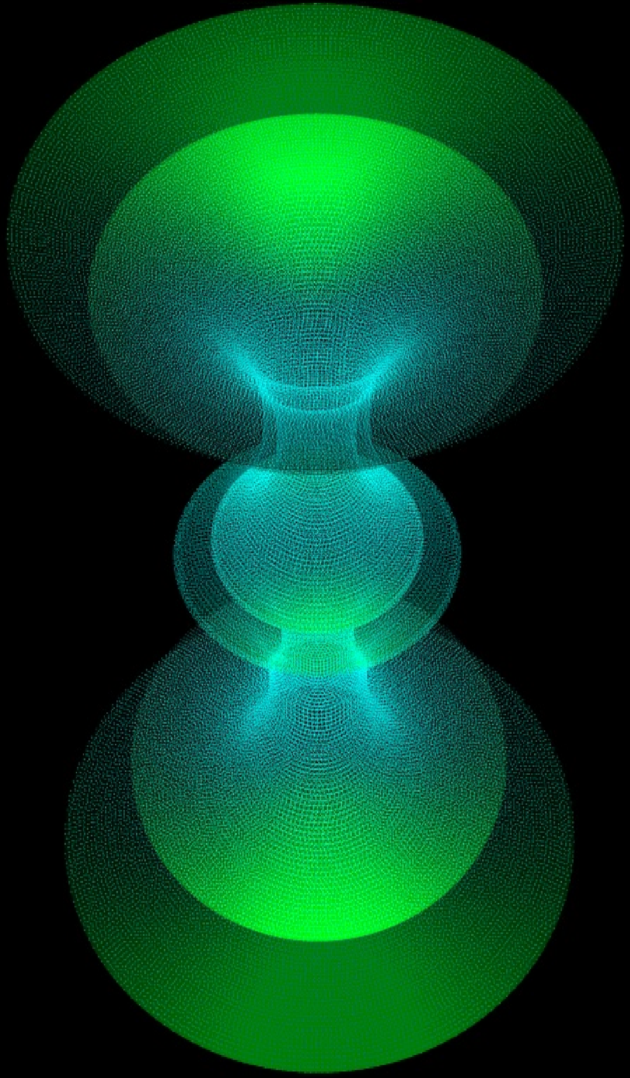

Universal Black Hole Microstates

ROBERTO EMPARAN ICREA+ICC UB

COPERNICUS WEBINAR

12 MARCH 2024

WITH ANA CLIMENT

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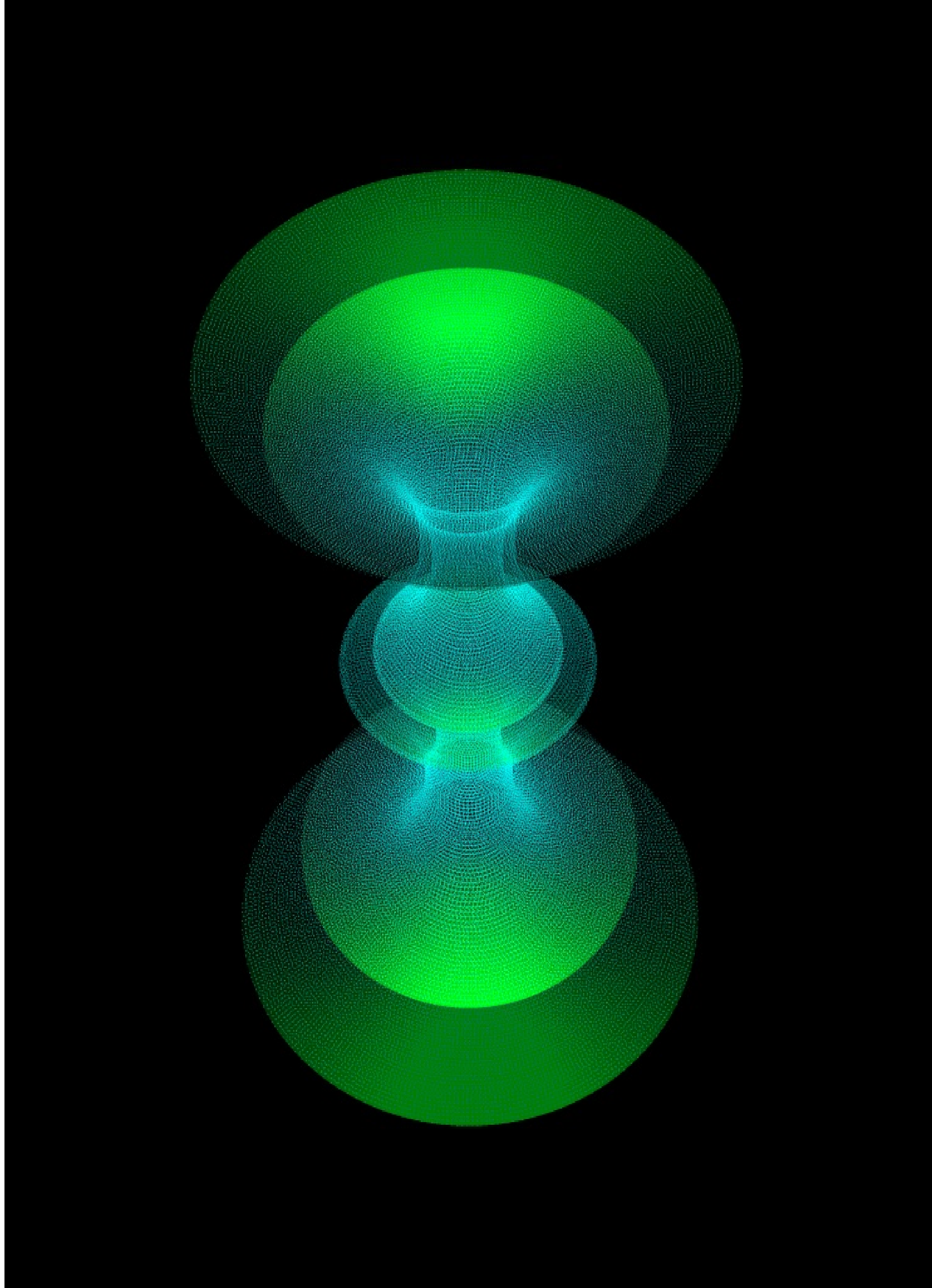


$$S = \frac{A}{4G\hbar}$$

and its discontents

Not counting states

$$S = \frac{A}{4G\hbar}$$

Bekenstein 1973

Hawking 1975

from phenomenological arguments

What microstates?

Not counting states

$$S = \frac{A}{4G\hbar}$$

Gibbons+Hawking 1976 from $Z(\beta) = \int_\beta \mathcal{D}g \; e^{-I[g]}$ Gravitational Path Integral

GPI

Entropy from *classical* saddle point?

Counting states – but not black holes

$$S = \frac{A}{4G\hbar}$$

Strominger+Vafa 1996 from D-branes in String theory

Non-gravitating states that are not black holes

Counting black hole states – with wormholes

$$S = \frac{A}{4G\hbar}$$

Balasubramanian+Lawrence
+Magan+Sasieta 2022
Penington+al (PSSY) 2019
and others

Black hole microstates from GPI-reloaded

$$\dim(\mathcal{H}) = e^S$$

Stochastic overlaps between states: from wormholes

Counting black hole states – with wormholes

$$S = \frac{A}{4G\hbar}$$

Stat-mech interpretation *from GPI*

Universal construction

Geometric microstates with smooth horizons

Black Hole Microstates

1. *Constructing* – with the Gravitational Path Integral (GPI)

2. *Computing* – overlaps from wormholes

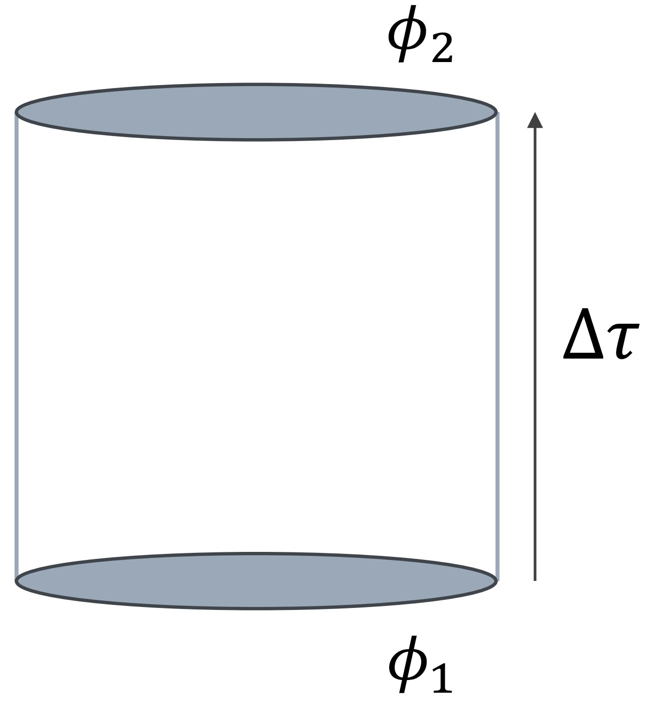
3. *Counting* – the dimension of the black hole Hilbert space

Quantum States from Path Integrals

FROM QUANTUM FIELD THEORY TO EUCLIDEAN QUANTUM GRAVITY

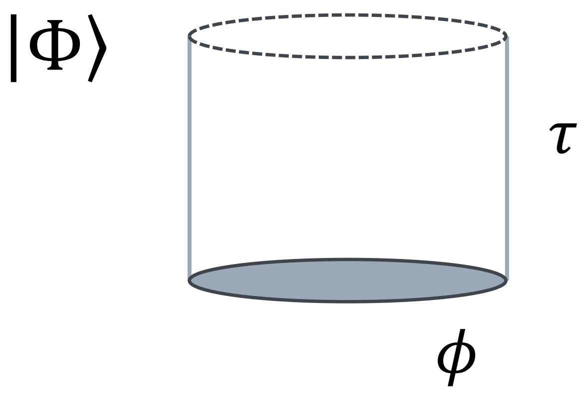
Amplitudes from Path Integral (PI)

$$\langle \phi_2 | e^{-\Delta\tau\, H} | \phi_1 \rangle = \int_{\phi(0)=\phi_1}^{\phi(\Delta\tau)=\phi_2} \mathcal{D}\phi \; e^{-I_E[\phi]}$$



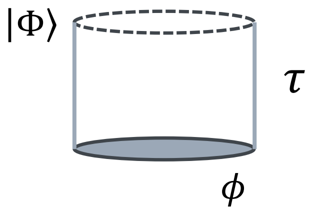
Cutting the PI: “State preparation”

$|\Phi\rangle = |\phi(\tau)\rangle = e^{-\tau H}|\phi\rangle$

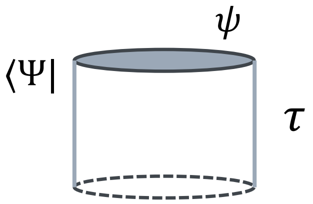


State overlaps

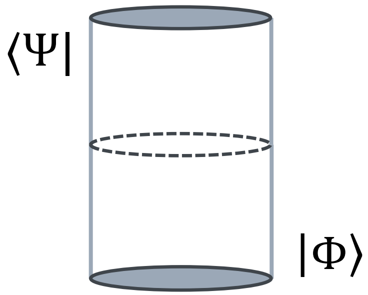
$$|\Phi\rangle = e^{-\tau H}|\phi\rangle$$



$$\langle\Psi| = \langle\psi|e^{-\tau H}$$



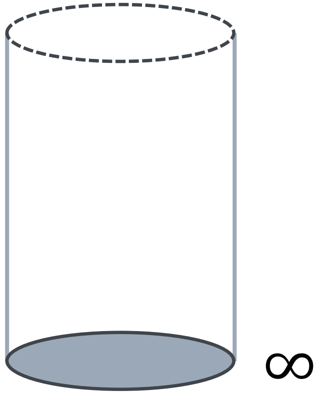
$$\langle\Psi|\Phi\rangle =$$



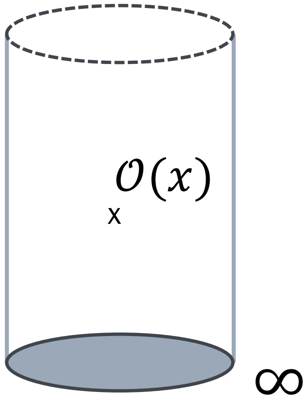
Ground state & States from operators

$$|0\rangle = \lim_{\tau\to\infty} e^{-\tau H}|\phi\rangle$$

$|0\rangle$

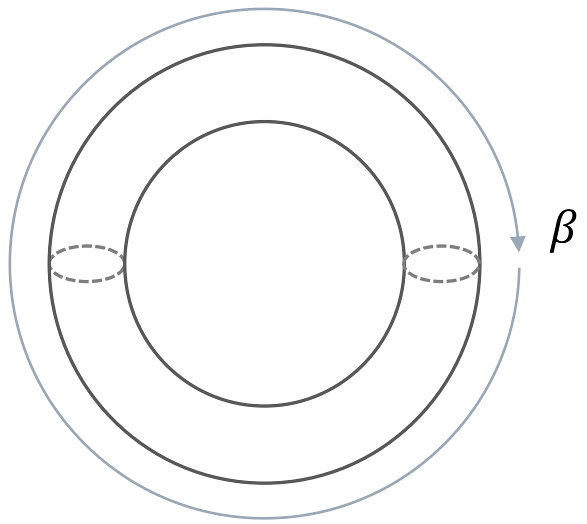


$$|\Psi\rangle = \mathcal{O}(x)|0\rangle$$



Thermal states

Imaginary time periodicity



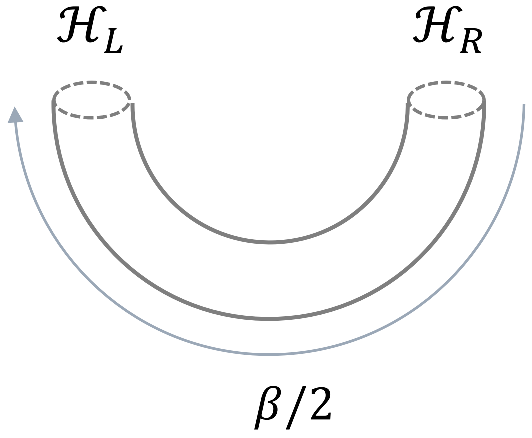
$$Z[\beta] = \int_{\phi(0)=\phi(\beta)} \mathcal{D}\phi \, e^{-I_E[\phi]}$$

$$= \sum_i \langle E_i | e^{-\beta H} | E_i \rangle$$

$$= \mathrm{Tr}\, e^{-\beta H}$$

Thermofield Double State – TFD

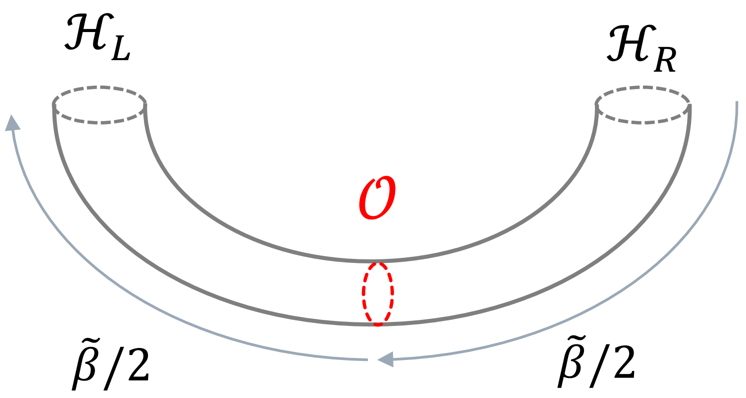
Cut open the path integral



$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Maximally entangled state

Partially Entangled Thermal States

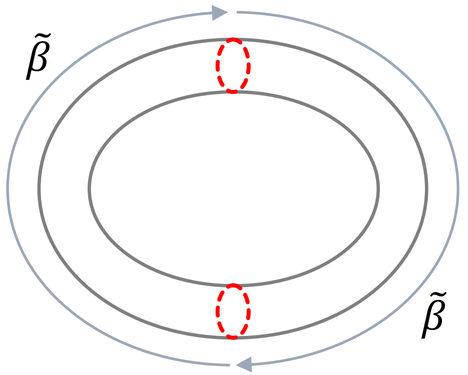


PETS

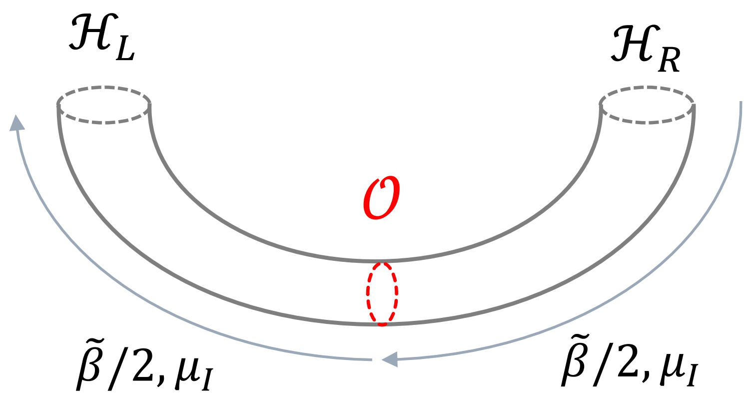
Goel+Lam+Turiaci+Verlinde

$$|\Psi\rangle = \frac{1}{\sqrt{Z_1}} \sum_i e^{-\tilde{\beta}H/2} \mathcal{O} e^{-\tilde{\beta}H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

$$Z_1 = \mathrm{Tr}\left(e^{-\tilde{\beta}H} \mathcal{O} e^{-\tilde{\beta}H} \mathcal{O}^\dagger\right) =$$



Partially Entangled Grand-canonical States

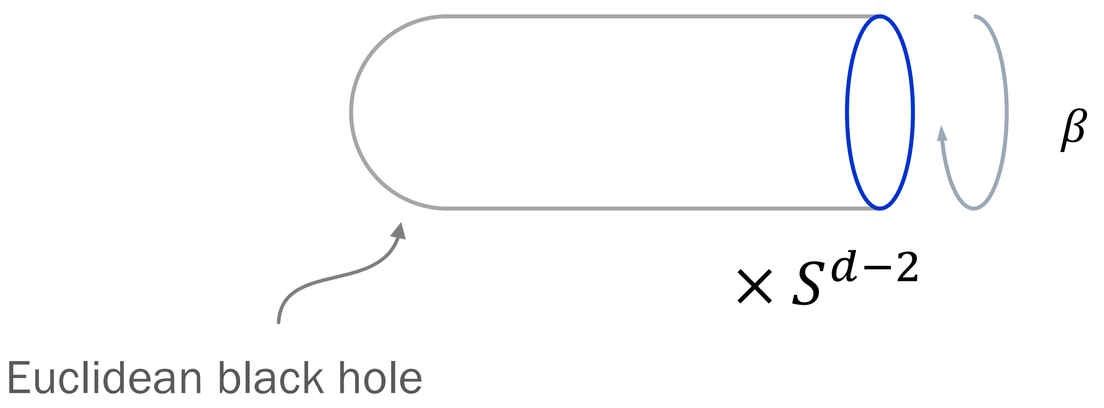


Add charge & rotation: **PEGS**

$$|\Psi\rangle = \frac{1}{\sqrt{Z_1}} \sum_i e^{-(\tilde{\beta}-\mu_I Q_I)H/2} \mathcal{O} e^{-(\tilde{\beta}-\mu_I Q_I)H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

Gravitational Partition Function

$$Z[\beta] = \int_{g(0)=g(\beta)} \mathcal{D}g \; e^{-I_{EH}[g]}$$



Black Magic

$$Z[\beta] = \int_{g(0)=g(\beta)} \mathcal{D}g \, e^{-I_{EH}[g]} \approx e^{-I_{EH}[\boldsymbol{g}_{cl}]}$$

semiclassical saddle-point
g_{cl} = Euclidean black hole

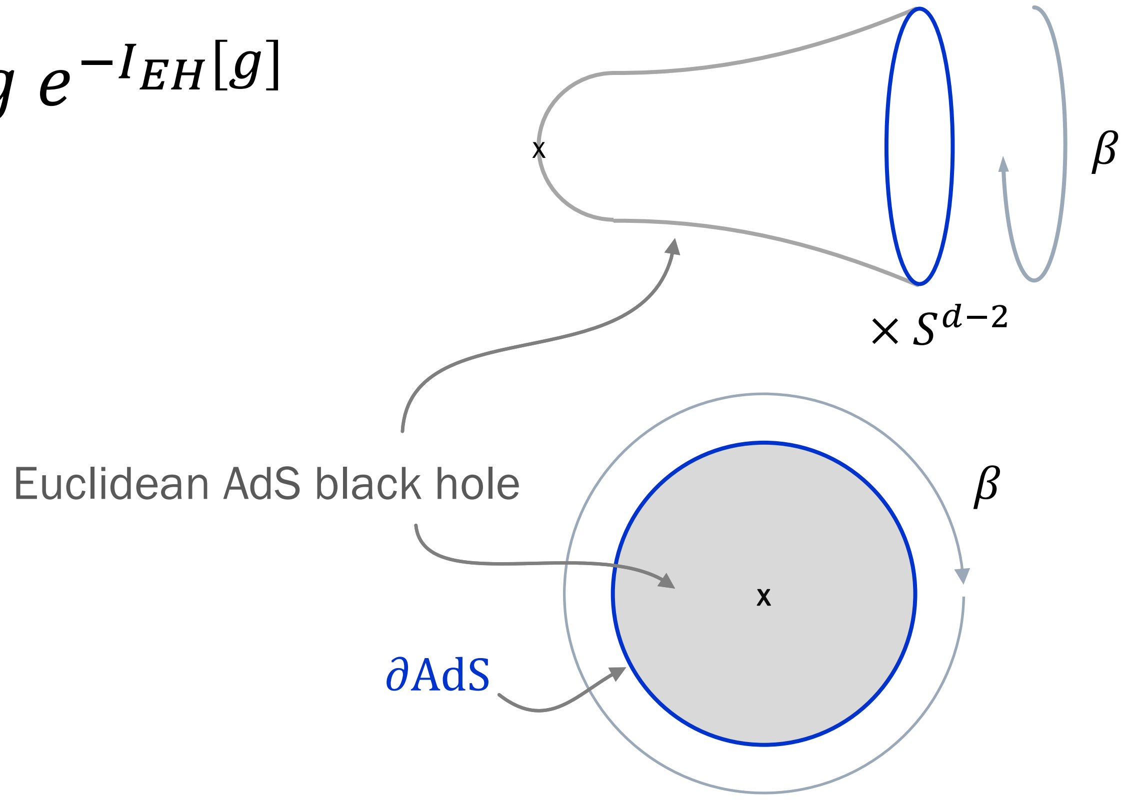
$$S = (\beta \partial_\beta - 1) I_{EH}[g_{cl}] = \frac{A}{4G}$$

Gibbons+Hawking

- $I_{EH}[g_{cl}]$ = Euclidean action of ***classical*** field configuration: zero-loop
- Not a trace over states: Trace = sum over all states running in a loop: one-loop contribution
- *Not a sum over microstates* – but still gives non-zero & correct $S = A/4G$

Gravitational Partition Function in AdS

$$Z_{\text{AdS}}[\beta] = \int_{g(0)=g(\beta)} \mathcal{D}g\, e^{-I_{EH}[g]}$$

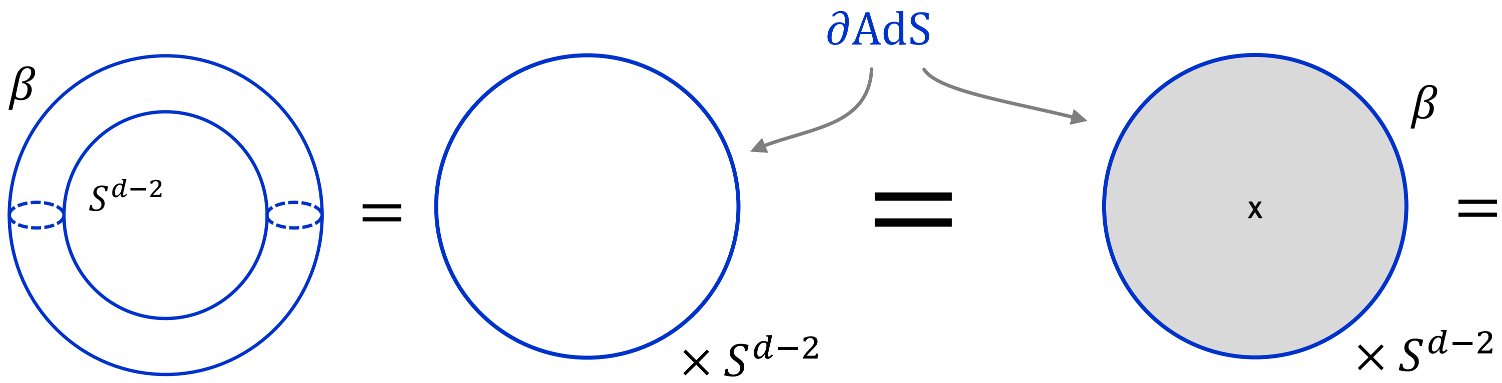


AdS/CFT

$$Z_{\mathrm{AdS}_d}[\beta] = Z_{\mathrm{CFT}_{d-1}}[\beta]$$

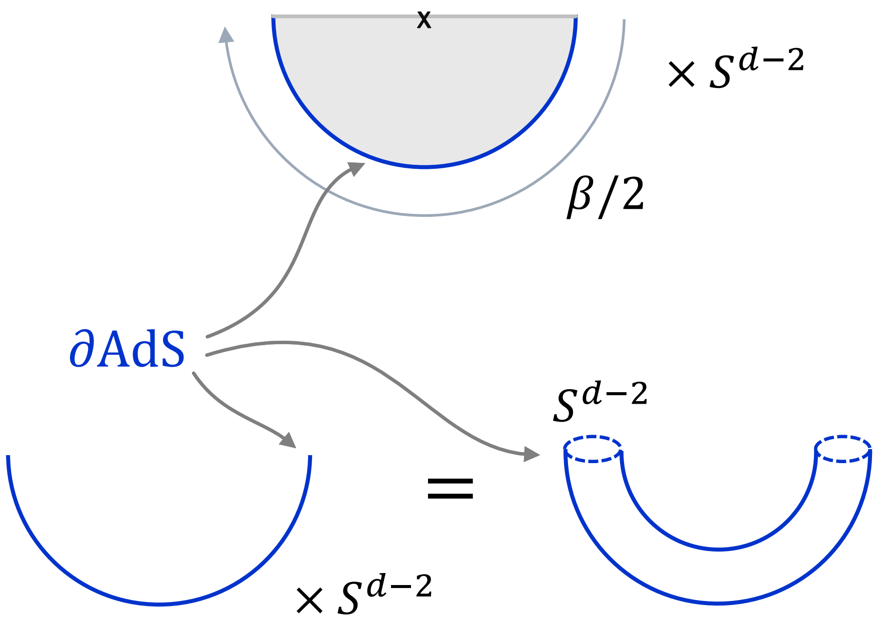
$Z[\beta] = \mathrm{Tr}\, e^{-\beta H}$

$Z_{\mathrm{CFT}_{d-1}}[\beta] =$

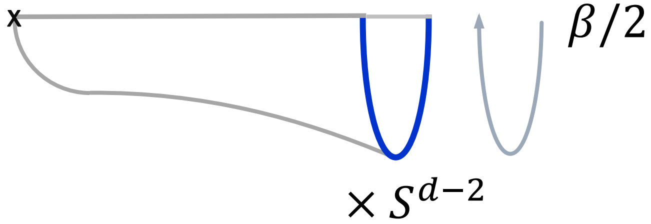


$= Z_{\mathrm{AdS}_d}[\beta]$

Thermal quantum states from cut GPI

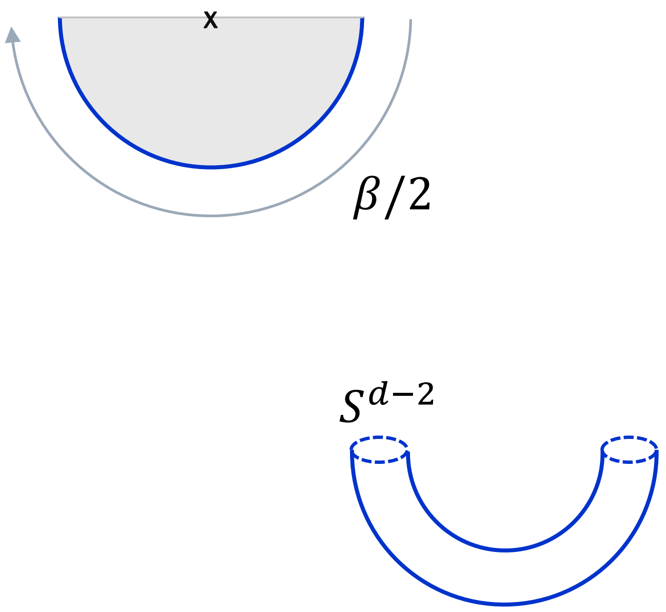


$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$



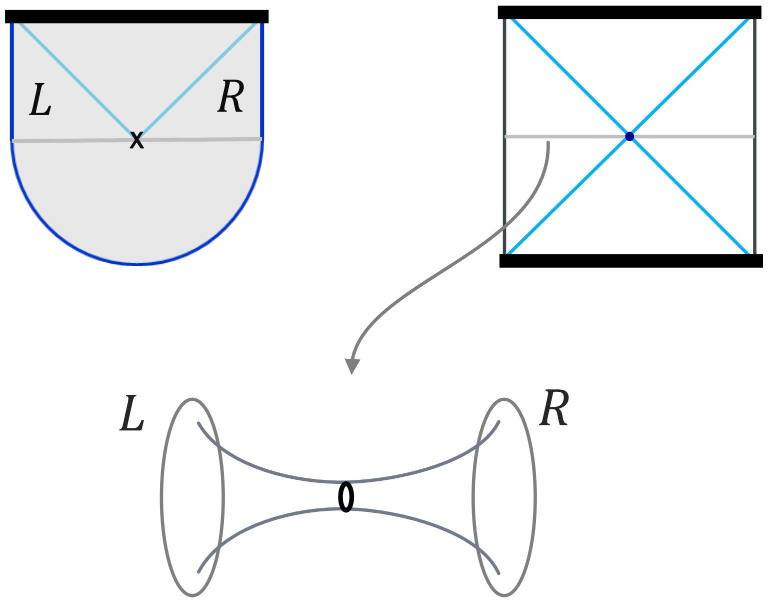
Thermofield double = Eternal black hole

$|\mathrm{TFD}\rangle =$



Lorentzian evolution

AdS black hole

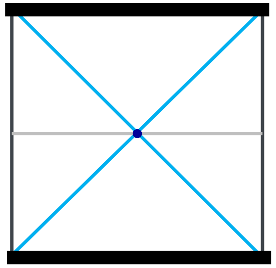


Maldacena 2001

Thermofield double = Eternal black hole

$$|\mathrm{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta E_i/2} |i\rangle_L |i\rangle_R$$

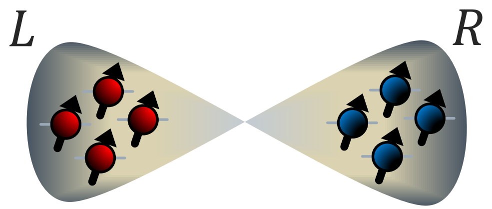
$=$



Eternal black hole

Bell/EPR pair

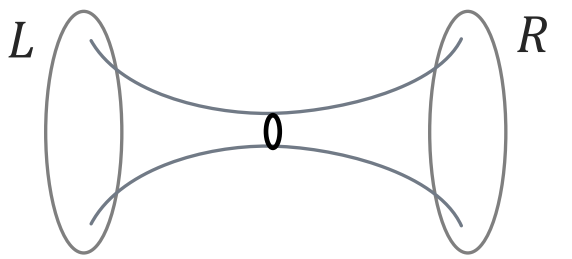
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_L\, |0\rangle_R + |1\rangle_L\, |1\rangle_R)$$



Correlation/connection, but no communication between sides

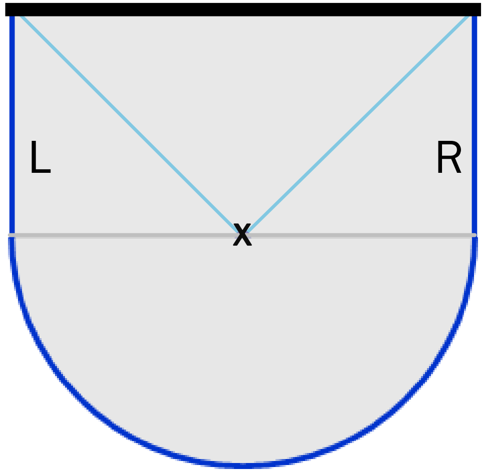
Thermal behavior when only one side is probed

ER bridge



Maldacena+Susskind 2013

Thermofield double



$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_i e^{-\beta H/2} |E_i\rangle_L \otimes |E_i\rangle_R$$

A specific (micro)state of the dual CFT

Dual geometry has a horizon and a singularity

Constructing

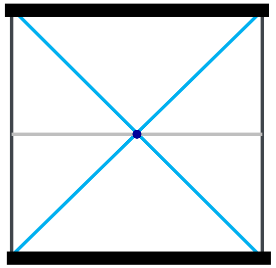
BLACK HOLE MICROSTATES FROM THE GRAVITATIONAL PATH INTEGRAL

Black Hole Microstates

Microscopic pure states $|\Psi\rangle$ that are almost indistinguishable (for simple observables) from thermal state ρ_{th}

$$\langle\Psi|\mathcal{O}(t)|\Psi\rangle \to \text{Tr}(\rho_{\text{th}}\mathcal{O}), \qquad \langle\Psi|\mathcal{O}(t)\mathcal{O}(0)|\Psi\rangle \to \text{Tr}(\rho_{\text{th}}\mathcal{O}(t)\mathcal{O}(0))$$

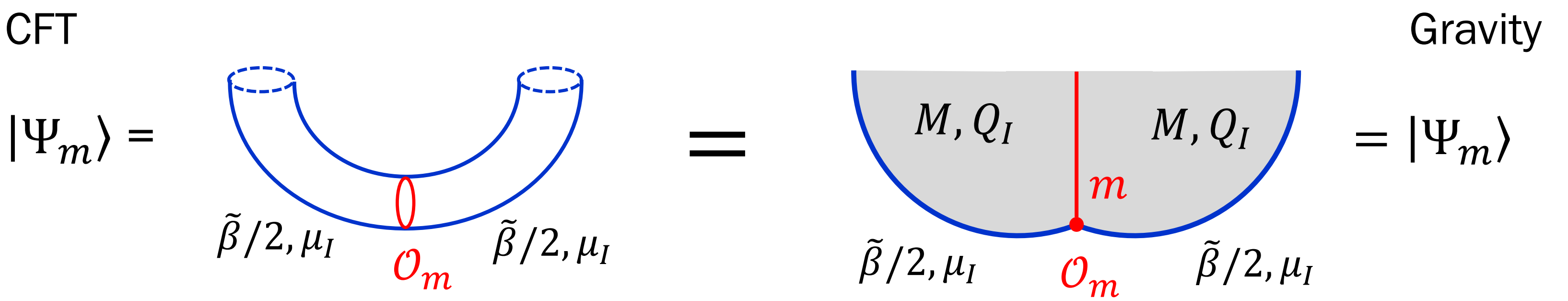
Geometric microstates look like a black hole when probed with simple operations



$$|\text{TFD}\rangle = \frac{1}{\sqrt{Z}}\sum_i e^{-\beta H/2}|E_i\rangle_L \otimes |E_i\rangle_R$$

A black hole microstate, though not very typical

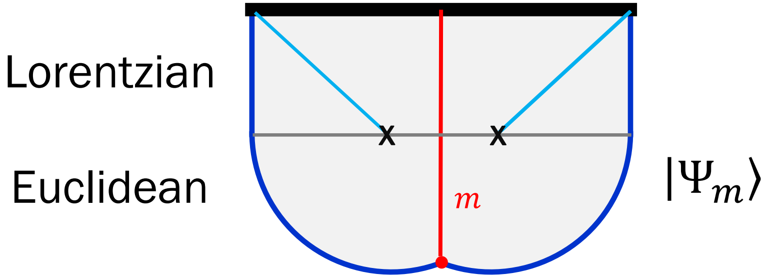
Geometric PEGS

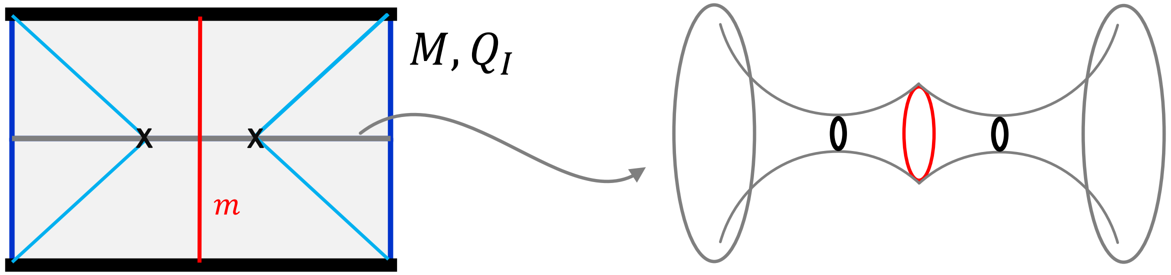


Operator $\mathcal{O}_m$ inserted at boundary creates particles in the bulk – a ‘shell of dust’ matter m

Make it heavy enough to backreact on geometry

Geometric PEGS

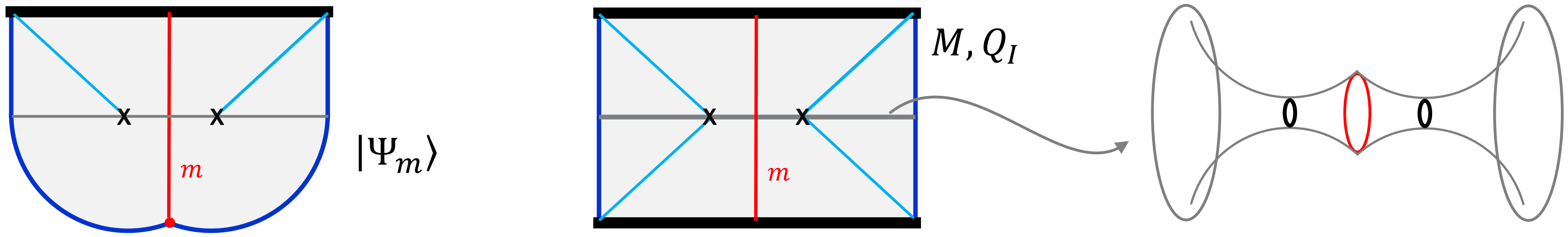




Shell moves inside black hole

Creates space within the black hole: ‘bag of gold’

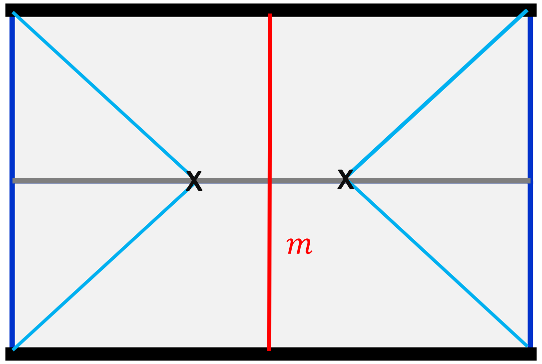
Geometric PEGS



Shell mass m can be arbitrarily large for fixed black hole M, Q_I

Huge (infinite!) number of states

Shell PEGS are Black Hole Microstates



$|\Psi_m\rangle$

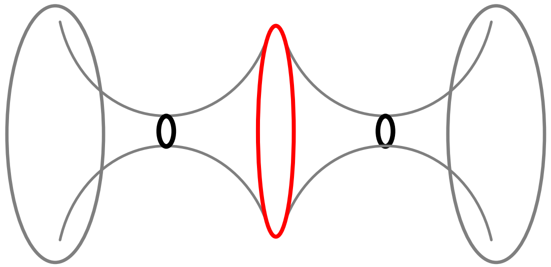
Almost indistinguishable (for simple observables) from grand-canonical state ρ_{th}

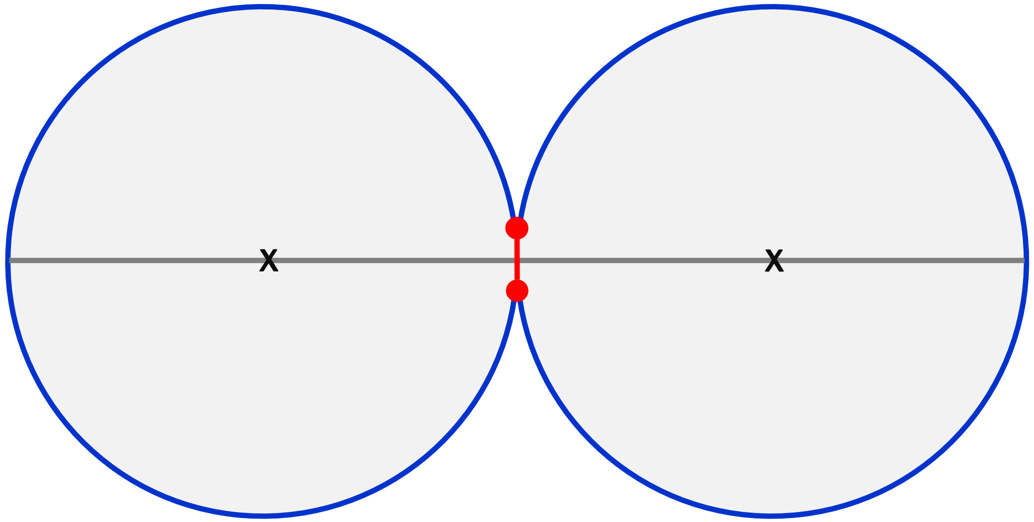
$$\langle\Psi_m|\mathcal{O}(t)|\Psi_m\rangle \to \text{Tr}(\rho_{\text{th}}\mathcal{O})$$

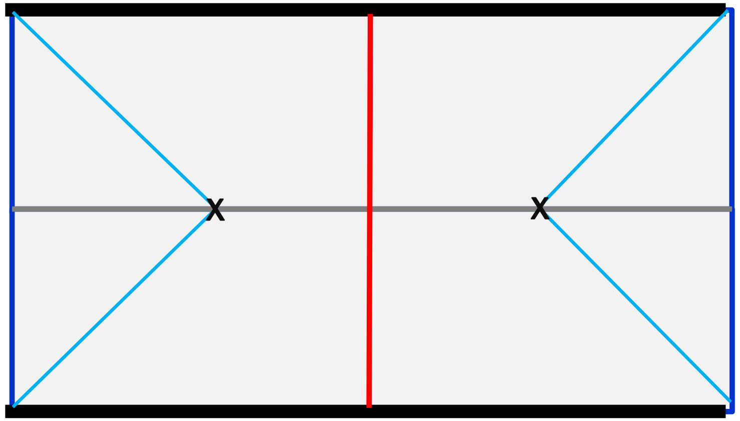
Looks like a black hole when probed with simple operations

Shell microstates have semiclassical description with horizons and singularities

Heavy-shell Microstates

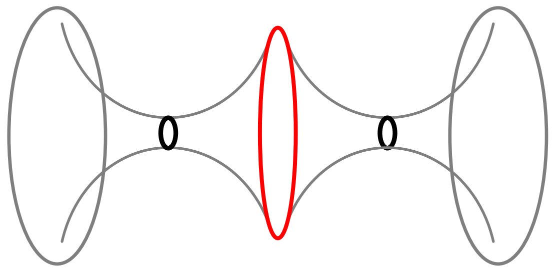


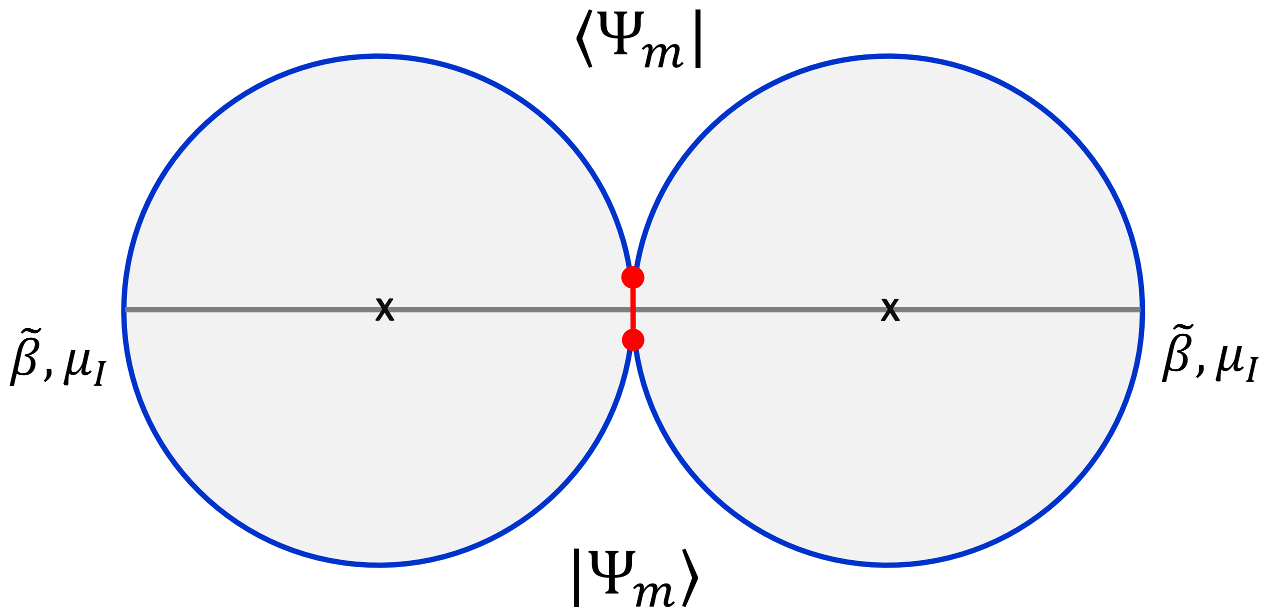




Shell close to the (would-be) boundary – little sensitivity to bulk black hole

Heavy-shell Microstates





$\langle \Psi_m | \Psi_m \rangle$ factorizes into $\approx Z[\tilde{\beta}, \mu_I]^2$

All calculations simplify a lot

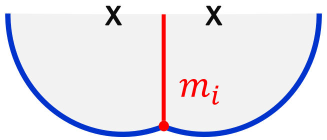
<u>Shell does not affect horizon properties</u>

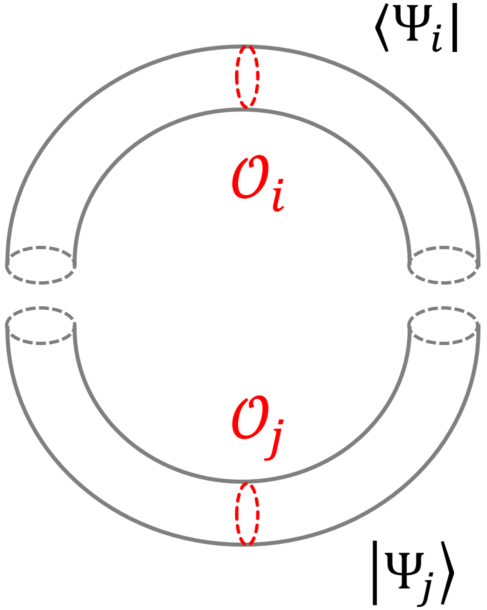
Dependence on shell m drops out $\rightarrow$ <u>universality</u>

Computing

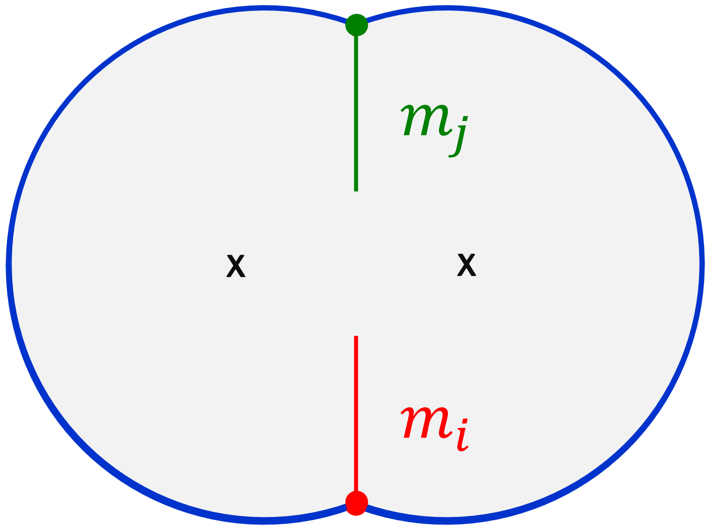
STATE OVERLAPS FROM WORMHOLES: UNIVERSALITY

How many states?

$|\Psi_i\rangle =$ 

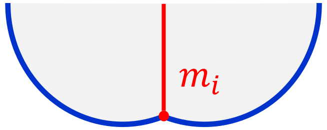


$$= G_{ij} = \langle\Psi_i|\Psi_j\rangle =$$

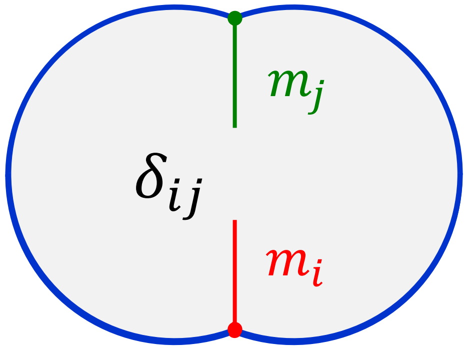


G: Gram matrix of state overlaps

Too many states?

$|\Psi_i\rangle =$ 

$$G_{ij} = \langle \Psi_i | \Psi_j \rangle = \delta_{ij}$$



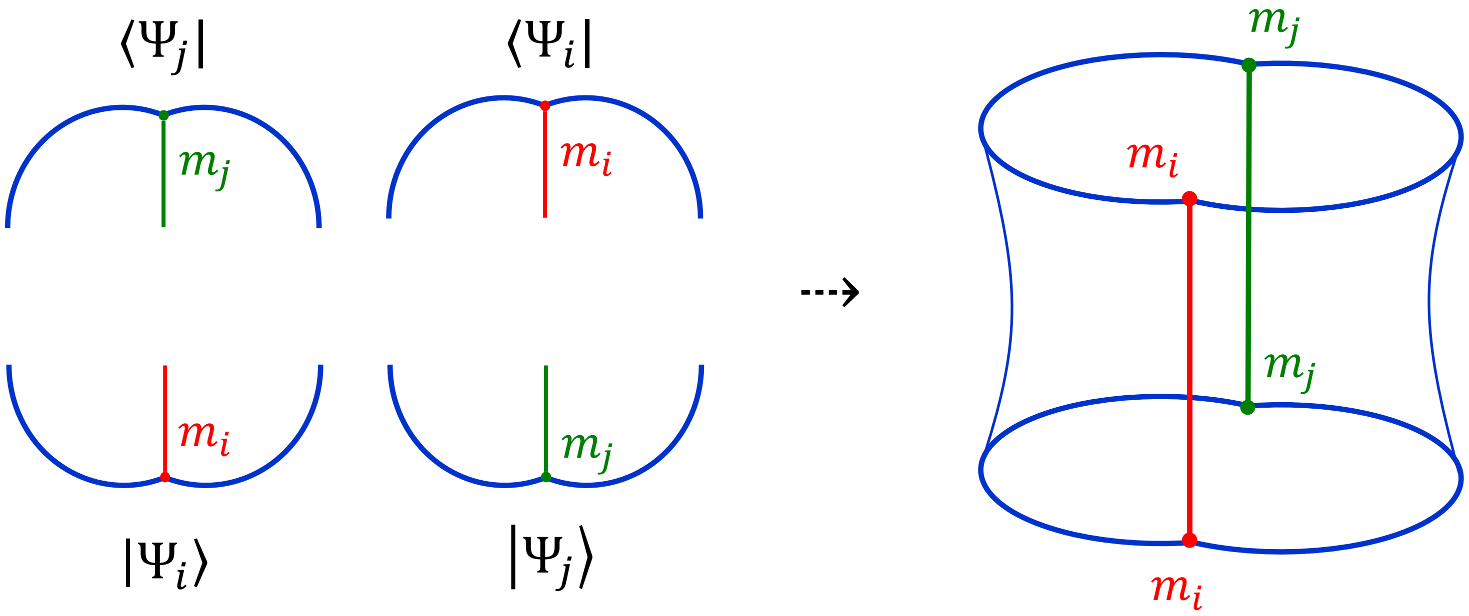
Infinite family of orthogonal states

$$\dim(\mathcal{H}_{BH}) = \infty \,!?$$

Products with Wormholes

average

$$\overline{G_{ij}G_{ji}} =$$

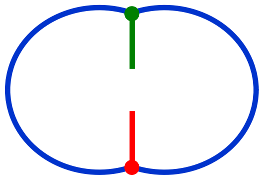


$$\neq 0$$

$$\overline{\langle\Psi_i|\Psi_j\rangle\langle\Psi_j|\Psi_i\rangle}$$

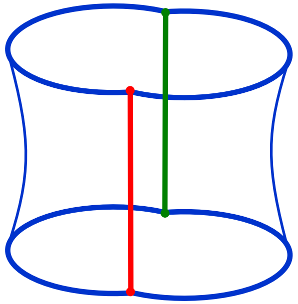
Wormholes ⇒ Statistical states

$\overline{G_{ij}} = 0$ for $i \neq j$



$= 0$

$\overline{G_{ij} G_{ji}} \neq 0$ for $i \neq j$



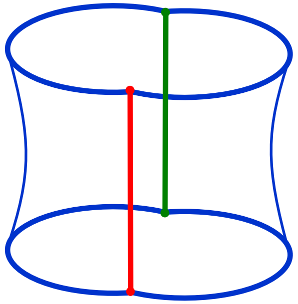
$\neq 0$

Not $\langle \Psi_i | \Psi_j \rangle = 0$

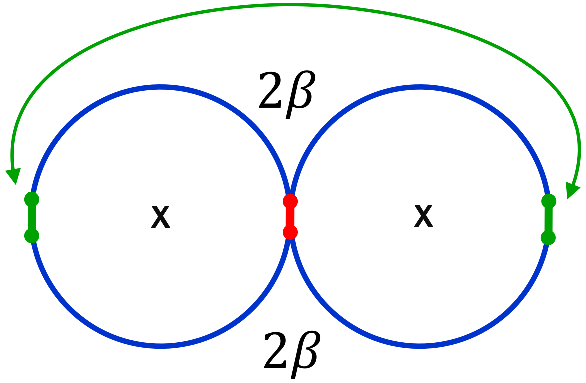
but $\overline{\langle \Psi_i | \Psi_j \rangle} = 0$

Heavy-Shell Wormholes

$$\overline{G_{ij}G_{ji}} =$$



$\longrightarrow$



$$= \frac{Z(2\beta, \mu_I)^2}{Z(\beta, \mu_i)^4}$$

Unaffected by shell m_i

Given by partition function of BH

Moments of G from wormholes

$$\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}} = \frac{Z(n\beta, \mu_I)^2}{Z(\beta, \mu_i)^{2n}}$$

Heavy-shell universality

Depends only on BH partition function

Counting

THE DIMENSION OF THE BLACK HOLE HILBERT SPACE

Dimension of set of states

$$F_\Omega = \{ |\Psi_i\rangle \in \mathcal{H}, i = 1, \dots, \Omega \}$$

$$d_\Omega = \dim F_\Omega = \min\{\Omega, \dim \mathcal{H}\}$$

$$= \text{rank}\, G_{ij}$$

$$G_{ij} = \langle \Psi_i | \Psi_j \rangle$$

Gram-Schmidt fails for BH microstates: $\overline{G_{ij}} = \delta_{ij}$

Statistical counting

From statistical moments $\overline{G^n}$

Statistics forced by GPI wormholes

Borrow from random matrix techniques: resolvent

$$R(\lambda) = \frac{\Omega}{\lambda} + \sum_{n=1}^{\infty} \frac{\mathrm{Tr}\,\overline{G^n}}{\lambda^{n+1}} \longrightarrow \overline{d_\Omega}$$

Penington+al (PSSY) 2019

Moments

From grand-canonical to microcanonical BH window

$$\left.\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}}\right|_{\text{grcan}} = \frac{Z(n\beta, \mu_I)^2}{Z(\beta, \mu_i)^{2n}}$$

inverse Laplace transform

$$\left.\overline{G_{i_1 i_2} G_{i_2 i_3} \dots G_{i_n i_1}}\right|_{\text{micro}} = e^{-(n-1)\frac{A}{4G}}$$

How many black hole states? $\exp A/4G$

We had $d_\Omega = \dim F_\Omega = \min\{\Omega, \dim \mathcal{H}\}$

Resolvent for $\overline{G^n}$ gives $\overline{d_\Omega} = \min\{\Omega, e^{A/4G}\}$

$$\Rightarrow \boxed{\dim \mathcal{H} = e^{A/4G}}$$

How many black hole states? $\exp S_{BH}$

More generally

$$\dim \mathcal{H} = e^{S_{BH}}$$

where S_{BH} is the value from Gibbons-Hawking GPI Partition Function

(through black magic)

Universality of $\dim \mathcal{H} = \exp S_{BH}$

Heavy shells can be constructed for

- Rotating and charged black holes
- Near-extremal, susy or not
- Quantum-corrected: $\log A$ and $\log T$
- Higher-curvature theories

Heavy-shell microstates ⇒ $\dim \mathcal{H} = e^{S_{BH}}$

Outlook

GPI BLACK MAGIC RELOADED – WITH WORMHOLE STATISTICS

Gravitational Path Integral can do a lot

- Construct microstate families and count their dimension
- Heavy-shell microstates ⇒ $\dim \mathcal{H} = e^{S_{BH}}$
- Works for all cases where Gibbons-Hawking gives an entropy

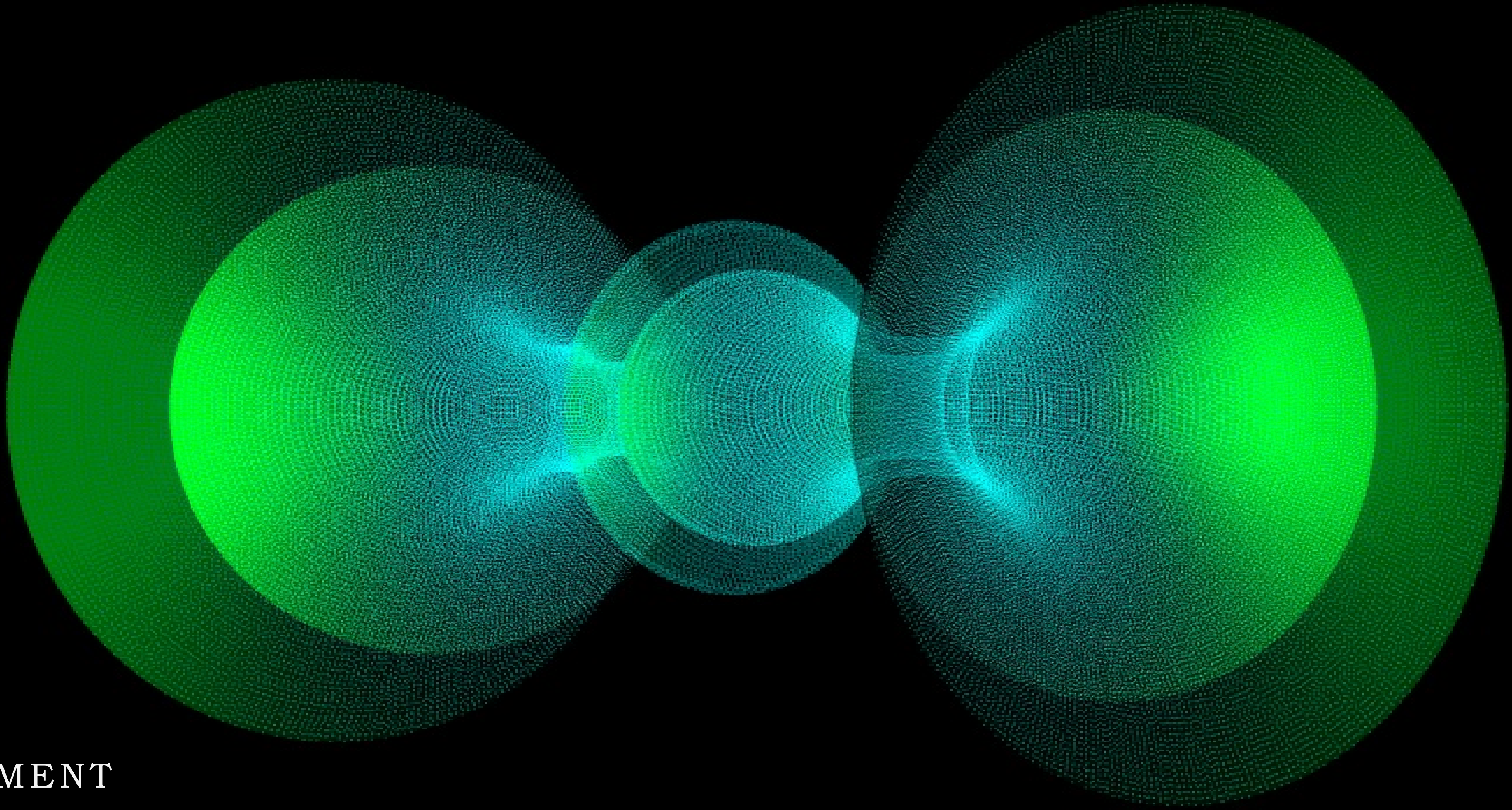
Universality: double-edged sword

- Extremely general, simple construction and result ☺
- Hides all microscopic distinctions ☹
- Works even when it should not (eg in the swampland)

Geometry and randomness

- Wormholes are how gravity knows about finite $\dim \mathcal{H}_{BH}$
- But they introduce intrinsic randomness
- Semiclassical BH geometry seems to need chaotic microscopics

Is this all one needs/can do for
(non-susy) BH microscopics?



ANA CLIMENT
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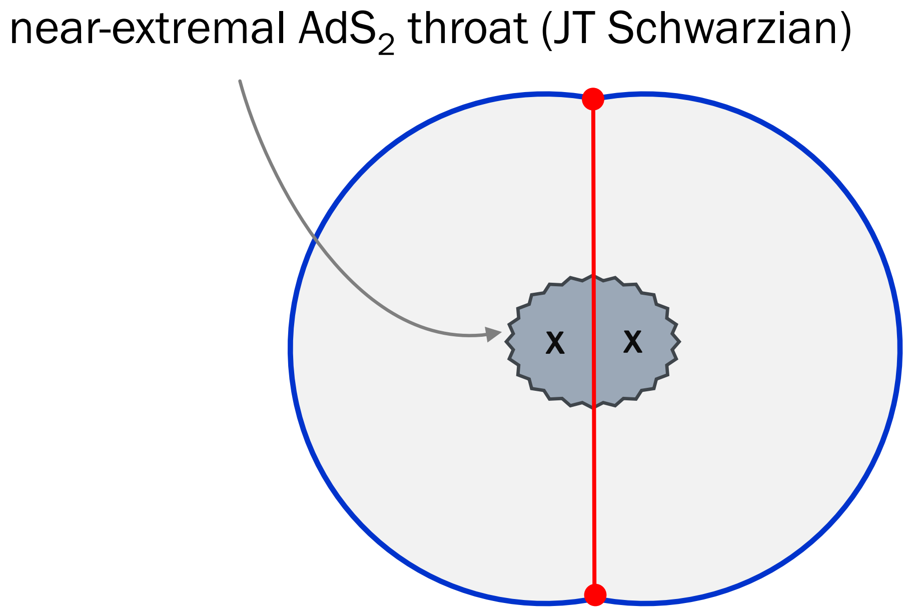
Thank you

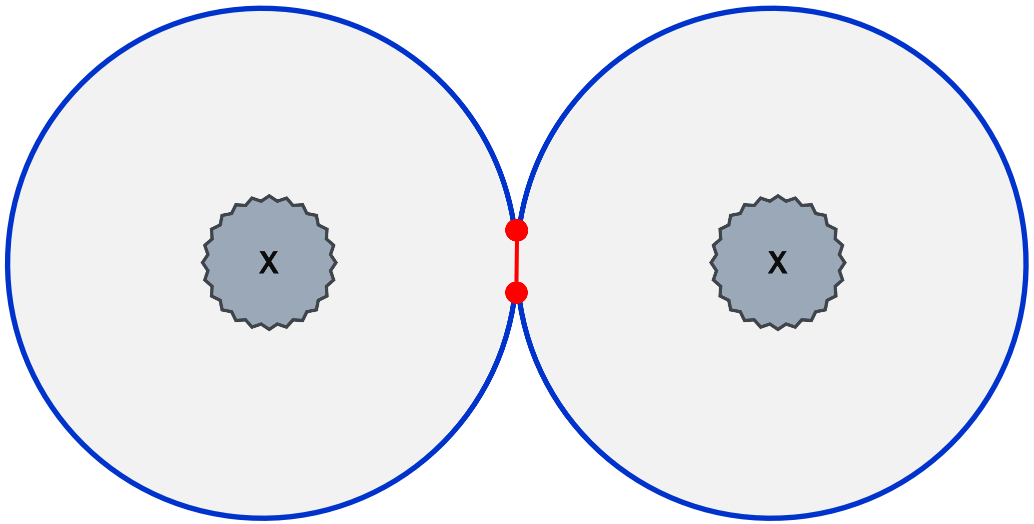
Backup material

Near-extremal Microstates



In-throat microstates (one JT Schwarzian)

Sensitive to throat



Out-throat microstates (two Schwarzians)

Universal