Cosmic expansion versus motion: Probing the difference

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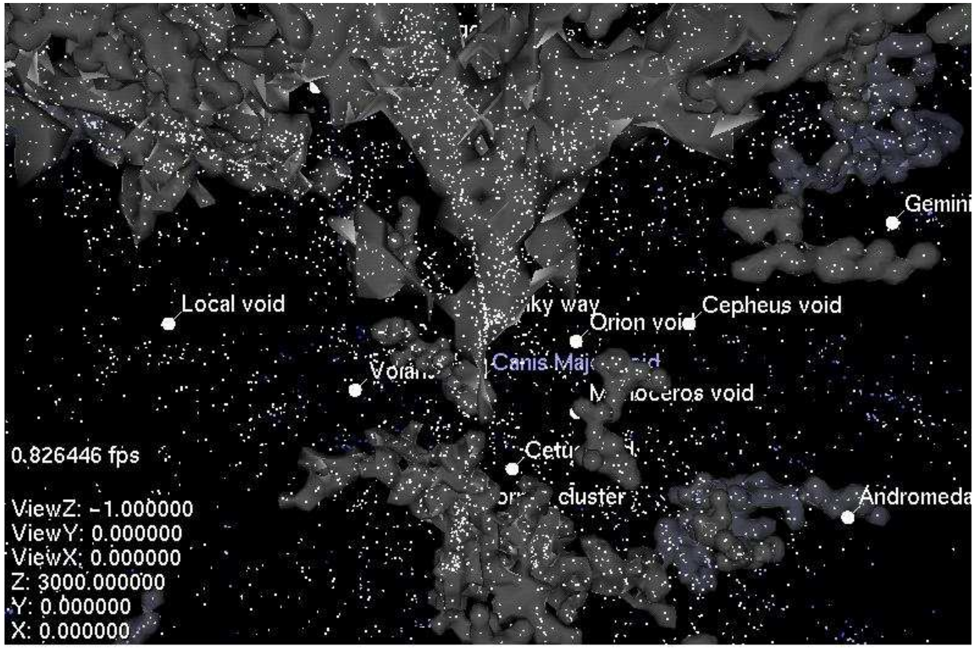

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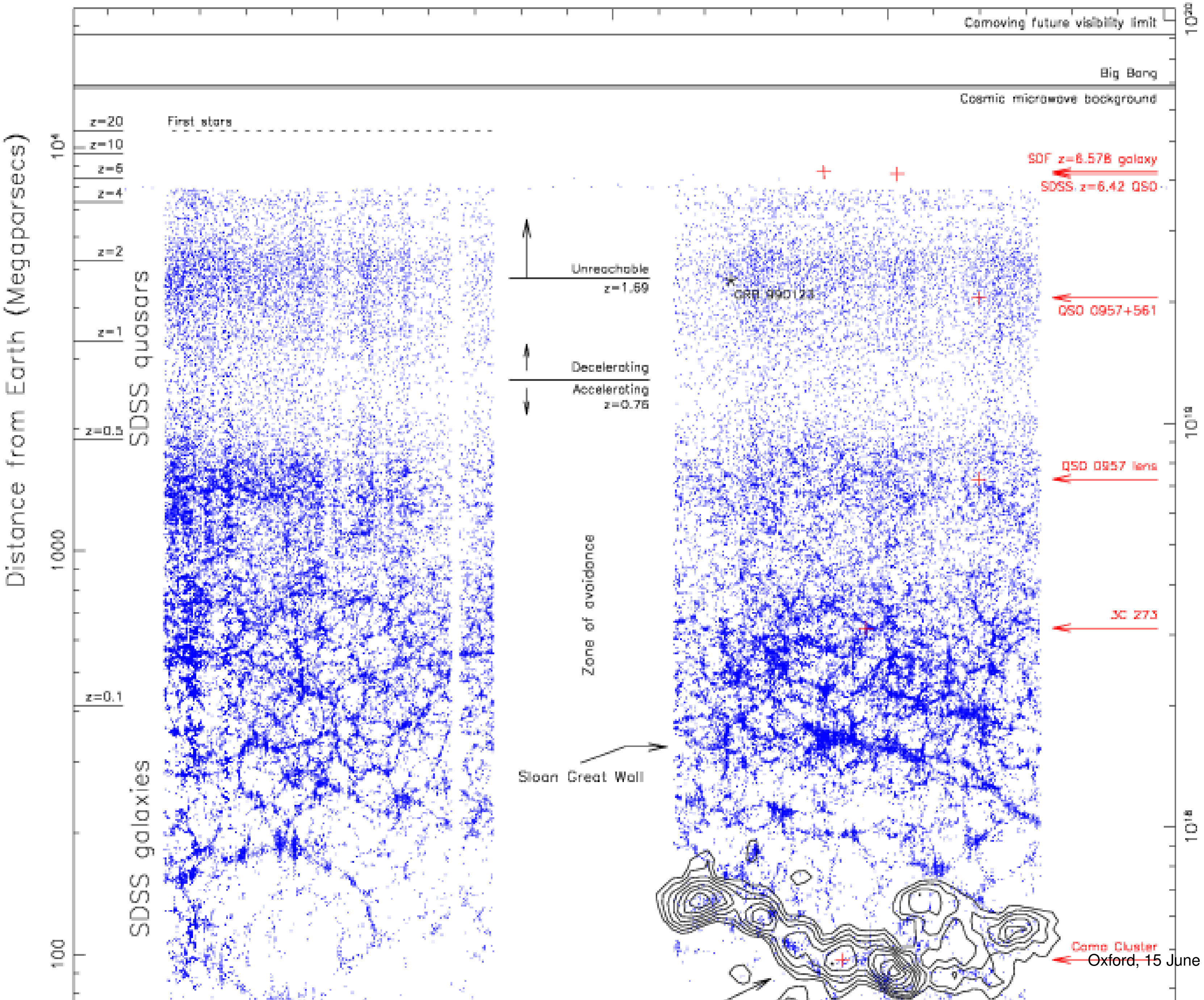
Outline of talk

- Cosmology: Quest for 2 numbers H_0, q_0 now a quest for 2 functions: $H(z)$, $D(z)$
- Can test foundations: e.g., Friedmann equation
- What is dark energy? Hypothesis:
  Dark energy is a misidentification of gradients in quasilocal kinetic gravitational energy in the geometry of an evolving structure of matter inhomogeneities
- Conceptual basis
- [*Present and future tests of timescape cosmology*:
  - *Supernovae, BAO, CMB, . . .* NOT TODAY]
  - Clarkson-Bassett-Lu test
- Variation of expansion
  - CMB anomalies and Ellis–Baldwin test

1. Important facts and concepts



Cosmic web: typical structures

- Galaxy clusters, 2 – 10 h^{-1}Mpc, form filaments and sheets or “walls” that thread and surround voids
- Universe is void dominated (60–80%) by volume, with distribution peaked at a particular scale (40% of total volume), with $\delta_\rho \equiv (\rho - \bar{\rho})/\bar{\rho}$, i.e., $\delta_\rho = -1$ if $\rho = 0$:

Survey	Void diameter	Density contrast
PSCz	$(29.8 \pm 3.5)h^{-1}$Mpc	$\delta_\rho = -0.92 \pm 0.03$
UZC	$(29.2 \pm 2.7)h^{-1}$Mpc	$\delta_\rho = -0.96 \pm 0.01$
2dF NGP	$(29.8 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$
2dF SGP	$(31.2 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$

Dominant void statistics in the Point Source Catalogue Survey (PSCz), the Updated Zwicky Catalogue (UZC), and the 2 degree Field Survey (2dF) North Galactic Pole (NGP) and South Galactic Pole (SGP), (Hoyle and Vogeley 2002,2004). More recent results of Pan et al. (2011) using SDSS Data Release 7 similar.

Statistical homogeneity scale (SHS)

- Modulo debate *some notion* of statistical homogeneity reached on 70–100 h^{-1}Mpc scales based on 2–point galaxy correlation function
- Also observe $\delta\rho/\rho \sim 0.07$ on scales $\gtrsim 100\,h^{-1}$Mpc (bounded) in largest survey volumes; no evidence yet for $\langle\delta\rho/\rho\rangle_{\mathcal{D}} \to \epsilon \ll 1$ as $\mathrm{vol}(\mathcal{D}) \to \infty$
- SHS close to Baryon Acoustic Oscillation (BAO) scale in galaxy clustering statistics.
- Explanation: evolution of $\delta\rho/\rho \sim 10^{-4} A_{\langle\ell<\ell_{\rm BAO}\rangle}/A_{\rm peak}$ since last scattering [PR D80 (2009) 123512].
- No direct evidence for FLRW spatial geometry below SHS (although assumed, e.g., defining boost of Local Group wrt CMB rest frame)

What is a cosmological particle (dust)?

- In FLRW one takes observers "comoving with the dust"
- Traditionally galaxies were regarded as dust. However,
  - Galaxies, clusters not homogeneously distributed today
  - Dust particles should have (on average) invariant masses over the timescale of the problem
- Must coarse-grain over expanding fluid elements larger than the largest typical structures [voids of diameter $30\,h^{-1}$Mpc with $\delta_\rho \sim -0.95$ are $\gtrsim 40\%$ of $z = 0$ universe]

$$\left.\begin{array}{c} g^{\rm stellar}_{\mu\nu} \to g^{\rm galaxy}_{\mu\nu} \to g^{\rm cluster}_{\mu\nu} \to g^{\rm wall}_{\mu\nu} \\ \vdots \\ g^{\rm void}_{\mu\nu} \end{array}\right\} \to g^{\rm universe}_{\mu\nu}$$

Averaging and backreaction

- *Fitting problem* (Ellis 1984):
  On what scale are Einstein's field equations valid?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- In general $\langle G^\mu{}_\nu(g_{\alpha\beta})\rangle \neq G^\mu{}_\nu(\langle g_{\alpha\beta}\rangle)$
- *Weak backreaction*: *Assume* global average is an exact (FLRW) solution of Einstein's equations on large scale
- *Strong backreaction*: Fully nonlinear
  - Einstein's equations are causal; no need for them on scales larger than light has time to propagate
  - Must extend principles of GR to explain and quantify non-Friedmann but near homogeneous, isotropic average expansion

That debate? Resolved?

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Class. Quantum Grav. **23** (2006) 235–250 doi:10.1088/0264-9381/23/1/012

Can the acceleration of our universe be explained by the effects of inhomogeneities?

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Abstract

No, it is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or 'dark energy'. We point out that our universe appears to be described very accurately on all scales by a Newtonianly perturbed FLRW metric. (This assertion is entirely

That debate? But I agreed!

It therefore is manifest that nonlinear corrections[4] to the dynamics of the universe will be negligible, i.e., there will be no important 'back-reaction' effects of the inhomogeneities on the observed expansion of the universe on large scales. In particular, accelerated expansion cannot occur if the smoothly distributed matter satisfies the strong–energy condition. However, our assertion that the metric, equation (1), very accurately describes our universe is merely an assertion, and we cannot preclude the possibility that other models (e.g., with large amplitude, long-wavelength gravitational waves or with matter density inhomogeneities of a different type) might also fit observations. Our main point of this paper, however, is that if one wishes to propose an alternative model, then it is necessary to show that all of the predictions of this model are compatible with observations such as the observed redshift–luminosity relation for type Ia supernovae and the various observed properties of the cosmological microwave background (CMB) radiation. As we shall illustrate in the next two sections, it does not suffice to show merely that the spatially averaged scale factor behaves in a desired way or that an effective stress–energy tensor is of a desired form.

That debate? But I agreed!

- Setting aside assertions, Ishibashi & Wald (2006) note Buchert's spatial averaging formalism is statistical
  - Quantifying magnitude of backreaction is not enough
  - What does time parameter (spatial foliations) mean?
  - How are observables related to spatial averages?
- Timescape model [New J. Phys. 9 (2007) 377; Phys. Rev. Lett. 99 (2007) 251101]
  - Addressed the above concerns
  - Returned to unanswered foundational questions in GR: fitting problem (G Ellis 1984); quasilocal energy; limits of strong equivalence principle
  - Provides a *phenomenological* model – confronts data (supernovae, GRBs, CMB, BAOs, . . . **bulk 'flows'** . . .

That debate? Continued

IOP Publishing Classical and Quantum Gravity

Class. Quantum Grav. **31** (2014) 234003 (16pp) doi:10.1088/0264-9381/31/23/234003

How well is our Universe described by an FLRW model?

Stephen R Green[1] **and Robert M Wald**[2]

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Abstract

Extremely well! In the ΛCDM model, the spacetime metric, g_{ab}, of our Universe is approximated by an FLRW metric, $g_{ab}^{(0)}$, to about one part in 10^4 or

That debate? The fine print

In an exactly similar manner, the *spacetime* metric of our Universe takes the form[4] $g_{ab} = g^{(0)}_{ab} + \gamma_{ab}$, where $g^{(0)}_{ab}$ has FLRW symmetry and the components of γ_{ab} are extremely small relative to $g^{(0)}_{ab}$—at the level of at most about one part in 10^4. This is true on all

[4] It is fair to ask how we 'know' the facts asserted in this paragraph. As with all scientific 'knowledge,' our beliefs are based on having a small set of simple assumptions that account for a vast amount of disparate data in a mathematically consistent manner. The ΛCDM model is based upon a simple set of assumptions and successfully accounts for a vast amount of disparate data. The results summarized in this article confirm that it is mathematically consistent. Our figure of 'one part in 10^4' comes from Newtonian cosmological simulations, which yield values of the Newtonian potential in the present Universe no larger than $\sim 10^{-4}$ (as occurs near the center of the richest galaxy clusters); our dictionary (see section 4) implies that the spacetime metric deviations from FLRW are of the same size as the Newtonian potential.

- Green and Wald provide a *self-consistency* check *assuming* that average cosmic expansion an *exact* solution of Einstein's field on *any* scale of averaging
- Unlike timescape their approach leaves *unaddressed* the foundational questions of GR (fitting problem, quasilocal energy, equivalence problem, . . .)

That debate? The other side

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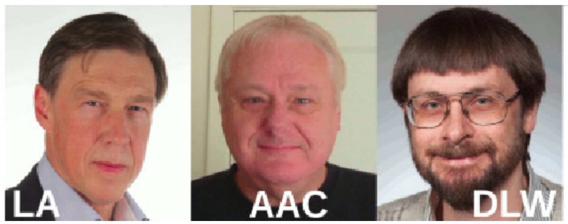
Class. Quantum Grav. **32** (2015) 215021 (38pp) doi:10.1088/0264-9381/32/21/215021

Is there proof that backreaction of inhomogeneities is irrelevant in cosmology?

Abstract

No. In a number of papers, Green and Wald argue that the standard FLRW model approximates our Universe extremely well on all scales, except close to strong-field astrophysical objects. In particular, they argue that the effect of inhomogeneities on average properties of the Universe (backreaction) is irrelevant. We show that this latter claim is not valid. Specifically, we



That debate? Resolved

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Class. Quantum Grav. **36** (2019) 205004 (33pp)   https://doi.org/10.1088/1361-6382/ab3a14

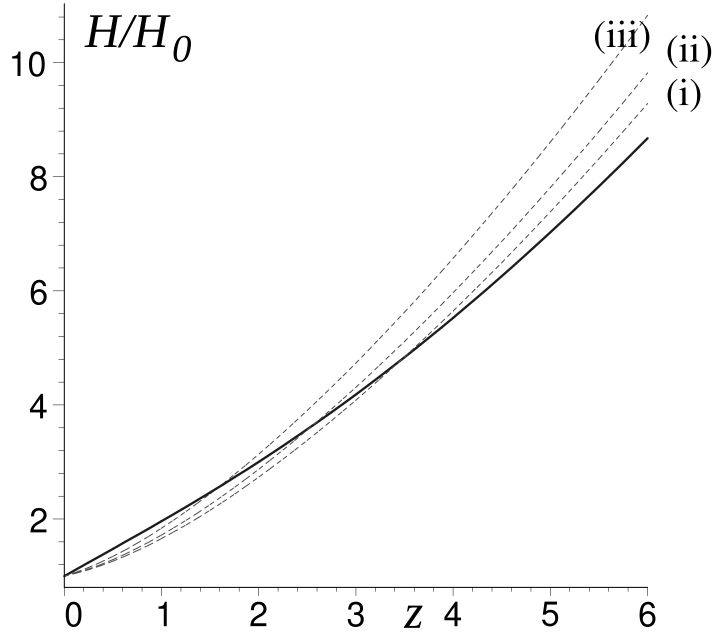
Cosmological backreaction in spherical and plane symmetric dust-filled space-times

Timothy Clifton[1] and Roberto A Sussman[2]

Abstract

We examine the implementation of Buchert's and Green & Wald's averaging formalisms in exact spherically symmetric and plane symmetric dust-filled cosmological models. We find that, given a cosmological space-time, Buchert's averaging scheme gives a faithful way of interpreting the large-scale expansion of space, and explicit terms that precisely quantify deviations from the behaviour expected from the Friedmann equations of homogeneous and isotropic cosmological models. The Green & Wald formalism, on the other hand, does not appear to yield any information about the large-scale properties of a given inhomogeneous space-time. Instead, this formalism is designed to calculate the back-reaction effects of short-wavelength fluctuations around a given 'background' geometry. We find that the inferred expansion of space in this approach is entirely dependent on the choice of this background, which is not uniquely specified for any given inhomogeneous space-time, and that the

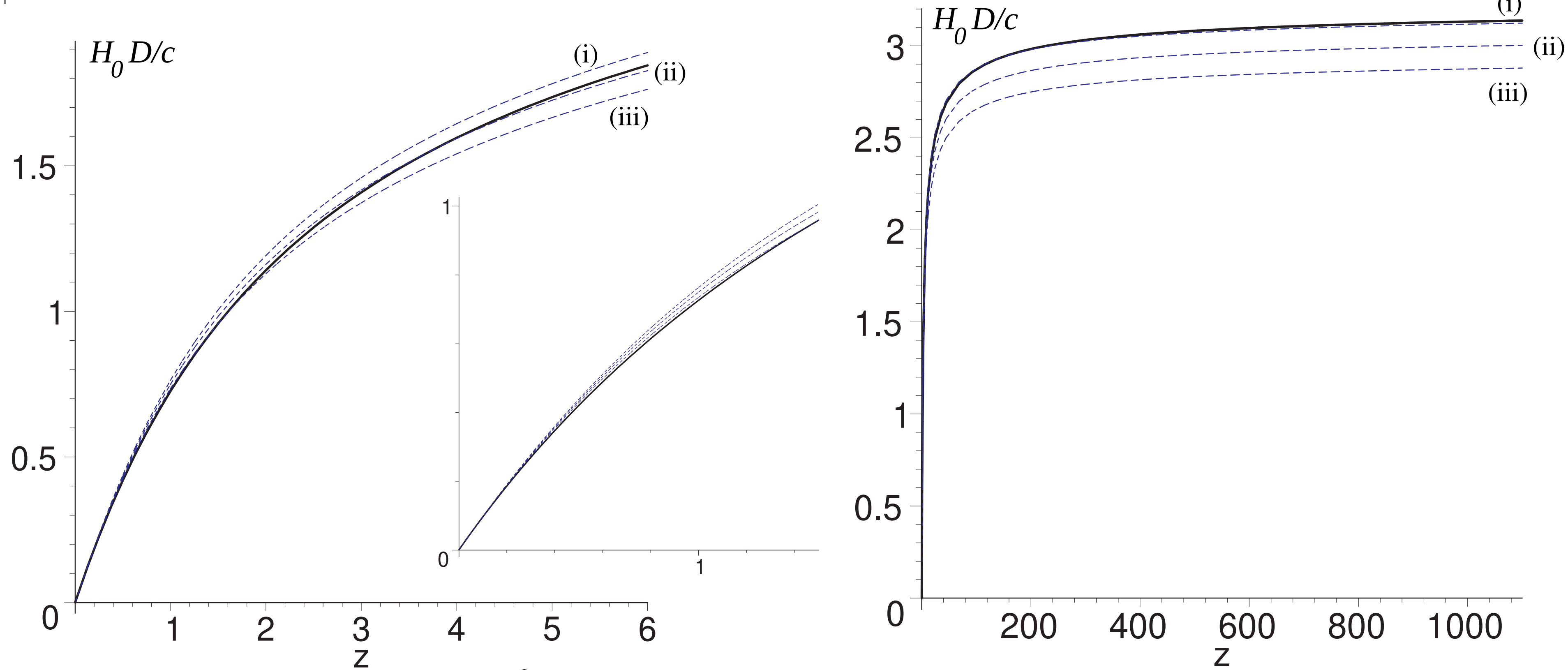
2. Average expansion: $H(z)/H_0$



$H(z)/H_0$ for $f_{v0} = 0.762$ (solid line) is compared to three spatially flat ΛCDM models: **(i)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.249, 0.751)$; **(ii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.279, 0.721)$ **(iii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.34, 0.66)$;.

- Function $H(z)/H_0$ displays quite different characteristics
- For $0 < z \lesssim 1.7$, $H(z)/H_0$ is larger for TS model, but value of H_0 assumed also affects $H(z)$ numerical value

Dressed "comoving distance" $D(z)$



TS model, with $f_{v0} = 0.695$, **(black)** compared to 3 spatially flat ΛCDM models (blue): **(i)** $\Omega_{M0} = 0.3175$ (best-fit ΛCDM model to Planck); (ii) $\Omega_{M0} = 0.35$; **(iii)** $\Omega_{M0} = 0.388$.

First principles: redshift

$$
\begin{aligned}
1+z_{\rm obs} &= \frac{-\mathbf{U}_{\rm em}\cdot\mathbf{k}}{-\mathbf{U}_{\rm obs}\cdot\mathbf{k}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \frac{E_{\rm em}}{E_{\rm obs}} \\
&\neq \sqrt{\frac{c+v}{c-v}} \simeq 1+\frac{v}{c}+\mathcal{O}\left(\frac{v^2}{c^2}\right) \\
&= \frac{\mathbf{U}_0\cdot\mathbf{k}}{\mathbf{U}_{\rm obs}\cdot\mathbf{k}}\,\frac{\mathbf{U}_1\cdot\mathbf{k}}{\mathbf{U}_0\cdot\mathbf{k}}\,\frac{\mathbf{U}_{\rm em}\cdot\mathbf{k}}{\mathbf{U}_1\cdot\mathbf{k}} \\
&= (1+z_{\rm pec,0})\,\frac{g_{\mu\nu}U_1^\mu k^\nu\big|_{\mathbf{x}_1}}{g_{\alpha\beta}U_0^\alpha k^\beta\big|_{\mathbf{x}_0}}\,(1+z_{\rm pec,1})
\end{aligned}
$$

Adapted to ideal observers at rest $\mathrm{d}x^i = 0$, define

(1) $$(1+z_{\rm cos}) \equiv \frac{g_{\mu\nu}U_1^\mu k^\nu\big|_{\mathbf{x}_1}}{g_{\alpha\beta}U_0^\alpha k^\beta\big|_{\mathbf{x}_0}} = \frac{g_{tt}U_1^t k^t\big|_{\mathbf{x}_1}}{g_{tt}U_0^t k^t\big|_{\mathbf{x}_0}}$$

First principles: redshift

For models with average isotropic expansion

$$
\begin{aligned}
(1+z_{\rm cos}) &\equiv (1+z_{\phi,0})\,(1+\bar z)\,(1+z_{\phi,1}) \\
\text{Interpret}\quad 1+z_{\phi,0} &= 1/(-g_{tt}U^t_0)\Big|_{\mathbf{x}_0} = 1/\sqrt{-g_{tt}(\mathbf{x}_0)} \\
1+z_{\phi,1} &= (-g_{tt}U^t_1)\Big|_{\mathbf{x}_1} = \sqrt{-g_{tt}(\mathbf{x}_1)} \\
1+\bar z &= k^t(\mathbf{x}_1)/k^t(\mathbf{x}_0)
\end{aligned}
$$

as “gravitational redshifts” versus “background expansion”

Notes

- FLRW: $1+\bar z = a(t_0)/a(t_1)$
- Standard cosmology: $z_{\phi,0},\ z_{\phi,1} \sim 10^{-5}$
- Λ-Szekeres: $z_{\phi,0},\ z_{\phi,1} \sim 10^{-3}$
- Timescape: Null cone conformal frame degeneracy
- Bondi-Metzner-Sachs group relevant generally

First principles: distances

Observational definition $d_L \equiv \sqrt{\dfrac{\mathcal{L}}{4\pi\mathcal{F}}}$

Etherington relation $D_A = \dfrac{D}{1+z} = \dfrac{d_L}{(1+z)^2}$

- d_L based on Euclidean $\mathcal{F} = \mathcal{L}/(4\pi {d_L}^2)$
- When non-Euclidean relation to null geodesics differs
- $d_L = (1+z)^2 D_A$ quite general, applies here.
- FLRW $D = \dfrac{c}{H_0\sqrt{|\Omega_{k0}|}} \operatorname{sinn}\left(\sqrt{|\Omega_{k0}|}\displaystyle\int_{1/(1+z)}^{1} \dfrac{\mathrm{d}y}{\sqrt{\Omega_{R0} + \Omega_{M0}y + \Omega_{k0}y^2 + \Omega_{\Lambda 0}y^4}}\right)$

  $\operatorname{sinn}(x) = \{\sinh(x),\ \Omega_{k0} > 0;\ x,\ \Omega_{k0} = 0;\ \sin(x),\ \Omega_{k0} < 0\}$
- Timescape $D = c\,(1+z)t^{2/3}\displaystyle\int_t^{t_0} \dfrac{2\,\mathrm{d}t'}{(2+f_{\rm v}(t'))(t')^{2/3}} = c\,(1+z)t^{2/3}(\mathcal{T}(t_0) - \mathcal{T}(t))$

  $\mathcal{T}(t) = 2t^{1/3} + \frac{b^{1/3}}{6}\ln\left(\frac{(t^{1/3}+b^{1/3})^2}{t^{2/3}-b^{1/3}t^{1/3}+b^{2/3}}\right) + \frac{b^{1/3}}{\sqrt{3}}\tan^{-1}\left(\frac{2t^{1/3}-b^{1/3}}{\sqrt{3}\,b^{1/3}}\right)$

  $b = \dfrac{2(1-f_{\rm v0})(2+f_{\rm v0})}{9f_{\rm v0}\bar{H}_0}, \qquad \text{bare } \bar{H}_0 = \dfrac{2(2+f_{\rm v0})H_0}{(4{f_{\rm v0}}^2 + f_{\rm v0} + 4)}$

First principles: distances

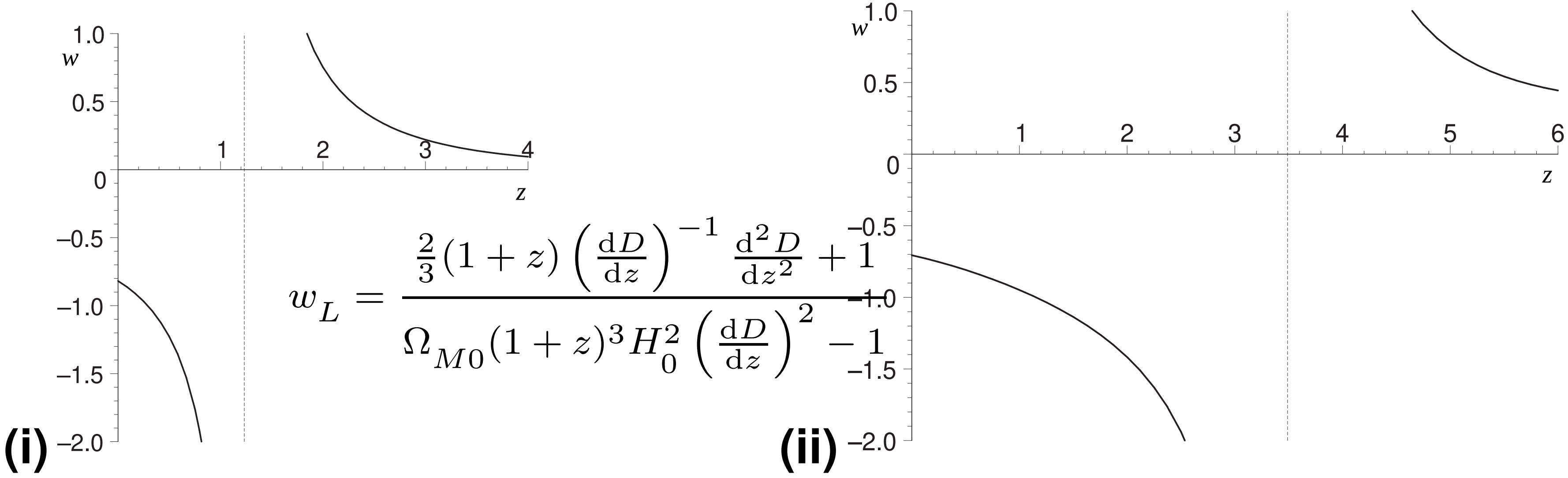
Derived quantities. Can define

Formal $P_{\rm d.e.} = w\rho_{\rm d.e.}$ $\qquad w(z) = \dfrac{\frac{2}{3}(1+z)D'^{-1}D'' + 1}{\Omega_{M0}(1+z)^3 {H_0}^2 D'^2 - 1}$

CBL test statistic $\qquad \Omega_k(z) = \dfrac{[H(z)D'(z)]^2 - 1}{[H_0 D(z)]^2}$

- FLRW $\Omega_{M0} = $ const
  - Different data sets give "tensions"
  - No unique counterpart in timescape, $w(z)$ ill-defined
- FLRW $\Omega_k(z) = \Omega_{k0} = $ const
  - ΛCDM Planck $\Omega_{k0} = $ tiny, some debate
  - Analytic $\Omega_k(z)$ prediction in timescape

Equivalent “equation of state” $P = w\,\rho c^2$?



$$w_L = \frac{\frac{2}{3}(1+z)\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^{-1}\frac{\mathrm{d}^2D}{\mathrm{d}z^2}+1}{\Omega_{M0}(1+z)^3H_0^2\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2-1}$$

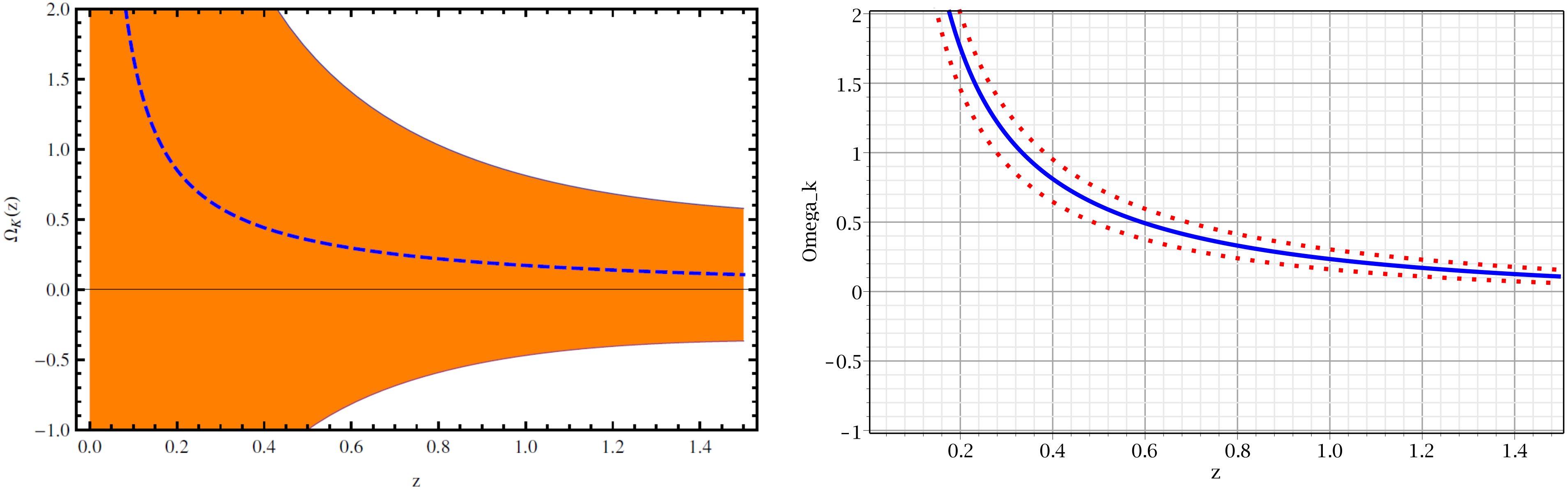
A formal “dark energy equation of state” $w_L(z)$ for the TS model, with $f_{v0} = 0.695$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.41$; **(ii)** $\Omega_{M0} = 0.3175$.

- Description by a “dark energy equation of state” makes no sense when there’s no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

Clarkson Bassett Lu test $\Omega_k(z)$

- For Friedmann equation a statistic constant for all z

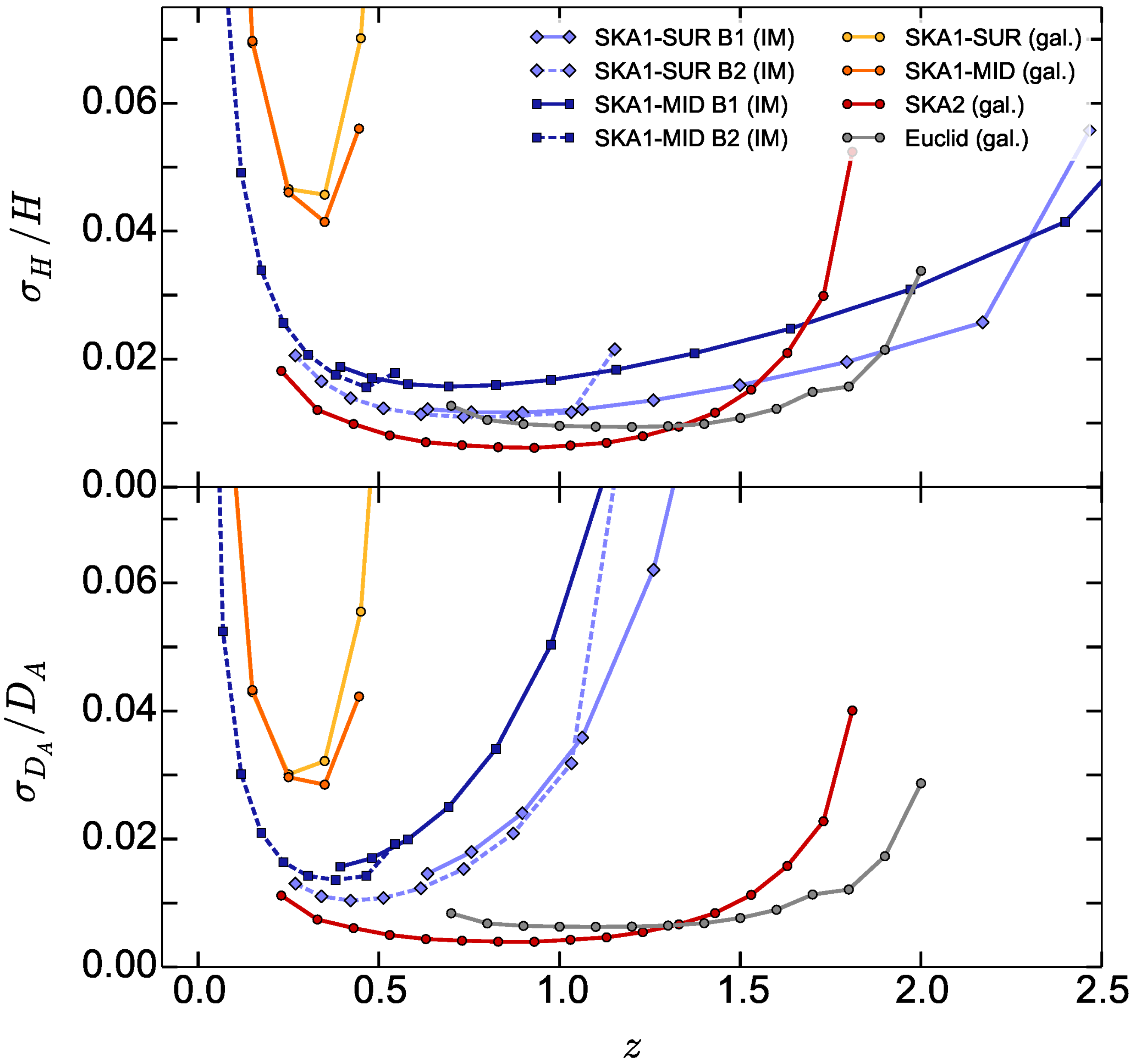
$$\Omega_{k0} = \Omega_k(z) = \frac{[c^{-1}H(z)D'(z)]^2 - 1}{[c^{-1}H_0 D(z)]^2}$$



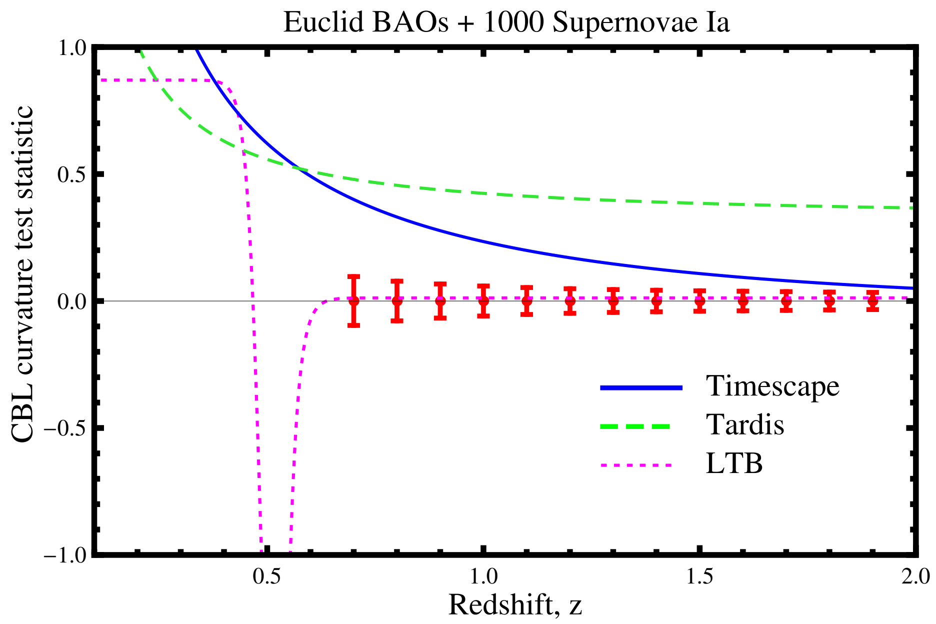
Left panel: CBL statistic from Sapone, Majerotto and Nesseris, PRD 90, 023012 (2014) Fig 8, using existing data from SneIa (Union2) and passively evolving galaxies for $H(z)$.

Right panel: TS prediction, with $f_{v0} = 0.695^{+0.041}_{-0.051}$.

Projections for Euclid, SKA

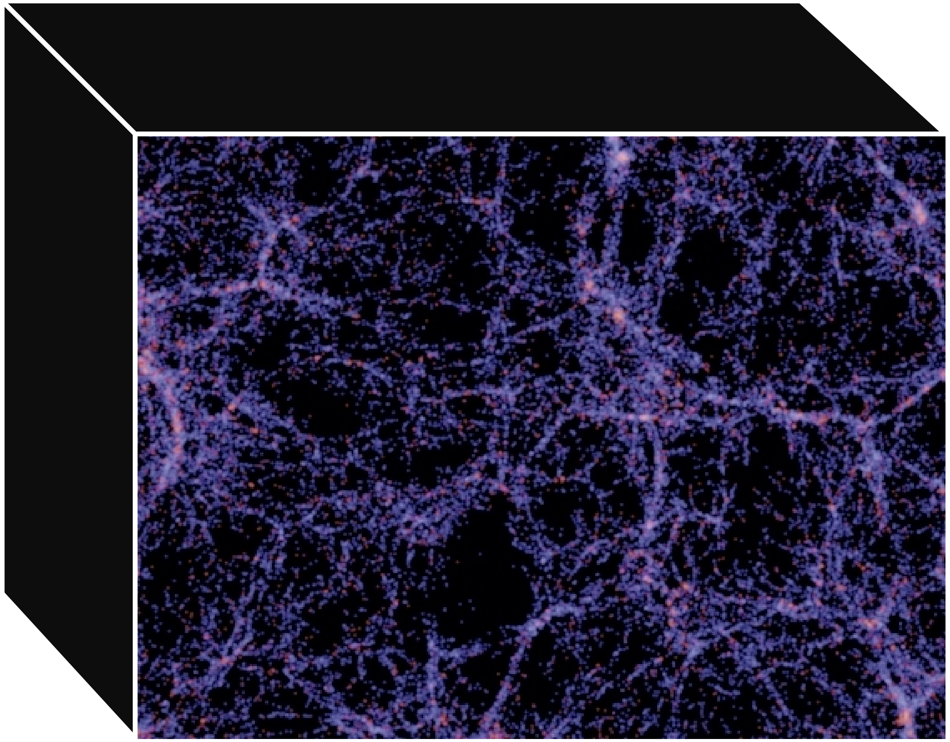


Clarkson Bassett Lu test with *Euclid*



- Projected uncertainties for ΛCDM, with *Euclid* $+$ 1000 SneIa, Sapone *et al*, PRD 90, 023012 (2014) Fig 10
- Timescape prediction (green), compared to non-Copernican Gpc void model (blue), and *tardis* cosmology, Lavinto *et al* JCAP 12 (2013) 051 (brown).
- Timescape prediction becomes greater than uncertainties for $z \lesssim 1.5$. (Falsfiable.)

3. Timescape concepts. . . SHS cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- Gradients in spatial curvature and gravitational energy can lead to calibration differences between rulers & clocks of bound structures and volume average

The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

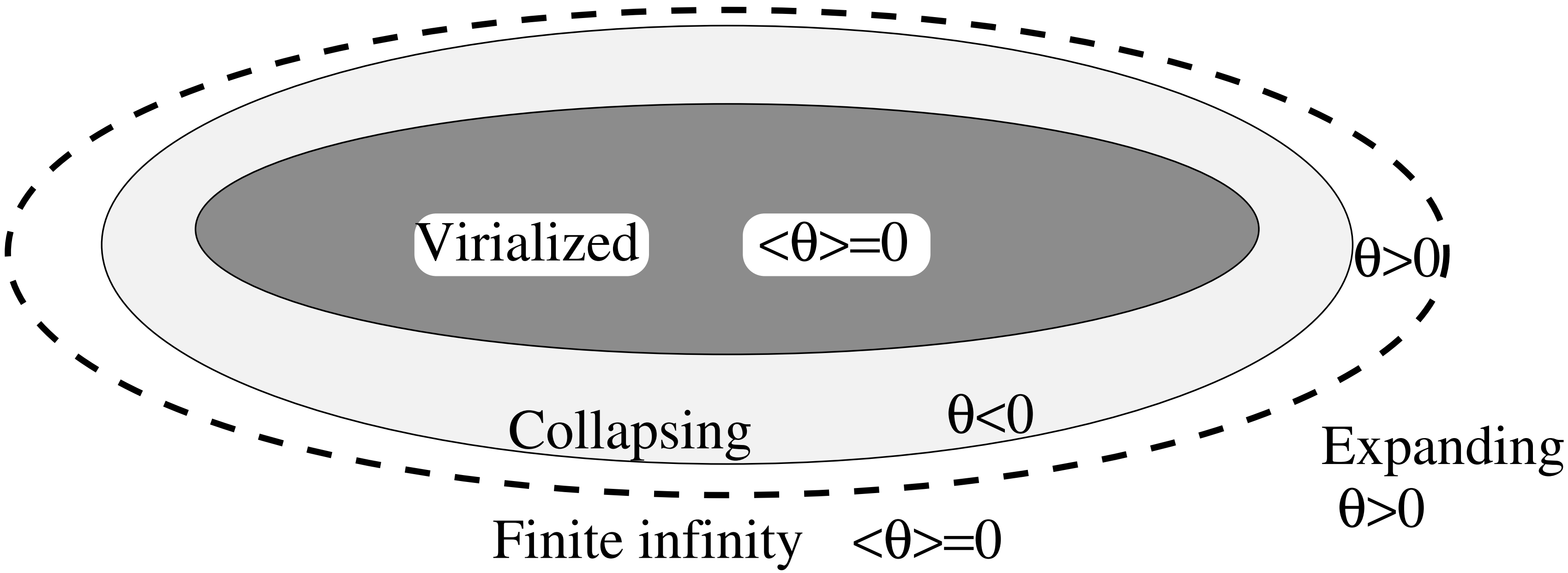
Cosmological Equivalence Principle

- *In cosmological averages it is always possible to choose a suitably defined spacetime region, the cosmological inertial region, on whose boundary average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation*,

$$\mathrm{d}s^2_{\mathrm{CIR}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

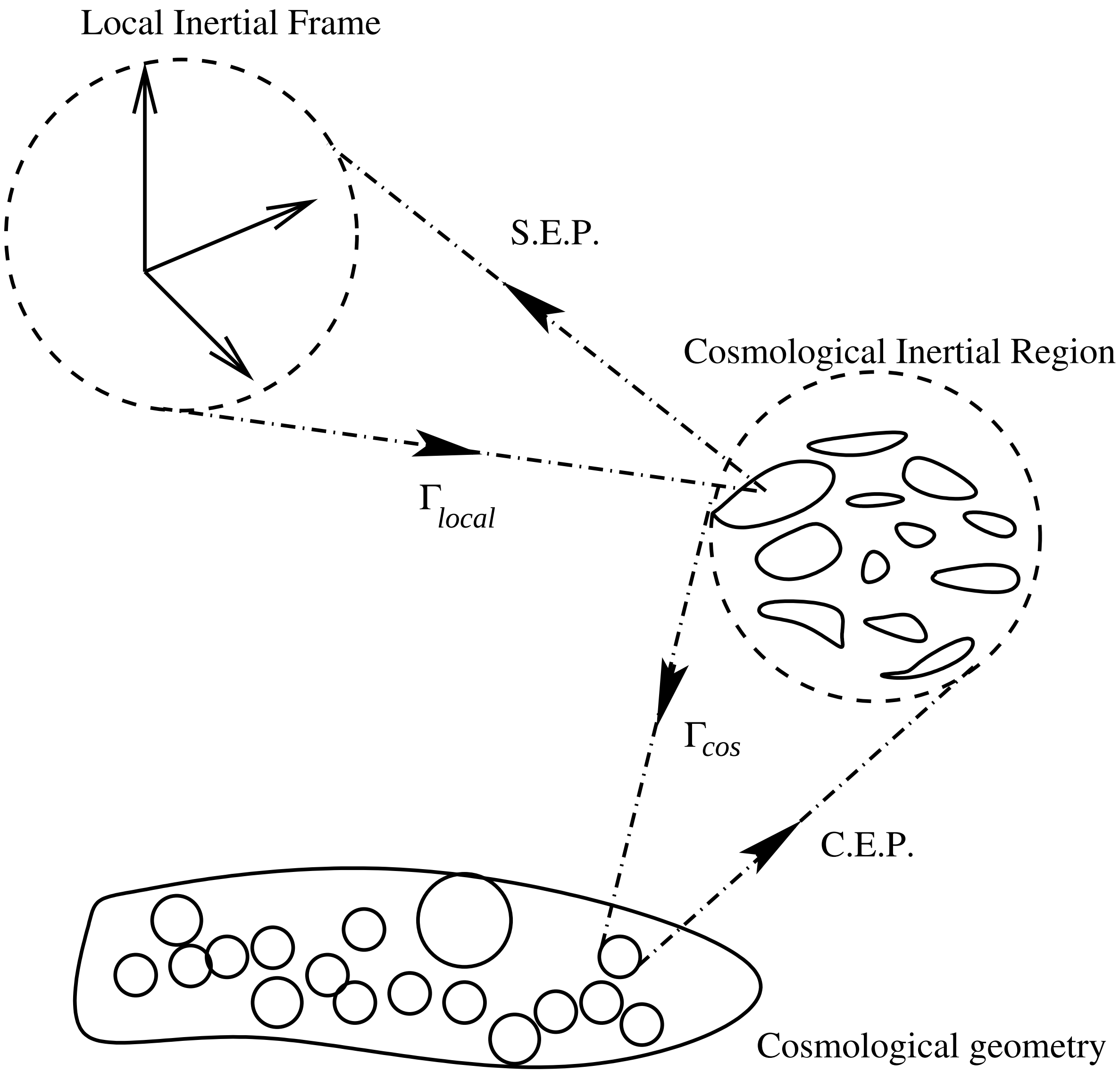
- Defines Cosmological Inertial Region (CIR) in which *regionally isotropic* volume expansion is equivalent to a velocity in special relativity
- Integrate on a bounding 2-sphere to define *"kinetic energy of expansion"*: globally it has gradients

Finite infinity

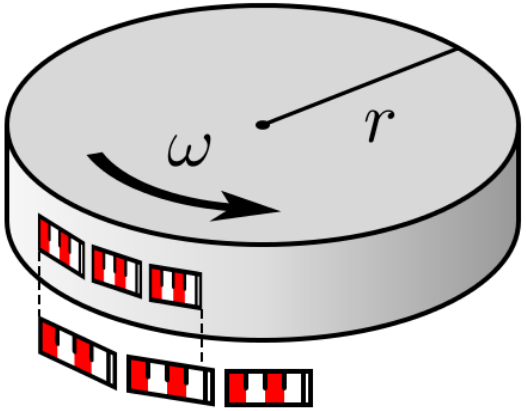


- Define *finite infinity*, "*fi*" as boundary to *connected* region within which *average expansion* vanishes $\langle\vartheta\rangle = 0$ and expansion is positive outside.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Statistical geometry…



Non-Euclidean kinematics: analogy



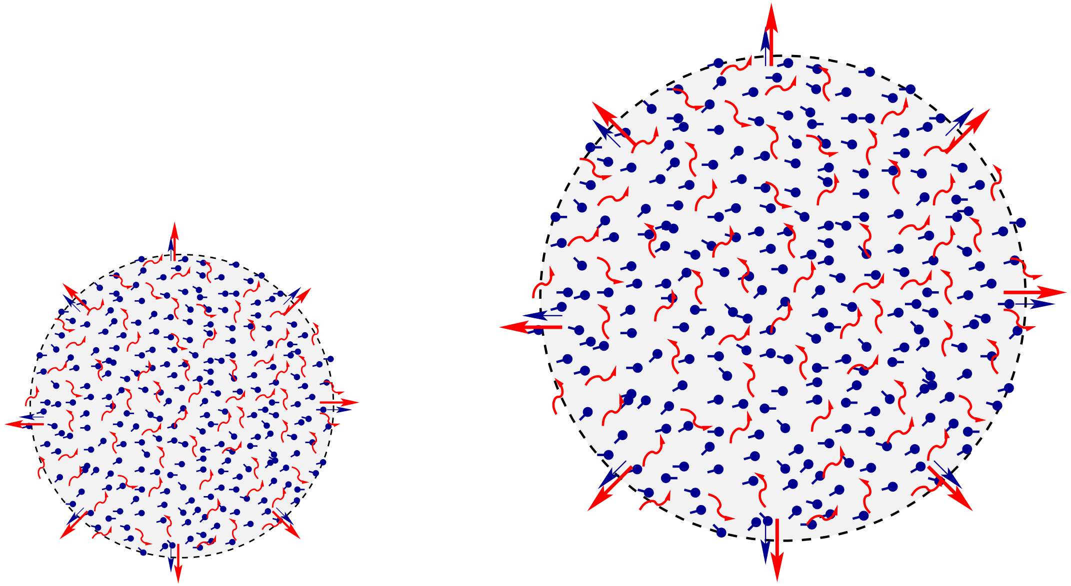
- Ehrenfest paradox / Born rigidity 1909
  - Global Minkowski observer, $\mathcal{C}_0 = 2\pi R$
  - Centre disk observer, $\mathcal{C}_{\rm in} = 2\pi\gamma R < \mathcal{C}_0$
  - Rotational kinetic energy “produces” non-Euclidean geometry even in special relativity
- CEP: kinetic energy of expansion analogous in universe
  - GR fitting problem, *integrated* over ~ 14 Gyr

Timescape phenomenology

$$\mathrm{d}s^2 = -(1+2\Phi)c^2\mathrm{d}t^2 + a^2(1-2\Psi)g_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

- Global statistical metric $\rightarrow \bar{a}$ by Buchert average
  - Unique solution for ensemble of disjoint voids and finite infinity (wall) regions
- **Uniform Hubble expansion condition on $< 100\,h^{-1}$Mpc scales**
  - Used to conformally match radial null geodesics of finite infinity and statistical geometries
  - Fit data: SNe, CMB, ... on $\gtrsim 100\,h^{-1}$Mpc scales
  - Relative regional volume deceleration integrates to a substantial difference in clock calibration of $\int \mathrm{d}\tau_{\rm w}$ c.f. $\int \mathrm{d}t = \int \mathrm{d}\tau_{\rm w}/\bar{a}$ over age of universe
  - Difference in *bare* (statistical or volume–average) and *dressed* (regional or finite–infinity) parameters

Relative volume deceleration. . .

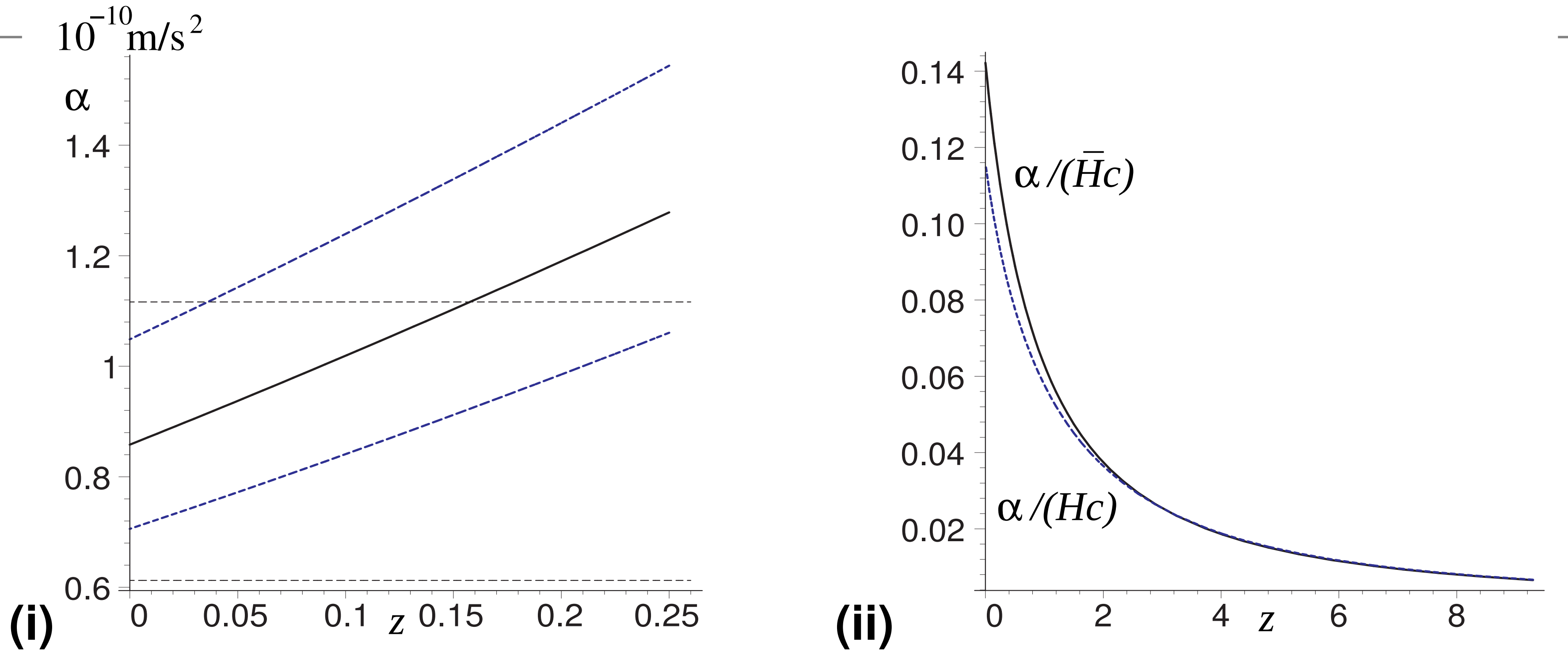


- Two fluids, 4-velocities U^μ, $\tilde{U}^\mu$, $U^\mu S_\mu = 0$, $\tilde{U}^\mu \tilde{S}_\mu = 0$, relative tilt $\gamma = (1-\beta^2)^{-1/2}$, $\beta \equiv v/c$,

$$U^\mu = \gamma(\tilde{U}^\mu + \beta\tilde{S}^\mu), \qquad S^\mu = \gamma(\tilde{S}^\mu + \beta U^\mu),$$

- Integrate on compact spherical boundary – average tilt $\langle\gamma\rangle$– time derivative relative volume deceleration.
- Integrated relative *clock rate drift*.

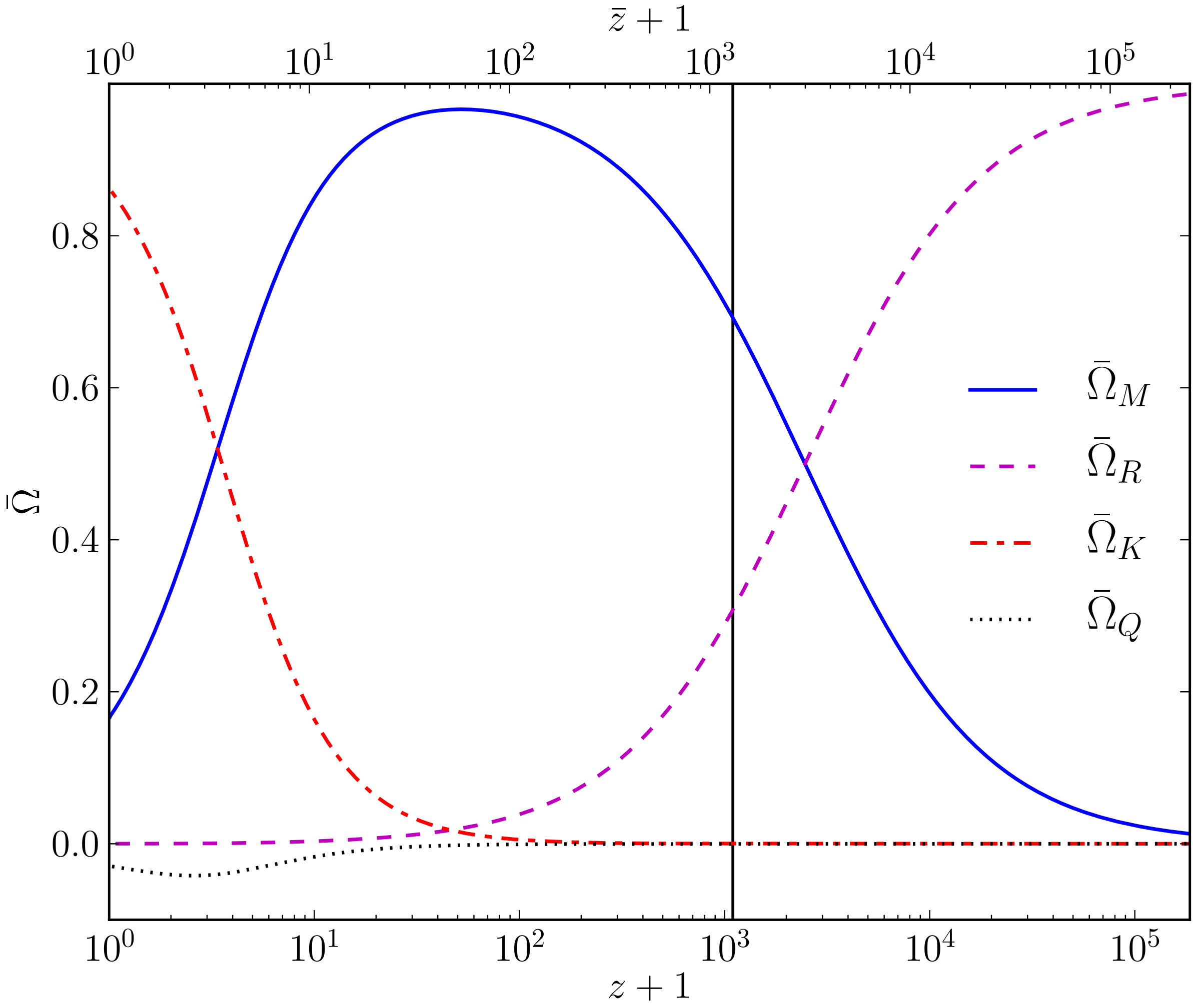
Relative deceleration scale



By cosmological equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c\bar{\gamma}\dot{\bar{\gamma}}/(\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z.

- Relative *volume* deceleration of expanding regions of different local density/curvature, leads cumulatively to canonical clocks differing by $\mathrm{d}t = \bar{\gamma}_{\rm w}\, \mathrm{d}\tau_{\rm w}$ ($\rightarrow\sim 35\%$)

Bare cosmological parameters



J.A.G. Duley, M.A. Nazer & DLW, CQG 30 (2013) 175006:

“curvature” $\bar{\Omega}_K \propto {f_{\rm v}}^{1/3}/(\bar{a}^2\bar{H}^2)$ dominates today ($z=0$)

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\,(1 - f_{\rm v})^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

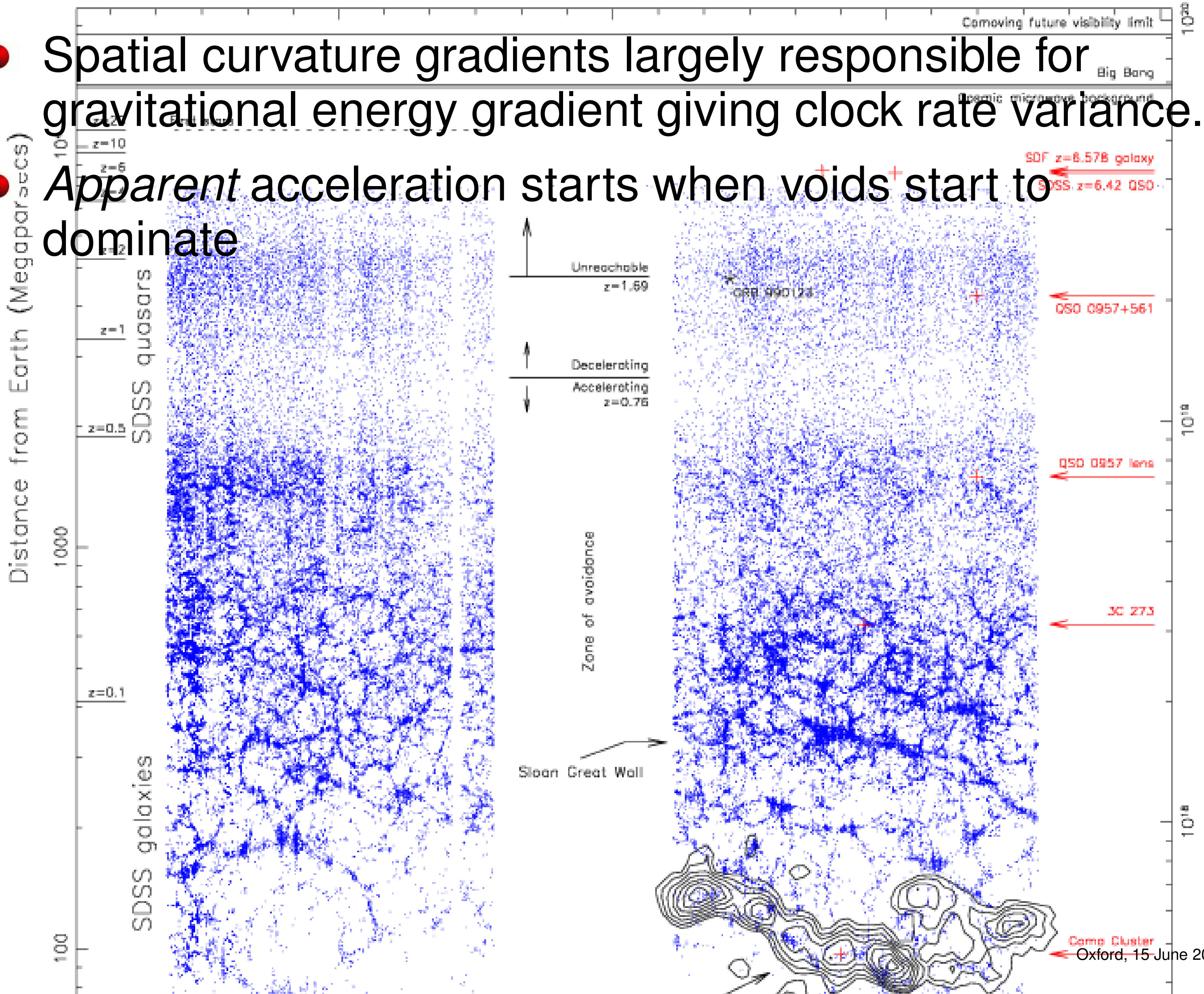
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\,(1 - f_{\rm v})\,(8f_{\rm v}{}^3 + 39f_{\rm v}{}^2 - 12f_{\rm v} - 8)}{\left(4 + f_{\rm v} + 4f_{\rm v}{}^2\right)^2}\,,$$

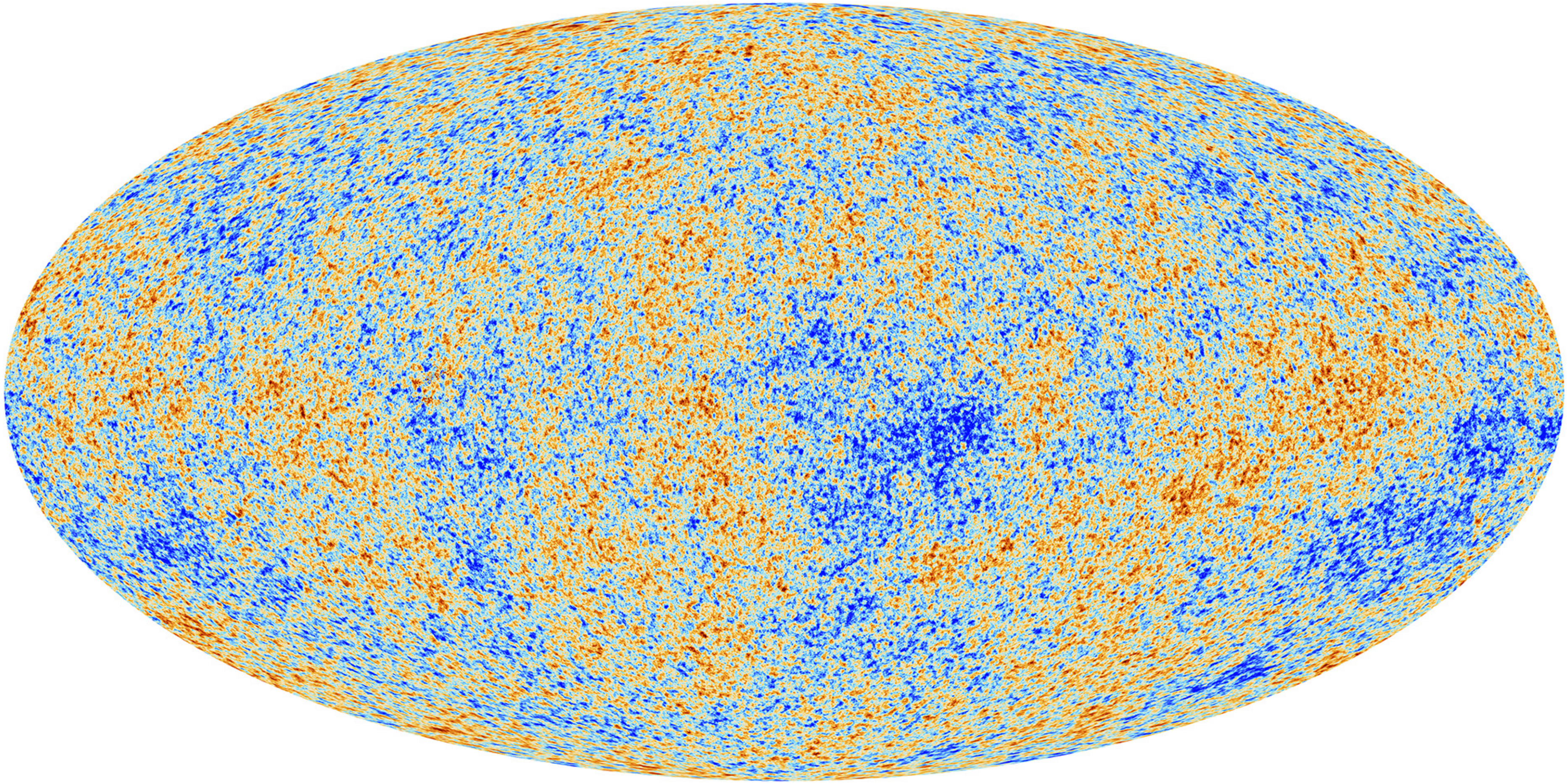
Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.5867\ldots$, and approaches $q \to 0^-$ at late times.

Cosmic coincidence not a problem

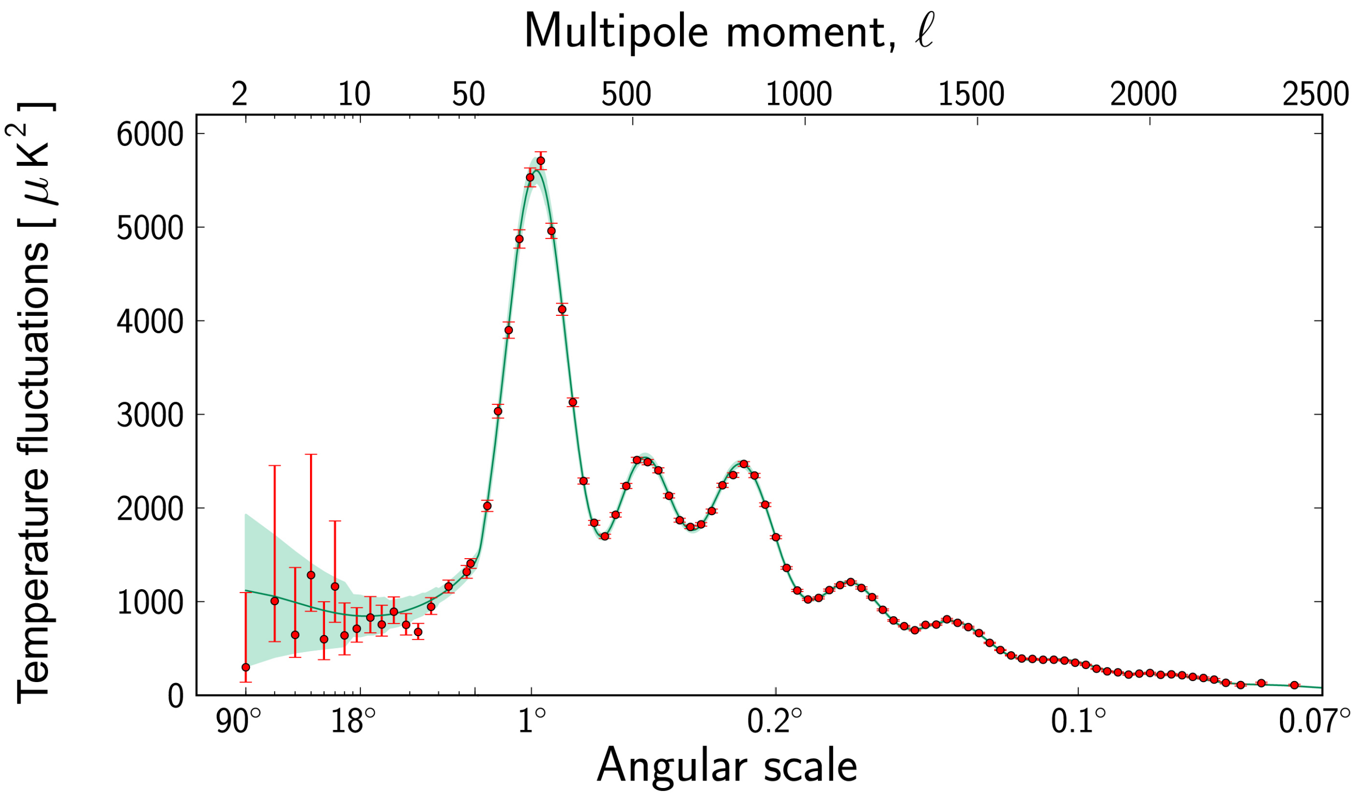
- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to dominate



Observational data fitting: CMB



Planck data ΛCDM parametric fit



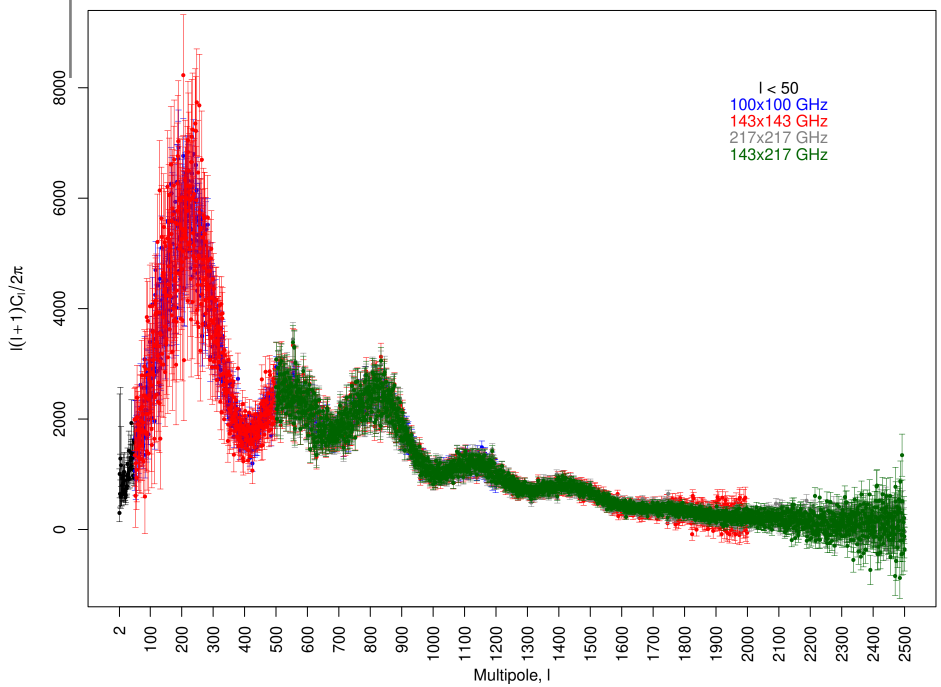
Duley, Nazer + DLW, CQG 30 (2013) 175006:

- Use angular scale, baryon drag scale from ΛCDM fit
- Baryon–photon ratio $\eta_{B\gamma} = 4.6$–5.6×10^{-10} within 2σ of all observed light element abundances (**including ^{7}Li**).

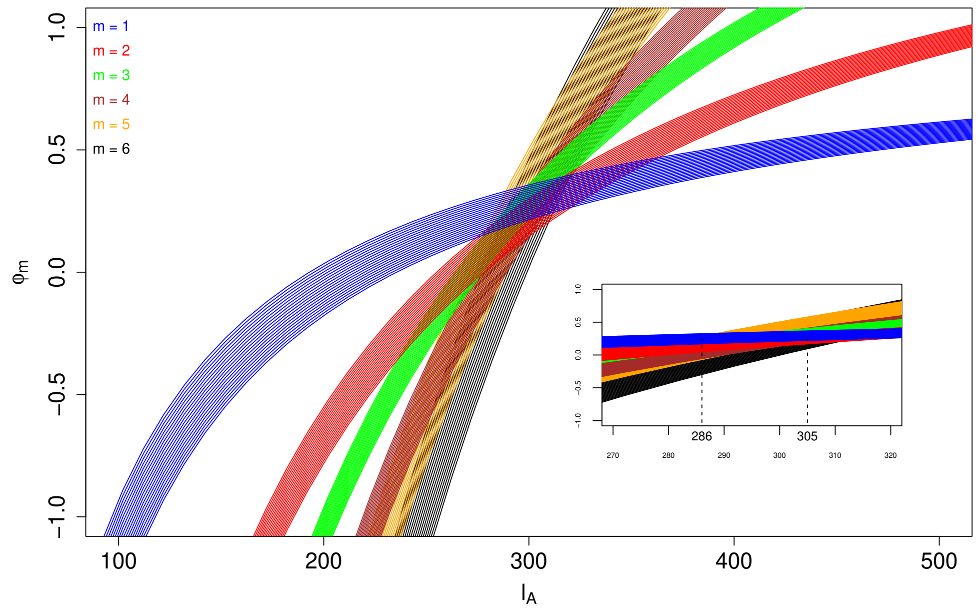
Planck constraints $D_A + r_{drag}$

- Dressed Hubble constant $H_0 = 61.7 \pm 3.0$ km/s/Mpc
- Bare Hubble constant $H_{w0} = \bar{H}_0 = 50.1 \pm 1.7$ km/s/Mpc
- Local max Hubble constant $H_{v0} = 75.2^{+2.0}_{-2.6}$ km/s/Mpc
- Present void fraction $f_{v0} = 0.695^{+0.041}_{-0.051}$
- Bare matter density parameter $\bar{\Omega}_{M0} = 0.167^{+0.036}_{-0.037}$
- Dressed matter density parameter $\Omega_{M0} = 0.41^{+0.06}_{-0.05}$
- Dressed baryon density parameter $\Omega_{B0} = 0.074^{+0.013}_{-0.011}$
- Nonbaryonic/baryonic matter ratio $\Omega_{C0}/\Omega_{B0} = 4.6^{+2.5}_{-2.1}$
- Age of universe (galaxy/wall) $\tau_{w0} = 14.2 \pm 0.5$ Gyr
- Age of universe (volume-average) $t_0 = 17.5 \pm 0.6$ Gyr
- Apparent acceleration onset $z_{acc} = 0.46^{+0.26}_{-0.25}$

Non-parametric CMB constraints



Raw Planck data



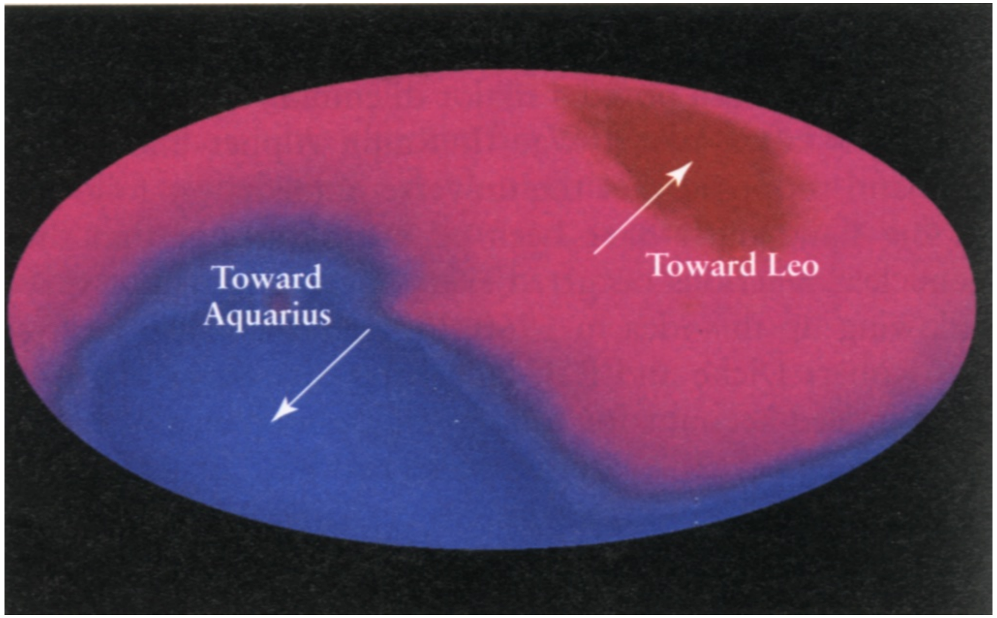
Fit to angular scale from 6 peaks

- What do we know without a cosmological model?
- $286 \leq \ell_A \leq 305$ at 95% confidence Aghamousa et al, JCAP 02(2015)007

4. Variation of expansion

- Scales $\lesssim 100\,h^{-1}$Mpc below “statistical homogeneity scale” most interesting
- Potential insights about
  - convergence of “bulk flows” (see also Kraljic & Sarkar, JCAP 10 (2016) 016)
  - H_0 tension
- Standard sirens (GW170817 etc): could test this!
- Toy model Λ–Szekeres solutions: Planck ΛCDM on $\gtrsim 100\,h^{-1}$Mpc, Szekeres inhomogeneity inside, K Bolejko, MA Nazer, DLW JCAP 06 (2016) 035
- Models for large angle CMB “anomalies” in future

Cosmic Microwave Background dipole



- Special Relativity: motion in a thermal bath of photons

$$T' = \frac{T_0}{\gamma(1 - (v/c)\cos\theta')}, \qquad \gamma = \left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

- 3.37 mK dipole: $v_{\text{Sun-CMB}} = 371$ km s^{-1} to $(264.14°, 48.26°)$; splits as $v_{\text{Sun-LG}} = 318.6$ km s^{-1} to $(106°, -6°)$ and $v_{\text{LG-CMB}} = 635 \pm 38$ km s^{-1} to $(276.4°, 29.3°) \pm 3.2°$

Planck mission: Doppler boosting

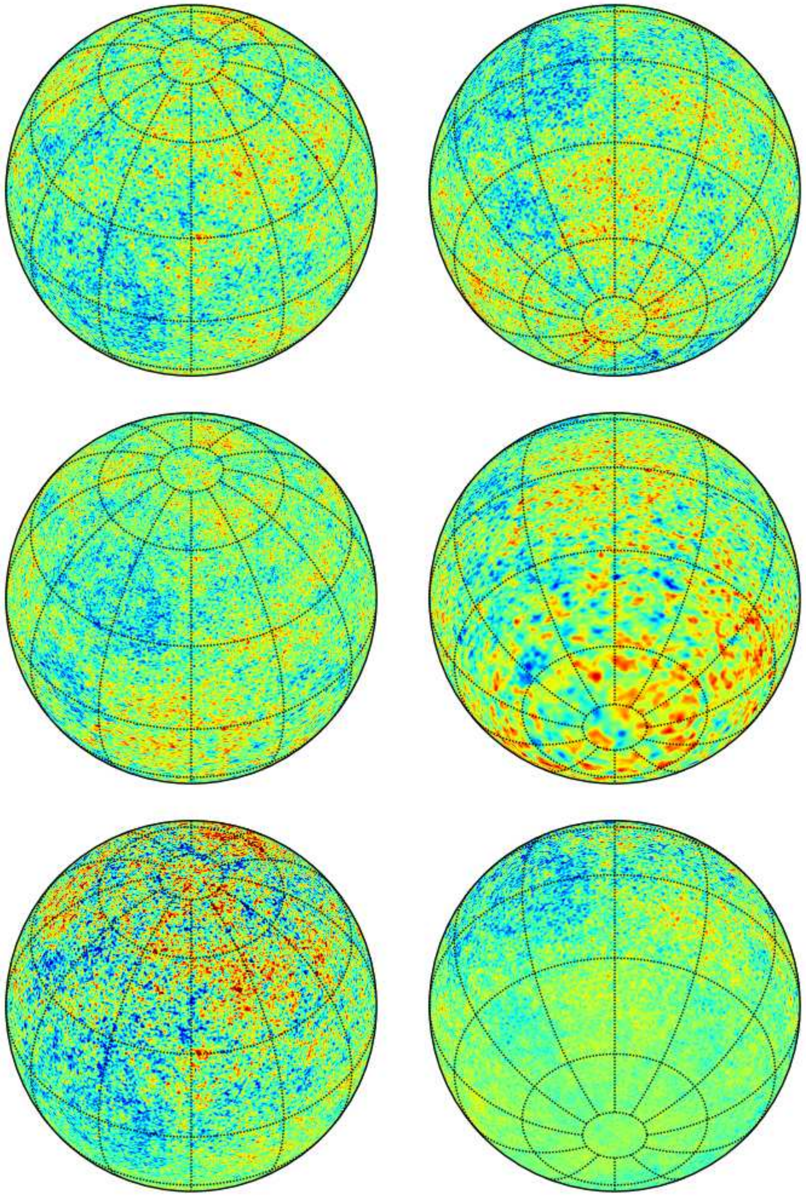
Boost dipole from second order effects

Original

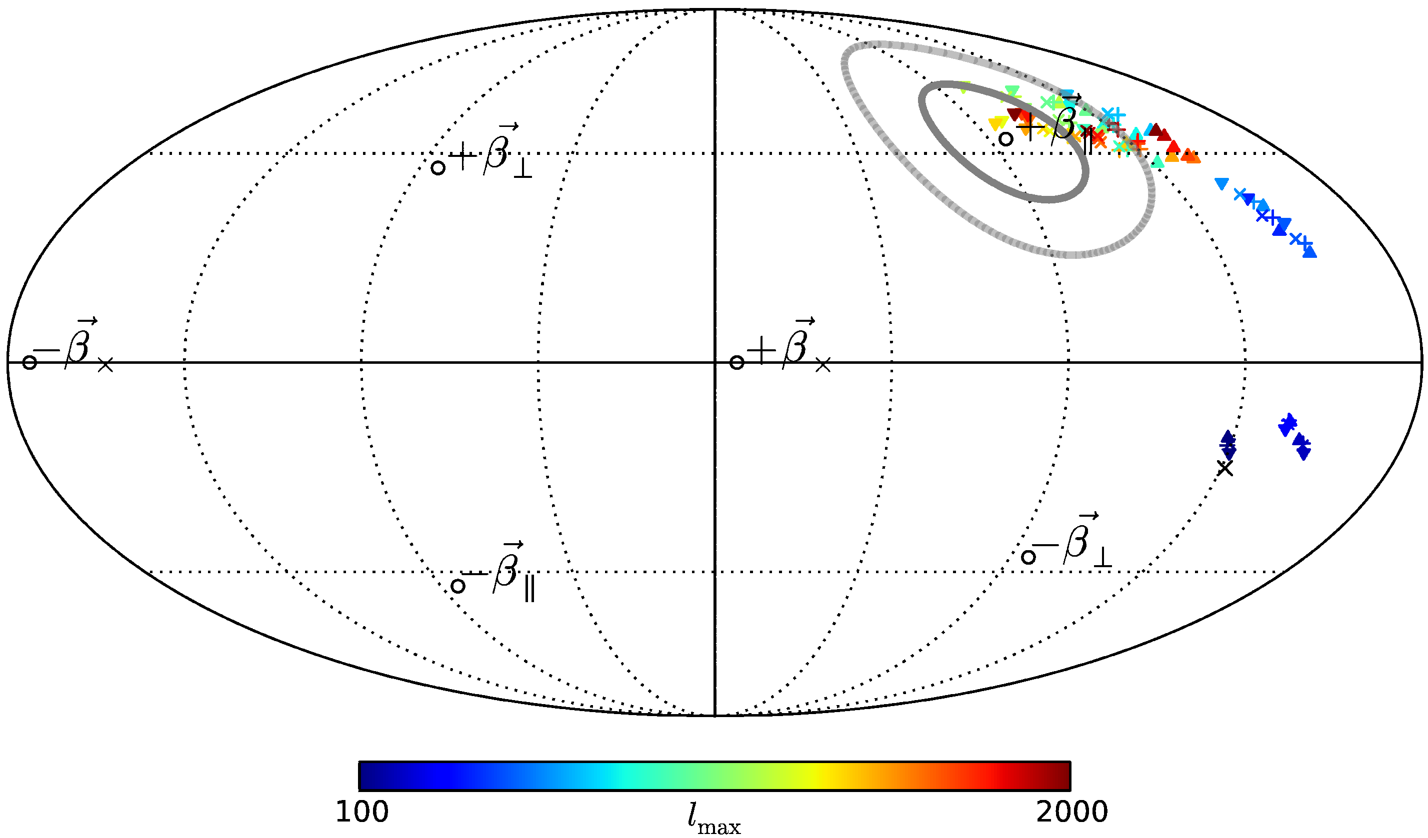
Aberration (Exaggerated)

Modulation (Exaggerated)

Eppur si muove?



Planck: A&A 571 (2014) A27



- Dipole direction consistent with CMB dipole $(\ell, b) = (264°, 48°)$ for small angles, $l_{\rm min} = 500 < l < l_{\rm max} = 2000$
- When $l < l_{\rm max} = 100$, shifts to WMAP power asymmetry modulation dipole $(\ell, b) = (224°, -22°) \pm 24°$

Large angle CMB anomalies?

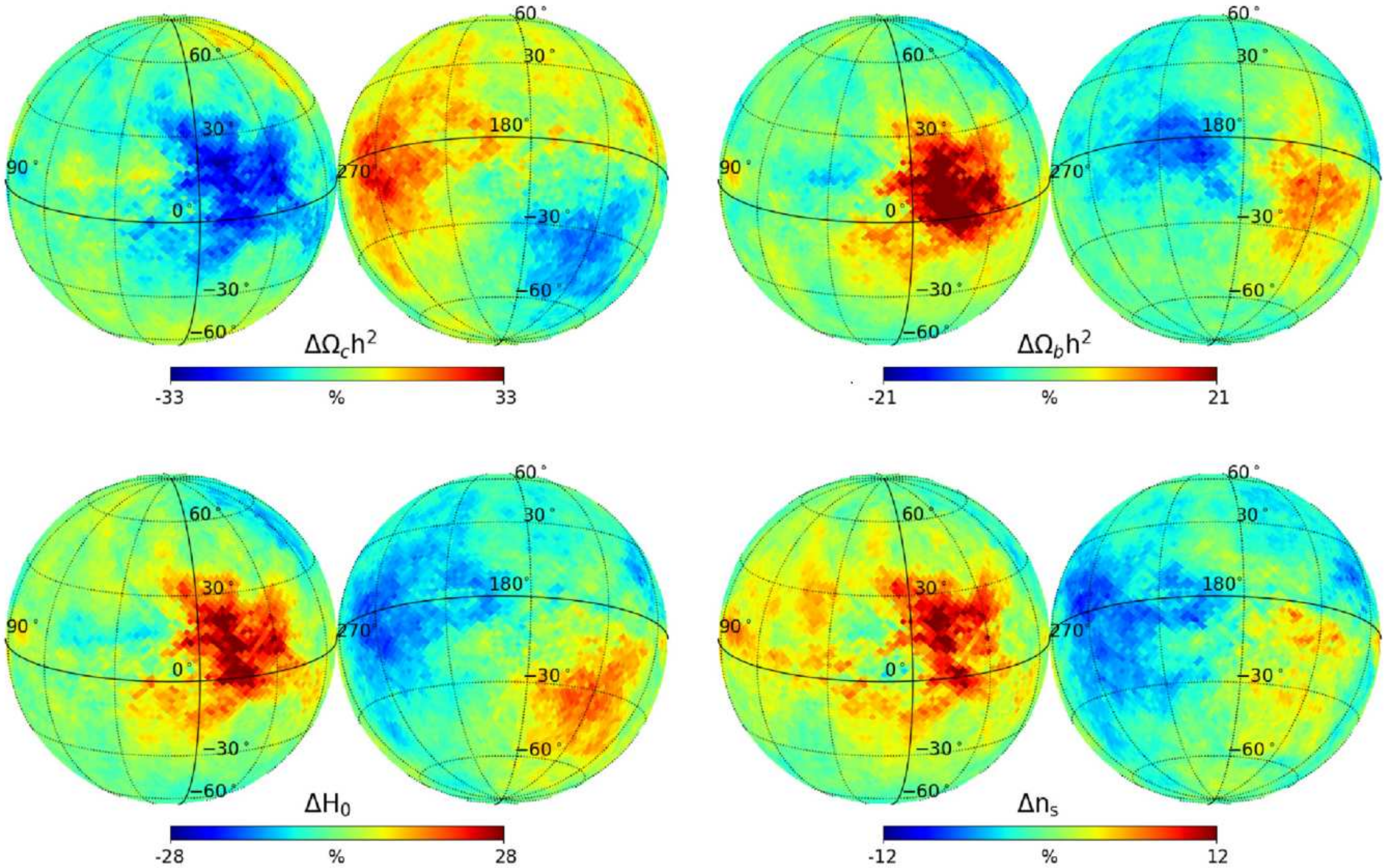
Anomalies (significance increased after Planck 2013):

- power asymmetry of northern/southern hemispheres
- alignment of the quadrupole and octupole etc;
- low quadrupole power;
- parity asymmetry; ...

Critical re-examination required; e.g.

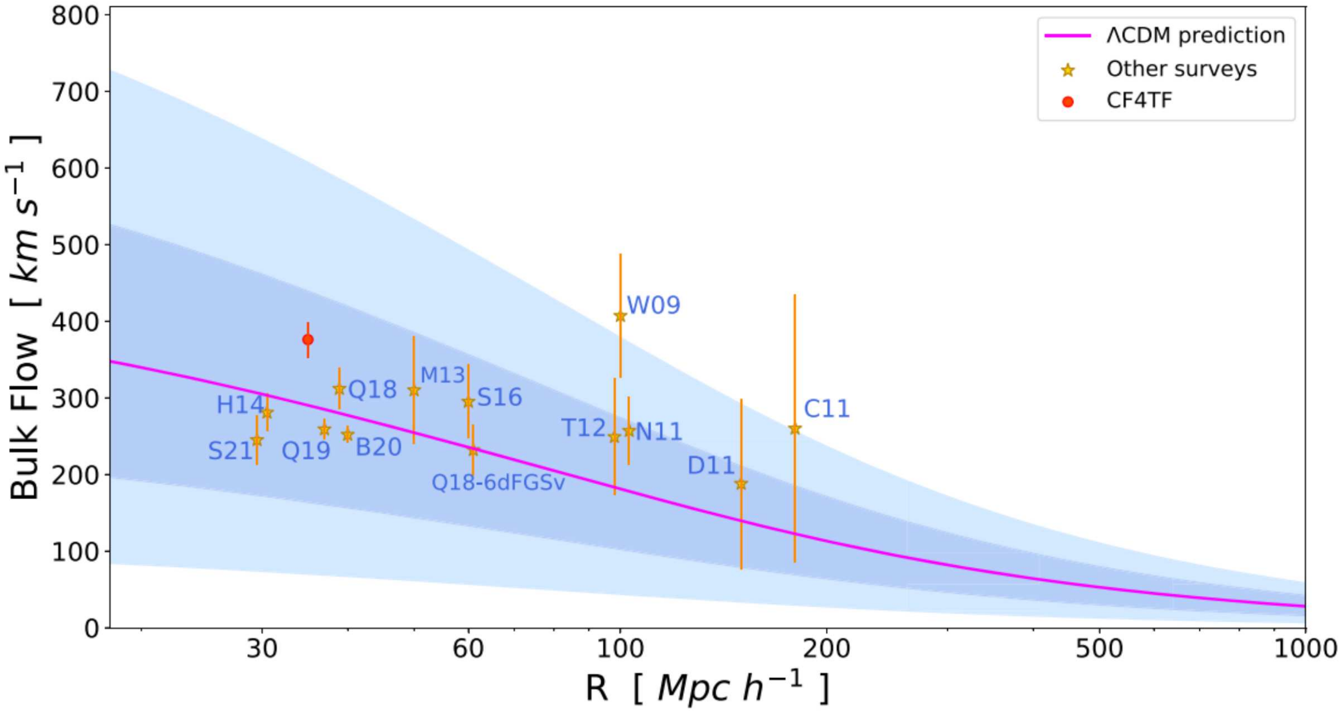
- light propagation through Hubble variation dipole foregrounds may differ subtly from Lorentz boost dipole
- dipole subtraction is an integral part of the map-making; is galaxy correctly cleaned?
- Freeman et al (2006): 1–2% change in dipole subtraction may resolve the power asymmetry anomaly.

Causal horizons in CMB



- Fosalba & Gaztañaga, anisotropic fit of ΛCDM parameters to Planck, MNRAS 504, 5840 (2021)

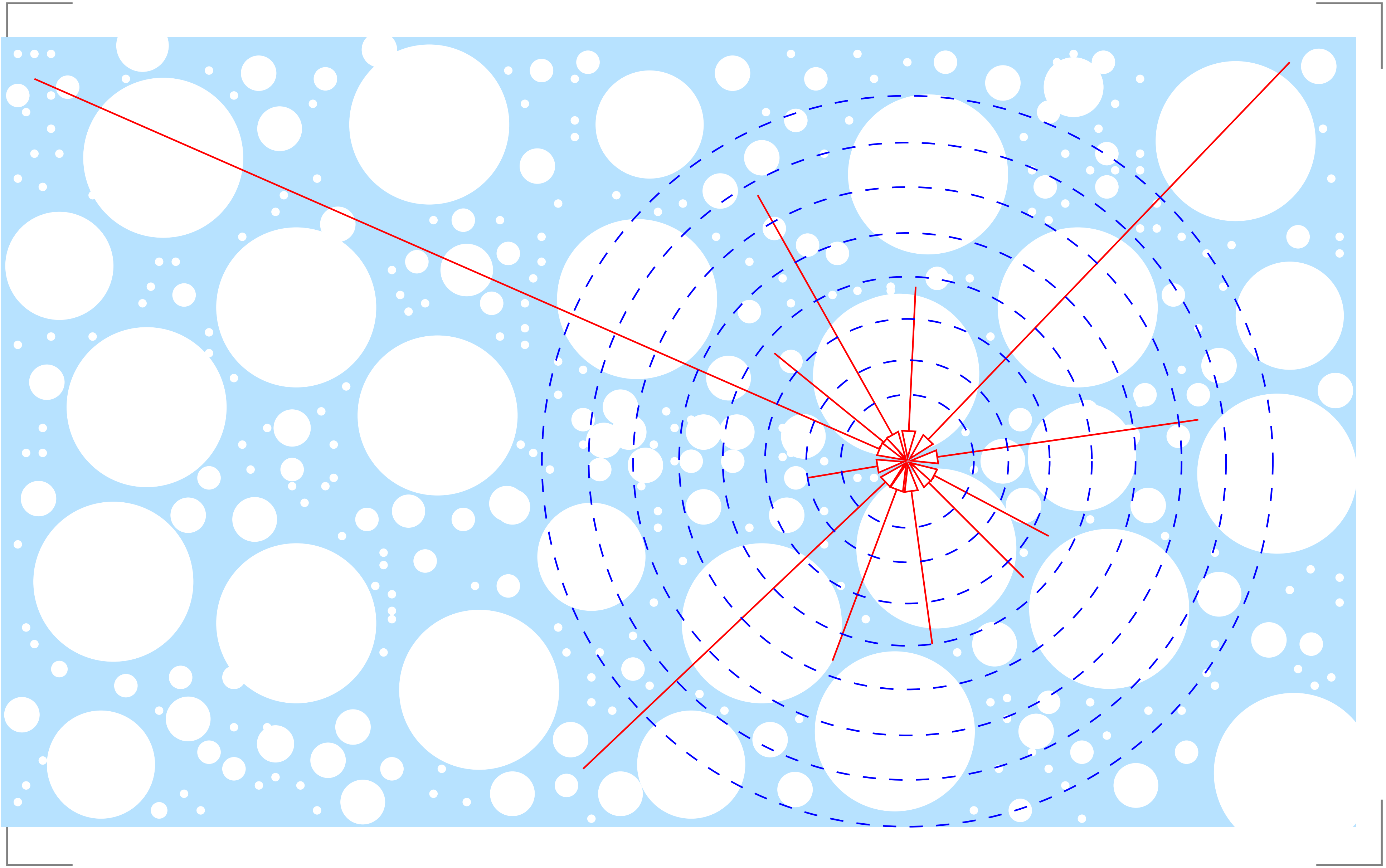
Convergence of bulk flows vs ΛCDM

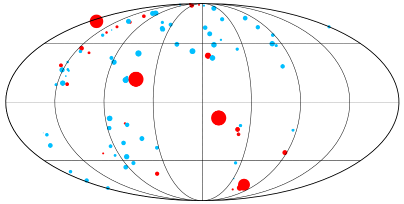


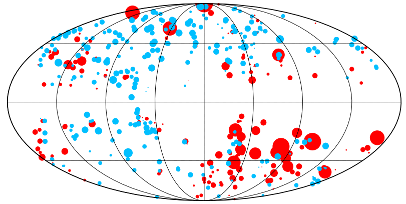
Qin et al, ApJ 922 (2021) 59

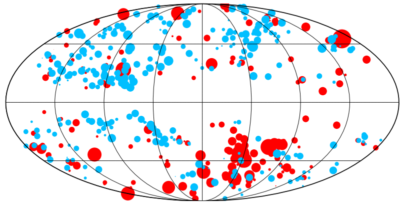
- Below SHS ok for convergence to ΛCDM expectation, larger scales problematic
- "Dark flow" kinematic Sunyaev-Zel'dovich effect debate
- See Cosmological Principle review: CQG 40 (2023) 094001 [arXiv: 2207.05765]

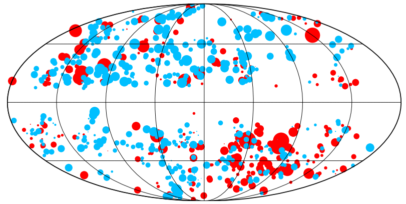
Apparent Hubble flow variation

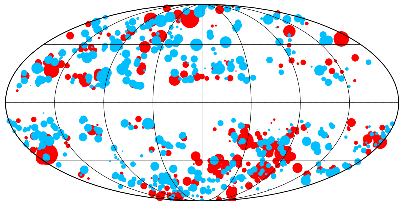


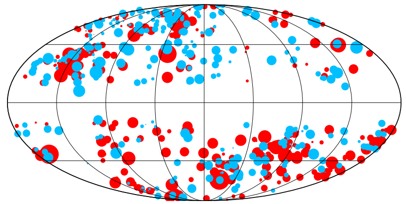

(a) 1: $0 - 12.5\ h^{-1}$ Mpc $N = 92$.

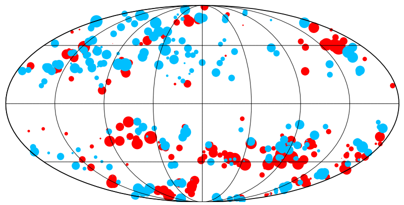

(b) 2: $12.5 - 25\ h^{-1}$ Mpc $N = 505$.

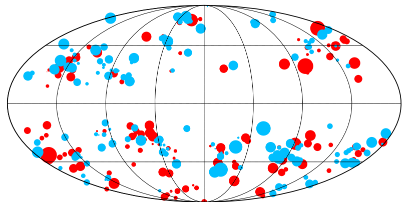

(c) 3: $25 - 37.5\ h^{-1}$ Mpc $N = 514$.

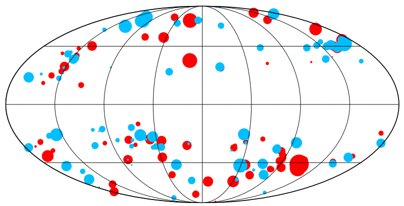

(d) 4: $37.5 - 50\ h^{-1}$ Mpc $N = 731$.

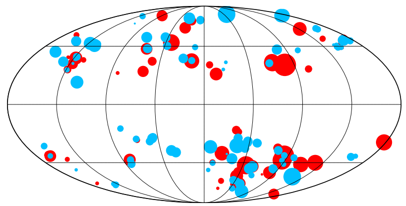

(e) 5: $50 - 62.5\ h^{-1}$ Mpc $N = 819$.

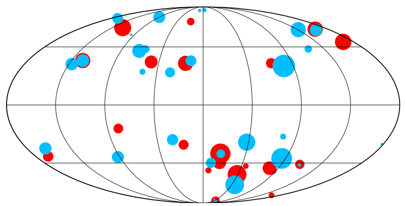

(f) 6: $62.5 - 75\ h^{-1}$ Mpc $N = 562$.


(g) 7: $75 - 87.5\ h^{-1}$ Mpc $N = 414$.

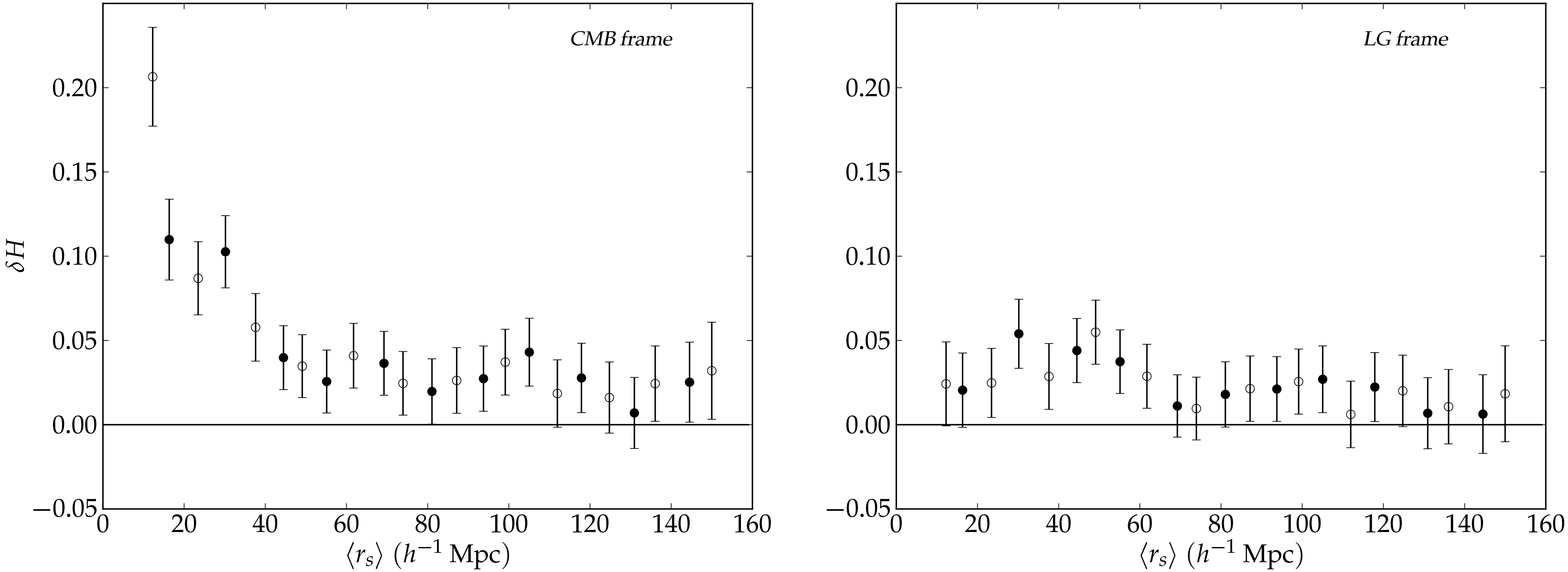

(h) 8: $87.5 - 100\ h^{-1}$ Mpc $N = 304$.


(i) 9: $100 - 112.5\ h^{-1}$ Mpc $N = 222$.


(j) 10: $112.5 - 156.25\ h^{-1}$ Mpc $N = 280$.


(k) 11: $156.25 - 417.4\ h^{-1}$ Mpc $N = 91$.

Radial variation $\delta H_s = (H_s - H_0)/H_0$



- Two choices of shell boundaries (closed & open circles)
- Result: Hubble expansion is very significantly more uniform in LG frame than in CMB frame: $\ln B > 5$; (except for $40 \lesssim r \lesssim 60\, h^{-1}$Mpc) [DLW, Smale, Mattsson & Watkins: Phys Rev D88 (2013) 083529].

Boosts and spurious monopole variance

- H_s determined by linear regression in each shell

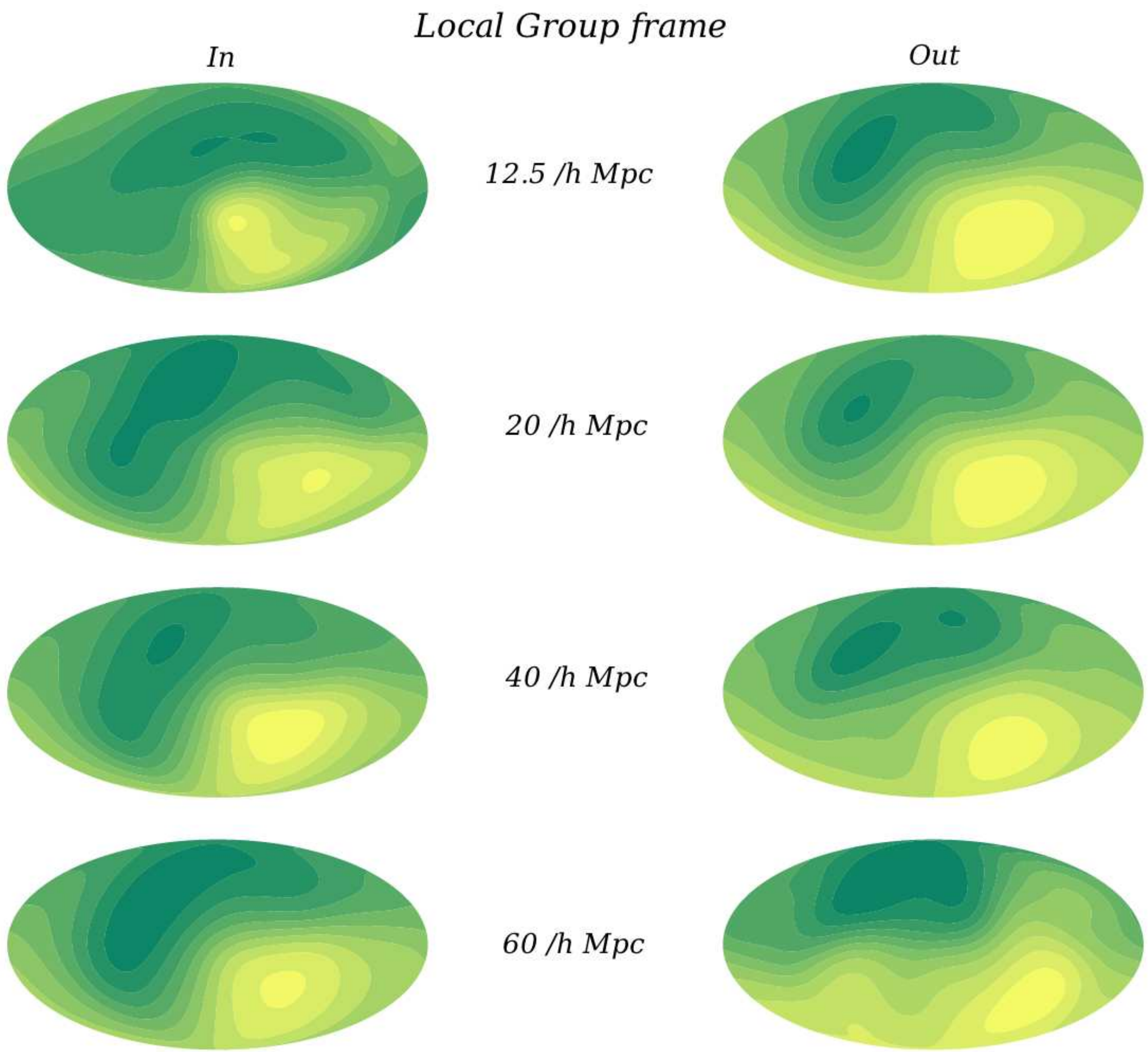
$$H_s = \left(\sum_{i=1}^{N_s} \frac{(cz_i)^2}{\sigma_i^2} \right) \left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2} \right)^{-1} ,$$

- Any boost $cz_i \to cz_i' = c(\gamma - 1) + \gamma \left[cz_i + v \cos \phi_i (1 + z_i) \right] \simeq cz_i + v \cos \phi_i$, then for uniformly distributed data, linear terms cancel on opposite sides of sky

$$H_s' - H_s \sim \left(\sum_{i=1}^{N_s} \frac{(v \cos \phi_i)^2}{\sigma_i^2} \right) \left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2} \right)^{-1}$$

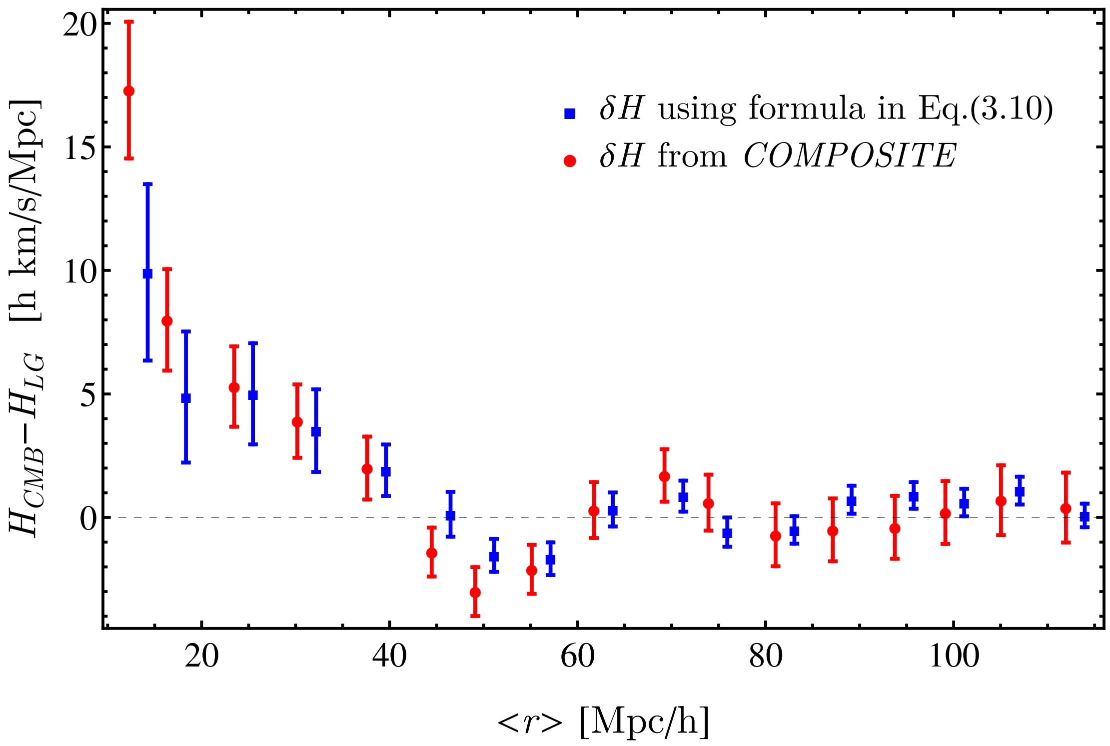
$$= \frac{\langle (v \cos \phi_i)^2 \rangle}{\langle cz_i r_i \rangle} \sim \frac{v^2}{3H_0 \langle r_i^2 \rangle}$$

Angular variation: LG frame



Note: $\ell = 0^\circ$, 180° , 360° on right, centre & left edge respectively

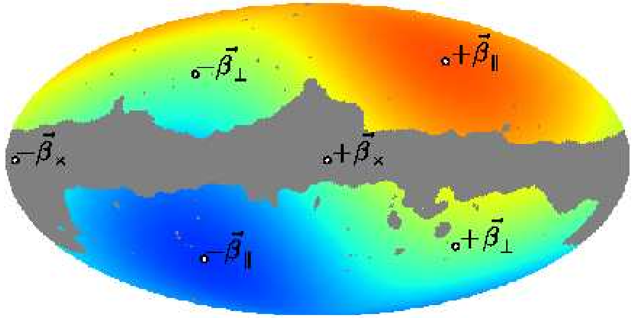
Boost offset and deviation



- Kraljic & Sarkar (JCAP 2016). FLRW + Newtonian N-body simulation with bulk flow $\mathbf{v}_{\text{bulk}}(r)$

$$H_s' - H_s \sim \frac{|\mathbf{v}|^2 - 2\mathbf{v}\cdot\mathbf{v}_{\text{bulk}}(r)}{3H_0\langle r^2\rangle}$$

Systematics for CMB



- Define nonkinematic foreground CMB anisotropies by

$$\Delta T_{\text{nk-hel}} = \frac{T_{\text{model}}}{\gamma_{\text{LG}}(1-\boldsymbol{\beta}_{\text{LG}}\cdot\hat{\mathbf{n}}_{\text{hel}})} - \frac{T_0}{\gamma_{\text{CMB}}(1-\boldsymbol{\beta}_{\text{CMB}}\cdot\hat{\mathbf{n}}_{\text{hel}})}$$

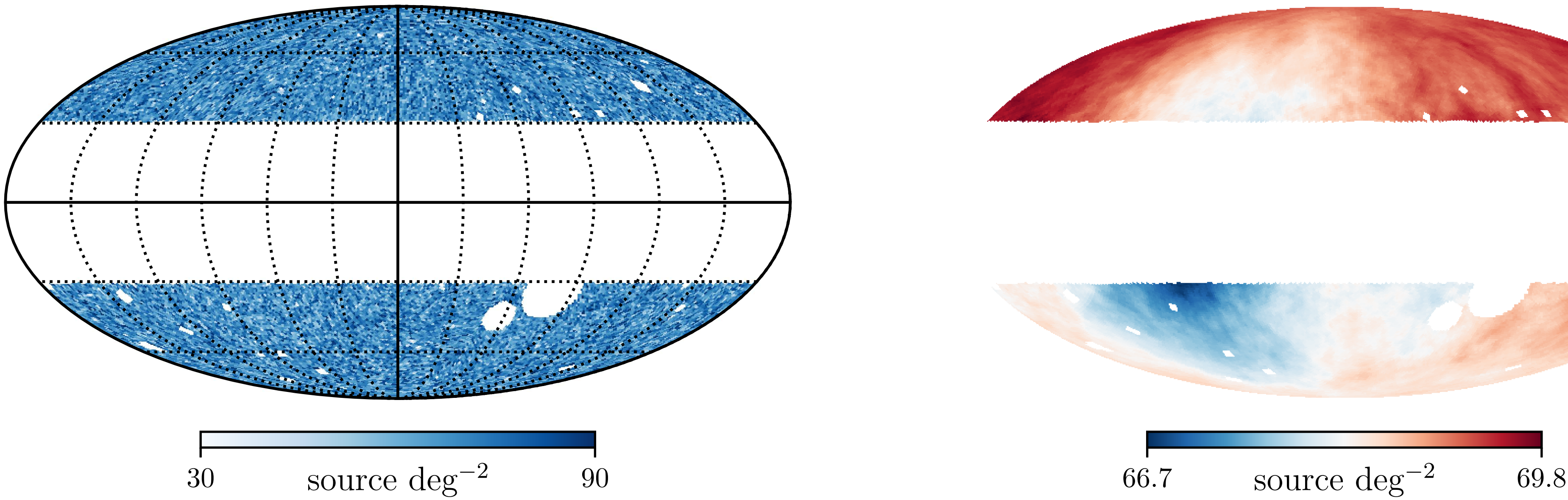
$$T_{\text{model}} = \frac{T_{\text{dec}}}{1+z_{\text{model}}(\hat{\mathbf{n}}_{\text{LG}})}, \quad T_0 = \frac{T_{\text{dec}}}{1+z_{\text{dec}}}$$

$z_{\text{model}}(\hat{\mathbf{n}}_{\text{LG}})$ = anisotropic model LG frame redshift;
T_0 = present mean CMB temperature
[Bolejko, Nazer & DLW, JCAP 06 (2016) 035]

Non-kinematic dipole in radio surveys

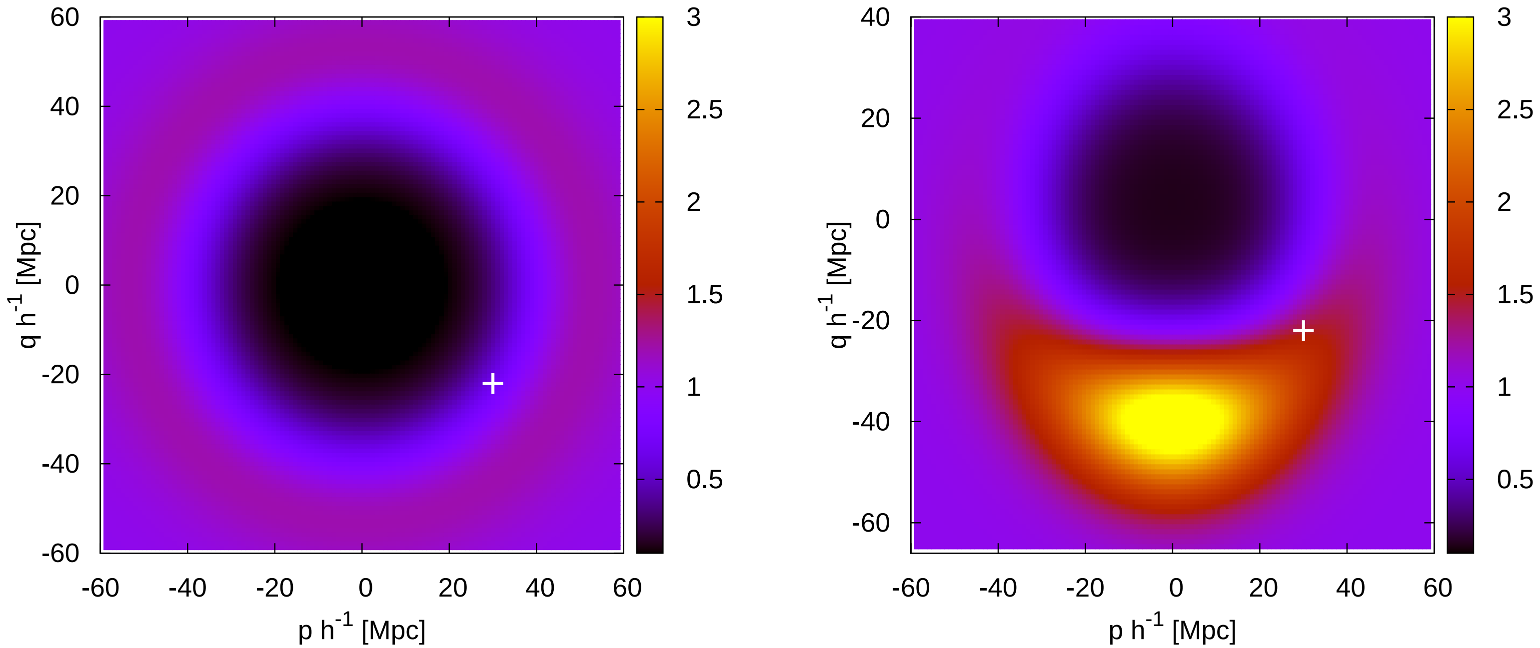
- Ellis–Baldwin test (1984): Aberration and modulation also testable in large galaxy number count surveys
- Rubart & Schwarz 2013: kinematic origin of radio galaxy dipole ruled out at 99.5% confidence
- Direction consistent with CMB dipole but amplitude differs from kinematic prediction by a factor 2 or more
- DLW *et al*. 2013 smoothed Hubble variance dipole in LG frame $(\text{RA}, \text{dec}) = (162^\circ \pm 4^\circ, -14^\circ \pm 3^\circ)$ for $r > r_o$ with $20\,h^{-1} \lesssim r_o \lesssim 45\,h^{-1}$Mpc, or lies within error circle of NVSS survey dipole found by Rubart & Schwarz, $(\text{RA}, \text{dec}) = (154^\circ \pm 21^\circ, -2^\circ \pm 21^\circ)$
- Unexpected anisotropies in other data, e.g., X–ray clusters: Migkas et al, A&A 649 (2021) A151

Non-kinematic dipole in quasar surveys



- Secrest *et al*. ApJ 908 (2021) L51: 1.36 million quasars out to large redshifts, $z < 3.6$, peaked at $z \sim 1.0$
- Kinematic origin of dipole *rejected* at 4.9 σ
- See: https://www.youtube.com/watch?v=eSYY9nnuNIo
- Combined with NRAO VLA Sky Survey – kinematic dipole *rejected* at 5.2 σ, ApJ 937 (2022) L31

LTB and Szekeres profiles



- Fix $\Delta r = 0.1 r_0$, $\varphi_{obs} = 0.5\pi$
- LTB parameters: $\alpha = 0$, $\delta_0 = -0.95$, $r_0 = 45.5\ h^{-1}$ Mpc; $r_{obs} = 28\,h^{-1}$Mpc, $\vartheta_{obs} =$ any
- Szekeres parameters: $\alpha = 0.86$, $\delta_0 = -0.86$; $r_{obs} = 38.5\ h^{-1}$ Mpc; $r_{obs} = 25h^{-1}$ Mpc, $\vartheta_{obs} = 0.705\pi$.

Szekeres model ray tracing constraints

- Require Planck satellite normalized FLRW model on scales $r \gtrsim 100\,h^{-1}$Mpc; i.e., spatially flat, $\Omega_m = 0.315$ and $H_0 = 67.3$ km/s/Mpc
- CMB temperature has a maximum $T_0 + \Delta T$, where

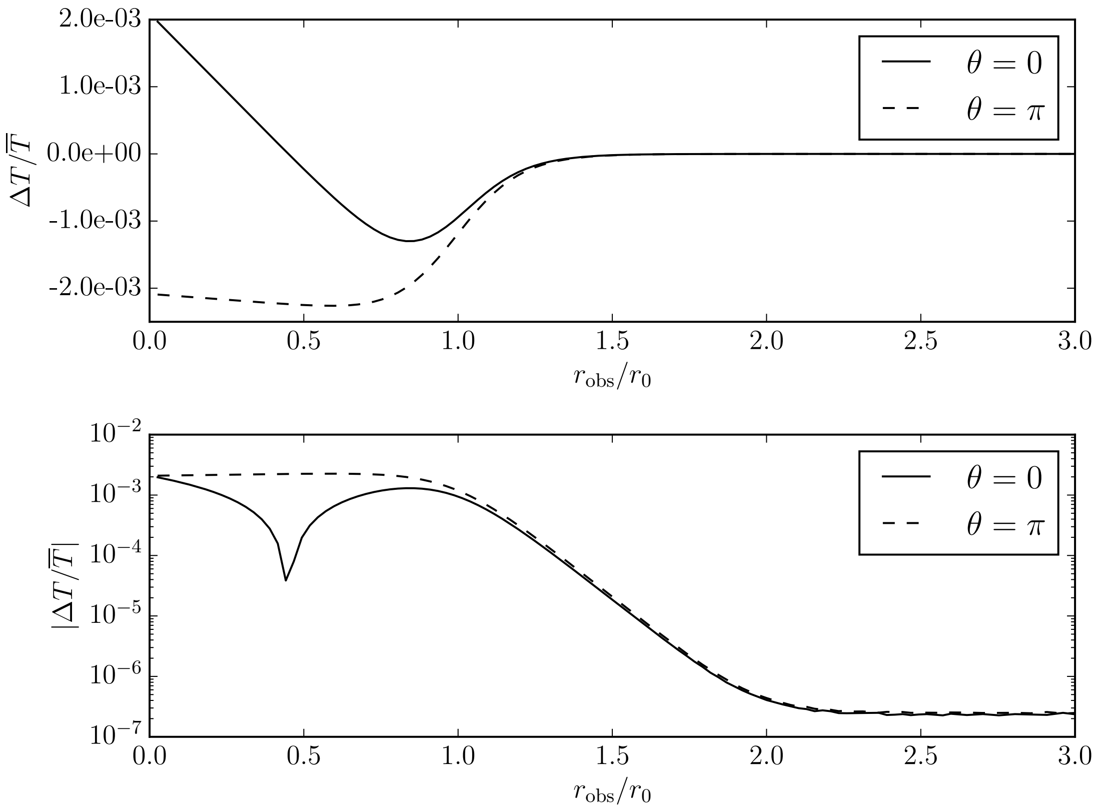
$$\Delta T(\ell = 276.4^\circ, b = 29.3^\circ) = 5.77 \pm 0.36 \text{ mK},$$

  matching dipole amplitude, direction in LG frame
- CMB quadrupole anisotropy lower than observed

$$C_{2,CMB} < 242.2^{+563.6}_{-140.1}\ \mu\text{K}^2.$$

- Hubble expansion dipole (LG frame) matches COMPOSITE one at $z \to 0$, if possible up to $z \sim 0.045$
- Match COMPOSITE quadrupole similarly, if possible

Peculiar potential not Rees–Sciama

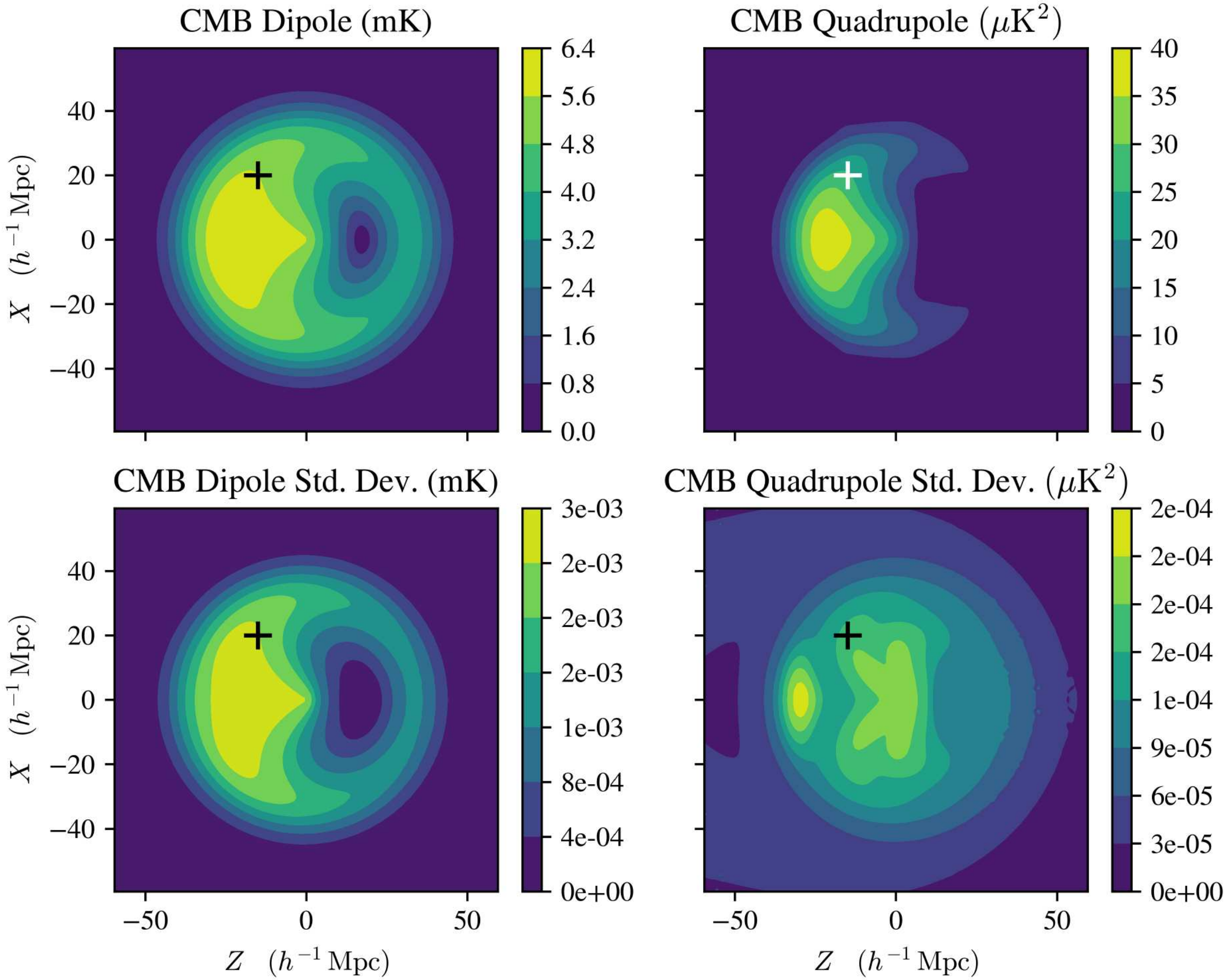


- Rees–Sciama (and ISW) consider photon starting and finishing from *average* point
- Across structure $|\Delta T|/T \sim 2 \times 10^{-7}$
- Inside structure $|\Delta T|/T \sim 2 \times 10^{-3}$

Thesis adventure in three

- AIM: Constrain effective Λ-Szekeres model by CMB & "peculiar velocity" data on $\lesssim 150\,h^{-1}$Mpc scales. Determine amplitude, direction of Ellis–Baldwin effect
- Lawerence Dam, 2016: Corrected bug in BNW Szekeres code. Deep exploration of notion of non-kinematic differential expansion in LTB
  - *https://ir.canterbury.ac.nz/handle/10092/13167*
- Morag Hills, 2022: Added Haantjes transform methods to better perform ray-tracing in Szekeres models
  - *https://ir.canterbury.ac.nz/handle/10092/103762*
- Finn O'Keeffe, 2023: Performed MCMC simulations on BNW & new models via non-trivial Haantjes transforms
  - *https://ir.canterbury.ac.nz/handle/10092/105565*

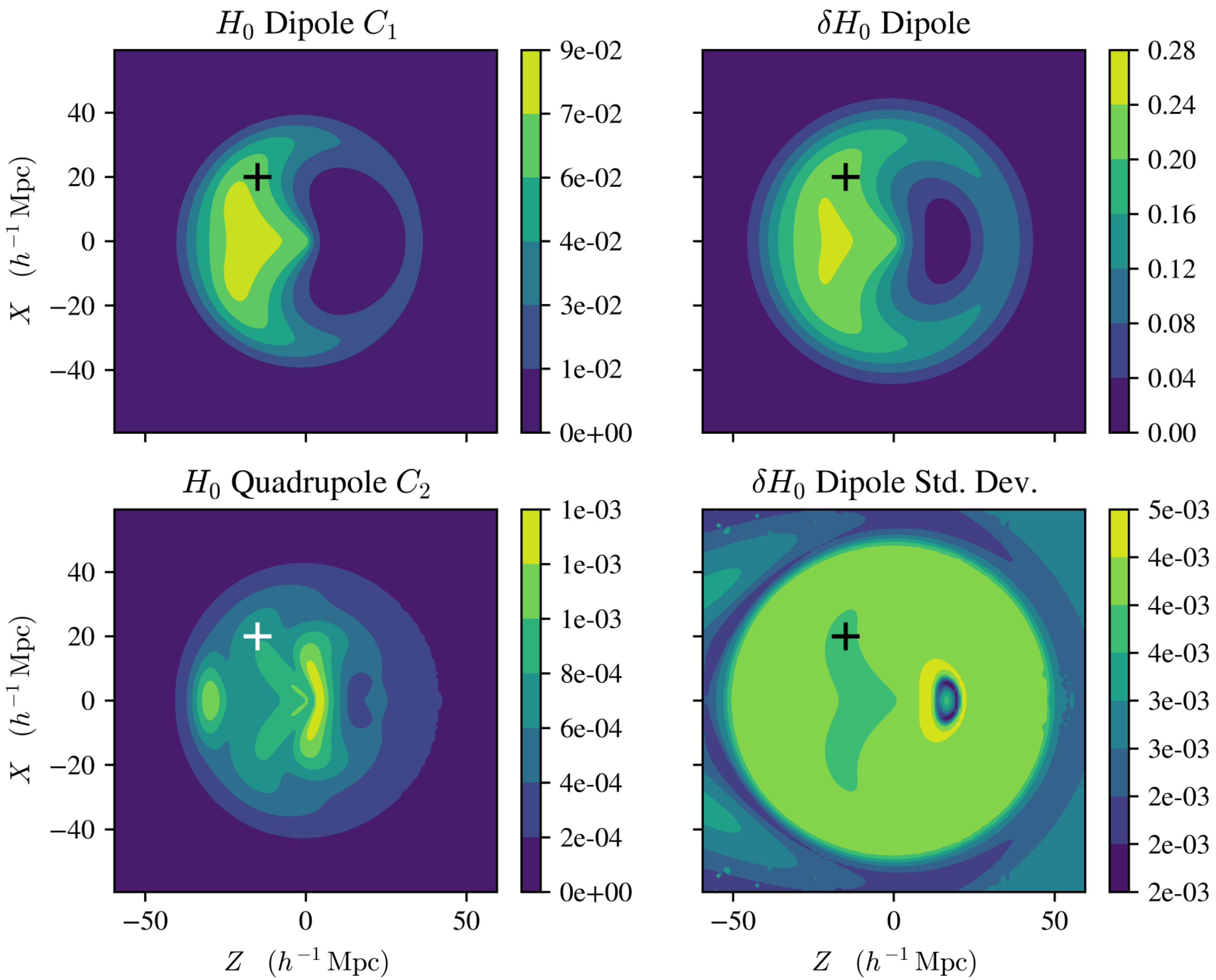
CMB dipole, quadrupole examples



Corrected BNW base model: CMB dipole, quadrupole amplitude, standard dev.

Morag A. Hills, MSc thesis, U Canterbury, 2022, https://ir.canterbury.ac.nz/handle/10092/103762

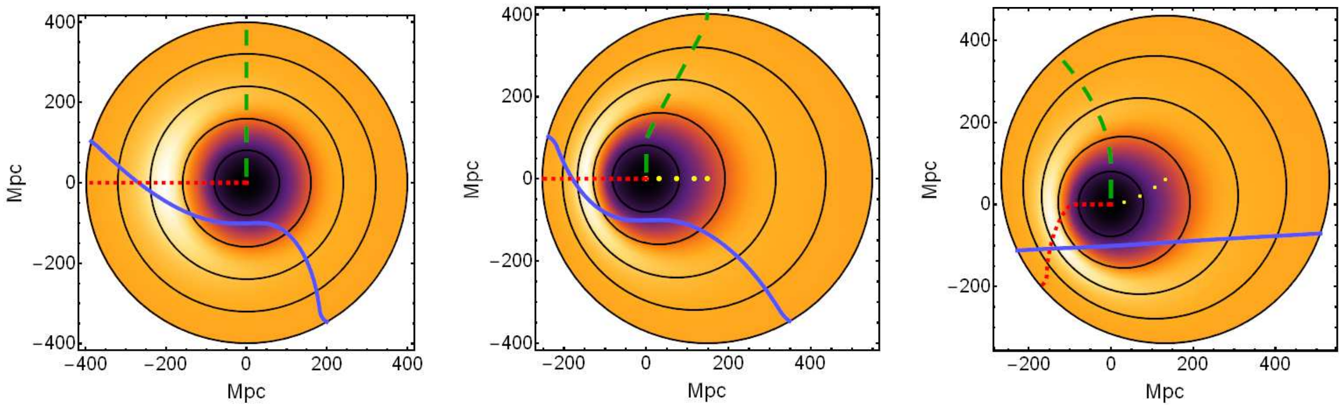
Hubble dipole, quadrupole examples

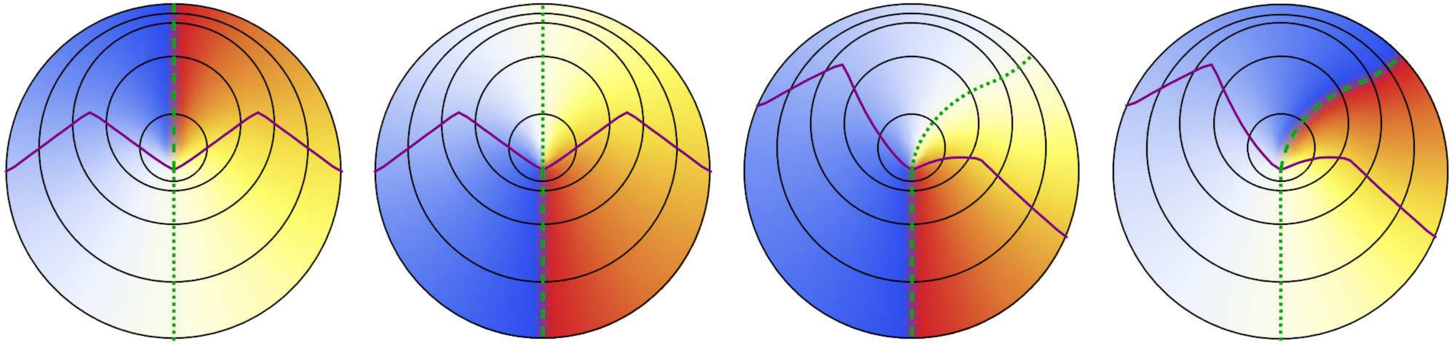


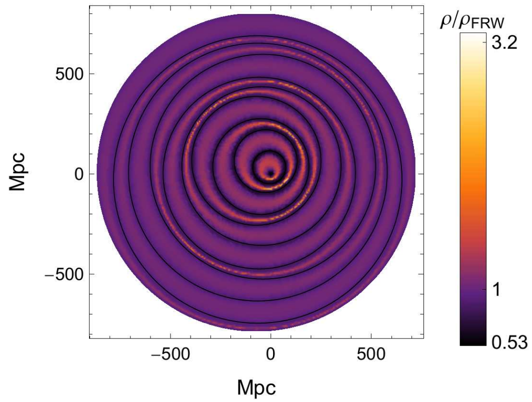
Corrected BNW base model: Hubble dipole, quadrupole, $\delta H_0 = (H_0 - \bar{H}_0)/\bar{H}_0$.

Morag A. Hills, MSc thesis, U Canterbury, 2022, https://ir.canterbury.ac.nz/handle/10092/103762

Szekeres, Haantjes transformations



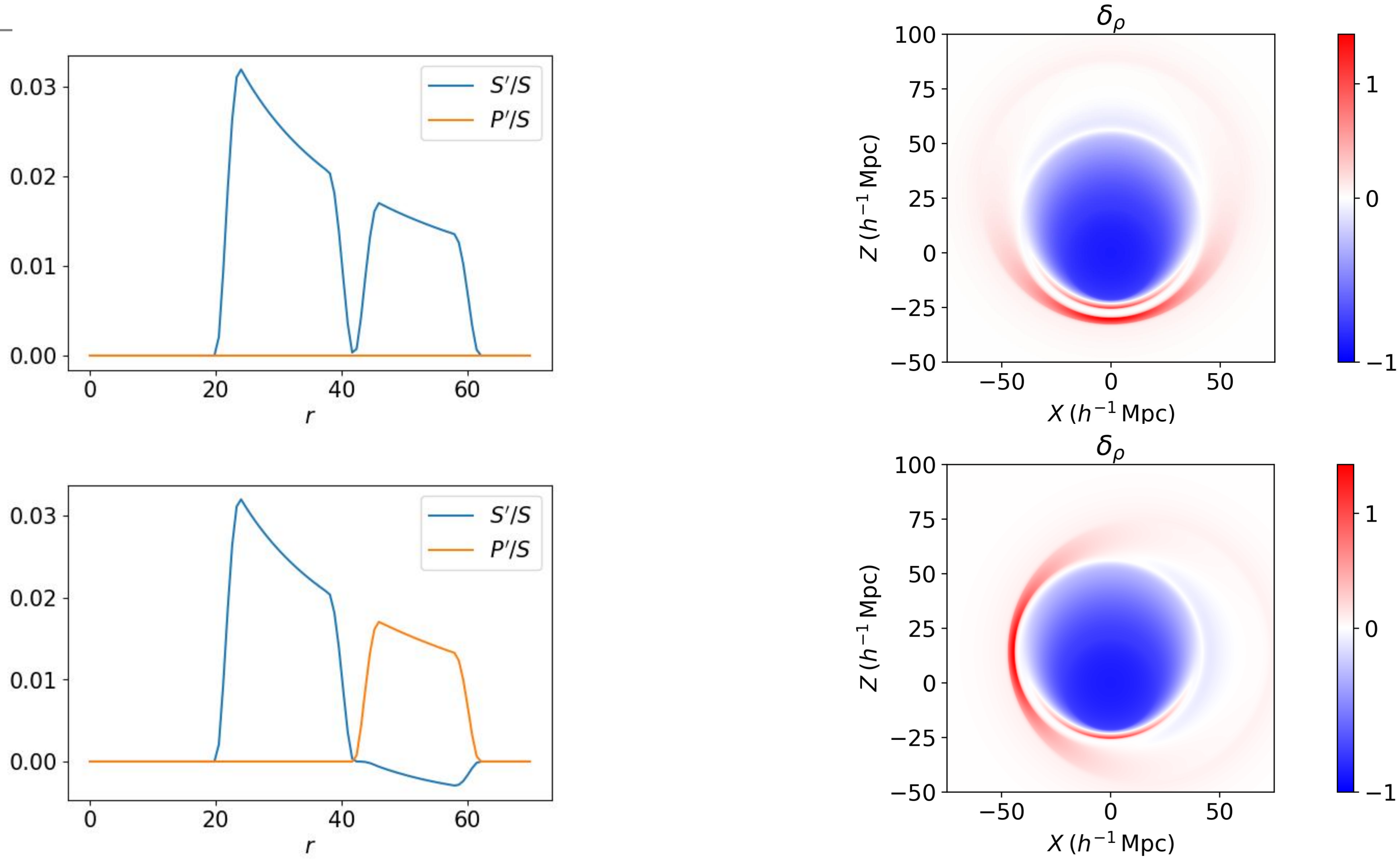




RG Buckley & EM Schlegel
Phys. Rev. D101 (2020) 023511

Top: Coordinate versus (almost) proper distances;
Middle: Haantjes coord transformations of dipole functions;
Right: Piecewise Haantjes transformations – new models

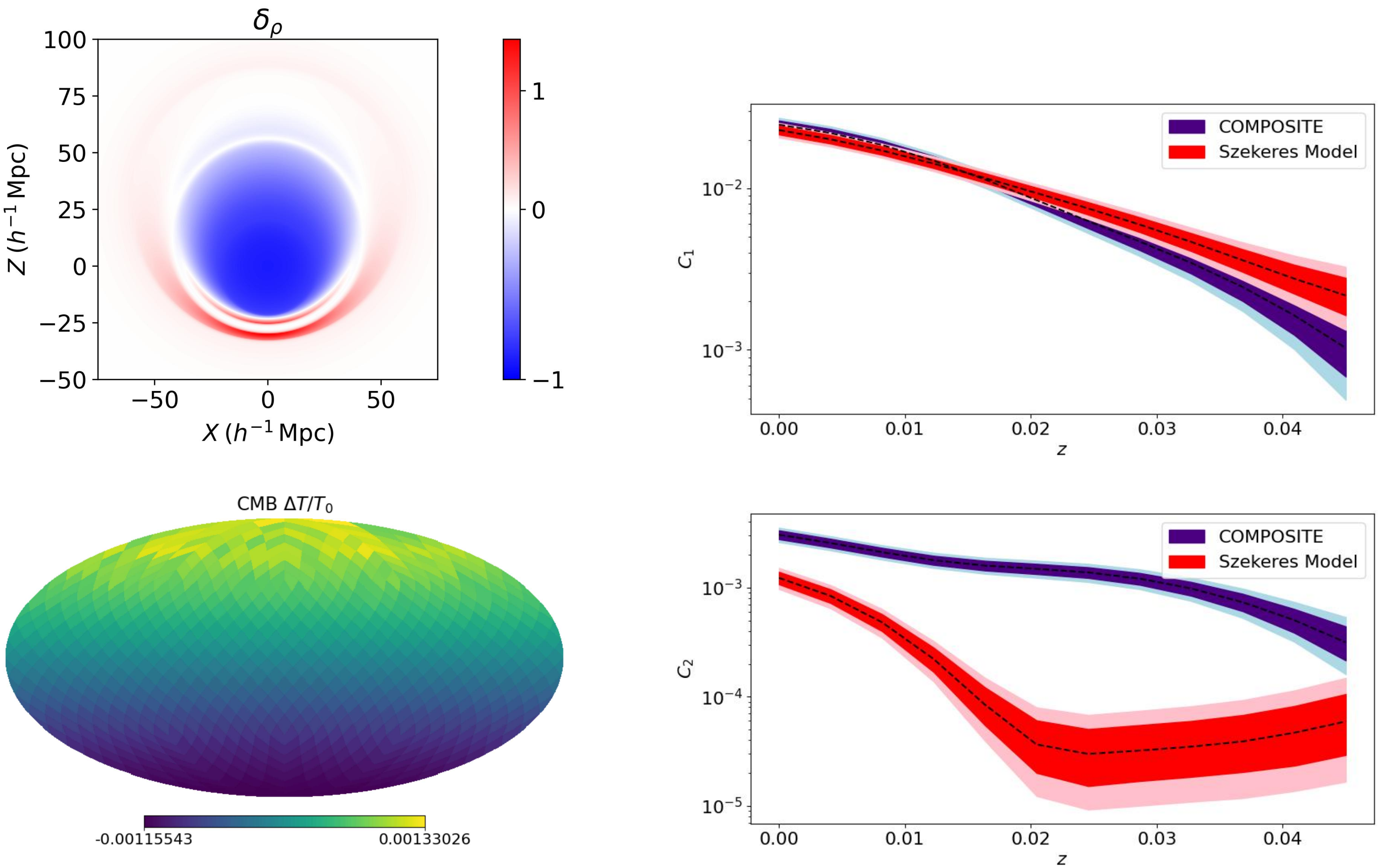
Λ-Szekeres, piecewise Haantjes



Dipole functions S'/S and P'/S and density contrast. Top unrotated ($\Theta = 0$), bottom rotated ($\Theta = \pi/2$). Parameters $\delta_0 = -0.9$, $\alpha_1 = \alpha_2 = 0.8$, and $r_0 = 40.0\,h^{-1}$ Mpc.

Finn O'Keeffe, MSc thesis, U Canterbury, 2023, https://ir.canterbury.ac.nz/handle/10092/105565

Λ-Szekeres, piecewise Haantjes



Case 1. $\Delta T = 3.14$ mK. *Left:* CMB dipole; *Right:* Hubble expansion dipole C_1, quadrupole C_2 for model (red) compared to COMPOSITE data in LG frame (blue)

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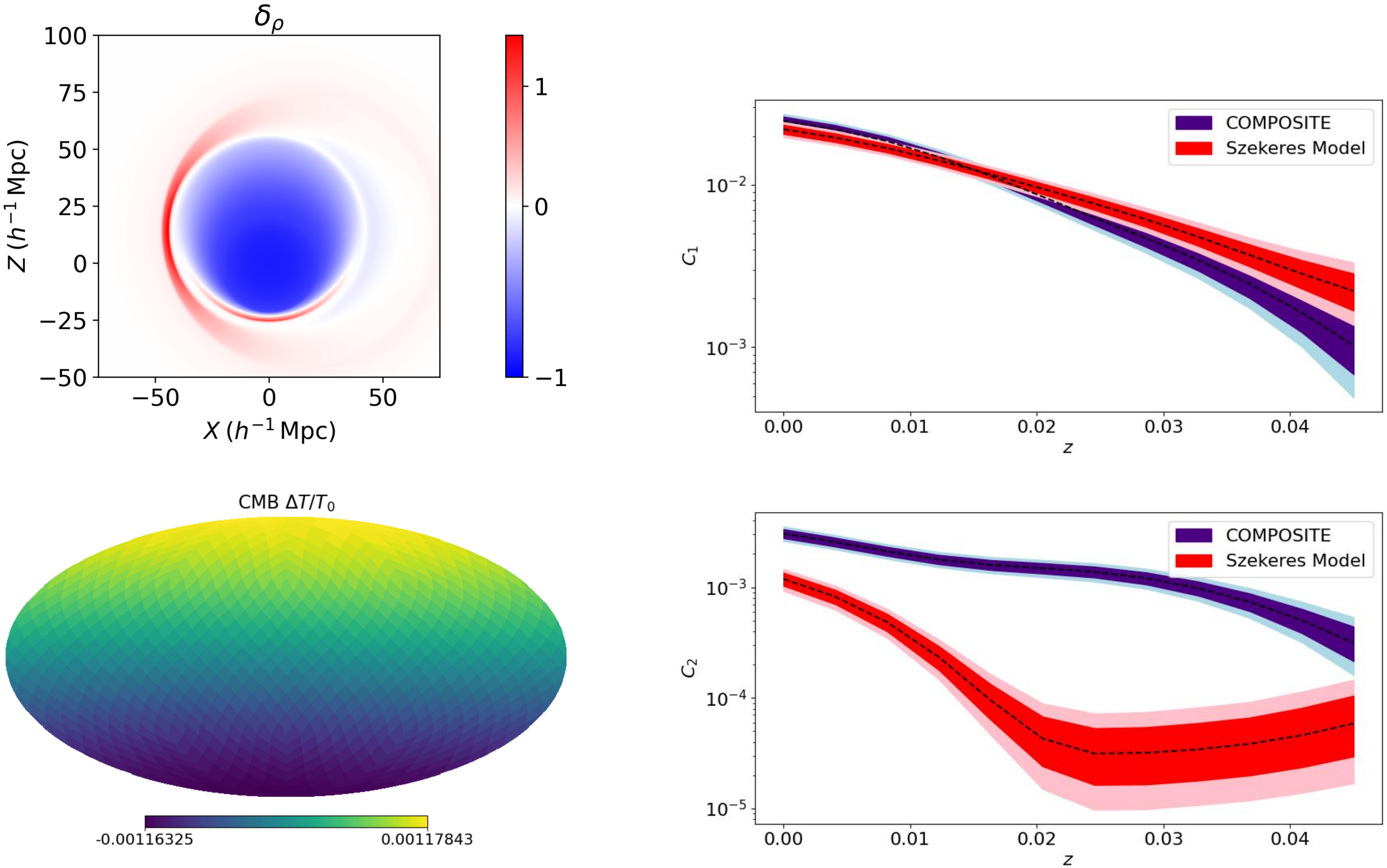

Case 2. $\Delta T = 3.17$ mK. *Left:* CMB dipole; *Right:* Hubble expansion dipole C_1, quadrupole C_2 for model (red) compared to COMPOSITE data in LG frame (blue)

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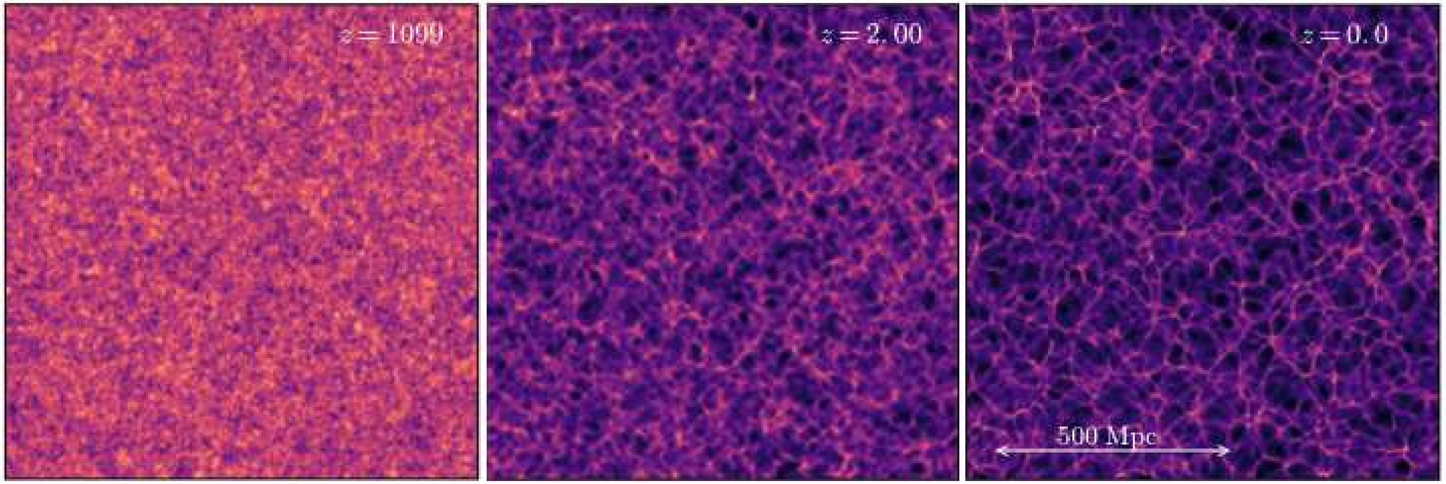
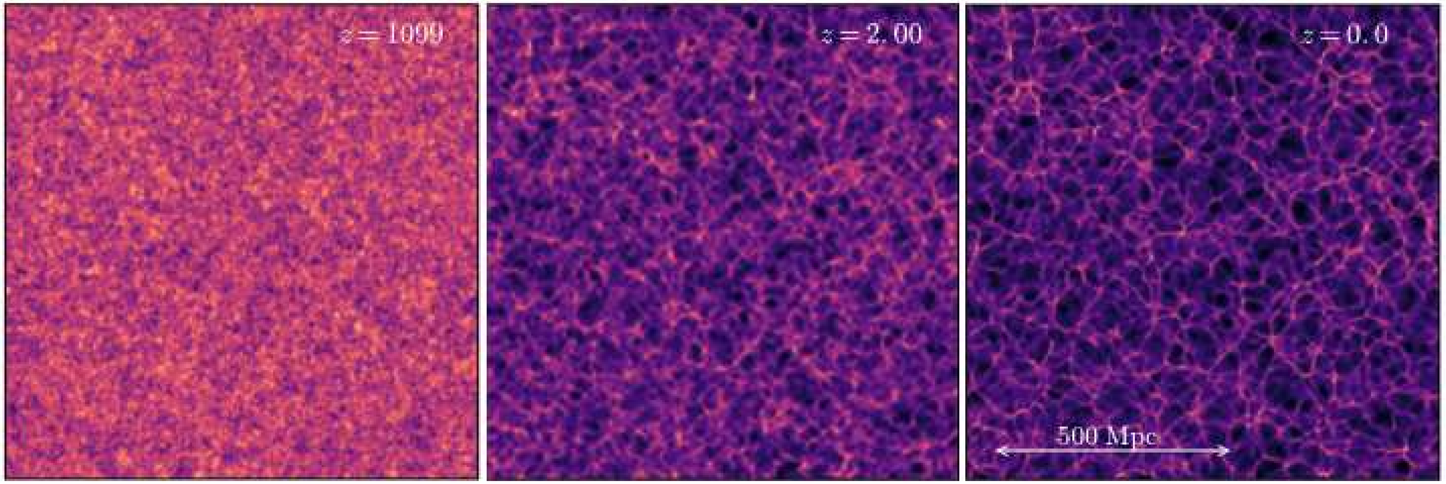
Full simulations: GRtoolkit



Macpherson et al, ApJ 865 (2018) L4, PRD 99 (2019) 063522

- Hayley Macpherson, Michael Williams:
  - Exploring void statistics in EdS and ΛCDM sims
- Simulations: self-consistency check of ΛCDM thus far
- “Torus condition” a concern even in this context
  - Numerical evidence for GR equivalent of Buchert–Ehlers no backreaction theorem
  - GR theorem: H. Macpherson, P. Mourier, in progress

Full simulations: goals



Macpherson et al, ApJ 865 (2018) L4, PRD 99 (2019) 063522

- With H Macpherson, K Bolejko, T Buchert:
  - Extremely large simulations to avoid torus condition
  - Other methods (silent universes etc – Bolejko)
  - Test self-consistency of open FLRW universe
  - Use timescape initial conditions that violate FLRW at 10^{-5}–10^{-6} on last scattering surface
- E P Snowden 1ST/2ND postdoc – AAS Job Register

Summary: Why is ΛCDM successful?

- Early Universe was extremely close to homogeneous and isotropic, leading to a simplifying principle – Cosmological Equivalence Principle
- Finite infinity geometry ($2 - 15\,h^{-1}$Mpc) is close to spatially flat (Einstein–de Sitter at late times) – N–body simulations successful *for bound structure*
- Hubble parameter (first derivative of statistical metric; i.e., connection) is to some extent observer dependent
- *Even on small scales there is a notion of uniform Hubble flow at expense of calibration of rulers AND CLOCKS*
- Full GR numerical relativity simulations to be explored (HJ Macpherson, M Williams, DLW)
- CBL test of Friedmann equation (Euclid etc) this decade
- FLRW/ΛCDM may fall sooner on Ellis–Baldwin test?

The future: personal view

- Modified geometry rather than “modified gravity”
- Rigorous definition of finite infinity key; causality!
- “Amplitudes”, BMS, gravitational memory, Strominger’s triangle, quantum gravity is finally coming together
- Theorists, modellers, observers must talk **about foundations**
- Different theorists (particles hep-th hep-ph) versus general relativity (gr-qc) versus astrophysics (astro-ph) must talk among themselves
- Quantum mechanics was effort of many
- Quantum gravity will be too!