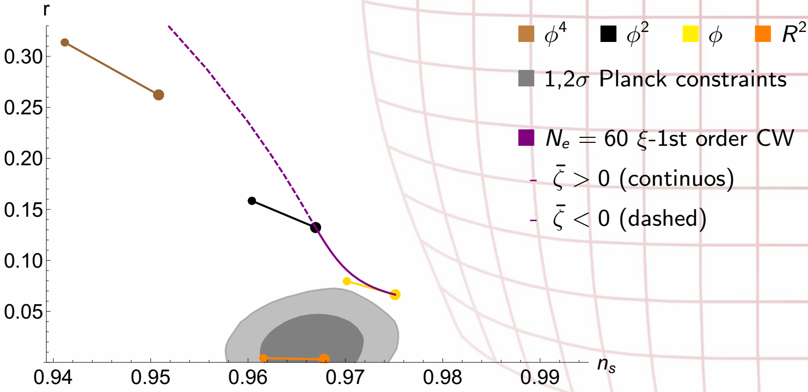
$$N_e = \int_{\bar{\zeta}_f}^{\bar{\zeta}_i} \mathrm{d}\bar{\zeta}\, \bar{U} \left(\frac{\mathrm{d}\bar{U}}{\mathrm{d}\bar{\zeta}}\right)^{-1} ,$$

where the field value at the end of inflation, $\bar{\zeta}_f$, is defined via $\epsilon_{\bar{U}}(\bar{\zeta}_f) = 1$. The field value $\bar{\zeta}_i$ at the time a given scale left the horizon is given by the corresponding N_e. To reproduce the correct A_s, the potential has to satisfy

$$\frac{\bar{U}(\bar{\zeta}_i)}{\epsilon_{\bar{U}}(\bar{\zeta}_i)} = 24\pi^2 A_s ,$$

and the other two main observables, i.e. the spectral index and the tensor-to-scalar ratio are expressed as

$$\bar{n}_s \simeq 1 + 2\eta_{\bar{U}} - 6\epsilon_{\bar{U}}$$
$$\bar{r} \simeq 16\epsilon_{\bar{U}}$$



Ruled out by data! $\rightarrow$ solution: add R^2

ξ-2nd order CW

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\xi\phi^2 R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- $V(\phi) = \frac{1}{4}\lambda(\phi)\phi^4$
- $\lambda(\phi) \simeq \frac{1}{2}\beta' \ln^2(\phi/v)$

Moving to the Einstein frame . . .

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\zeta}\partial_\nu\bar{\zeta} - \bar{U}(\bar{\zeta})\right]$$

- $\bar{U}(\bar{\zeta}) = \frac{m^2}{2}\bar{\zeta}^2 \to$ quadratic inflation
- $\phi = e^{\bar{\zeta}/v}v$
- $m^2 = m_2^2 = \frac{\beta' v^2}{4}$

Ruled out by data! $\to$ solution: add R^2

• R^2 in the Palatini formulation •

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{\alpha}{2}R^2 + \frac{1}{2}\left(\partial\bar{\zeta}\right)^2 - \bar{U}(\bar{\zeta})\right]$$

- very well studied by V. Enckell et al., JCAP 02 (2019) 022
- here only the relevant details
- in the metric case, R^2 introduces a new scalar dof, aka scalaron
- in the Palatini case there is no additional dof.
- replacing the R^2 term with the auxiliary field action, we obtain

$$S = \int d^4x\sqrt{-g}\left[-\frac{1+\chi}{2}R + \frac{1}{2}\left(\partial\bar{\zeta}\right)^2 - \frac{\chi^2}{8\alpha} - \bar{U}(\bar{\zeta})\right]$$

- performing now a Weyl transformation with $\Omega^2 = 1+\chi$ we obtain

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2\left(1+\chi\right)}\left(\partial\bar{\zeta}\right)^2 - \frac{1}{(1+\chi)^2}\left(\frac{\chi^2}{8\alpha} + \bar{U}(\bar{\zeta})\right)\right]$$

R^2 in the Palatini formulation

- solving the EoM for χ and doing some algebra we get

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}(\partial\zeta)^2\left[1+\alpha\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^2\right] - U(\zeta)\right]$$

$$U = \frac{\bar{U}}{1+8\alpha\bar{U}} \qquad \& \qquad \frac{d\bar{\zeta}}{d\zeta} = \sqrt{1+8\alpha\bar{U}}$$

Main features (see JCAP 02 (2019) 022):

- during SR $\alpha\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^2 \ll 1 \Rightarrow$ neglected
- $\bar{U} \gg 1 \Rightarrow U \approx \frac{1}{8\alpha}$: naturally asymptotically flat U
- at the 1st order α affects only r, leaving everything else unchanged

$$r = \frac{\bar{r}}{1+12\pi^2 A_s\bar{r}\alpha} \quad \rightarrow \text{we can flatten any } \bar{U} : r_{\text{limit}} = r(\alpha\rightarrow\infty) = \frac{1}{12\pi^2 A_s\alpha}$$

$$n_s = 1 - 6\epsilon_U + 2\eta_U = 1 - 6\epsilon_{\bar{U}} + 2\eta_{\bar{U}} \qquad 24\pi^2 A_s = \frac{U}{\epsilon_U} = \frac{\bar{U}}{\epsilon_{\bar{U}}}$$

Future(?) satellites $\delta_r \approx 10^{-4}$. We ensure detectability by imposing $r_{\text{limit}} > \delta_r$
$\Rightarrow \alpha < 4\times 10^{10}$.

• $A(\phi)$ & R^2 in Palatini •

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}A(\phi)R + \frac{\alpha}{2}R^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Key properties under $g_{\mu\nu} \to \Omega^2 g_{\mu\nu}$ in Palatini gravity:

- $\Gamma^\lambda_{\rho\sigma}$ and $g_{\mu\nu}$ are independent variables
- $R^\lambda{}_{\mu\nu\sigma}(\Gamma, \partial\Gamma)$ is invariant under any transformation of the sole $g_{\mu\nu}$
- $R_{\mu\nu}(\Gamma, \partial\Gamma) = \delta^\nu{}_\lambda R^\lambda{}_{\mu\nu\sigma}(\Gamma, \partial\Gamma)$ invariant as well
- $R = g^{\mu\nu}R_{\mu\nu}(\Gamma, \partial\Gamma)$ scales inversely: $R \to \frac{R}{\Omega^2}$
- $\boxed{\int d^4x\sqrt{-g}R^2 \text{ is invariant}}$

Therefore is convenient to first move to an "intermediate frame" where we recover immediately GR for $\alpha = 0$:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{\alpha}{2}R^2 + \frac{1}{2}\left(\partial\bar{\zeta}\right)^2 - \bar{U}(\bar{\zeta})\right]$$

and then we just proceed as just shown before.

ξ-CW & R^2

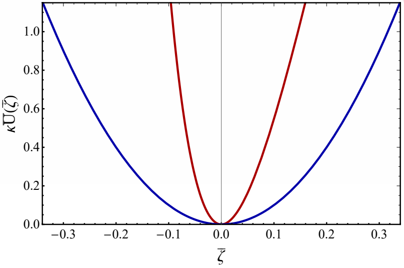
$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}R+\frac{1}{2}(\partial\zeta)^2\left[1+\alpha\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^2\right]-U(\zeta)\right]$$

$$U=\frac{\bar{U}}{1+8\alpha\bar{U}} \qquad \& \qquad \frac{d\bar{\zeta}}{d\zeta}=\sqrt{1+8\alpha\bar{U}}$$

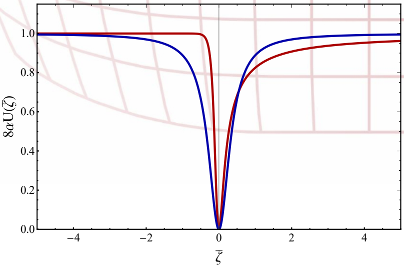
where

- ξ-1st order CW: $\bar{U}(\bar{\zeta})=\Lambda^4\left(4\,\frac{\bar{\zeta}}{v}+e^{-4\,\frac{\bar{\zeta}}{v}}-1\right)$
- ξ-2nd order CW: $\bar{U}(\bar{\zeta})=\frac{m^2}{2}\bar{\zeta}^2$

$\alpha=0$



$\alpha\neq0$



• ξ-CW & R^2: improvements •

- all the general results have been derived in JCAP 02 (2019) 022
- however the small contribution coming from the end of inflation has not been taken into account
- WE DO IT!
  - $\alpha\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^2 \lll 1$ at the end of inflation
  - we compute its effect to ρ_{end}
  - we compute N_e using the instantaneous RH approx.

• End of inflation •

- $\bar{\zeta}(\zeta)$ not always solvable $\Rightarrow$ we use $\bar{\zeta}(t)$
- From I. D. Gialamas, A.B. Lahanas, PRD 101 (2020) 8, 084007

$$\begin{aligned}\rho(\bar{\zeta}) &= K(\bar{\zeta})X + 3L(\bar{\zeta})X^2 + U(\bar{\zeta}) \\ p(\bar{\zeta}) &= K(\bar{\zeta})X + L(\bar{\zeta})X^2 - U(\bar{\zeta})\end{aligned}$$

where

$$X = \frac{1}{2}(\partial\bar{\zeta})^2 = \frac{1}{2}\dot{\bar{\zeta}}^2 \qquad U(\bar{\zeta}) = \frac{\bar{U}(\bar{\zeta})}{1 + 8\alpha\bar{U}(\bar{\zeta})}$$

$$K(\bar{\zeta}) = \frac{1}{(1 + 8\alpha\bar{U})} \qquad L(\bar{\zeta}) = 2\alpha K(\bar{\zeta})$$

- end of inflation: $\epsilon_H = -\frac{\dot{H}}{H^2} = 1$, i.e. $\rho = -3p \Rightarrow$ 1 good sol.:

$$\Rightarrow \ X_{\rm end} = \frac{-K(\bar{\zeta}_{\rm end}) + \sqrt{K(\bar{\zeta}_{\rm end})^2 + 3L(\bar{\zeta}_{\rm end})U(\bar{\zeta}_{\rm end})}}{3L(\bar{\zeta}_{\rm end})} = \frac{-1 + \sqrt{1 + 6\alpha\bar{U}}}{6\alpha}$$

- $\alpha \to 0 \Rightarrow X_{\rm end} = \frac{\bar{U}(\bar{\zeta}_{\rm end})}{2} \to$ OK!
- $\bar{\zeta}_{\rm end}$ determined numerically from $\epsilon_H = 1$

The reheating temperature in case of instantaneous reheating

$$T_{\rm reh} = \left(\frac{30}{\pi^2} \frac{\rho_{\rm end}}{g^*_{\rm reh}} \right)^{1/4}$$

- $g^*_{\rm reh} = 106.75$ are the effective dof for $T_{\rm reh} \geq$ TeV (justified a posteriori)

Moreover, from J. Ellis et. al., JCAP 07 (2015) 050

$$N_e \simeq 61.1 + \frac{1}{4} \ln \left(\frac{U_*^2}{\rho_{\rm end}} \right)$$

Performing the computations we obtain

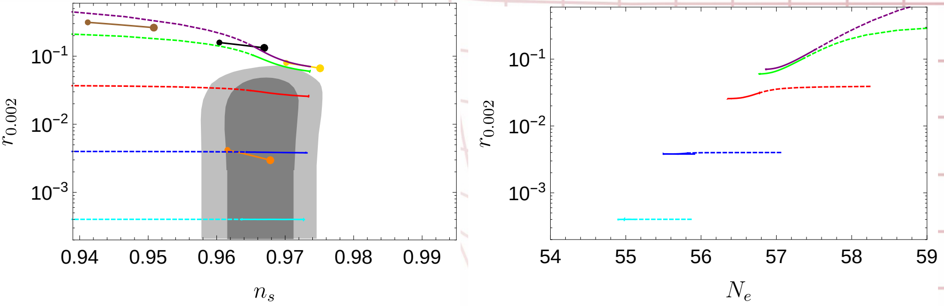
$$N_e \simeq 61.1 + \frac{1}{4} \ln \left(\frac{6\alpha \bar{U}_*^2 \left(1 + 8\alpha \bar{U}_{\rm end}\right)}{\left(1 + 8\alpha \bar{U}_*\right)^2 \left(1 - \sqrt{1 + 6\alpha \bar{U}_{\rm end}} + 12\alpha \bar{U}_{\rm end}\right)} \right)$$

and taking the leading order term for $\alpha \to \infty$ we get

$$N_{\rm limit} \simeq 60.4 - \frac{1}{4} \ln \alpha$$

Therefore the number of e-folds is decreasing with α increasing.
Applying the limit $\alpha < 4 \times 10^{10}$ we obtain $N_e \gtrsim 54.3$.

• ξ-1st order CW & R^2: r vs. n_s •

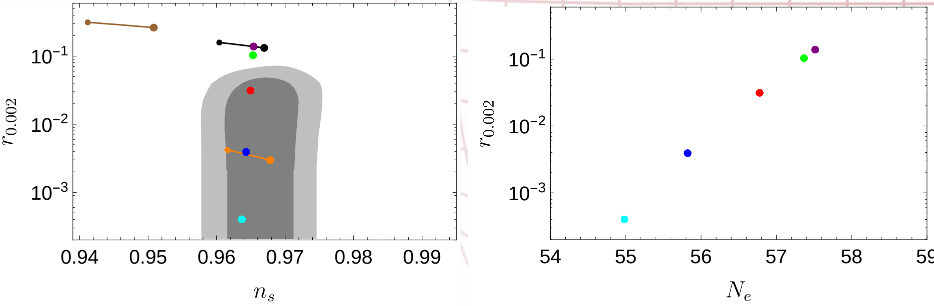


■ ϕ^4 ■ ϕ^2 ■ ϕ ■ R^2 ■ 1,2σ Planck constraints

■ $\alpha = 0$ ■ $\alpha = 10^7$ ■ $\alpha = 10^8$ ■ $\alpha = 10^9$ ■ $\alpha = 10^{10}$

$\bar{\zeta} > 0$ (continuos) $\bar{\zeta} < 0$ (dashed)

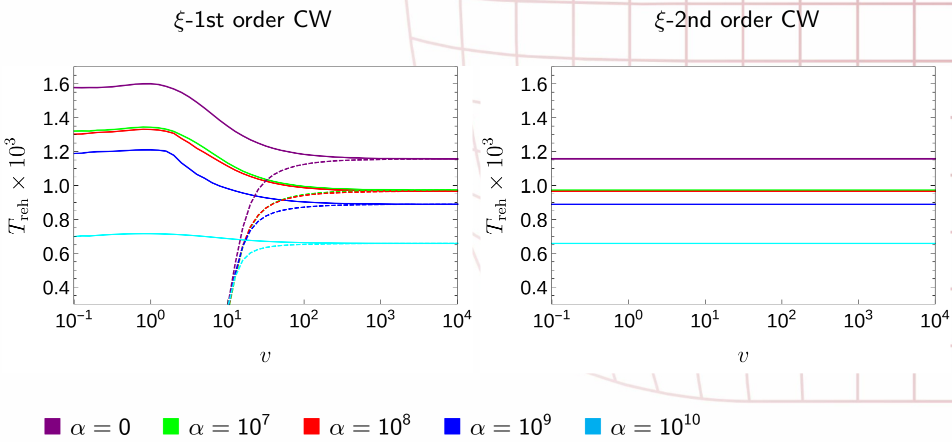
• ξ-2nd order CW & R^2: r vs. n_s •



■ ϕ^4 ■ ϕ^2 ■ ϕ ■ R^2 ■ 1,2σ Planck constraints

■ $\alpha = 0$ ■ $\alpha = 10^7$ ■ $\alpha = 10^8$ ■ $\alpha = 10^9$ ■ $\alpha = 10^{10}$

ξ-CW & R^2: T_{RH}


ξ-1st order CW
ξ-2nd order CW
T_reh × 10^3
1.6
1.4
1.2
1.0
0.8
0.6
0.4
10^-1
10^0
10^1
10^2
10^3
10^4
v
α = 0
α = 10^7
α = 10^8
α = 10^9
α = 10^10

• Conclusions •

- Planck Legacy data motivated us to reconsider ξ-CW inflation in presence of an R^2 term in Palatini gravity.
  - M_P is dynamically generated from v via $A(\phi)$.

- we have identified two type of CW models
  1st order CW: $\lambda(\phi) \approx \ln(\phi)$
  2nd order CW: $\lambda(\phi) \approx \ln^2(\phi)$
  which are both ruled out with the contribution of αR^2

- αR^2 allows to reduce r in order to be in agreement with data.
  As α gets larger, r decreases:
  - detectability of r future(?) satellites: $\alpha < 4 \times 10^{10}$

- n_s plays then a key-role in compatibility with data

- we assumed instantaneous reheating: $T_{\rm reh} \sim 10^{-3} M_P$.

- Future(?) experiments (LiteBIRD, CORE+CMB-Bharat, PIXIE, PICO) will constrain even more r vs. n_s plane and therefore α

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

• Gravity preliminaries •

The properties of spacetime are essentially described by two tensors:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$
- the metric tensor: $g_{\mu\nu}$

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- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$
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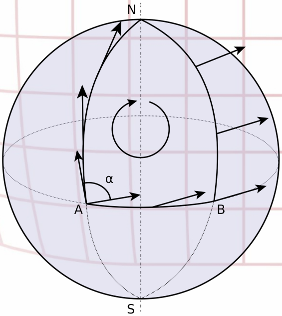
$\Gamma^{\lambda}_{\alpha\beta}$ describes the parallel transport of tensor fields along a given curve. If the spacetime is curved, parallel transport around a closed path, after a full cycle, results in a finite mismatch. The curvature is uniquely determined by the Riemann tensor $R^{\mu}_{\ \alpha\nu\beta}(\Gamma)$ whose contraction $R_{\alpha\beta}(\Gamma) \equiv R^{\mu}_{\ \alpha\mu\beta}(\Gamma)$ gives the Ricci tensor[1]



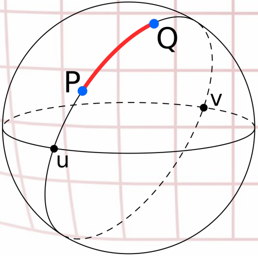
[1] We consider only $\Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$ i.e. torsion free space-time.

Gravity preliminaries

The properties of spacetime are essentially described by two tensors:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$
- the metric tensor: $g_{\mu\nu}$

$g_{\mu\nu}$ allows us to introduce the notion of distance.



The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship, and if they are to have any it must derive from additional constraints (metric formalism) or geometrodynamics (Palatini formalism).

• Our setup •

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}A(\phi)R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \frac{\alpha}{2}R^2\right]$$

- non-minimal coupling to Palatini gravity $A(\phi)$
- CW potential $V(\phi)$
- dynamical induced Plank mass: constraint between inflaton vev and ξ

- unfortunately results will be ruled out at 2σ
- solution: add Palatini R^2 term

STRATEGY

- It has been proven that the cosmological perturbations are invariant under a change of frame (e.g. T. Prokopec & J. Weenink, JCAP09(2013)027).
- On the other hand, the quantum equivalence of the Einstein and Jordan frames is still an unsolved issue. Many different point of views, articles and authors
- Here we adopt the following computational strategy:
  - We compute the effective potential in the Jordan frame.
  - Once we have the final expression of the 1-loop Jordan frame scalar potential, we move to the Einstein frame, where the calculation of the slow-roll parameters is easier.
  - Given a scalar potential in the Jordan frame, the cosmological perturbations are then independent, in the slow-roll approximation, from the choice of the frame in which we perform the inflationary calculations

• Model independent approximation •

- the running of $\lambda(\phi)$ is described by its beta function

$$\beta_\lambda(\mu) = \frac{d\lambda}{d\ln\mu}$$

where μ is the renormalization scale.

- Ignoring all the details of the theory completion etc., we can still solve the RGE as a Taylor series

$$\lambda(\phi) = \lambda(v) + \beta(v)\ln\frac{\phi}{v} + \frac{1}{2!}\beta'(v)\ln^2\frac{\phi}{v} + \frac{1}{3!}\beta''(v)\ln^3\frac{\phi}{v} + \cdots\,,$$

where $\beta'(\mu)$ and $\beta''(\mu)$ are respectively the first and second derivative of $\beta(\mu)$ with respect to $t = \ln\mu$ and we assumed without loss of generality that $\phi > 0$.

- Therefore for 2nd order CW we have that

$$\lambda^a(\phi) \simeq \frac{\beta'(v)}{2}\ln^2\frac{\phi}{v}$$

while for 1st order CW

$$\lambda^b(\phi) \simeq \lambda(v) + \beta(v)\ln\frac{\phi}{v}$$

• Freedom in the choice of μ_0 •

The choice $\mu_0 = M_P$ is just a convenient parametrization.

In the region of validity of the first order approximation (where β_λ is essentially constant), the result is independent on the choice of μ_0. The parametrization using $\mu_0 = M_P$, is related to another one using $\mu_0 = \mu^* \neq M_P$ via the RGE solution

$$\begin{aligned} \lambda(M_P) &= \lambda(\mu^*)\left[1 + \delta(\mu^*)\ln\left(\frac{M_P}{\mu^*}\right)\right], \\ \delta(M_P) &= \frac{\beta_\lambda(M_P)}{\lambda(M_P)} = \frac{\beta_\lambda(\mu^*)}{\lambda(M_P)}, \end{aligned}$$

where we used $\beta_\lambda(M_P) \simeq \beta_\lambda(\mu^*)$.

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• No ξ running •

We can approximate the running of ξ with

$$\begin{aligned}\xi &= \xi(M_P) + \beta_\xi(M_P)\ln\left(\frac{\phi}{M_P}\right) \\ &= \xi(M_P)\left[1 + \delta_\xi(M_P)\ln\left(\frac{\phi}{M_P}\right)\right]\end{aligned}$$

where β_ξ is the beta function of the non-minimal coupling

$$\beta_\xi(\phi) \simeq \frac{1}{16\pi^2}\left[6\lambda_\phi(\phi)\left(\xi(\phi) + \frac{1}{6}\right)\right]$$

Then, because of the constraint on the amplitude of scalar perturbations

$$A_s = (2.14 \pm 0.05) \times 10^{-9}$$

and the $16\pi^2$ suppression factor, it is $\beta_\xi \ll \xi$. The quantum corrections are therefore negligible and $\xi \simeq \xi(M_P)$.

• ξ-1st order CW •

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \bar{\zeta} \partial_\nu \bar{\zeta} - \bar{U}(\bar{\zeta}) \right]$$

- $\bar{U}(\bar{\zeta}) = \Lambda^4 \left(4 \, \frac{\bar{\zeta}}{v} + e^{-4 \, \frac{\bar{\zeta}}{v}} - 1 \right)$
- $\phi = e^{\bar{\zeta}/v} v$

We can immediately appreciate two relevant limit cases:

- For $v \ll 1$ ($\xi \gg 1$) and $\bar{\zeta} > 0$, the potential becomes linear

$$\bar{U}(\bar{\zeta}) \approx a_\zeta \, \bar{\zeta} \qquad \text{with } a_\zeta = 4 \frac{\Lambda^4}{v}$$

- On the other hand for $v \gg 1$ ($\xi \ll 1$), the potential becomes quadratic

$$\bar{U}(\bar{\zeta}) \approx \frac{m^2}{2} \bar{\zeta}^2 \qquad \text{with } m = m_1 = 4 \frac{\Lambda^2}{v}$$

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}(\partial\zeta)^2\left[1+\alpha\left(1+8\alpha\bar{U}(\zeta)\right)(\partial\zeta)^2\right] - U(\zeta)\right]$$

$$U = \frac{\bar{U}}{1+8\alpha\bar{U}} \qquad \& \qquad \frac{d\bar{\zeta}}{d\zeta} = \sqrt{1+8\alpha\bar{U}}$$

where

- ξ-1st order CW: $\bar{U}(\bar{\zeta}) = \Lambda^4\left(4\,\frac{\bar{\zeta}}{v} + e^{-4\,\frac{\bar{\zeta}}{v}} - 1\right)$
- ξ-2nd order CW: $\bar{U}(\bar{\zeta}) = \frac{m^2}{2}\bar{\zeta}^2$

some relevant limits:

- ξ-1st order CW with $v \ll 1$ ($\xi \gg 1$) and $\bar{\zeta} > 0$,

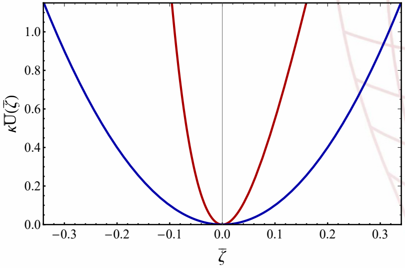
$$\bar{U}(\bar{\zeta}) \approx a_\zeta\,\bar{\zeta} \quad \Rightarrow \quad U(\zeta) \simeq \frac{1}{8\alpha}\frac{1+\frac{1}{2\alpha a_\zeta \zeta}}{\left(1+\frac{1}{4\alpha a_\zeta \zeta}\right)^2}$$

- ξ-1st order CW with $v \gg 1$ ($\xi \ll 1$) or exact for ξ-2nd order CW

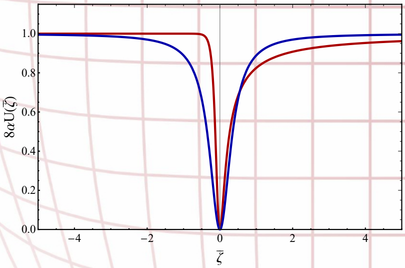
$$\bar{U}(\bar{\zeta}) \approx \frac{m^2}{2}\bar{\zeta}^2 \quad \Rightarrow \quad U(\zeta) \simeq \frac{\tanh^2\left(2\sqrt{\alpha}m\zeta\right)}{8\alpha}$$

• ξ-CW & R^2: U plots •

$\xi = 10,\ \alpha = 0$



$\xi = 10,\ \alpha = 10^{10}$



- 1st order CW with $\kappa = \Lambda^{-4}$ — with $\Lambda = 0.0015$
- 2nd order CW with $\kappa = \xi^2/\beta'$ — with $\beta' = 10^{-9}$

• End of inflation: like the paper •

- for numerical convenience we use the original field definition $\phi(\bar{\zeta}(\zeta(t)))$
- In a flat FRW metric where $\phi = \phi(t)$

$$\rho(\phi) = K(\phi)X + 3L(\phi)X^2 + U(\phi)$$
$$p(\phi) = K(\phi)X + L(\phi)X^2 - U(\phi)$$

$$X = \frac{1}{2}(\partial\phi)^2 = \frac{1}{2}\dot{\phi}^2$$
$$K(\phi) = \frac{1}{A(1+8\alpha\bar{U})}$$
$$L(\phi) = \frac{2\alpha}{A^2(1+8\alpha\bar{U})}$$

- end of inflation: $\epsilon_H = -\frac{\dot{H}}{H^2} = 1$, i.e. $\rho = -3p \Rightarrow$

$$\Rightarrow \text{ 1 good sol.: } X_{\rm end} = \frac{-K(\phi_{\rm end}) + \sqrt{K(\phi_{\rm end})^2 + 3L(\phi_{\rm end})U(\phi_{\rm end})}}{3L(\phi_{\rm end})}$$

- $K \to 1$ & $L \to 0 \Rightarrow X_{\rm end} = \frac{U(\phi_{\rm end})}{2} \to$ OK!
- $\phi_{\rm end}$ determined numerically from $\epsilon_H = 1 \Rightarrow \epsilon_U \simeq \left(1 + \sqrt{1 - \frac{\eta_U}{2}}\right)^2$

• Validity of the theory •

- We assume that gravitational radiative corrections are negligible. We make a reasonable guess starting from the Einstein frame and assuming that quantum gravity effects are negligible when the inflaton energy is much below the Planck scale. Since we are dealing with SR, we get

$$U_* \ll 1$$

- It is easy to check that this is always ensured because of $A_s \simeq 2.1 \times 10^{-9}$

$$U_* = \frac{3}{2}\pi^2 A_s r \simeq 3.1 \times 10^{-8} r \ll 1$$

- Analogously, the constraint the Jordan in frame is

$$\frac{V_*^{\mathrm{J}}}{f^2} \ll 1$$

$$V^{\mathrm{J}} = V(\phi) + \frac{\chi^2}{8\alpha} \qquad f = \chi + A(\phi)$$

$$\mathrm{SR} \rightarrow \chi \simeq \frac{8\alpha V(\phi)}{A(\phi)}$$

$$\Rightarrow \qquad \frac{V_*}{f^2} \simeq \frac{V_*}{A(\phi_*)^2 + 8\alpha V_*} = U_* \ll 1$$

• Validity of the theory II •

- We assume that the equations of the running quartic couplings are valid at least during the duration of inflation. This is ensured by assuming the existence of an ad hoc dark sector which provides the dominant contribution to the effective potential while the self-correction to the quartic coupling remains subdominant.
- The 1st order CW potential is linear in the logarithmic term, therefore it corresponds to a 1-loop effective potential. As such the validity of the approximation is ensured by the requirement

$$\beta_\lambda(\phi) \approx \frac{\lambda(\phi)^2}{\pi^2} \ll \beta$$

  We can prove that it is always satisfied in the considered range of v.
- On the other hand, the 2nd order CW potential in is quadratic in the logarithmic term, therefore it corresponds to a 2-loop effective potential. As such the validity of the approximation is ensured by the requirement

$$\beta_\lambda^2(\phi) \approx \frac{\lambda(\phi)^4}{\pi^4} \ll \beta'$$

  We can prove that it is satisfied for $v \gtrsim 0.1$.