

Baryogenesis and baryon isocurvature perturbation from primordial hypermagnetic fields

Relevant papers: T. Fujita (Waseda) & KK, PRD93 (2016) 083520 [arXiv:1602.02109 (hep-ph)]

KK & A.J.Long (Rice), PRD94 (2016) 063501 [arXiv:1606.08891 (astro-ph.CO)]

KK & A.J.Long (Rice), PRD94 (2016) 123509 [arXiv:1610.03074 (hep-ph)]

KK, F. Uchida, J. Yokoyama (Tokyo), JCAP 04 (2021) 034 [arXiv: 2012.14435 (astro-ph.CO)]

See also: F. Uchida (Tokyo), M. Fujiwara (TUM), KK, J. Yokoyama (Tokyo), PLB843 (2023) 138002

[arXiv: 2212.14355 (astro-ph.CO)]



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Copernicus webinar series

27/02/2024 @ on-line

1. Introduction

- Primordial magnetic fields (PMFs) -

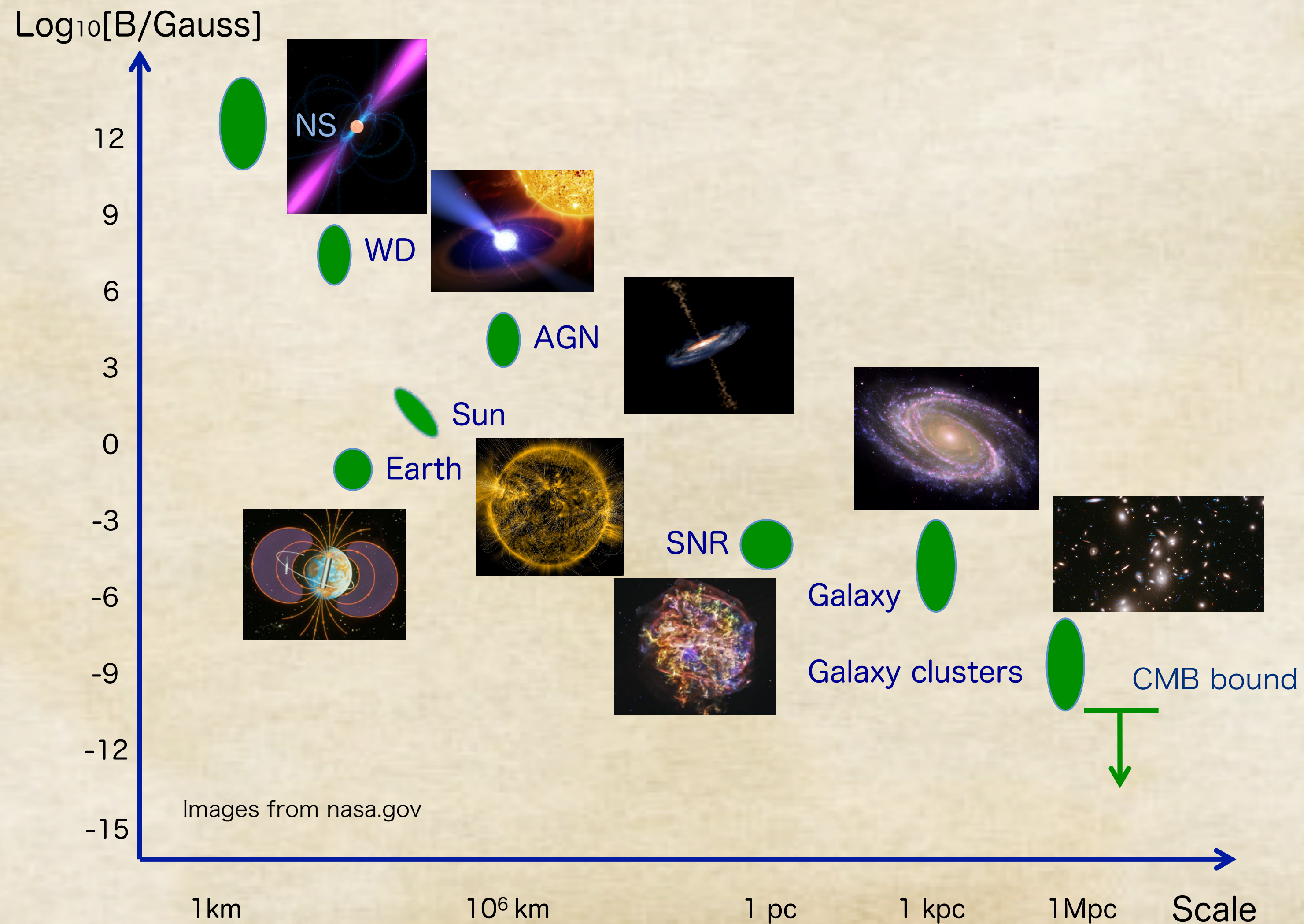
2. Baryon asymmetry from PMFs

3. Baryon isocurvature constraints

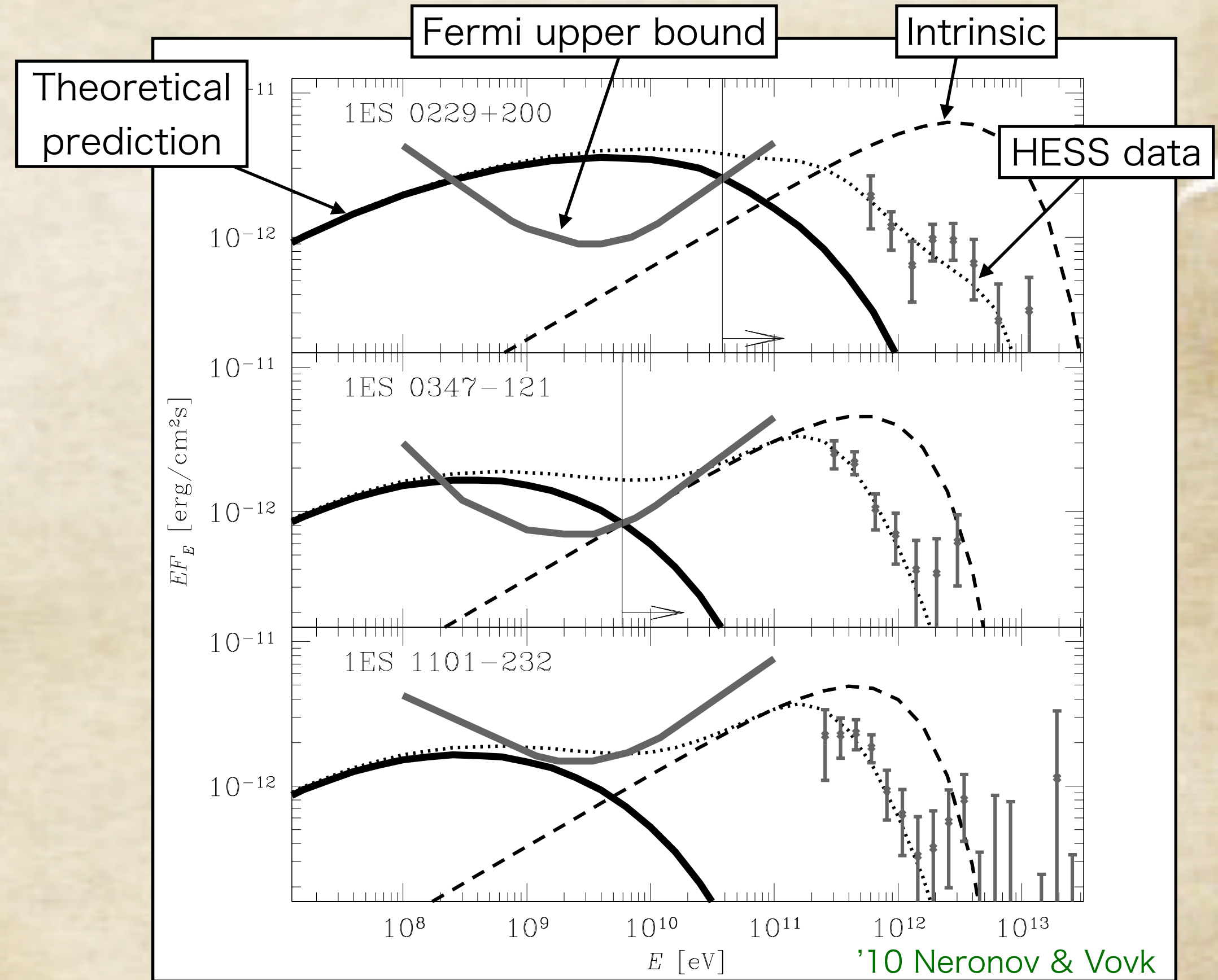
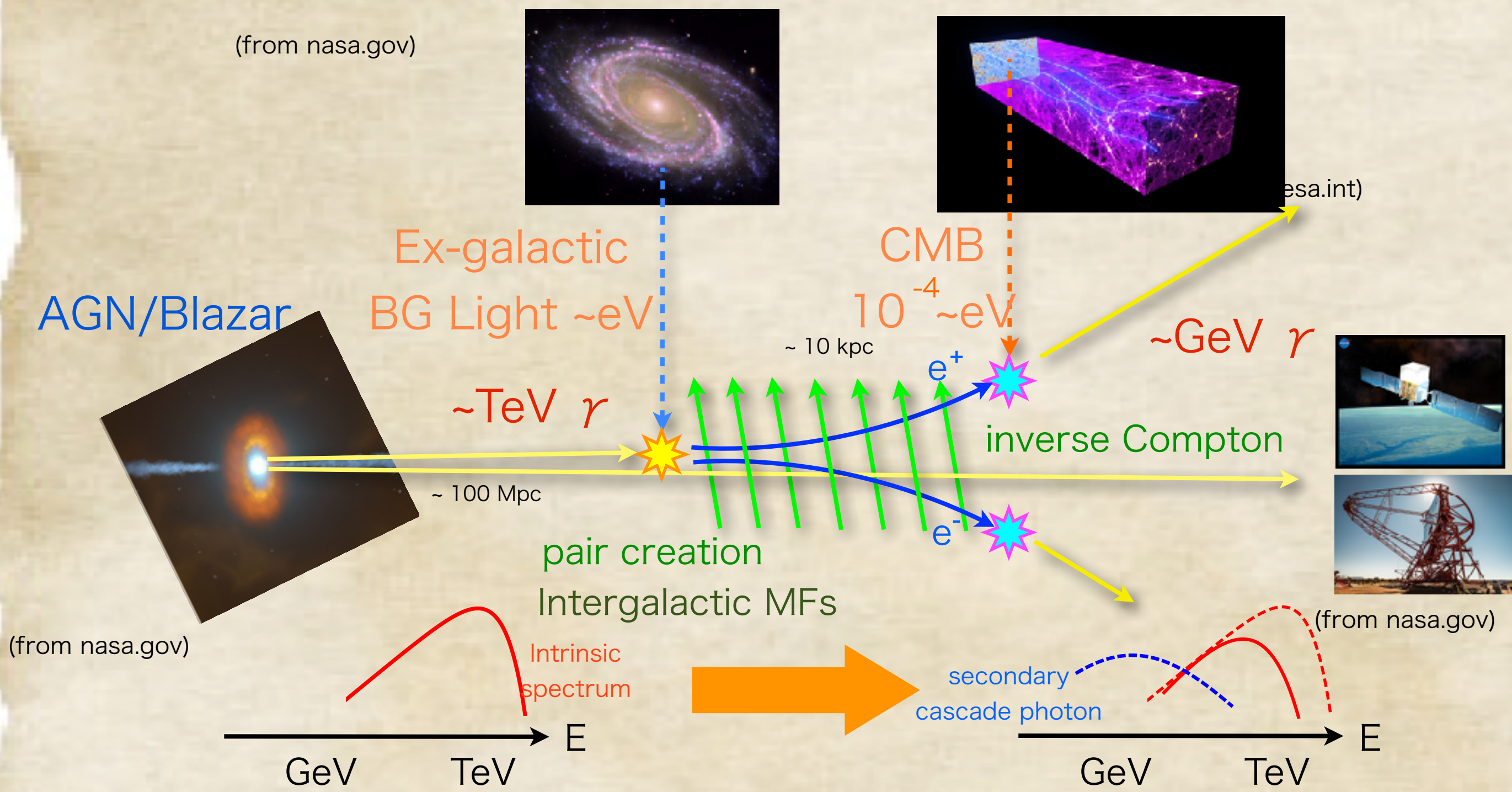
4. Summary

Introduction

Magnetic fields are ubiquitous in the Universe.

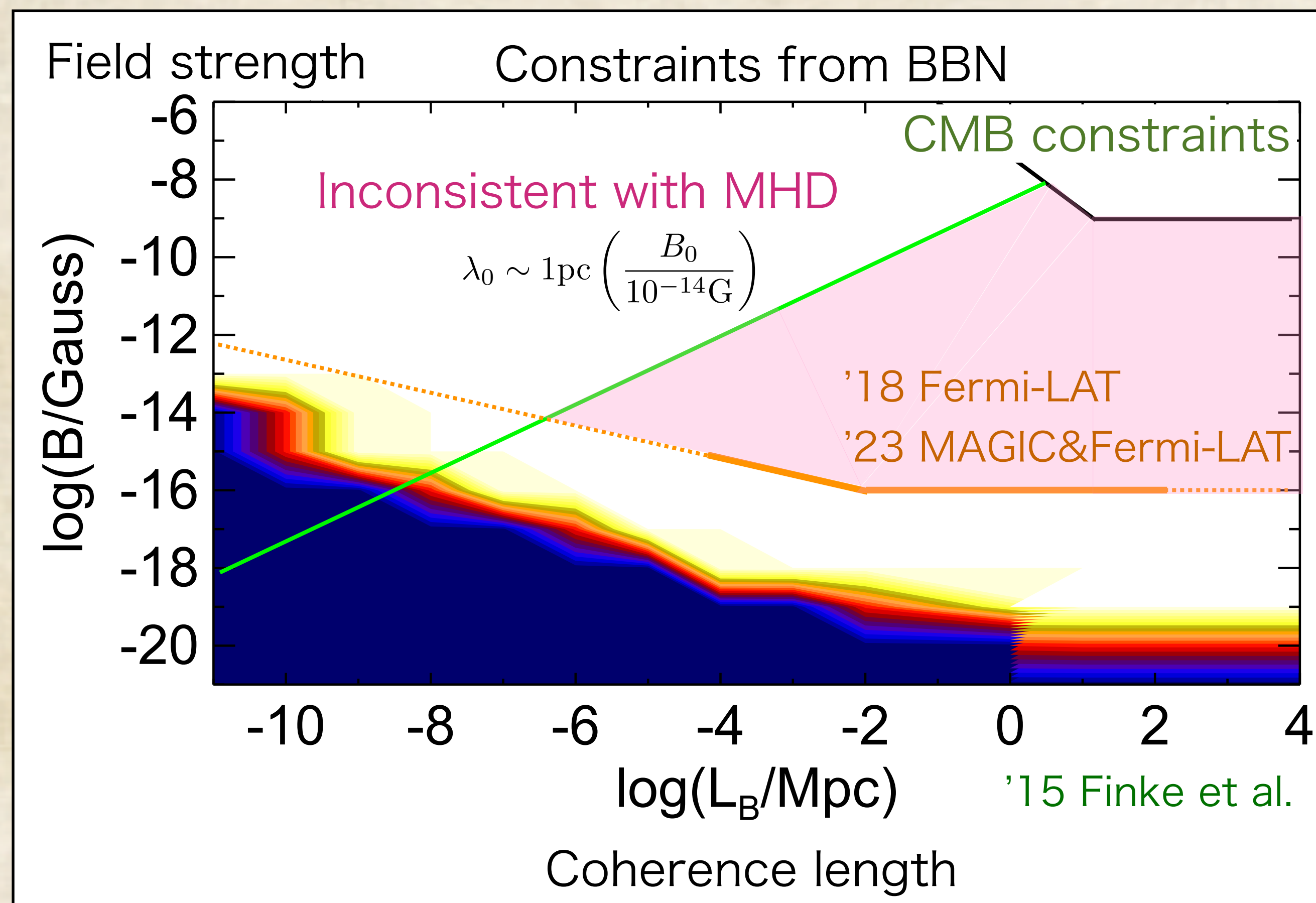


Observations of the intergalactic magnetic fields

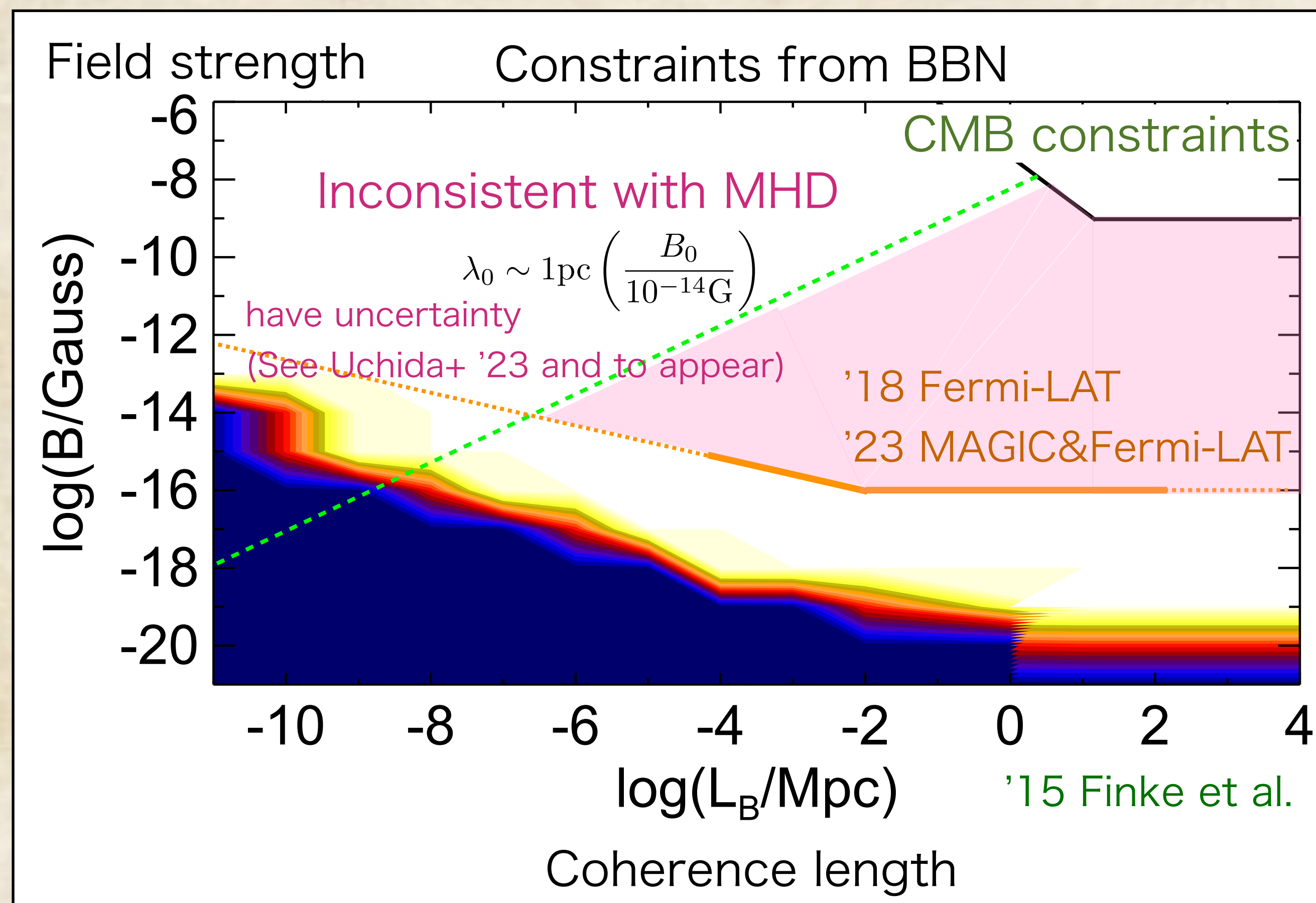


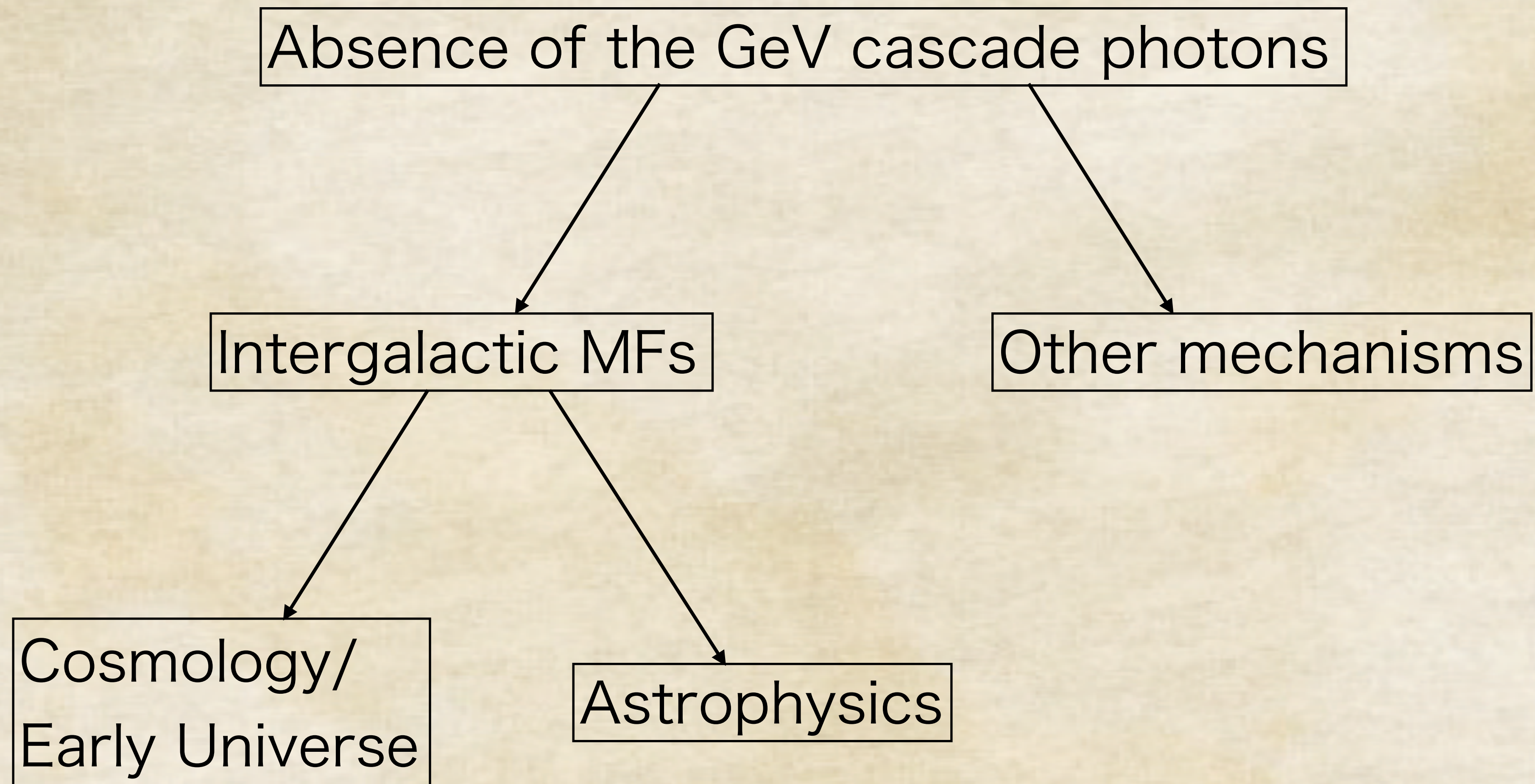
Non-observation of the secondary cascade GeV photon can give the lower bound of the intergalactic magnetic fields (indirect implication)

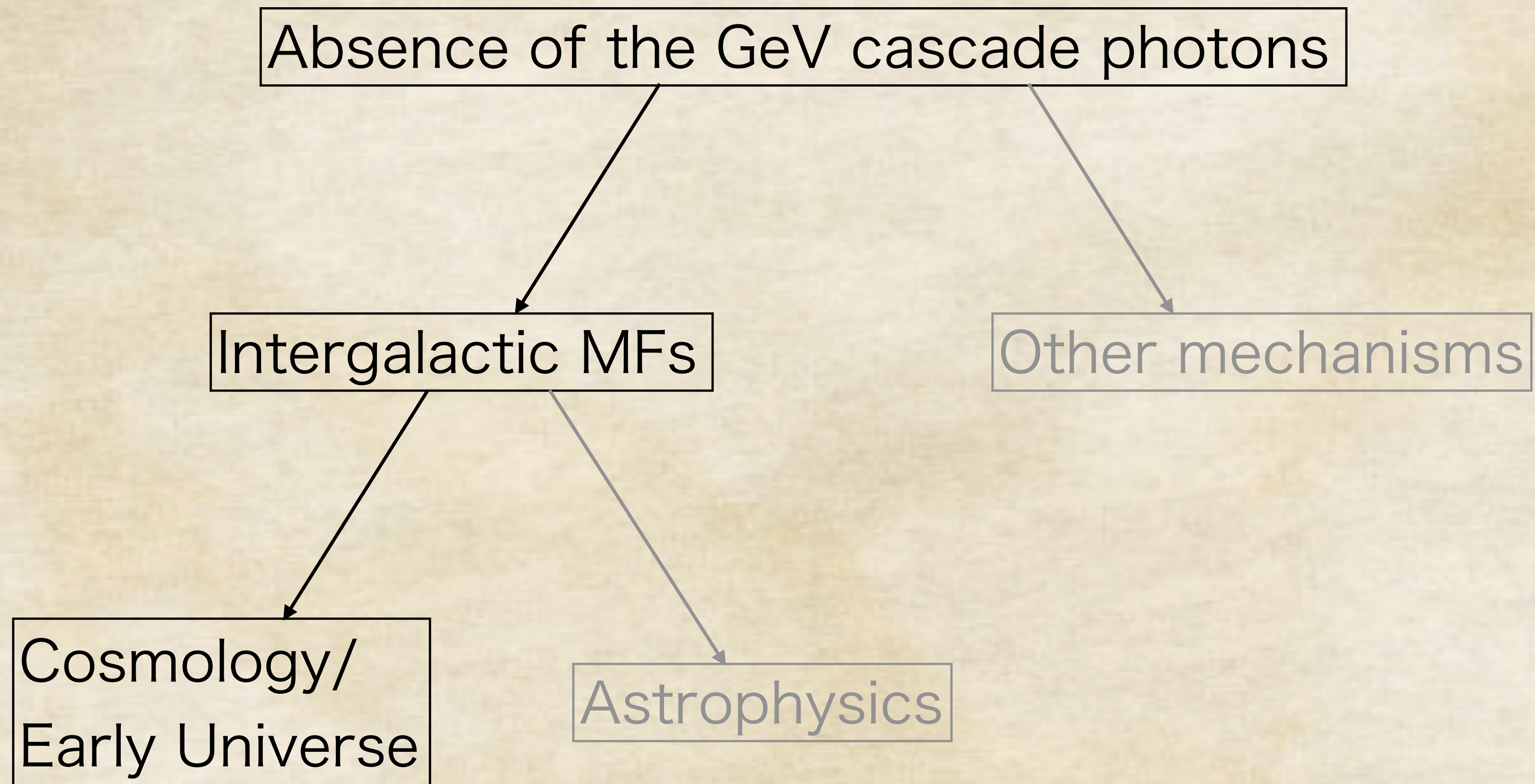
Latest constraints from Fermi



Latest constraints from Fermi

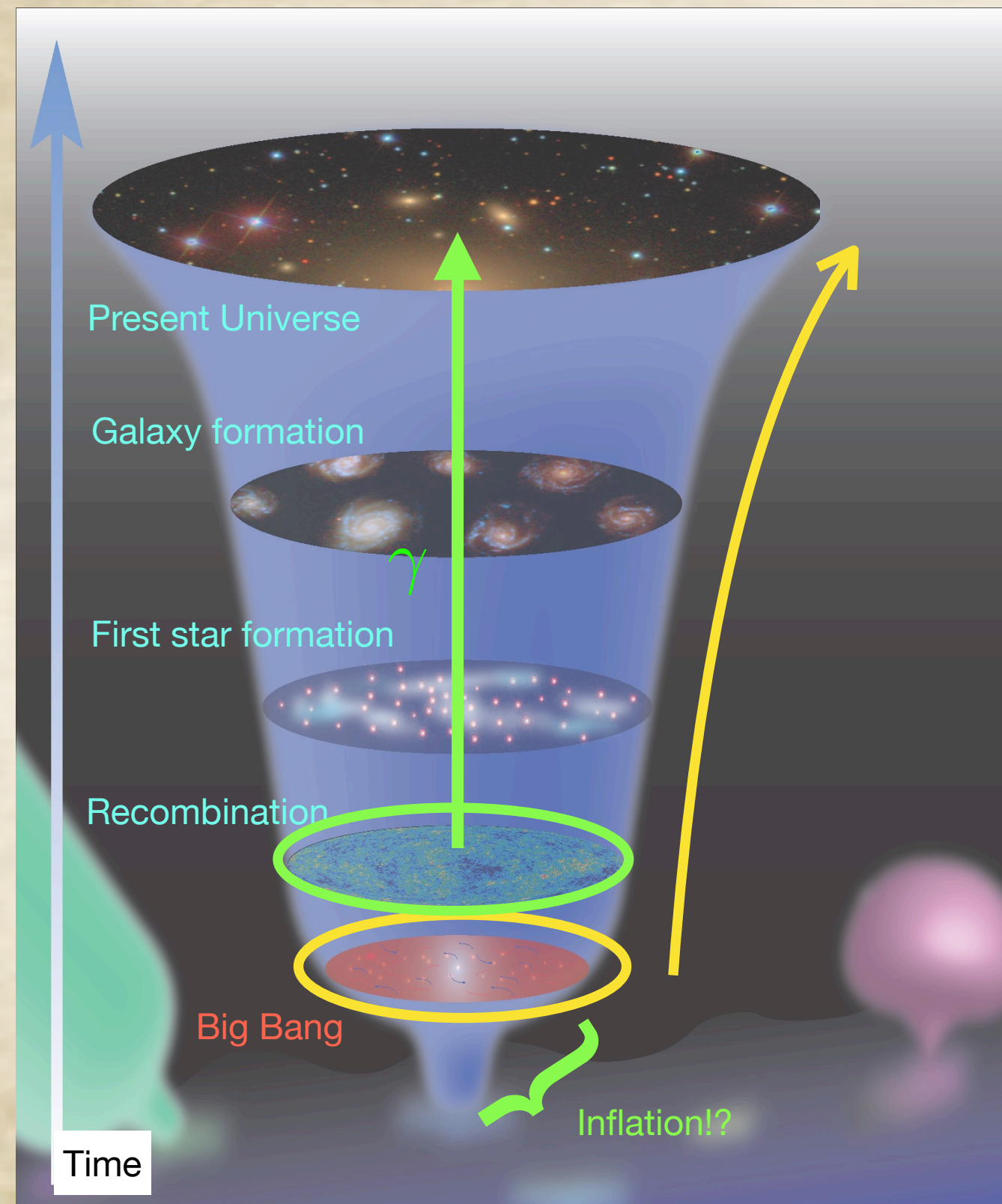






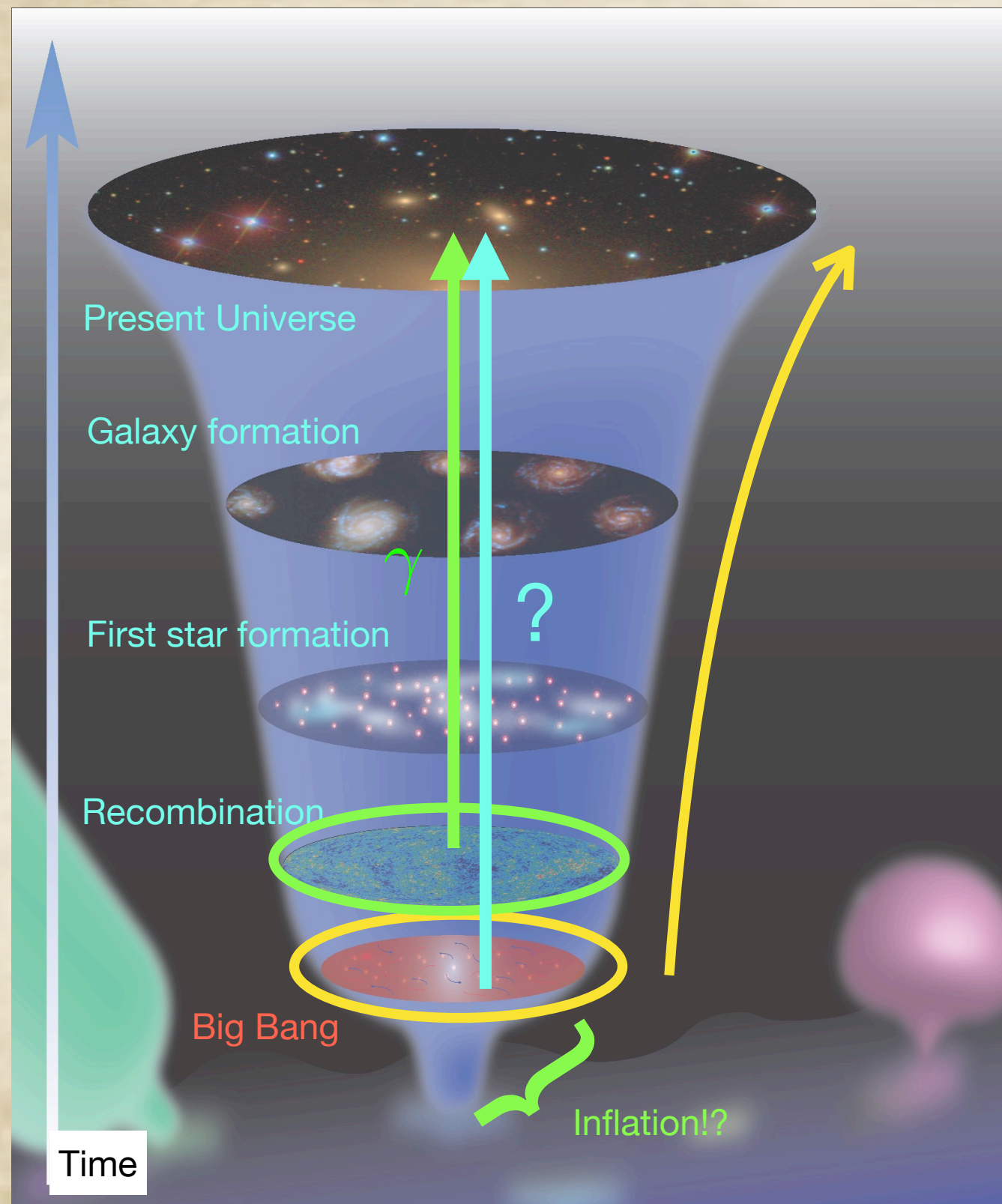
We might expect that they are relics from the early Universe.

1. Long range MFs are not in thermal equilibrium but keep their long-range spectrum (no “thermal” mass for the MFs). => Carry the information before the recombination?



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2. Generation mechanism (magnetogenesis) may need new physics beyond the SM.
=> Target for the phenomenological model builders, such as axion inflation or phase transition.
3. Chiral effects may play an important role of their generation and/or evolution.
=> Interest for field theorists.

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Baryon asymmetry of the Universe can be also explained!

(’98 Giovannini & Shaposhnikov, ’16 Fujita & KK, KK & Long)

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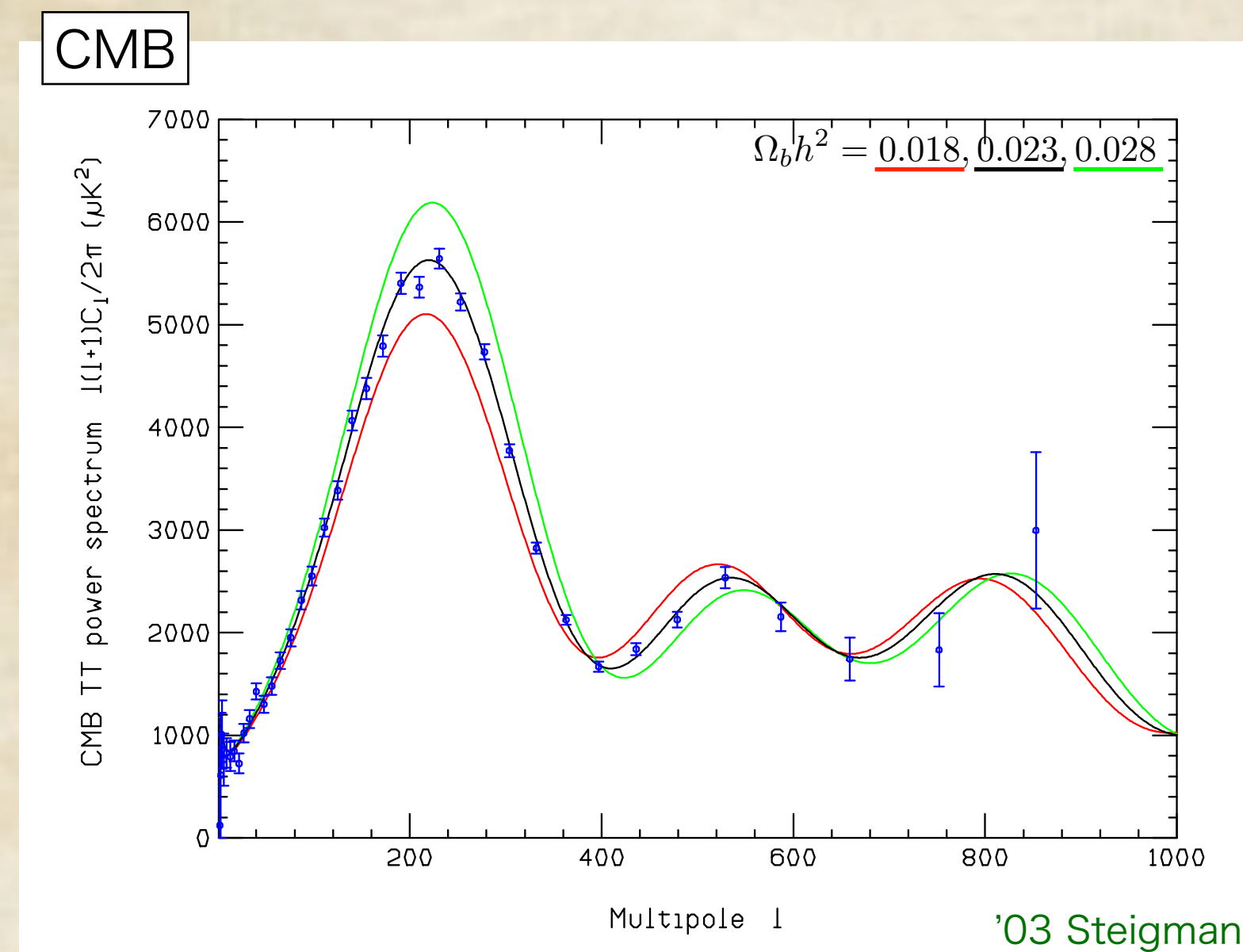
(’98 Giovannini & Shaposhnikov,’16 Fujita & KK, KK & Long)

We however show that any magnetogenesis mechanisms before the EWSB are almost impossible to explain the IGMFs suggested by blazar observations.

Baryogenesis from PMFs

Baryon Asymmetry of the Universe

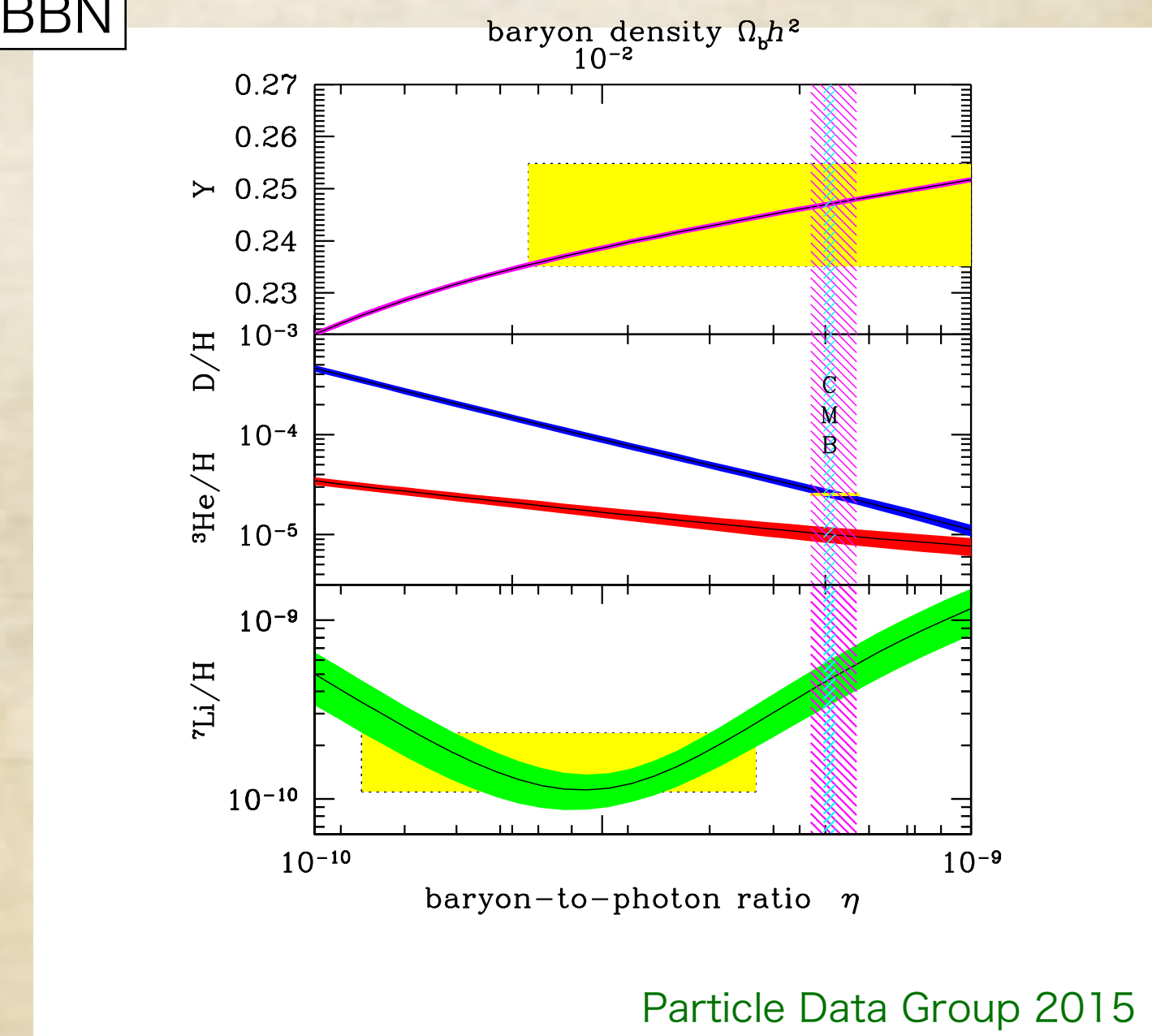
- We live in a matter-antimatter asymmetric Universe.
- BBN and CMB can evaluate it quantitatively.



$$\eta \equiv \frac{n_B}{n_\gamma} = (6.09 \pm 0.06) \times 10^{-10}$$

(Planck 2015)

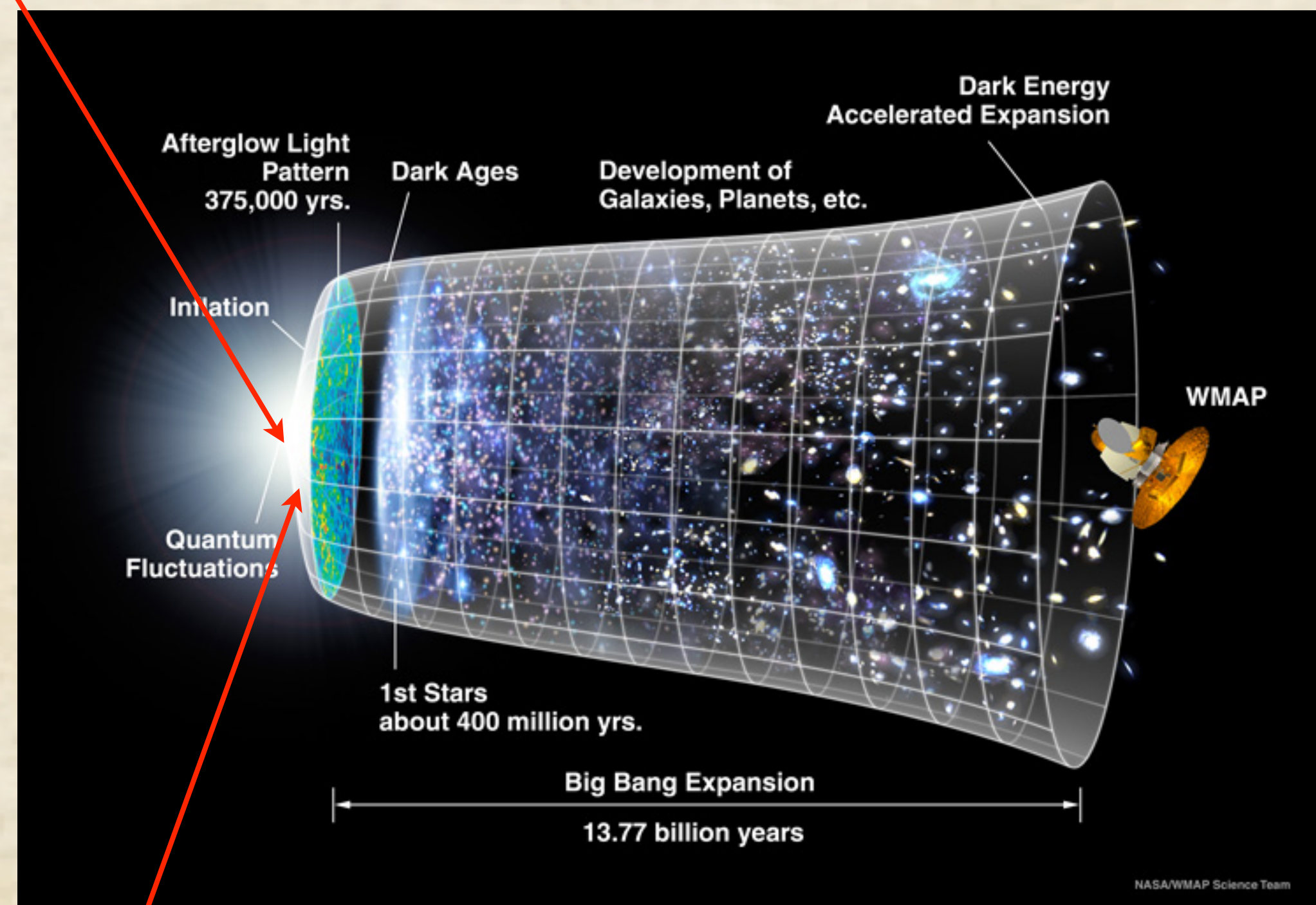
BBN



$$\eta = (6.180 \pm 0.195) \times 10^{-10}$$

BBN+D ; '15 Cyburt+

Inflation dilutes the preexisting asymmetry.



WMAP team

After inflation before BBN, asymmetry must be generated.

{	matter :	~100000000000+1	$\xrightarrow[T \sim 100 \text{ MeV}]{\text{annihilation}}$	{	matter :	~1
	anti matter :	~100000000000			anti matter :	~0
	photon :	~100000000000			photon :	~100000000000

In order to generate baryon asymmetry...

Sakharov's condition is required. ('67 Sakharov)

1. B-violation
2. C & CP-violation
3. Deviation from thermal equilibrium

BSM is required!?

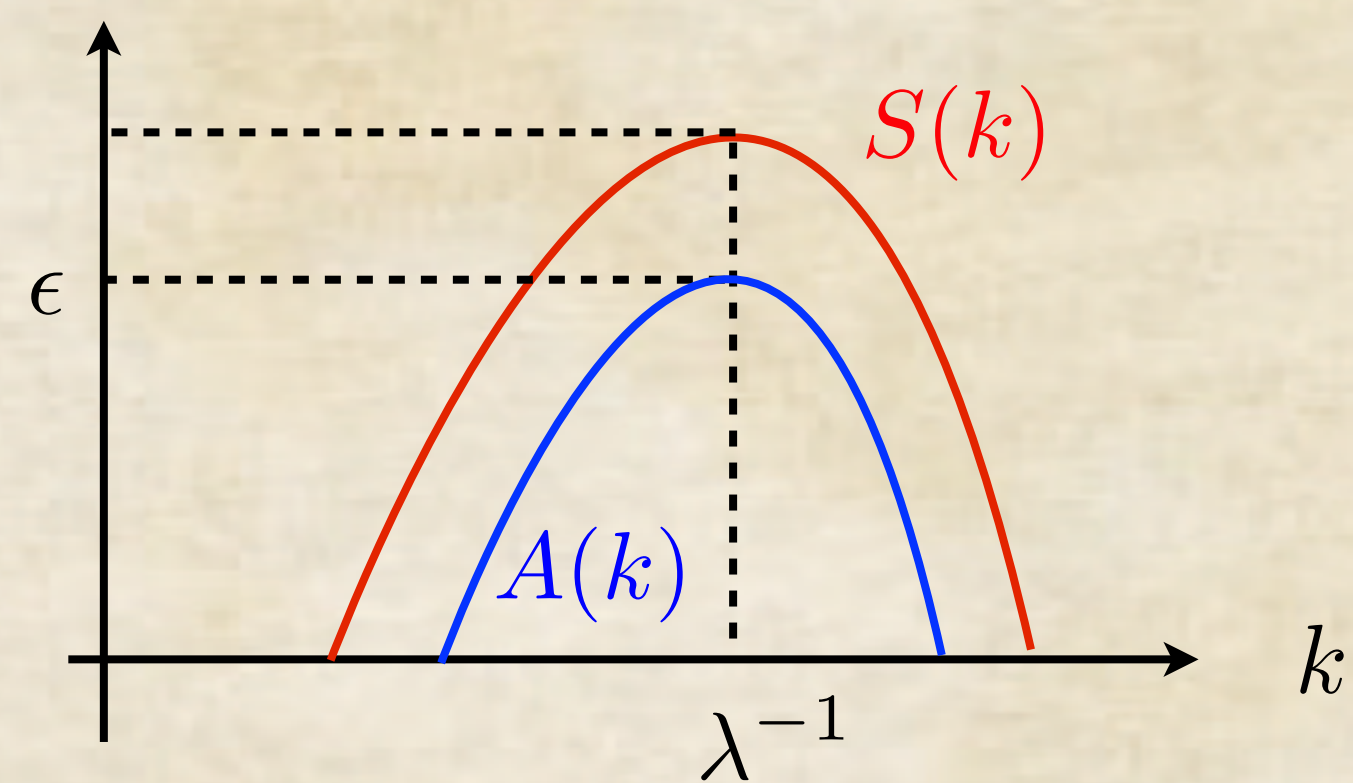
- Leptogenesis ('85 Fukugita&Yanagida) : RH neutrinos
- Affleck-Dine ('85 Affleck&Dine, '95 Dine,Randall&Thomas) : SUSY with ~~B~~ and ~~CP~~ op.
- EW baryogenesis ('85 Kuzmin, Rubakov&Shaposhnikov) : 1st order EWPT + ~~CP~~ op.

How helical hypermagnetic fields can satisfy the Sakharov's condition?

Giovannini&Shaposhnikov ('98), KK& Fujita, KK&Long ('16)

Imagine that the hypermagnetic fields exist before the EWSB as Gaussian stochastic fields,

$$\langle B_i(\mathbf{k}) \rangle = 0 \quad \langle B_i(\mathbf{k}) B_j(\mathbf{k}') \rangle = (2\pi)^3 \left((\delta_{ij} - \hat{k}_i \hat{k}_j) \underline{S(k)} + i \epsilon_{ijk} \hat{k}_k \underline{A(k)} \right) \delta(\mathbf{k} - \mathbf{k}') \quad (S(k) \geq A(k))$$

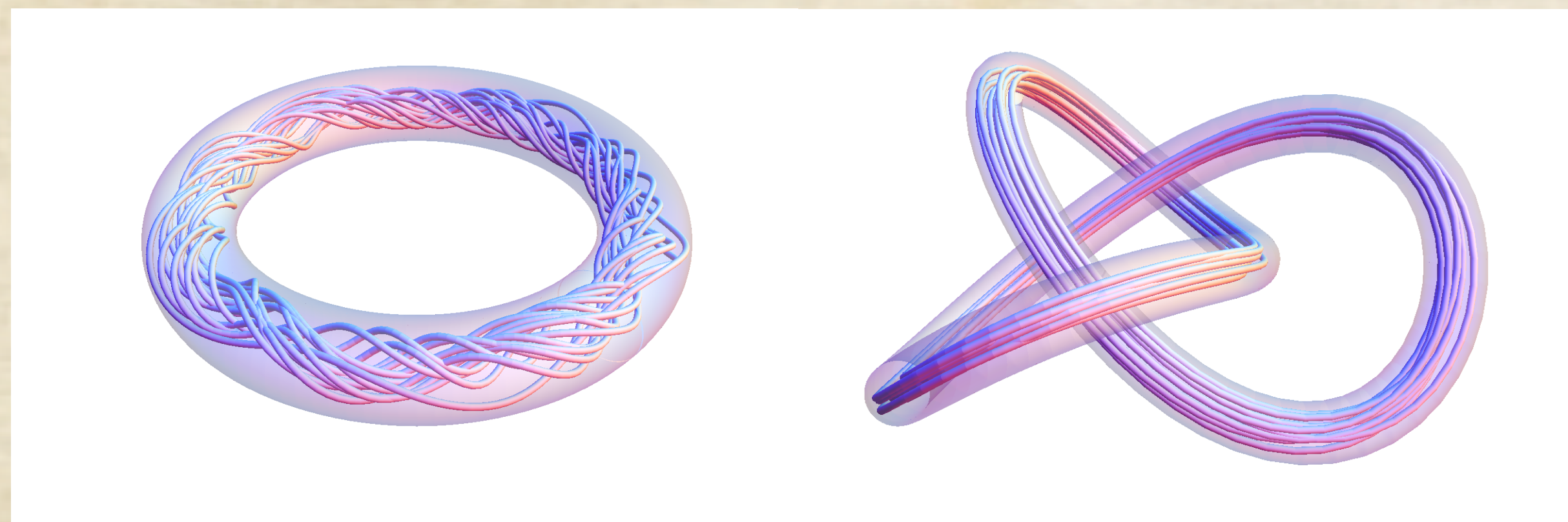
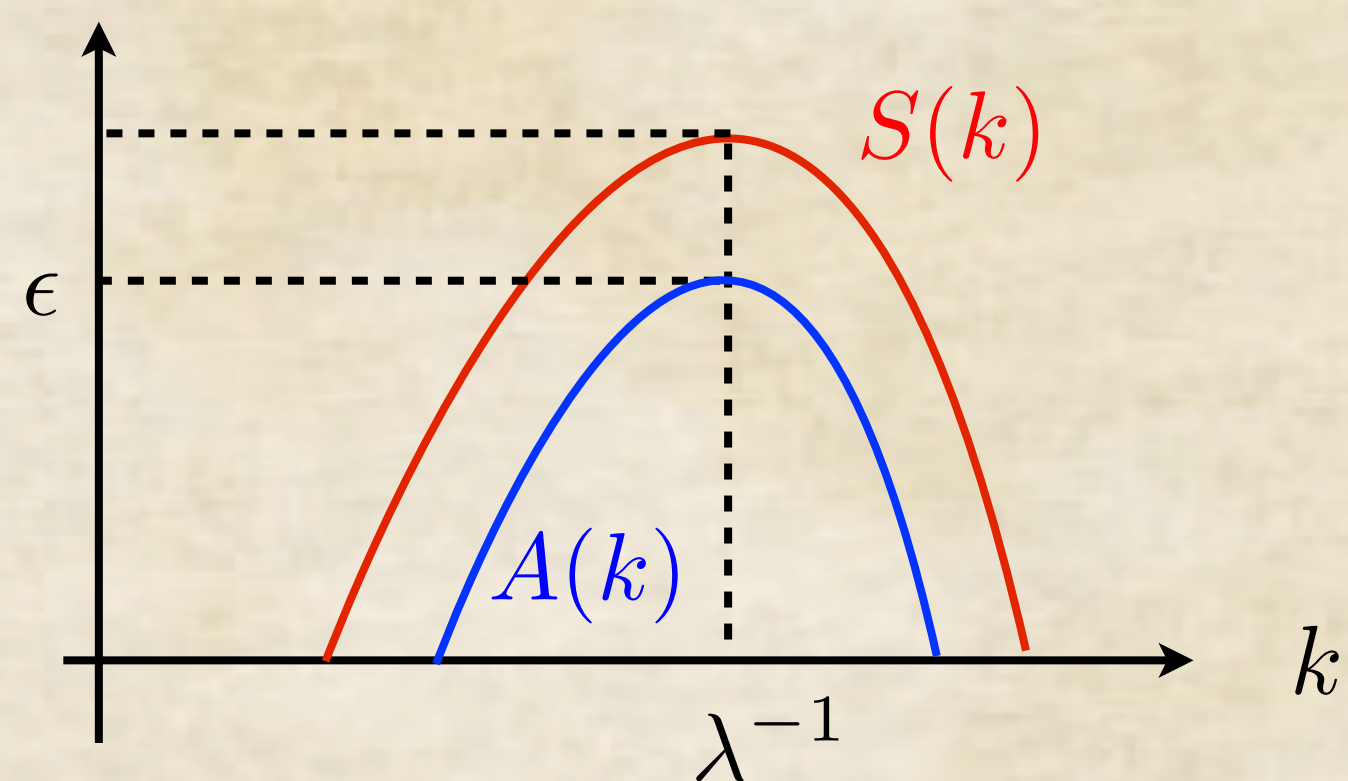


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From these notations, characteristics of the magnetic fields are given by

$$\rho_B = \int \frac{d^3 k}{(2\pi)^3} k^2 \underline{S(k)} \Rightarrow \bar{B} = \sqrt{2\rho_B} \quad \underline{\mathcal{H}} = 2 \int \frac{d^3 k}{(2\pi)^3} k \underline{A(k)} \quad \lambda = \frac{\int dk k^3 S(k)}{\int dk k^4 S(k)}$$



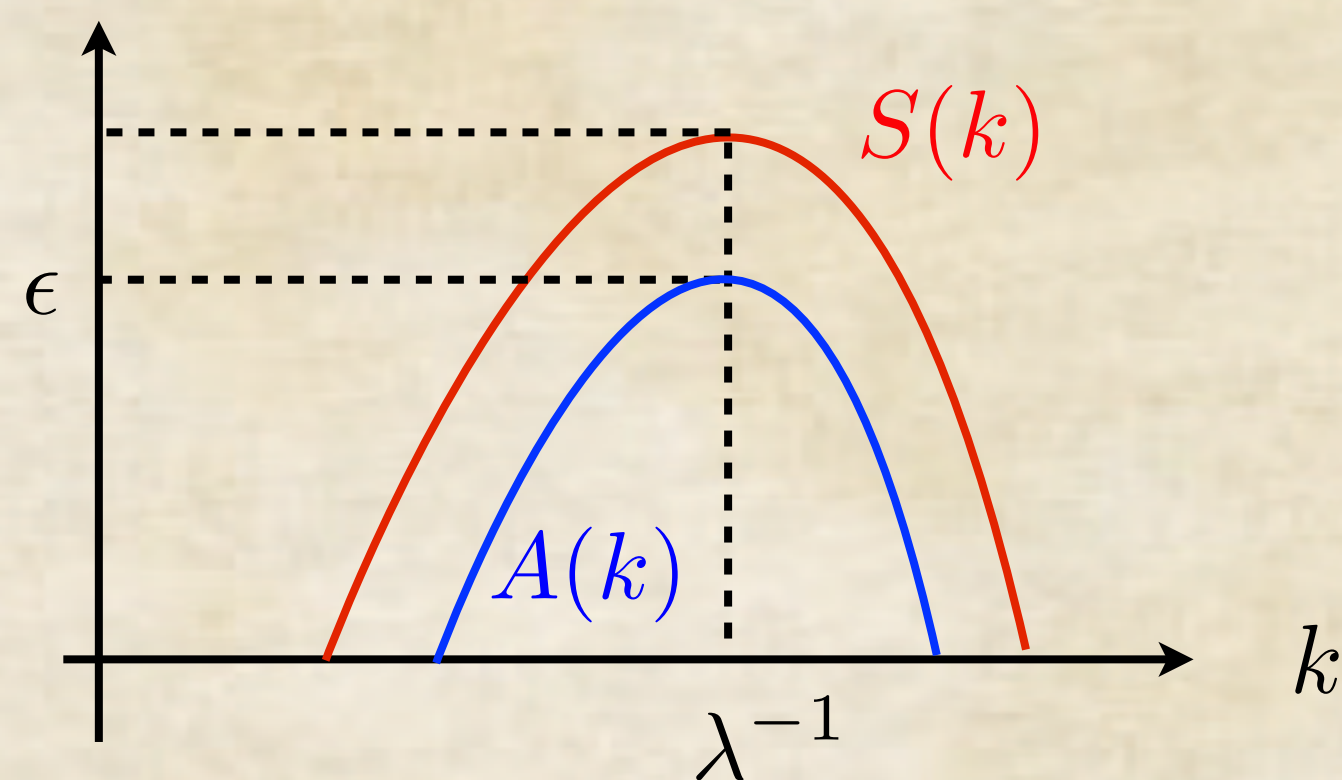
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$$\underline{\mathcal{H}} \simeq \epsilon \lambda \bar{B}^2 \quad (\epsilon: \text{ helicity fraction})$$

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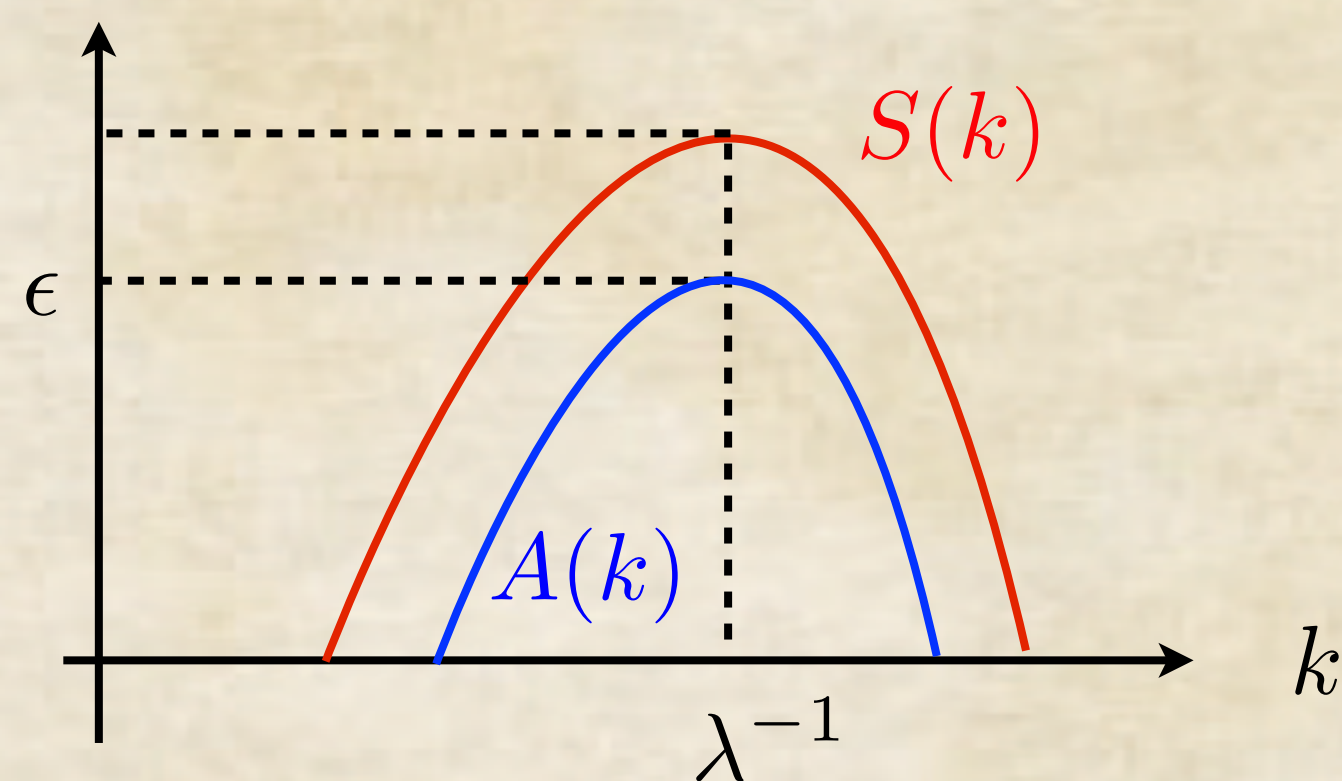
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If the average of helicity density \mathcal{H} decays, B+L asymmetry is generated in the Universe?



$$\Delta Q_B = \Delta Q_L = N_g \left(\Delta N_{CS} - \frac{g'^2}{16\pi^2} \underline{\Delta \mathcal{H}_Y} \right)$$

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$$(S(k) \geq A(k))$$

If there were BG large-scale helical hyperMFs,

B-violation: SM chiral anomaly

C&CP-violation/non-equilibrium

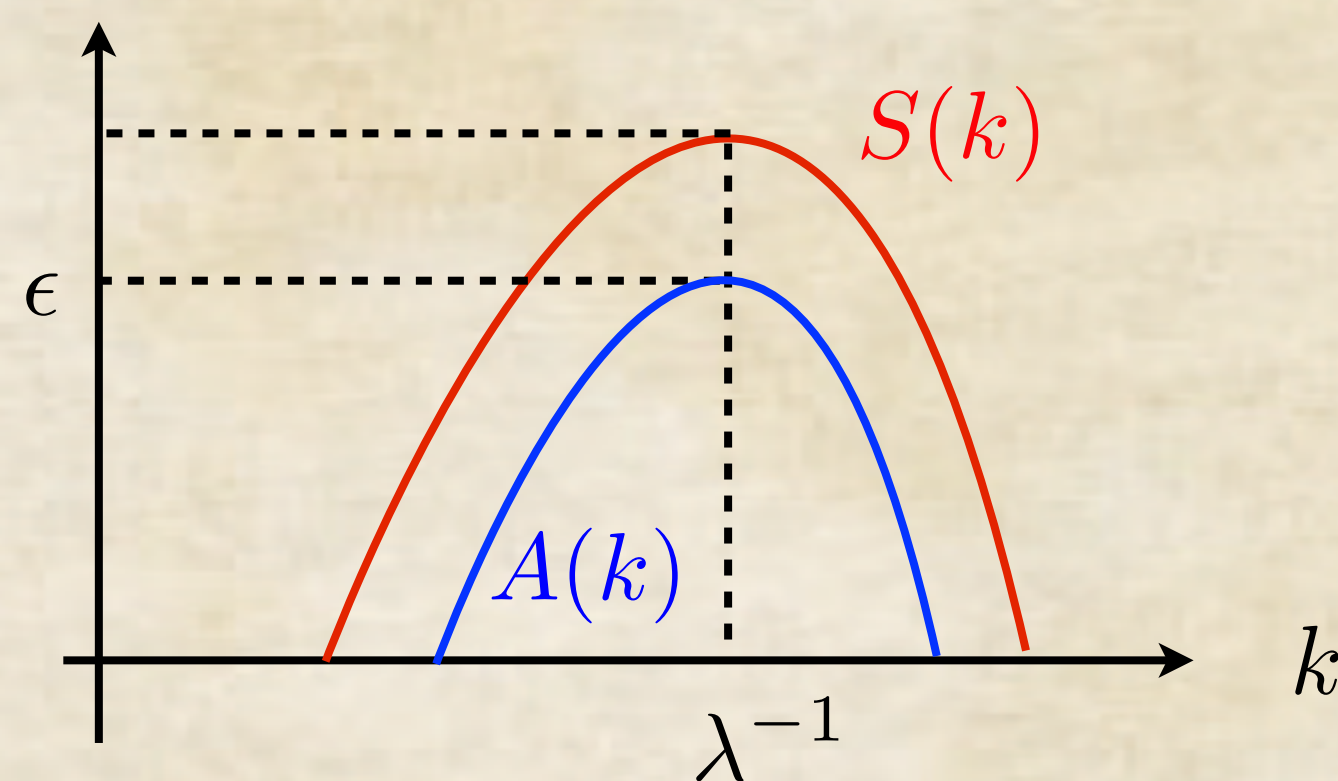
:existence of large-scale magnetic helicity

Sakharov's conditions are satisfied!

From these notat

$$\rho_B = \int \frac{d^3k}{(2\pi)^3}$$

$$\frac{^3 S(k)}{^4 S(k)}$$



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How to realize helicity decay?

1. Decay due to MHD with finite conductivity ('98 Giovannini&Shaposhnikov)

The characteristic MF strength obeys the Maxwell eq. with the MHD approximation.

$$\nabla \times \overline{\mathbf{B}} = \mathbf{J} = \sigma \overline{\mathbf{E}} \quad \Rightarrow \quad \overline{\mathbf{E}} = \frac{1}{\sigma} \nabla \times \overline{\mathbf{B}}$$

$$\partial_t \mathcal{H} = -2 \overline{\mathbf{E}} \cdot \overline{\mathbf{B}} = -\frac{2}{\sigma} \overline{(\nabla \times \mathbf{B}) \cdot \mathbf{B}} \simeq -\frac{2}{\sigma} \epsilon \frac{\overline{B}^2}{\lambda}$$

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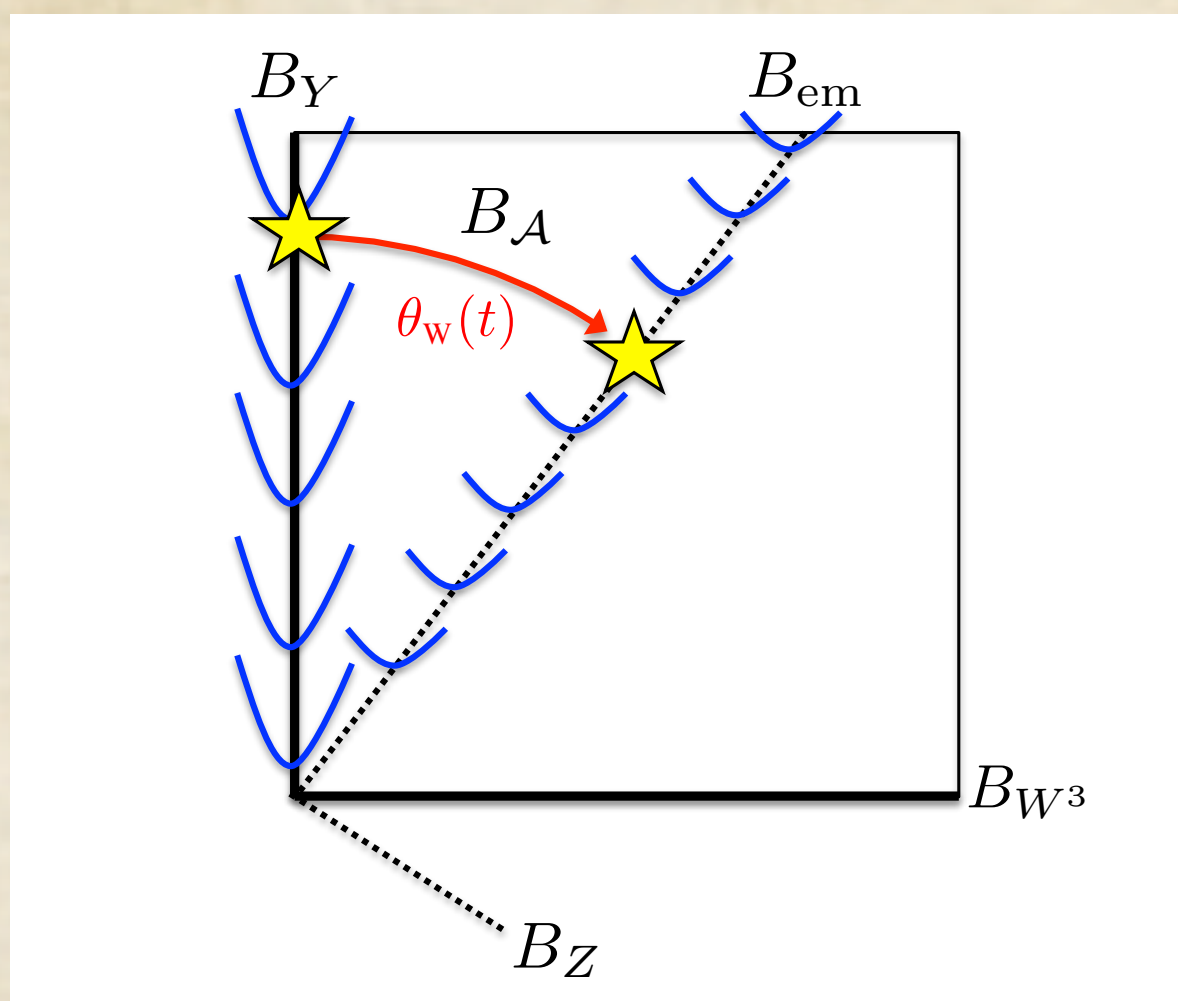
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2. Electroweak symmetry breaking (16 KK&Long)



Gauge group

$$SU(2)_W \times U(1)_Y \rightarrow U(1)_{em}$$

Large-scale (massless) MFs

$$B_Y \rightarrow B_{em} = \cos \theta_W B_Y + \sin \theta_W B_{W^3}$$

Magnetic helicity

$$H_Y^{\text{before}} \rightarrow H_{em}^{\text{after}} = H_Y^{\text{before}}$$

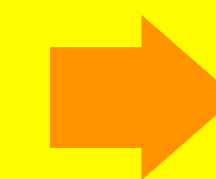
$$H_Y^{\text{after}} = \cos^2 \theta_W H_{em}^{\text{after}} = \cos^2 \theta_W H_Y^{\text{before}}$$

$$N_{CS, W^3}^{\text{after}} \sim \sin^2 \theta_W H_{em}^{\text{after}} = \sin^2 \theta_W H_Y^{\text{before}}$$

BAU:

$$\Delta H_Y = -\sin^2 \theta_W H_Y^{\text{before}}$$

$$\Delta N_{CS} \sim \sin^2 \theta_W H_Y^{\text{before}}$$



$$\Delta Q_B = \# \Delta N_{CS} - \# \Delta H_Y \sim \sin^2 \theta_W H_Y^{\text{before}}$$

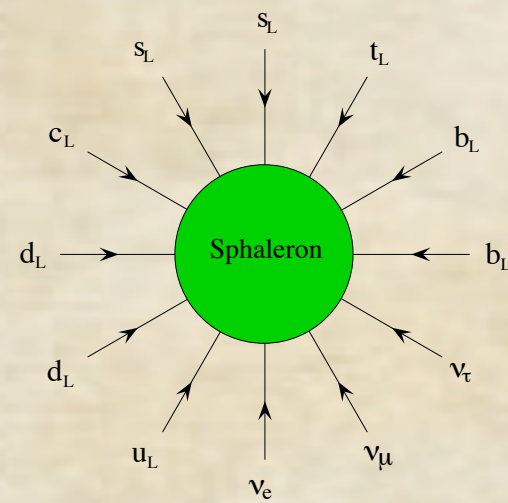
To evaluate the baryon asymmetry from the hypermagnetic helicity decay, we need to evaluate the washout effect.

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EW sphalerons+chirality flip by electron Yukawa



W. Buchmüller, 1212.3554

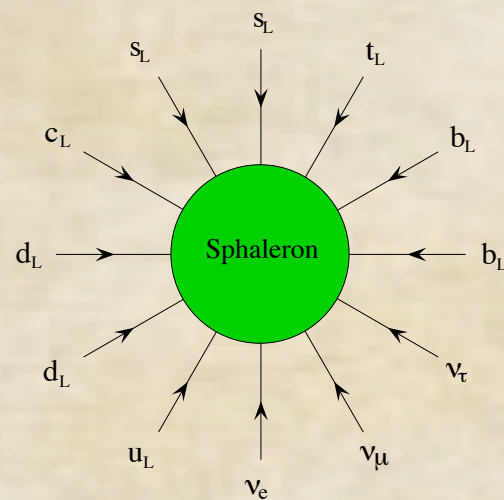
Thermal fluctuations of W-boson induces ΔN_{CS}

Together with Yukawa int., try to washout Q_B

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Chiral Magnetic Effect

(’80 Vilenkin, ’08 Fukushima, Kharzeev, & Warringa)

Ampere’s law

$$\nabla \times \mathbf{B}_Y = \mathbf{J} = \underbrace{\sigma(\mathbf{E}_Y + \mathbf{v} \times \mathbf{B}_Y)}_{\text{Ohm's current}} + \underbrace{\frac{2\alpha_Y}{\pi} \mu_5 \mathbf{B}_Y}_{\text{Chiral magnetic current}}$$

$$\Rightarrow \mathbf{E}_Y = -\mathbf{v} \times \mathbf{B}_Y + \frac{1}{\sigma} \left(\nabla \times \mathbf{B}_Y - \frac{2\alpha_Y}{\pi} \mu_5 \mathbf{B}_Y \right)$$

$$\begin{aligned} \frac{d}{dt} n_f &\ni \# \langle Y_{\mu\nu} \tilde{Y}^{\mu\nu} \rangle (= -4 \langle \mathbf{E}_Y \cdot \mathbf{B}_Y \rangle) \\ &= \# \frac{1}{\sigma} \left(\langle \mathbf{B}_Y \cdot (\nabla \times \mathbf{B}_Y) \rangle - \frac{2\alpha_Y}{\pi} \mu_5 |\mathbf{B}_Y|^2 \right) \\ &= \# \frac{1}{\sigma} \left(\frac{B_p^2}{\lambda_B} - \frac{2\alpha_Y}{\pi} \mu_5 B_p^2 \right) \end{aligned}$$

$$\mu_5 = \sum_{f'} (-)^{q_{R/L}} 6y_{f'}^2 n_{f'} / T^2$$

Schematically evolution equation is given by

$$\frac{dn_B}{dt} = \left(\underbrace{\# \frac{B^2}{\sigma \lambda}}_{\text{MHD decay}} + \underbrace{\# \dot{\theta}_W \lambda B^2}_{\text{EWSB}} \right) - \underbrace{\Gamma_{\text{w.o.}} n_B}_{\text{washout term}}$$

EW sphaleron
chirality-flip
CME

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Source term
washout term

MHD decay
EWSB
EW sphaleron
chirality-flip
CME

Reaches at “terminal” asymmetry...

$$n_B \simeq \frac{\# B^2 / \sigma \lambda + \# \dot{\theta}_W \lambda B^2}{\Gamma_{\text{w.o.}}}$$

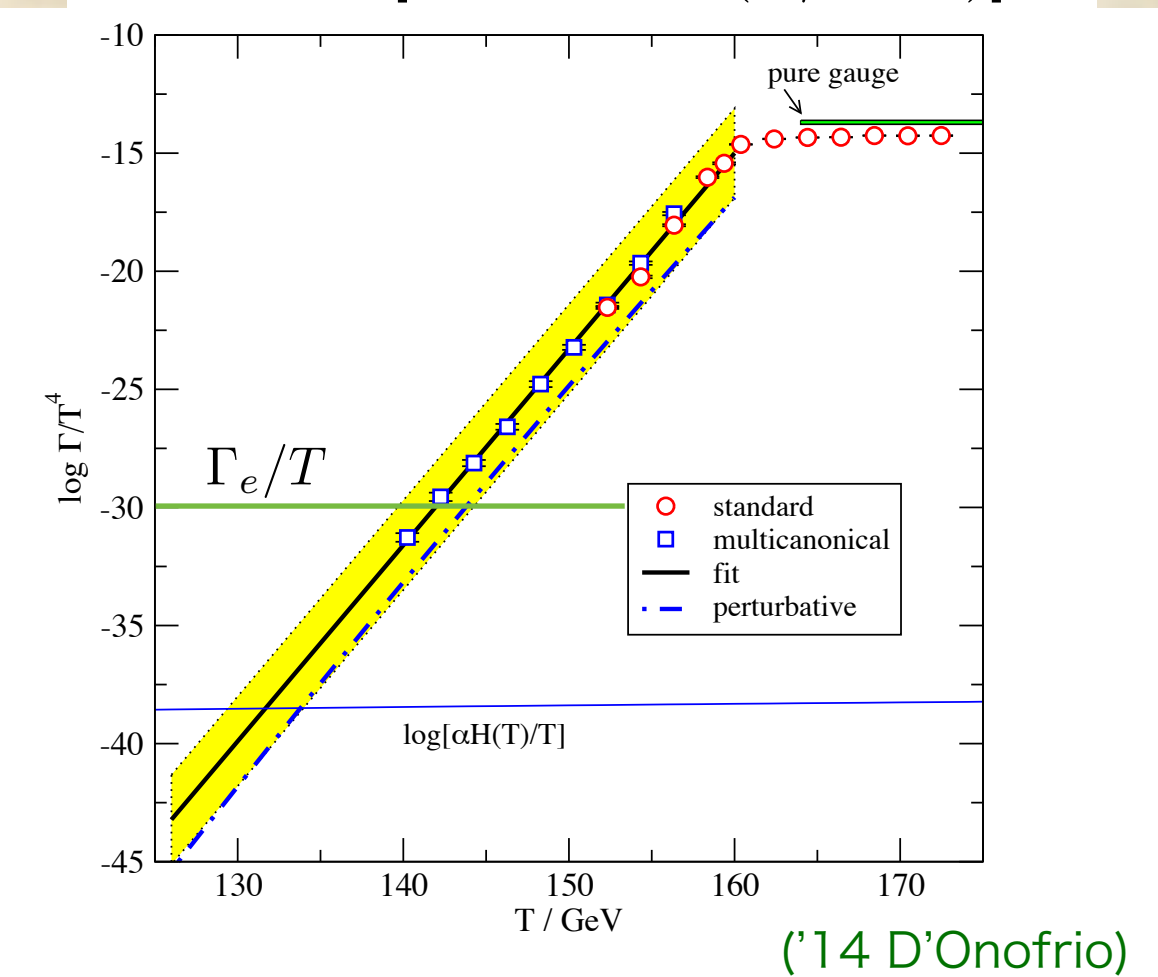
Washout term $\Gamma_{\text{w.o.}}$

High temperature ($T > 140$ GeV): electron Yukawa or CME

Low temperature ($T < 140$ GeV): EW sphaleron

Sphaleron rate

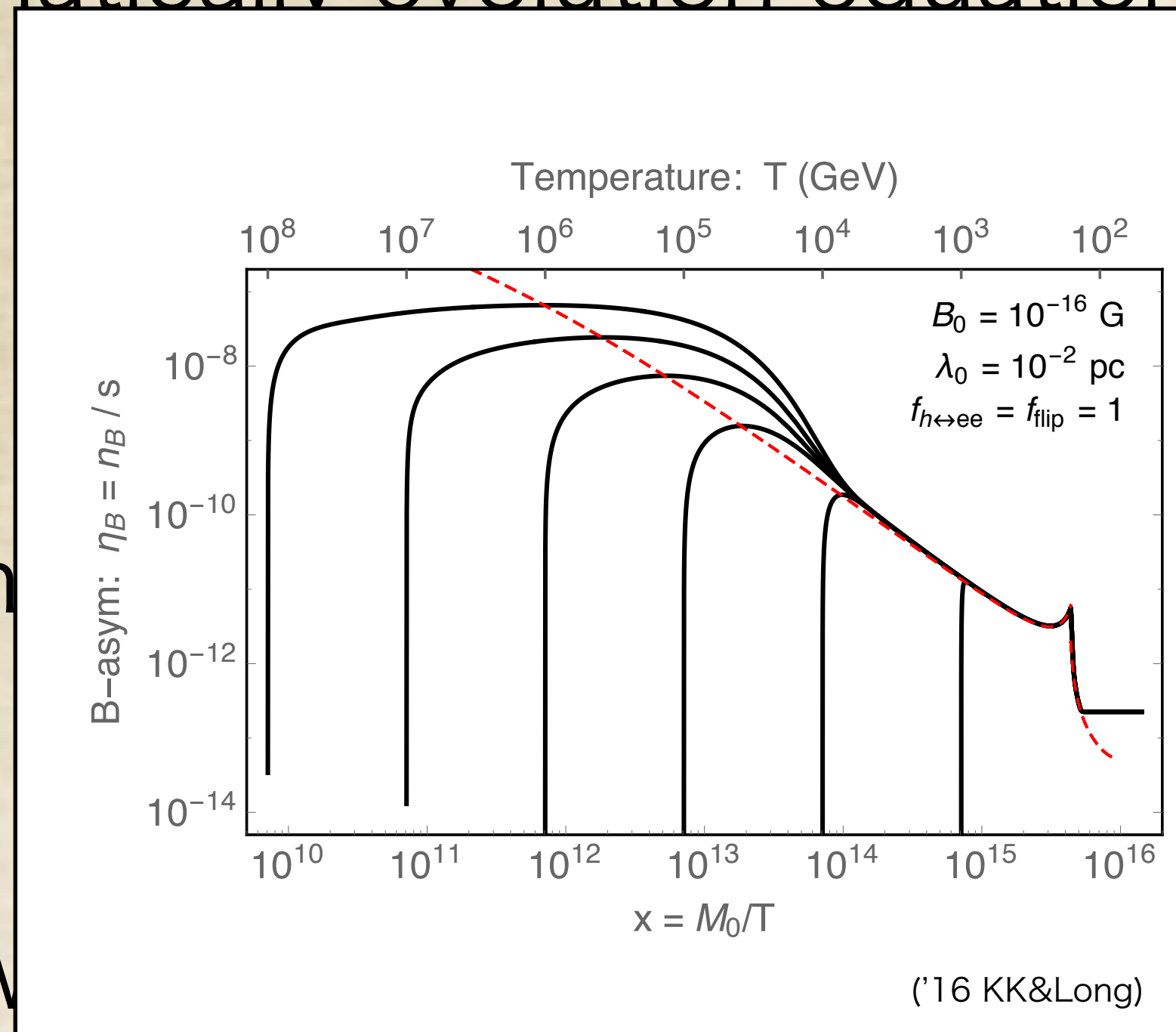
$$\Gamma_W \simeq \exp[-145 + 0.8(T/\text{GeV})]T$$



Schematically evolution equation is given by

Reach

W



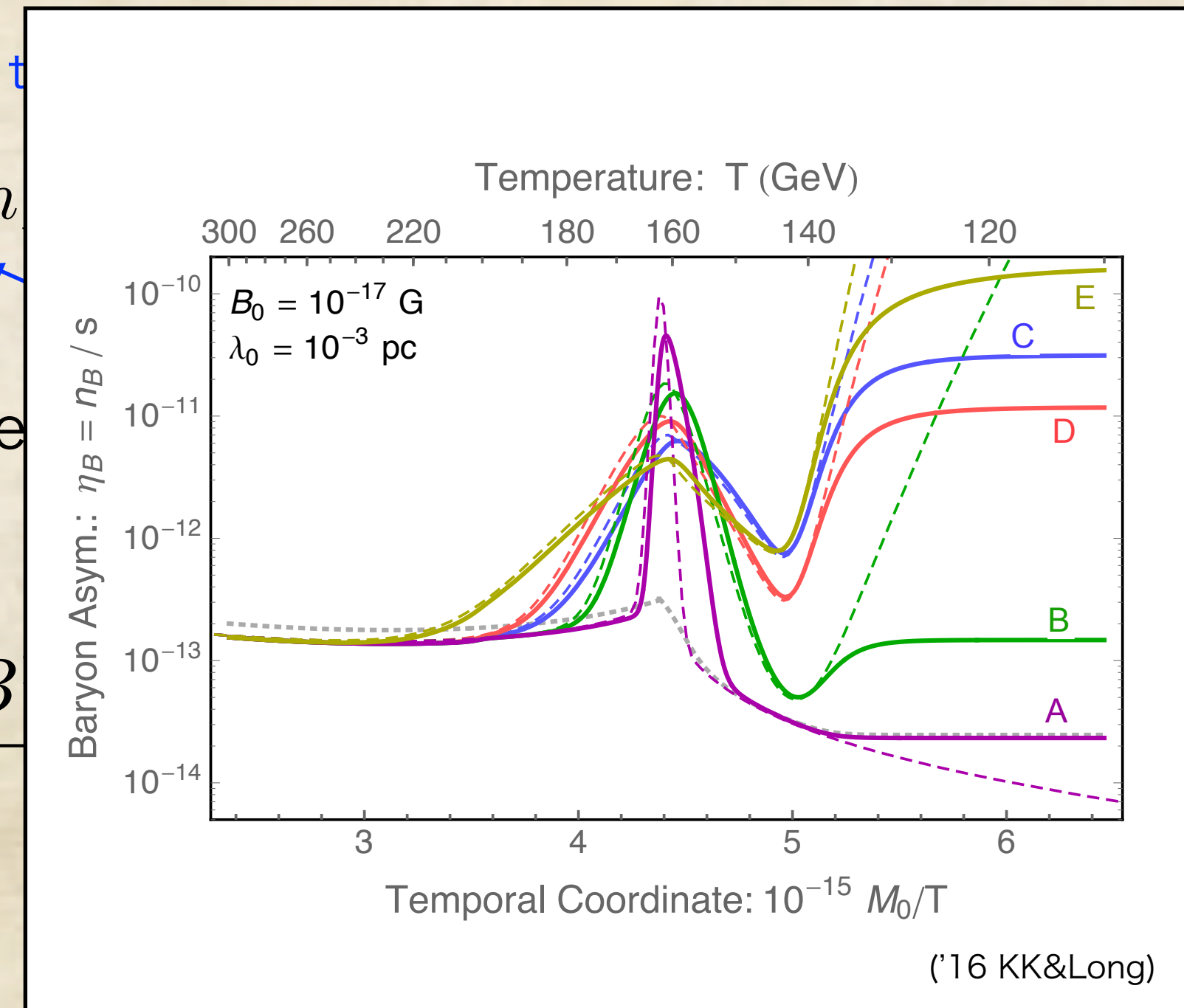
term washout t

$$\left(\dot{\theta}_W \lambda B^2 \right) - \Gamma_{w.o.} n$$

 EWSB EW sphaleron

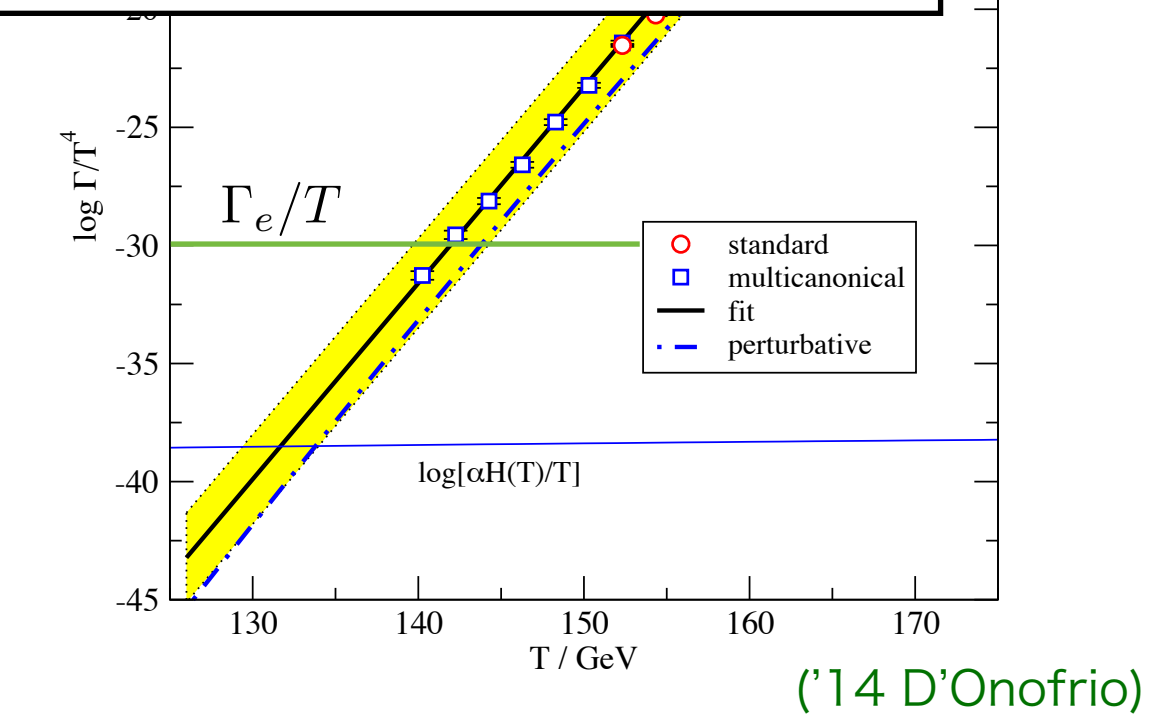
$$B^2 / \sigma \lambda + \dot{\theta}_W \lambda B$$

$$\Gamma_{w.o.}$$



High temperature ($T > 140$ GeV): electron Yukawa or CME

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Subtle issue behind that...

Early EWSB (crossover) completion, late sphaleron freeze out

=> Net BAU is suppressed ('98 Giovannini&Shaposhnikov)

Early sphaleron freeze out, late EWSB (crossover) completion

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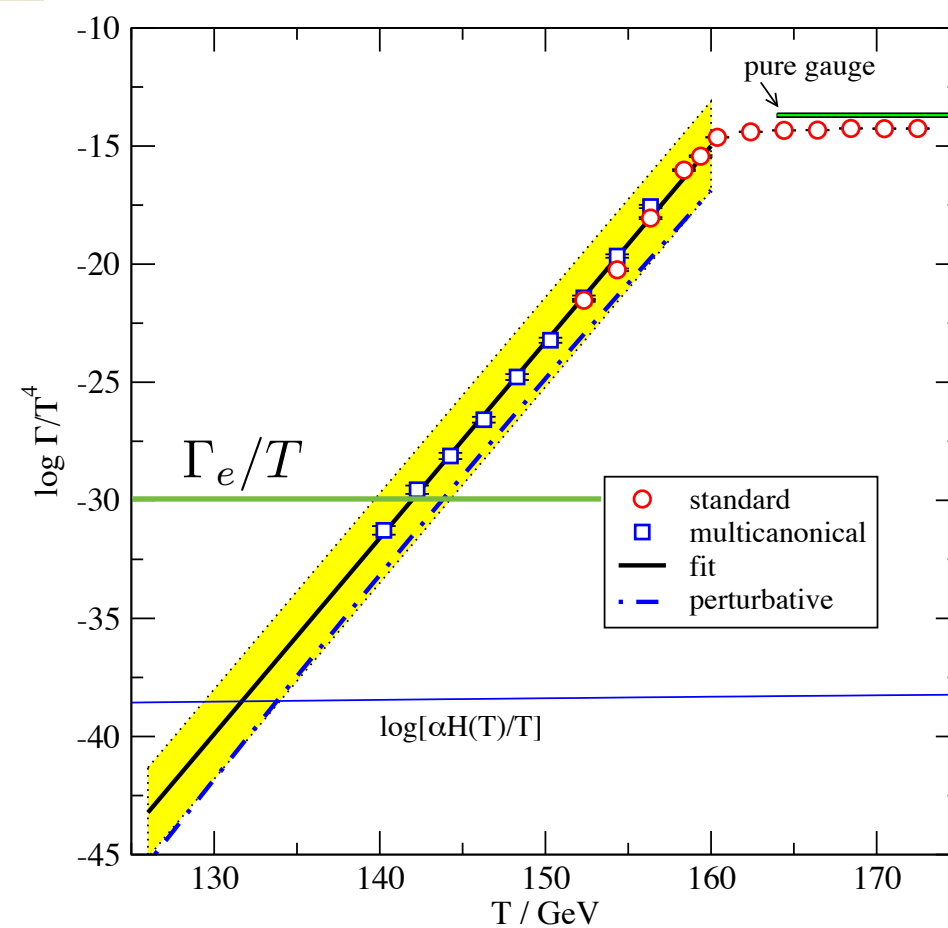
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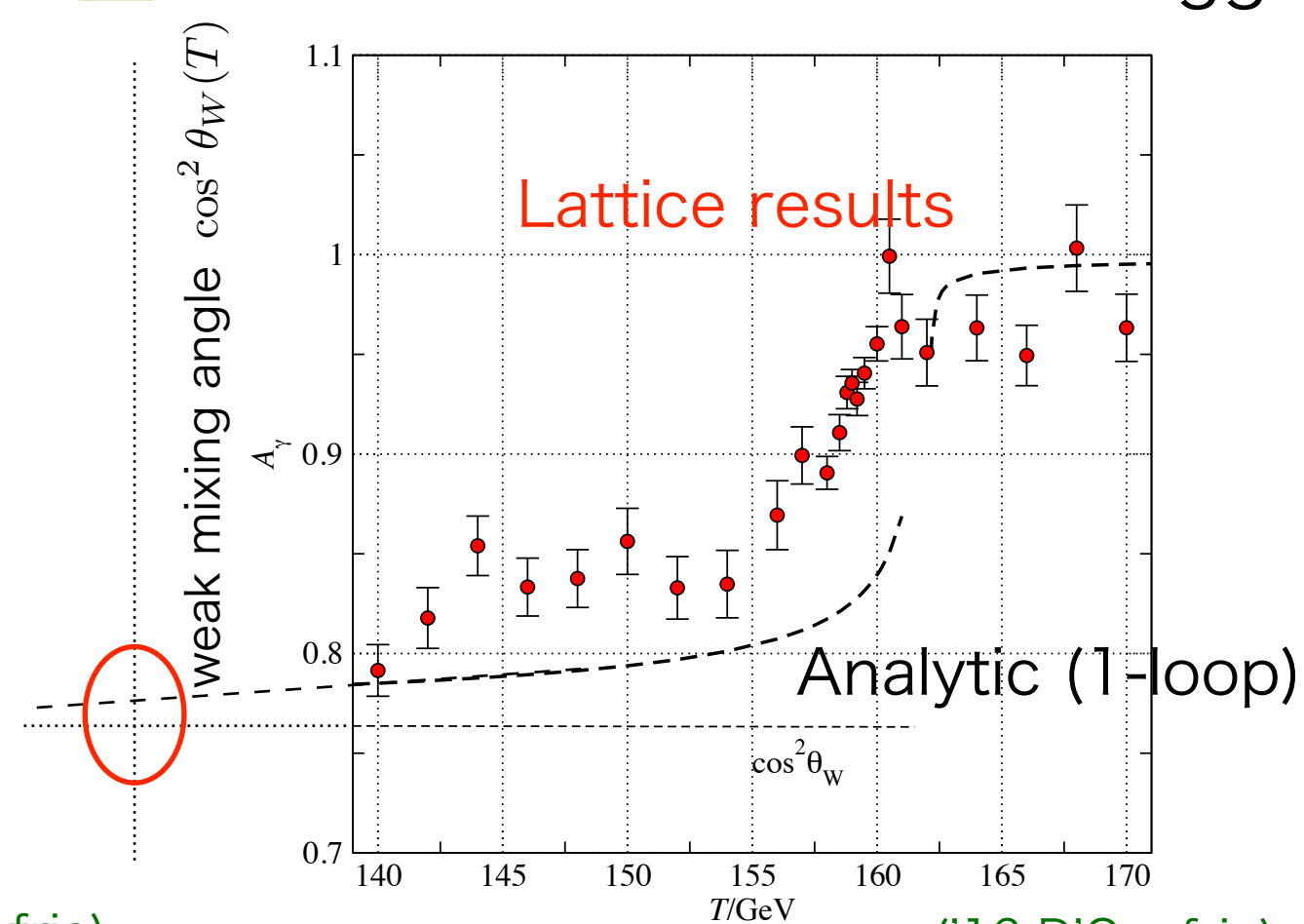
Sphaleron rate

$$\Gamma_W \simeq \exp[-145 + 0.8(T/\text{GeV})]T$$



('14 D'Onofrio)

lattice simulations for the EW crossover with 125 GeV Higgs



('16 D'Onofrio)

BAU is very likely to remain!

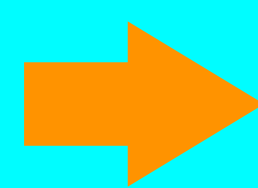
Quantitative results are sensitive to $\theta_W(t)$

Finally analytic formula for the average baryon asymmetry is given.

$$\bar{\eta}_B \simeq 10^{-10} \epsilon f(T, \theta_w) \left(\frac{\lambda}{10^6 \text{GeV}^{-1}} \right) \left(\frac{\bar{B}}{10^{-3} \text{GeV}^2} \right)^2 \Big|_{T=135 \text{GeV}}$$

$$f(T, \theta_w) \equiv -\sin 2\theta_w T \frac{d\theta_w}{dT} (\simeq 0.1) \quad \text{at} \quad T \simeq 135 \text{GeV}$$

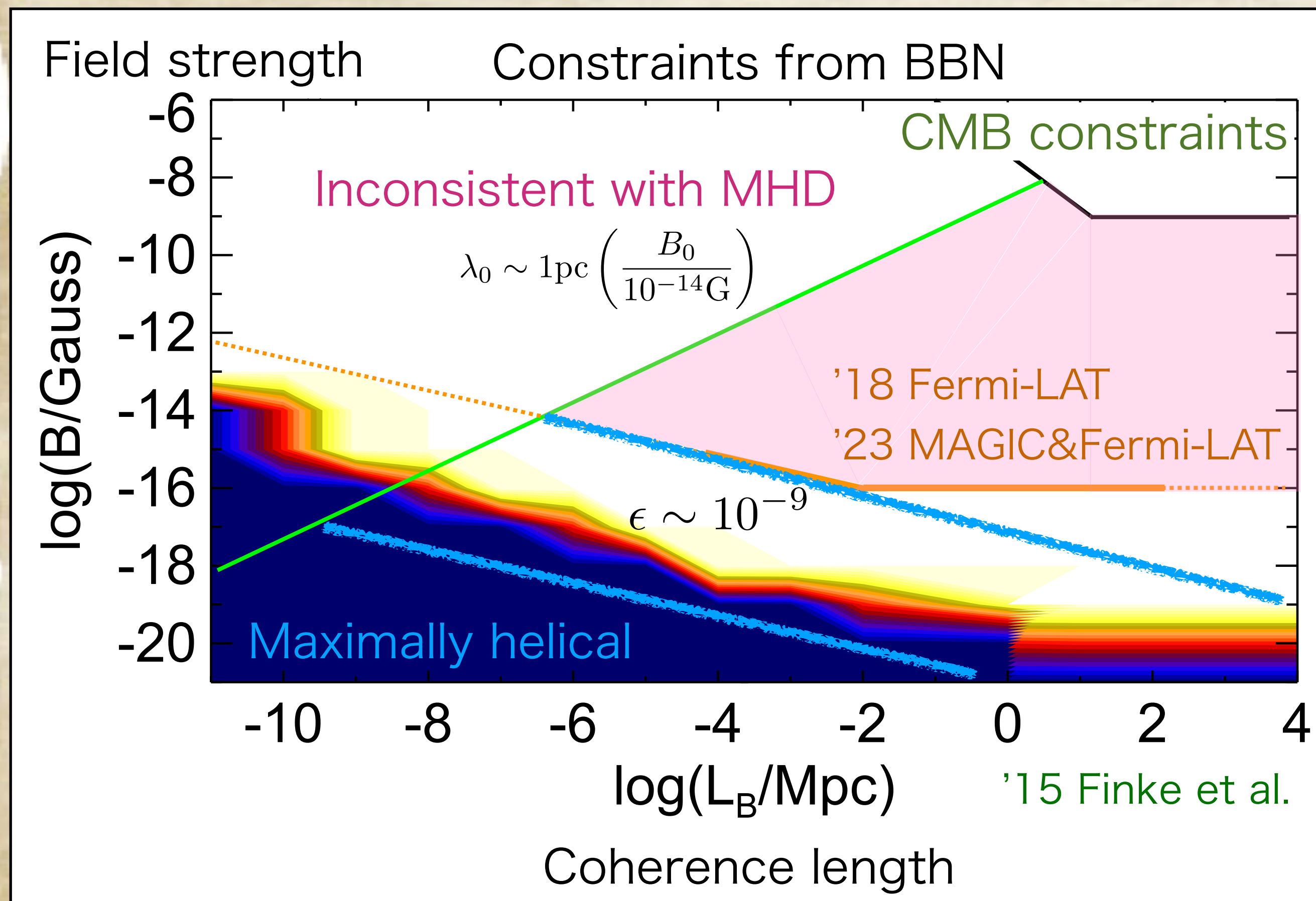
Magnetogenesis with positive helicity before EWSB.



With appropriate properties of hyper MFs, present BAU can be explained.

※ Since helicity is just the difference between the right and left helicity modes, the sign of helicity can be the same beyond the coherence length of MFs.

IGMFs that can explain the BAU '16 KK & Long



IGMFs with tiny helicity ratio $\epsilon < 10^{-9}$ seems to explain the BAU.

IGMFs suggested by blazars with larger helicity should have generated after EWSB otherwise BAU is generated too much.

Baryon isocurvature constraints

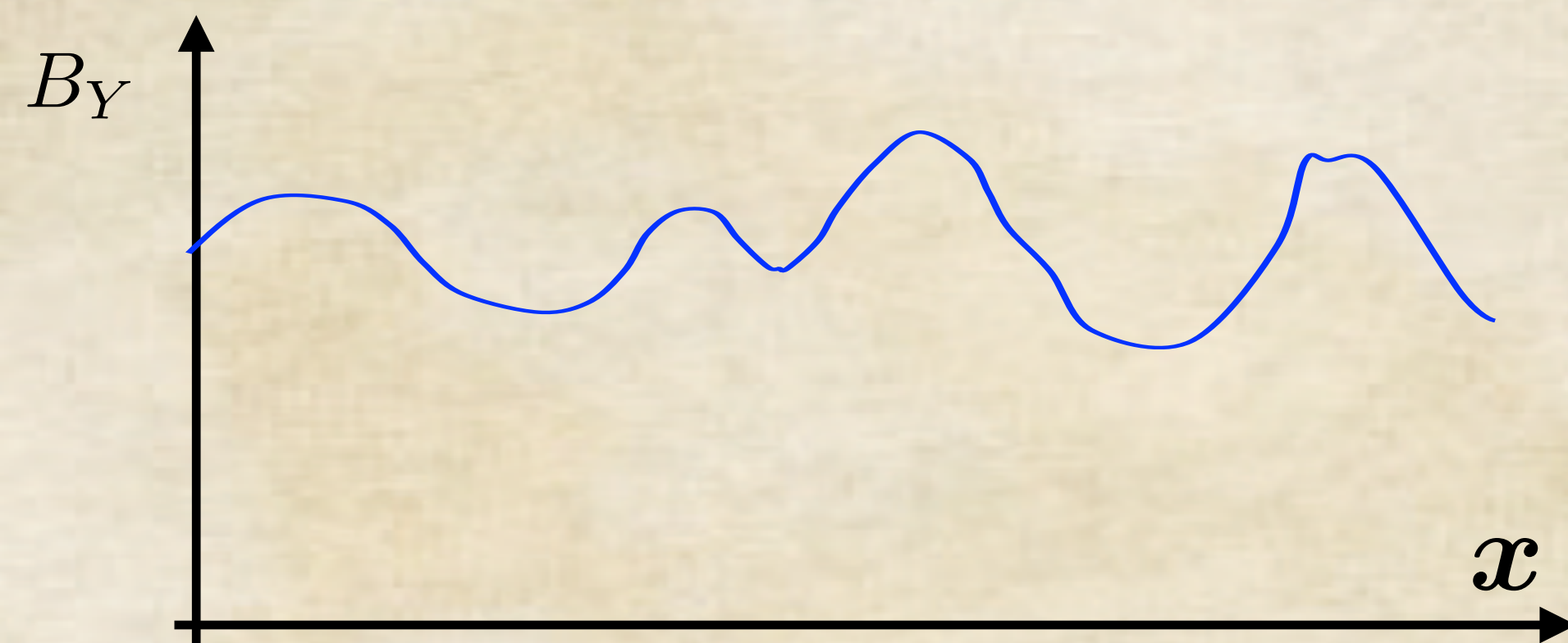
KK, F. Uchida, J. Yokoyama (Tokyo), JCAP 04 (2021) 034 [arXiv: 2012.14435 (astro-ph.CO)]



Basic idea

Baryon asymmetry evaluated thus far is the spatially-averaged one

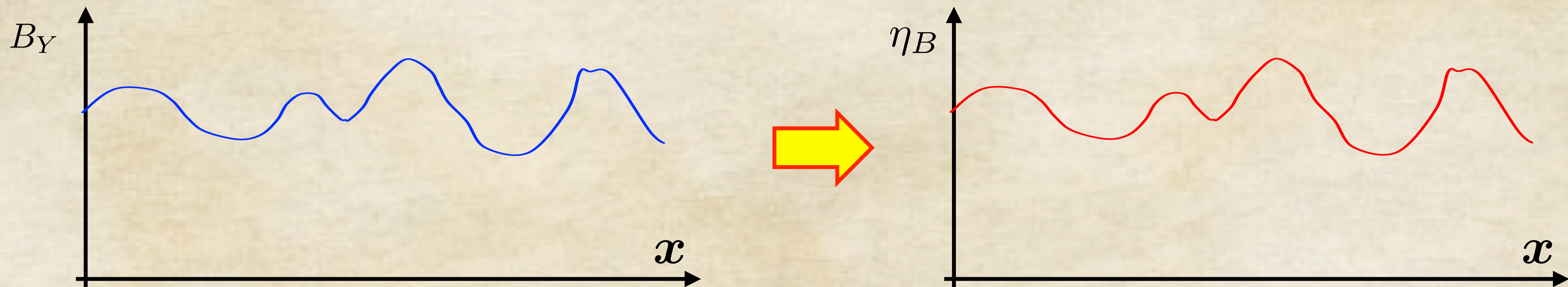
=> We expect that it has spatial dependence (“baryon isocurvature perturbation”) according to the spatial distributions of hypermagnetic fields.



Basic idea

Baryon asymmetry evaluated thus far is the spatially-averaged one

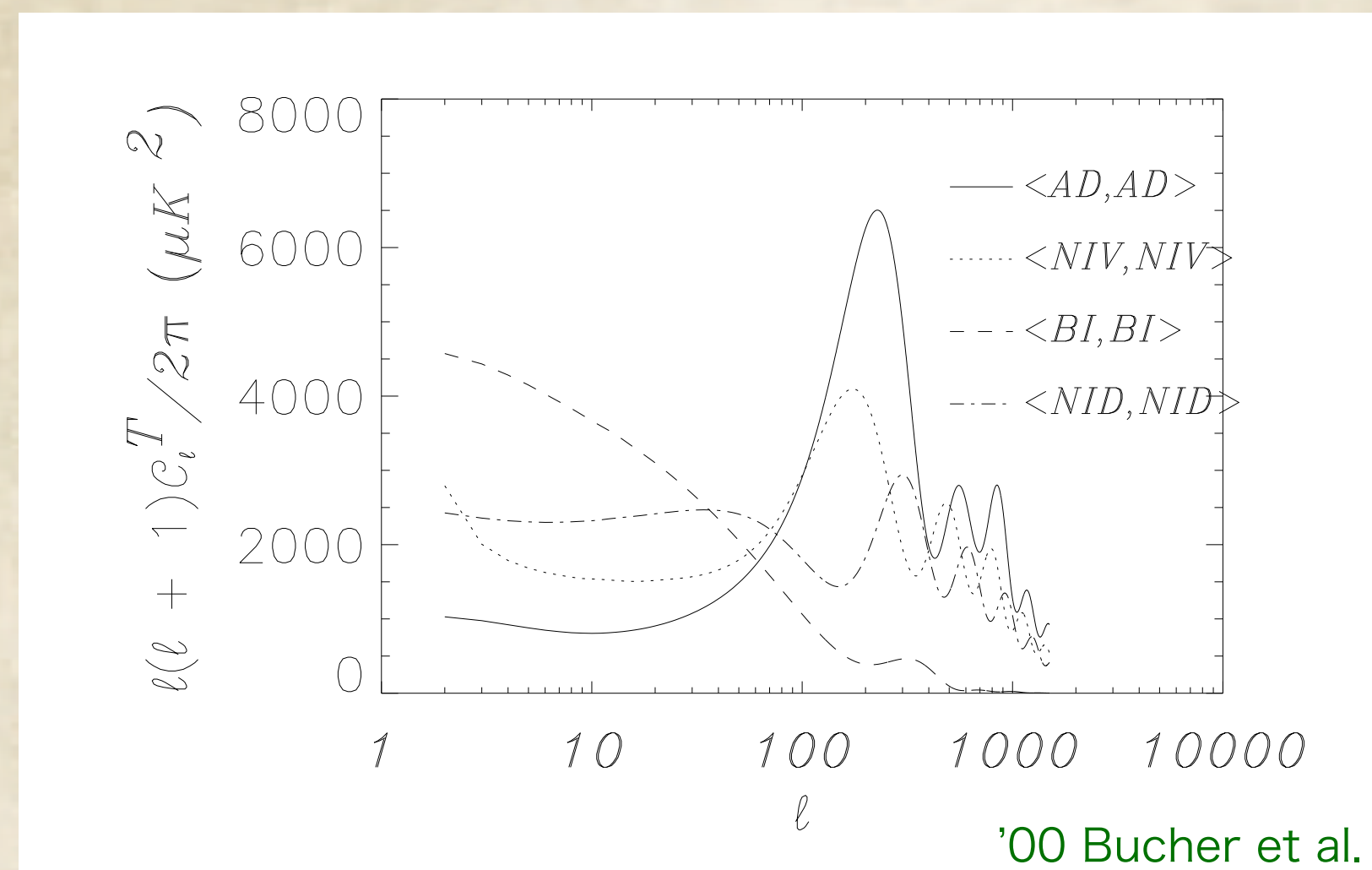
=> We expect that it has spatial dependence (“baryon isocurvature perturbation”) according to the spatial distributions of hypermagnetic fields.



constrained by observations?

Observational constraints on the baryon isocurvature perturbations

Mpc scales: CMB gives constraints.

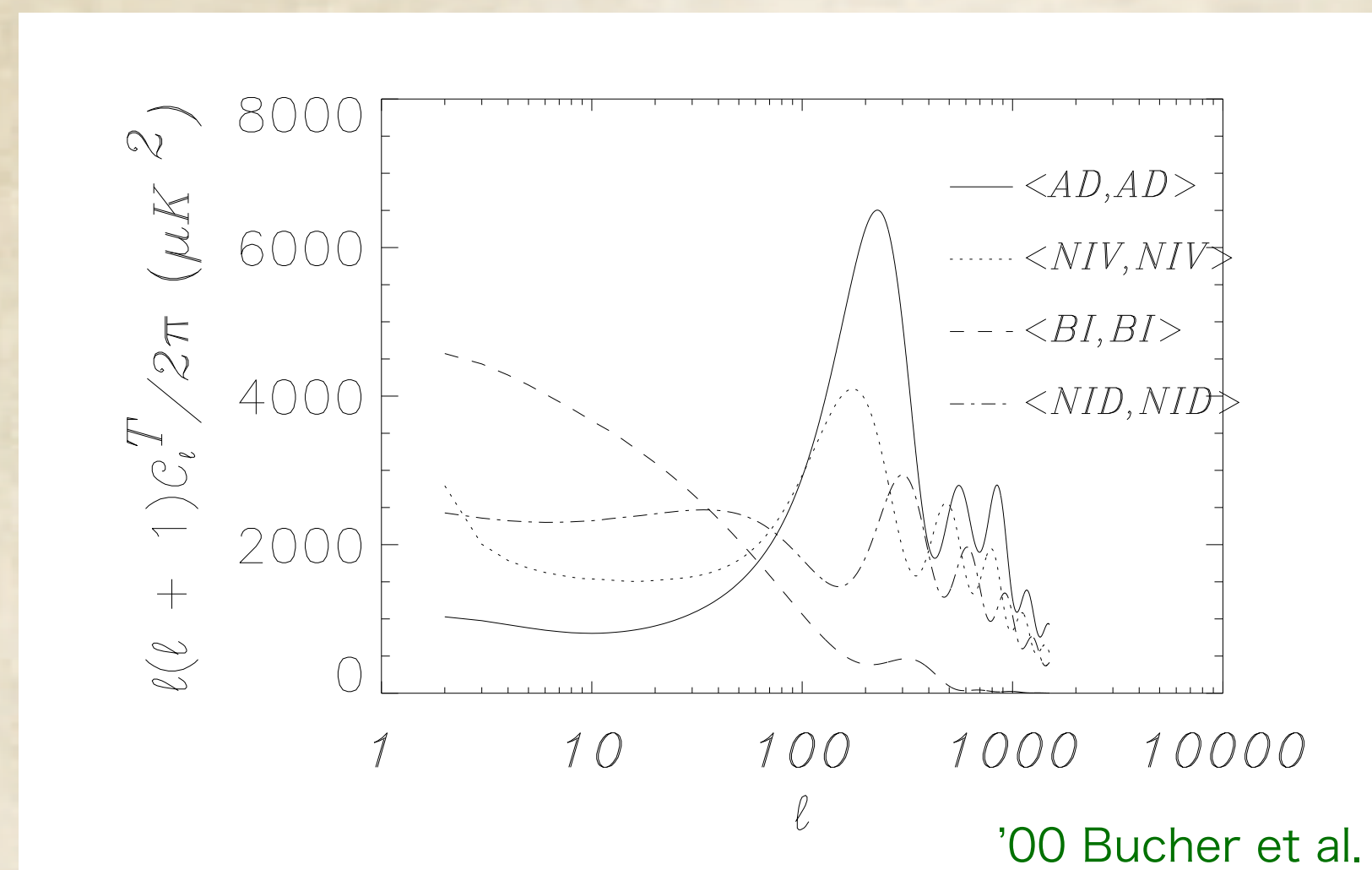


$$\beta_{\text{iso}} \equiv \frac{\mathcal{P}_{II}}{\mathcal{P}_{RR} + \mathcal{P}_{II}} \lesssim 0.49 \quad @k = 0.1 \text{Mpc}^{-1}$$

'18 Planck

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'18 Planck

Much smaller scales: Inhomogeneous BBN

'87 Applegate+, Alock+

Baryon fluctuation with the scale larger than the neutron diffusion scale remains at BBN epoch and changes the prediction of light elements.

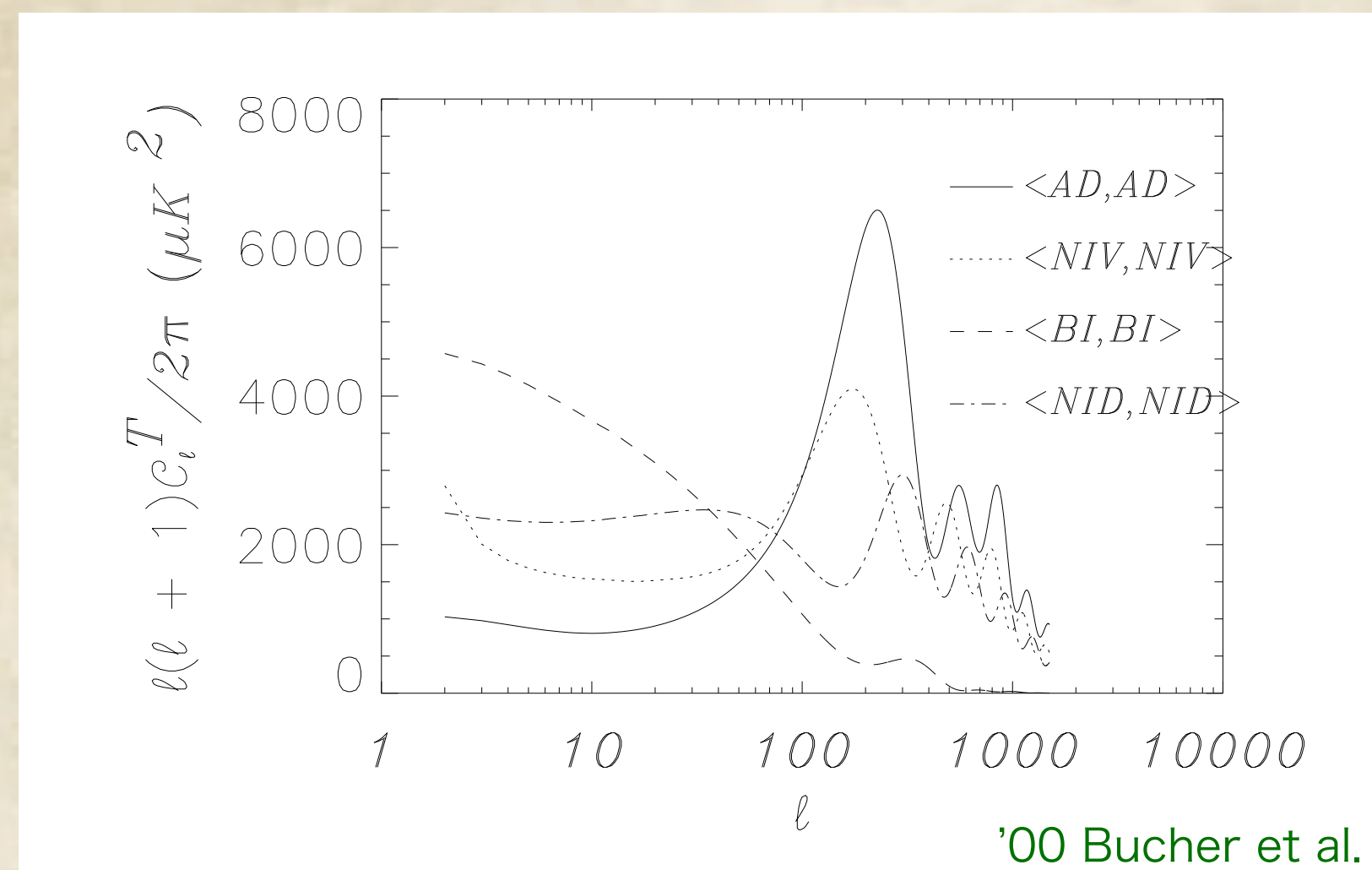
'08 Pisanti+, '15 Planck

$$10^5 (D/H)_p = 18.754 - 1534.4 \omega_B + 48656 \omega_B^2 - 552670 \omega_B^3, \quad \omega_B = \Omega_B h^2$$

$$\bar{\omega}_B^2 + \langle \delta\omega_B^2 \rangle$$

Observational constraints on the baryon isocurvature perturbations

Mpc scales: CMB gives constraints.

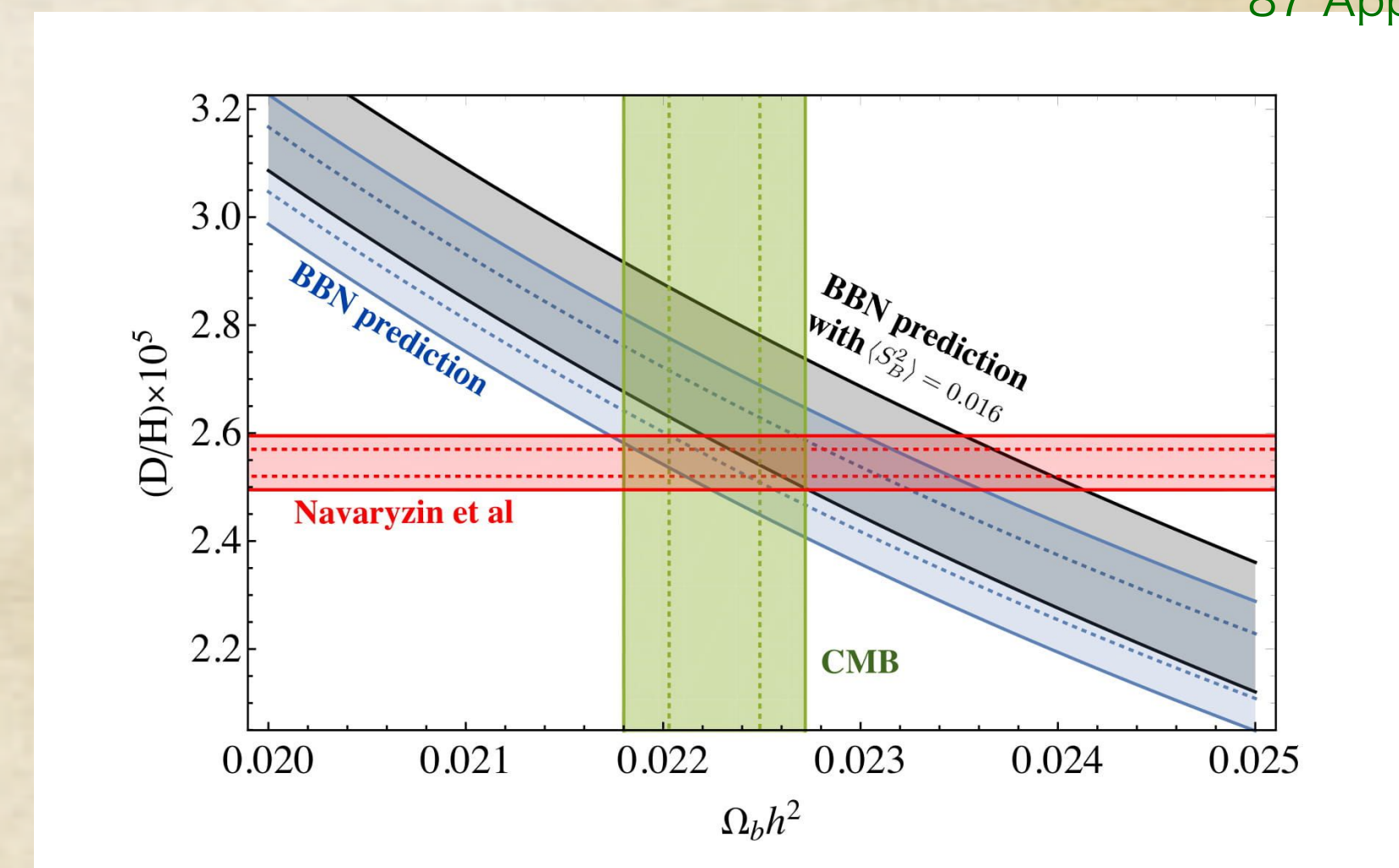


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'18 Planck

Much smaller scales: Inhomogeneous BBN

'87 Applegate+, Alock+



$$\langle S_{B,\text{BBN}}^2 \rangle = \frac{\langle \delta\eta_{B,\text{BBN}}^2(\mathbf{x}) \rangle}{\bar{\eta}_B^2} = \frac{1}{V} \int \frac{d^3k}{(2\pi)^3} \mathcal{P}_{\delta B}^{\text{BBN}} < 0.016$$

'18 Inomata +

Baryon isocurvature perturbations from hypermagnetic fields at EWSB

$$\eta_{B,EW}(\mathbf{x}) = \mathcal{C}\mathbf{Y}(\mathbf{x}) \cdot \mathbf{B}_Y(\mathbf{x}) (= \mathcal{C}\mathcal{H}_Y(\mathbf{x})) \quad \Rightarrow \quad \langle \delta\eta_{B,EW}(\mathbf{x})\delta\eta_{B,EW}(\mathbf{x} + \mathbf{r}) \rangle = \mathcal{C}^2 \langle \mathbf{Y}(\mathbf{x}) \cdot \mathbf{B}_Y(\mathbf{x}) \mathbf{Y}(\mathbf{x} + \mathbf{r}) \cdot \mathbf{B}_Y(\mathbf{x} + \mathbf{r}) \rangle - \overline{\eta_{B,EW}}^2$$

Fourier transform

$$\mathcal{G}(\mathbf{k}) = \frac{\mathcal{C}^2}{\overline{\eta_B}^2} \int \frac{d^3p}{(2\pi)^3} [p^2 S(|\mathbf{k} - \mathbf{p}|)S(p) + |\mathbf{k} - \mathbf{p}|p A(|\mathbf{k} - \mathbf{p}|)A(p)] \times \left[1 - \frac{2(\mathbf{k} - \mathbf{p}) \cdot \mathbf{p}}{p^2} + \frac{((\mathbf{k} - \mathbf{p}) \cdot \mathbf{p})^2}{|\mathbf{k} - \mathbf{p}|^2 p^2} \right].$$

Two-point function has nonzero value even for non-helical magnetic fields!

'98 Giovannini & Shaposhnikov

Baryon isocurvature perturbations from hypermagnetic fields at EWSB

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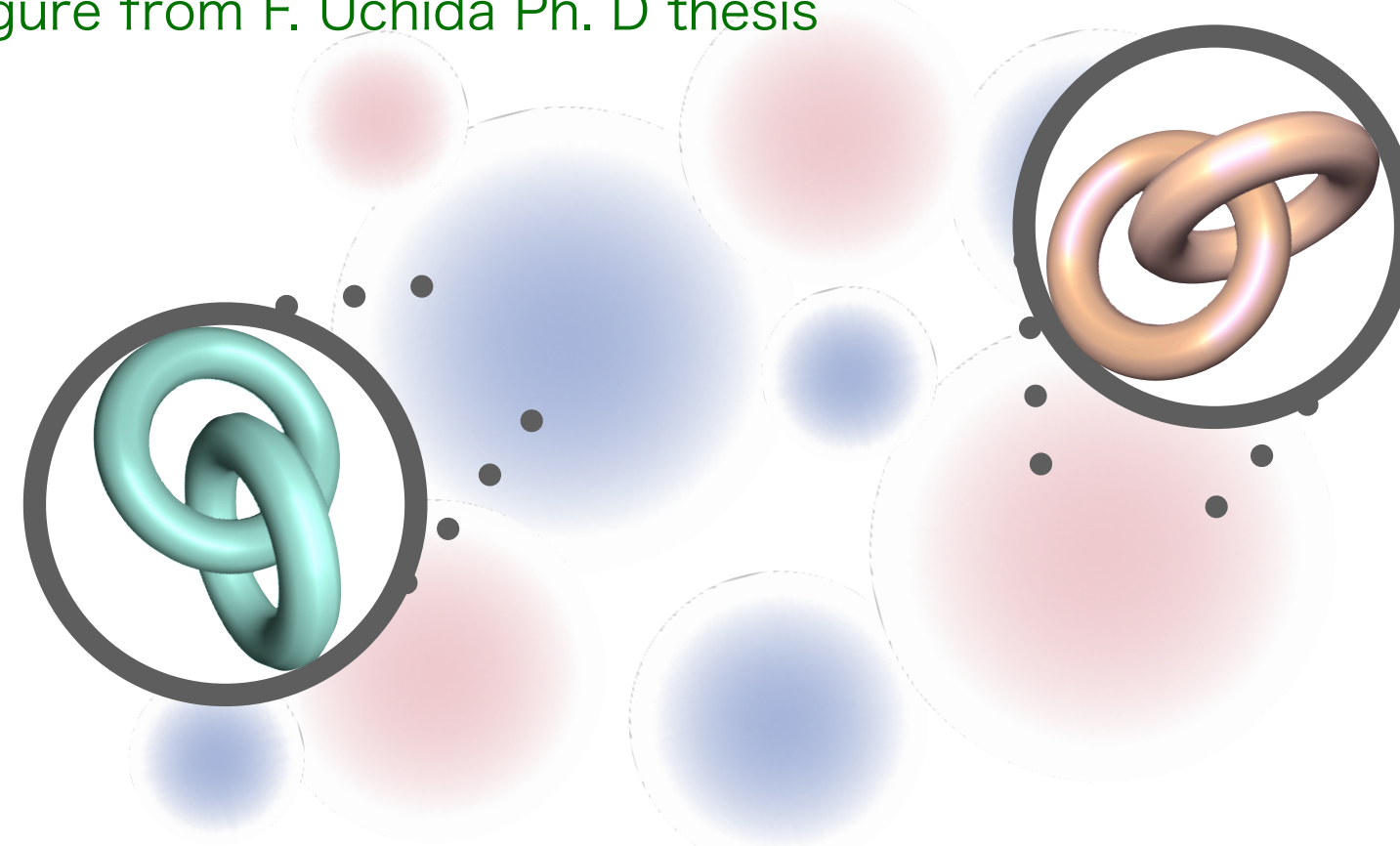
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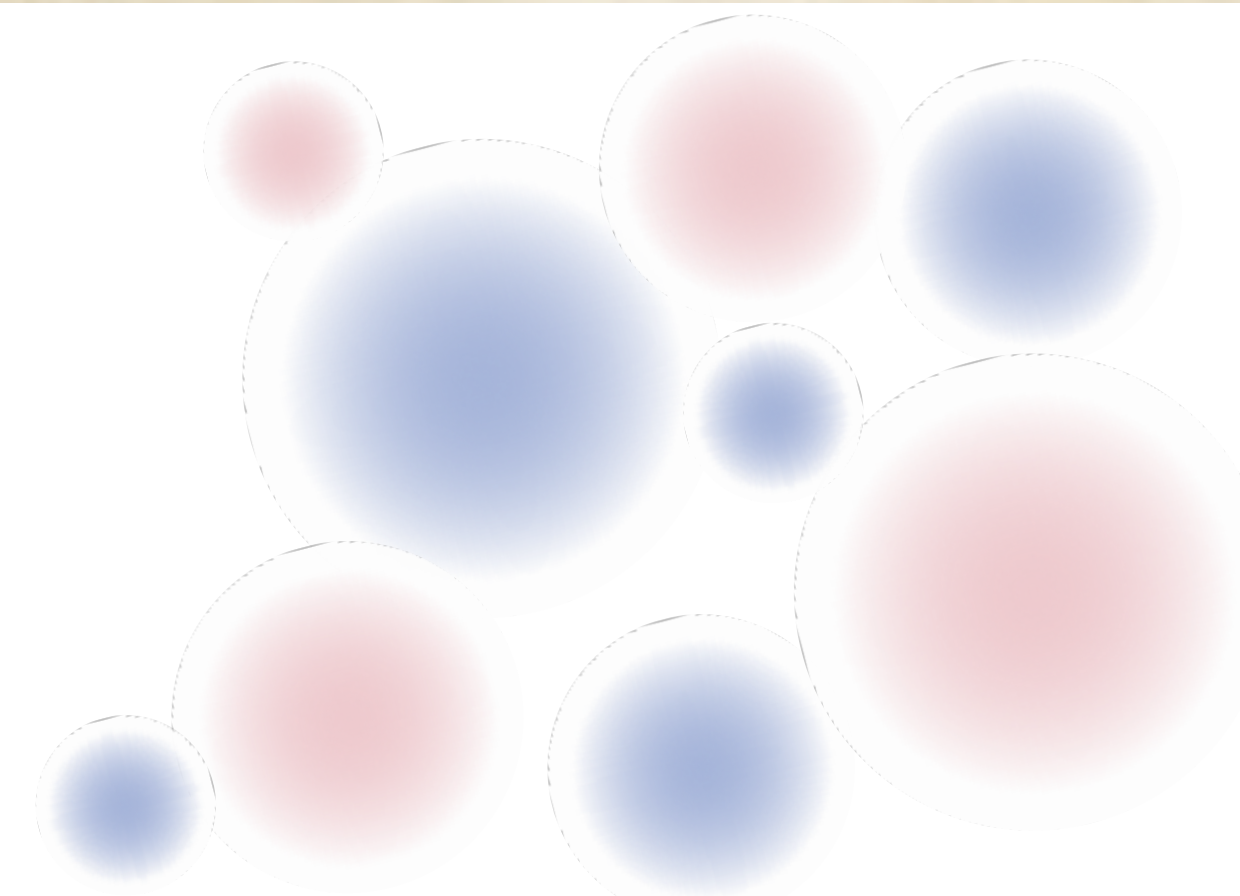
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Figure from F. Uchida Ph. D thesis



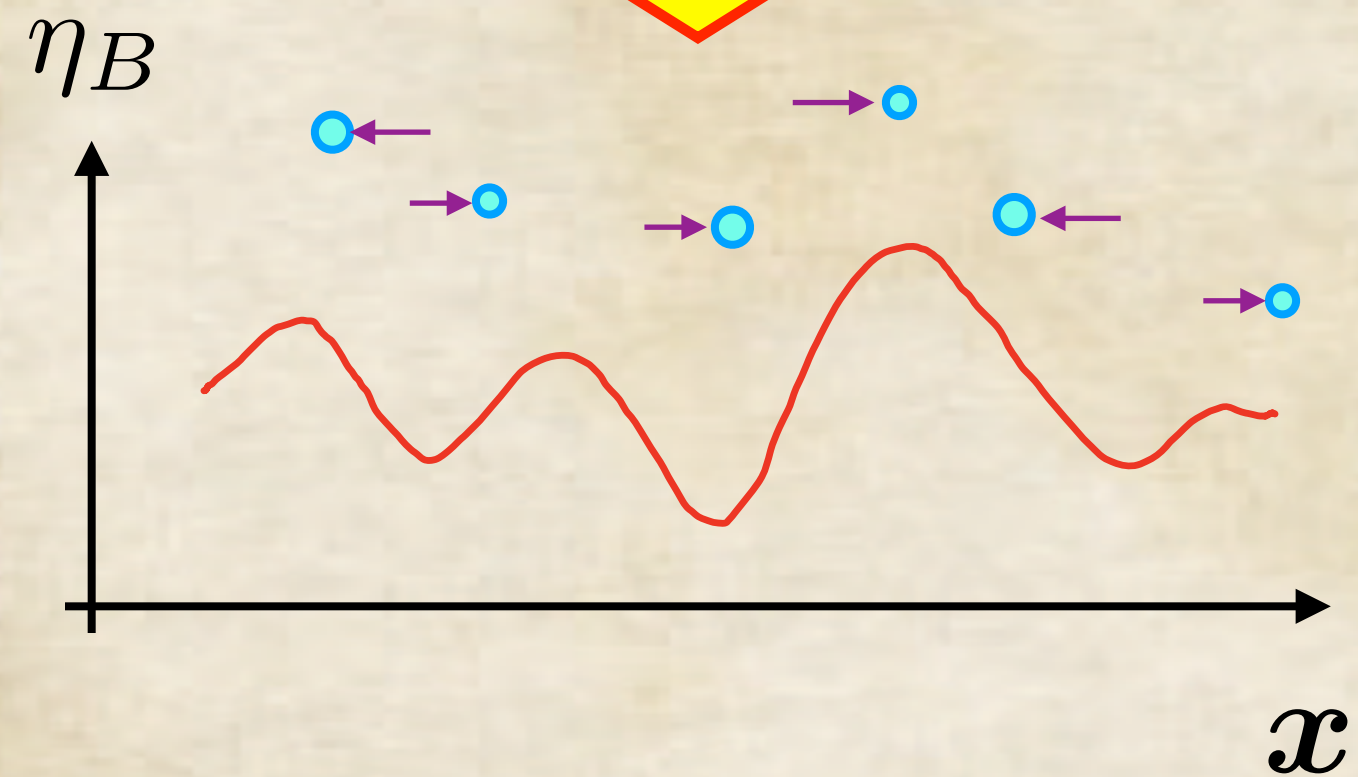
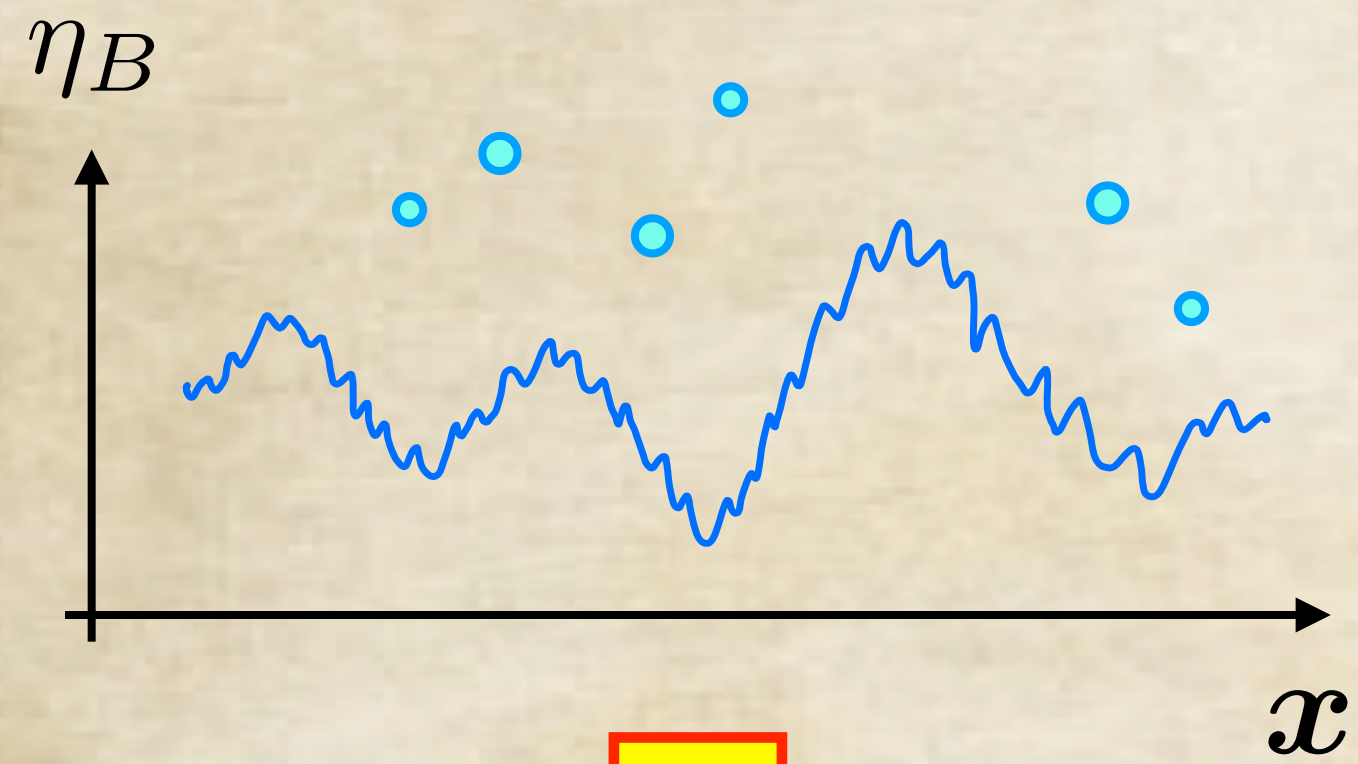
magnetic helicity density



baryon number density at EWSB

Baryon isocurvature perturbations at BBN

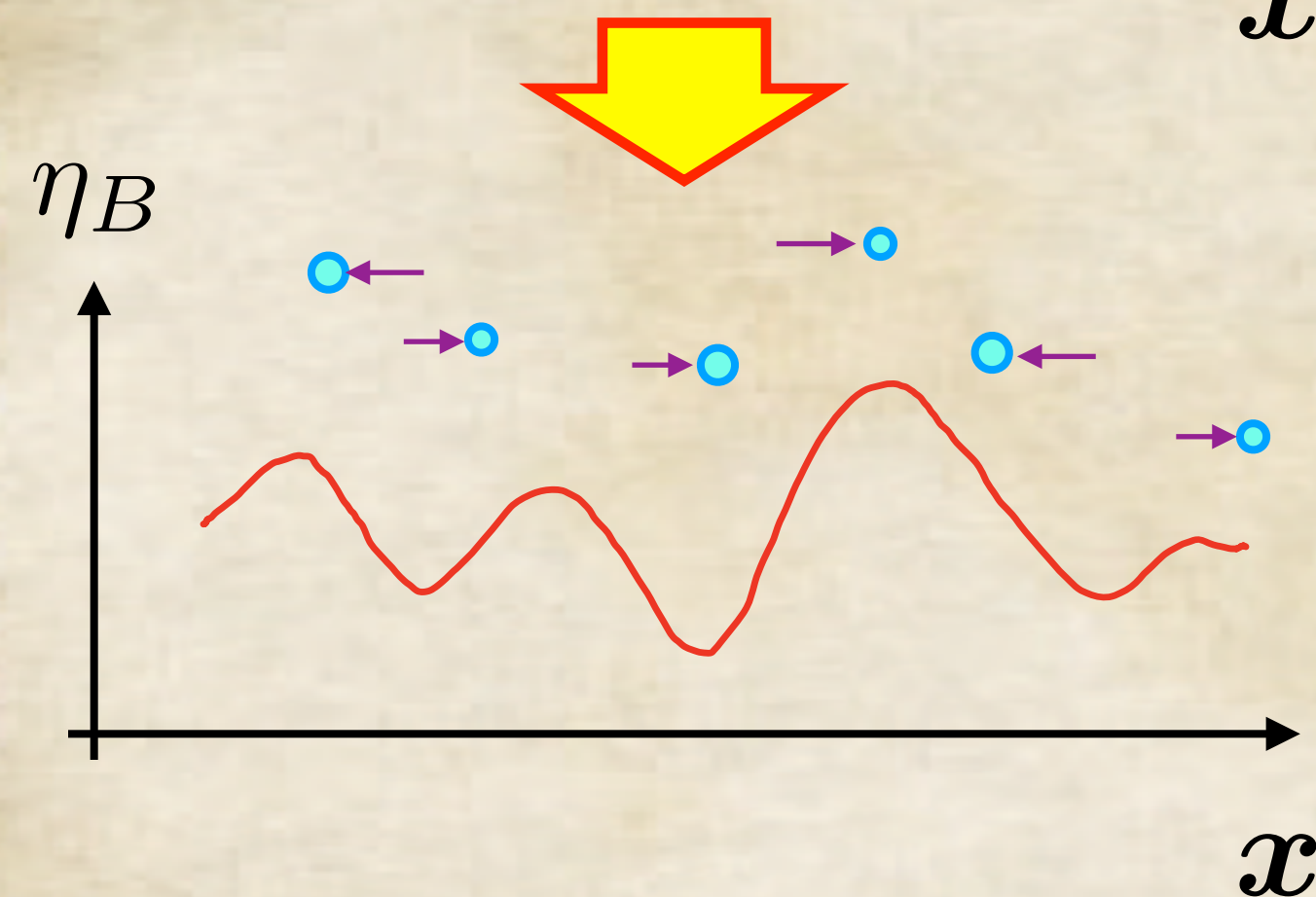
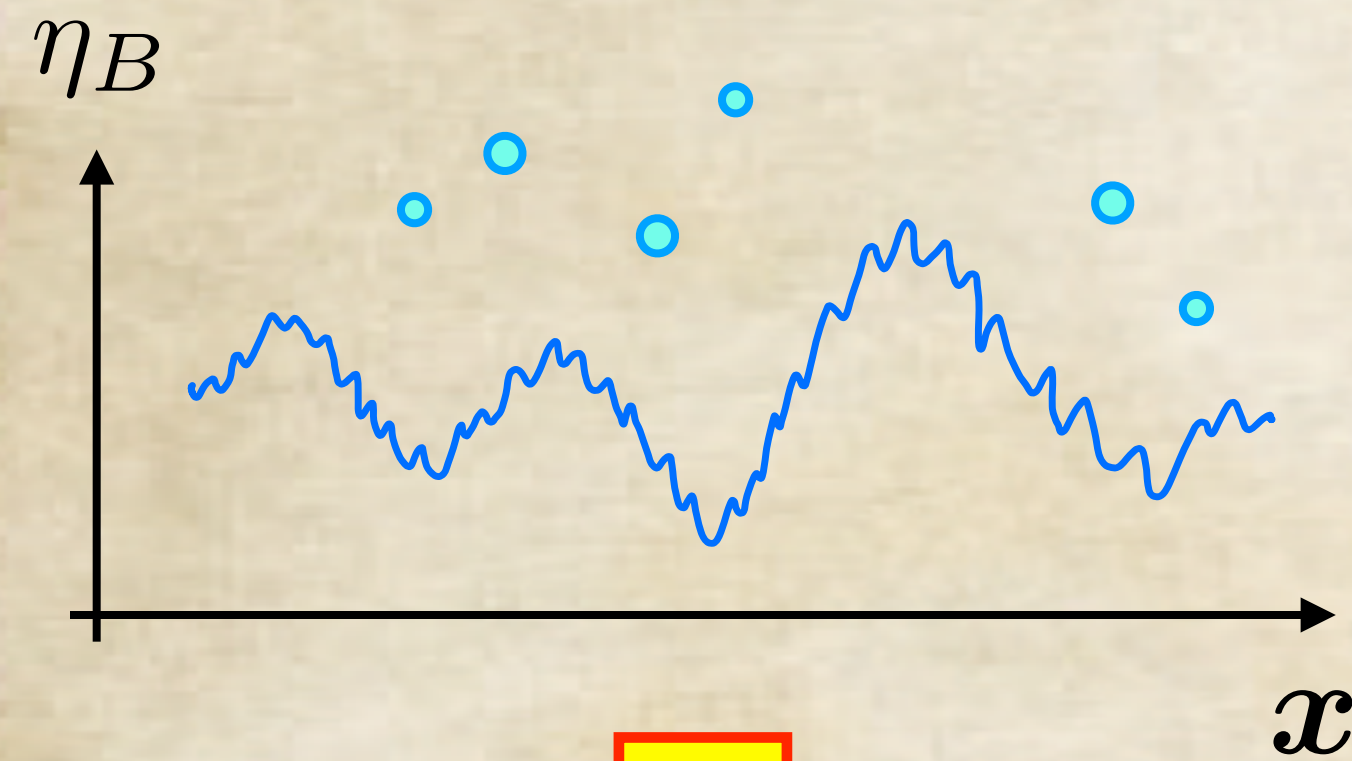
... Neutron diffusion erases the small scale inhomogeneities.



Baryon isocurvature perturbations at BBN

... Neutron diffusion erases the small scale inhomogeneities.

=> Corresponds to the treatment that the baryon asymmetry is convoluted with the Gaussian window function.



$$\langle S_{B,\text{BBN}}^2 \rangle = \int \frac{d^3 k}{(2\pi)^3} e^{-\frac{k^2}{3k_d^2}} \mathcal{G}(\mathbf{k}) \quad \text{neutron diffusion scale:} \quad k_d^{-1} = 0.0025 \text{ pc}$$

$$= \frac{c^2}{4\pi^4 \bar{\eta}_B^2} \int dk_1 dk_2 k_1^2 k_2^2 \sum_{\pm} \left(\pm \left\{ \frac{(k_1 \pm k_2)^2}{2} [S(k_1)S(k_2) \pm A(k_1)A(k_2)] \frac{3k_d^2}{3k_1 k_2} \left(1 \mp \frac{3k_d^2}{2k_1 k_2} \right) \right. \right.$$

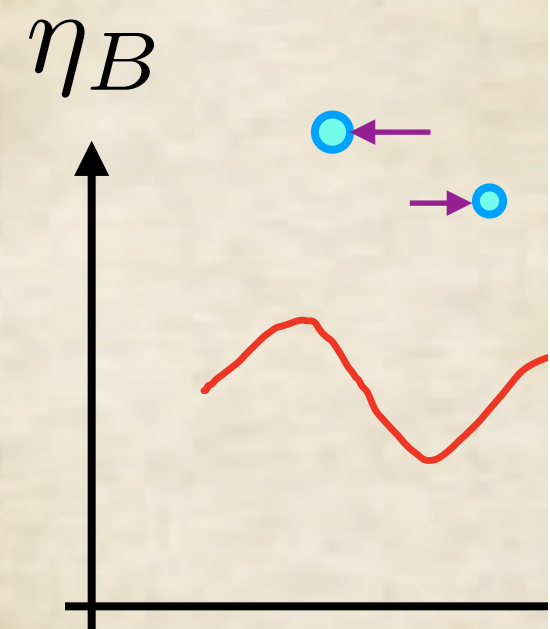
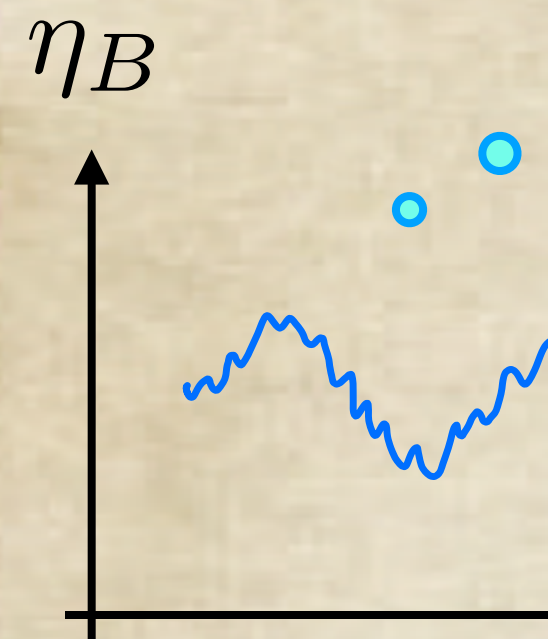
$$\left. \left. + \left[\frac{k_1^2 + k_2^2}{2} S(k_1)S(k_2) + k_1 k_2 A(k_1)A(k_2) \right] \left(\frac{3k_d^2}{2k_1 k_2} \right)^3 \right\} \exp \left[-\frac{2(k_1 \mp k_2)^2}{3k_d^2} \right] \right)$$

For given the MF spectra ($S(k)$, $A(k)$), we can evaluate the baryon isocurvature perturbation at BBN.

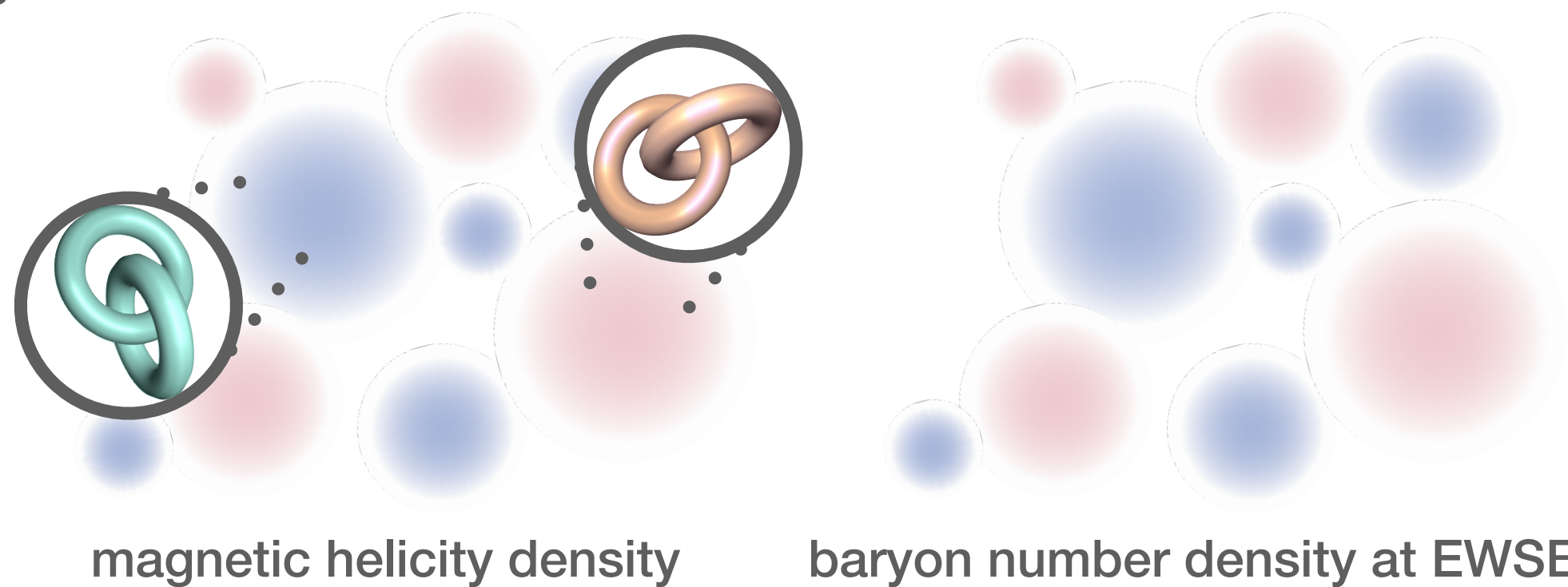
=> BBN constraint $\langle S_{B,\text{BBN}}^2 \rangle < 0.016$ can be given with respect to any MF spectra :)

Baryon iso

... Ne



real space configurations

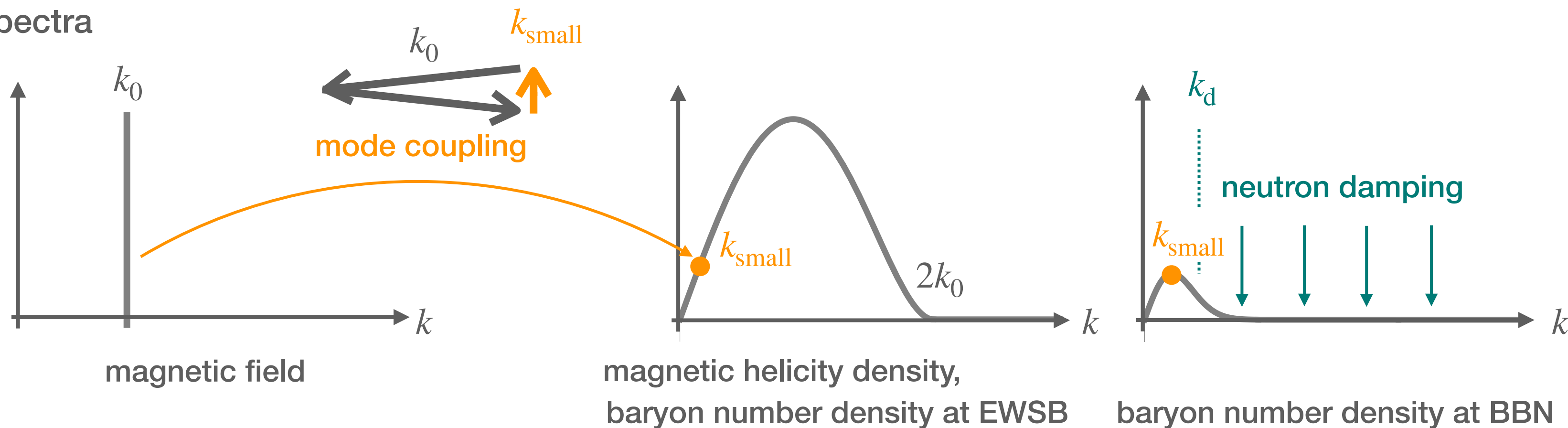


magnetic helicity density

baryon number density at EWSB

baryon number density at BBN

power spectra



magnetic field

magnetic helicity density,
baryon number density at EWSB

baryon number density at BBN

Figure from F. Uchida Ph. D thesis

\mathcal{X}

=> BBN constraint $\langle S_{B, \text{BBN}}^2 \rangle < 0.016$ can be given with respect to any MF spectra :)

try

$$\tau_d^{-1} = 0.0025 \text{pc}$$

$$\left(1 \mp \frac{3k_d^2}{2k_1 k_2} \right)$$

$$\left. \vphantom{\left(1 \mp \frac{3k_d^2}{2k_1 k_2} \right)} \right\} \exp \left[-\frac{2(k_1 \mp k_2)^2}{3k_d^2} \right]$$

Some general features:

- BBN constrains the ensemble average of baryon isocurvature perturbations

$$\langle S_{B,\text{BBN}}^2 \rangle < 0.016$$

=> perturbations at all the scales up to the present Hubble scale matters.

- Baryon isocurvature perturbation at small scale, $k > k_d$, at the EWSB becomes smaller by the neutron diffusion until BBN, but is not completely washed out.

Constraints on peaky MF spectra

$$\rho_{B,c} \simeq \frac{1}{2} B_{c,fo}^2, \quad \lambda_{c,fo} \simeq k_{\sigma}^{-1}, \quad \mathcal{H}_Y = \epsilon_{fo} \lambda_{c,fo} B_{c,fo}$$

- delta-function model: $S(k) = \pi^2 \frac{B_{c,fo}^2}{k_{\sigma}^4} \delta(k - k_{\sigma}), \quad A(k) = \epsilon_{fo} S(k),$

- power-law with exponential UV cutoff: $S(k) = \frac{2\pi^2}{\Gamma(\frac{5+\alpha}{2})} \frac{B_{c,fo}^2}{k_{\sigma}^5} \left(\frac{k}{k_{\sigma}}\right)^{\alpha} \exp\left[-\left(\frac{k}{k_{\sigma}}\right)^2\right], \quad A(k) = \epsilon_{fo} S(k).$
 $(\alpha > -5/2)$

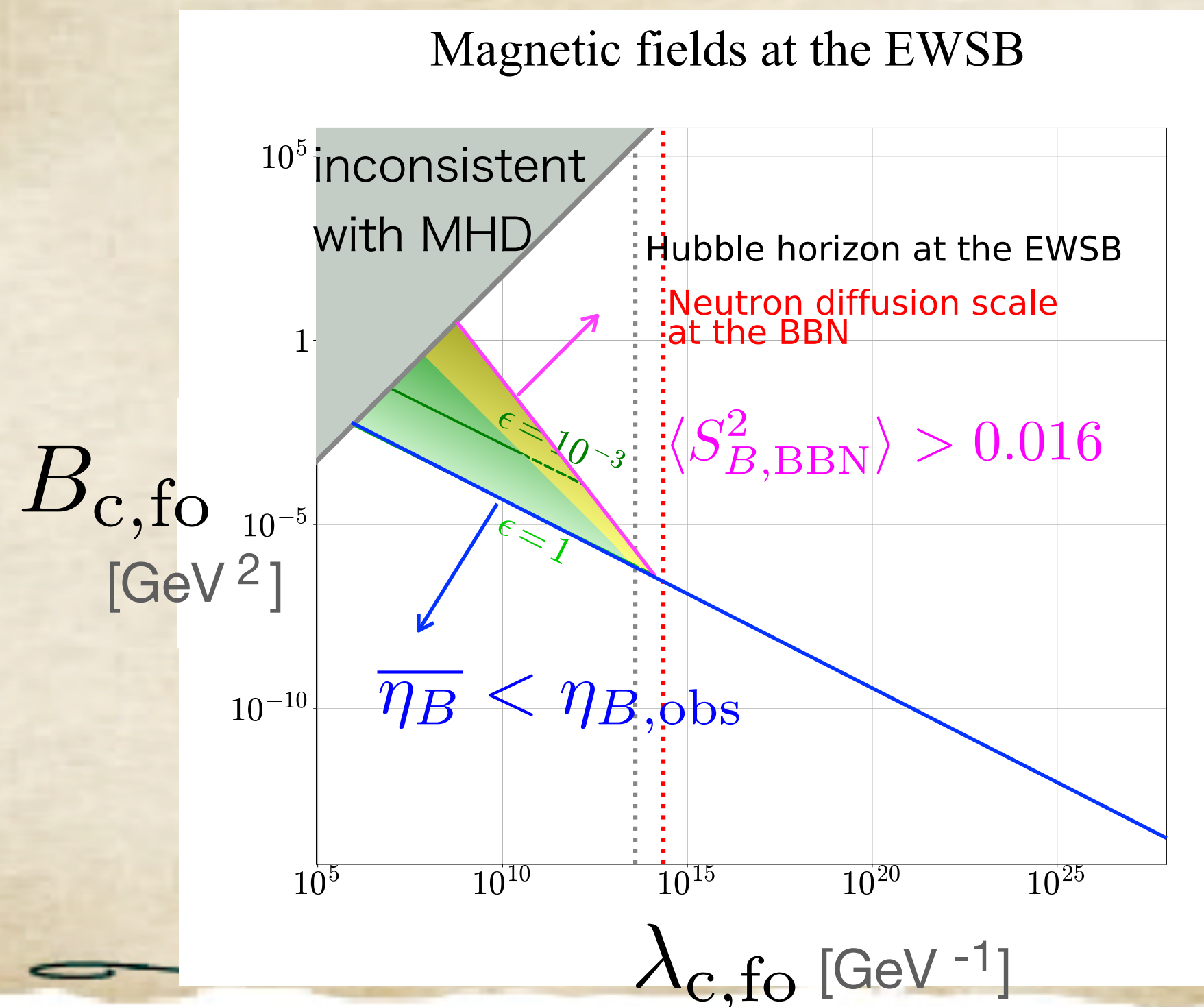
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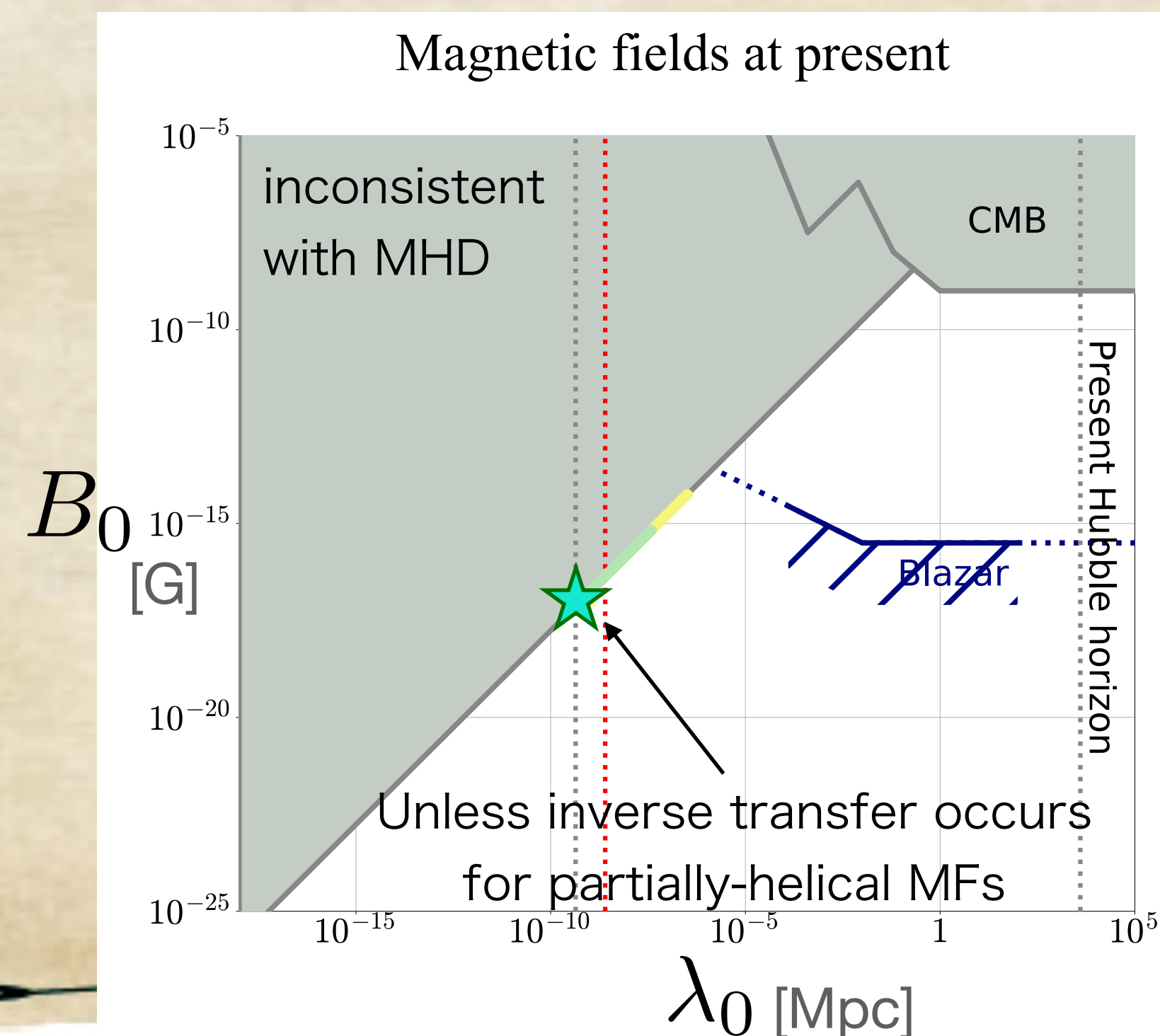
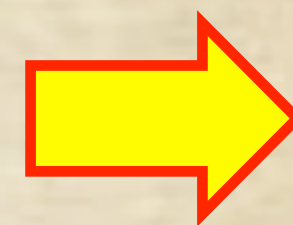
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If you would like to explain the BAU...



MF evolution with cascade being taken into account



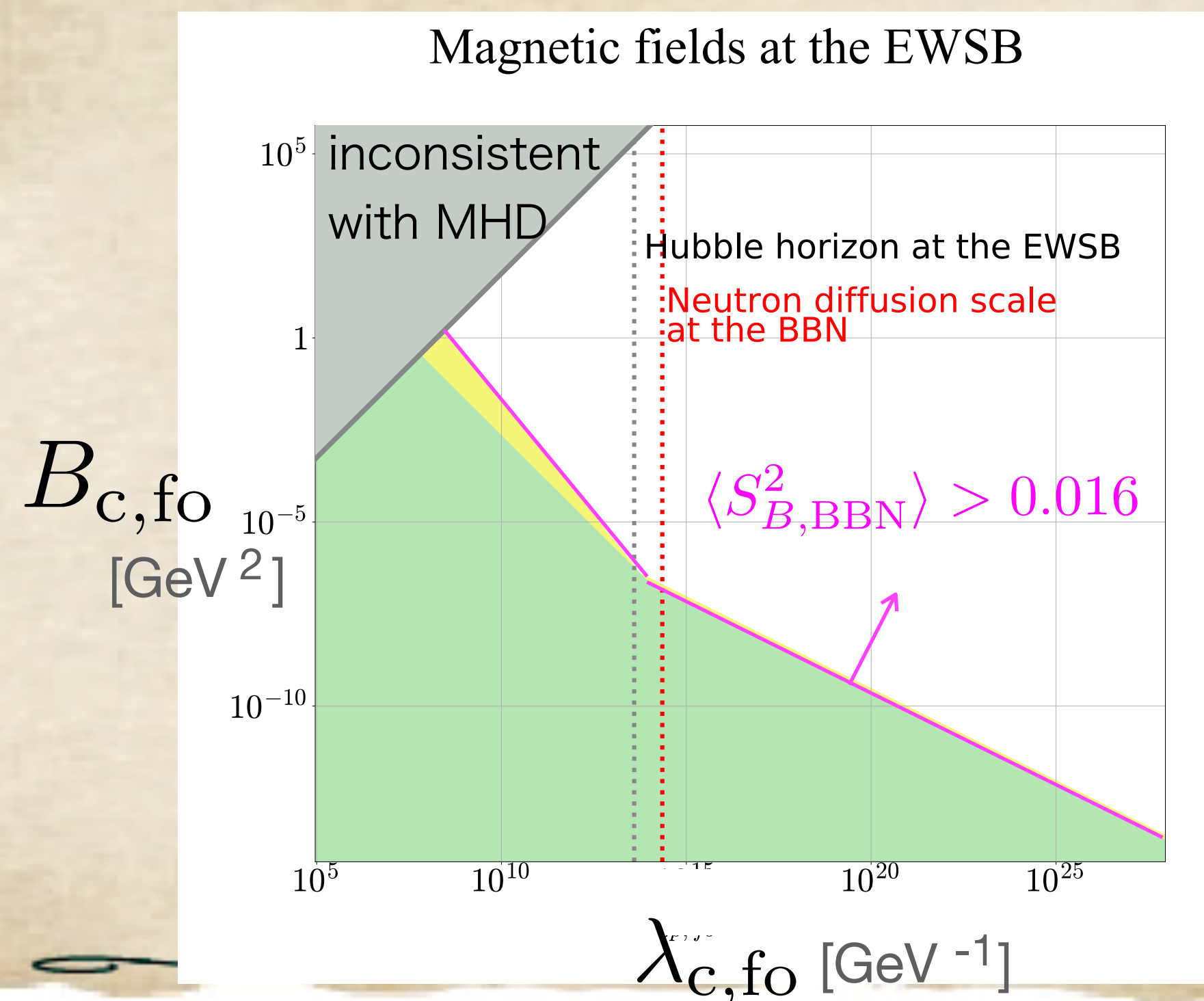
Constraints on peaky MF spectra

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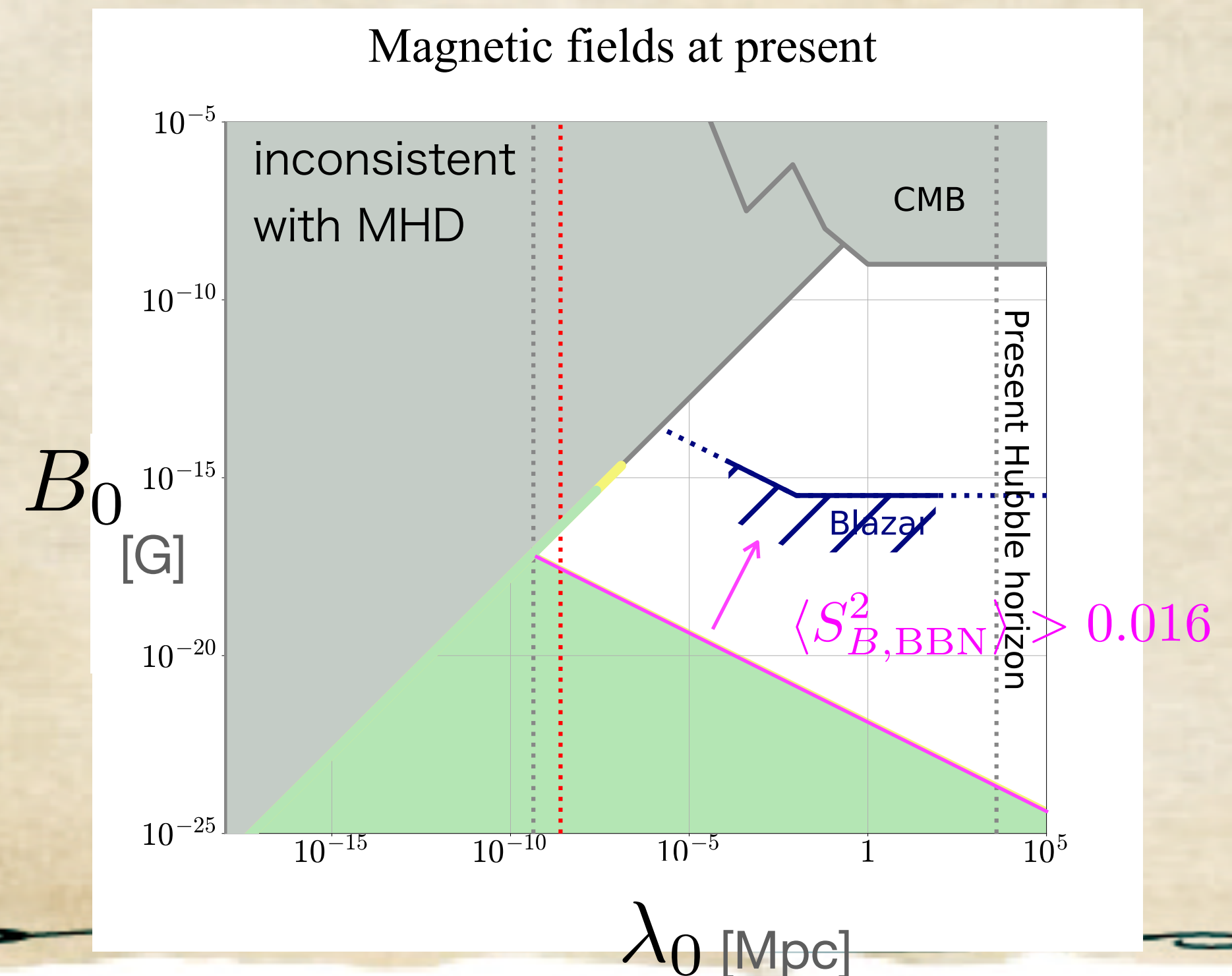
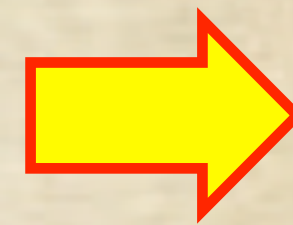
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 $(\alpha > -5/2)$

For non-helical MFs, forgetting about BAU



MF evolution with cascade being taken into account



For more flat spectrum such as those from inflationary magnetogenesis?

Just taking

$$S(k) = \frac{2\pi^2}{\Gamma(\frac{5+\alpha}{2})} \frac{B_{c,fo}^2}{k_\sigma^5} \left(\frac{k}{k_\sigma}\right)^\alpha \exp\left[-\left(\frac{k}{k_\sigma}\right)^2\right] \text{ with } \alpha \simeq -5 \quad \Rightarrow \quad \langle S_{B, \text{BBN}}^2 \rangle : \text{IR divergent?}$$

Reparameterize as

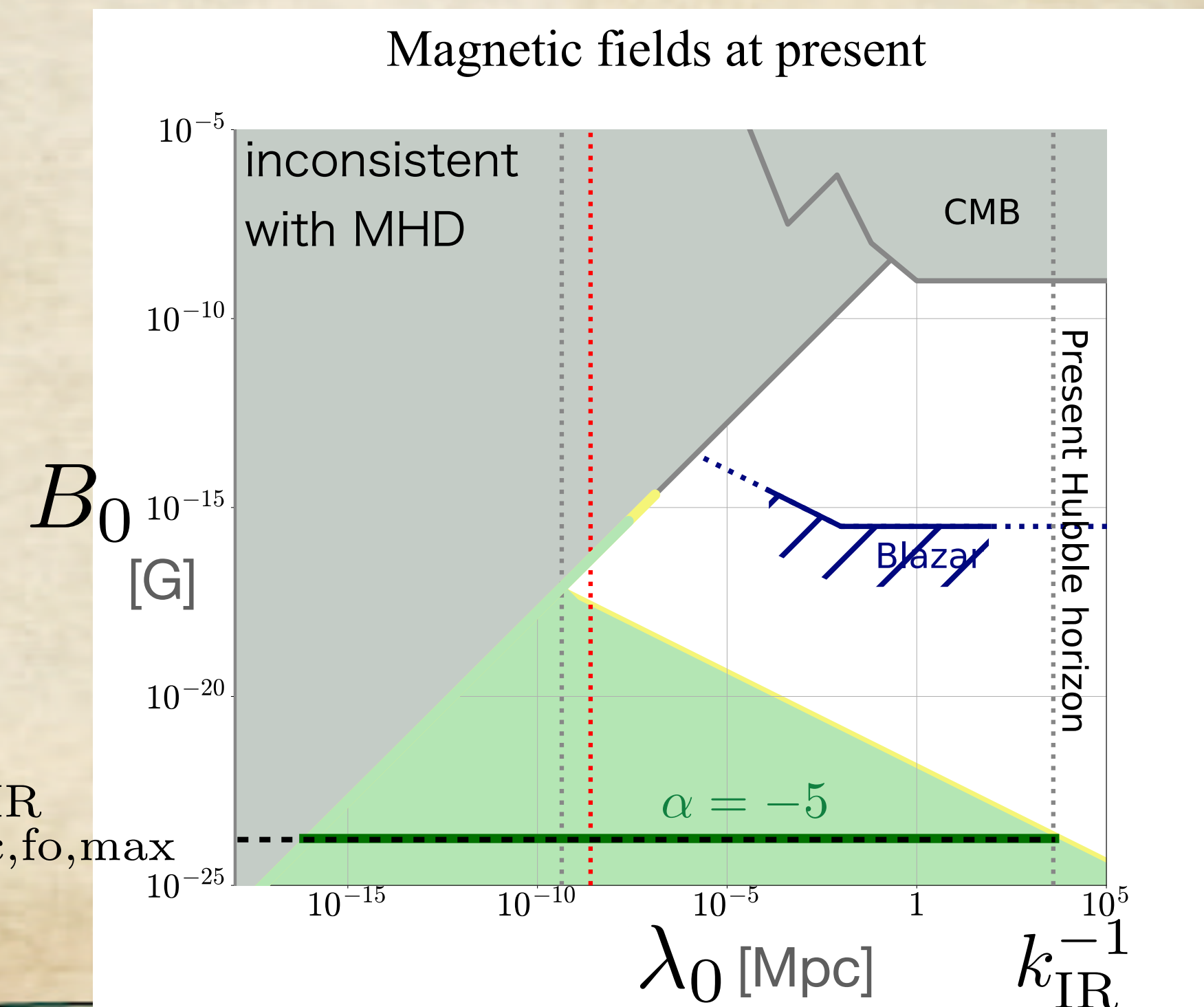
$$S(k) = \frac{(B_{c,fo}^{\text{IR}})^2}{k_{\text{IR}}^5} \left(\frac{k}{k_{\text{IR}}}\right)^\alpha \quad \text{with IR cutoff } k_{\text{IR}}$$

For long enough magnetogenesis during inflation, the IR cutoff k_{IR} should be taken as H_0

$$\Rightarrow \langle S_{B, \text{BBN}}^2 \rangle \sim \frac{C^2 (B_{c,fo}^{\text{IR}})^4}{\bar{\eta}_{B, \text{obs}}^2 k_{\text{IR}}^2} < 0.016$$

$B_{c,fo, \text{max}}^{\text{IR}}$

Constraint on flat MF spectrum



Summary

1. Absence of GeV cascade photon from blazars gave a motivation to study PMFs.
2. Hypermagnetic helicity decay is found to be able to generate BAU.
3. Baryon isocurvature constraint is so strong that magnetogenesis before the EWSB is impossible to explain the IGMFs suggested by blazar observations.
 1. 1st order (EW/QCD) PT would be more well-motivated.
 2. Inflationary magnetogenesis is possible only if its scale is smaller than the EW scale.
 3. EWSB is modified so that the EW sphaleron is more effective.
4. BAU can be still explained by the primordial hyper MFs, but we need to remove the “foreground” IGMFs to test the scenario.

