Copernicus Webinar, 2023.7.4

Can we detect gravitons or HFGWs?

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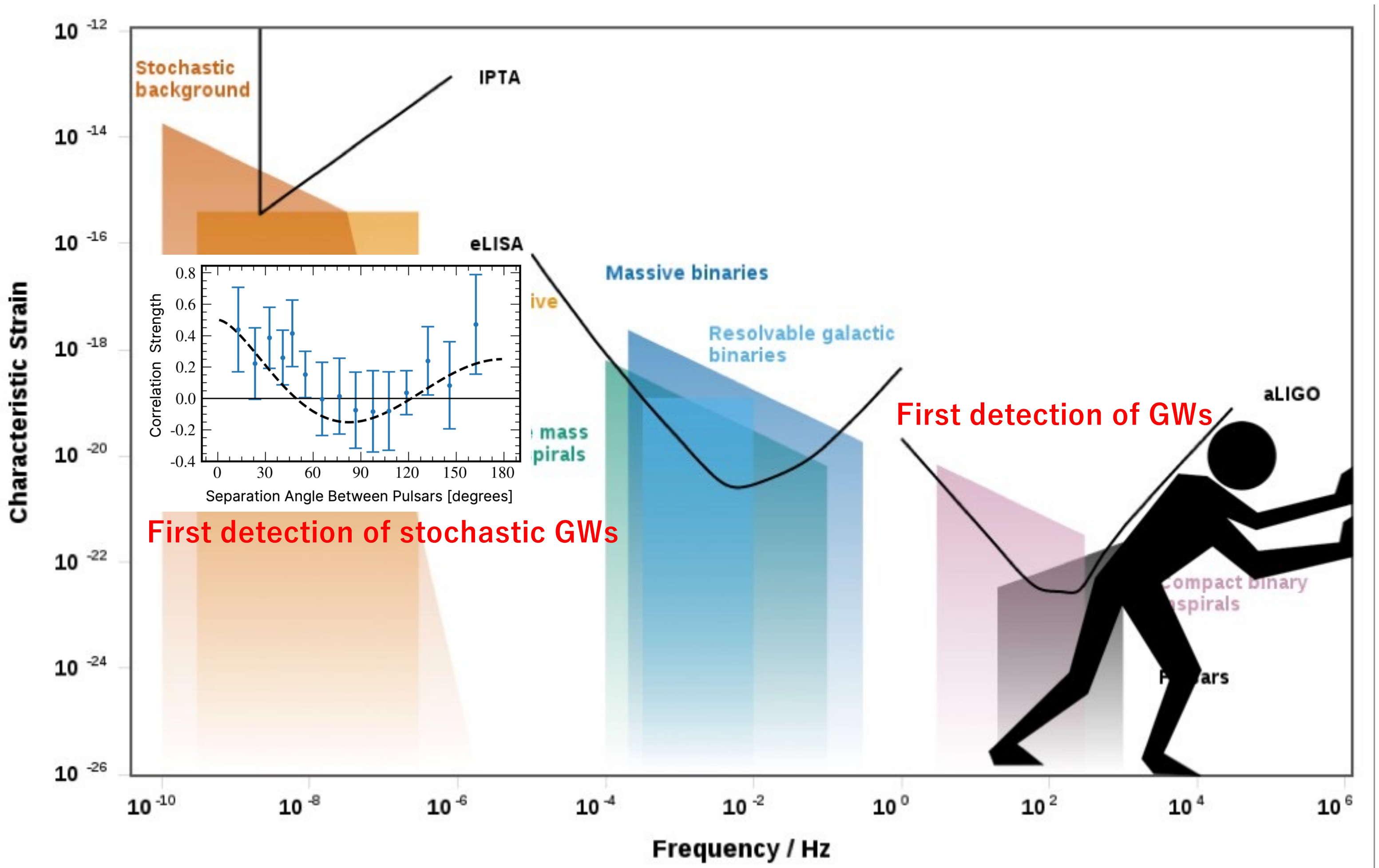
Frequency frontier of GWs

It is important to extend frequency ranges we can observe!



gravitons?

HFGWs?

Can we detect a graviton with LIGO?

Astrophysical sources can create a bunch of gravitons!

However, it is difficult to identify a single graviton. Dyson 2013

The reason is simple.

GWs detected at LIGO have
a frequency 1kHz and an amplitude $h \approx 10^{-21}$.

energy density $\rho = \dfrac{c^2}{32\pi G}\omega^2 h^2$ $\omega = 1\text{kHz}, \ \ \text{h}=10^{-21} \quad \Rightarrow \quad 10^{-10}\text{erg/cm}^3$

energy density of a single graviton $\rho_s = \dfrac{\hbar\omega^4}{c^3} \Rightarrow 10^{-47}\text{erg/cm}^3$

number of gravitons detected $n_g = \dfrac{1}{32\pi}\dfrac{c^3}{G\hbar}\left(\dfrac{c}{\omega}\right)^2 h^2 = 10^{37}$

Hence, it is difficult to resolve a single graviton with LIGO.

There remain two possible directions

Indirect detection

It might be possible to detect gravitons indirectly
through the noise of gravitons like as
a discovery of molecules through the Brownian motion.

Quantum information can be utilized.
We study the decoherence process of entanglement due to gravitons.

Detection with conversion

High frequency gravitational waves

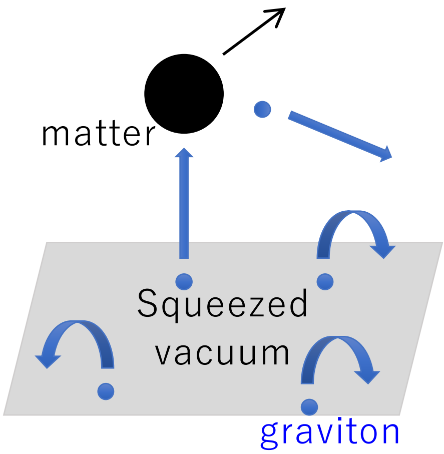
Quantum sensing would be important.

A basic picture of the first part

The present universe originates from inflation,
which is supported by CMB and galaxy observations.

The inflation predicts primordial gravitational waves.
This can be regarded as the condensation of gravitons.
Indeed, the vacuum state of gravitons is squeezed.

We consider graviton fluctuations in this background.

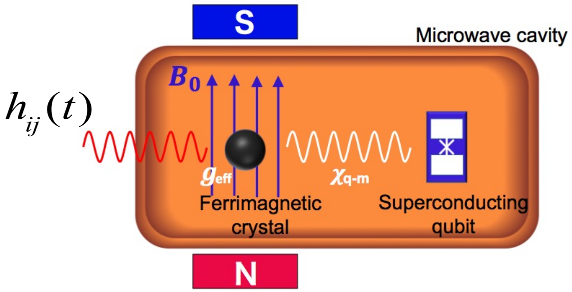


Suppose a massive quantum object is moving.
The object is interacting with gravitons.
This induces decoherence of the quantum entanglement.

Thus, we need to know
primordial gravitational waves
and how matter couples with gravity.

A basic picture of the second part

The basic idea for detecting high frequency GWs is to use
the interaction between quantum matter and high frequency GWs.
We take magnons as the quantum matter in this talk.



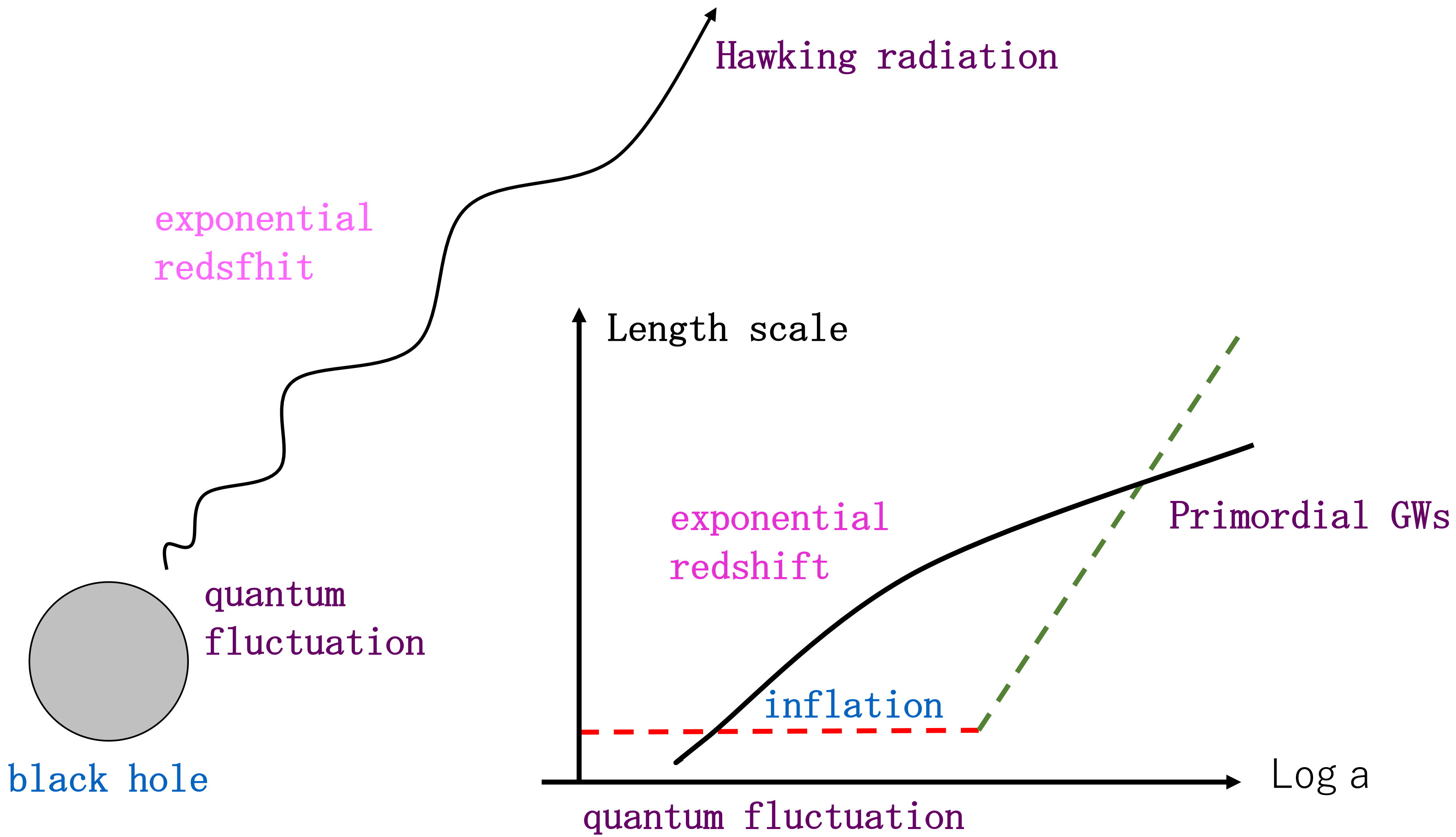
As a working hypothesis , we use the duality between axion and gravity.
More precisely, we use the existing data of axion experiments
to give constraints on the amplitude of GWs at GHz.
However, our aim is to detect high frequency GWs by utilizing
recent development of quantum sensing
with the hope detecting high frequency GWS leads to detecting a graviton.

Plan of this talk

- Basics of primordial GWs
- How matter interact with gravity?
- Quantum matter in graviton background
- Indirect detection of gravitons
- Duality of axion and graviton experiments
- Graviton search with magnons   Detection with conversion
- Summary

Basics of primordial GWs

PGWs as Quantum fluctuations of spacetime!

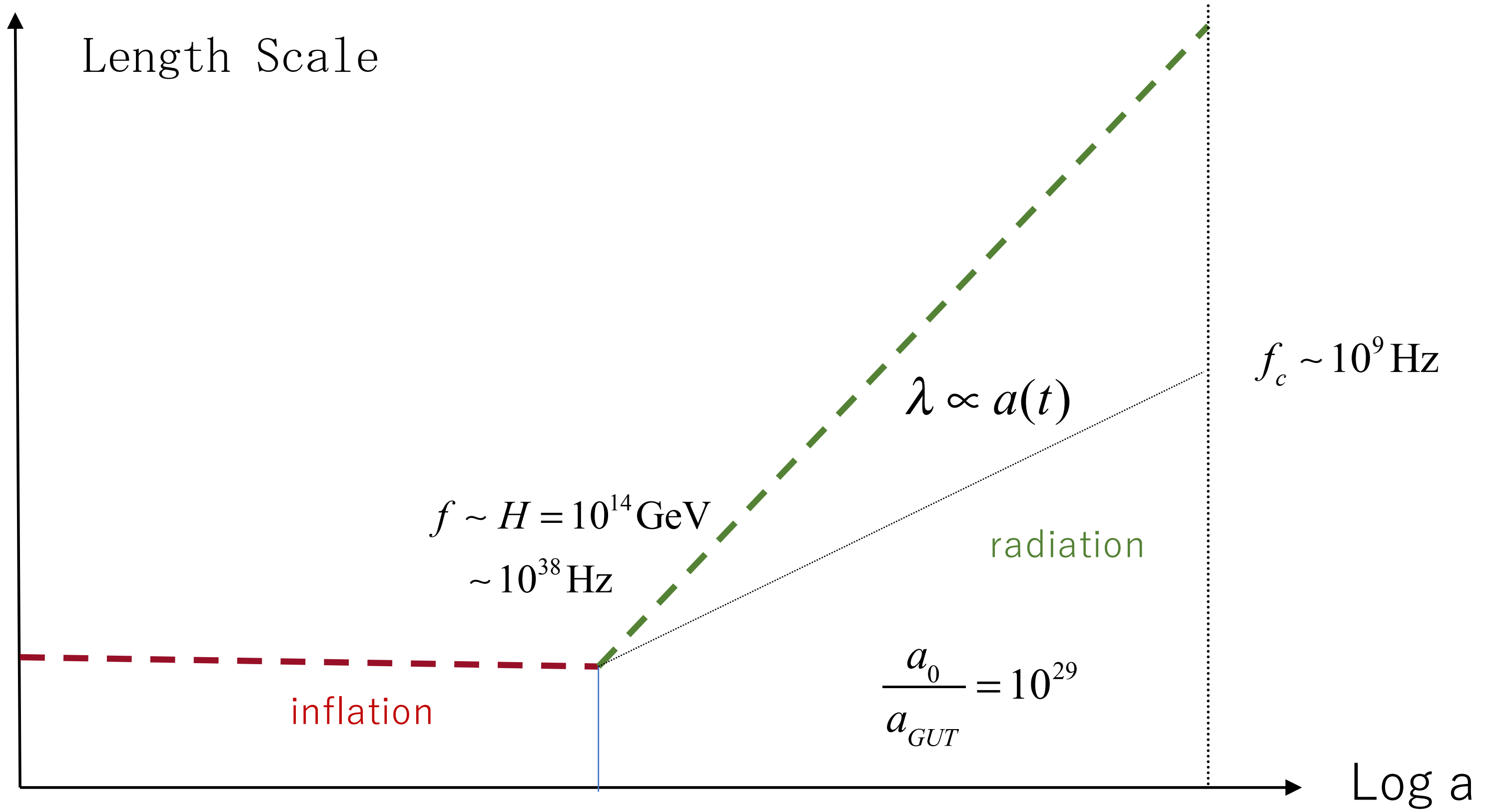


Maximal frequency of PGWs

Free fall time $t_{ff} \sim 1/\sqrt{G\rho}$ → frequency $f \sim \sqrt{G\rho} \sim H$

We observe GWs with a red shifted frequency $f_{obs} \sim H\frac{a(t_e)}{a(t_0)} \propto a(t_e)$

Hence, GWs with the maximum frequency are created at the end of inflation.



Squeezed state

Grishchuk 1998

Graviton production is described by the Bogoliubov transformation

$$b_k = \alpha_k a_k + \beta_k^* a_k^\dagger \quad \Longrightarrow \quad a_k = \alpha_k^* b_k - \beta_k^* b_k^\dagger$$

Thus, the cosmic expansion creates the two mode squeezed state.

$$\left| BD \right\rangle \propto \exp\left[\sum_k \tanh r_k b_k^\dagger b_{-k}^\dagger \right] \left| 0_R \right\rangle \qquad \tanh r_k = \frac{\beta_k^*}{\alpha_k^*}$$

$$\propto \left| 0_{\mathbf{k}} \right\rangle \otimes \left| 0_{-\mathbf{k}} \right\rangle + \tanh r_k \left| 1_{\mathbf{k}} \right\rangle \otimes \left| 1_{-\mathbf{k}} \right\rangle + \tanh^2 r_k \left| 2_{\mathbf{k}} \right\rangle \otimes \left| 2_{-\mathbf{k}} \right\rangle + \cdots$$

$$\Omega_g = \frac{1}{\rho_c} \frac{d\rho_g}{d \log f} = \frac{16\pi^2}{\rho_c} f^4 \left| \beta_k \right|^2 = 10^{-14} \left(\frac{H}{10^{-4} M_p} \right)^2$$

The squeezing parameter r_k can be calculated as

$$\sinh r_k = \frac{1}{2} \left(\frac{f_c}{f} \right)^2 \qquad f_c = 10^9 \sqrt{\frac{H}{10^{-4} M_p}}\,\mathrm{Hz}$$

We assume the present graviton state is kept to be squeezed.

How Matter interact with gravity?

Equivalence principle

Any metric can be set to that of Minkowski spacetime along a geodesic.

Thus, the geodesic motion of a single particle
is always decoupled with a geometry.

$$S_M = -M \int_{\gamma_\tau} d\tau = -M \int d\tau \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}$$

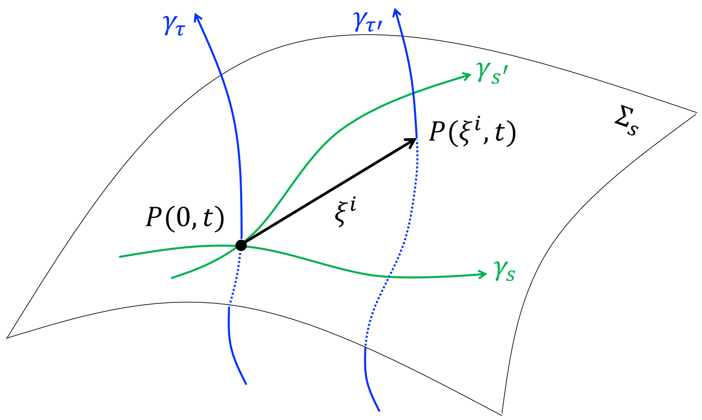
In this local Lorentz coordinate system, we can expand the metric as

$$g_{\mu\nu}\left(x^\alpha\right) = \eta_{\mu\nu} + \underline{\frac{1}{2!} g_{\mu\nu,\alpha\beta}\Big|_{\gamma_\tau}} \left(x^\alpha - x_0^\alpha\right)\left(x^\beta - x_0^\beta\right) + \cdots$$

described by the spacetime curvature

Fermi normal coordinates

Because of the equivalence principle,
only the relative motion has physical meaning.



Along a geodesic, we can construct the local Lorentz frame, so-called Fermi-normal coordinate

Action of a geodesics deviation reads

$$S_m = -M\int_{\gamma_\tau} d\tau - m\int_{\gamma_{\tau'}} d\tau' = -m\int d\tau' \sqrt{-g_{\mu\nu}(x^\alpha(\tau'))\frac{dx^\mu(\tau')}{d\tau'}\frac{dx^\nu(\tau')}{d\tau'}}$$

$$= -m\int d\tau' \left[1 + \frac{1}{2}R_{0i0j}(t)\xi^i(t)\xi^j(t) - \frac{1}{2}\left(\frac{d\xi^i(t)}{dt}\right)^2 + \cdots\right]$$

$$R_{0i0j} = -\frac{1}{2}\frac{\partial^2 h_{ij}}{\partial t^2}$$ in TT gauge

Interaction between a particle and graviton

$$S_m = \int dt \left[\frac{m}{2}\left(\frac{d\xi^i}{dt}\right)^2 + \frac{m}{4}\ddot{h}_{ij}(x^i = 0, t)\xi^i\xi^j\right]$$

Quantum matter
in graviton background

S.Kanno, J.S. and J.Tokuda,
``Noise and decoherence induced by gravitons,''
Phys. Rev. D 103, 044017 (2021)

Geodesic motion in graviton BG

Equations of motion

$$\ddot{h}^A(\mathbf{k},t)+k^2h^A(\mathbf{k},t)=\frac{m}{2M_p\sqrt{V}}e_{ij}^{*A}(\mathbf{k})\frac{d^2}{dt^2}\left\{\xi^k(t)\xi^l(t)\right\}$$

$$\ddot{\xi}(t)=\frac{1}{M_p\sqrt{V}}\sum_{\mathbf{k},A}e_{ij}^A(\mathbf{k})\ddot{h}^A(k,t)\xi^j(t)$$

Quantum langevin equation

Parikh, Wilczek, Zahariade 2020, 2021
Kanno, Tokuda, Soda 2021

$$\ddot{\xi}^i(t)+\underbrace{\frac{m}{40\pi M_p^2}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right)\xi^j\frac{d^5}{dt^5}\left\{\xi^k\xi^l\right\}}_{\text{reaction force}}=-N_{ij}\xi^j$$

Noise: $N_{ij}=\frac{2}{M_p\sqrt{V}}\sum_{\mathbf{k},A}k^2h_I^A(\mathbf{k},t)e^{i\mathbf{k}\cdot\mathbf{x}}e_{ij}^A(\mathbf{k})$

$$h_I^A(\mathbf{k},t)=a_A(\mathbf{k})u_k(t)+a_A^\dagger(-\mathbf{k})u_k^*(t)$$

$$\left[a_A(\mathbf{k}),a_{A'}^\dagger(\mathbf{k}')\right]=\delta_{AA'}\delta_{\mathbf{k},\mathbf{k}'}$$

Noise of gravitons in squeezed state

Noise correlation

$$\langle\psi|\{N_{ij}(t),N_{kl}(0)\}|\psi\rangle=\frac{1}{10\pi^2M_p^2}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\right)F(\Omega_m t)$$

Ω_m : cutoff

$$F(\Omega_m t)=\int_0^{\Omega_m}dk\,k^6\,\mathrm{Re}\left(u_k^{sq}(t)u_k^{sq*}(0)\right)$$

$$u_k^{sq}(t)=u_k^M(t)\cosh r_k-u_k^{M*}(t)\sinh r_k$$

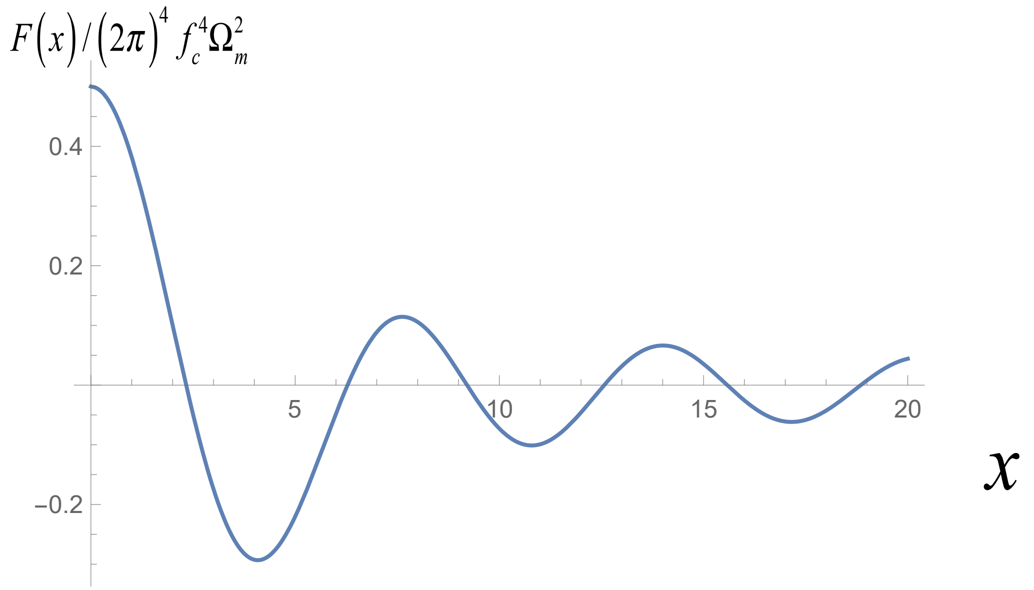
$$u_k^M(\eta)=\frac{1}{\sqrt{2k}}e^{-ik\eta}$$

Recall the formula

$$\sinh r_k=\frac{1}{2}\left(\frac{f_c}{f}\right)^2,\quad k=2\pi f$$

For $r_k\gg 1$, we obtain

$$F(x)=(2\pi)^4 f_c^4\Omega_m^2\frac{x\sin x+\cos x-1}{x^2}$$



Thus, squeezed state significantly enhances the noise of gravitons.

Indirect detection of gravitons

S.Kanno, J.S. and J.Tokuda,
``Indirect detection of gravitons through quantum entanglement,''
Phys.Rev.D 104 (2021) 083516.

Quantum mirrors and a single photon

By using laser cooling, we can make a ground state of mirrors.

$$|\psi(t_i)\rangle = \underline{|0\rangle \otimes |0\rangle} \otimes |g\rangle$$

Yu et al. 2020

mirror graound state

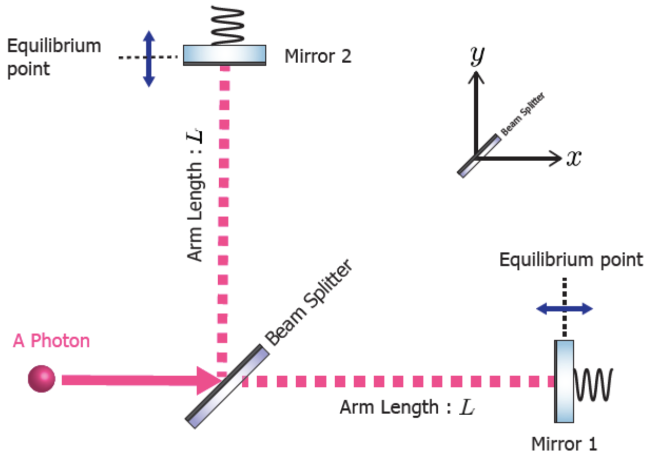
squeezed graviton state

40kg quantum mirrors

A single photon goes through a beam splitter or reflected by it, which is described by a superposition state.

Marshall et al. 2003

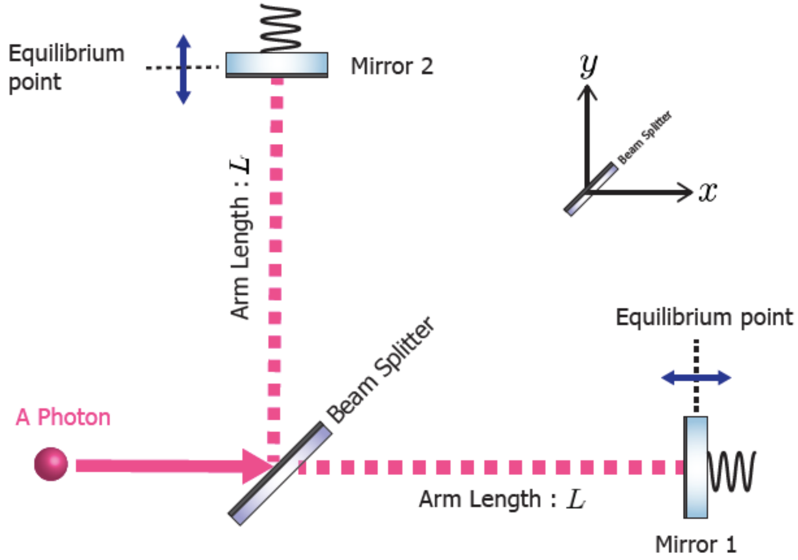
$$|\Phi(t_i)\rangle = |\mathrm{arm}:1\rangle + |\mathrm{arm}:2\rangle$$



Entangled mirrors

When the photon hit a mirror, the mirror starts to oscillate.
Thus, there arises a superposition state of mirrors.

$$|\psi(t_i)\rangle = \left\{ \frac{1}{\sqrt{2}} |\xi_1\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} |0\rangle \otimes |\xi_2\rangle \right\} \otimes |g\rangle$$



Reduced density matrix

$$\rho(t_i) = \mathrm{Tr}_g |\psi(t_i)\rangle\langle\psi(t_i)|$$

$$= \left\{ \frac{1}{\sqrt{2}} |\xi_1\rangle \otimes |0\rangle + \frac{1}{\sqrt{2}} |0\rangle \otimes |\xi_2\rangle \right\} \left\{ \frac{1}{\sqrt{2}} \langle\xi_1| \otimes \langle 0| + \frac{1}{\sqrt{2}} \langle 0| \otimes \langle\xi_2| \right\}$$

$$= \frac{1}{2} |\xi_1\rangle\langle\xi_1| \otimes |0\rangle\langle 0| + \underbrace{\frac{1}{2} |0\rangle\langle\xi_1| \otimes |\xi_2\rangle\langle 0| + \frac{1}{\sqrt{2}} |\xi_1\rangle\langle 0| \otimes |0\rangle\langle\xi_2|}_{\text{interference terms}} + \frac{1}{\sqrt{2}} |0\rangle\langle 0| \otimes |\xi_2\rangle\langle\xi_2|$$

Can a single photon excite a mirror?

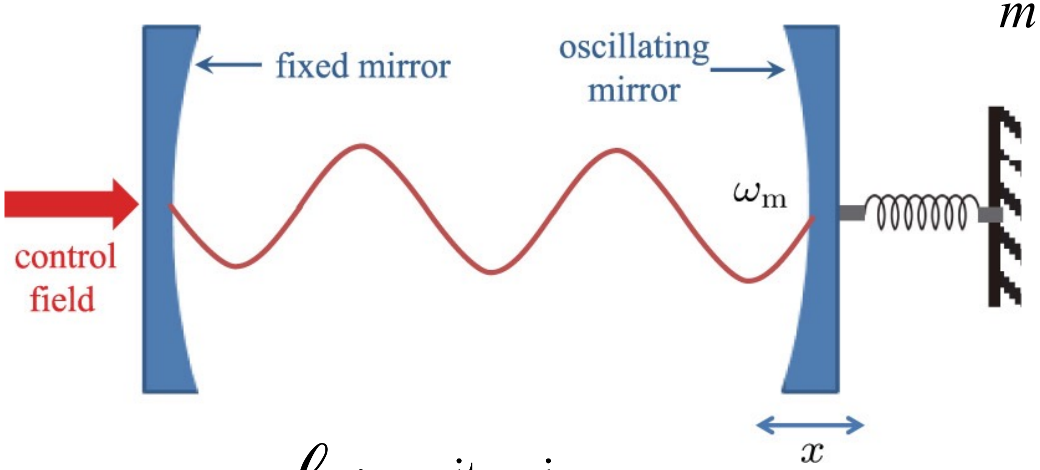
Condition for displacement by grand state size:

Momentum imparted by the photon has to be larger than the initial uncertainty of the mirror momentum.

photon momentum | period of mirror oscillation | time of one round trip | momentum uncertainty

$$\frac{\Omega_c}{c}\frac{2\pi}{\omega_m}\frac{1}{\frac{\ell}{c}} \geq \sqrt{m\omega_m}$$

$$\Longrightarrow \quad \kappa = \frac{\Omega_c}{\omega_m}\frac{x_{zp}}{\ell} \geq 1 \qquad x_{zp} = \sqrt{\frac{\hbar}{2m\omega_m}}$$



ℓ : cavity size

$$m = 40\text{kg} \qquad \omega = 1\text{kHz} \qquad \ell = 10\text{cm}$$

$$x_{zp} = 10^{-16}\text{cm}$$

$$\kappa = 1 \quad \Rightarrow \quad \Omega_c = 10^{20}\text{Hz}$$

Decoherence of entangled mirrors

After a while, the mirros are interacting with gravitons

$$S \simeq \sum_{A=1,2} \int dt \left[\frac{m}{4} \ddot{h}_{11}(0,t) \xi_A^1 \xi_A^1 + \frac{m}{4} \ddot{h}_{22}(0,t) \xi_A^2 \xi_A^2 \right]$$

Consequently, there appears the entanglement
between the mirrors and gravitons.

$$\left|\psi(t_i)\right\rangle = \frac{1}{\sqrt{2}} \left|\xi_1\right\rangle \otimes \left|0\right\rangle \otimes \left|g;\xi_1\right\rangle + \frac{1}{\sqrt{2}} \left|0\right\rangle \otimes \left|\xi_2\right\rangle \otimes \left|g;\xi_2\right\rangle$$

Thus, mirrors will be decohered due to noises of gravitons.

$$\rho(t) = \mathrm{Tr}_g \left|\psi(t)\right\rangle \left\langle\psi(t)\right|$$

$$= \frac{1}{2}\left|\xi_1\right\rangle\left\langle\xi_1\right| \otimes \left|0\right\rangle\left\langle 0\right| + \underbrace{\frac{1}{2}\left\langle g;\xi_1 \middle| g;\xi_2\right\rangle \left|0\right\rangle\left\langle\xi_1\right| \otimes \left|\xi_2\right\rangle\left\langle 0\right| + \frac{1}{2}\left\langle g;\xi_2 \middle| g;\xi_1\right\rangle \left|\xi_1\right\rangle\left\langle 0\right| \otimes \left|0\right\rangle\left\langle\xi_2\right|}_{\text{interference terms}} + \frac{1}{2}\left|0\right\rangle\left\langle 0\right| \otimes \left|\xi_2\right\rangle\left\langle\xi_2\right|$$

Entanglement Negativity

Initial density matrix of mirrors

$$\rho_m = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{11} & \rho_{12} & 0 \\ 0 & \rho_{21} & \rho_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

After evolution

$$\rho_m = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_{11} & \exp\left(i\Phi\right)\rho_{12} & 0 \\ 0 & \exp\left(-i\Phi^*\right)\rho_{21} & \rho_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

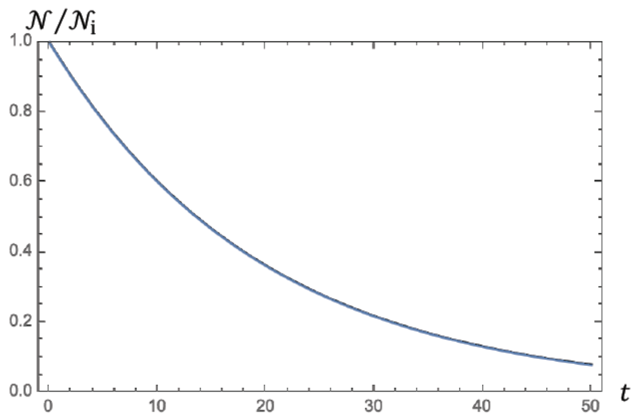
Taking a partial transpose,

$$\rho_m = \begin{pmatrix} 0 & 0 & 0 & \exp\left(i\Phi\right)\rho_{12} \\ 0 & \rho_{11} & 0 & 0 \\ 0 & 0 & \rho_{22} & 0 \\ \exp\left(-i\Phi^*\right)\rho_{21} & 0 & 0 & 0 \end{pmatrix}$$

We can calculate the negativity

$$\Gamma = \mathrm{Im}\,\Phi$$

$$\mathcal{N} = \left|\rho_{12}(t_i)\right| \exp\left(-\Gamma\right)$$



Decoherence time

By solving non-Markovian master equation, we obtain the decoherence rate

$$\Gamma = \frac{4\pi^3}{5}\underbrace{\left(\frac{m}{M_p}\right)^2}_{\text{effective coupling}}\underbrace{\left(L f_c\right)^4}_{\text{squeezing factor}}\left(\frac{A}{L}\right)^2\omega \qquad M_p \approx 10^{-5} g$$

Substituting

$$m = 40\text{kg}\,,\quad \text{L=40 km}\,,\quad \text{A}=10\frac{\hbar}{\sqrt{2m\omega}} = 10^{-15}\text{cm}\,,\quad \omega\text{=1kHz}\,,\quad \text{f}_c = 1\text{GHz}$$

we see the decoherence time is 20s.

Note that the decoherence due to air molecule is dominant among other decoherence sources.

In the present case, the time scale becomes

$$t_d = 1200\left(\frac{a}{0.17\text{m}}\right)^{-2}\left(\frac{T}{10\text{K}}\right)^{-3/2}\text{s} \qquad P = 10^{-10}\,\text{Pa}$$

This time scale is much larger than the decoherence time due to noise of gravitons.
Therefore, there is a chance to detect gravitons indirectly.

Duality of axion and graviton experiments

--- A working hypothesis ---

Duality of axion and graviton experiments

Both axions and graviton are coherently oscillating.

$$S = -\frac{1}{2}\int \sqrt{-g}\left[\left(\partial_\mu a\right)^2 + m^2 a^2\right]$$

graviton

$$h_{ij} \propto \cos\left(\mathbf{k}\cdot\mathbf{x} - \omega t\right), \quad \omega = |\mathbf{k}|$$

axion

$$a = a_0 \cos mt$$

matter

Remarkably, there exists a duality between axion and graviton experiments.

Detectors useful for axion search
is also useful for graviton search

Detectors useful for axion search
is also useful for graviton search

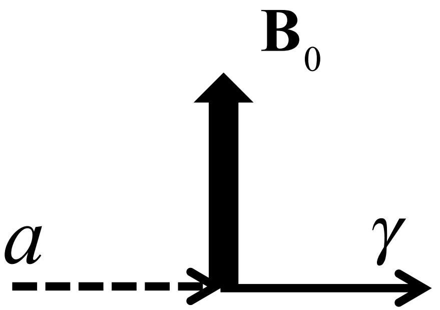
Axion - photon - graviton conversion

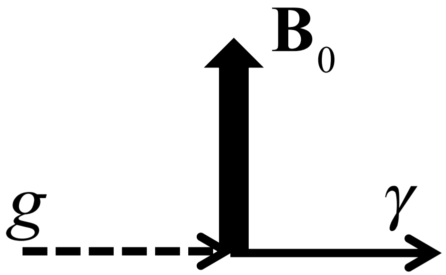
We often use photons to probe axions and gravitons.

$$S=\int\sqrt{-g}\left[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}+\frac{1}{4}a\,\varepsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right]$$

$L\supset a\,\mathbf{E}\cdot\mathbf{B}_0$ Sikivie 1983

$L\supset h_{ij}\,E^iB_0^j$ Gertsenshtein 1962





The photons could be other excitations such as phonons, magnons, … .

An application of duality

A useful application of duality is
to use existing data from axion experiments
for giving constraints on gravitons.

Here, we take magnons and show how to use duality to probe gravitons.

Axion - magnon - graviton conversion

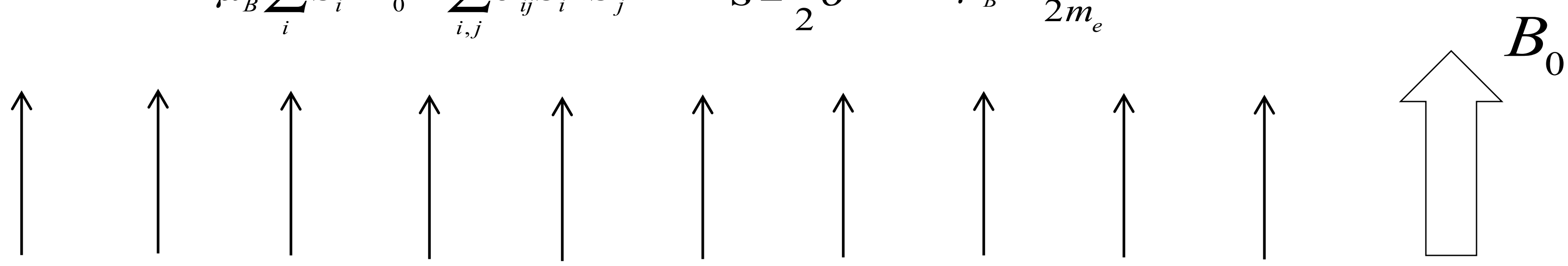
Remarkably, it is possible to detect a single magnon with the superconducting qubit.
In the future, we will be able to create the squeezed magnon system.
Thus, we can utilize the recent development of quantum sensing.

Spin waves

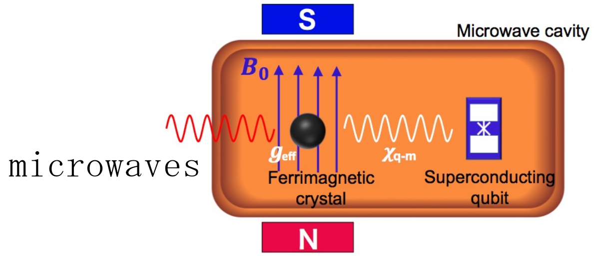
The homogeneous magnetic field is imposed.

Bohr magneton

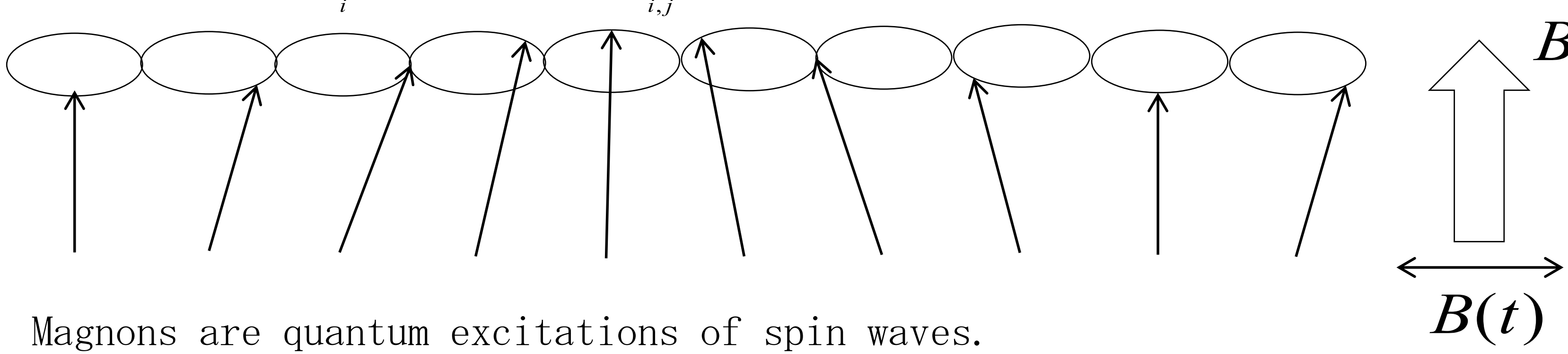
$$H = -2\mu_B \sum_i \hat{S}_i \cdot \vec{B}_0 - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j \qquad \mathbf{S} = \frac{1}{2}\sigma \qquad \mu_B = \frac{e}{2m_e}$$



The above is the ground state.
Microwaves are used for generating spin waves.



$$H = -2\mu_B \sum_i \hat{S}_i \cdot \left(\vec{B}_0 + B(t)\right) - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$



Magnons are quantum excitations of spin waves.
Their creation and annihilation are described by $c^\dagger$ and c.

$$H_{\text{magnon}} = \hbar\omega c^\dagger c \qquad \hbar\omega = 2\mu_B B_0 \qquad \text{Larmor frequency}$$

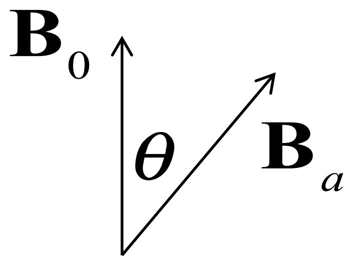
Axion – magnon interaction

Interaction term $$L_{\text{int}} = -ig_{aee} a \bar{\psi} \gamma_5 \psi = \frac{g_{aee}}{2m} \partial_\mu a \bar{\psi} \gamma^\mu \gamma_5 \psi$$

In a nonrelativistic approximation,

$$H \simeq \frac{(\mathbf{p} - e\mathbf{A})^2}{2m_e} - 2\mu_B \hat{S} \cdot \mathbf{B}_0 - 2\mu_B \hat{S} \cdot \mathbf{B}_a(t)$$

The coherently oscillating axion can plays a roll of microwaves.



$$\vec{B}_a(t) = \frac{g_{aee}}{e} \vec{\nabla} a(t)$$
$$= \left(\frac{1}{2} B_a \sin\theta \left(e^{-i\omega_a t} + e^{i\omega_a t} \right), 0, 0 \right)$$

$$f_a = \frac{\omega_a}{2\pi} = \frac{m_a c^2}{h} = 0.24 \left(\frac{m_a}{1.0\mu\text{eV}} \right) \text{GHz}$$

We consider only Kittel mode(k=0)

$$H_{m-a} = \hbar\omega c^\dagger c + g_{eff} \left(c^\dagger e^{-i\omega_a t} + c e^{i\omega_a t} \right)$$

Number of electron density

Case of YIG $N \approx 10^{22}$

$$g_{eff} = \frac{1}{2} \mu_B B_a \sin\theta \sqrt{N}$$

Graviton search with magnons

A. Ito, T. Ikeda, K. Miuchi and J. Soda,
``Probing GHz gravitational waves with graviton-magnon resonance,''
Eur. Phys. J. C80, no.3, 179 (2020) [arXiv:1903.04843 [gr-qc]].

A. Ito and J. Soda,
``A formalism for magnon gravitational wave detectors,''
Eur. Phys. J. C80, no.6, 545 (2020) [arXiv:2004.04646 [gr-qc]].

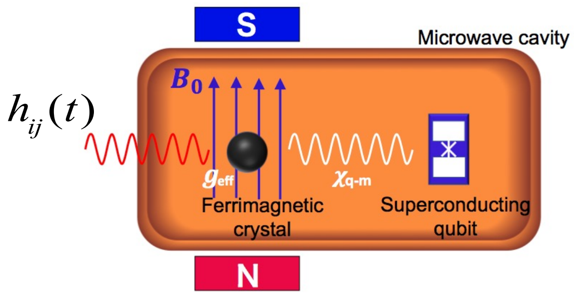
A. Ito and J. Soda,
Exploring HighFrequency Gravitational Waves with Magnons,
[arXiv:2212.04094 [gr-qc]].

Graviton – magnon interaction

GWs

$$h_{ij}(t) = h^{+}(t) e_{ij}^{+} + h^{\times}(t) e_{ij}^{\times}$$

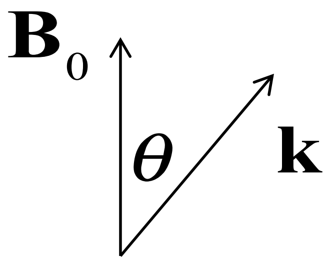
$$h^{+,\times}(t) = \frac{h^{+,\times}}{2}\left(e^{-i\omega_h t} + e^{i\omega_h t}\right)$$

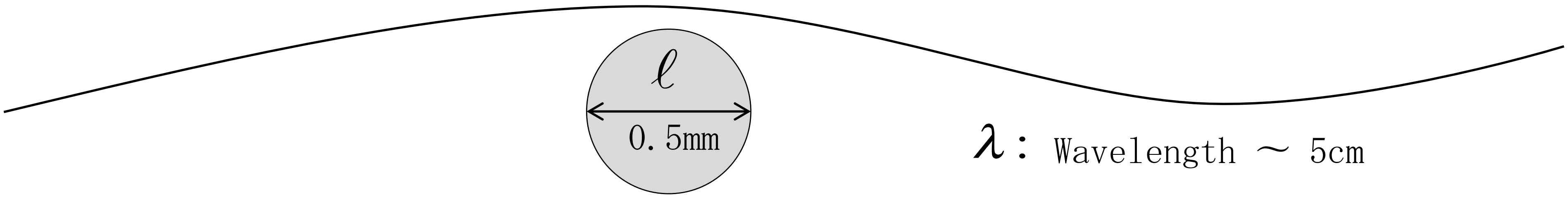


$$H = -2\mu_B \sum_i \hat{S}_i \cdot \vec{B}_0 - \mu_B \sum_i S_i^a h_{az}(t) B_0 - \sum_{i,j} J_{ij} \hat{S}_i \cdot \hat{S}_j$$

$$H_{m-g} = \hbar\omega c^{\dagger} c + g_{eff}\left(c^{\dagger} e^{-i\omega_h t} + c e^{i\omega_h t}\right)$$

$$g_{eff} = \frac{\sqrt{2}}{240}\left(\frac{2\pi\ell}{\lambda}\right)^2 \mu_B B_0 \sin\theta \sqrt{N}\left[\cos^2\theta \left(h^{(+)}\right)^2 + \left(h^{(\times)}\right)^2\right]^{1/2}$$





λ: Wavelength ~ 5cm

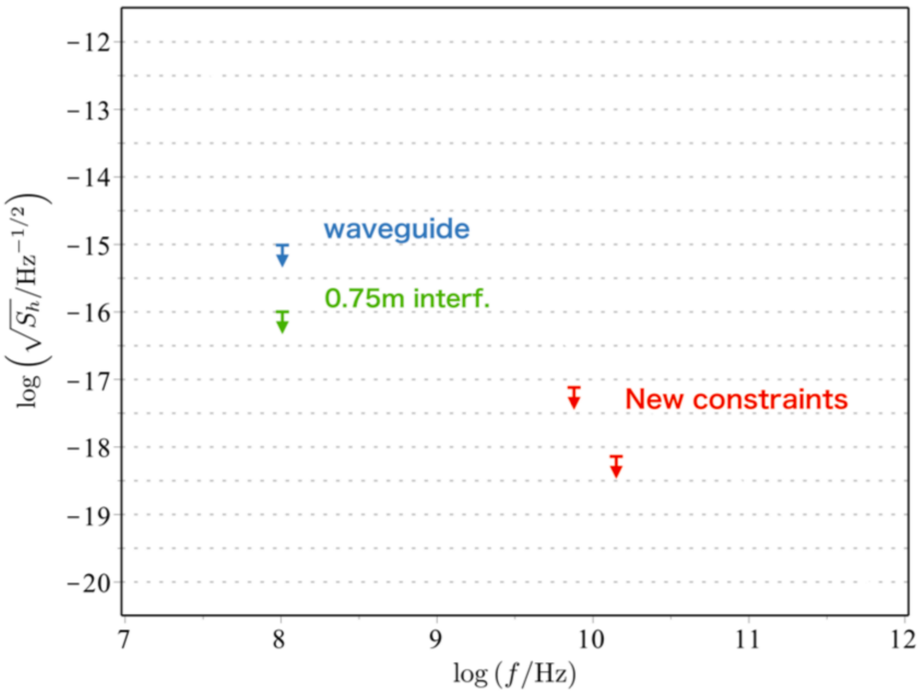
Magnon limits on GHz GWs

Ito et al. 2019
Ito & Soda 2020

Limits on axions

$$g_{eff} < \begin{cases} 3.5\times10^{-12}\,\mathrm{eV} & \text{QUAX, Crescini et al. 2018} \\ 3.1\times10^{-11}\,\mathrm{eV} & \text{Flower et al. 2018} \end{cases}$$

$$g_{eff} = \frac{\sqrt{2}}{240}\left(\frac{2\pi\ell}{\lambda}\right)^2 \mu_B B_0 \sin\theta\sqrt{N}\left[\cos^2\theta\left(h^{(+)}\right)^2+\left(h^{(\times)}\right)^2\right]^{1/2}$$



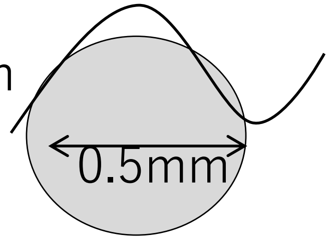
$$\sqrt{S_h} \approx h < \begin{cases} 7.5\times10^{-19}\,\mathrm{Hz}^{-1/2} & \text{at } 14\ \mathrm{GHz} \\ 8.7\times10^{-18}\,\mathrm{Hz}^{-1/2} & \text{at } 8.2\ \mathrm{GHz} \end{cases}$$

$$h_c < \begin{cases} 1.3\times10^{-13} & \text{at } 14\ \mathrm{GHz} \\ 1.1\times10^{-12} & \text{at } 8.2\ \mathrm{GHz} \end{cases}$$

Normal coordinates revisited

Fortini & Gualdi 1982
Marzlin 1994
Rakhmanov 2014
Licht 2004
Berlin et al. 2022

Wavelength ～ 0.5mm



$$g_{00} = -1 - 2 R_{0i0j}\Big|_{x^i=0} x^i x^j \,\mathrm{Re}\left[\frac{1-e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^2} - \frac{i}{\mathbf{k}\cdot\mathbf{x}}\right]$$

$$g_{0i} = 2 R_{0ijk}\Big|_{x^i=0} x^j x^k \,\mathrm{Re}\left[i\frac{1-e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^3} + \frac{i}{2\mathbf{k}\cdot\mathbf{x}} + \frac{e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^2}\right]$$

$$g_{ij} = \delta_{ij} + 2 R_{ikjl}\Big|_{x^i=0} x^k x^l \,\mathrm{Re}\left[\frac{1+e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^2} + 2i\frac{1-e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^3}\right]$$

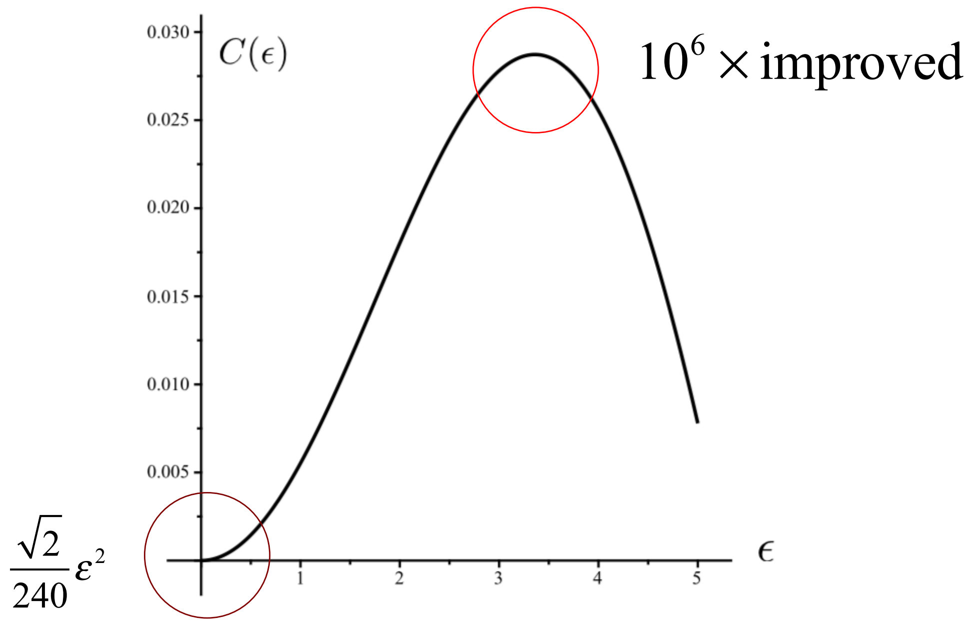
$$H_{GW} = -\mu_B B^i \hat{S}^j Q_{ij}$$

$$Q_{ij} = 2\delta_{ij} R_{0k0l}\Big|_{x^i=0} x^k x^l \,\mathrm{Re}\left[\frac{1-e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^2} - \frac{i}{\mathbf{k}\cdot\mathbf{x}}\right]$$
$$-2\left[\delta_{ij} R_{mkml}\Big|_{x^i=0} x^k x^l - R_{ikjl}\Big|_{x^i=0} x^k x^l\right]\mathrm{Re}\left[\frac{1+e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^2} + 2i\frac{1-e^{-i\mathbf{k}\cdot\mathbf{x}}}{(\mathbf{k}\cdot\mathbf{x})^3}\right]$$

Effeciency is improved

$$g_{eff} = C(\ell,\lambda)\mu_B B_z \sin\theta\sqrt{N}\left[\cos^2\theta\left(h^+\right)^2 + \left(h^\times\right)^2\right]^{1/2}$$

$$C(\ell,\lambda) = -\frac{3}{16\sqrt{2}\pi\ell^3}\int d^3x\left[1+\cos kx - \frac{2\sin kx}{kx}\right]$$
$$= -\frac{\sqrt{2}}{8} + \frac{3\sqrt{2}Si(\varepsilon)}{8\varepsilon} + \frac{3\sqrt{2}\cos\varepsilon}{4\varepsilon^2} - \frac{3\sqrt{2}\sin\varepsilon}{4\varepsilon^3} \approx \frac{\sqrt{2}}{240}\varepsilon^2 + \cdots \qquad \varepsilon = k\ell$$



Sensitivity

Ito & Soda 2022

$$h_c = 2.5\times10^{-20}\left(\frac{T_N}{1\mathrm{K}}\right)^{1/2}\left(\frac{V_m}{(4\pi/3)10^3\mathrm{cm}^3}\right)^{-1/2}\left(\frac{\eta}{1/2}\right)^{-1/2}\left(\frac{C}{0.029}\right)^{-1}$$

$$\times\left(\frac{\omega_h/2\pi}{1.6\times10^9\,\mathrm{Hz}}\right)^{-3/4}\left(\frac{Q_e}{10^5}\right)^{1/2}\left(\frac{Q_m}{10^4}\right)^{-1}\left(\frac{\tau}{1\mathrm{day}}\right)^{-1/4}$$

T_N : effective temperature

V_m : Volume

η : conversion efficiency

ω_h : GW angular frequency

Q_e : quality factor of cavity

Q_m : quality factor of magnon

τ : observation time

Summary

- We obtained quantum Langevin equation for a massive particle with graviton noise.

- We have discussed indirect detection of gravitons.

  The decoherence time is 20s much shorter than that of other decoherence sources. In principle, we can detect gravitons by observing the quantum entanglement negativity.

- Aiming at detecting a graviton, we developed a detection method for high frequency GWs.

- By borrowing the existing data from axion search, we have succeeded in giving constraints on the amplitude of GHz HFGWs.

- We showed the sensitivity can be improved significantly.