Particle Production by Gravitational Fields and Black Hole Evaporation

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M.F. Wondrak, W.D. van Suijlekom, and H. Falcke, Phys. Rev. Lett. 130 (2023) 221502.

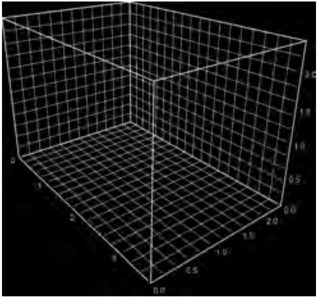
Outline

- Particle production from vacuum fluctuations
- Mathematical treatment
- Application to electric and gravitational fields
- Discussion & conclusions

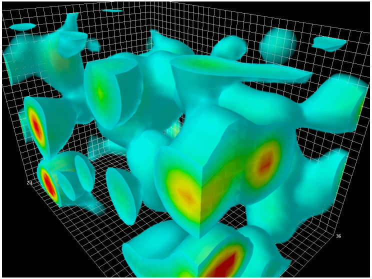
How Do Particles Arise From Vacuum Fluctuations?

Vacuum Fluctuations

Classical vacuum



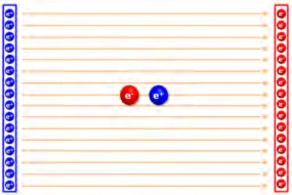
Quantum vacuum



- The quantum vacuum state is populated by short-lived excitations, staying undetectable according to the Heisenberg uncertainty relation, $\Delta E \, \Delta t \geq \hbar/2$.
- The simulation on the right shows gluons in quantum chromodynamics (QCD) arising, interacting, and annihilating.
  (lattice QCD simulation, box size: $2.4 \times 2.4 \times 3.6 \ \mathrm{fm}^3$).

http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/ImprovedOperators/index.html [last access: March 8, 2023].

Effects in an Electric Background



Particle production in a homogeneous electric field $\vec{E}$ (Schwinger effect).
Here: Free complex scalar field of charge q and mass m.

Does a vacuum state evolve into a non-vacuum state?

$$\langle 0_{\text{out}} | 0_{\text{in}} \rangle = \int \mathcal{D}\phi \; \exp\left(\frac{\mathrm{i}}{\hbar} S[\phi, A] \right) =: \exp\left(\frac{\mathrm{i}}{\hbar} W[A] \right) .$$

$W = \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}_{\text{eff}}$: effective action; A: e.m. background field

The probability for particle production reads

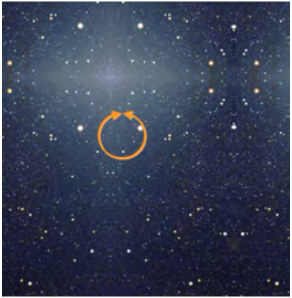
$$1 - |\langle 0_{\text{out}} | 0_{\text{in}} \rangle|^2 = 1 - \exp(-2/\hbar \, \Im W[A]) .$$

Effective Lagrangian for a constant homogeneous electric background field:

$$\Im(\mathcal{L}_{\text{eff}}) = \frac{(q\vec{E})^2}{16\pi^3\hbar} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(- \frac{n\pi m^2}{\hbar \sqrt{q^2 \vec{E}^2}} \right) . \tag{1}$$

H.H. Euler, W. Heisenberg, Zeitschr. Physik 98 (1936) 714; J.S. Schwinger, Phys. Rev. 82 (1951) 664.

Effects in Flat Spacetime



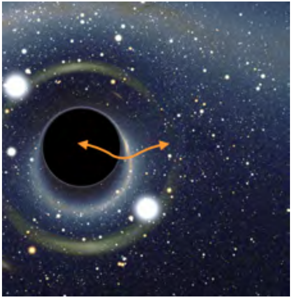
- Diagrammatic language for a vacuum fluctuation: closed loop. Creation of a particle–anti-particle pair followed by recombination.
- In flat spacetime, there is no obstruction to annihilation.

Effects in Curved Spacetime: Presence of a Horizon

- A pair creation event in the close vicinity to the event horizon: One particle may cross the event horizon.
- No annihilation possible, loop does not close. Outside partner turns into a real particle.
- A distant observer perceives a flux of particles.
  $\rightarrow$ Heuristic description of Hawking radiation.

S.W. Hawking, Commun. Math. Phys. **43** (1975) 199

Effects in Curved Spacetime: Presence of a Horizon



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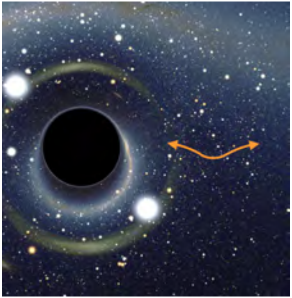
  → Heuristic description of Hawking radiation.

S.W. Hawking, Commun. Math. Phys. **43** (1975) 199

Effects in Curved Spacetime: Tidal Forces

- A pair creation event in a gravitational field: The particle paths are subject to tidal forces.
- Reduced probability for annihilation, loop does not close. Both partners turn into real particles.
- A distant observer perceives a flux of particles.

Effects in Curved Spacetime: Tidal Forces



- A pair creation event in a gravitational field: The particle paths are subject to tidal forces.
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- A distant observer perceives a flux of particles.

Mathematical Treatment

Does a vacuum state evolve into a non-vacuum state?
Special case of the generating functional

$$\langle 0_{\text{out}} | 0_{\text{in}} \rangle = \int \mathcal{D}\phi \, \exp\left(\frac{\mathrm{i}}{\hbar} S[\phi, g] \right) =: \exp\left(\frac{\mathrm{i}}{\hbar} W[g] \right)$$

introducing the effective action W. The probability for particle production reads

$$1 - |\langle 0_{\text{out}} | 0_{\text{in}} \rangle|^2 = 1 - \exp(-2/\hbar \, \Im W[g]) \, .$$

The action for a free neutral scalar field in Euclidean spacetime (Wick-rotated) is given by

$$S_{\text{E}}[\phi, g_{\text{E}}] = \frac{1}{2} \int \mathrm{d}^D x \sqrt{g_{\text{E}}} \, \left(\hbar^2 \, \partial_\mu \phi \, \partial^\mu \phi + m^2 \phi^2 + \hbar^2 \, \xi R \, \phi^2 \right) =: \frac{1}{2} \int \mathrm{d}^D x \sqrt{g_{\text{E}}} \, \phi H \phi.$$

ξ: coupling parameter to Ricci scalar.
$\xi = 0$: minimal coupling, i.e. the only coupling term between matter and curvature is the metric determinant.
$\xi = 1/6$: conformal coupling, i.e. classical field equations invariant under conformal transformations.

Mathematical Treatment

The Gaussian form of the path integral shows that the effective action is 1-loop exact, and has the formal solution

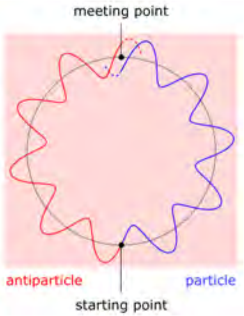
$$\begin{aligned} W_{\mathrm{E}}[g_{\mathrm{E}}] &= -\hbar \ln \int \mathcal{D}\phi \, \exp\left(-\frac{1}{\hbar} S_{\mathrm{E}}[\phi, g_{\mathrm{E}}]\right) \\ &= \frac{\hbar}{2} \ln \det H/\tilde{\mu}^2 \\ &= \frac{\hbar}{2} \operatorname{Tr} \ln H/\tilde{\mu}^2 \\ &= -\frac{\hbar}{2} \left(\frac{\tilde{\mu}}{\hbar}\right)^{2z} \int_0^\infty \frac{\mathrm{d}s}{s^{1-z}} \underbrace{\operatorname{Tr}\left(e^{-s(\Delta+\xi R)}\right)}_{\text{heat trace}} e^{-s(m^2-\mathrm{i}\epsilon)/\hbar^2}. \end{aligned}$$

ζ-function regularization: arbitrary mass scale $\tilde{\mu}$, parameter $z \in \mathbb{C}$.

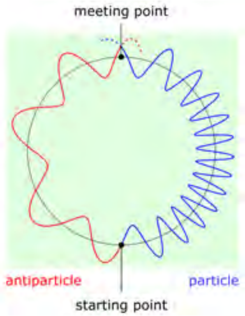
Interpretation in Terms of Interference

When re-writing the heat trace in the Schwinger representation (first quantization), the effective action in $D = 4$ dimensions becomes

$$W_{\mathrm{E}}[g_{\mathrm{E}}] = -\frac{\hbar}{2}\left(\frac{\tilde{\mu}}{\hbar}\right)^{2z}\int \mathrm{d}^4x\,\sqrt{g_{\mathrm{E}}}\int_0^\infty \frac{\mathrm{d}s}{s^{1-z}}\int_{x(0)=x(s)}\mathcal{D}x(\tau)\,\exp\left(-\frac{1}{\hbar}\,S_{\mathrm{e}}[x(\tau)]\right).$$



Destructive interference.



Constructive interference.

Covariant Perturbation Theory

Expand the heat trace for the scalar field in curvatures (covariant perturbation theory)

$$\mathrm{Tr}\, e^{-s(\Delta+\xi R)} \simeq \frac{1}{(4\pi s)^2}\int \mathrm{d}^4x\,\sqrt{g_\mathrm{E}}\,\Bigg[1 - s\left(\xi - \frac{1}{6}\right)R + s^2\Big(R_{\mu\nu\rho\sigma}\tilde{f}_1(s\Delta)R^{\mu\nu\rho\sigma} - R_{\mu\nu}\tilde{f}_1(s\Delta)R^{\mu\nu} + Rf_R(s\Delta)R + \Omega_{\mu\nu}f_5(s\Delta)\Omega^{\mu\nu}\Big)\Bigg] + \dots$$

including for the moment the case that the particle was also electrically charged.

f: form factors;

$\Omega_{\mu\nu}$: curvature of e.m. gauge connection, in flat spacetime: $\Omega_{\mu\nu} = [\nabla_\mu, \nabla_\nu] = \mathrm{i}qF_{\mu\nu}/\hbar$

Next, the effective action becomes

$$W_\mathrm{E} = -\frac{\hbar}{32\pi^2}\left(\frac{\tilde{\mu}}{\hbar}\right)^{2z}\int \mathrm{d}^4x\,\sqrt{g_\mathrm{E}}\,\Bigg[\Gamma(-2+z)\left(\frac{m^2}{\hbar^2}-\mathrm{i}\epsilon\right)^{2-z} - \Gamma(-1+z)\left(\frac{m^2}{\hbar^2}-\mathrm{i}\epsilon\right)^{1-z}\left(\xi-\frac{1}{6}\right)R + \Gamma(z)\left(\frac{m^2}{\hbar^2}-\mathrm{i}\epsilon\right)^{-z}\left(\frac{1}{180}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}\right) + \frac{1}{2}\left(\xi-\frac{1}{6}\right)^2R^2 + \frac{1}{12}\Omega_{\mu\nu}\Omega^{\mu\nu}\right)\Bigg] + \dots$$

A.O. Barvinsky, G.A. Vilkovisky, Nucl. Phys. B333 (1990) 471; A. Codello, O. Zanusso, J. Math. Phys. 54 (2013) 013513.

Covariant Perturbation Theory

When sending the regularization parameter $z \to 0$, we find

$$\left(\frac{\tilde{\mu}}{\hbar}\right)^{2z}\left(\frac{m^2}{\hbar^2} - \mathrm{i}\epsilon\right)^{-z} = 1 - z\,\ln\left(\frac{m^2}{\tilde{\mu}^2} - \mathrm{i}\epsilon\right) + \mathcal{O}(z^2). \qquad (2)$$

Thus the effective action actually presents an imaginary part in the massless case!

The particle production probability reads:

$$\mathcal{P} = \frac{1}{32\pi}\int \mathrm{d}^4x\,\sqrt{-g}\left[\frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 R^2 + \frac{1}{180}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}\right) + \frac{1}{12}\,\Omega_{\mu\nu}\Omega^{\mu\nu}\right].$$

Particle Production in Purely Electric Fields

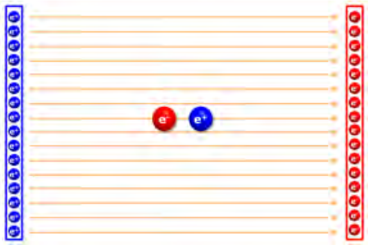
Particle production in a homogeneous electric field $\vec{E}$ (Schwinger effect).

From the treatment above, the probability for particle production events for a massless complex scalar field with charge q reads:

$$\begin{aligned}\mathcal{P} &= \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{1}{12} \, \Omega_{\mu\nu}\Omega^{\mu\nu} \\ &= \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{(q\vec{E})^2}{6\hbar^2}\end{aligned}$$

The massless limit of Schwinger's closed-form expression gives:

$$\begin{aligned}\mathcal{P} = \frac{2}{32\pi} \int \mathrm{d}^4x \, \sqrt{-g} \, \frac{(q\vec{E})^2}{6\hbar^2} \, \frac{12}{\pi^2} \\ \times \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp\left(-\frac{n\pi m^2}{\hbar\sqrt{q^2\vec{E}^2}}\right)}_{\to \pi^2/12}\end{aligned}$$



Our prediction is consistent with the closed-form expression in the massless limit.

Particle Production in the Schwarzschild Spacetime

Particle production in a Schwarzschild spacetime:

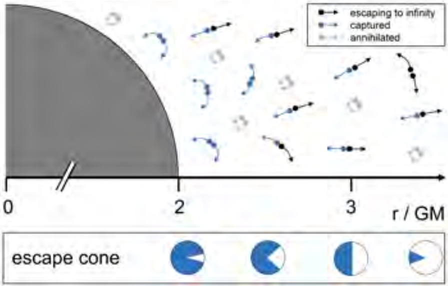
$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)$$

Ricci-flat solution ($R_{\mu\nu}=0$).
Kretschmann scalar:

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} = 48(GM)^2/r^6$$

Particle production events for a massless real scalar field:

$$\mathcal{P} = \frac{1}{32\pi}\int d^4x\sqrt{-g}\,\frac{4\,(GM)^2}{15r^6}$$



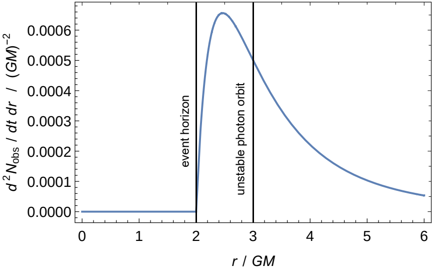
Escape cone similar to graybody factor.
Event horizon at $r=2GM$: escape probability = 0%,
photon orbit at $r=3GM$: escape probability = 50%.

Outside the photon orbit, even two (entangled) particles may escape to infinity.
→ hint for non-thermal emission.

Radial Distribution

Increasing pair production rate towards $r \to 0$,
increasing escape probability towards $r \to \infty$.

The highest production rate of escaping particles per spherical shell occurs at $\sim 2.5\,GM$.



Adjustment Factors

From analogy with the Schwinger effect:

- In a particle production event, an arbitrary number of pairs can be created. In the electric case, $12/\pi^2 \approx 1.22$ pairs are produced per event on average.
- Transferring the typical energy scale from the Schwinger effect to the gravitational sector. Requiring that the electric and the gravitational field strength lead to an equal particle production event rate density, one finds a formal replacement rule $\Omega_{\mu\nu}\Omega^{\mu\nu} \to 2/30 \times R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ such that

$$E_{\text{curv}} = \hbar\,(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}/30)^{1/4} = \underbrace{6.11 \times 10^{-30}\,\text{J}}_{38\,\text{peV}} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{r}{2.5\,GM}\right)^{-3/2}$$

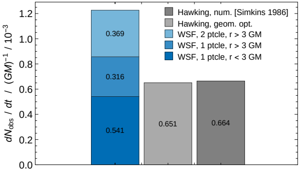
- A more intuitive scale is given by the effective temperature for blackbody radiation from matching $\langle E \rangle = E_{\text{curv}}$:

$$T_{\text{eff}}(r) = \frac{30\zeta(3)}{\pi^4}\,\frac{E_{\text{curv}}(r)}{k_B} \approx 164\,\text{nK} \left(\frac{M}{M_\odot}\right)^{-1} \left(\frac{r}{2.5\,GM}\right)^{-3/2}$$

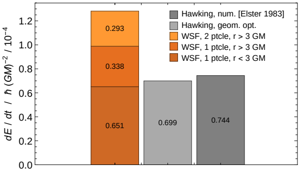
Comparison

	WSF	Hawking effect
Particle Flux	$12.3 \times 10^{-4}\,(GM)^{-1}$ $250\,\mathrm{s}^{-1}\,(M_\odot/M)$	$6.51 \times 10^{-4}\,(GM)^{-1}$ $132\,\mathrm{s}^{-1}\,(M_\odot/M)$
Energy Flux	$12.8 \times 10^{-5}\,\hbar\,(GM)^{-2}$ $5.57 \times 10^{-28}\,\mathrm{J\,s}^{-1}\,(M_\odot/M)^2$	$6.99 \times 10^{-5}\,\hbar\,(GM)^{-2}$ $3.04 \times 10^{-28}\,\mathrm{J\,s}^{-1}\,(M_\odot/M)^2$
Local temperature $(r = 2.5\,GM,\ M = M_\odot)$	$164\,\mathrm{nK}$	$138\,\mathrm{nK}$

$\Rightarrow$ The predictions correspond to each other up to a relative factor ~ 1.9.



Particle flux.



Energy flux.

Discussion

- A neutral massless scalar field is a model for 1 degree of freedom, photons and gravitons have two polarizations. Photons might not be created, but gravitons do (consider relation to anomalies).
- For mathematical consistency, we worked in Euclidean signature. A Wick rotation is naturally assumed to lead to a stationary Lorentzian spacetime.
- The curvature expansion of the effective action extends to 2nd order. Consideration of the impact of a magnetic field on the Schwinger effect (from the parity-odd term $\Omega_{\mu\nu}\tilde{\Omega}^{\mu\nu}$) requires higher orders.
- The relation of the energy of produced pairs to curvature (Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$) has not been derived stringently, but from analogy to Schwinger. It might miss terms for more complex spacetimes.
- The local emission is assumed isotropic for a static observer, while the tidal forces are anisotropic.

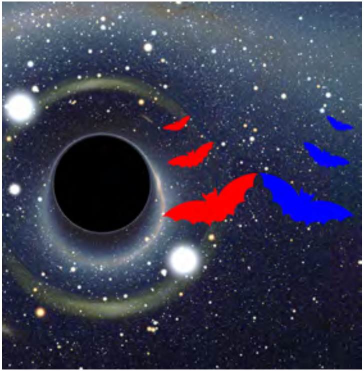
M.N. Chernodub, arXiv:2306.03892 [hep-th]; A. Ferreiro, J. Navarro-Salas, S. Pla, arXiv:2306.07628 [gr-qc]; M.P. Hertzberg, A. Loeb, arXiv:2307.05243 [gr-qc]; M.F. Wondrak, W.D. van Suijlekom, H. Falcke, 2308.12326 [gr-qc].

Conclusions

- Strong gravitational fields generate particle–anti-particle pairs.
  A virtual particle pair does not recombine because of tidal forces.
- The predicted particle and energy fluxes are compatible with the exact expression for the Schwinger effect, and with the Hawking emission within a factor ~ 2.
- The presence of a black hole event horizon does not enter the calculations.

$\Rightarrow$ This indicates that normal matter (e.g. neutron stars) would decay, not only black holes.

Particle Production by Gravitational Fields



Halloween edition: Bat production.

Questions?

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