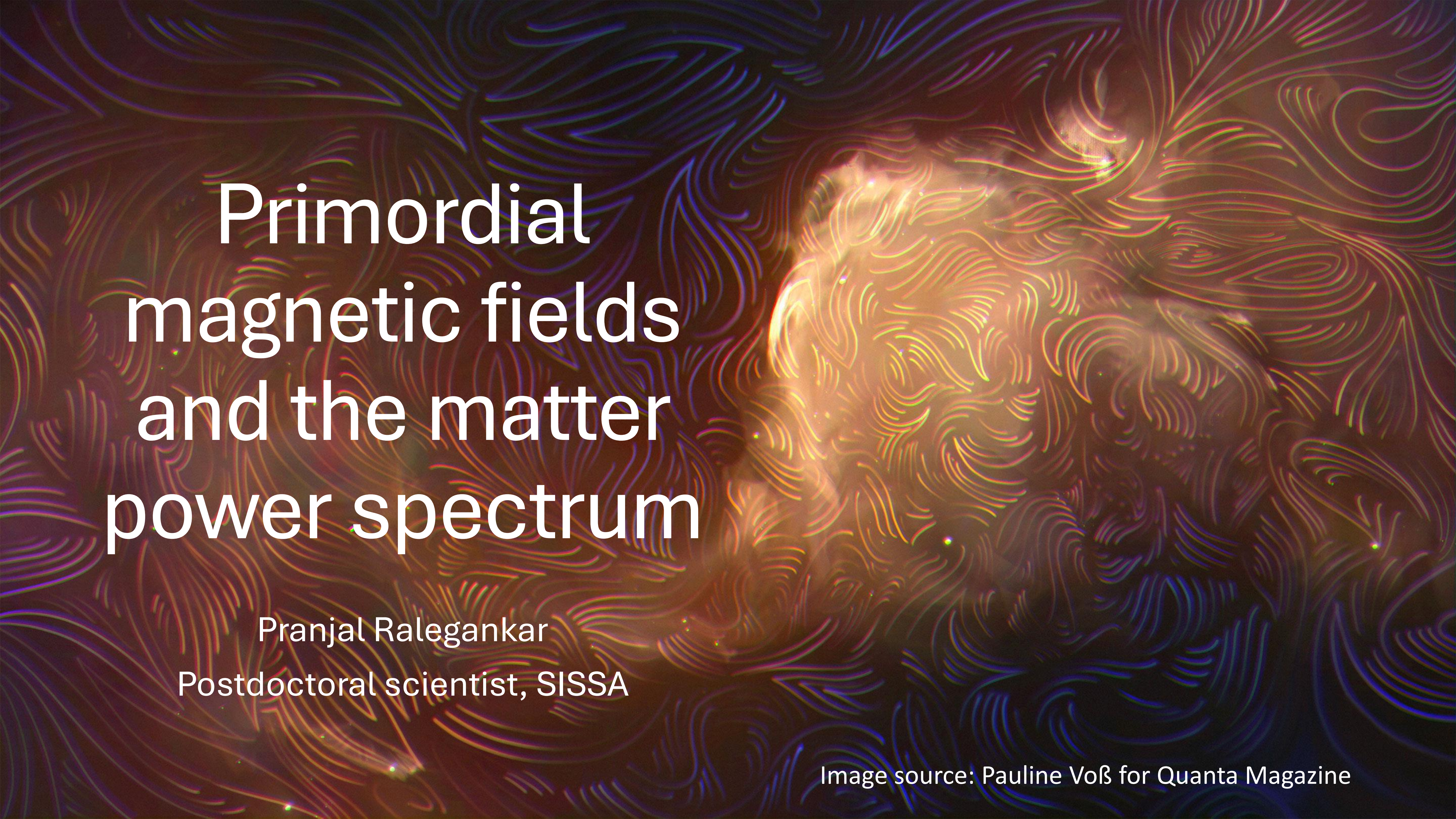


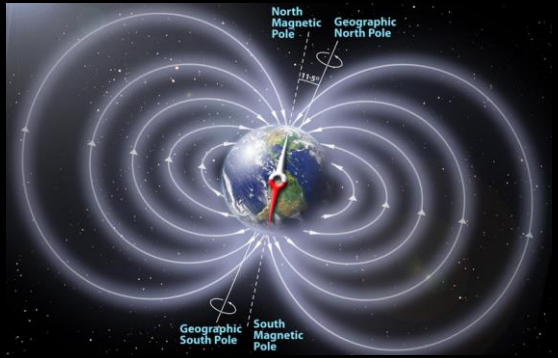
Primordial magnetic fields and the matter power spectrum

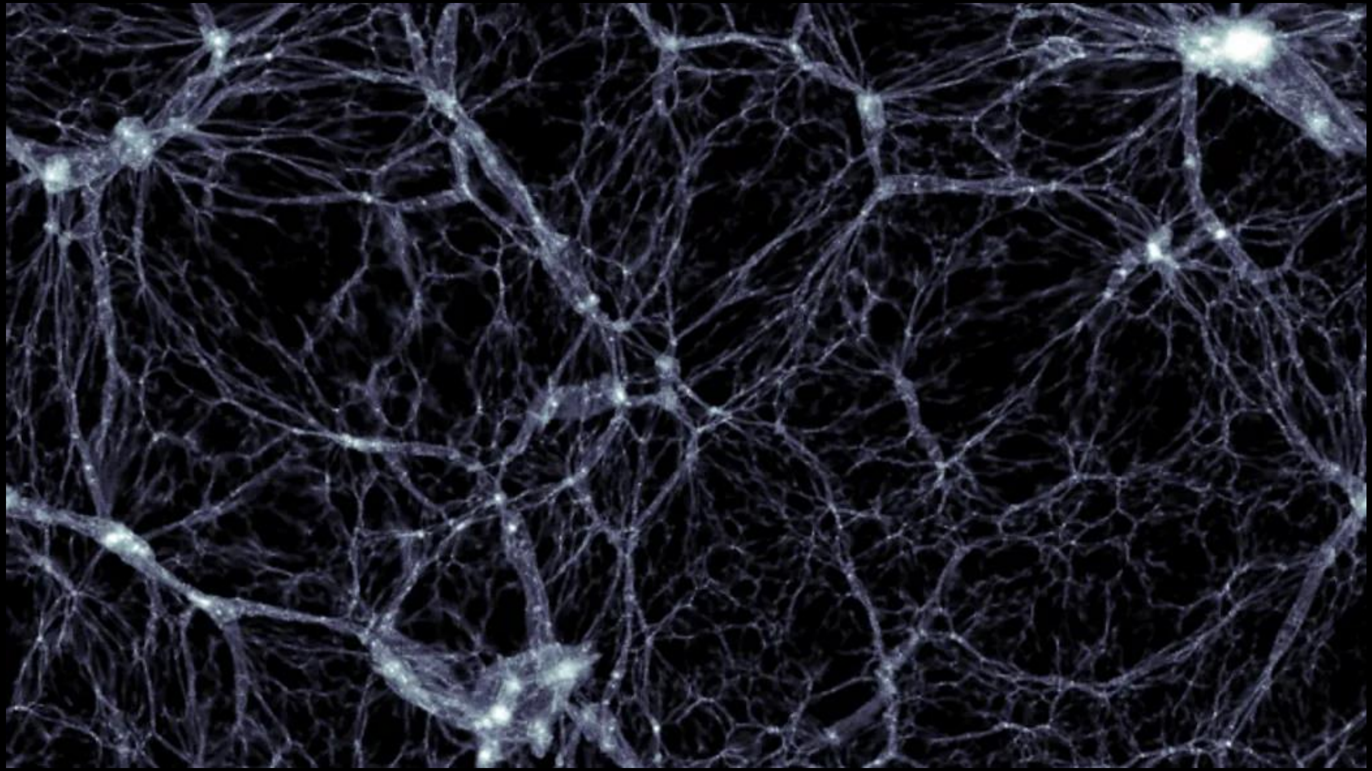
Pranjal Ralegankar

Postdoctoral scientist, SISSA

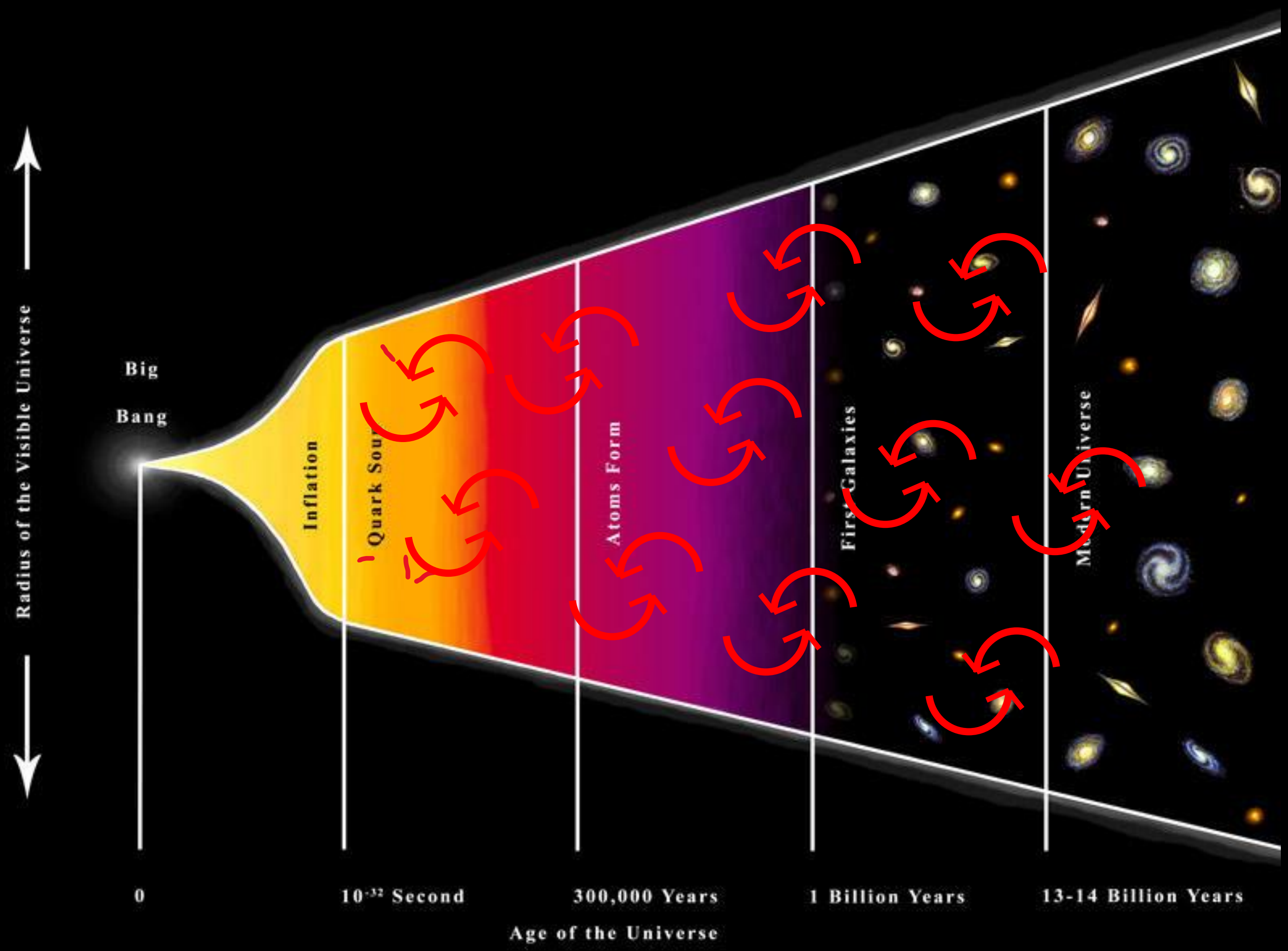
Image source: Pauline Voß for Quanta Magazine

Ubiquitous Magnetic Fields



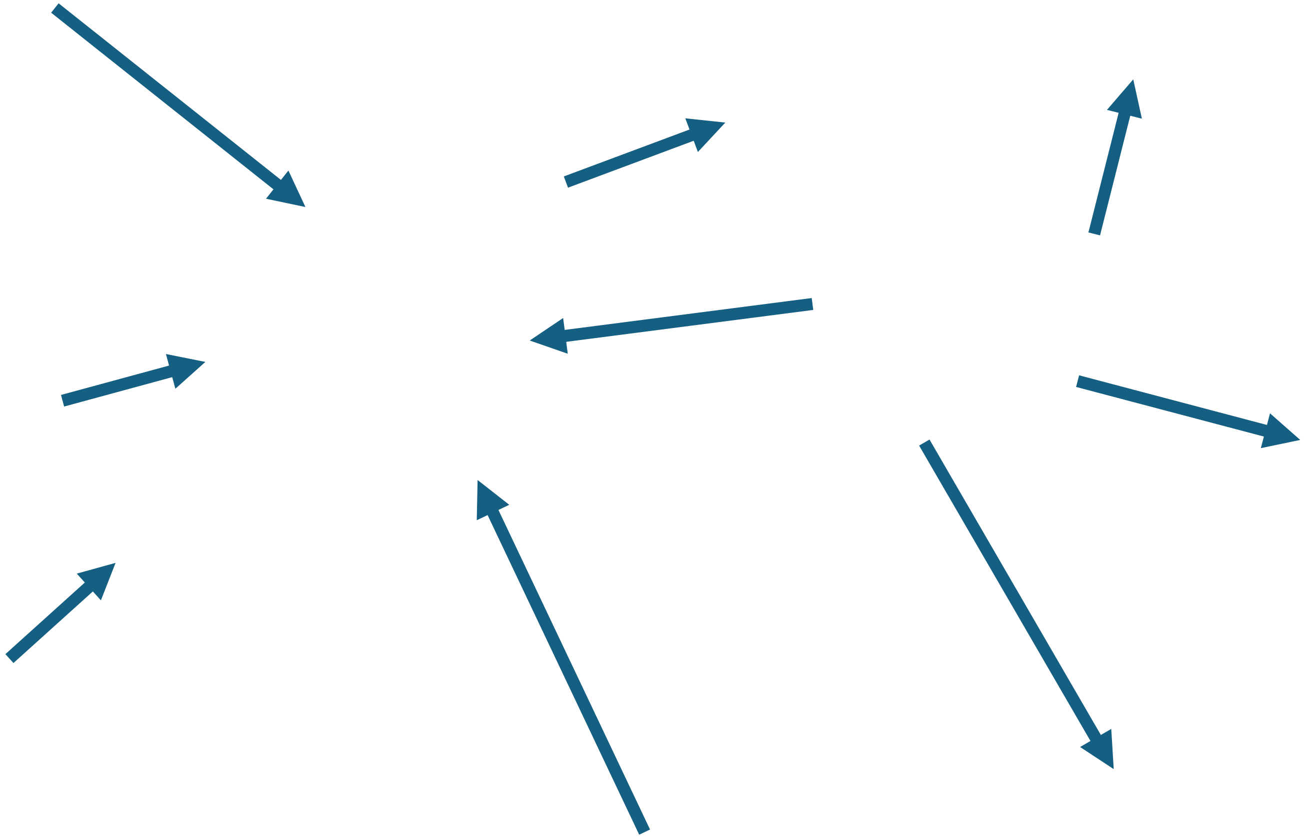


Primordial: Produced by Big Bang plasma

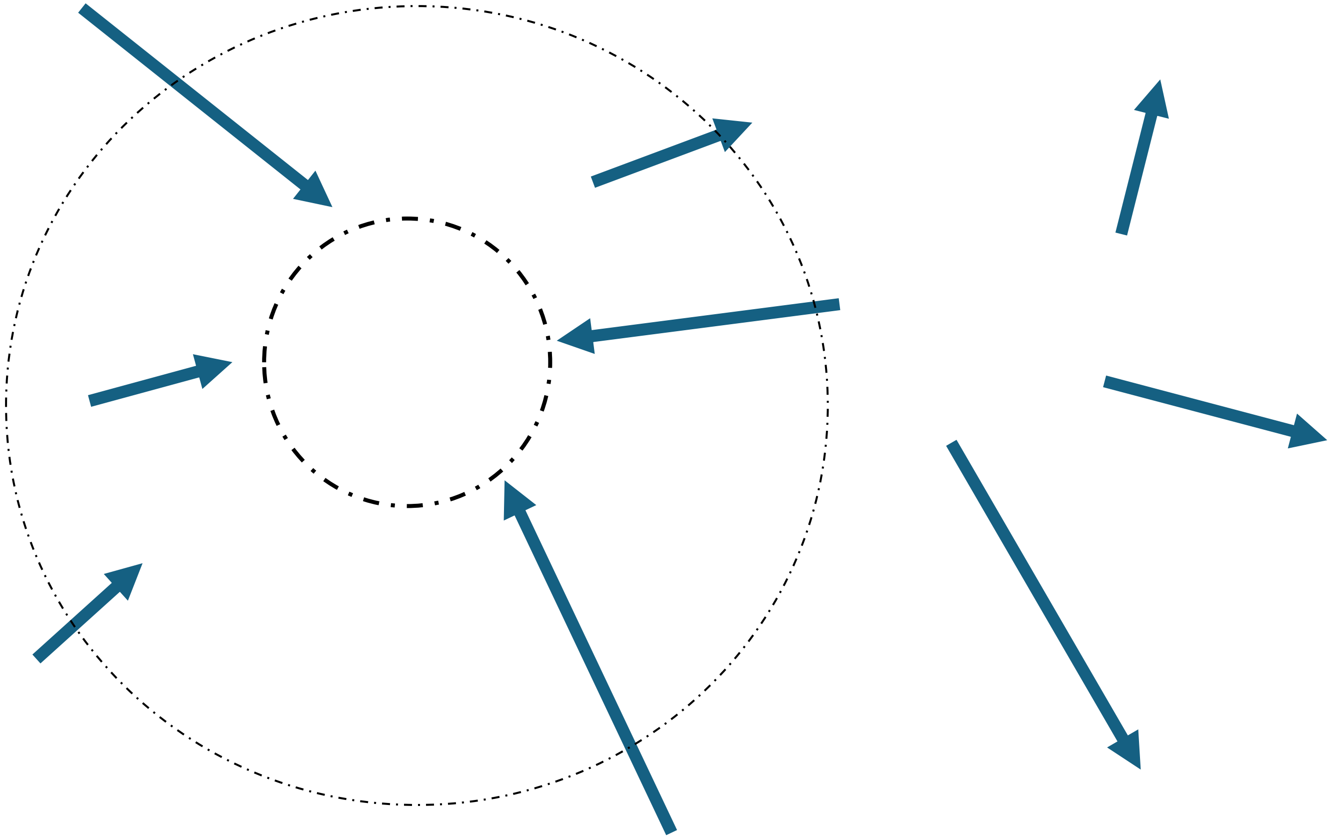


Primordial Magnetic Fields enhance density perturbations

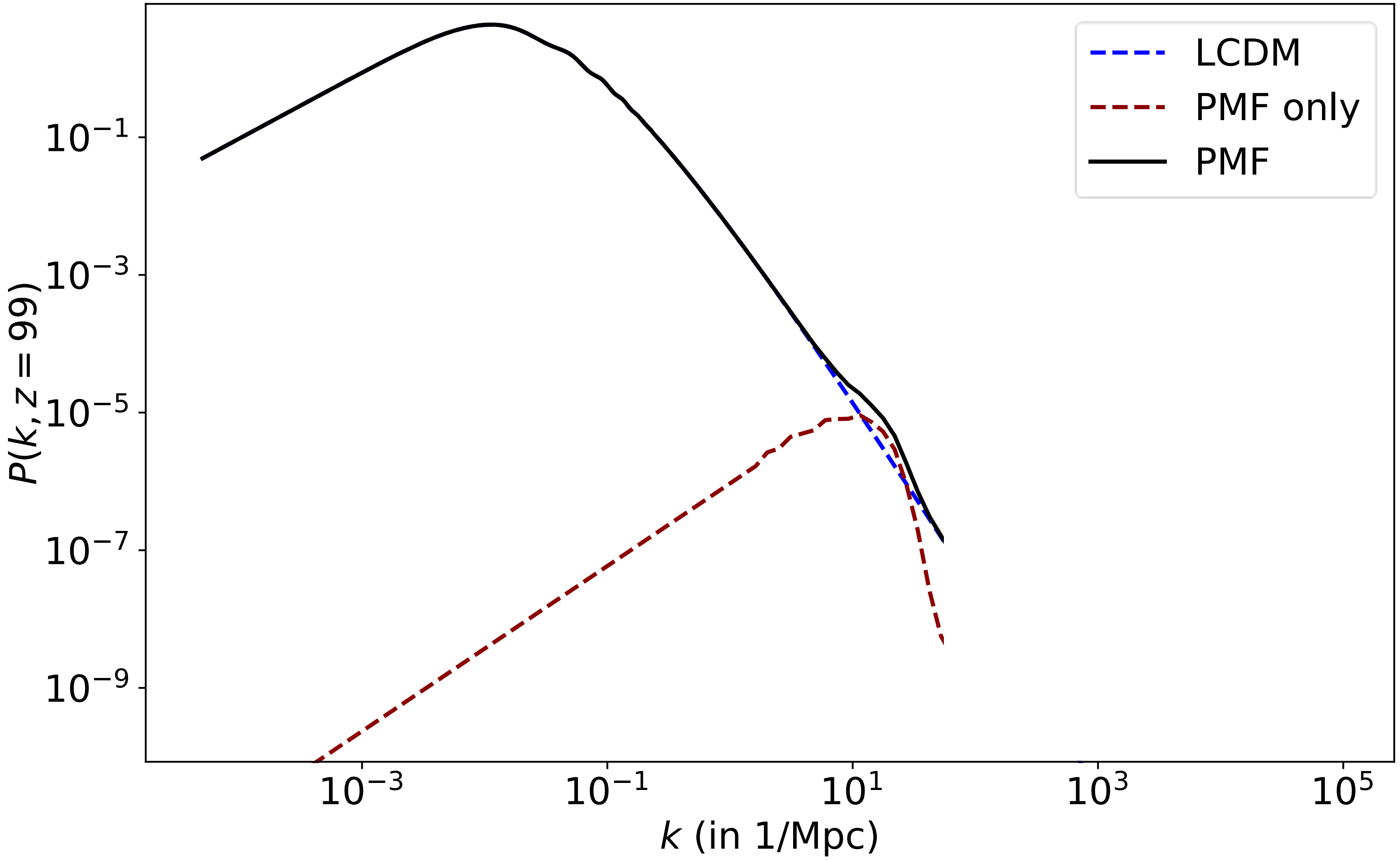
Primordial Magnetic Fields enhance density perturbations



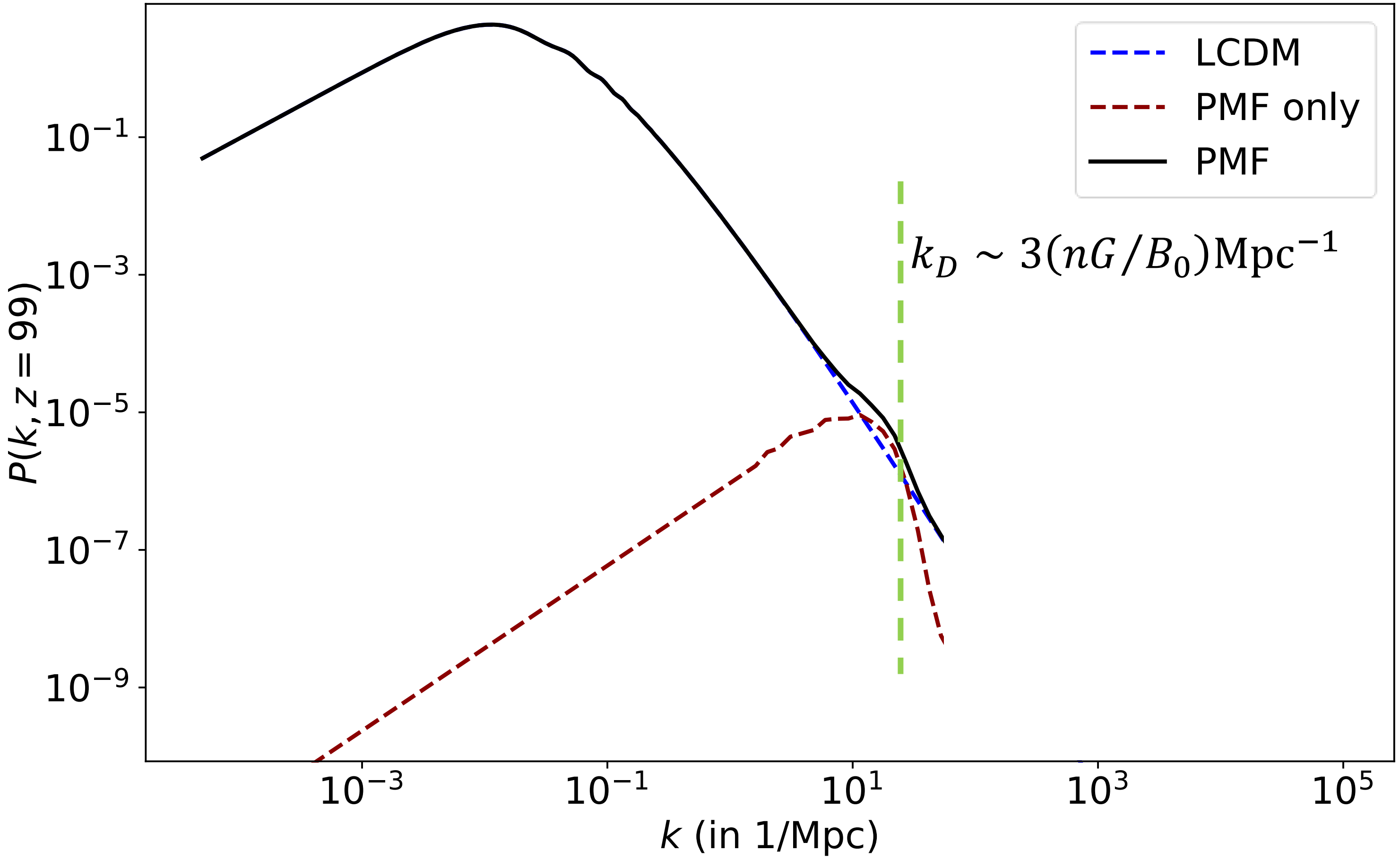
Primordial Magnetic Fields enhance density perturbations



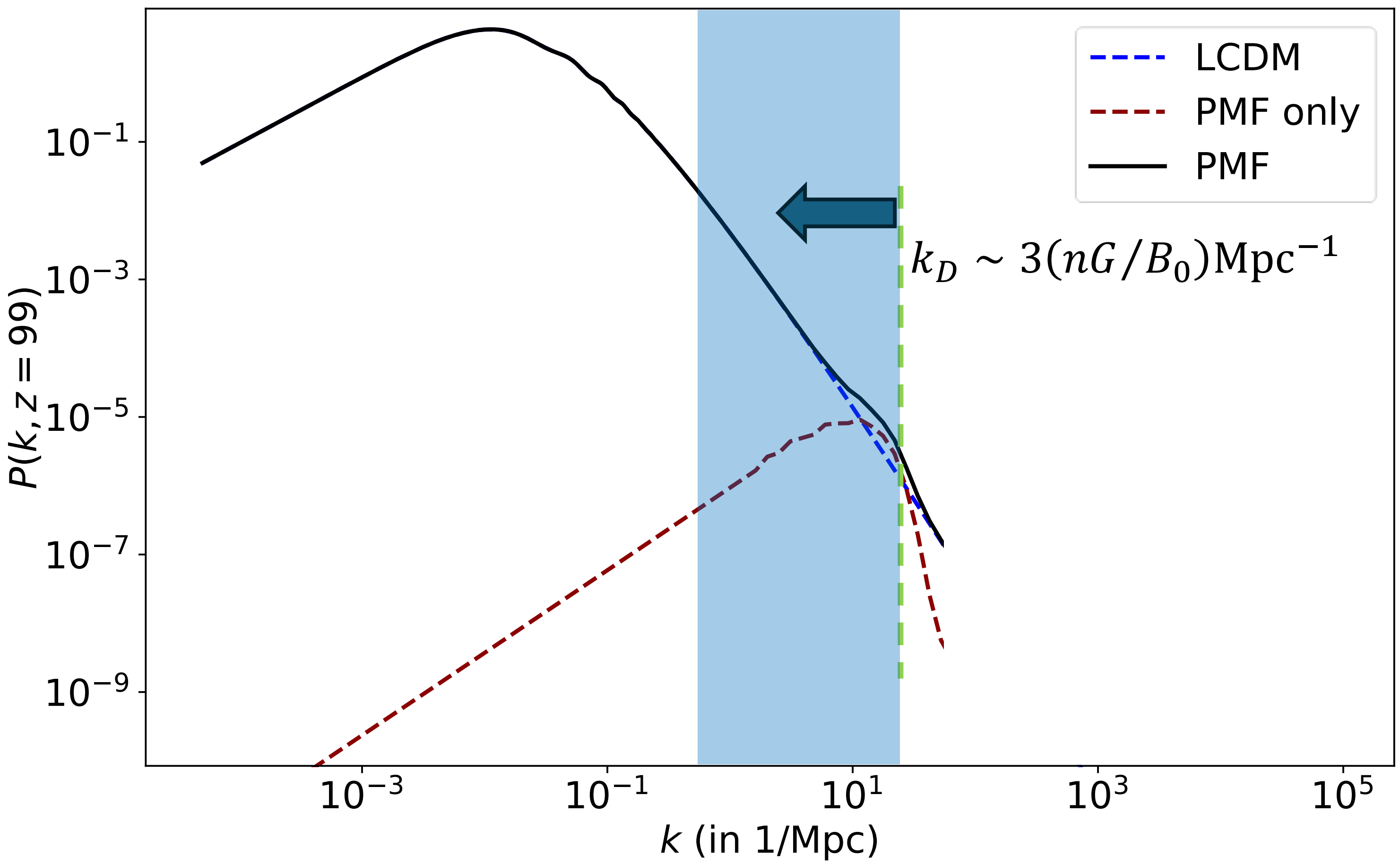
Primordial Magnetic Fields enhance power spectrum on small scales



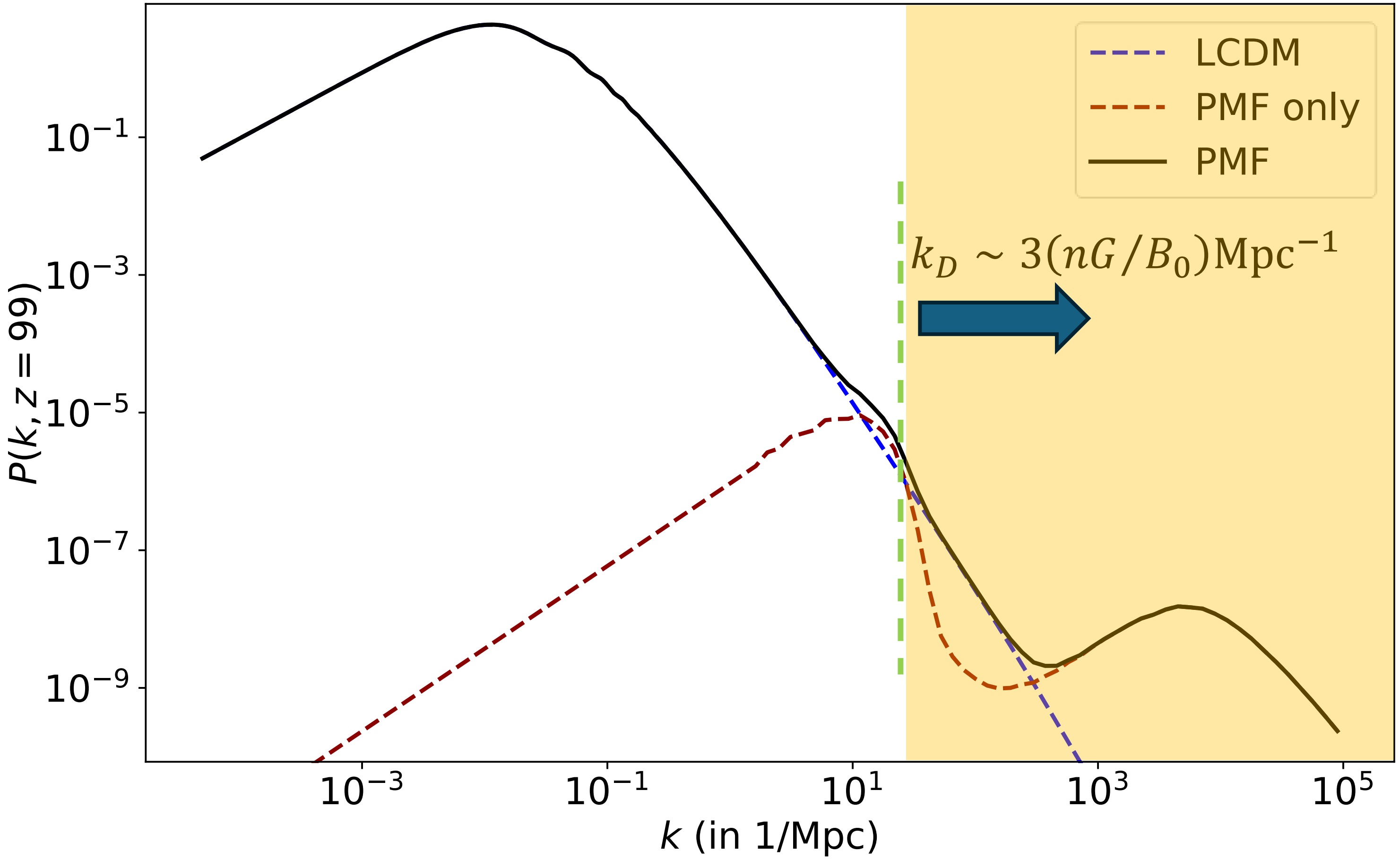
Backreaction from baryons suppresses baryon density perturbations below Magnetic damping (Jeans) scale



Part 1: Enhanced baryon fraction above jeans scale



Part 2: Dark matter minihalos below jeans scale



Part 1

Enhancing baryon fraction through Primordial magnetic fields

Arxiv: 2402.14079

Post-recombination Ideal MHD

$$\frac{\partial\ (\vec{B})}{\partial t} = \frac{\nabla \times (\vec{v}_b \times \vec{B})}{a}$$

$$\frac{\partial \vec{v}_b}{\partial t} + H\vec{v}_b + \frac{(\vec{v}_b . \nabla)\vec{v}_b}{a} = \frac{\left(\nabla \times \vec{B}\right) \times \vec{B}}{4\pi a^5 \rho_b} - \frac{\mathrm{c}_\mathrm{b}^2 \nabla \delta_b}{a} - \frac{\nabla \phi}{a}$$

$$\frac{\partial \delta_b}{\partial t} = -\frac{\nabla . \vec{v}_b}{a} - \frac{\nabla . (\delta_b \vec{v}_b)}{a}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM} \delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \left[\frac{\partial \ln(a^2 H)}{\partial \ln a} + 1\right] \frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Post-recombination Ideal MHD

$$\frac{\partial (\vec{B})}{\partial t} = \frac{\nabla \times (\vec{v}_b \times \vec{B})}{a}$$

$$\frac{\partial \vec{v}_b}{\partial t} + H\vec{v}_b + \frac{(\vec{v}_b.\nabla)\vec{v}_b}{a} = \frac{\left(\nabla \times \vec{B}\right) \times \vec{B}}{4\pi a^5 \rho_b} - \frac{\mathrm{c}_\mathrm{b}^2 \nabla \delta_b}{a} -$$

$$\frac{\partial \delta_b}{\partial t} = -\frac{\nabla.\vec{v}_b}{a} - \frac{\nabla.(\delta_b \vec{v}_b)}{a}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM}\delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \left[\frac{\partial \ln(a^2 H)}{\partial \ln a} + 1\right]\frac{\partial \delta_{DM}}{a \partial a} =$$

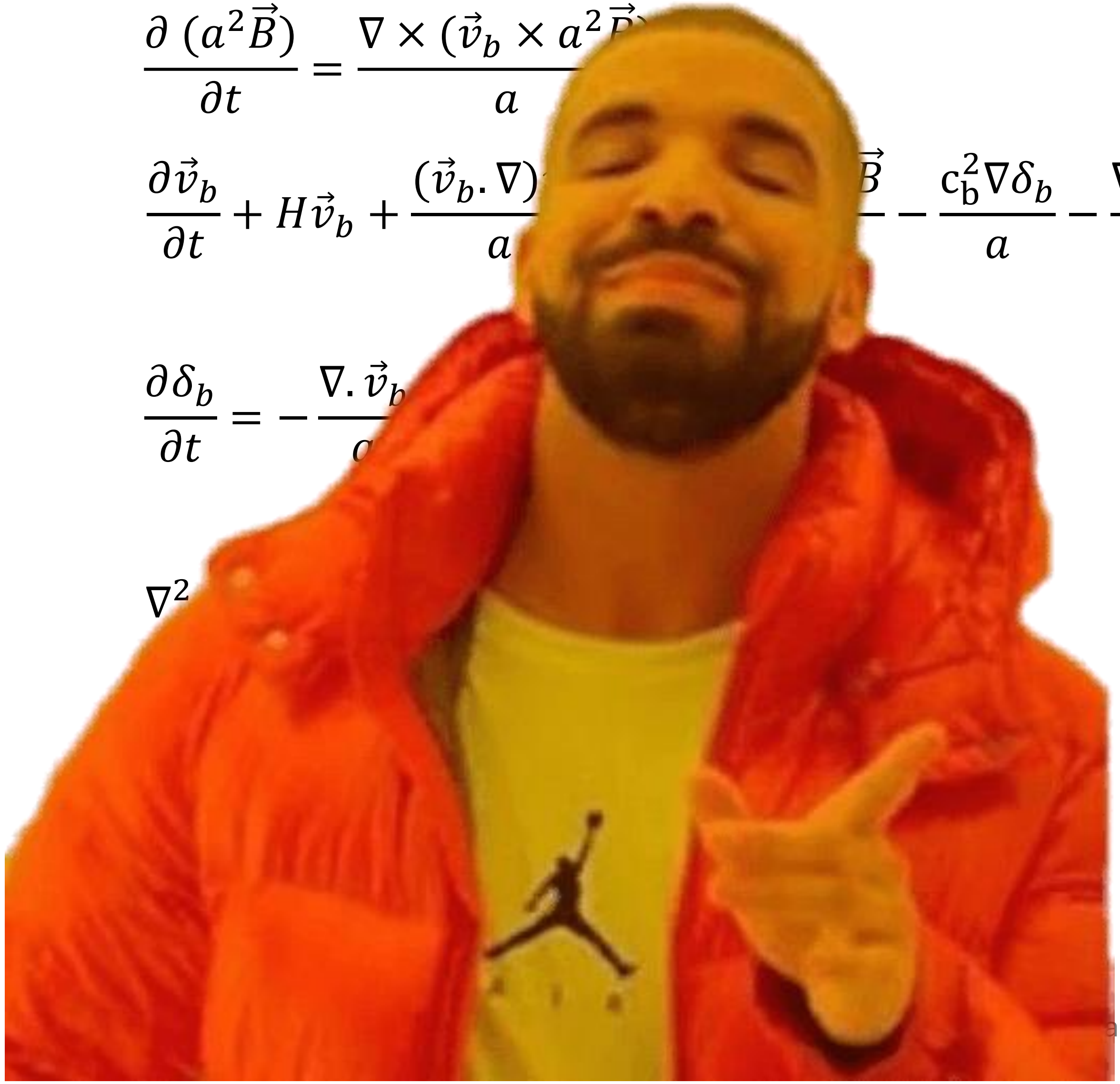
Post-recombination Ideal MHD

$$\frac{\partial\,(a^2\vec{B})}{\partial t} = \frac{\nabla\times(\vec{v}_b\times a^2\vec{B}}{a}$$

$$\frac{\partial\vec{v}_b}{\partial t} + H\vec{v}_b + \frac{(\vec{v}_b.\nabla)}{a} \quad \vec{B} - \frac{c_b^2\nabla\delta_b}{a} - \frac{\nabla\phi}{a}$$

$$\frac{\partial\delta_b}{\partial t} = -\frac{\nabla.\vec{v}_b}{a}$$

∇^2



Focus on large scales, linear limit

$$\delta \ll 1, v_b \ll aH$$

Post-recombination Ideal MHD linear limit

$$\frac{\partial\ (\vec{B})}{\partial t} = 0$$

$$\frac{\partial^2 \delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_b}{a \partial a} = -\frac{\nabla \cdot \left(\nabla \times \vec{B}\right) \times \vec{B}}{(4\pi a^3 \rho_b) a^5 H^2} + \frac{\nabla^2 \phi}{(a^2 H)^2}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM}\delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Magnetic fields are flux frozen

$$\frac{\partial\,(\vec{B})}{\partial t} = 0$$

$$\frac{\partial^2 \delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_b}{a \partial a} = -\frac{\nabla \cdot \left(\nabla \times \vec{B}\right) \times \vec{B}}{(4\pi a^3 \rho_b) a^5 H^2} + \frac{\nabla^2 \phi}{(a^2 H)^2}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM} \delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Baryons driven by Lorentz force and gravity

$$\frac{\partial\,(\vec{B})}{\partial t} = 0$$

$$\frac{\partial^2 \delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_b}{a\partial a} = -\frac{\nabla \cdot \left(\nabla \times \vec{B}\right) \times \vec{B}}{(4\pi a^3 \rho_b) a^5 H^2} + \frac{\nabla^2 \phi}{(a^2 H)^2}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}\left(\rho_b \delta_b + \rho_{DM}\delta_{DM}\right)$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_{DM}}{a\partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Dark matter only influence by gravity

$$\frac{\partial\,(\vec{B})}{\partial t} = 0$$

$$\frac{\partial^2 \delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_b}{a \partial a} = -\frac{\nabla \cdot \left(\nabla \times \vec{B}\right) \times \vec{B}}{(4\pi a^3 \rho_b) a^5 H^2} + \frac{\nabla^2 \phi}{(a^2 H)^2}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM}\delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Star of the show: S_0 term

$$\frac{\partial\,(\vec{B})}{\partial t} = 0$$

$$\frac{\partial^2 \delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_b}{a \partial a} = -\frac{\nabla \cdot \left(\nabla \times \vec{B}\right) \times \vec{B}}{(4\pi a^3 \rho_b) a^5 H^2} + \frac{\nabla^2 \phi}{(a^2 H)^2}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM}\delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Star of the show: S_0 term

$$\frac{\partial\,(\vec{B})}{\partial t} = 0$$

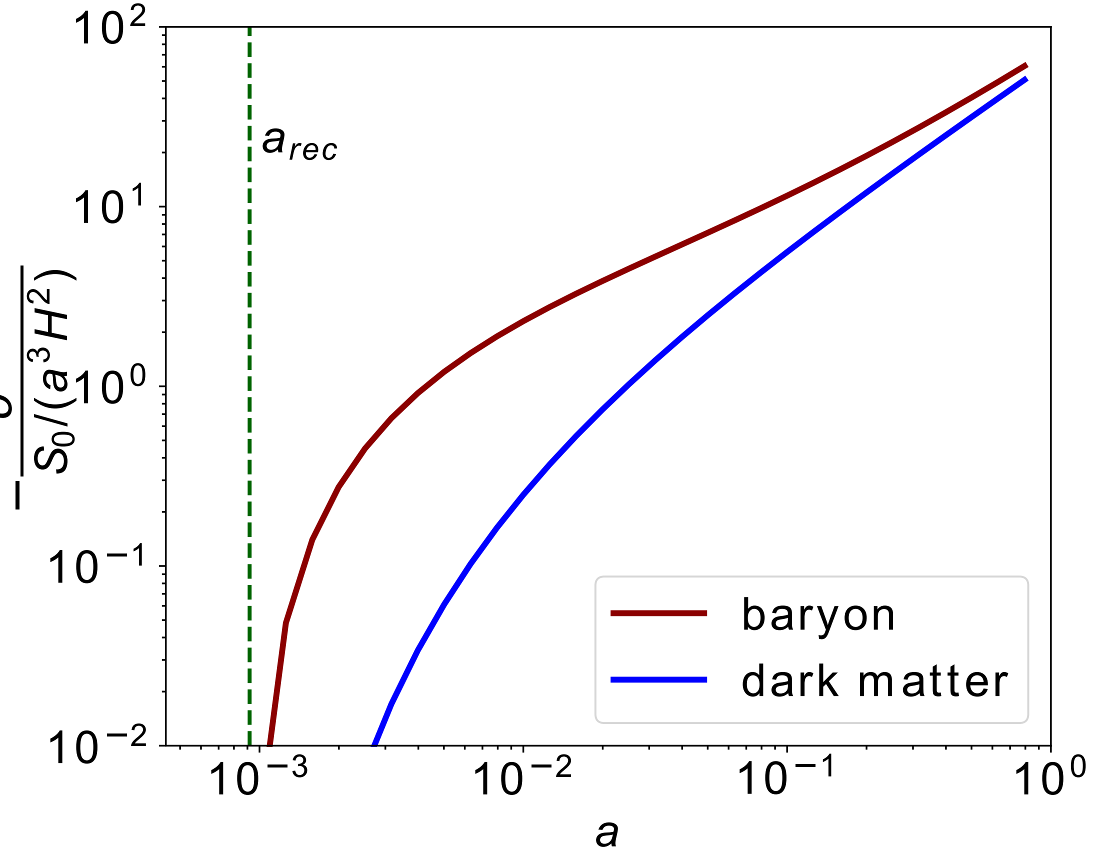
$$\frac{\partial^2\delta_b}{\partial a^2} + \frac{3}{2}\frac{\partial\delta_b}{a\partial a} = -\frac{S_0}{a^2(a^3H^2)} + \frac{\nabla^2\phi}{(a^2H)^2}$$

$$\nabla^2\phi = \frac{a^2}{2M_{Pl}^2}(\rho_b\delta_b + \rho_{DM}\delta_{DM})$$

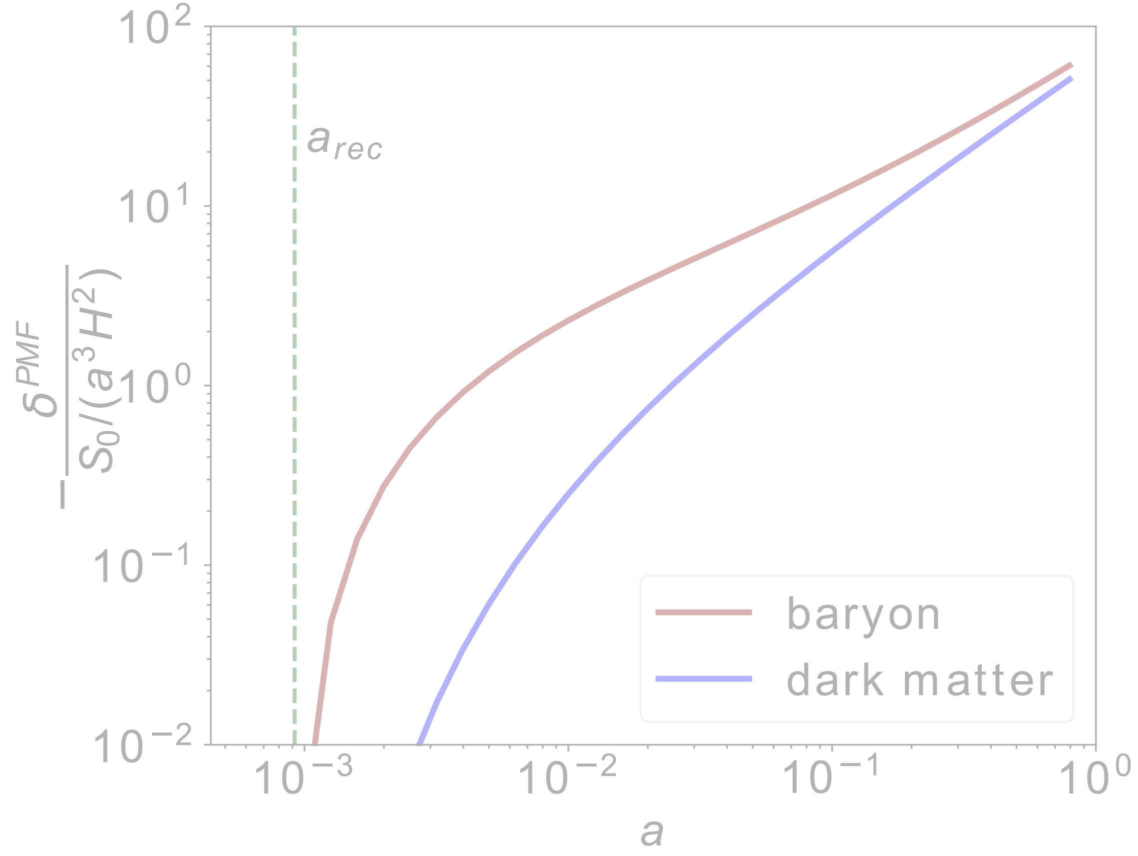
$$\frac{\partial^2\delta_{DM}}{\partial a^2} + \frac{3}{2}\frac{\partial\delta_{DM}}{a\partial a} = \frac{\nabla^2\phi}{(a^2H)^2}$$

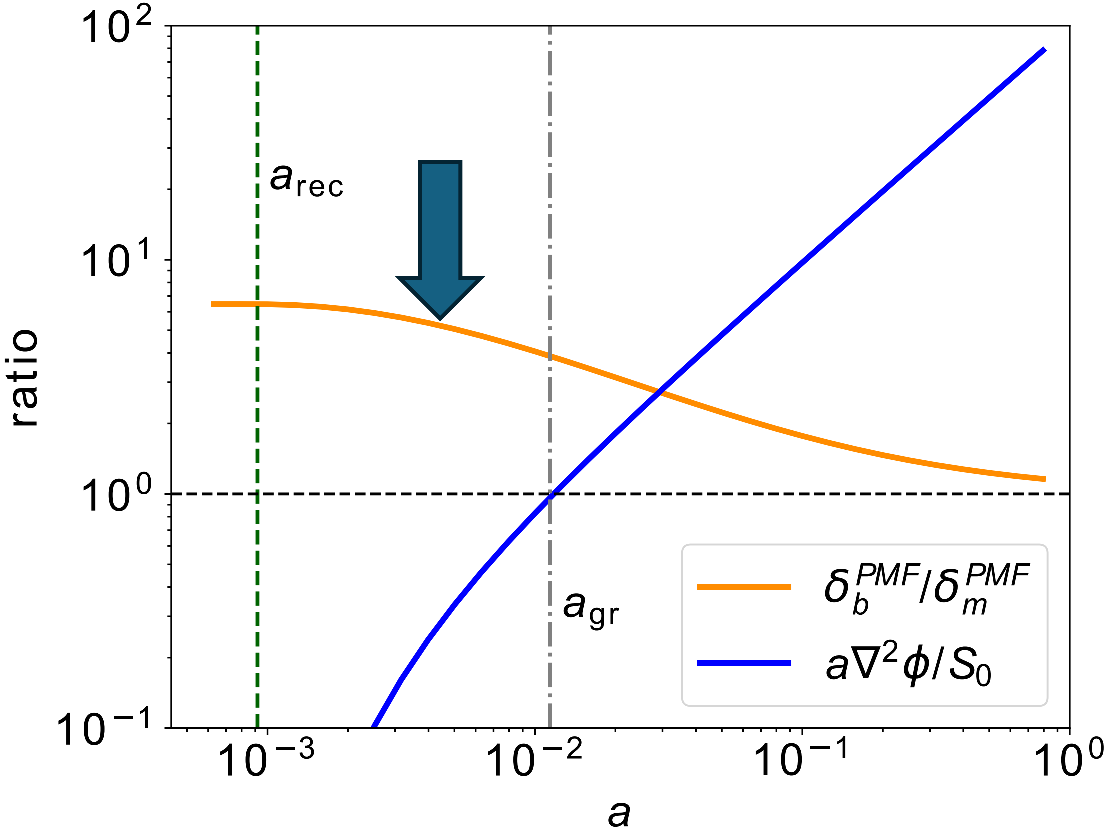
$S_0 = \frac{\nabla\cdot(\nabla\times\vec{B})\times\vec{B}}{4\pi a^3\rho_b}$=constant

S_0 sources baryon perturbations

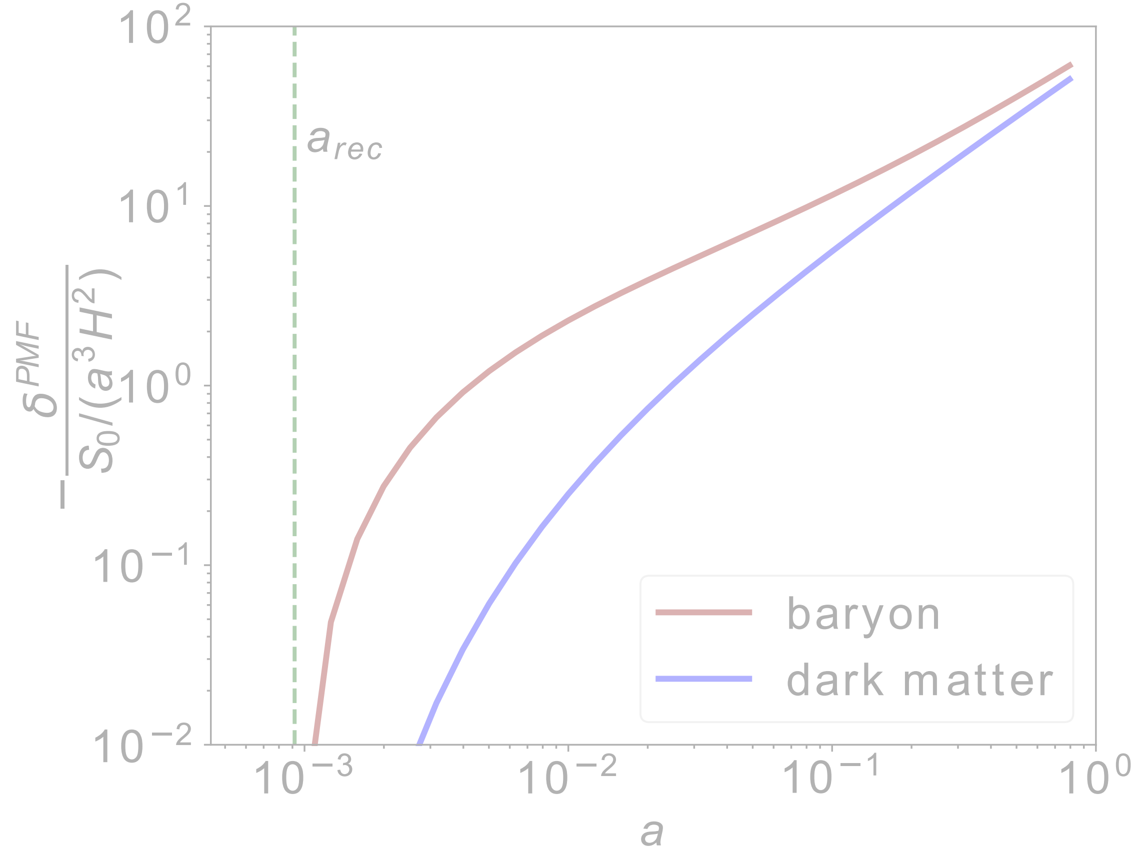


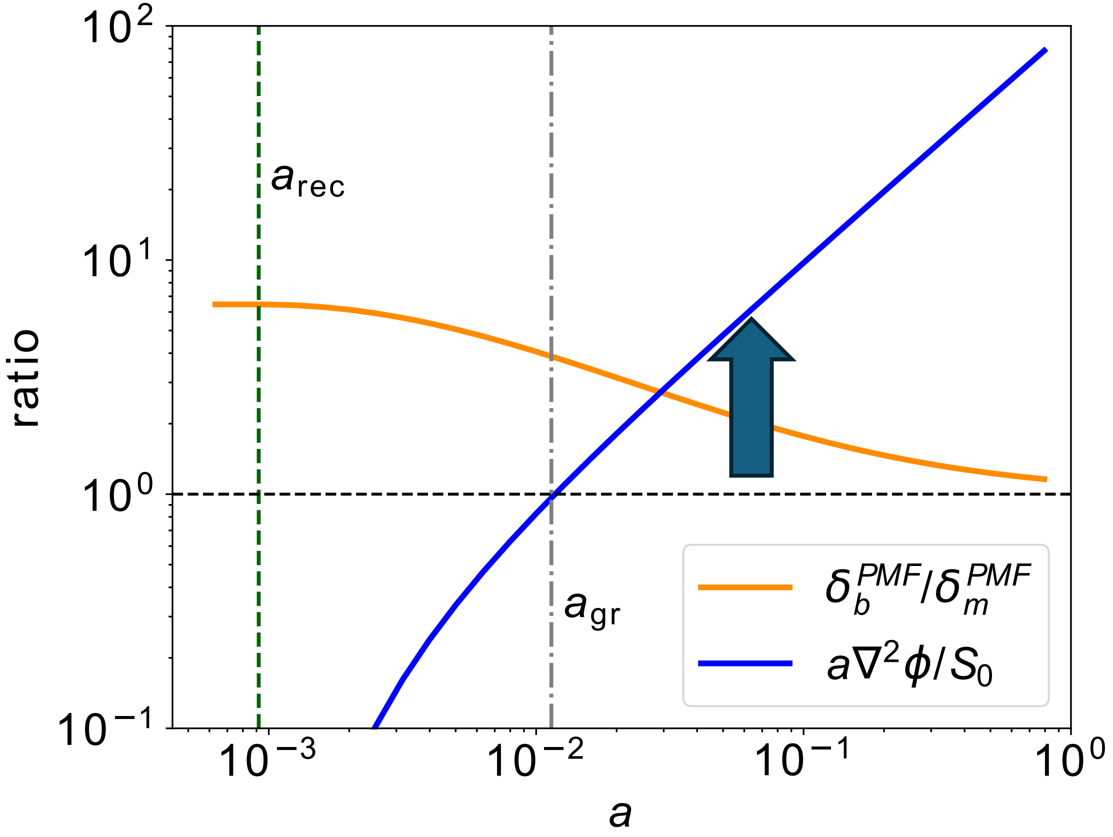
Baryon fraction decreases with time



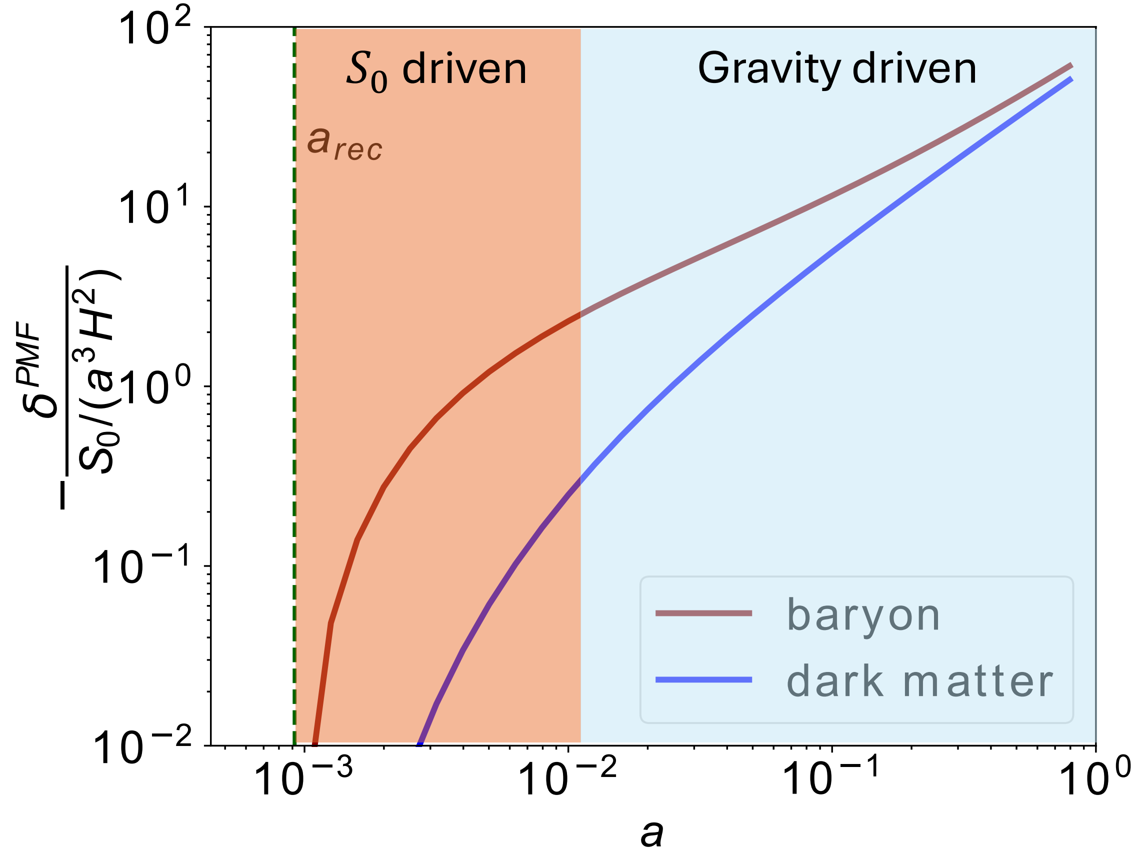


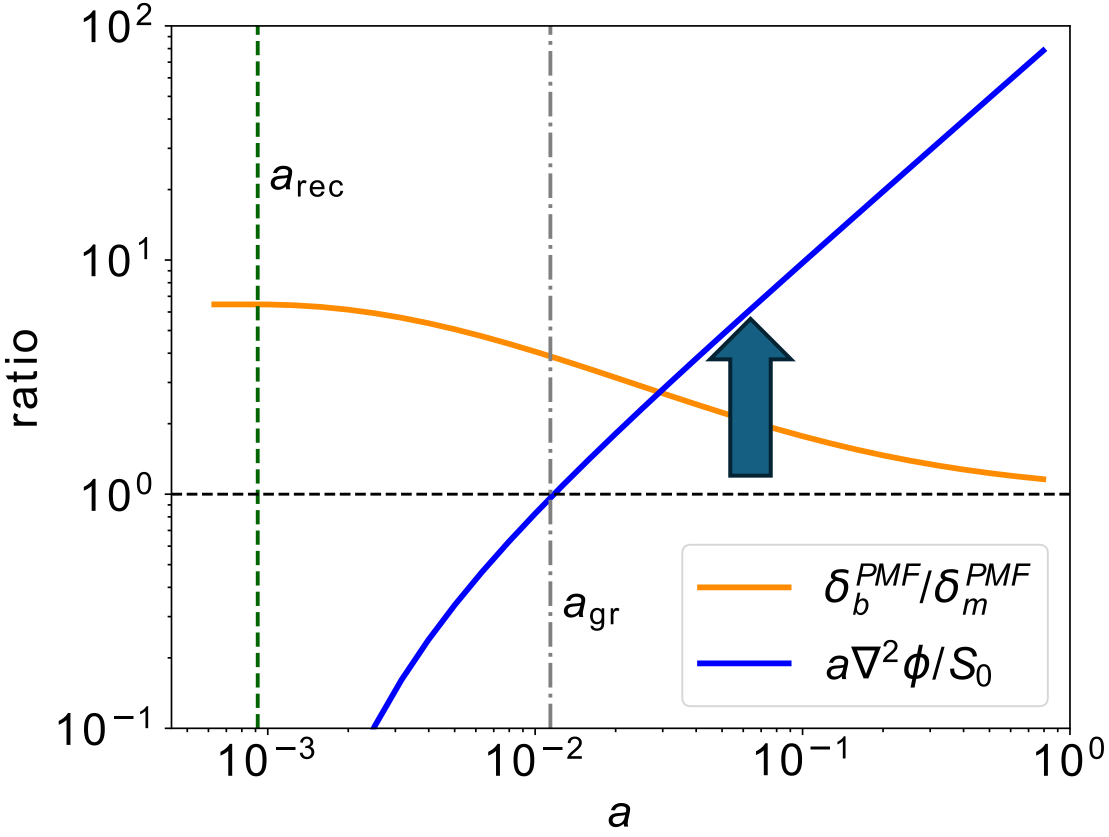
Gravity quickly overcomes Lorentz force





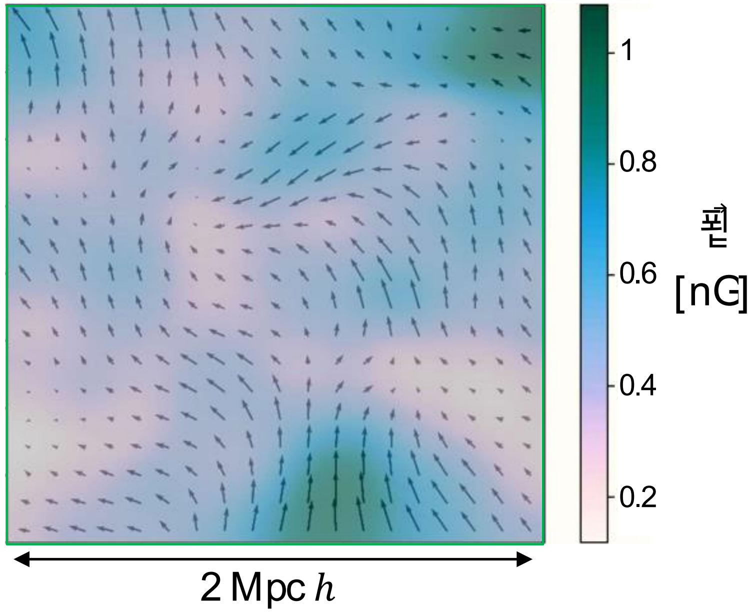
Gravity quickly overcomes Lorentz force



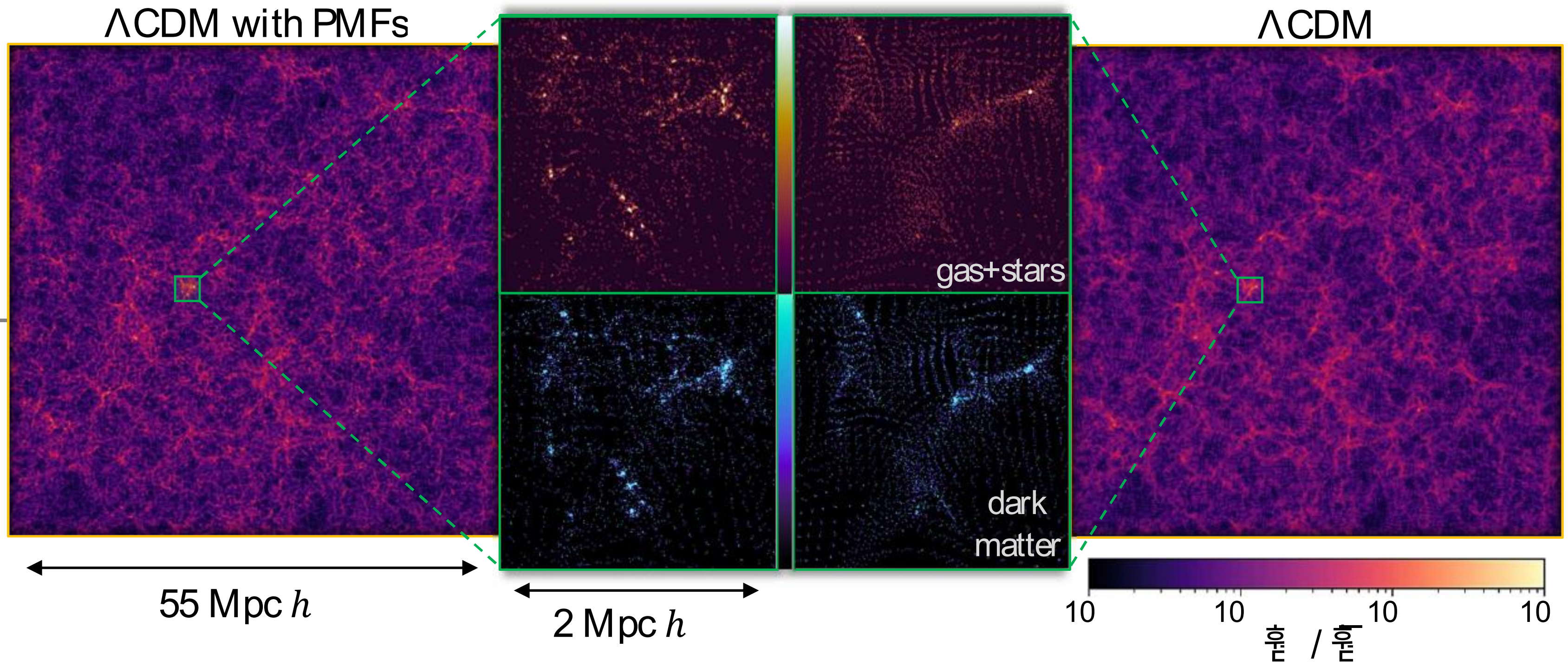


Simulations

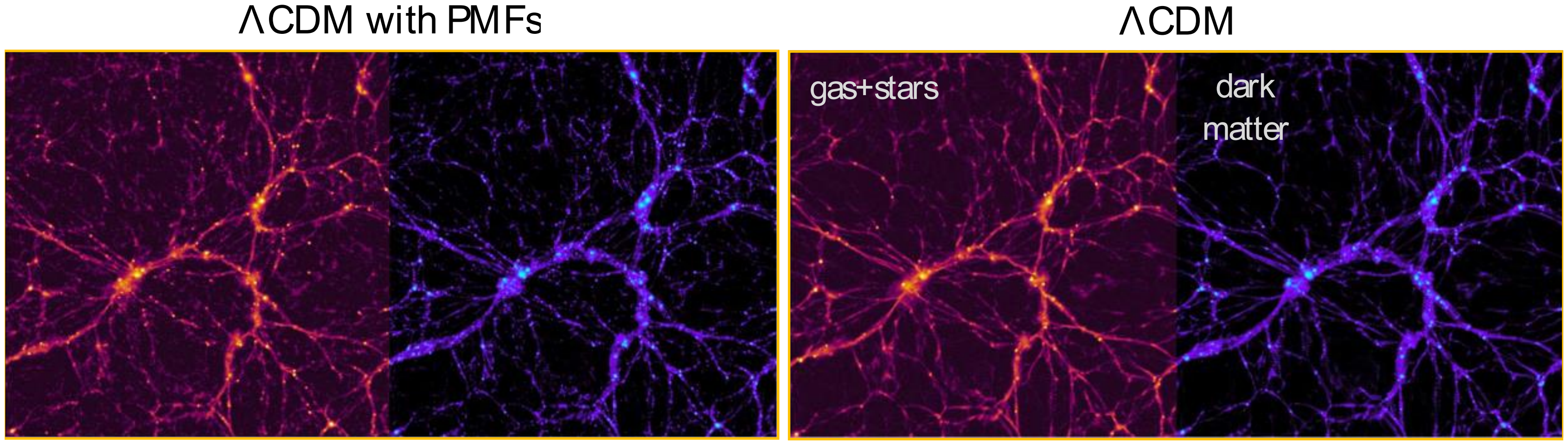
Initial conditions: redshift = 99



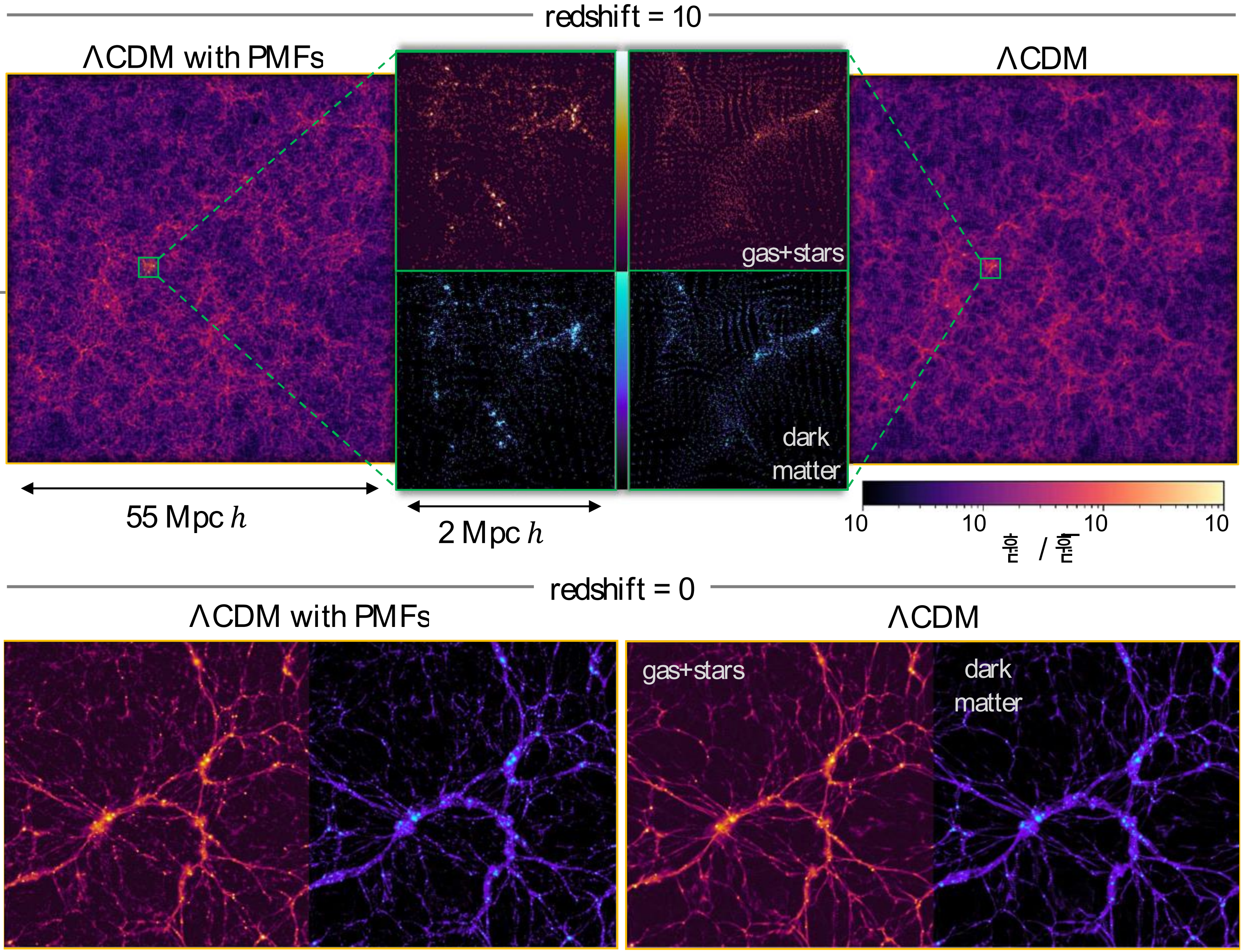
redshift = 10



redshift = 0

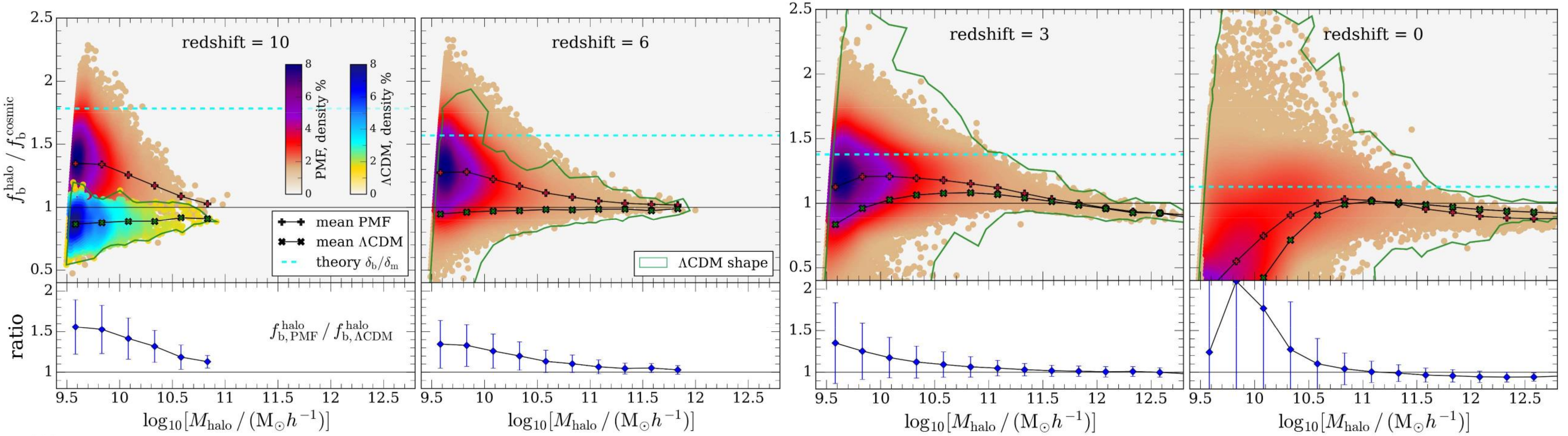


Simulations

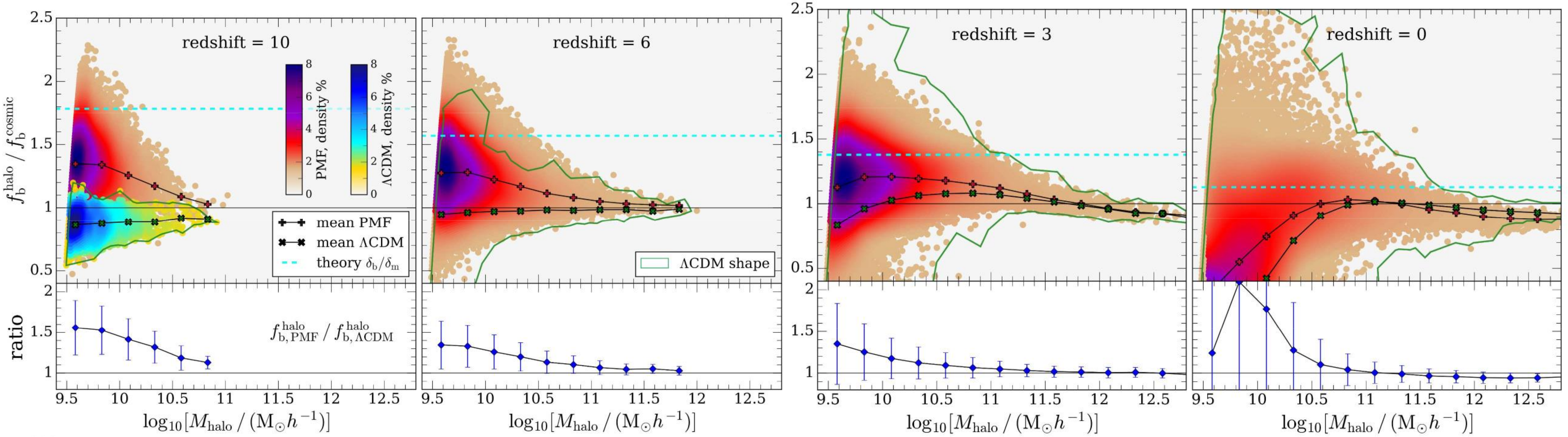


Baryon fraction in halos: enhanced by PMFs

Baryon fraction in halos: enhanced by PMFs

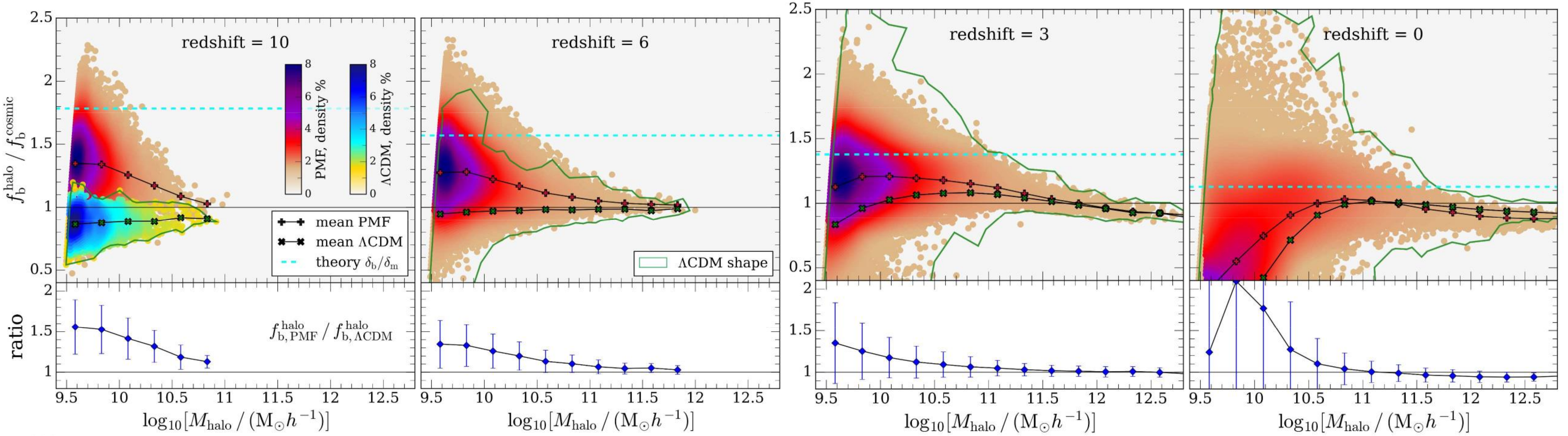


Baryon fraction in halos: stochastic nature



$$\frac{f_b^{halo}}{f_b^{cosmic}} = \frac{\delta_b^{PMF} + \delta_b^{\Lambda CDM}}{\delta_m^{PMF} + \delta_m^{\Lambda CDM}}$$

Possible implications for black hole formation, star formation efficiency etc



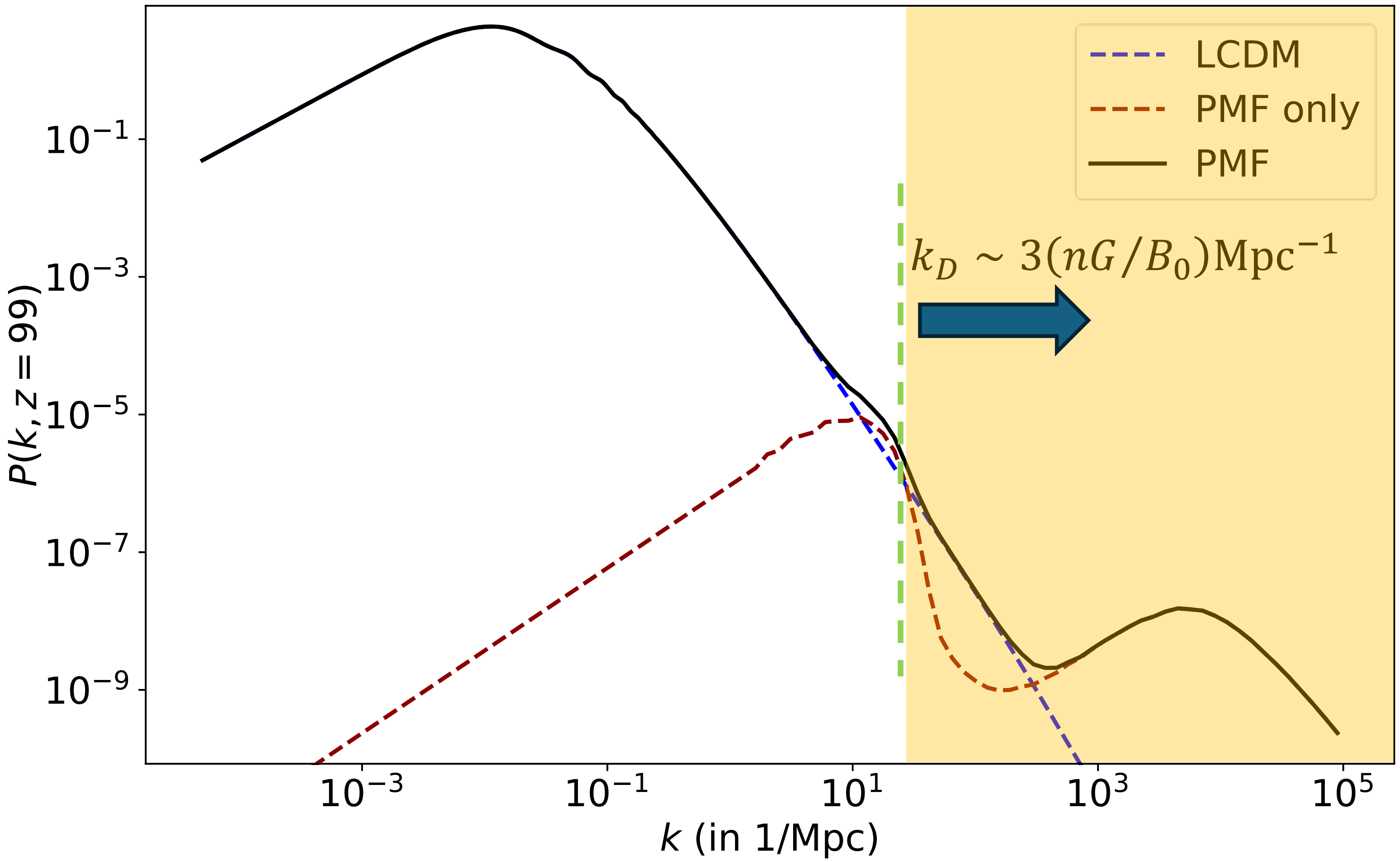
$$\frac{f_b^{halo}}{f_b^{cosmic}} = \frac{\delta_b^{PMF} + \delta_b^{\Lambda CDM}}{\delta_m^{PMF} + \delta_m^{\Lambda CDM}}$$

Part 2

Enhancing Dark matter minihalos through Primordial magnetic fields

ARXIV: 2303.11861

Part 2: Dark matter minihalos below jeans scale



Pre-recombination Ideal MHD.. With non-linear terms

$$\frac{\partial\ (\vec{B})}{\partial t} = \frac{\nabla \times (\vec{v}_b \times \vec{B})}{a}$$

$$\frac{\partial \vec{v}_b}{\partial t} + (H + \alpha)\vec{v}_b + \frac{(\vec{v}_b . \nabla)\vec{v}_b}{a} = \frac{\left(\nabla \times \vec{B}\right) \times \vec{B}}{4\pi a \rho_b} - \frac{\mathrm{c}_\mathrm{b}^2 \nabla \delta_b}{a} - \frac{\nabla \phi}{a}$$

$$\frac{\partial \delta_b}{\partial t} = -\frac{\nabla . \vec{v}_b}{a} - \frac{\nabla . (\delta_b \vec{v}_b)}{a}$$

$$\nabla^2 \phi = \frac{a^2}{2 M_{Pl}^2} (\rho_b \delta_b + \rho_{DM} \delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \left[\frac{\partial \ln(a^2 H)}{\partial \ln a} + 1\right] \frac{\partial \delta_{DM}}{a \partial a} = \frac{\nabla^2 \phi}{(a^2 H)^2}$$

Pre-recombination Ideal MHD.. With non-linear terms

$$\frac{\partial\,(\vec{B})}{\partial t} = \frac{\nabla \times (\vec{v}_b \times \vec{B})}{a}$$

$$\frac{\partial \vec{v}_b}{\partial t} + (H + \alpha)\vec{v}_b + \frac{(\vec{v}_b.\nabla)\vec{v}_b}{a} = \frac{\left(\nabla \times \vec{B}\right) \times \vec{B}}{4\pi a \rho_b} - c_b^2$$

$$\frac{\partial \delta_b}{\partial t} = -\frac{\nabla.\,\vec{v}_b}{a} - \frac{\nabla.\,(\delta_b \vec{v}_b)}{a}$$

$$\nabla^2 \phi = \frac{a^2}{2M_{Pl}^2}(\rho_b \delta_b + \rho_{DM}\delta_{DM})$$

$$\frac{\partial^2 \delta_{DM}}{\partial a^2} + \left[\frac{\partial \ln(a^2 H)}{\partial \ln a} + 1\right]\frac{\partial \delta_{DM}}{a \partial a} =$$

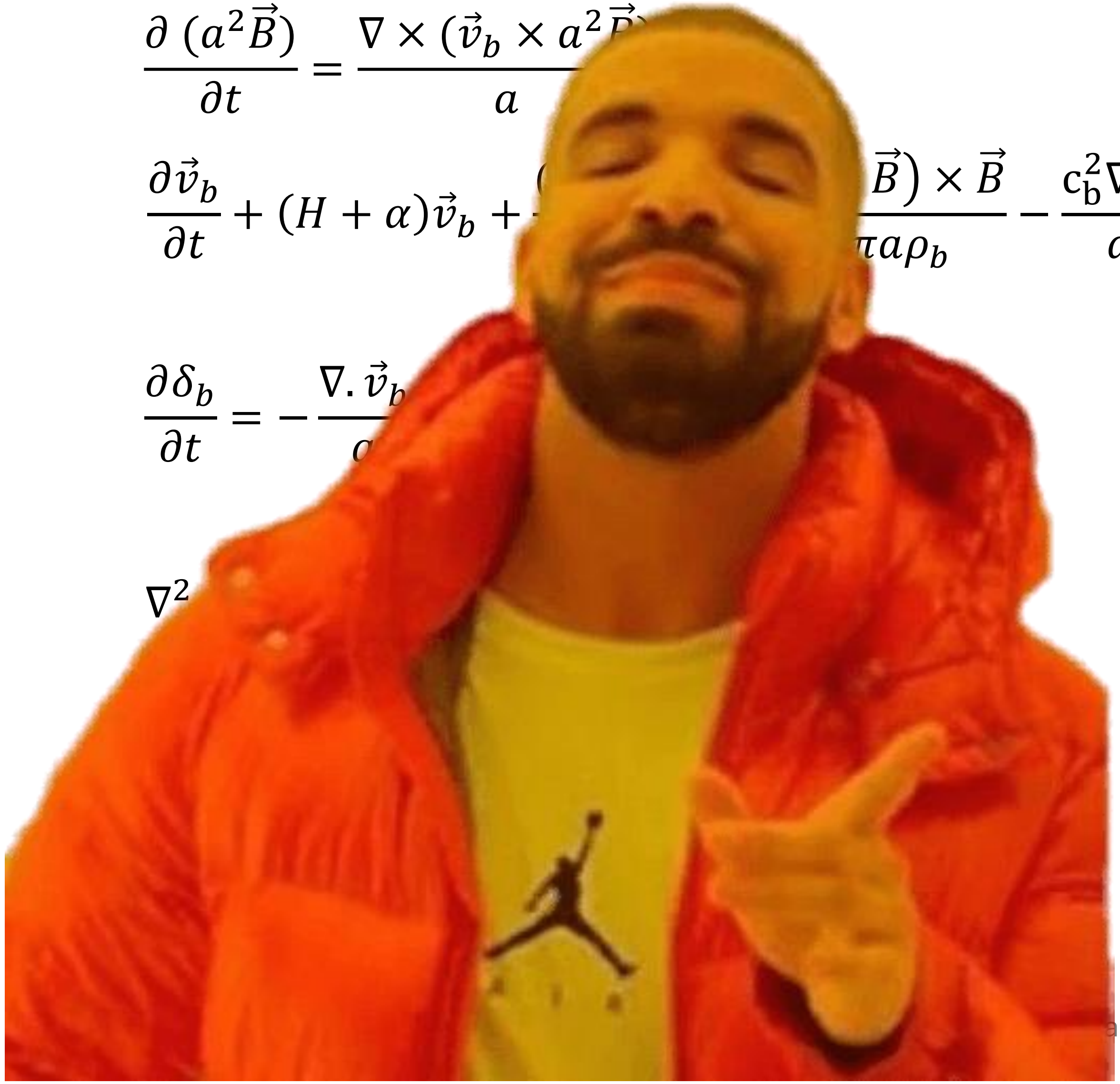
Non-Relativistic Ideal MHD in photon Drag regime

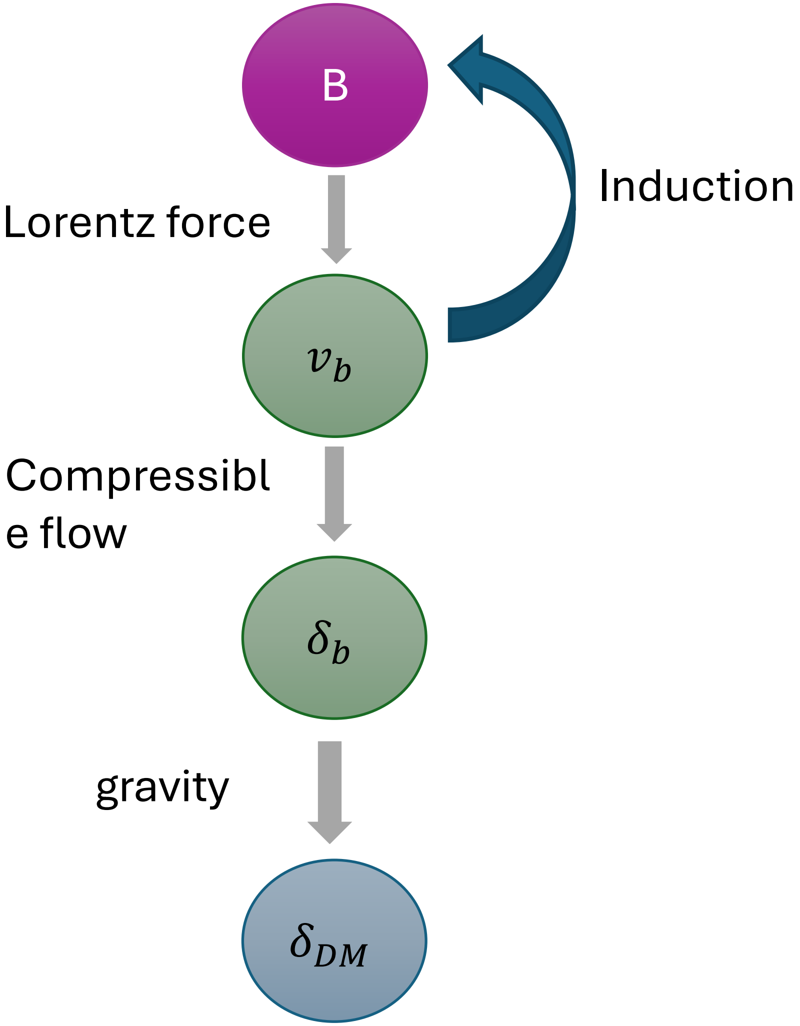
$$\frac{\partial\,(a^2\vec{B})}{\partial t} = \frac{\nabla\times(\vec{v}_b\times a^2\vec{B}}{a}$$

$$\frac{\partial\vec{v}_b}{\partial t} + (H+\alpha)\vec{v}_b + \quad \vec{B})\times\vec{B} \quad \pi a\rho_b - \frac{c_b^2\nabla\delta_b}{a} - \frac{\nabla\phi}{a}$$

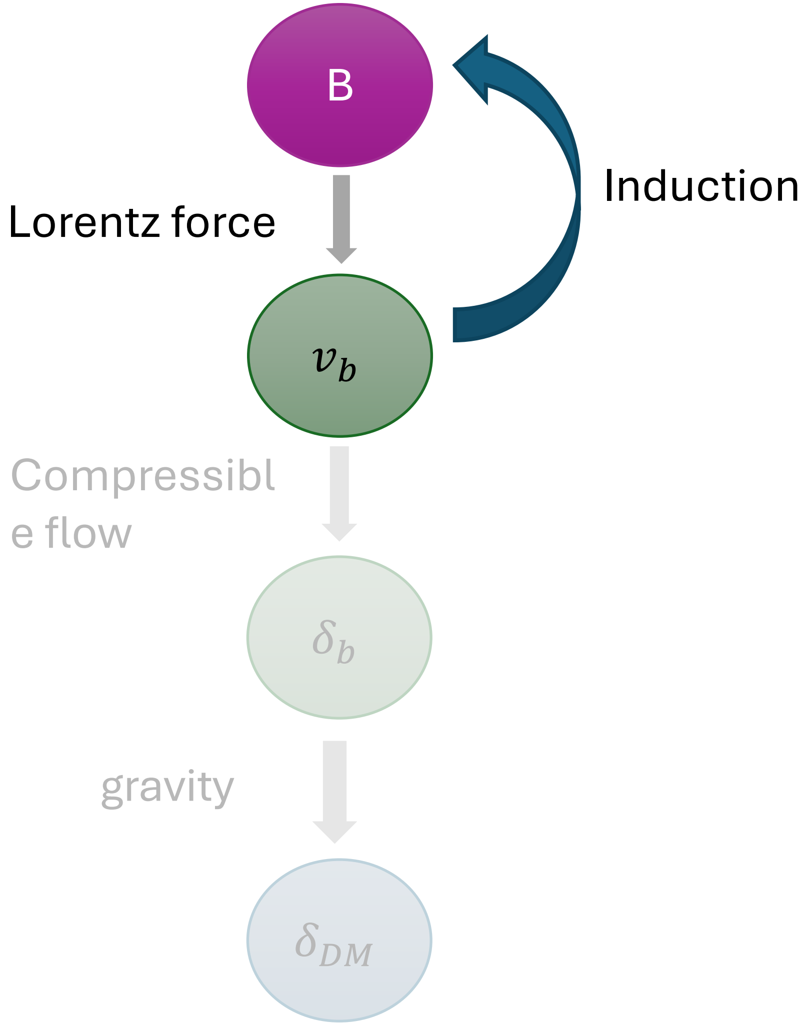
$$\frac{\partial\delta_b}{\partial t} = -\frac{\nabla.\vec{v}_b}{a}$$

$$\nabla^2$$





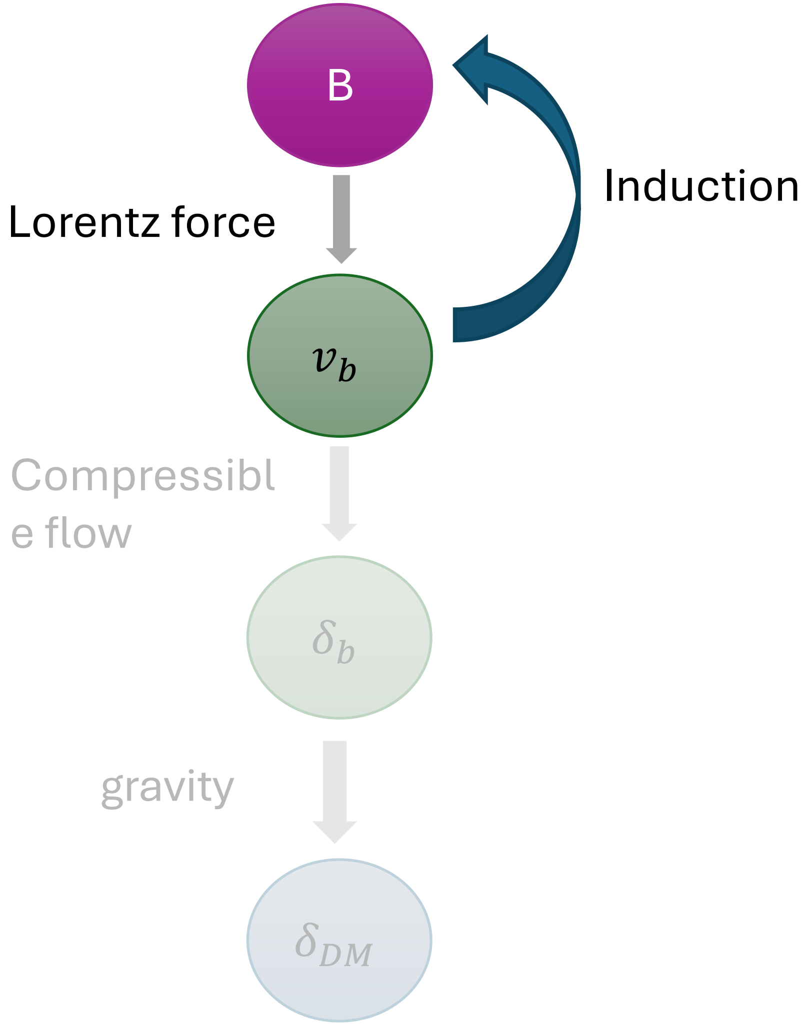
Non-Relativistic Ideal MHD in photon Drag regime: Can Solve B analytically!!



magnetic damping scale

$$P_B(k,t) = P_B(k,t_I)e^{-\frac{k^2}{k_D^2}}$$

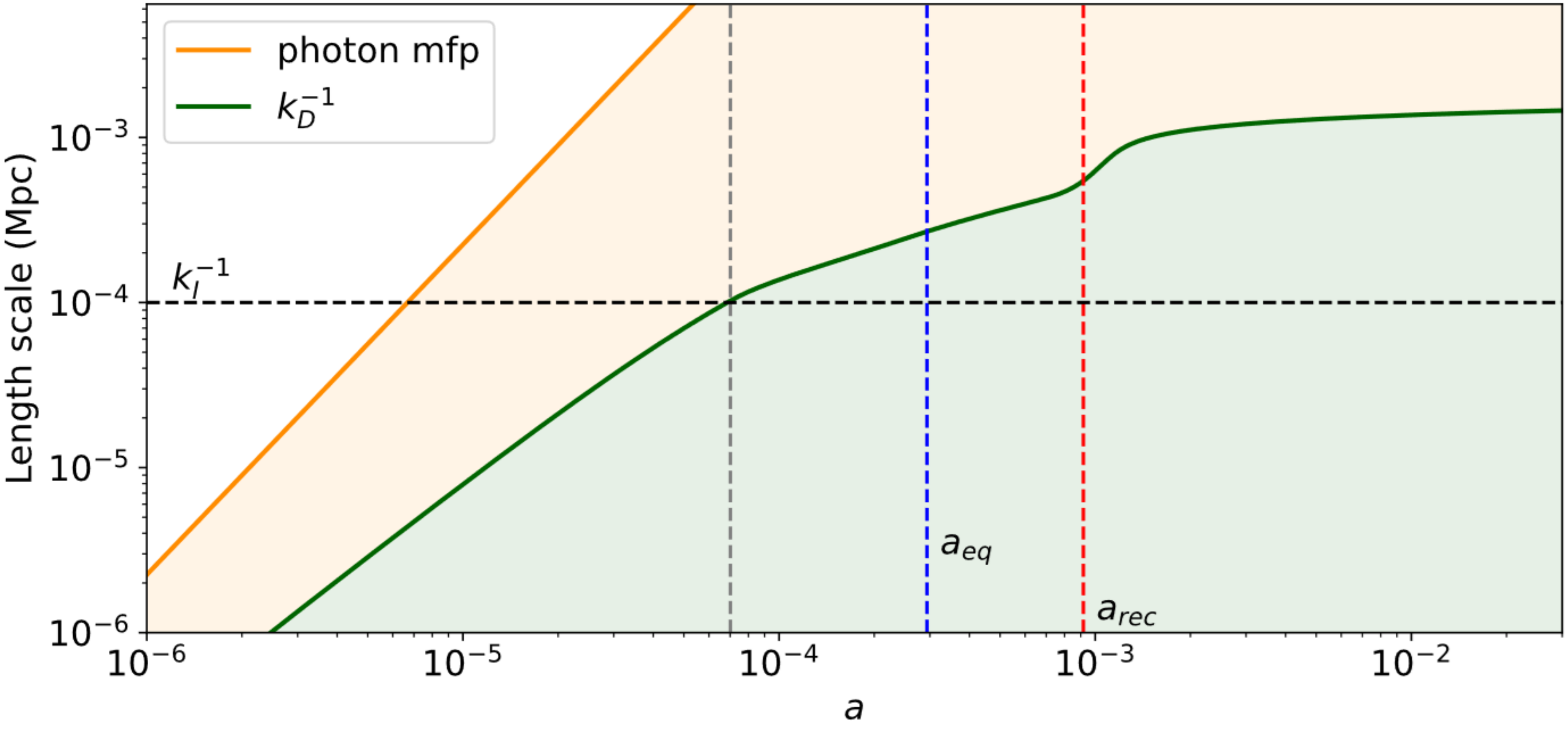
$$k_D^{-1}(a) \sim \tau v_b$$



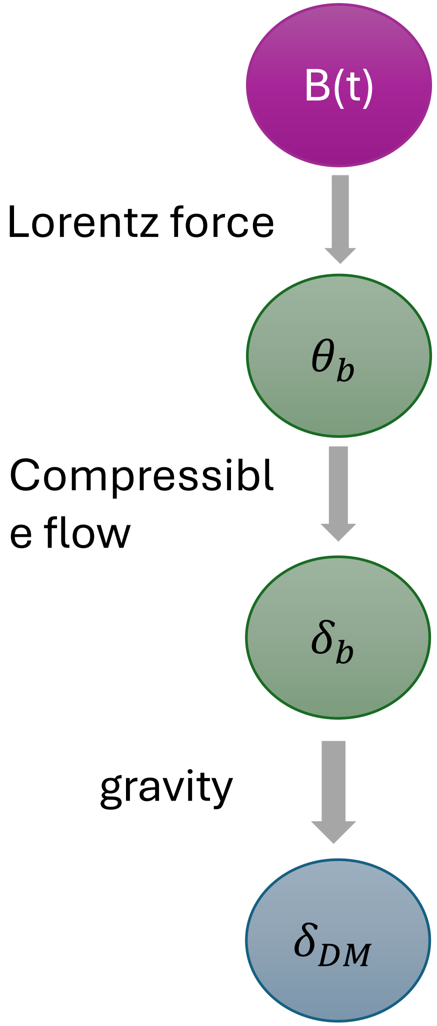
magnetic damping scale Evolution

$$P_B(k,t) = P_B(k,t_I)e^{-\frac{k^2}{k_D^2}}$$

$$k_D^{-1}(a) \sim \tau v_b$$



Solve Perturbations with magnetic fields as external source



B(t)

Lorentz force

θ_b

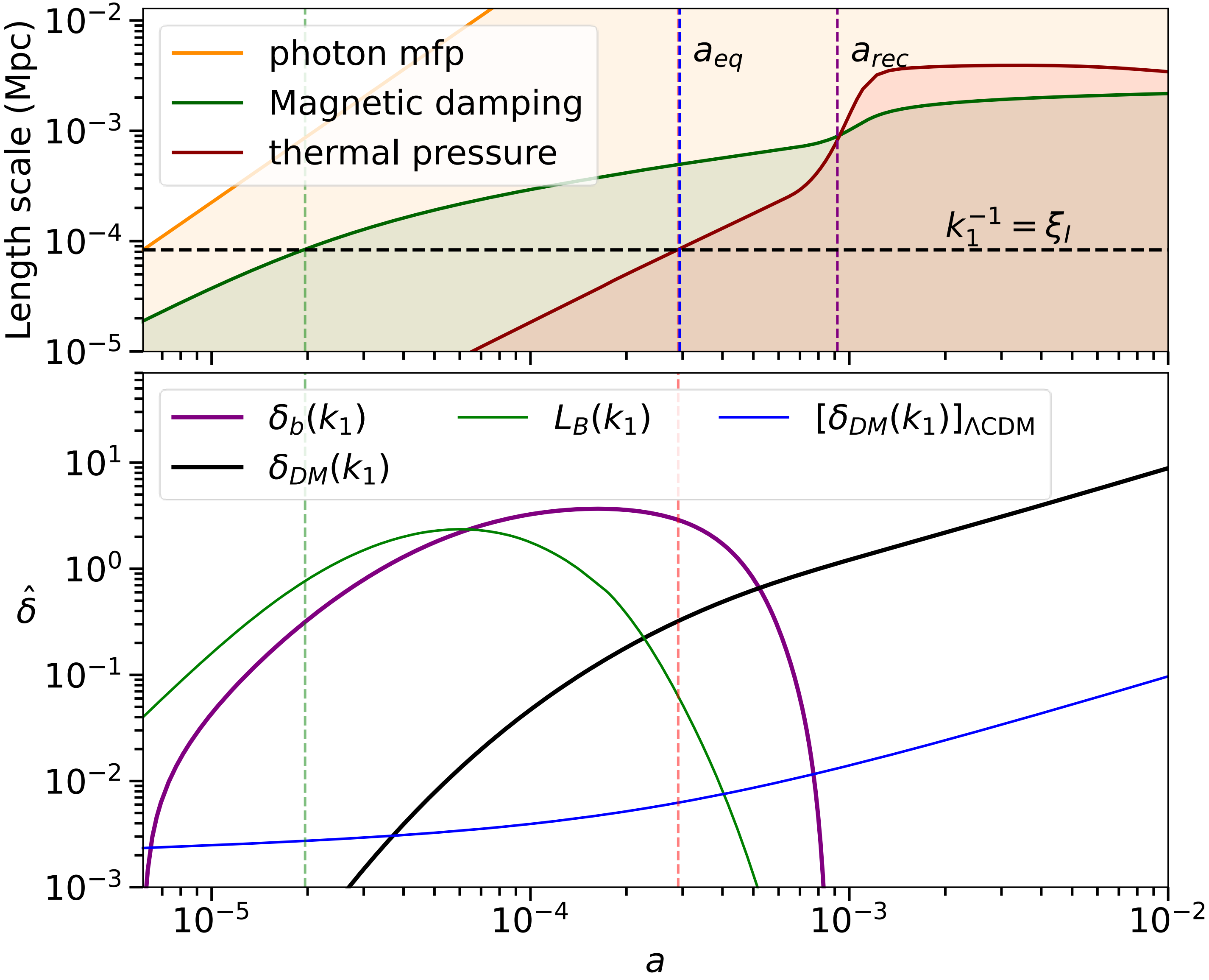
Compressible flow

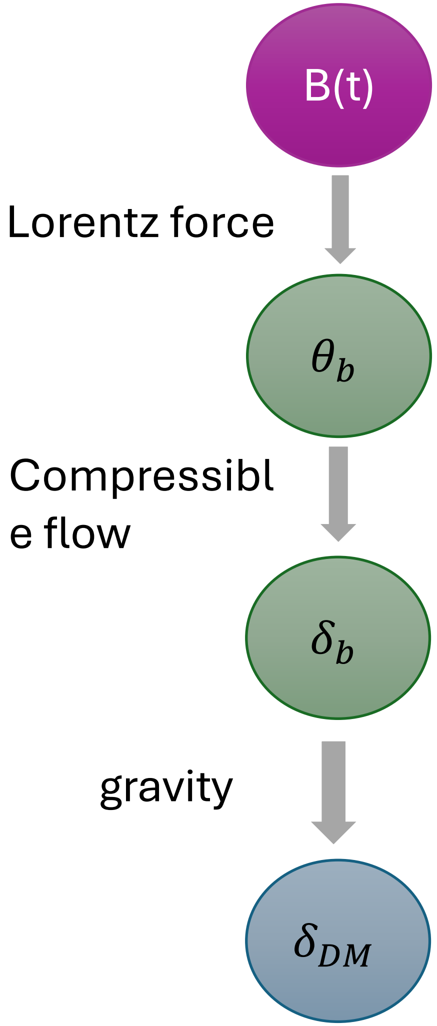
δ_b

gravity

δ_{DM}

Perturbation evolution plot





B(t)

Lorentz force

θ_b

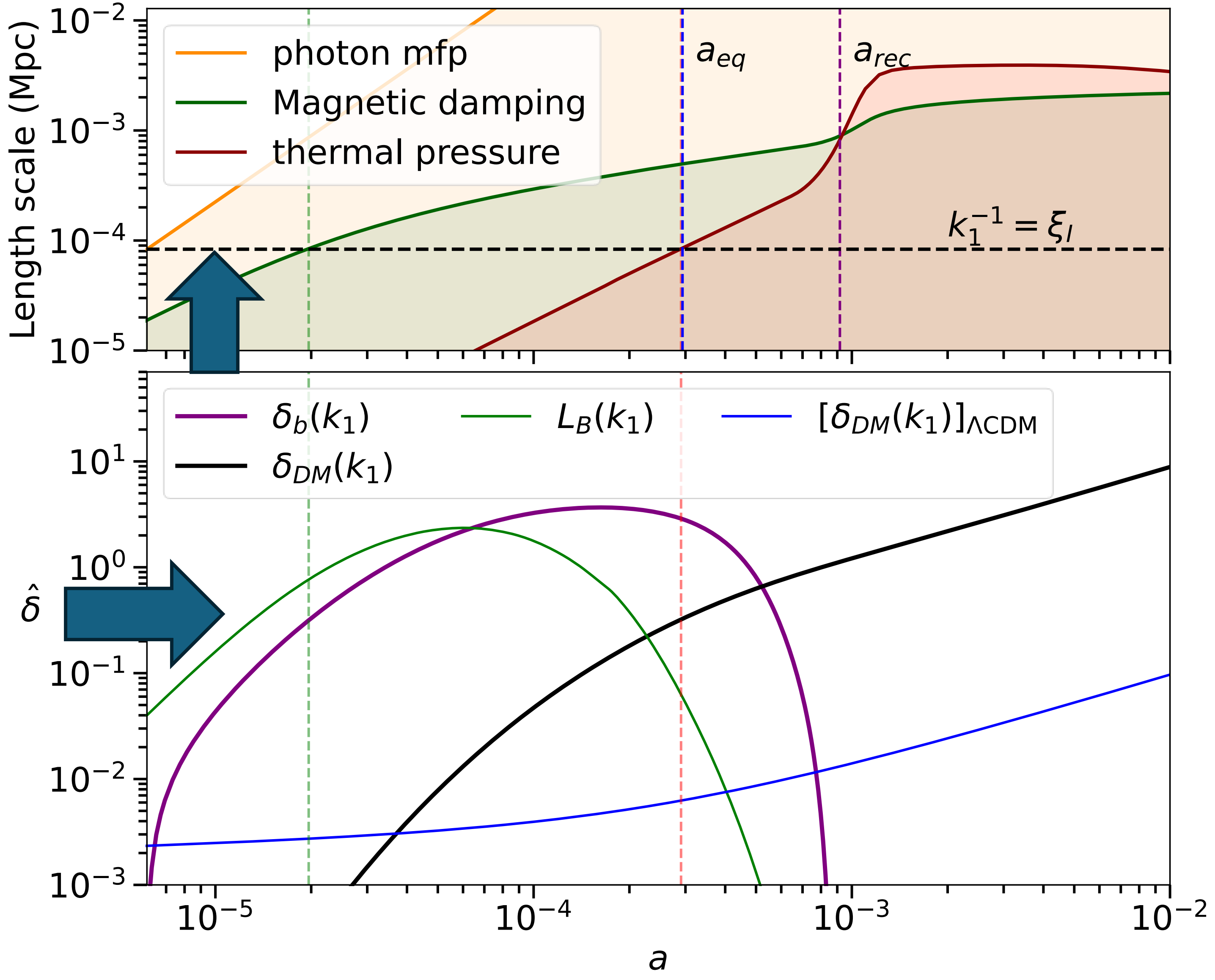
Compressibl
e flow

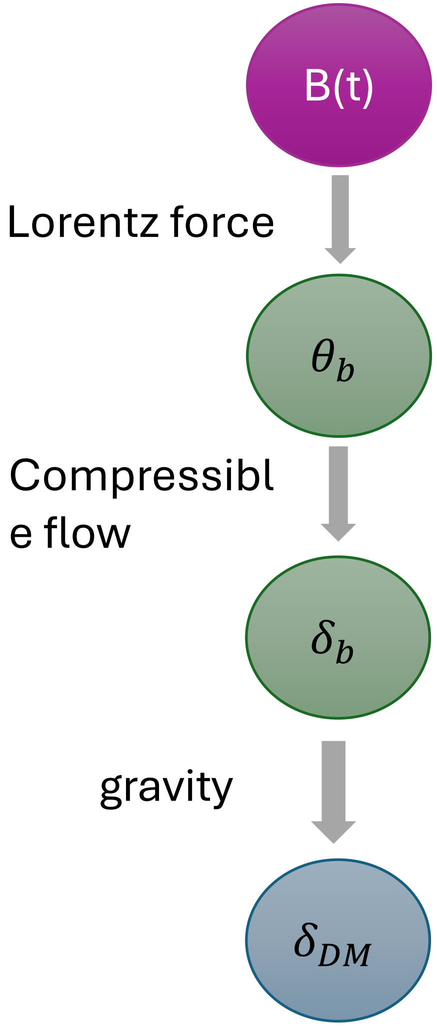
δ_b

gravity

δ_{DM}

Lorentz force enhances baryon perturbations for modes outside k_D^{-1}





B(t)

Lorentz force

θ_b

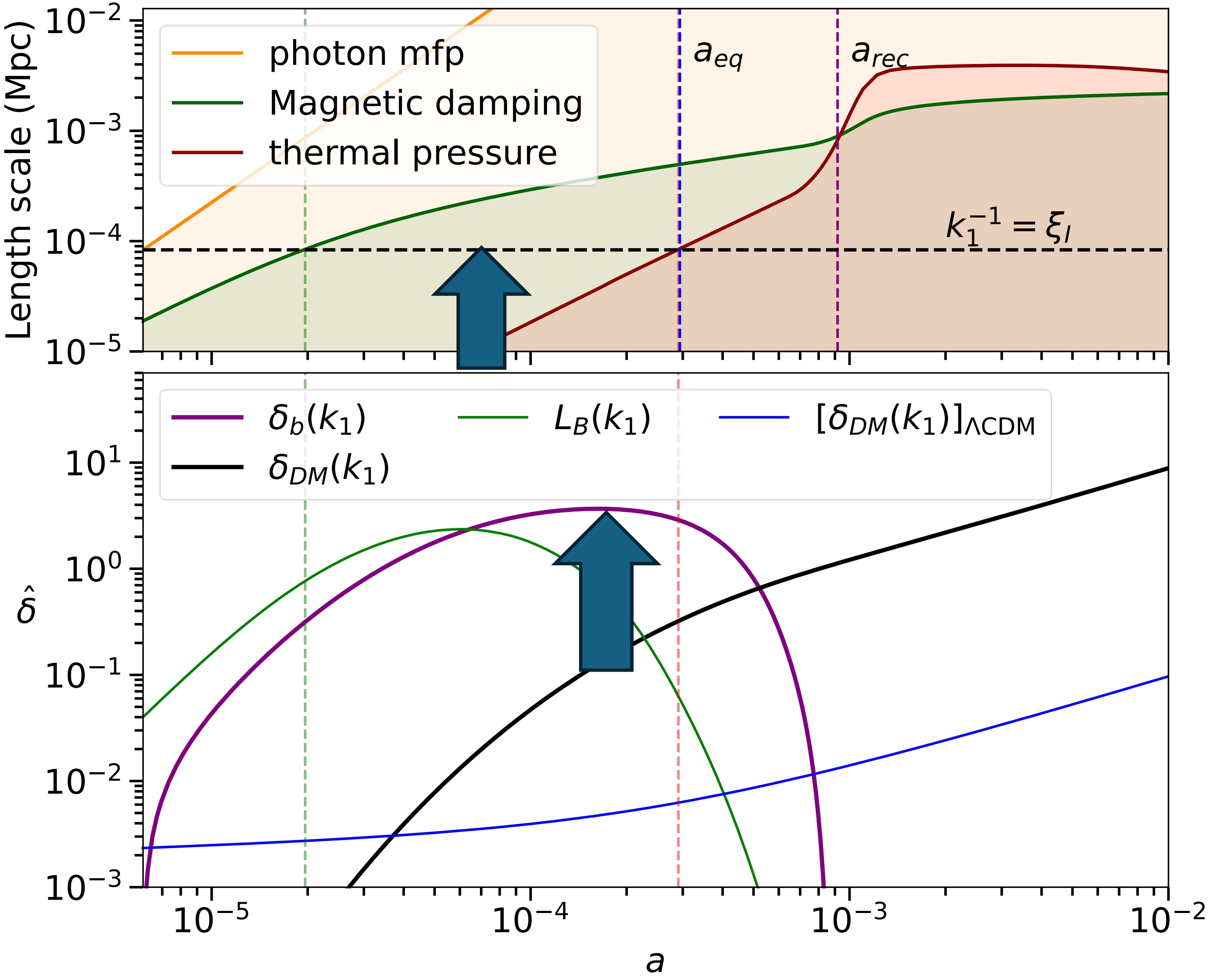
Compressibl
e flow

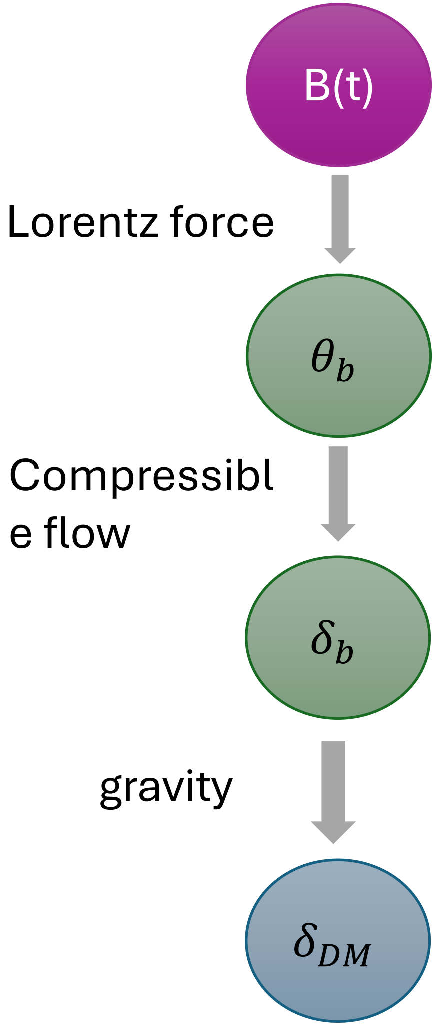
δ_b

gravity

δ_{DM}

baryon perturbations asymptote once mode enters k_D^{-1}





B(t)

Lorentz force

θ_b

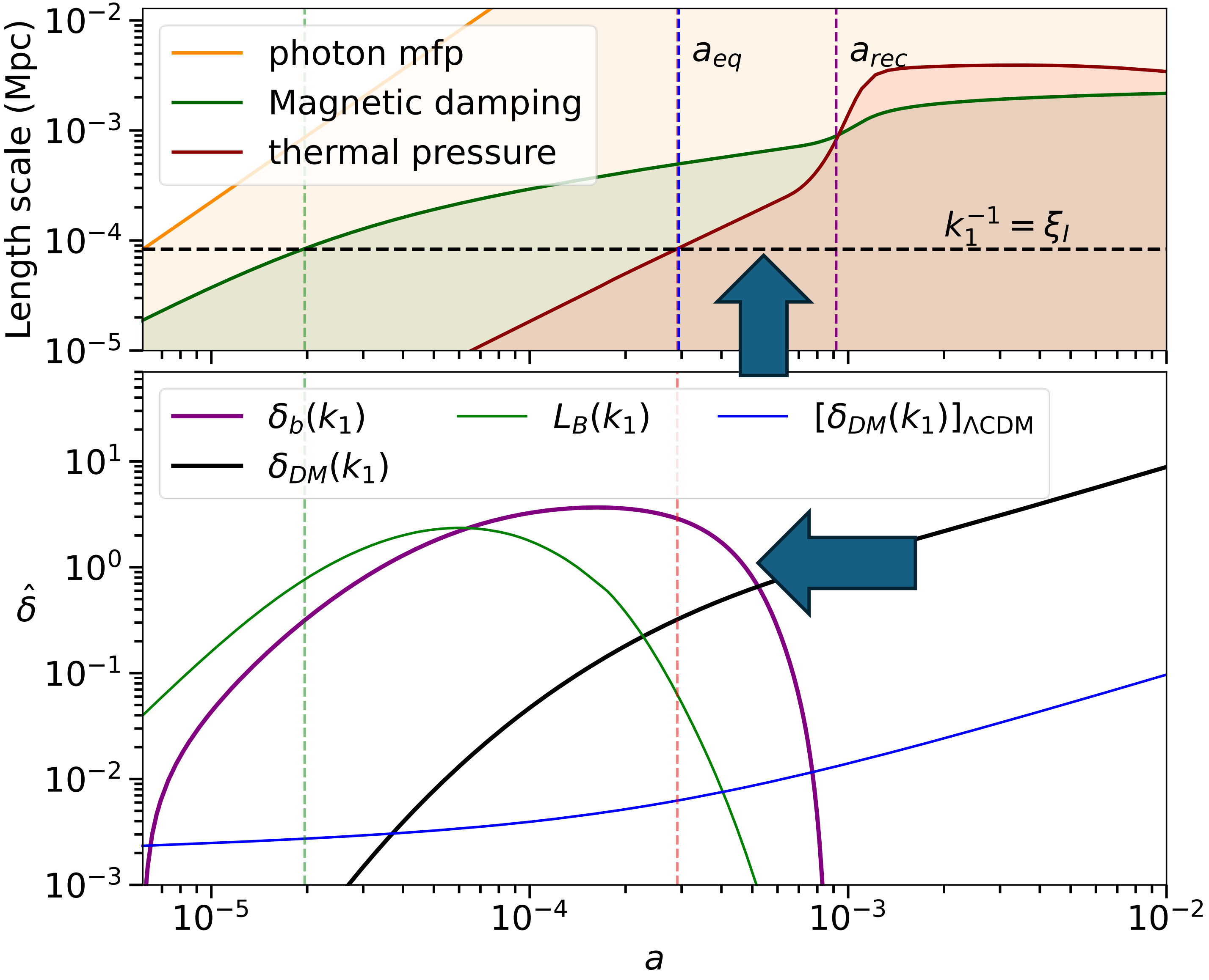
Compressible flow

δ_b

gravity

δ_{DM}

baryon perturbations damped by thermal pressure



B(t)

Lorentz force

θ_b

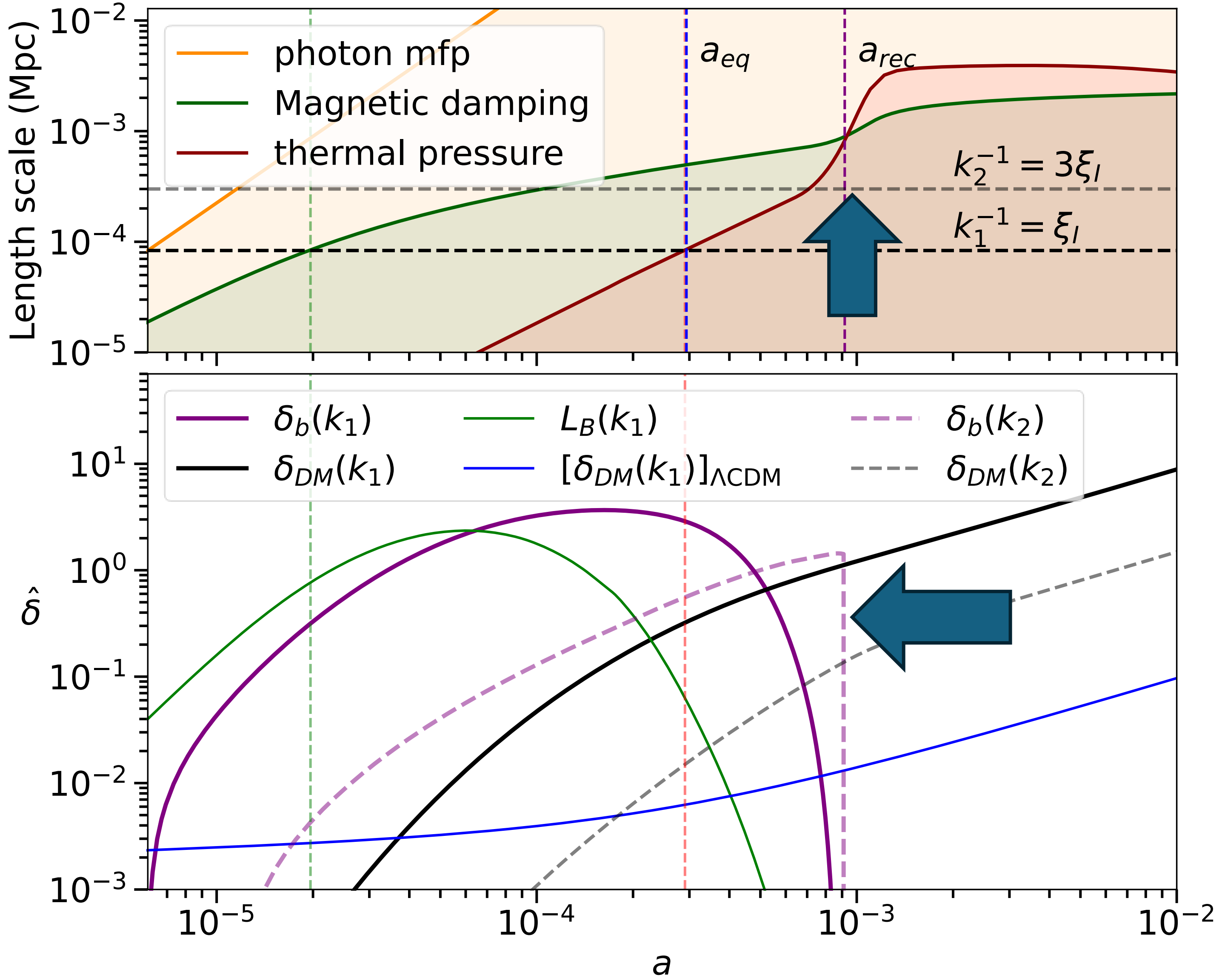
Compressibl
e flow

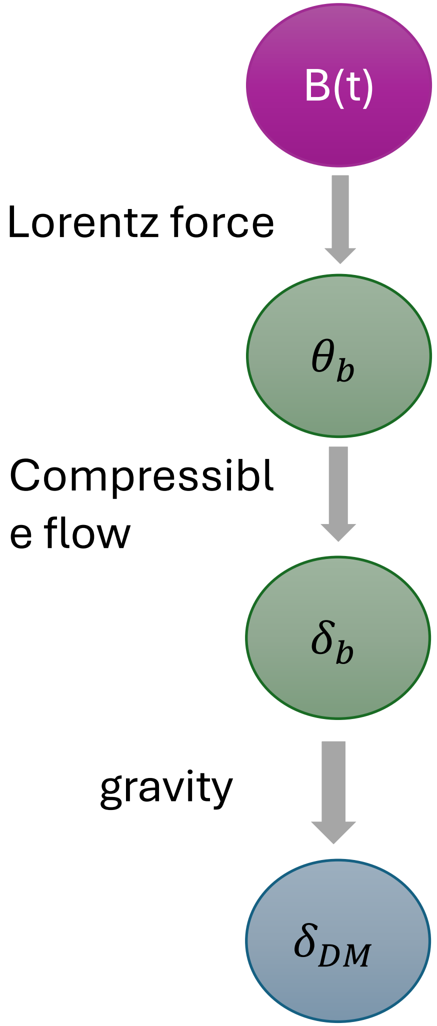
δ_b

gravity

δ_{DM}

baryon perturbations damped by turbulence at recombination





B(t)

Lorentz force

θ_b

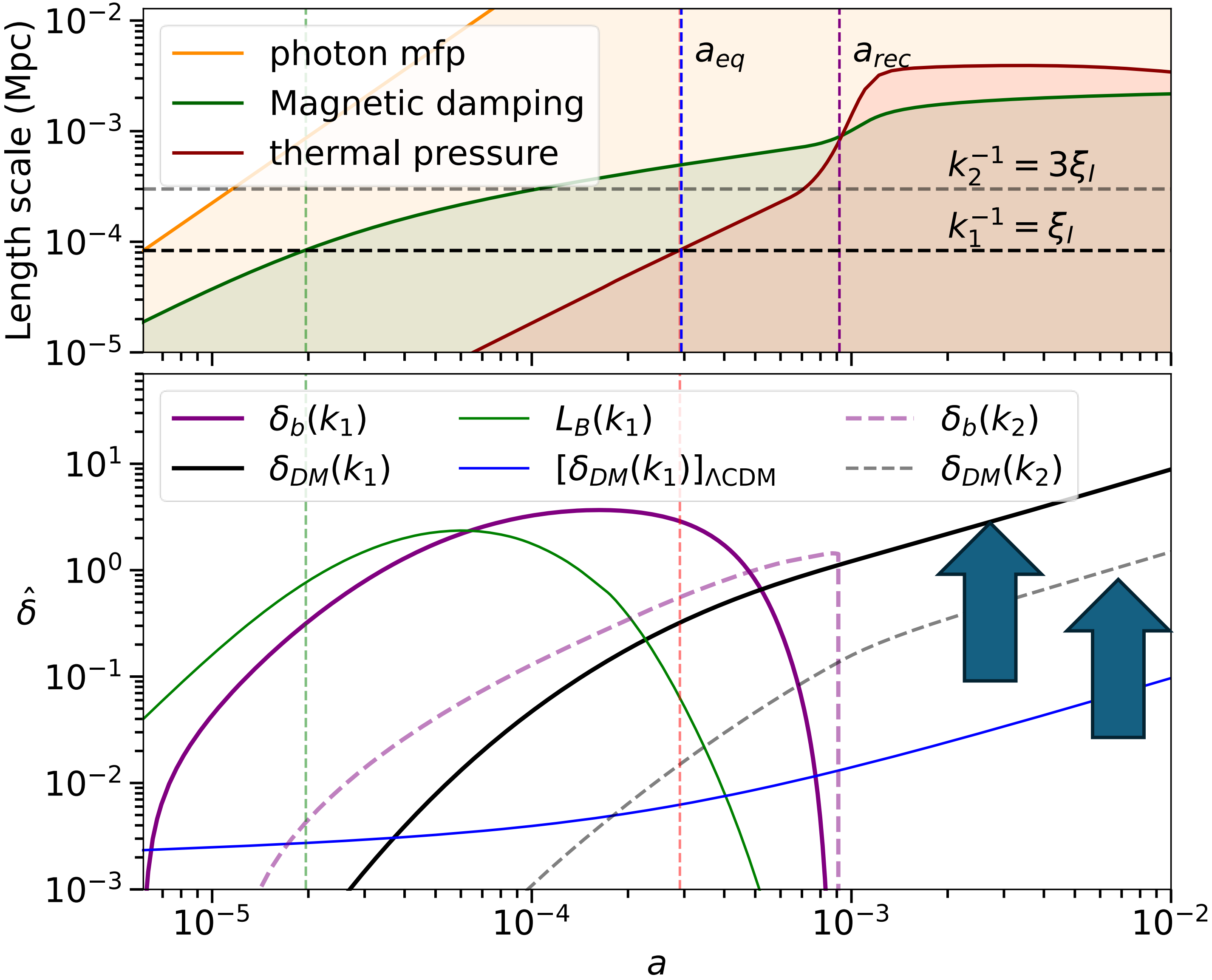
Compressibl e flow

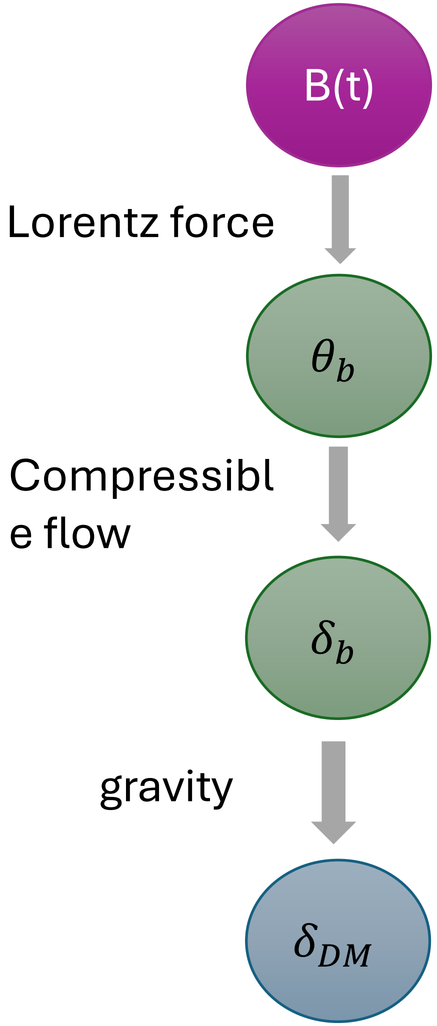
δ_b

gravity

δ_{DM}

Dark matter perturbations continues to grow!





B(t)

Lorentz force

θ_b

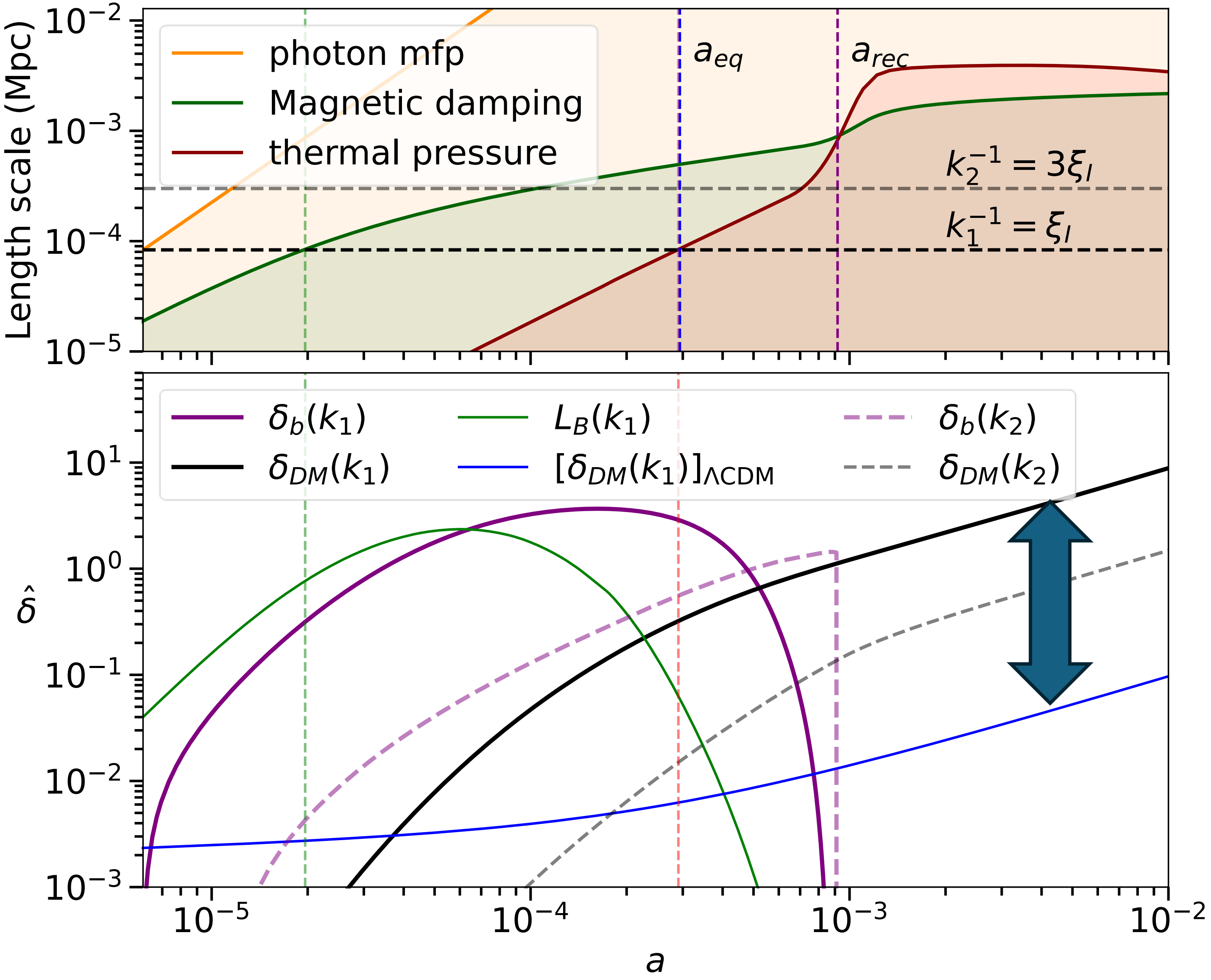
Compressible flow

δ_b

gravity

δ_{DM}

Dark matter perturbations enhanced by orders of magnitude compared to ΛCDM



B(t)

Lorentz force

θ_b

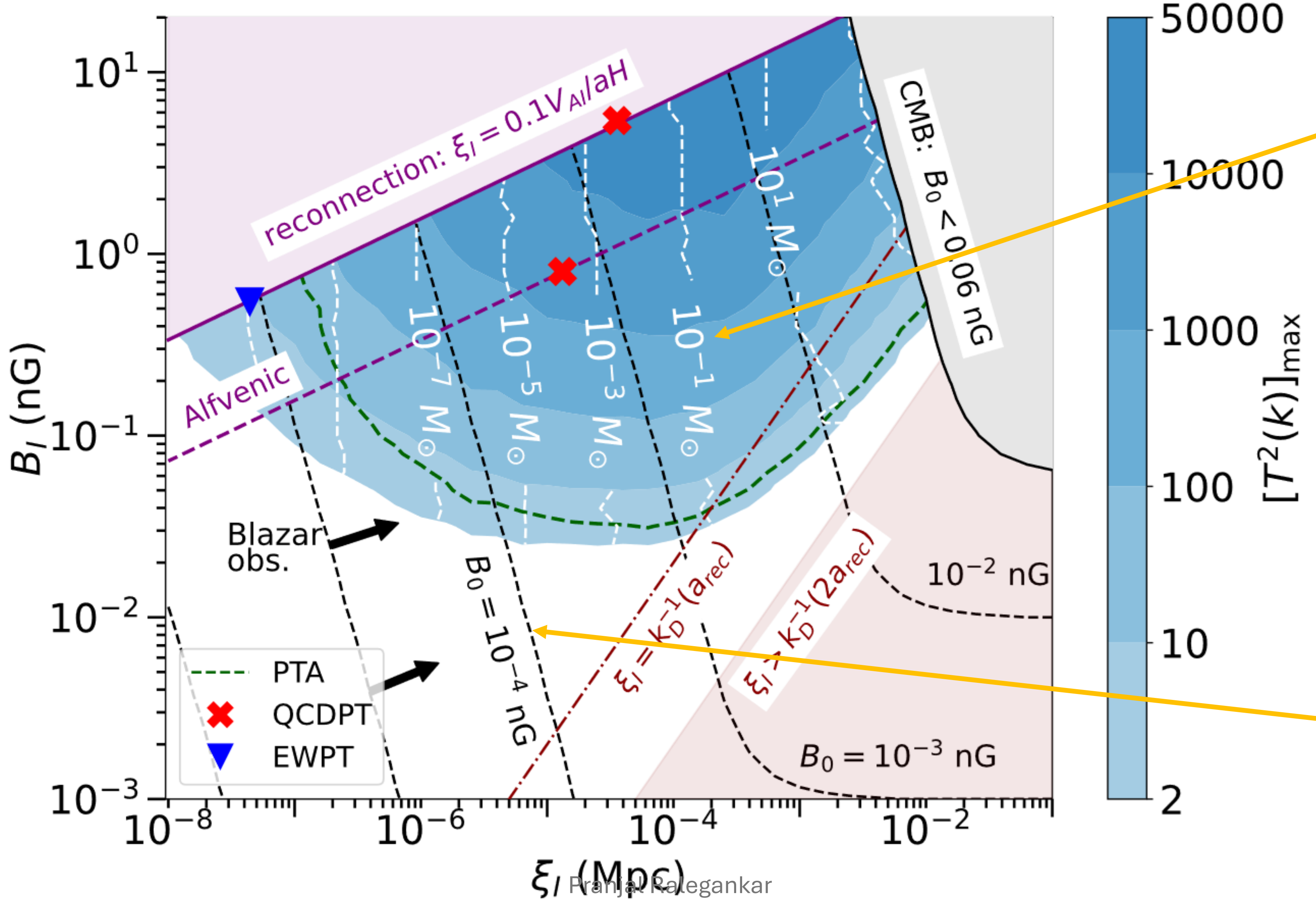
Compressible flow

δ_b

gravity

δ_{DM}

Parameter Space with Enhanced Power on Small scales

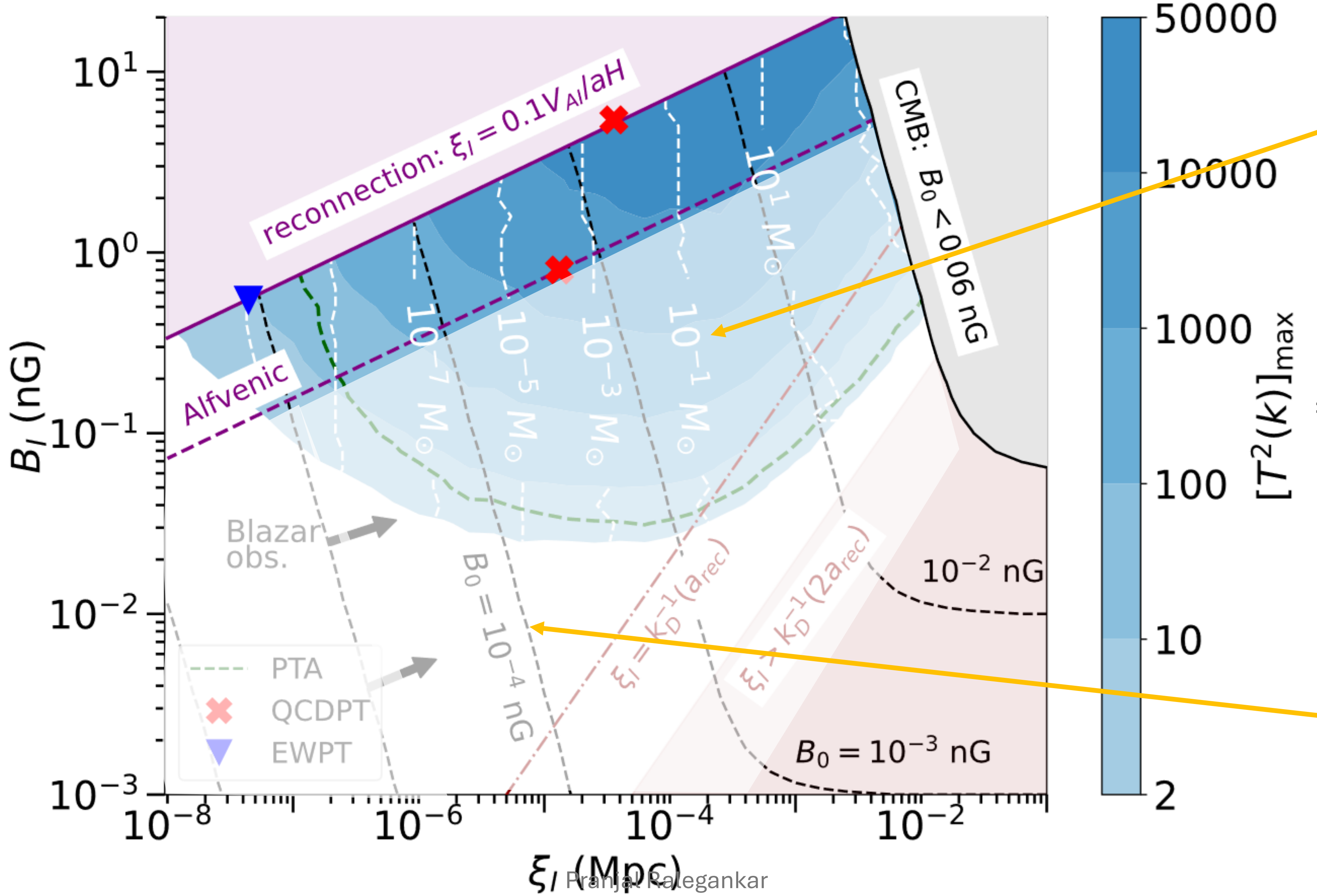


$$= \frac{P(k)}{P_{\Lambda CDM}(k)}$$

DM minihalo mass

Present-day magnetic field strength

Parameter Space Expected from Magnetogenesis from phase transitions

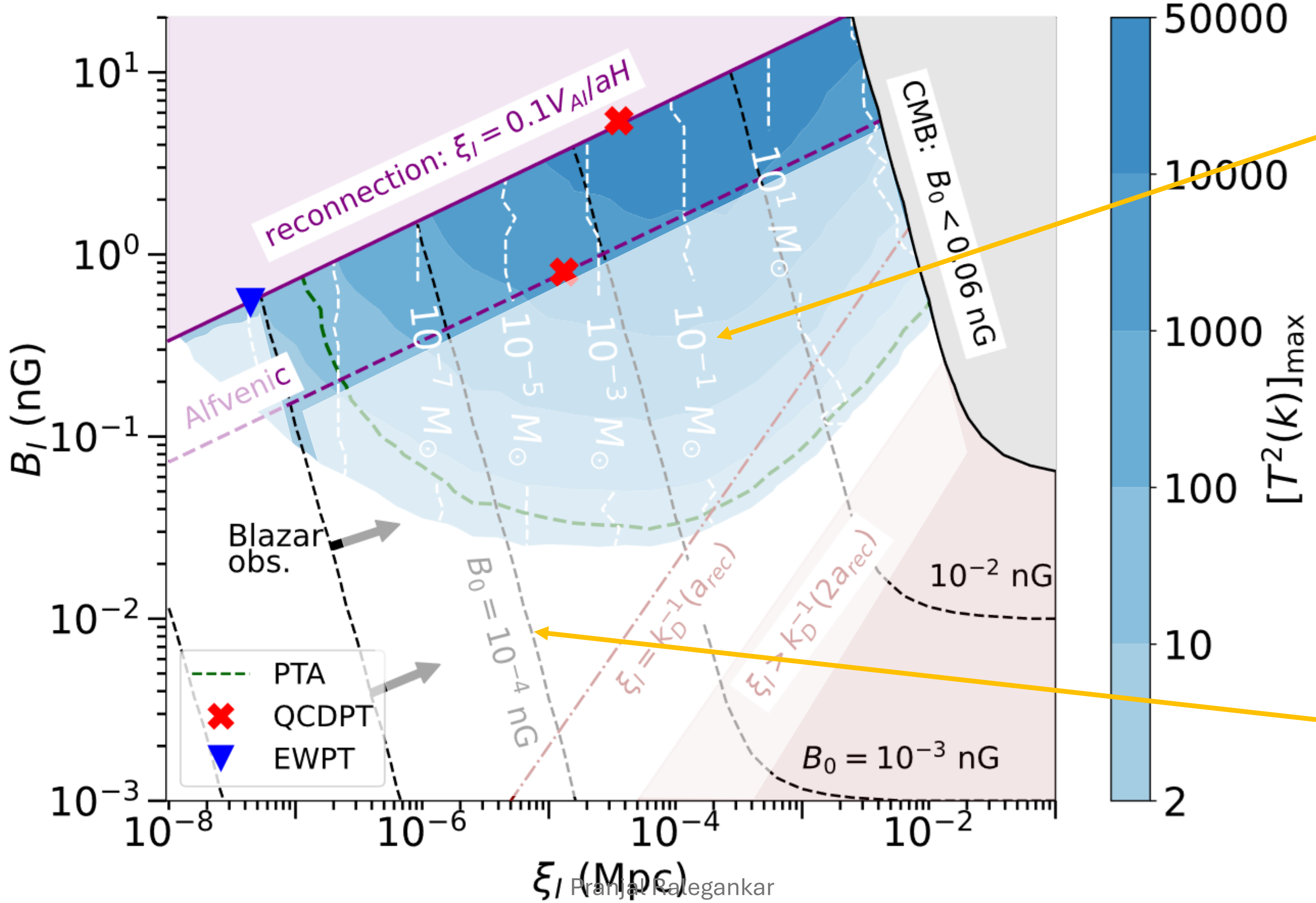


$= \frac{P(k)}{P_{\Lambda CDM}(k)}$

DM minihalo mass

Present-day magnetic field strength

Parameter Space To explain blazar observations

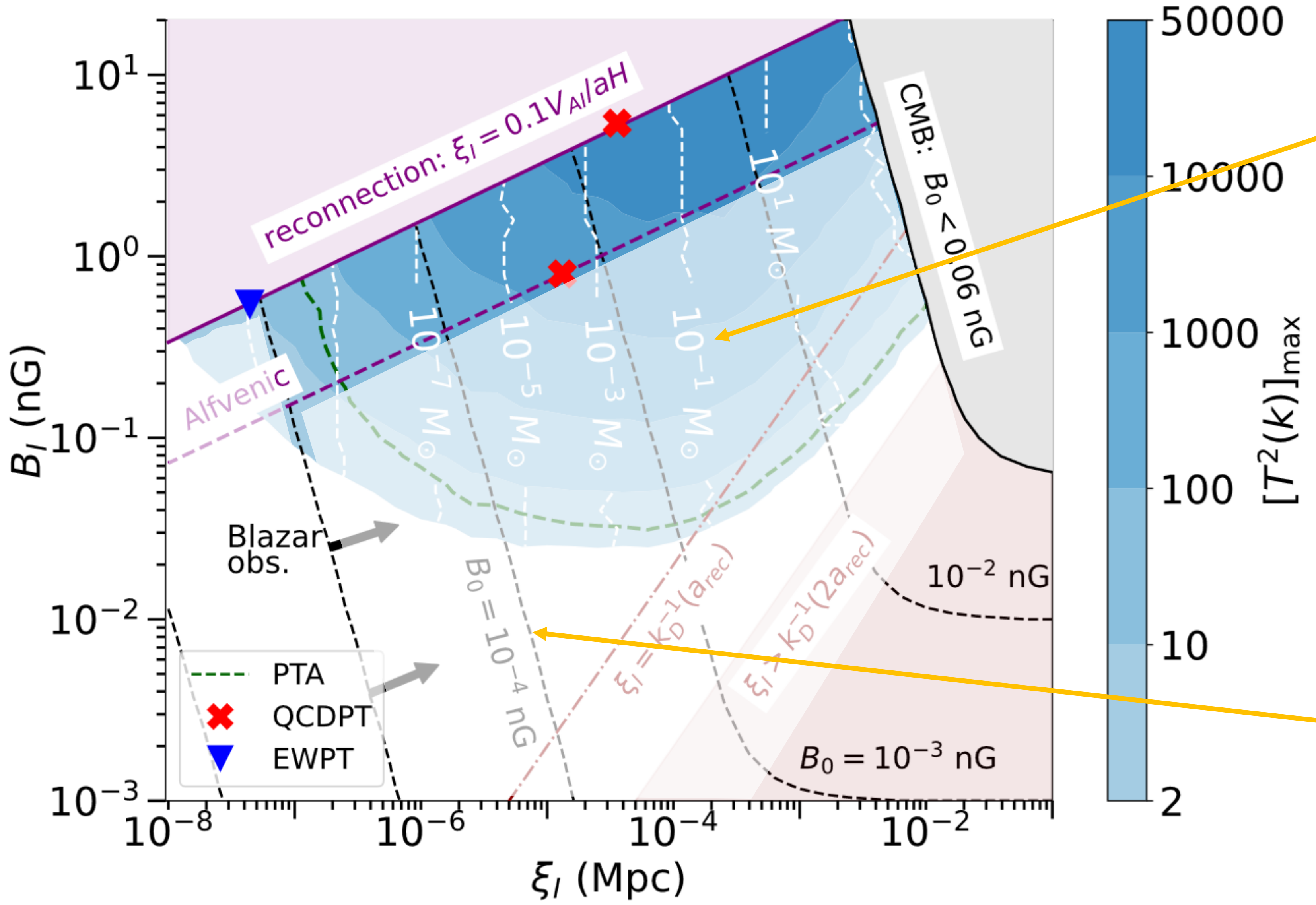


$$= \frac{P(k)}{P_{\Lambda CDM}(k)}$$

DM minihalo mass

Present-day magnetic field strength

Universe maybe filled with dark matter minihalos!!



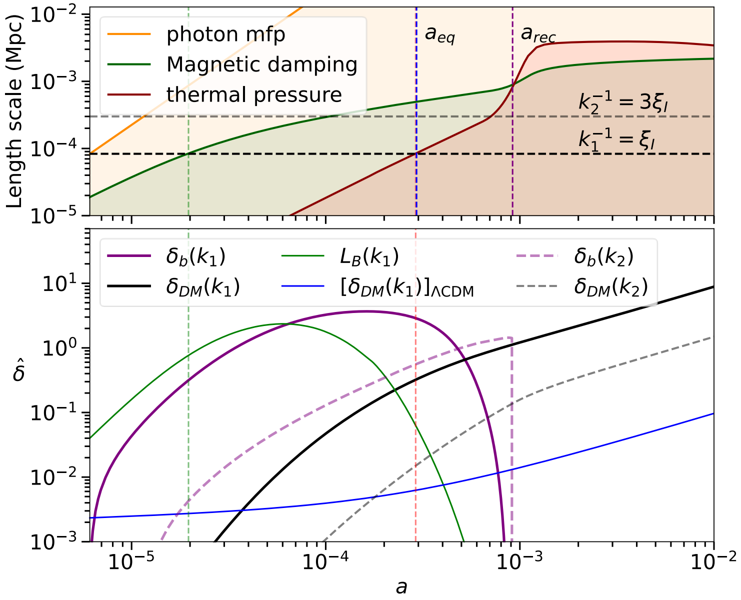
$$= \frac{P(k)}{P_{\Lambda CDM}(k)}$$

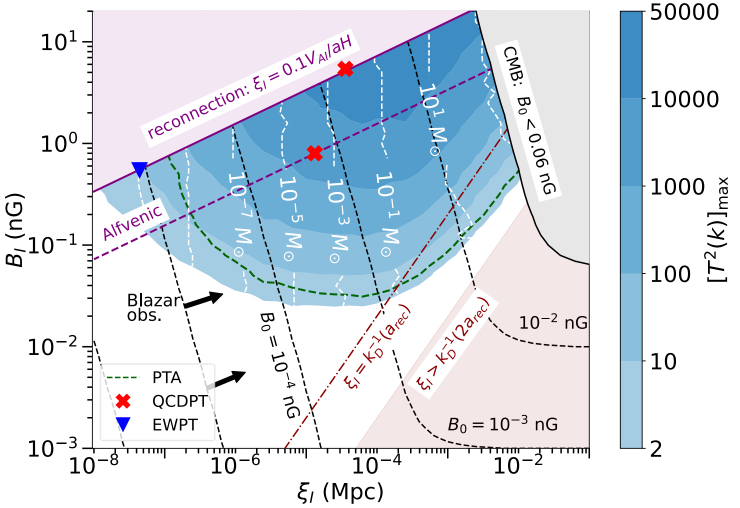
DM minihalo mass

Present-day magnetic field strength

Summary and Concluding remarks

- Magnetic fields can enhance power on small scale dark matter distribution gravitationally.
- PTA/GAIA detection of DM minihalos can provide best probe of primordial magnetic fields
- Results are qualitative: Need MHD simulations to get accurate quantitative answers.
- Ironic: how invisible dark matter can help look for visible entity: magnetic fields





Backup

Back to power spectrum