# Renormalization Group in Six-derivative Quantum Gravity

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and based on the paper ArXiv:hep-th/2104.13980

# Motivation for HD Gravity

## Motivation:

Let's first quantize matter, put it on curved spacetime background, only later quantize gravitation (Utiyama, De Witt, Shapiro)

## Observation:

1-loop off-shell divergences of standard matter theory (with two derivatives) are proportional to $R^2$ and $C^2$ on a curved spacetime background. Counterterms needed to be added to the divergent matter effective action are of these types $R^2$ and $C^2$ (in $d = 4$) even if the gravitational theory was Einstein-Hilbert Quantum Gravity with $R$ in the action

## Conclusion:

These counterterms contain higher derivatives of the background metric. Higher derivatives are inevitable!

# Higher Derivative Quantum Gravity

## Four-derivative theory (Stelle '77)

$$S_{\rm QG} = \int d^4x \sqrt{|g|} \left( \omega_\kappa R + \theta_R R^2 + \theta_C C^2 \right)$$

## General higher-derivative theory (Asorey, Lopez, Shapiro '96)

$$S_{\rm QG} =$$

$$= \int d^4x \sqrt{|g|} \left( \omega_\Lambda + \omega_\kappa R + \sum_{n=0}^{N} \omega_{R,n} R \Box^n R + \sum_{n=0}^{N} \omega_{C,n} C \Box^n C + O\left(\mathcal{R}^3\right) \right)$$

## 6-derivative theories

Here we consider the case $N = 1$.
We quantize the theory and study RG flow at one-loop level
We generalize Stelle's gravity and quantum results from it

# Why 6-derivative Quantum Gravity?

## Theoretical motivations

- quantum super-renormalizability
- possibility of UV-finiteness
- exact and unambiguous expressions for $\beta$-functions of running coupling parameters of the theory
- gauge- and scheme-independence of UV-divergences
- possibility of Lee-Wick (LW) pair of complex conjugate poles of the propagator (to ameliorate the problem of unitarity in HD QG)
- *amazingly* simple and analytic final results for $\beta$-functions
- very good theoretical laboratory for study RG flows in QG

## 6-der QG

is better behaved on the quantum level than 4-der HD QG of Stelle!

# The theory

## Classical theory

Action:

$$S_{\mathrm{QG}} = \int d^4x \sqrt{|g|}\, \mathcal{L}\,,$$

Lagrangian (density):

$$\mathcal{L} = \omega_C C_{\mu\nu\rho\sigma} \Box C^{\mu\nu\rho\sigma} + \omega_R R \Box R + \theta_C C^2 + \theta_R R^2 + \theta_{\mathrm{GB}} E_4 + \omega_\kappa R + \omega_\Lambda$$

(some) GR invariant scalar terms:

$$C^2 = C^2_{\mu\nu\rho\sigma} = R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3} R^2,$$

$$E_4 = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2.$$

Fundamental ratio of the theory:

$$x = \frac{\omega_C}{\omega_R}$$

# Super-renormalizable Quantum Gravity

**Propagator of all quantum modes in UV regime (monomial asymptotics $\Box^3$) Modesto**

$\Pi \sim k^{-6}$

**Superficial degree of divergence $\Delta$ of $L$-loop graph $G$**

$\Delta = 4L + V[\mathrm{vertex}] - I[\mathrm{propagator}]$

**Graviton $h_{\mu\nu}$ and FP ghost fields $C_\mu$ are dimensionless $\Rightarrow$ the same maximal number of derivatives in vertices as in propagators in UV**

$[\mathrm{vertex}] = -[\mathrm{propagator}] = k^6$

**Bound on $\Delta$**

$\Delta \leqslant 4 - 2(L-1); \quad \text{for } L \geqslant 4 \quad \Delta < 0$

$\Longrightarrow$ no loop divergences for higher loops (quantum corrections are finite)

# Consequences of power counting of UV divergences

## Structure of divergences

- the only possible divergent structures are $C^2$, $R^2$, $E_4$, $R$ and $\Lambda$
- The divergences $C^2$, $R^2$, $E_4$ receive contributions only at one-loop level ($L = 1$)
- The $R$ (Newton's gravitational constant) divergence receive contributions also at $L = 2$ level
- The $\Lambda$ (cosmological constant) divergence receive contributions also at $L = 2, 3$ levels
- from 4-loop level the theory is finite
- terms $\theta_C$, $\theta_R$, $\theta_{\mathrm{GB}}$, $\omega_\kappa$ and $\omega_\Lambda$ do not affect the counterterms $C^2$, $R^2$, $E_4$
- terms $O\left(\mathcal{R}^3\right)$ may affect above $\implies$ possibility of complete UV-finiteness of the model

Here we concentrate on the most difficult to get counterterms $C^2$, $R^2$ and $E_4$

# Minimal model

## Minimal working model

$$S_{\rm min} = \int d^4x \sqrt{|g|} \{ \omega_C \, C_{\mu\nu\rho\sigma} \Box C^{\mu\nu\rho\sigma} \; + \; \omega_R \, R \Box R \}$$

## The expected form of exact one-loop divergences

$$S_{\rm div} = \int d^4x \sqrt{|g|} \{ c_C \, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + c_R \, R^2 \; + c_{\rm GB} E_4 \}$$

## Expected dependence

due to dimensional reasons

$$[c_C] = [c_R] = [c_{\rm GB}] = E^0, \quad [x] = \left[ \frac{\omega_C}{\omega_R} \right] = E^0$$

$$c_C = c_C(x), \quad c_R = c_R(x), \quad c_{\rm GB} = c_{\rm GB}(x)$$

# Universality of UV-divergences in effective action $\Gamma$

## Power counting for $L = 1$ $\Delta = 4$

- counterterm action $S_{\rm div}$ contains up to four derivatives on the metric
- classical minimal action $S_{\rm min}$ contains precisely six derivatives on the metric $\Longrightarrow$ classical EOM $\varepsilon^{\mu\nu}$ are with six derivatives

## Parametrization independence theorem (Kallosh, Tyutin, Tarasov)

$$\Gamma(\alpha_i) - \Gamma(\alpha_i^0) = \int d^4x \sqrt{|g|}\, \varepsilon^{\mu\nu} f_{\mu\nu} \quad \text{with} \quad f_{\mu\nu} = f_{\mu\nu}(g_{\kappa\lambda}, \alpha_i, \alpha_i^0)$$

## Independence of $S_{\rm div}$

- of gauge choices
- of gauge-fixing choices
- of parametrization ambiguities for quantum field
- of scheme choice for renormalization

# One-loop computation

## Method of computation

- covariant Barvinsky-Vilkovisky trace technology (generalized Schwinger-DeWitt method)
- quantum variable $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$
- based on the simple one-loop formula

$$\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln \hat{H}, \quad \text{with} \quad \hat{H} = \frac{\delta^2 S}{\delta\phi^2}$$

- minimal gauge fixing choice for gauge parameters
- simplified contributions from Faddeev-Popov and third ghosts quantum fields

$$\Gamma^{(1)} = \frac{i}{2}\mathrm{Tr}\ln \hat{H} - i\mathrm{Tr}\ln \hat{M} - \frac{i}{2}\mathrm{Tr}\ln \hat{C}$$

- very difficult computation (needed to be done using Mathematica xTensor package for symbolic tensor algebra)
- various checks on it were succesfully performed

# Quantum results

## Results in 6-derivative gravitational theory in $d = 4$

$$\Gamma^{(1)C,R,E}_{\rm div} = -\frac{1}{2\epsilon(4\pi)^2}\int d^4x\sqrt{|g|}\left\{\left(\frac{2x}{9}+\frac{397}{40}\right)C^2-\frac{7}{36}R^2+\frac{1387}{180}E_4\right\}$$

with $\epsilon = \frac{4-d}{2}$ as a parameter of DIMREG scheme
and the fundamental ratio $x = \frac{\omega_C}{\omega_R}$

## Results in 4-derivative Stelle theory in $d = 4$

$$\Gamma^{(1)C,R,E}_{\rm div} = -\frac{1}{2\epsilon(4\pi)^2}\int d^4x\sqrt{|g|}\left\{-\frac{133}{20}C^2+\left(-\frac{5}{2}x'^2+\frac{5}{2}x'-\frac{5}{36}\right)R^2 +\frac{196}{45}E_4\right\} \quad \text{with} \quad x' = \frac{\theta_R}{\theta_C}$$

# Beta functions

## System of $\beta$ functions

$$\beta_C = \mu \frac{d\theta_C}{d\mu} = \frac{1}{(4\pi)^2}\left(\frac{2}{9}\frac{\omega_C}{\omega_R} + \frac{397}{40}\right), \quad \text{exact}$$

$$\beta_R = \mu \frac{d\theta_R}{d\mu} = -\frac{1}{(4\pi)^2}\,\frac{7}{36}\,, \quad \text{exact}$$

$$\beta_{\mathrm{GB}} = \mu \frac{d\theta_{\mathrm{GB}}}{d\mu} = \frac{1}{(4\pi)^2}\,\frac{1387}{180}\,, \quad \text{exact}$$

$$\beta_\kappa = \mu \frac{d\omega_\kappa}{d\mu} = -\frac{1}{(4\pi)^2}\left[\frac{5\theta_C}{6\omega_C} + \frac{\theta_R}{2\omega_R} - \frac{5\theta_R}{\omega_C}\right],$$

$$\beta_\Lambda = \mu \frac{d\omega_\Lambda}{d\mu} = \frac{1}{(4\pi)^2}\left[\frac{5\omega_\kappa}{2\omega_C} - \frac{\omega_\kappa}{6\omega_R} - \frac{5}{2}\left(\frac{\theta_C}{\omega_C}\right)^2 - \frac{1}{2}\left(\frac{\theta_R}{\omega_R}\right)^2\right].$$

# Solutions for RG flows

$$\theta_C(t) = \theta_C(0) + \beta_C\, t = \theta_C(0) + \frac{1}{(4\pi)^2}\left(\frac{2x}{9} + \frac{397}{40}\right) t\,,$$

$$\theta_R(t) = \theta_R(0) + \beta_R\, t = \theta_R(0) - \frac{1}{(4\pi)^2}\frac{7}{36}\, t\,,$$

$$\theta_{\rm GB}(t) = \theta_{\rm GB}(0) + \beta_{\rm GB}\, t = \theta_{\rm GB}(0) + \frac{1}{(4\pi)^2}\frac{1387}{180}\, t\,,$$

$$\omega_\kappa(t) = \omega_\kappa(0) + a_\kappa\, t + b_\kappa\, t^2\,,$$

$$\omega_\Lambda(t) = \omega_\Lambda(0) + a_\Lambda\, t + b_\Lambda\, t^2 + c_\Lambda\, t^3\,.$$

## Observation

For $t \to +\infty$ couplings $\theta_R$ and $\theta_{\rm GB}$ tend to $-\infty$ and $+\infty$ respectively (we have asymptotic freedom in them); the coupling $\theta_C$ tends also to AF in UV, if not the special value of ratio $x$: $x_* = -\frac{3573}{80} = -44.6625$.
For $x = x_*$ the coupling $\theta_C$ sits at the non-trivial FP (asymptotic safety)

# Conclusions

## Six-derivative Gravity

- super-renormalizability and options for UV-finiteness
- exact and universal beta functions for $\theta_C$, $\theta_R$ and $\theta_{\rm GB}$ couplings
- gauge- and parametrization-independence of UV divergences
- exact RG flows and asymptotic freedom in UV

## Further developments

- conditions for AF in UV and AS
- dominance of free propagation over interactions
- rescaling of the graviton field (like Fradkin, Tseytlin)
- quantum stability of the Lee-Wick complex conjugate pairs
- addition of terms $O(\mathcal{R}^3)$ for UV-finiteness
- spectrum around flat Minkowski and around (A)dS spacetimes

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Dziękuję!

Thank you!

Obrigado!