On the instability of axion inflation with strong backreaction from gauge modes

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Copernicus Webinar

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UMass Amherst

Peloso, LS, 2209.08131
Garcia-Bellido, Papageorgiou, Peloso, LS, 2303.13425
von Eckardstein, Peloso, Schmitz, Sobol, LS, 2309.04254
Corba', LS 2401.NNNNN

Axion inflation

Pseudoscalar, shift symmetric inflaton, radiatively stable

theoretically very attractive

"natural" coupling to U(1) gauge fields:

$$\mathcal{L}(\varphi,\, A^\mu) = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\alpha}{4f}\varphi F^{\mu\nu}\tilde{F}^{\mu\nu}$$

EOM for helicity-λ modes of photon

$$A_\lambda'' + \left(\mathbf{k}^2 + \lambda\frac{\alpha\,\phi'}{f}|\mathbf{k}|\right) A_\lambda = 0$$

Amplification of chiral vectors

$$A''_\lambda + \left(\mathbf{k}^2 + \lambda \frac{\alpha\,\phi'}{f} |\mathbf{k}| \right) A_\lambda = 0$$

for λ=-, the "mass term" is negative
and large for ~*1* Hubble time:

Exponential amplification of **left-handed modes only**
(parity violation)

$$A_- \propto \exp\left\{ \frac{\pi}{2} \frac{\alpha\,\dot\phi}{f\,H} \right\}$$

Cosmological magnetic fields

(Observed up to $\sim Mpc$ scales, $\sim 10^{-17}G$, uncertain origin)

Anber, LS 06

Blue spectrum, $B(k) \propto k^2$
too weak at large scales

…despite *inverse cascade*
(MHD effect for chiral gauge fields,
amplifying large scale spectrum)

Phenomenology

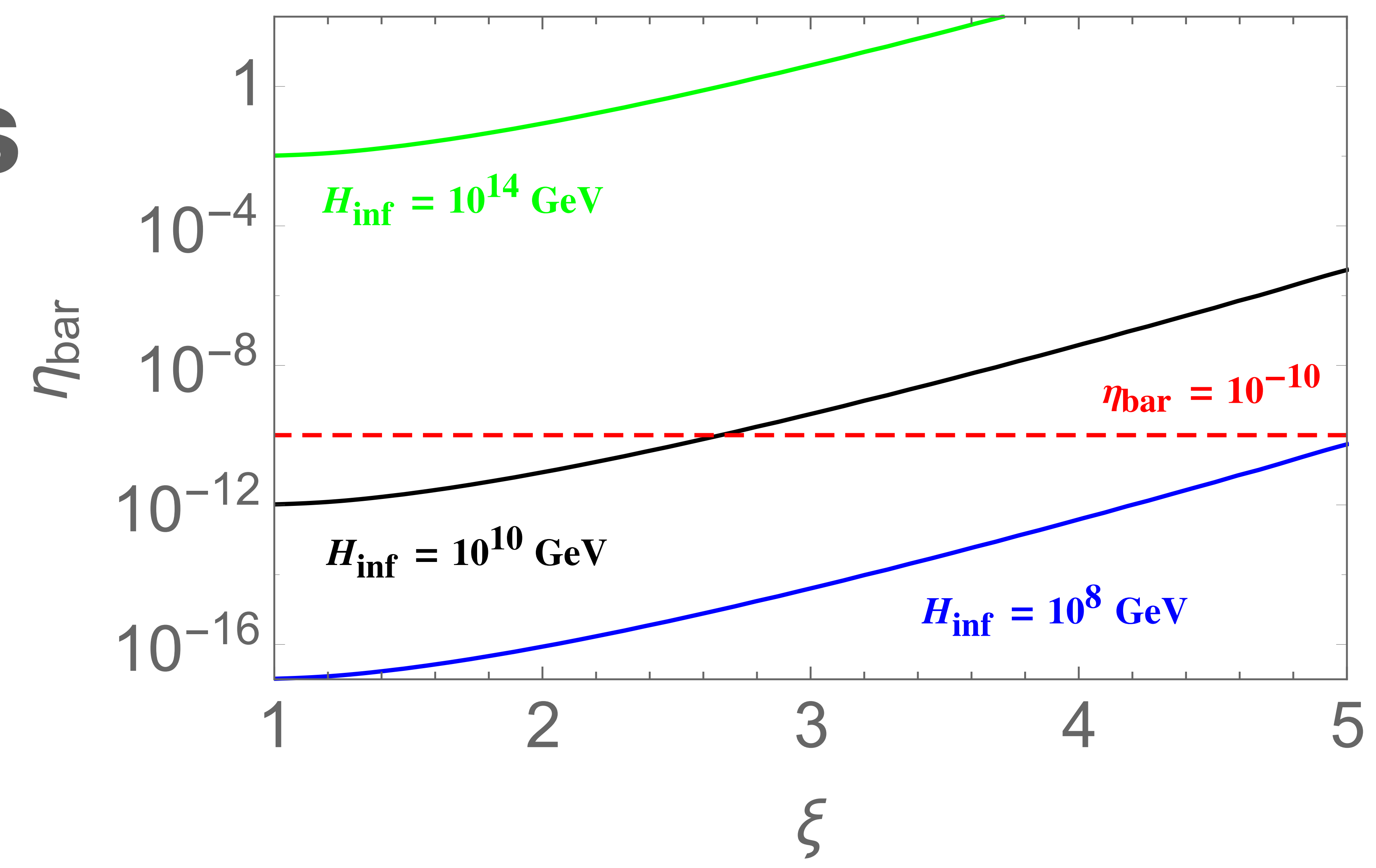
Nonvanishing net helicity

+

chiral anomaly

Baryogenesis

Anber Sabancilar 16
Domcke von Harling Morgante Mukaida 19

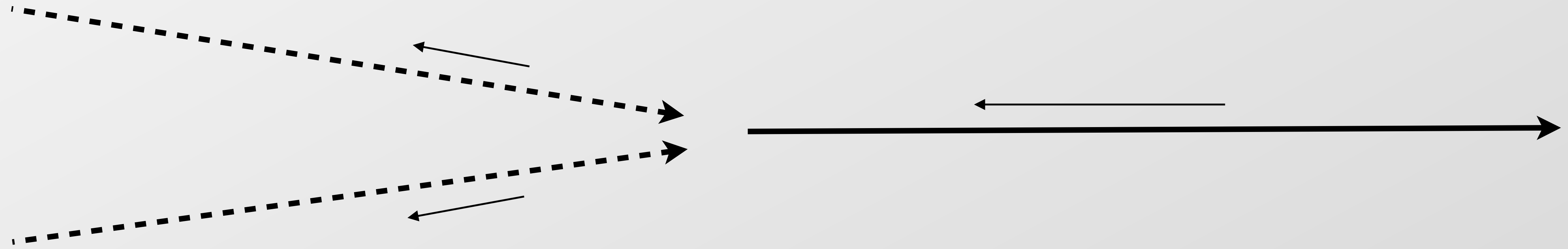


Chiral gravitational waves

LS 11

$$A_\mu + A_\nu \to \delta g_{\mu\nu}$$

in the limit of small transverse momentum two LH photons cannot create a RH graviton



Chiral gravitational waves

$$A_\mu + A_\nu \to \delta g_{\mu\nu}$$

$$\mathcal{P}_L(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 9\times 10^{-7}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

$$\mathcal{P}_R(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 2\times 10^{-9}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

"standard"
parity-invariant part

parity-violation

$$\xi \equiv \frac{\alpha\dot{\phi}}{2\,f\,H} \gtrsim 1$$

Phenomenology

...but also, very large f_{NL}

Barnaby Peloso 10

$$A_\mu + A_\nu \to \delta\varphi$$

When effect of photons is large enough, $f_{NL}{\sim}10^4$

RULED OUT

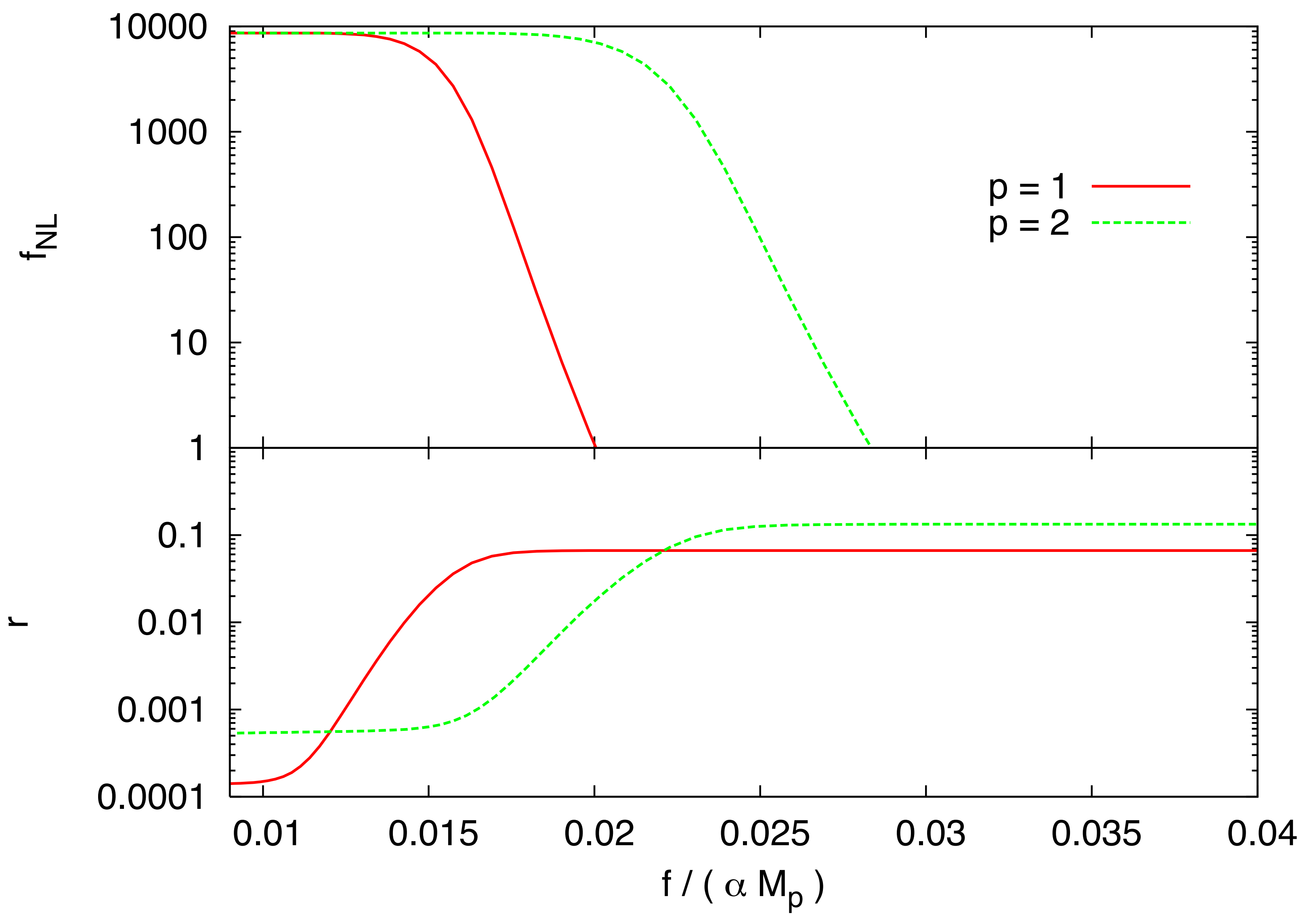


FIG. 2: Observational predictions for the large-field power-law inflation model (11) with $p = 1, 2$ and assuming $N_e \cong 60$. The spectral index is $n_s = 0.975, 0.967$ for $p = 1, 2$. At small f/α the coupling of ϕ to $F\tilde{F}$ is stronger and nongaussianity is large. The tensor-to-scalar ratio decreases at strong coupling; however, the decrease is important only at values of f/α which are ruled out by the current bound on $f_{NL}^{\rm equil}$.

Phenomenology

But constraints on f_{NL} on CMB scales only!

Inflationary gravitational waves for LIGO (LISA…)?

$$\mathcal{P}_L(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 9\times 10^{-7}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$
$$\mathcal{P}_R(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 2\times 10^{-9}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

Cook LS 11

$$\xi \equiv \frac{\alpha\dot{\phi}}{2\,f\,H} \gtrsim 1$$

ξ *increases* during inflation

↓

GWs produced towards the end of inflation (i.e. at smaller scales) have larger amplitude

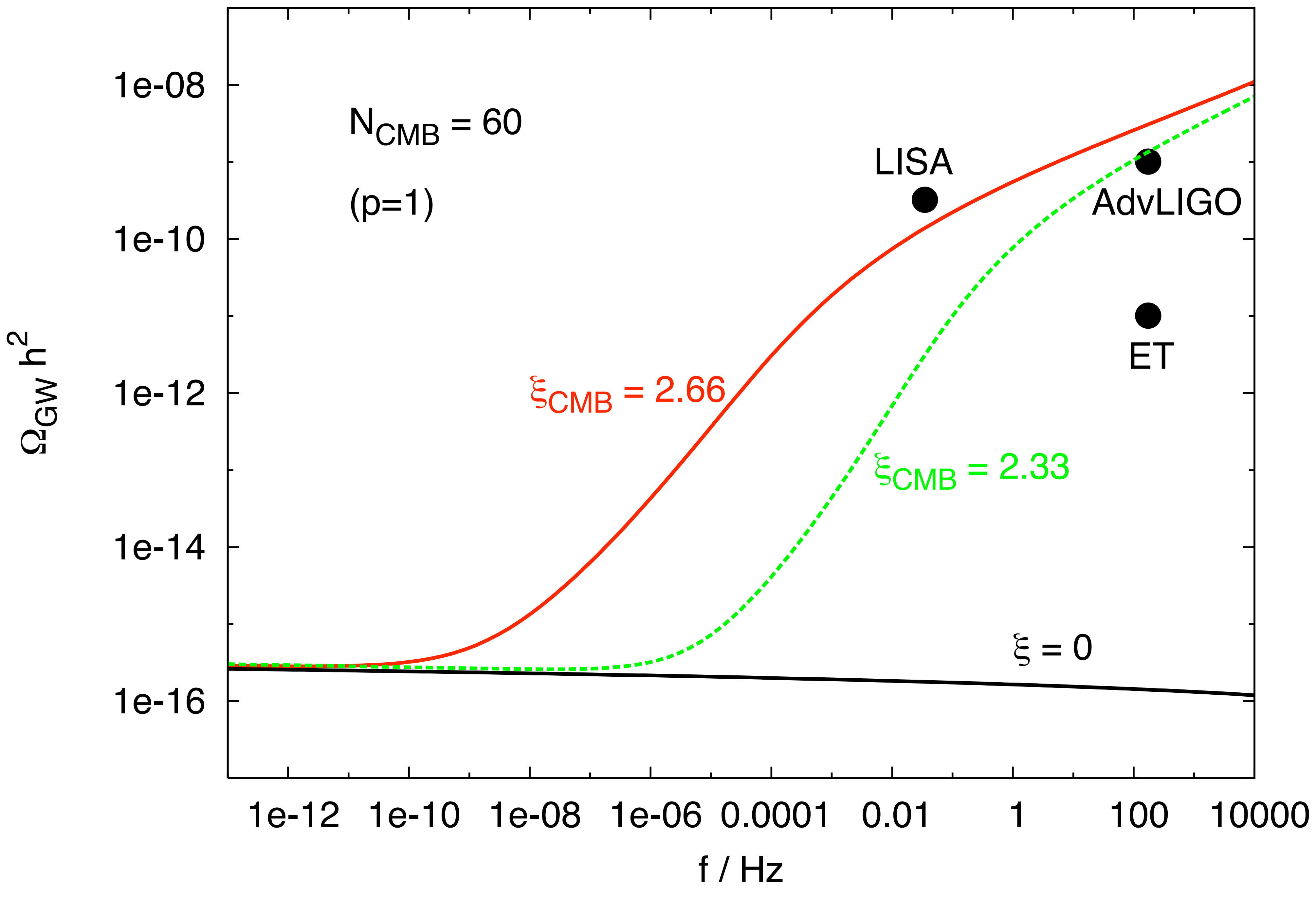
↓

might be detected by GW interferometers!

Phenomenology

But constraints on f_{NL} on CMB scales only!

Inflationary gravitational waves for LIGO (LISA…)?



Barnaby Pajer Peloso 11

(Those GWs might be correlated with CMB perturbations!)

Corba', LS 2401.NNNNN

Phenomenology

How about an axion in a transient roll?

Namba, Peloso, Shiraishi, LS, Unal 15

Field σ $(\neq\varphi)$ coupled to gauge fields rolls only for a finite number of efoldings

$$V_{-}(\sigma) = \frac{\Lambda^4}{2}\left[\cos\left(\frac{\sigma}{f}\right) + 1\right]$$

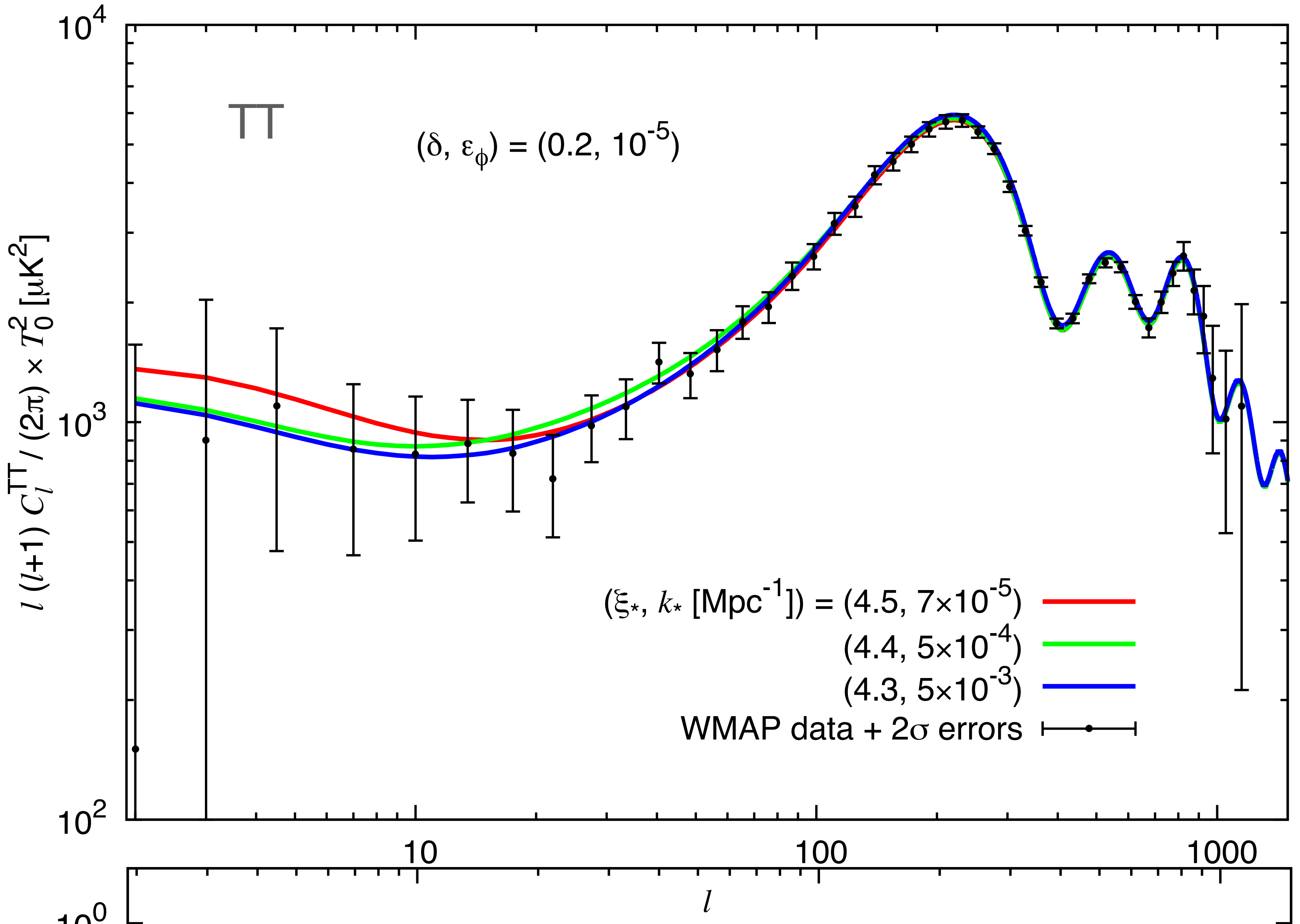
↓

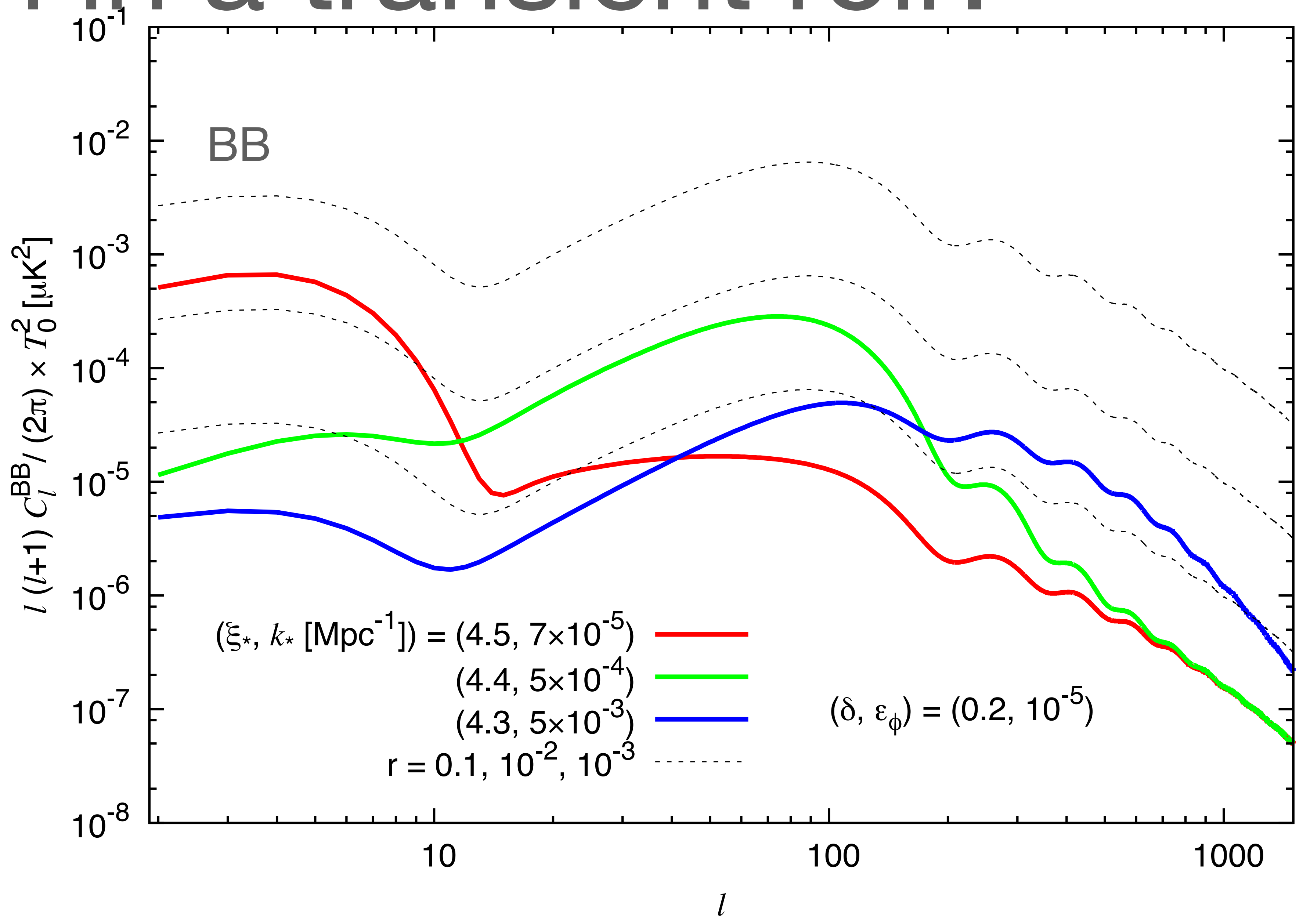
its effects will be visible only on a finite range of multipoles

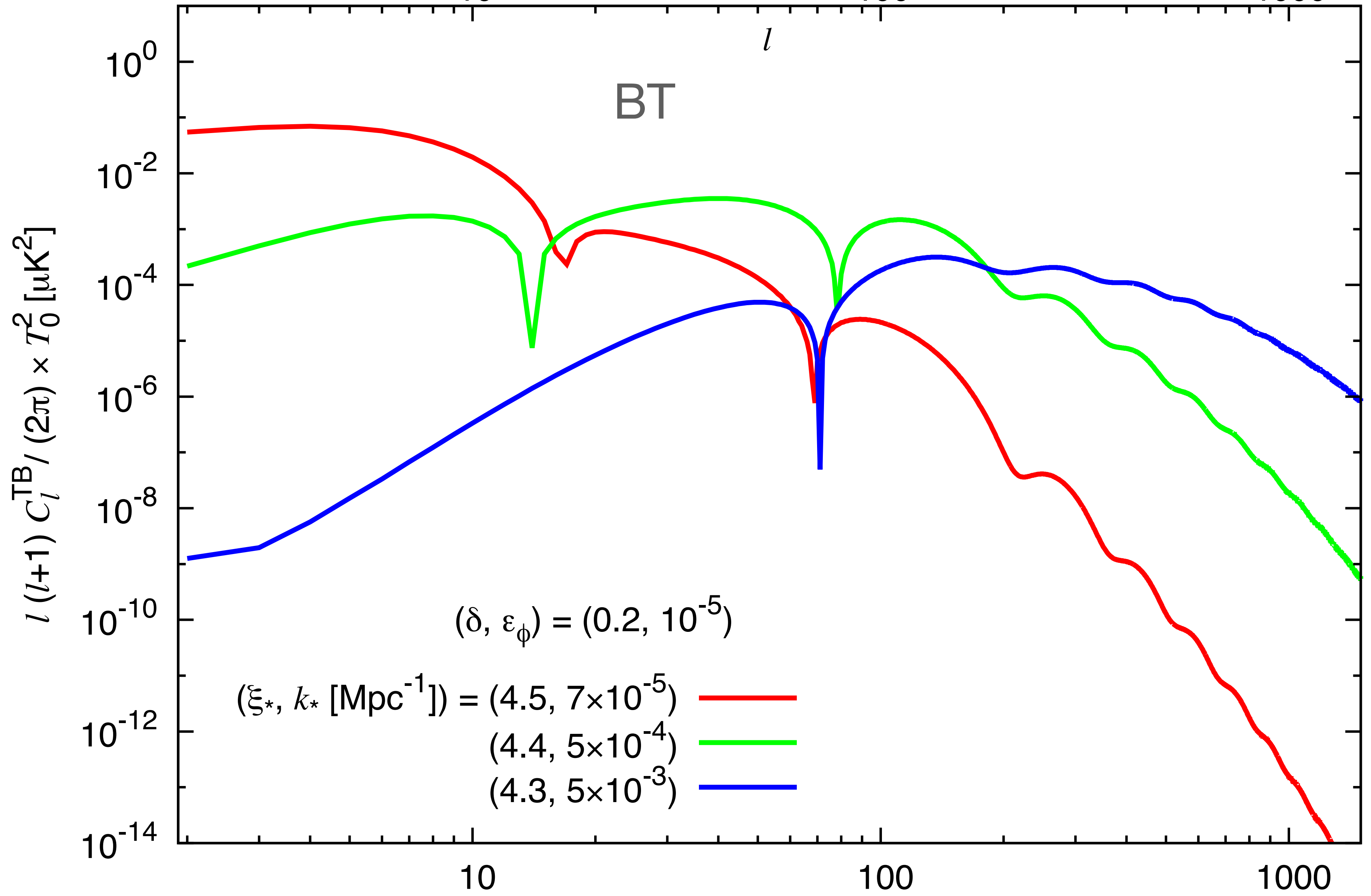
Choose those multipoles to be $l = O(1 \div 100)$, where effects of tensors are important, but nongaussianities in the T fluctuations are weakly constrained because of cosmic variance

Phenomenology

How about an axion in a transient roll?







Should vanish in parity-even Universe

Phenomenology

Linde, Mooij, Pajer 12
Garcia-Bellido, Peloso, Unal 16

Many vectors at small scales ⟹
very large scalar perturbations

Primordial black holes

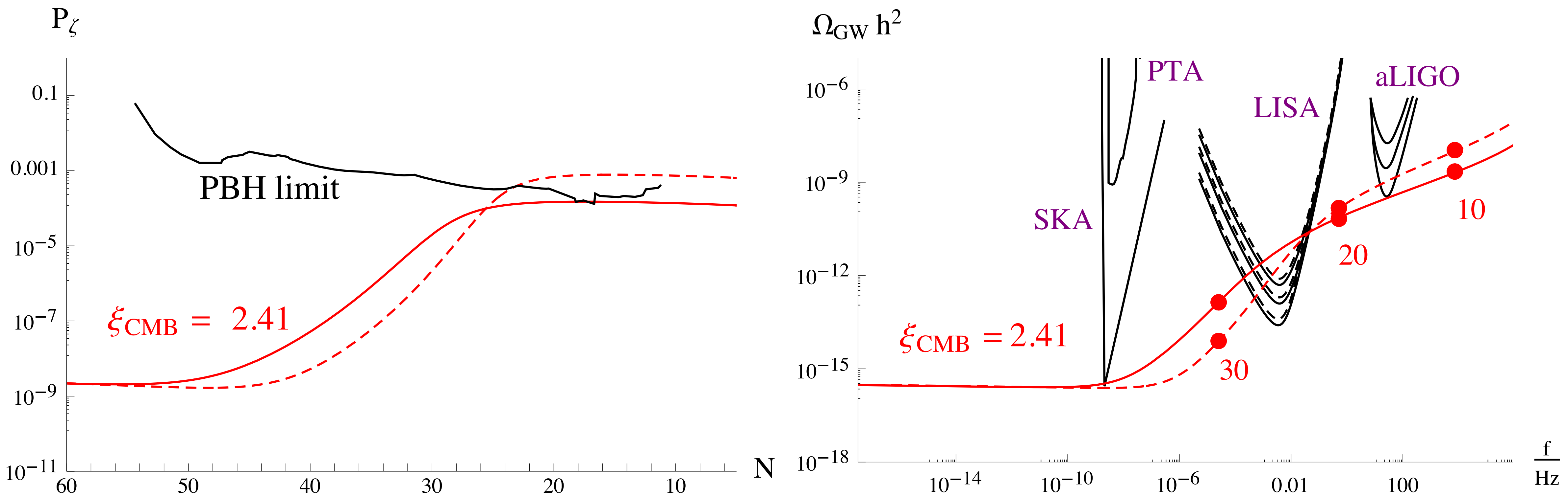


Figure 5. Scalar and tensor signals for a linear inflation potential. The solid lines show the signal if $\mathcal{N} = 6$ gauge fields are amplified. For comparison, the dashed lines show the signal when 1 gauge field is amplified.

A realization of trapped inflation

Anber LS 09

Accounting for backreaction of vectors

$$\ddot{\phi} + 3\,H\,\dot{\phi} + V'(\phi) = -\quad \frac{\alpha}{f}\,\langle \vec{E}\cdot\vec{B}\rangle$$

with

$$\langle \mathbf{E}\cdot\mathbf{B}\rangle \propto e^{\pi\,\frac{\alpha}{f}\frac{|\dot{\phi}|}{H}}$$

Strong backreaction regime:

$$V'(\phi) = -\quad \frac{\alpha}{f}\,\langle \vec{E}\cdot\vec{B}\rangle$$

$$\dot{\phi} \simeq \frac{f\,H}{\alpha\pi}\log\left(\ldots\right)$$

A realization of trapped inflation

Slow roll does not rely on potential flatness:

$$V(\phi) = \Lambda^4 \left[\cos\left(\frac{\phi}{f}\right) + 1\right]$$

with $f < M_P$ can support inflation!

Interesting!

$f > M_P$ conjectured to be forbidden in UV-complete theories of gravity

A realization of trapped inflation

How many efoldings?

$$N_e \simeq \int_{\phi_i}^{\phi_f} \frac{H\, d\phi}{\dot{\phi}} \simeq H \frac{\phi_f - \phi_i}{\dot{\phi}} \lesssim \frac{\pi\, f\, H}{\dot{\phi}} \simeq \frac{\alpha\, \pi^2}{\log(\ldots)}$$

$\gtrsim \mathcal{O}(10)$

For this to work, need $\alpha > 10^2$

(Can be realized with two gauge groups, similar to KNP)

Fraser Reece 19

[Other consistency conditions checked and satisfied]

Strong backreaction

NOTE: strong backreaction happens quite generally towards the end of inflation in phenomenologically interesting models

IMPORTANT

that we understand it well!

Strong backreaction

Looking more carefully into the backreacted equations…

$$\ddot{\phi} + 3\,H\,\dot{\phi} + V'(\phi) = -\frac{\alpha}{f}\langle \mathbf{E}\cdot\mathbf{B}\rangle$$

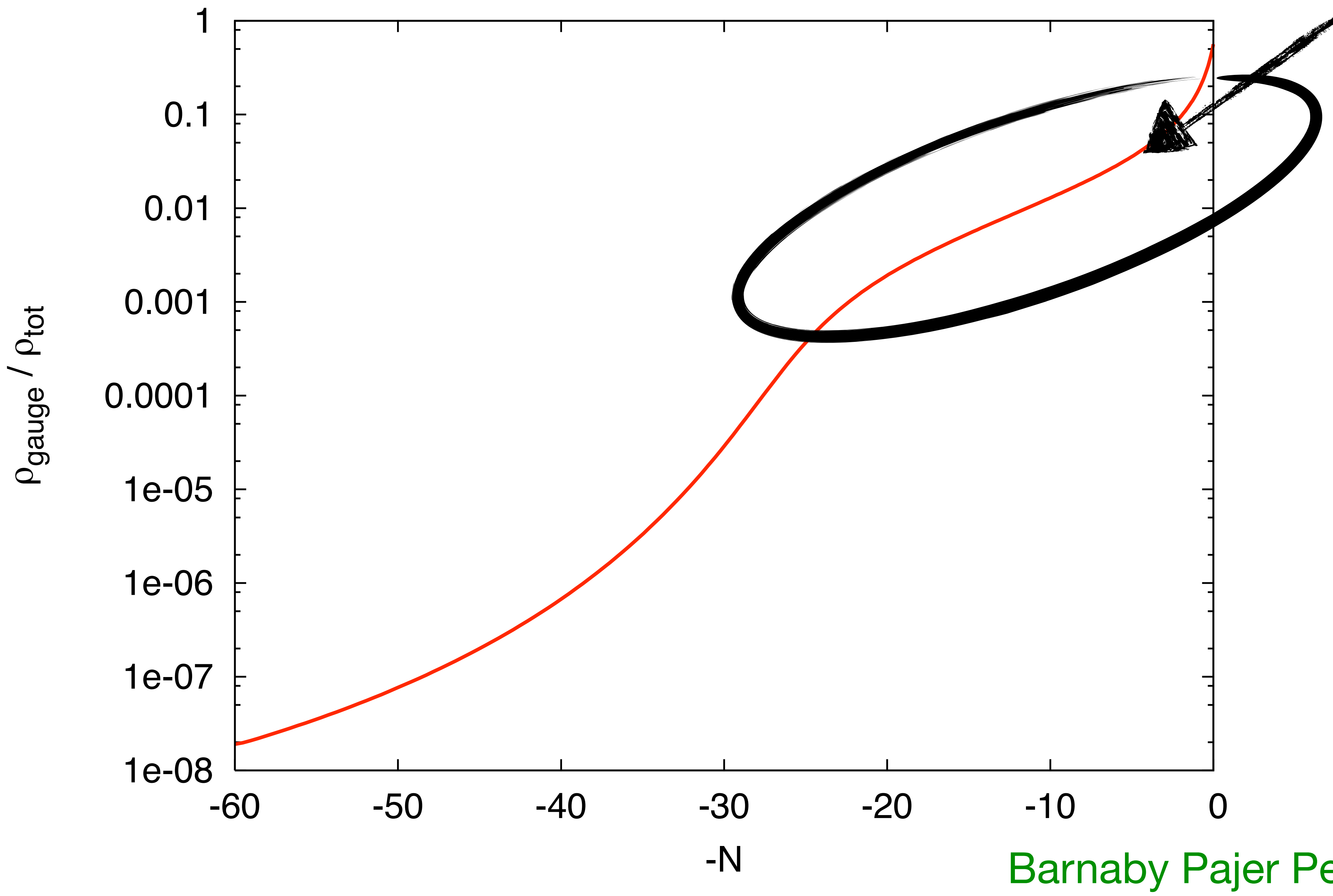
Backreaction!

with $\langle \mathbf{E}\cdot\mathbf{B}\rangle \propto e^{\pi\,\frac{\alpha}{f}\frac{|\dot{\phi}|}{H}}$

Increases during inflation

Backreaction in action

…an equation for ϕ only?



Barnaby Pajer Peloso 11

Strong backreaction

But remember $\langle \mathbf{E} \cdot \mathbf{B} \rangle = \int \mathbf{E}(\mathbf{k}) \cdot \mathbf{B}(\mathbf{k})\, d^3\mathbf{k}$

where $\mathbf{E}(\mathbf{k}, \mathrm{t})$ and $\mathbf{B}(\mathbf{k}, \mathrm{t})$ depend on $\mathbf{E}(\mathbf{k}, \mathrm{t'}<\mathrm{t})$, $\mathbf{B}(\mathbf{k}, \mathrm{t'}<\mathrm{t})$

Cannot use single equation local in time, **need numerics!**

$$\Phi'' + 2aH\Phi' + a^2 V' = -\frac{\alpha^2}{4\pi^2 a^3 f} \int dk\, k^2 \frac{\partial}{\partial \tau} |A_+|^2$$

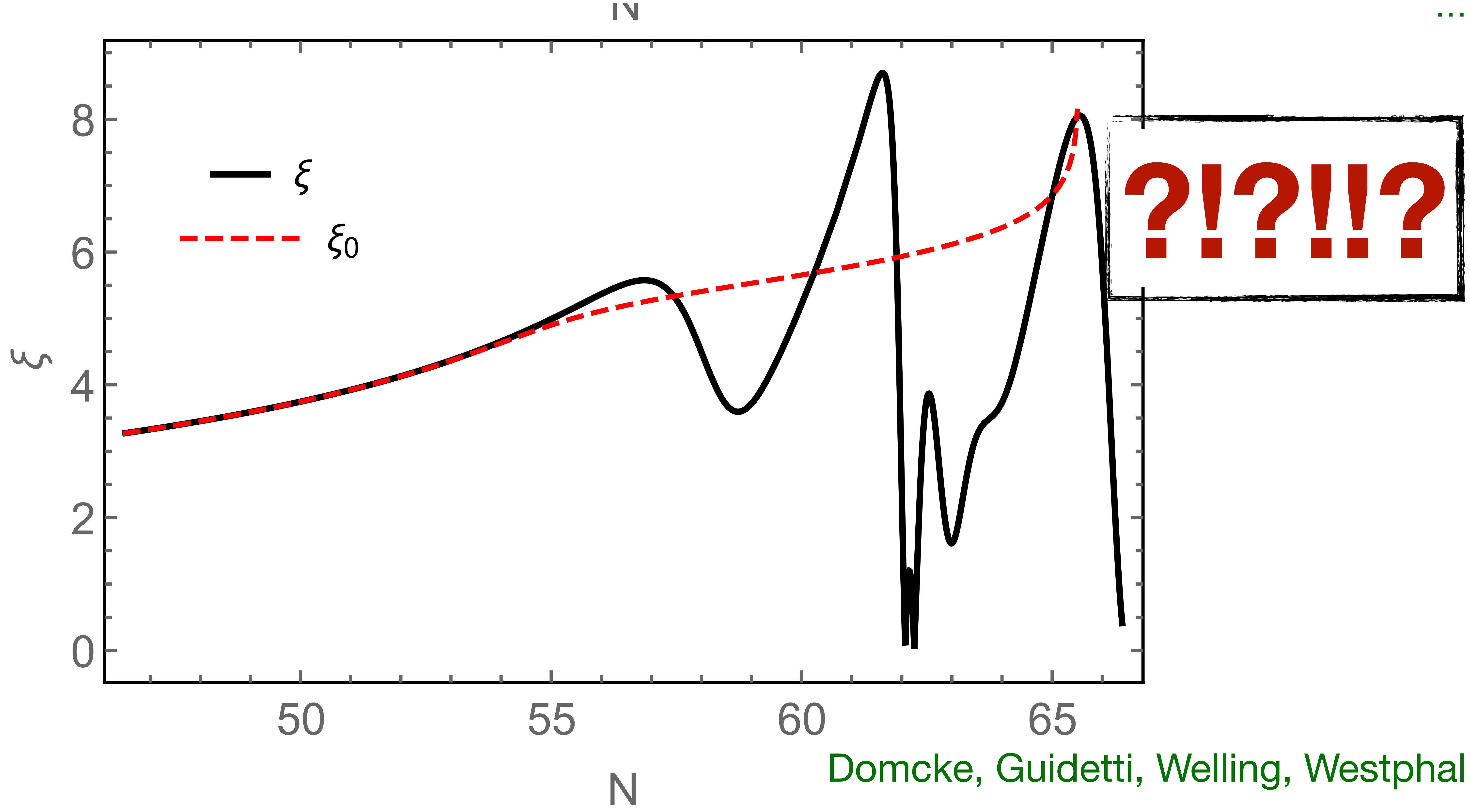
$$A_+'' + k^2 A_+ - \frac{\alpha\, \Phi'}{f} A_+ = 0$$

(neglecting inflation gradients and non-amplified helicity of gauge field)

Strong backreaction

Numerical result with uniform inflaton and one helicity of photon only

Cheng, Lee, Ng, 15
Notari, Tywoniuk 16
Dall'Agata, Gonzalez-Martin, Papageorgiou, Peloso 19
Domcke, Guidetti, Welling, Westphal 20
Gorbar, Schmitz, Sobol, Vilchinwskii 21
…



Domcke, Guidetti, Welling, Westphal 20

Strong backreaction

Where is this coming from?

Notari, Tywoniuk 16
Domcke, Guidetti, Welling, Westphal 20

$\langle \mathbf{E} \cdot \mathbf{B} \rangle$ does not react instantly to change in ξ

$$\ddot{\phi}(t) + 3\,H\,\dot{\phi}(t) + V'(\phi(t)) = -\frac{\alpha}{f}\langle \mathbf{E} \cdot \mathbf{B} \rangle(t)$$

$$\ddot{\phi}(t) + 3\,H\,\dot{\phi}(t) + V'(\phi(t)) = -\frac{\alpha}{f}\int^{t} K(t,\,t')\langle \mathbf{E} \cdot \mathbf{B} \rangle(t')\,dt' \simeq -\frac{\alpha}{f}\langle \mathbf{E} \cdot \mathbf{B} \rangle(t - \Delta t)$$

So…. why oscillations?

Baby example: try to solve $f'(t) = f(t+q)$, with q real

solution $f(t)=e^{a\,t}$, where a must satisfy $a=e^{a\,q}$

The function $e^{a\,q}-a$ has a negative minimum for $q<e^{-1}$

Two real roots a_1, a_2 for $q<e^{-1}$

Looking for complex $a=a_R+i\,a_I$…

Infinite solutions!

(with $q\,a_I \simeq \pi/2+n\,\pi$ at large n)

Of course,
NOT a Cauchy problem!

Strong backreaction

Incidentally…

If the gauge field were to react *instantaneously* to a change in $\phi(t)$

↓

effective equation local in time and no oscillations

Creminelli, Kumar, Salehian, Santoni 23

Is it possible to find a model where this happens?
Yes, if backreacting field has fully subhorizon dynamics

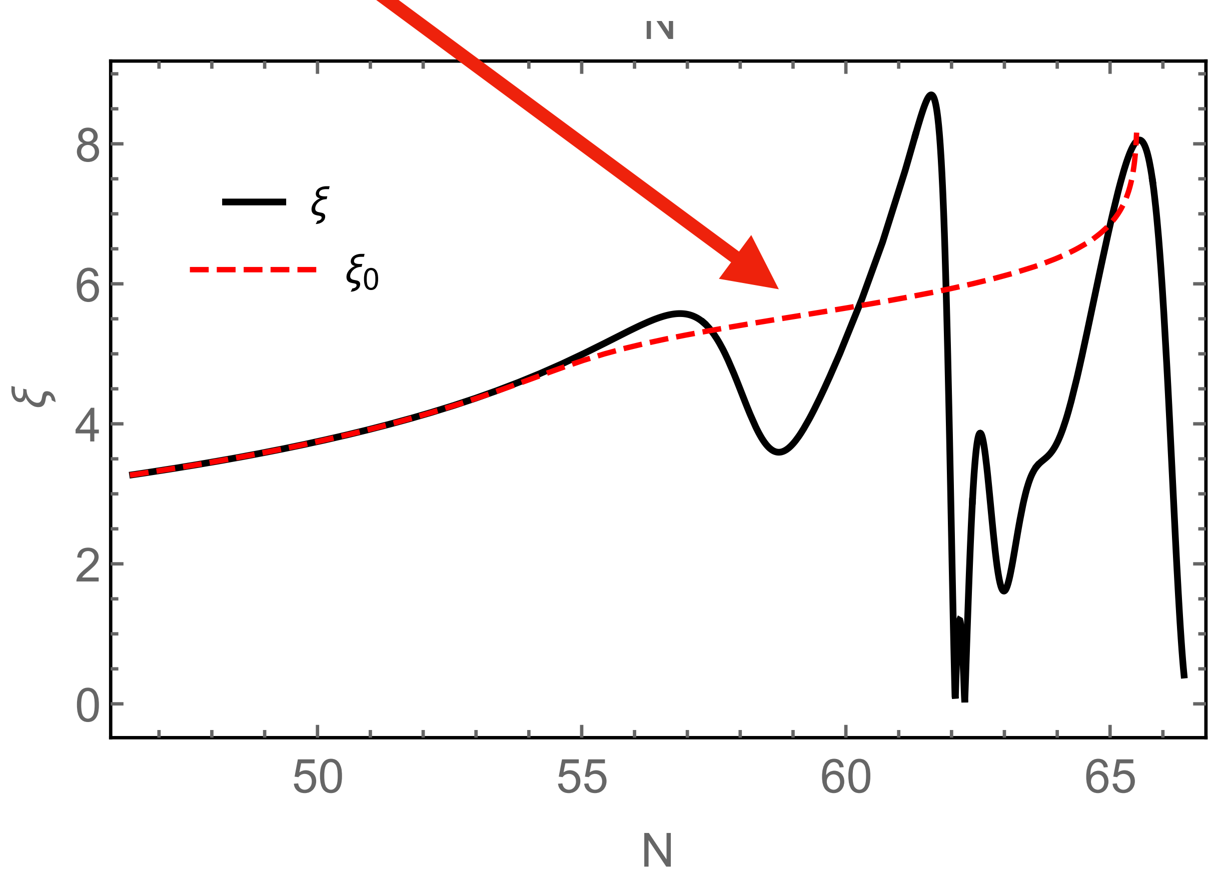
An analytical study

Analytical study for small perturbations around $\phi(t)=\overline{\Phi}(t)$…

Peloso, LS, 2209.08131

where… $$\ddot{\bar{\Phi}} + 3\,H\,\dot{\bar{\Phi}} + V'(\bar{\Phi}) = -\frac{\alpha}{f}\langle \mathbf{E}\cdot\mathbf{B}\rangle(H,\,\dot{\bar{\Phi}})$$

with the RHS computed assuming H, $\dot{\overline{\Phi}}(t)=const.$



…and around $A_+(t)=\overline{A_+}(t)$, also computed under the same assumption

An analytical study

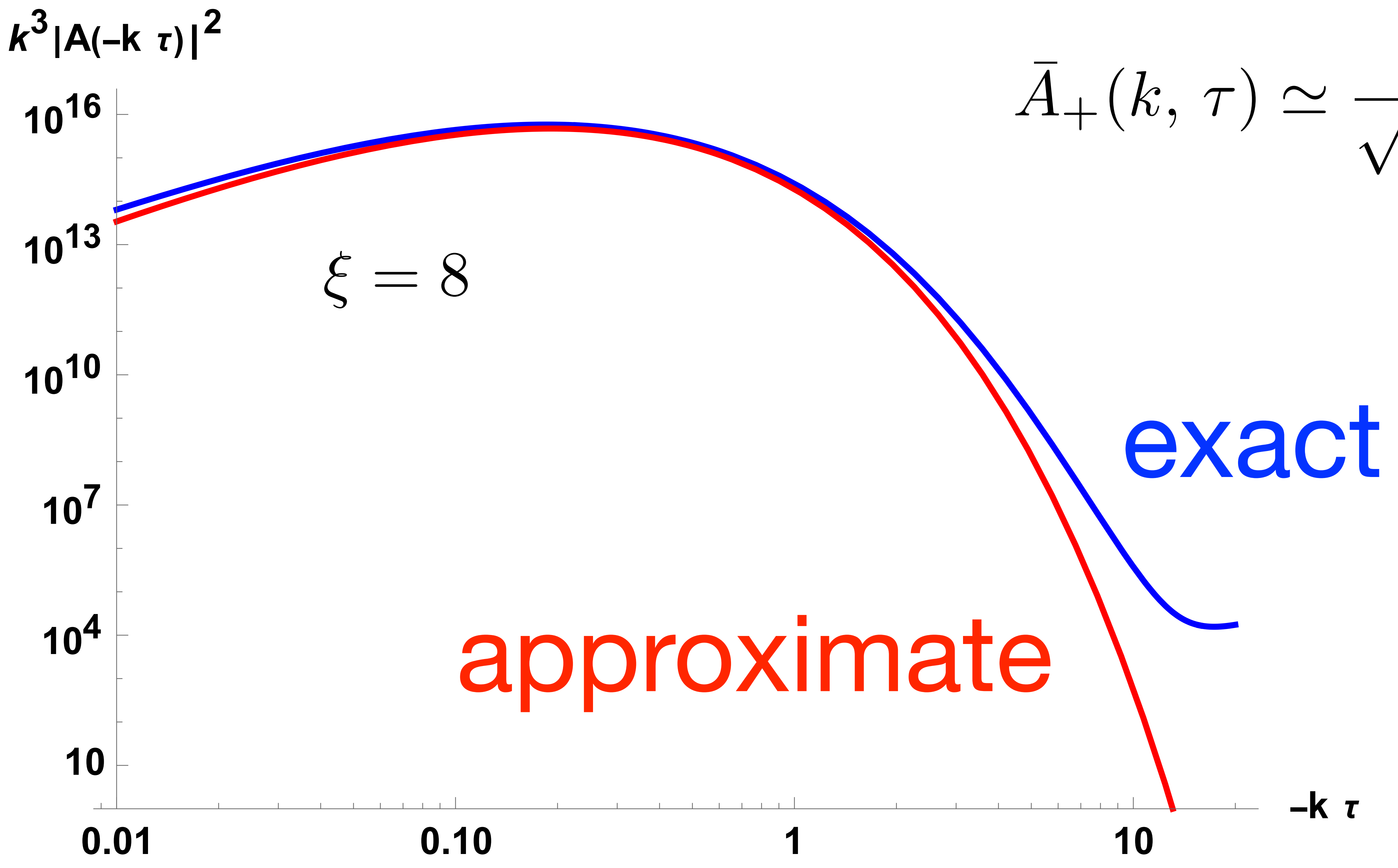
$$\xi \equiv \frac{\alpha\, \dot{\bar{\Phi}}}{2\, f\, H}$$

The unperturbed photon mode function…

$$\bar{A}_+(k,\,\tau) = \frac{1}{\sqrt{2\,k}}\,\frac{2^{1+i\xi}\,e^{-\pi\xi/2}}{\Gamma(i\xi+1)}\,e^{ik\tau}k\tau\left[\pi\xi\mathrm{csch}(\pi\xi)\,{}_1F_1(1-i\xi;2;-2ik\tau)+e^{\pi\xi}\Gamma(1-i\xi)U(1-i\xi,2,-2ik\tau)\right]$$ 😳

…approximated by…

$$\bar{A}_+(k,\,\tau) \simeq \frac{1}{\sqrt{2\,k}}\left(-\frac{k\tau}{2\xi}\right)^{1/4} e^{\pi\xi-2\sqrt{-2\,\xi\,k\,\tau}}$$ 😅



A linearized analysis

writing $\Phi = \bar{\Phi} + \delta\Phi \quad , \quad A = \bar{A} + \delta A$

linearized equations

$$\delta\Phi'' + 2aH\delta\Phi' + a^2V''\delta\Phi = -\frac{\alpha}{f a^2}\int \frac{d^3k}{(2\pi)^3}\frac{k}{2}\frac{\partial}{\partial\tau}\left[\bar{A}\,\delta A^* + \bar{A}^*\,\delta A\right] \; ,$$

$$\delta A'' + \left(k^2 - \frac{k\,\bar{\Phi}'}{f}\right)\delta A = \frac{\alpha\,\bar{A}}{f}\,\delta\Phi' \, ,$$

the second one solved with retarded propagator

$$\delta A\,(\tau,\, k) = \frac{\alpha\, k}{f}\int^{\tau} d\tau'\, G_k\left(\tau,\, \tau'\right)\, \bar{A}\left(\tau',\, k\right)\delta\Phi'\left(\tau'\right)$$

and then inserted into the first one

A linearized analysis

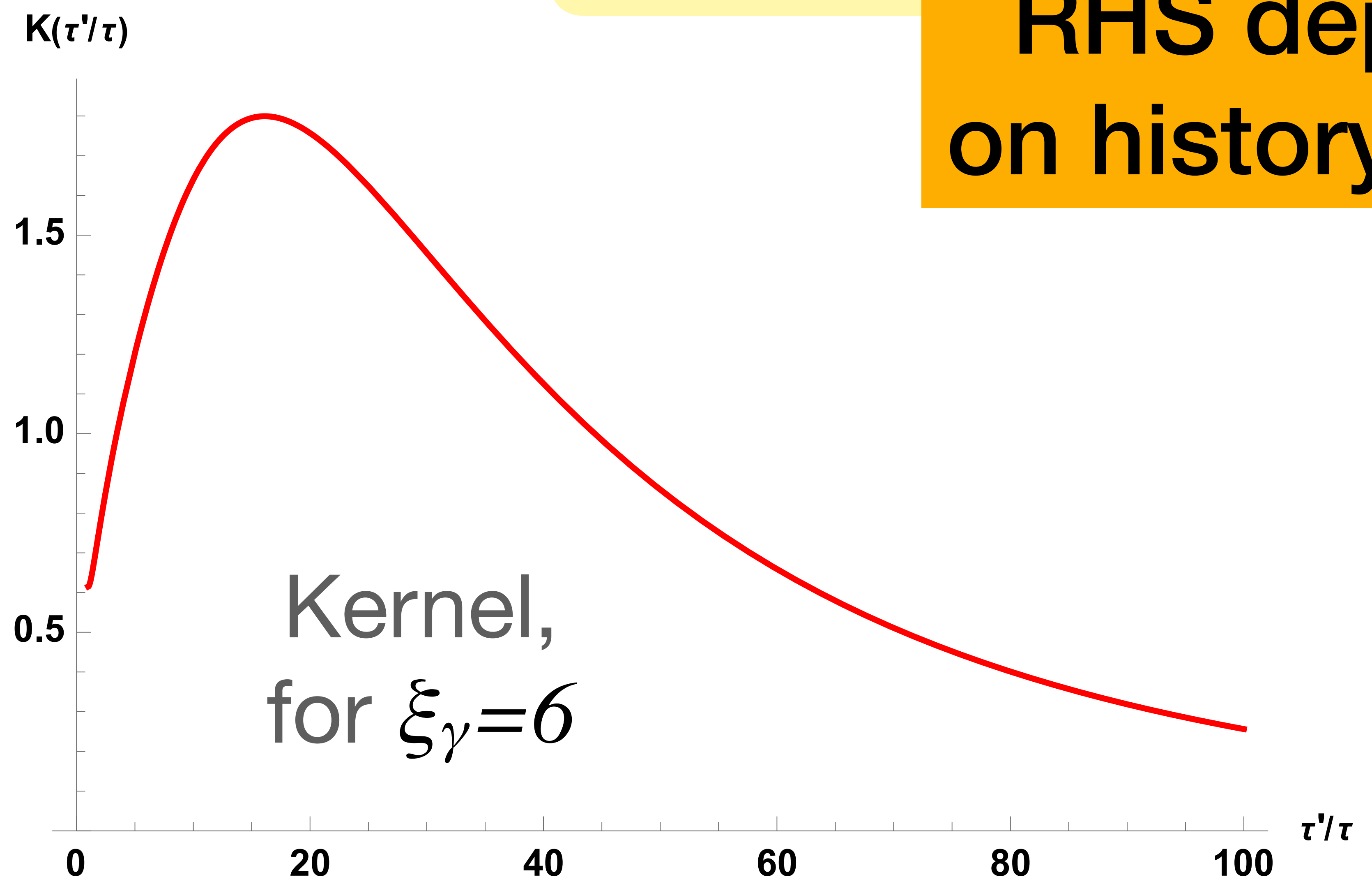
after some manipulation…

UV cutoff, ξ x $O(1)$ number

$$\delta\Phi'' + 2aH\delta\Phi' + a^2V''\delta\Phi = \frac{\alpha^2}{f^2a^2}\frac{e^{2\pi\xi}}{2^8\pi^2\xi^5}\int^\tau \frac{d\tau'}{(-\tau')^4}\delta\Phi'(\tau')\,\frac{\partial}{\partial\tau}\int_0^{4\xi_\gamma^2} dy\,y^3\sqrt{\tau\tau'}\left[e^{-4\sqrt{y}} - e^{-4\sqrt{y}\sqrt{\frac{-\tau}{-\tau'}}}\right]$$

RHS depends on history of $\delta\Phi$!

kernel peaked at $|\tau'|>|\tau|$!



A linearized analysis

after some manipulation…

$$\delta\Phi'' + 2aH\delta\Phi' + a^2V''\delta\Phi = \frac{\alpha^2}{f^2a^2}\frac{e^{2\pi\xi}}{2^8\pi^2\xi^5}\int^{\tau}\frac{d\tau'}{(-\tau')^4}\delta\Phi'\left(\tau'\right)\frac{\partial}{\partial\tau}\int_0^{4\xi_\gamma^2}dy\, y^3\sqrt{\tau\tau'}\left[e^{-4\sqrt{y}} - e^{-4\sqrt{y}\sqrt{\frac{-\tau}{-\tau'}}}\right]$$

UV cutoff, ξ x $O(1)$ number

RHS depends on history of $\delta\Phi$!

kernel peaked at $|\tau'|>|\tau|$!

look for solution $\delta\Phi \propto (-\tau)^{-\frac{1+\zeta}{2}}$

A linearized analysis

look for solution $\delta\Phi \propto (-\tau)^{-\frac{1+\zeta}{2}}$

where ζ must satisfy $\frac{\xi\, f\, V''}{\alpha\,(-V')} \simeq \frac{(1+\zeta)\,(7+\zeta)}{\zeta\,(8+\zeta)} \left[\frac{1}{(8\,\xi_\gamma)^\zeta}\,\frac{\Gamma\,(9+\zeta)}{\Gamma\,(9)} - 1\right]$

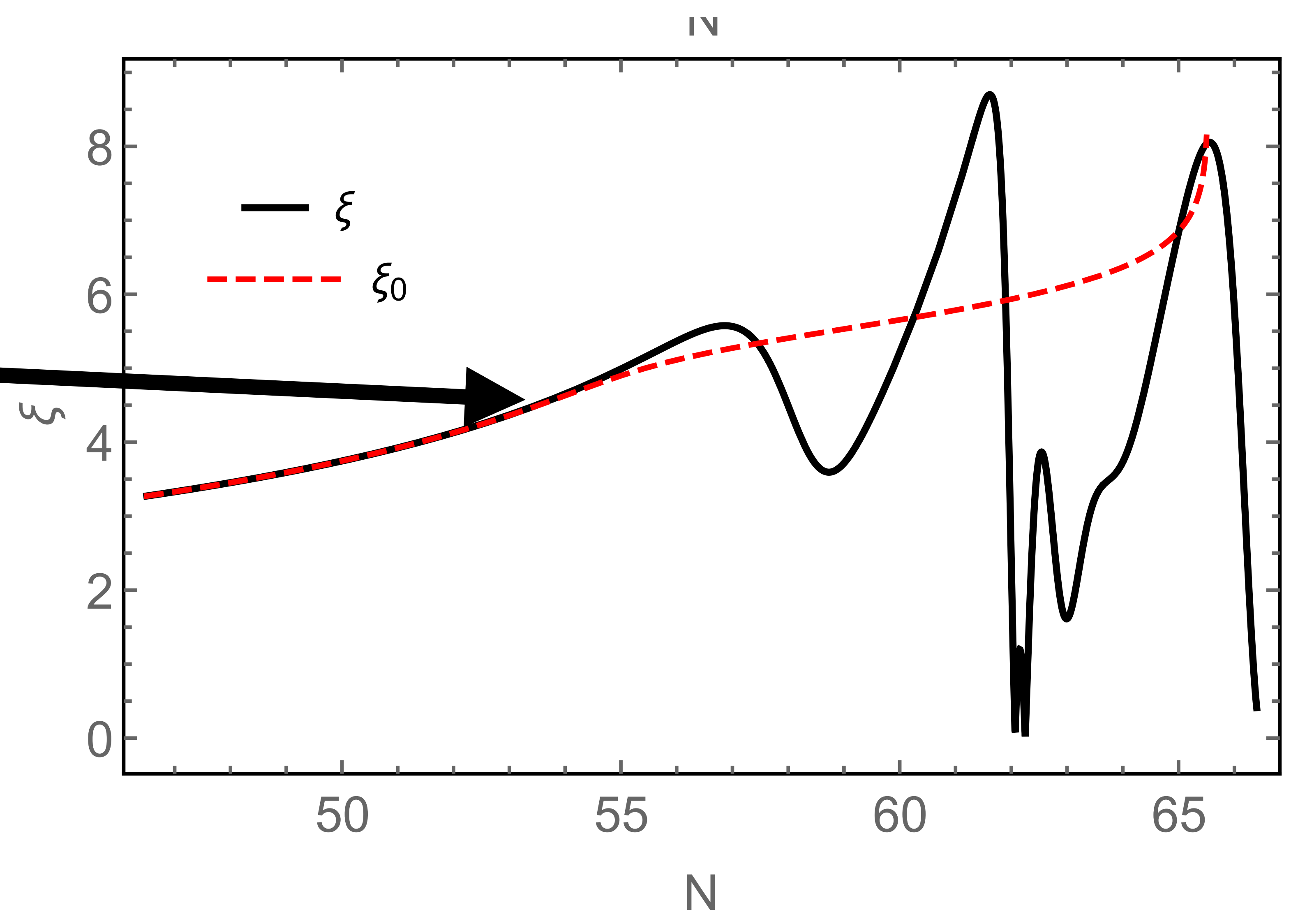
$\simeq 0$

Always at least one with ζ complex
and $Re[\zeta]<-1$!

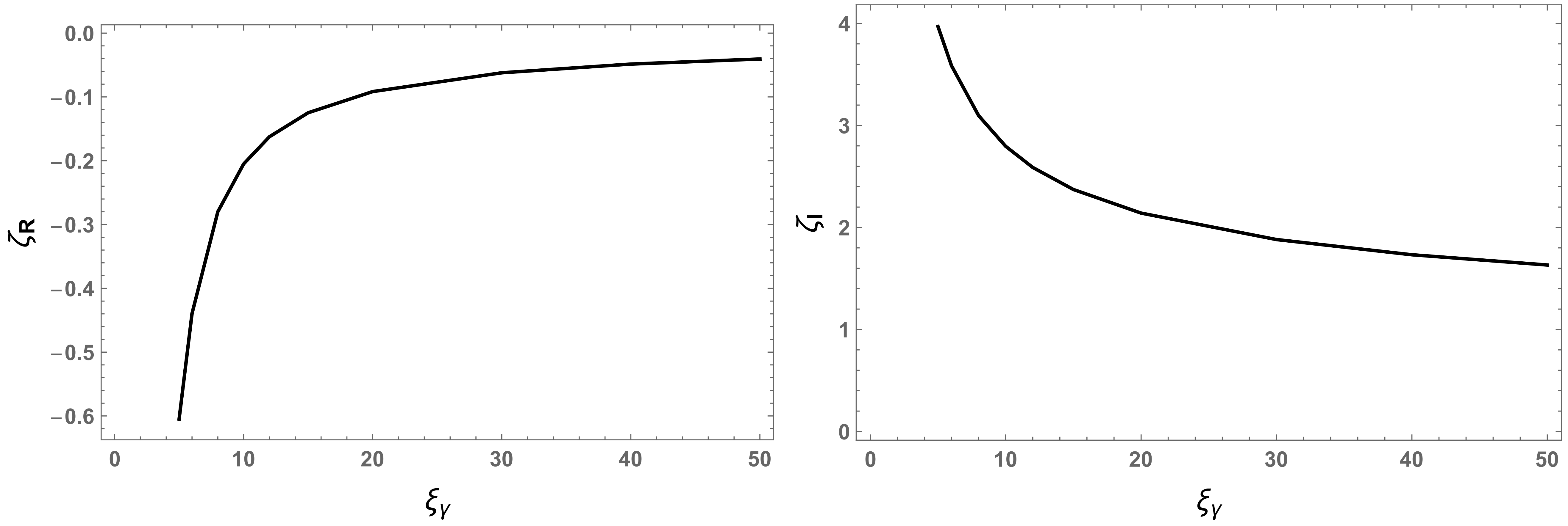
A linearized analysis

Always at least one with ζ complex
and $Re[\zeta]<-1$!

LINEARIZED SOLUTION GROWS AND OSCILLATES



A linearized analysis



Values of $\mathrm{Re}(\zeta)$ (left panel) and of $\mathrm{Im}(\zeta)$ (right panel) of the most unstable mode.

Properties

- weak dependence of ζ on ξ
- instability stronger with larger ξ
- instability always present
- period of oscillations, about 5 efoldings

Gravitational waves for interferometers

Inflationary gravitational waves for LIGO (LISA...)?

$$\mathcal{P}_L(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 9\times 10^{-7}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

$$\mathcal{P}_R(\mathbf{k}) = \frac{H^2}{\pi^2 M_P^2}\left(1 + 2\times 10^{-9}\,\frac{H^2}{M_P^2}\,\frac{e^{4\pi\xi}}{\xi^6}\right)$$

LS 2011

ξ increases during inflation

large amplitude at short (interferometer) scales

Cook, LS 2010

How does this change with more realistic $\xi(t)$?

Flashes of gravitational waves from axion inflation

Garcia-Bellido, Papageorgiou, Peloso, LS, 2303.13425

Numerical study of axion/gauge field system, then…

…integrate numerically with Green's function

$$h_{ij}(\mathbf{q},\,\tau) = \frac{2}{M_P^2}\int \underbrace{d\tau'\,G_q(\tau,\,\tau')}_{\text{Green's function}} \underbrace{\int \frac{d\mathbf{p}}{(2\pi)^{3/2}}\left(F_{0i}(\mathbf{p},\,\tau')F_{0j}(\mathbf{q}-\mathbf{p},\,\tau') - F_{ik}(\mathbf{p},\,\tau')F_{jk}(\mathbf{q}-\mathbf{p},\,\tau')\right)}_{\text{gauge field stress-energy}}$$

Green's function

gauge field stress-energy

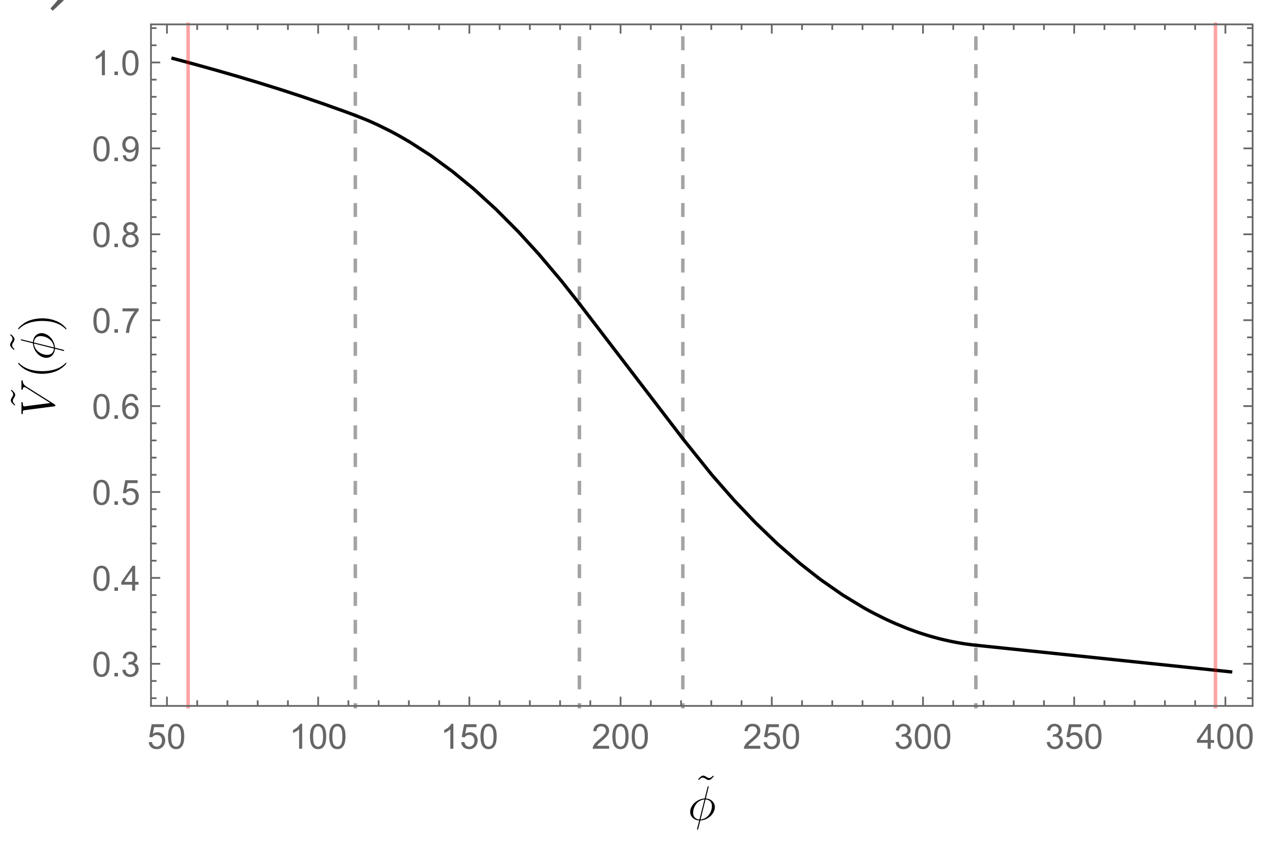
Gravitational waves for interferometers

Need numerical solution of background

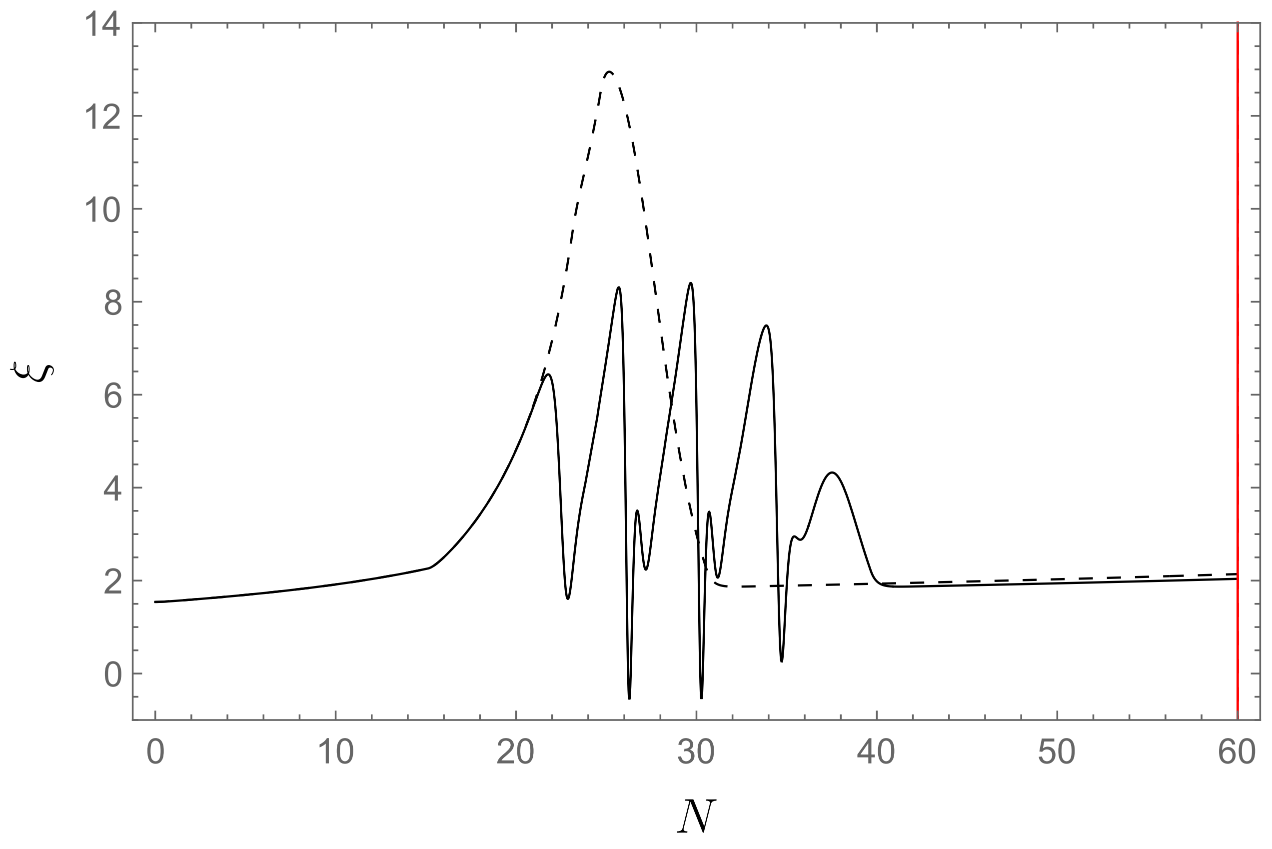
Garcia-Bellido, Papageorgiou, Peloso, LS, 2303.13425

Example for steep-ish potential @ intermediate times

$V(\phi)$



$\xi(N)$

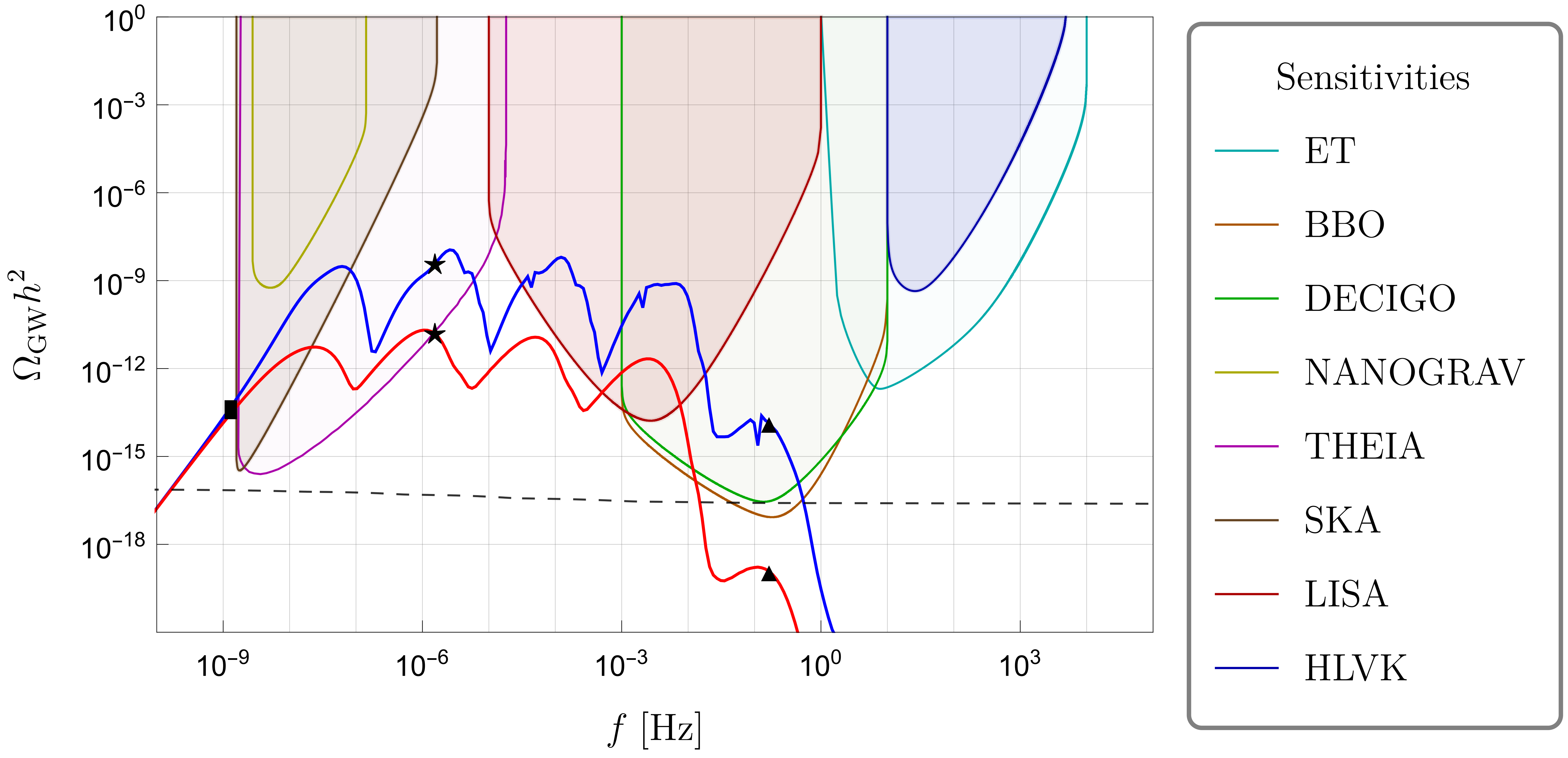


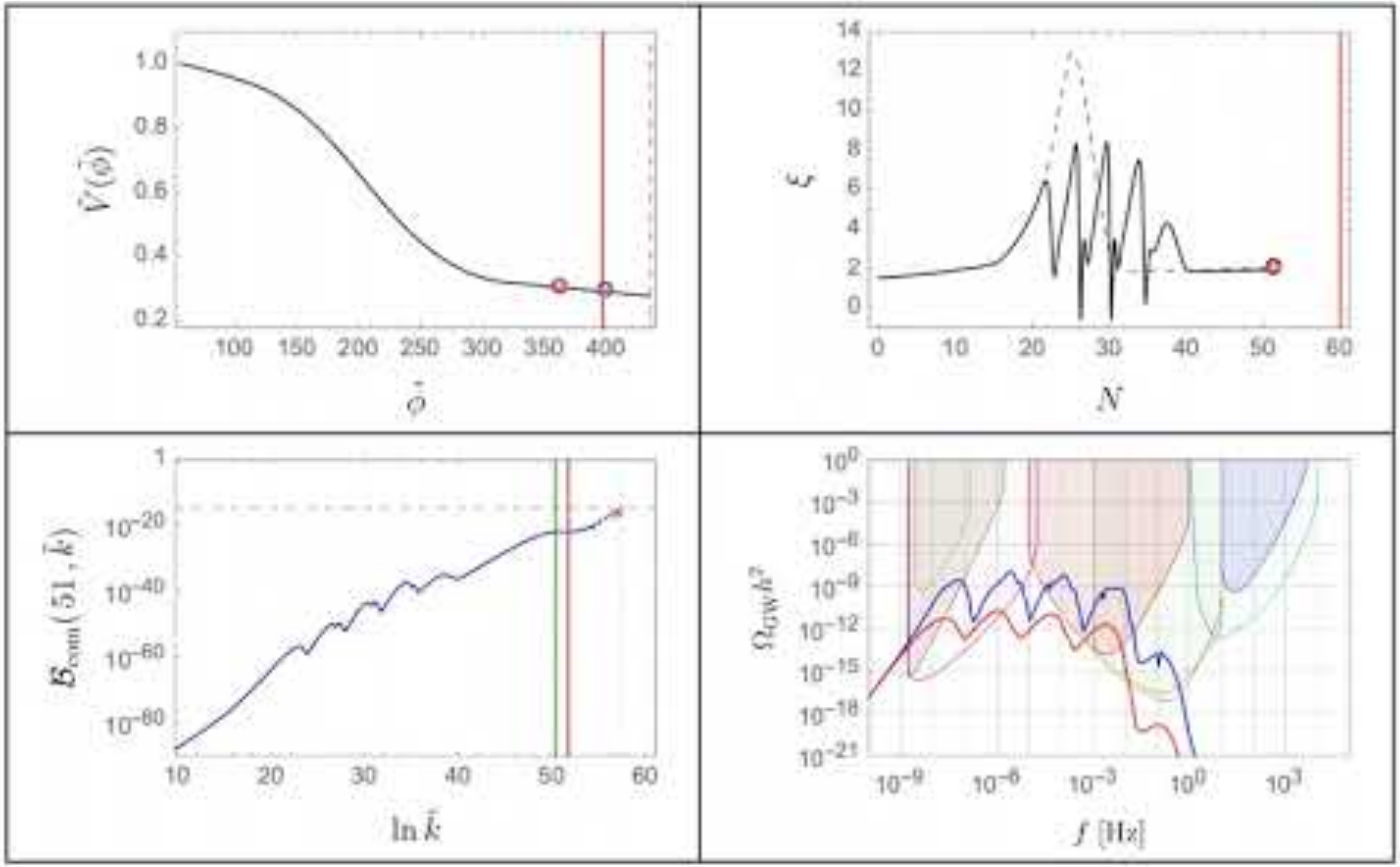
Gravitational waves for interferometers

Flashes of gravitational waves from axion inflation

Garcia-Bellido, Papageorgiou, Peloso, LS, 2303.13425

Example for steep-ish potential @ intermediate times





1.0
0.8
0.6
0.4
0.2
$\tilde{V}(\tilde{\phi})$
100 150 200 250 300 350 400
$\tilde{\phi}$
14
12
10
8
6
4
2
0
ξ
0 10 20 30 40 50 60
N
1
10^{-20}
10^{-40}
10^{-60}
10^{-80}
$\tilde{B}_{\rm com}(51, \tilde{k})$
10 20 30 40 50 60
$\ln \tilde{k}$
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-18}
10^{-21}
$\Omega_{\rm GW} h^2$
10^{-9} 10^{-6} 10^{-3} 10^{0} 10^{3}
f [Hz]

A more detailed study of the development of the instability

von Eckardstein, Peloso, Schmitz, Sobol, LS, 2309.04254

Gradient expansion method

Define

$$\mathcal{E}^{(n)} \equiv \frac{1}{a^n}\,\langle \boldsymbol{E} \cdot \mathrm{rot}^n\, \boldsymbol{E} \rangle \ ,$$
$$\mathcal{G}^{(n)} \equiv -\frac{1}{2a^n}\,\langle \boldsymbol{E} \cdot \mathrm{rot}^n\, \boldsymbol{B} + \mathrm{rot}^n\, \boldsymbol{B} \cdot \boldsymbol{E} \rangle \ ,$$
$$\mathcal{B}^{(n)} \equiv \frac{1}{a^n}\,\langle \boldsymbol{B} \cdot \mathrm{rot}^n\, \boldsymbol{B} \rangle \ ,$$

and rewrite Maxwell's equations as the tower

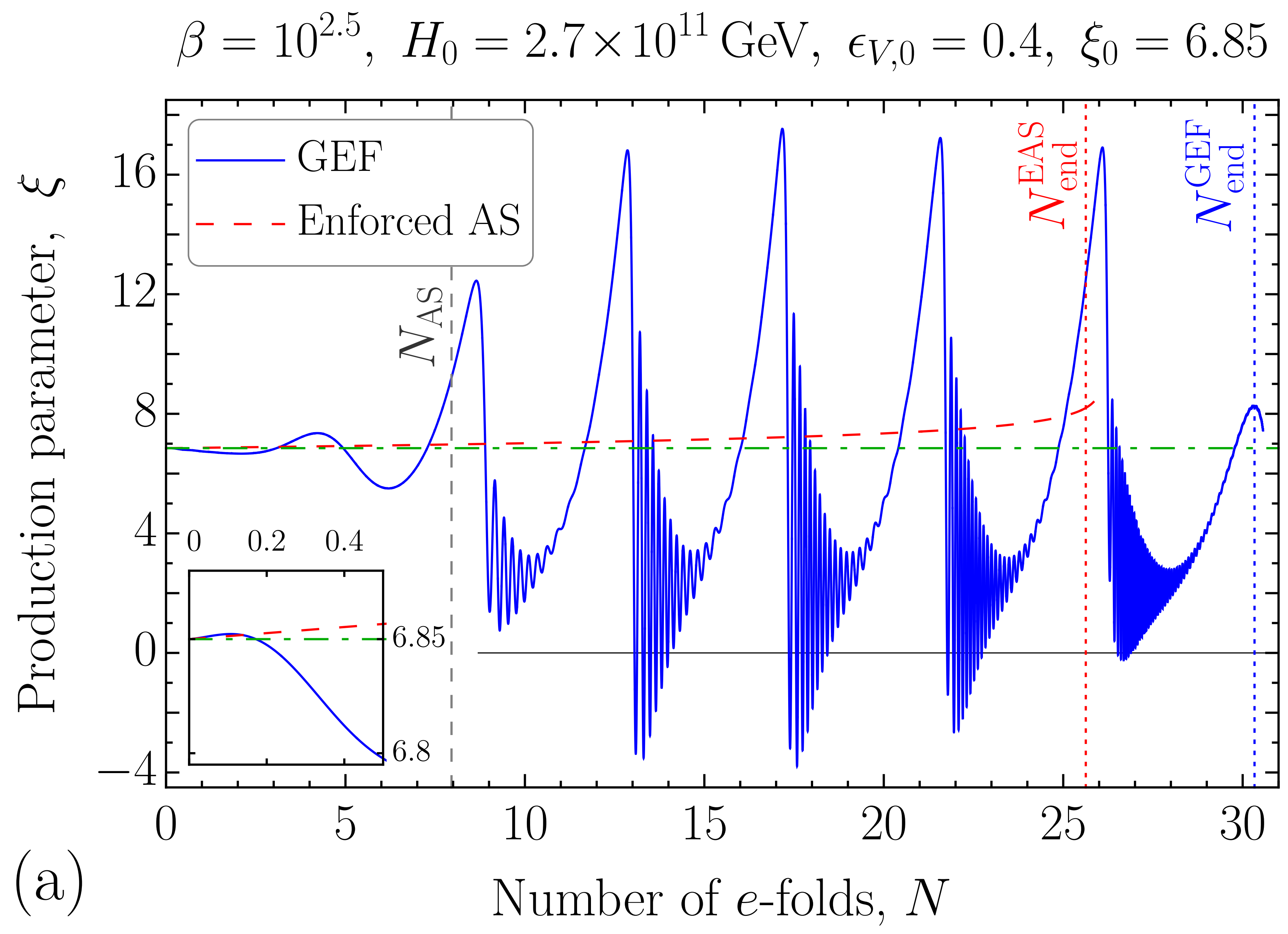
$$\dot{\mathcal{E}}^{(n)} + (n+4)H\,\mathcal{E}^{(n)} - \frac{2\beta}{M_{\mathrm{P}}}\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = S_{\mathcal{E}} \,,$$
$$\dot{\mathcal{G}}^{(n)} + (n+4)H\,\mathcal{G}^{(n)} - \frac{\beta}{M_{\mathrm{P}}}\dot{\phi}\,\mathcal{B}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} = S_{\mathcal{G}} \,,$$
$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = S_{\mathcal{B}} \,.$$

boundary terms, can be accounted for

Gradient expansion

A more detailed study of the development of the instability

von Eckardstein, Peloso, Schmitz, Sobol, LS, 2309.04254



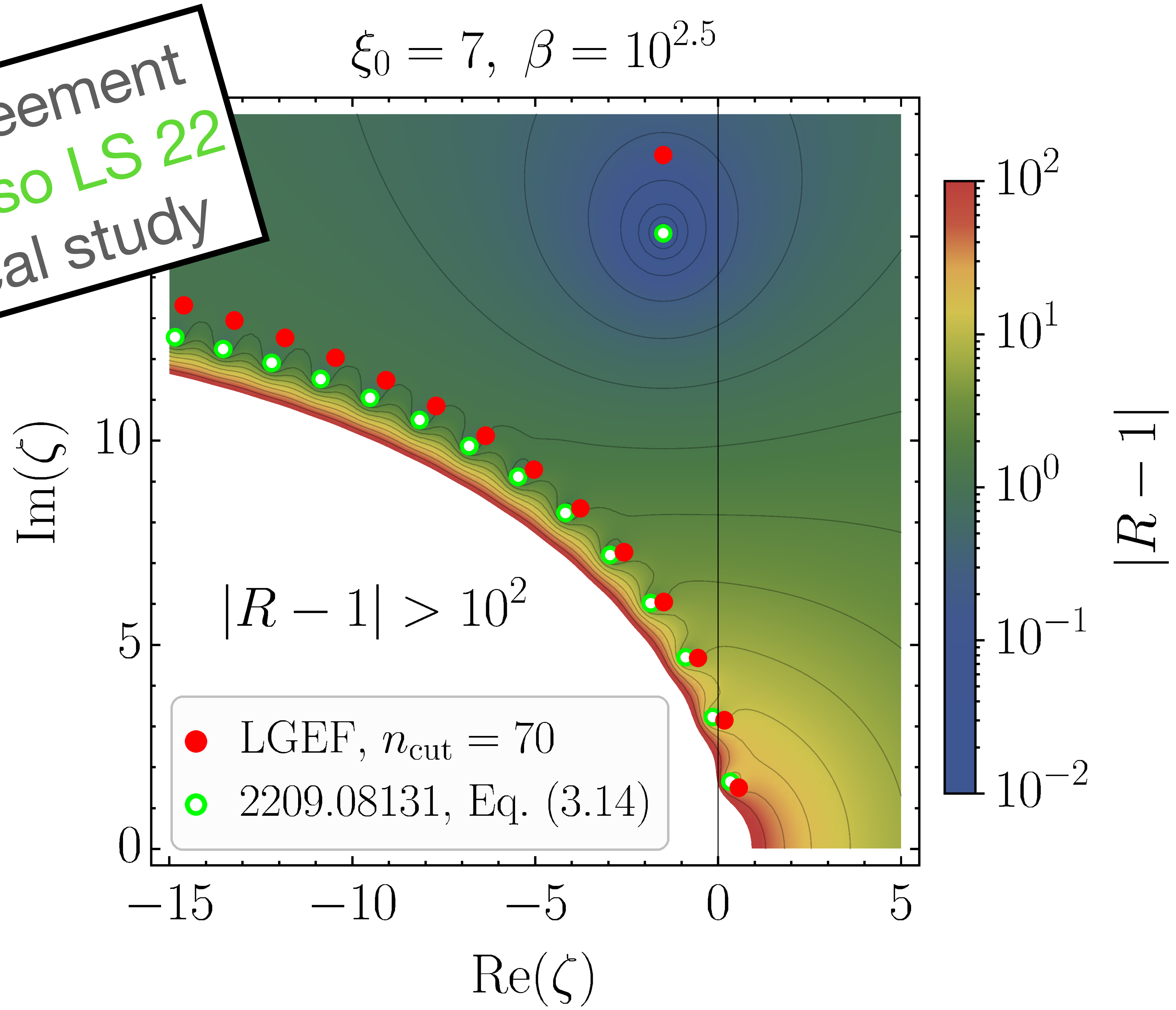
Instability excited by slow-roll evolution of background

A more detailed study of the development of the instability

von Eckardstein, Peloso, Schmitz, Sobol, LS, 2309.04254

Linearizing the tower of gradient equations…

good agreement with Peloso LS 22 analytical study

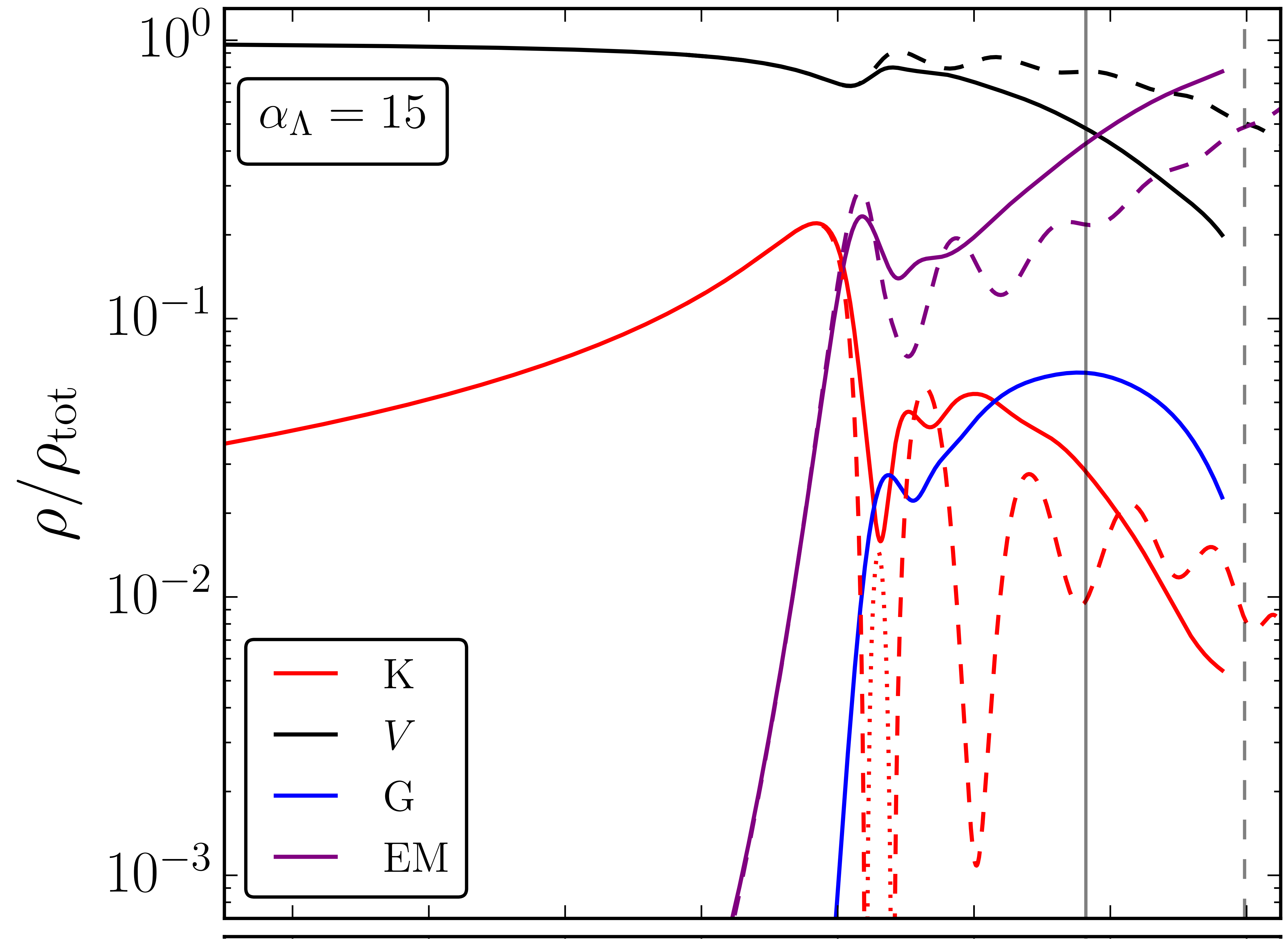


Lyapunov exponents

How about the inflaton gradients?

Only two (lattice) studies so far deal with inflaton gradients

Caravano, Komatsu, Lozanov, Weller 22
Figueroa, Lizarraga, Urio, Urrestilla 23



Inflaton gradients appear to be large and to affect the dynamics a lot!

Maybe a small gradient expansion can be treated analytically?

To sum up…

Axion inflaton/gauge dynamics even richer than thought

- Need mostly numerical work to analyze
- But some analytical results are possible!
- **IMPORTANT** role of $\nabla\phi$ here not fully clear yet!
- …or maybe more semi-analytical methods?