Copernicus webinar, March 28 2023

Universal Nature of Black Hole Ringdown: Overtone Excitation and Graybody Factors

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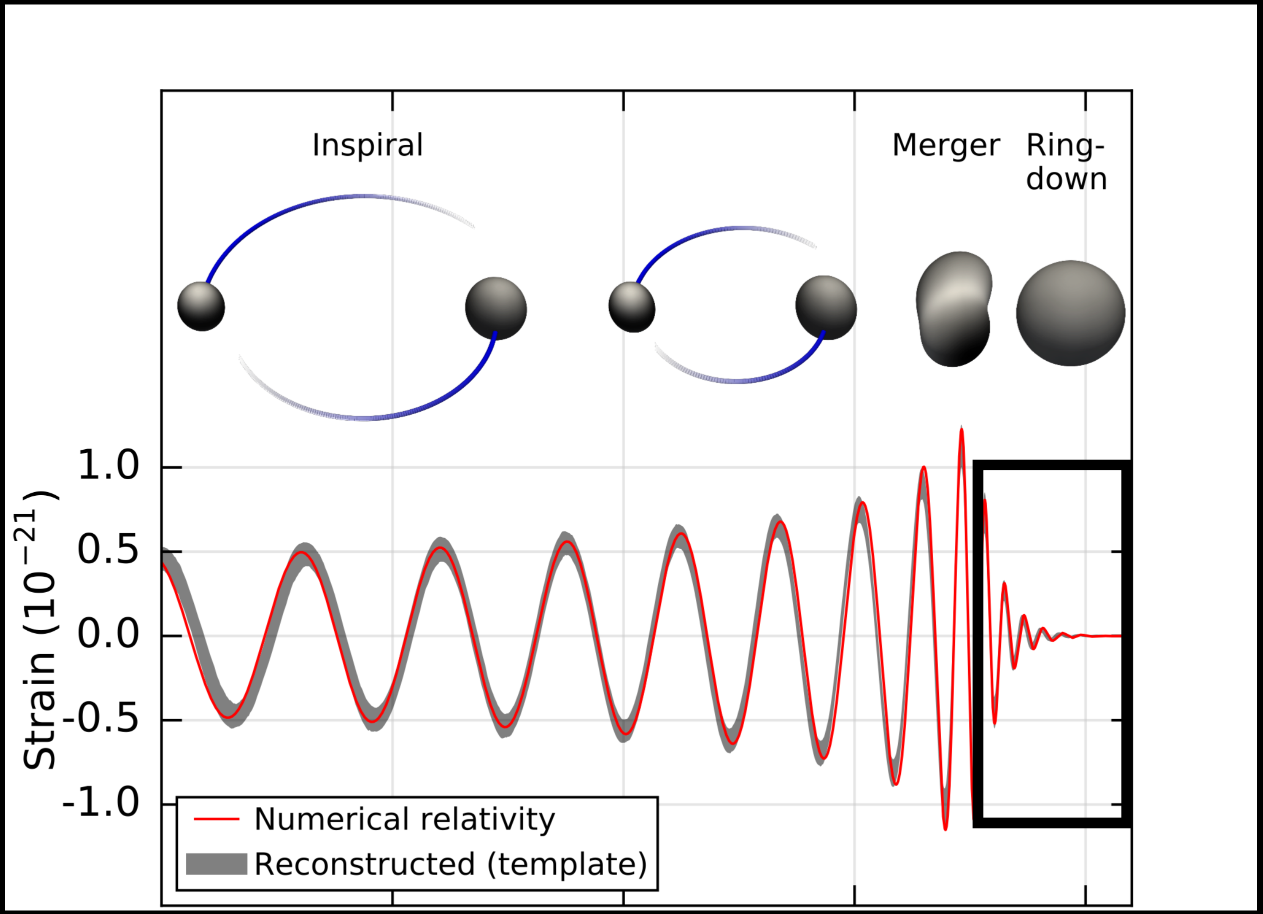
RIKEN Interdisciplinary
Theoretical and Mathematical
Sciences Program

RIKEN

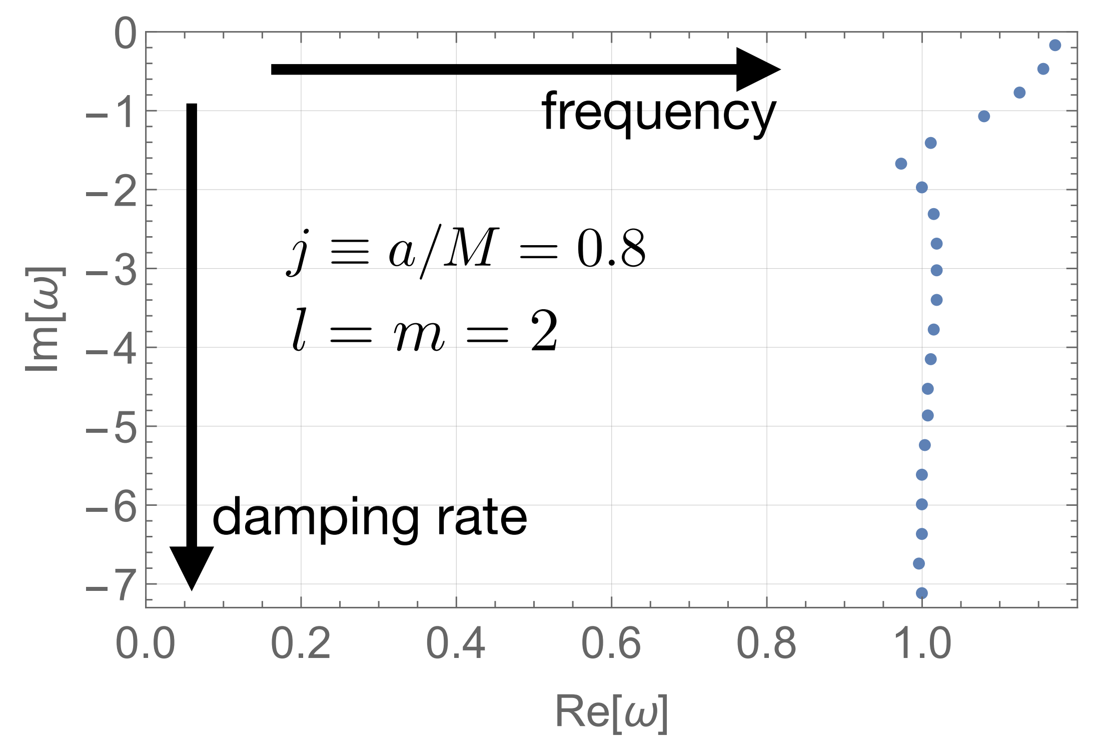
NO arXiv: 2109.09757
NO arXiv: 2208.02923
NO and Daichi Tsuna arXiv: 2210.14049

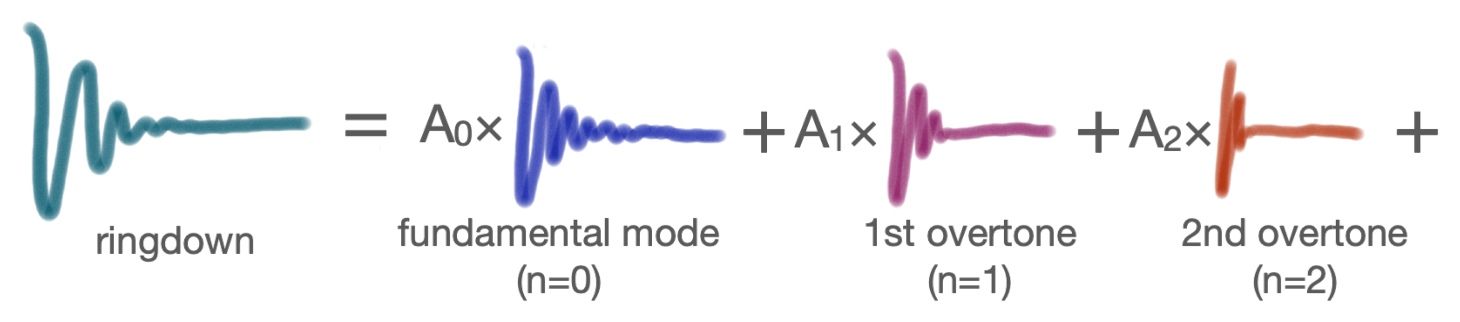
Quasi-Normal (QN) Modes of BHs and Ringdown

B. P. Abbott et al. (2016)



relaxation process of a BH ringing
=ringdown phase
= superposition of quasi-normal modes



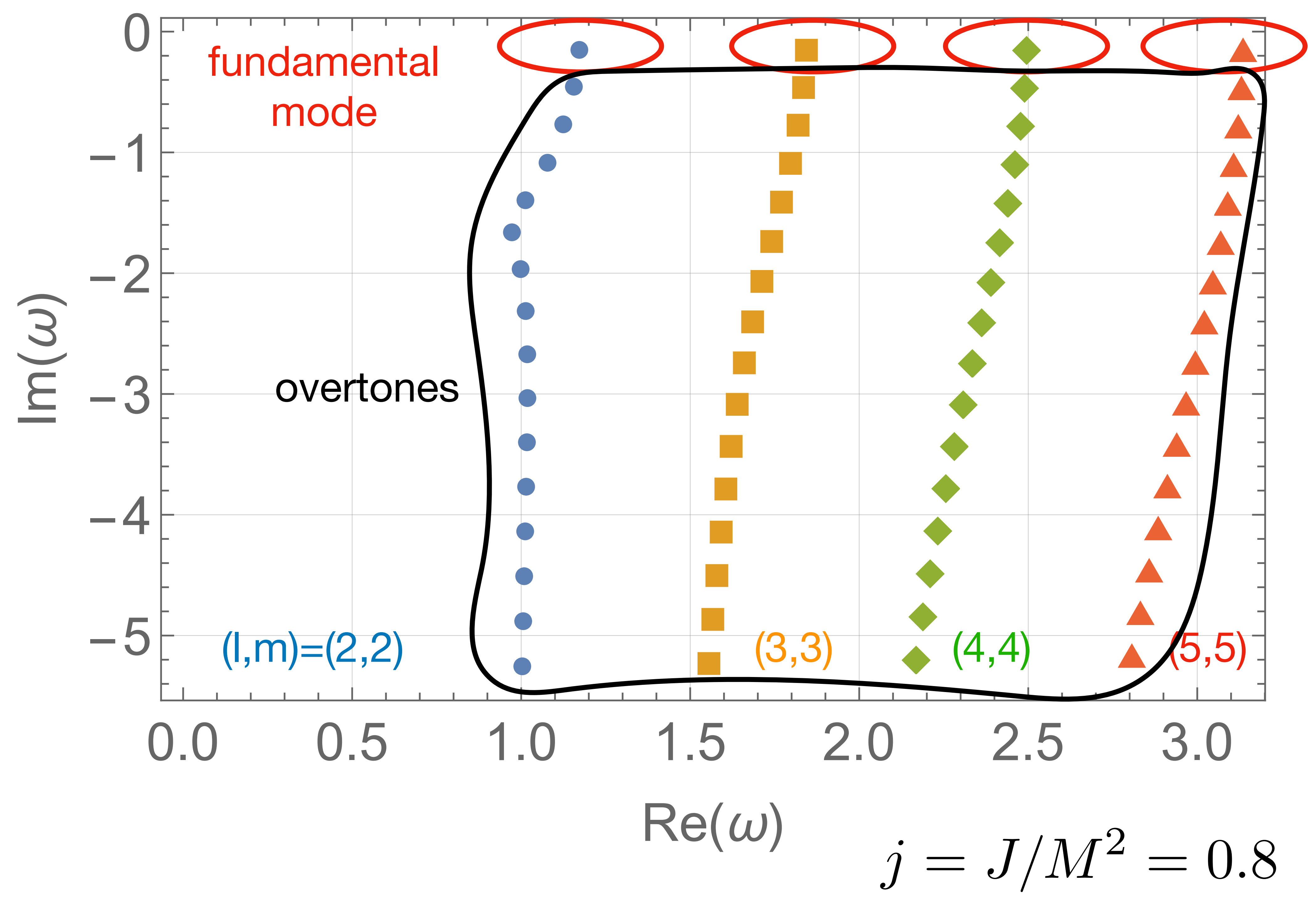


$$h_{\text{ringdown}} \sim \sum_n A_n e^{-t/\tau_n} \cos[f_n(t - r^* - t_0) + \delta_n]$$

determined only by mass and angular momentum (no-hair theorem)

Credit: EHT collaboration

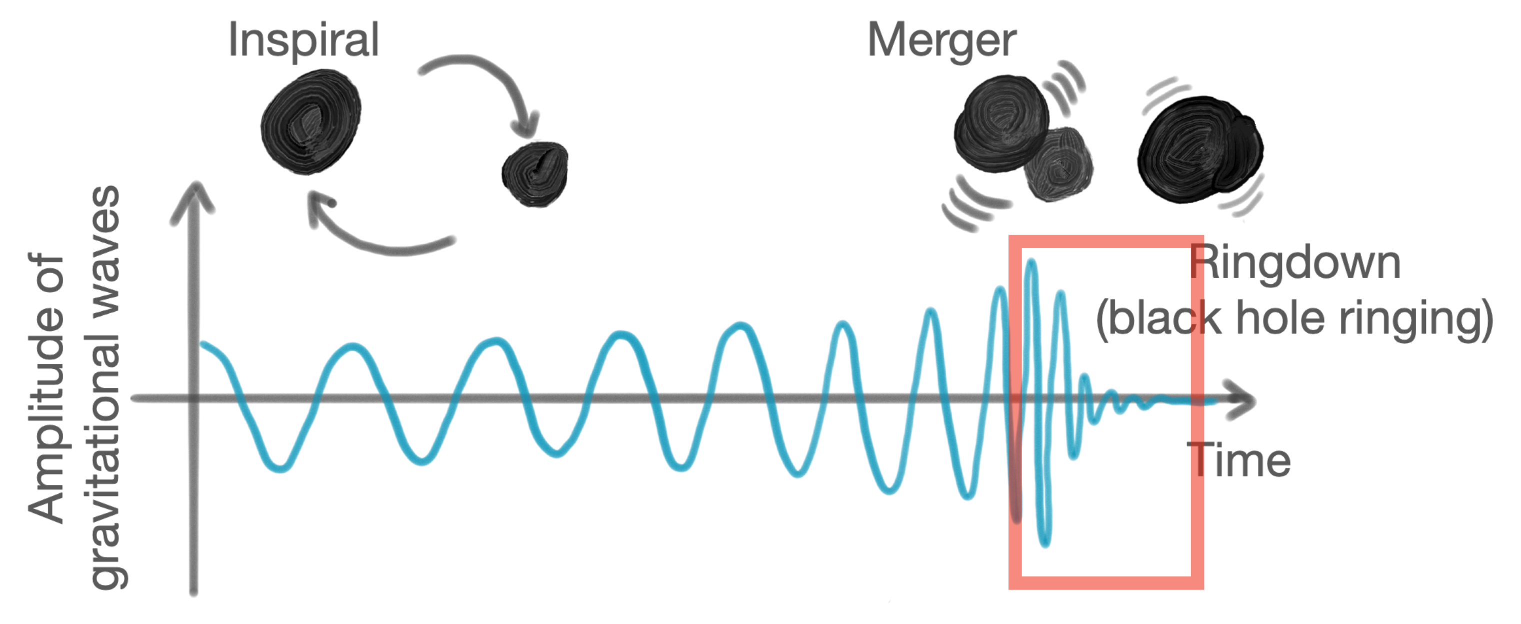
Quasi-Normal Modes (QNMs)

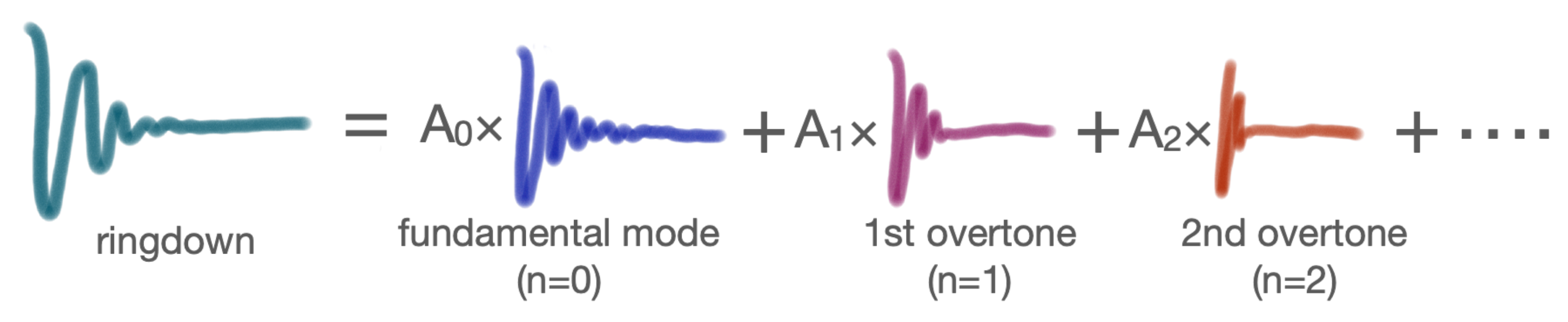


$j = J/M^2 = 0.8$

$$\omega_q = \underbrace{\mathrm{Re}[\omega_q]}_{\text{(frequency)}} + i\underbrace{\mathrm{Im}[\omega_q]}_{\text{(damping rate)}}$$

Why is a BH ringing important?

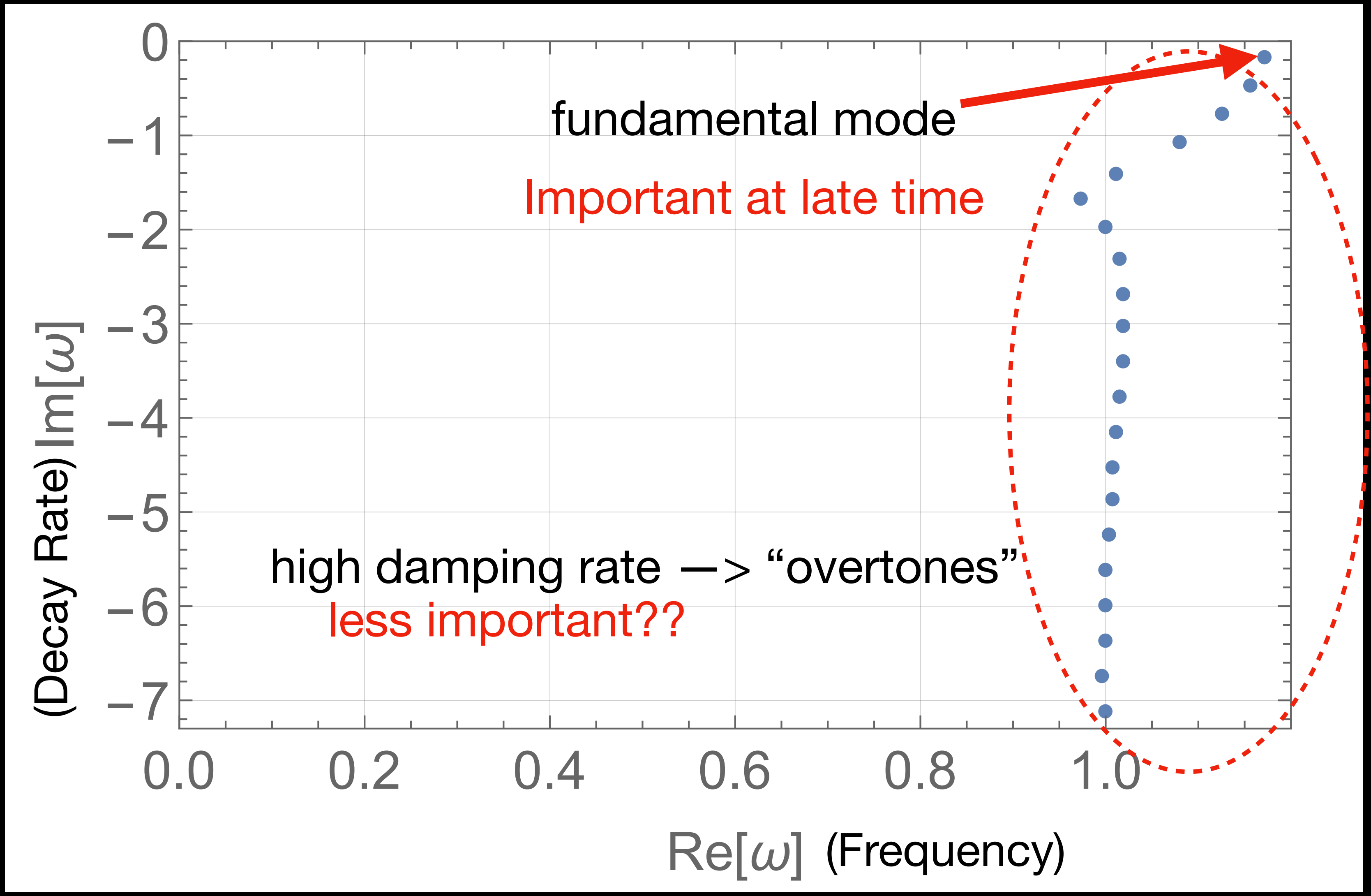




→Measurement of each QN mode

→Test of GR in strong-gravity regimes

Overtone QN modes



fundamental mode

Important at late time

high damping rate —> “overtones”

less important??

Kerr BH (j=0.8, M=0.5)

1 Ringdown of comparable mass ratio BBH mergers

NO arXiv: 2109.09757

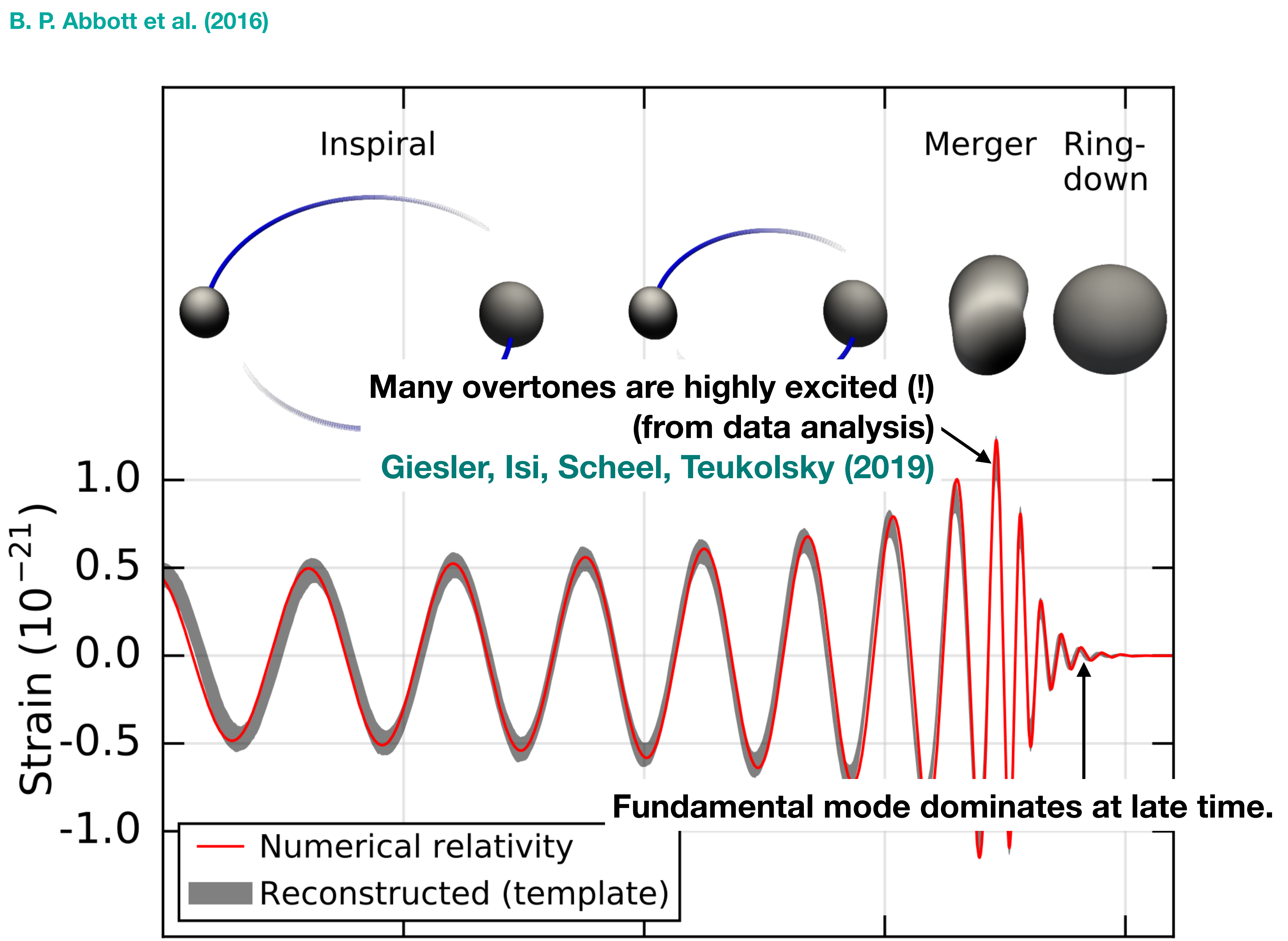
2 Ringdown of extreme mass ratio mergers

NO arXiv: 2208.02923

3 Alternative modeling of ringdown for extreme mass ratio mergers

NO arXiv: 2208.02923

When does ringdown start?



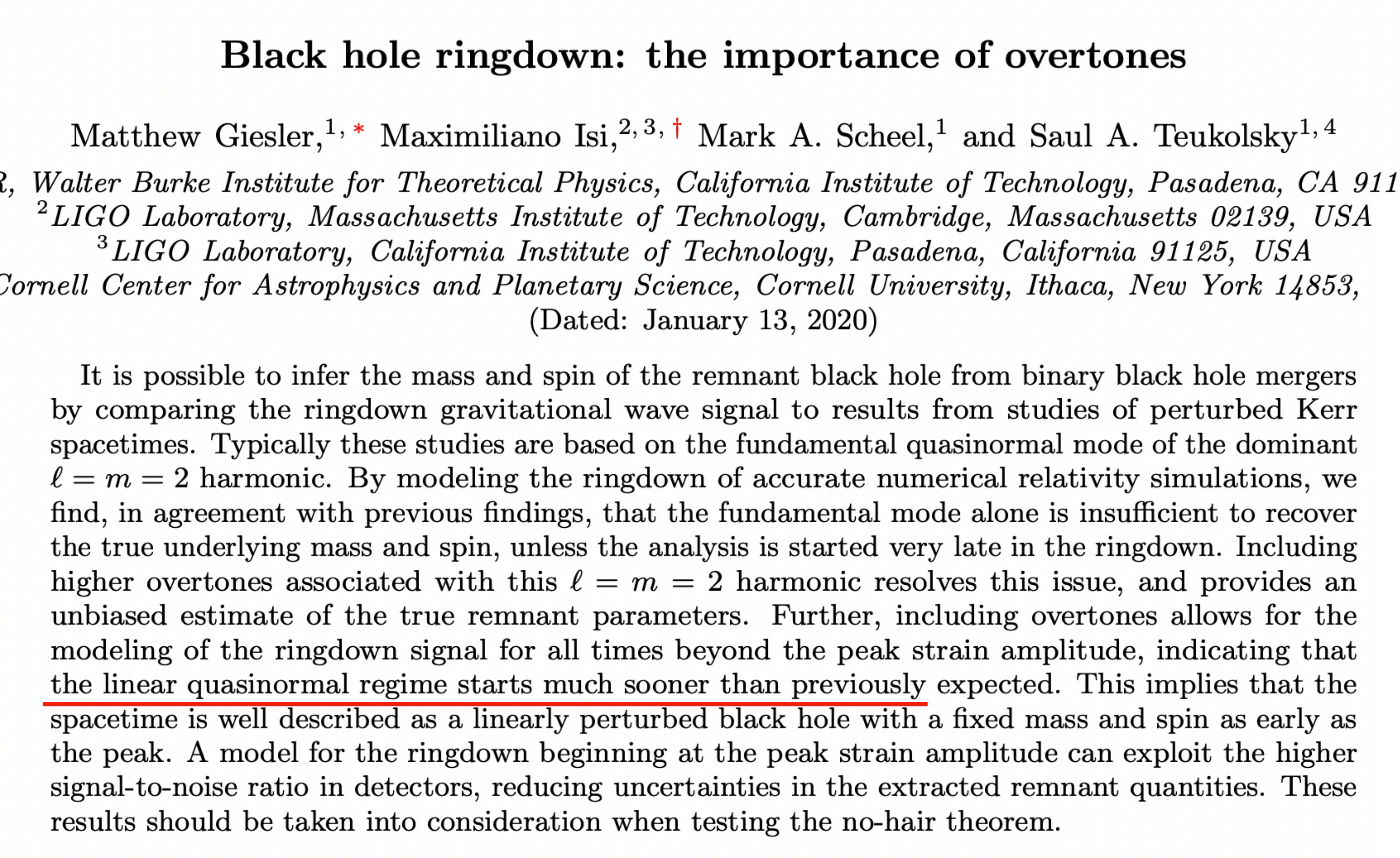
B. P. Abbott et al. (2016)

Many overtones are highly excited (!) (from data analysis)

Giesler, Isi, Scheel, Teukolsky (2019)

Fundamental mode dominates at late time.

Binary Black Hole with comparable mass ratio



Black hole ringdown: the importance of overtones

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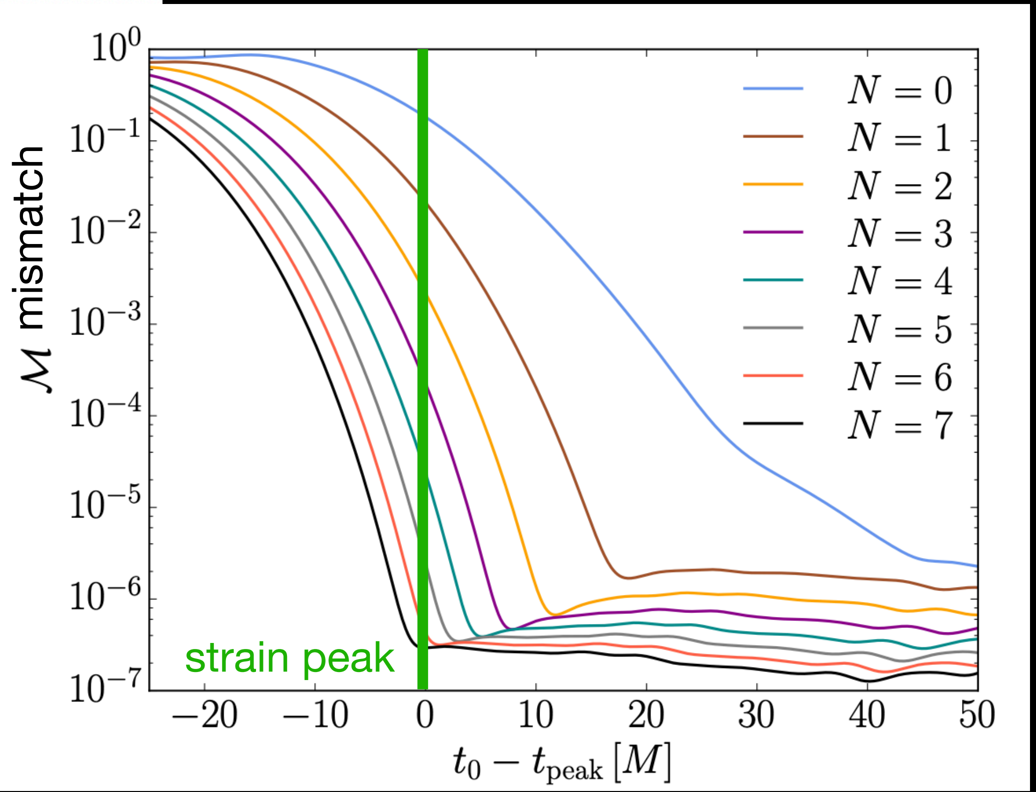
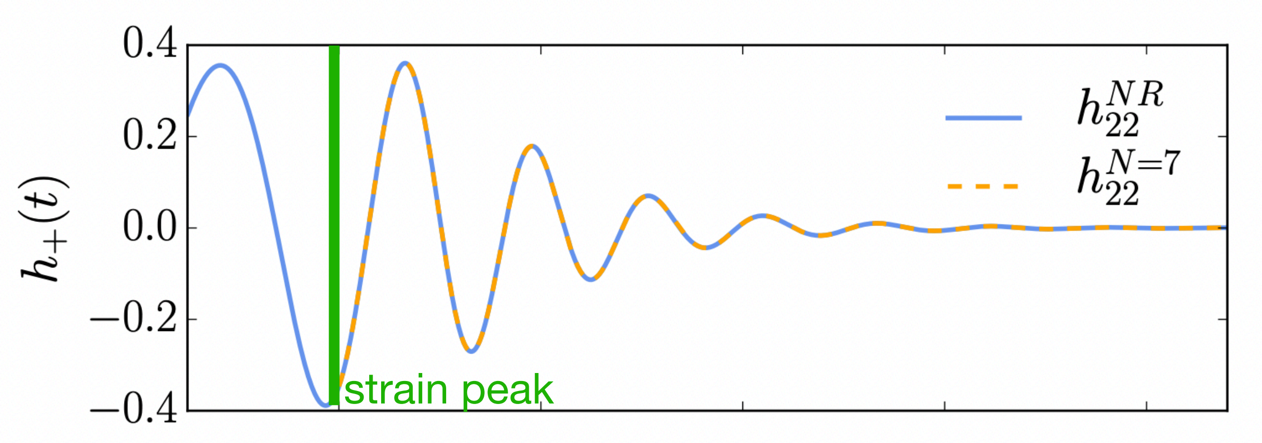
[2]*LIGO Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[3]*LIGO Laboratory, California Institute of Technology, Pasadena, California 91125, USA*
Cornell Center for Astrophysics and Planetary Science, Cornell University, Ithaca, New York 14853,
(Dated: January 13, 2020)

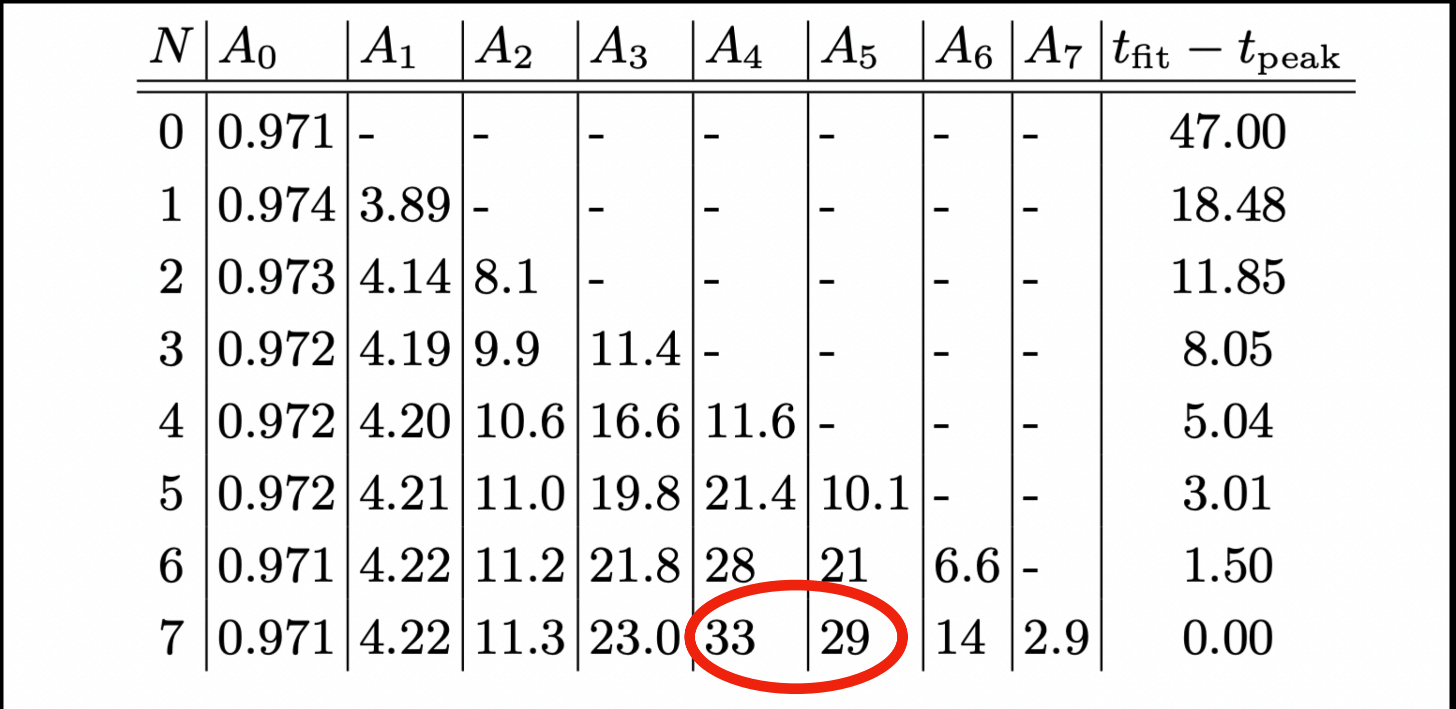
It is possible to infer the mass and spin of the remnant black hole from binary black hole mergers by comparing the ringdown gravitational wave signal to results from studies of perturbed Kerr spacetimes. Typically these studies are based on the fundamental quasinormal mode of the dominant $\ell = m = 2$ harmonic. By modeling the ringdown of accurate numerical relativity simulations, we find, in agreement with previous findings, that the fundamental mode alone is insufficient to recover the true underlying mass and spin, unless the analysis is started very late in the ringdown. Including higher overtones associated with this $\ell = m = 2$ harmonic resolves this issue, and provides an unbiased estimate of the true remnant parameters. Further, including overtones allows for the modeling of the ringdown signal for all times beyond the peak strain amplitude, indicating that the linear quasinormal regime starts much sooner than previously expected. This implies that the spacetime is well described as a linearly perturbed black hole with a fixed mass and spin as early as the peak. A model for the ringdown beginning at the peak strain amplitude can exploit the higher signal-to-noise ratio in detectors, reducing uncertainties in the extracted remnant quantities. These results should be taken into consideration when testing the no-hair theorem.



Giesler, Isi, Scheel, Teukolsky (2019)



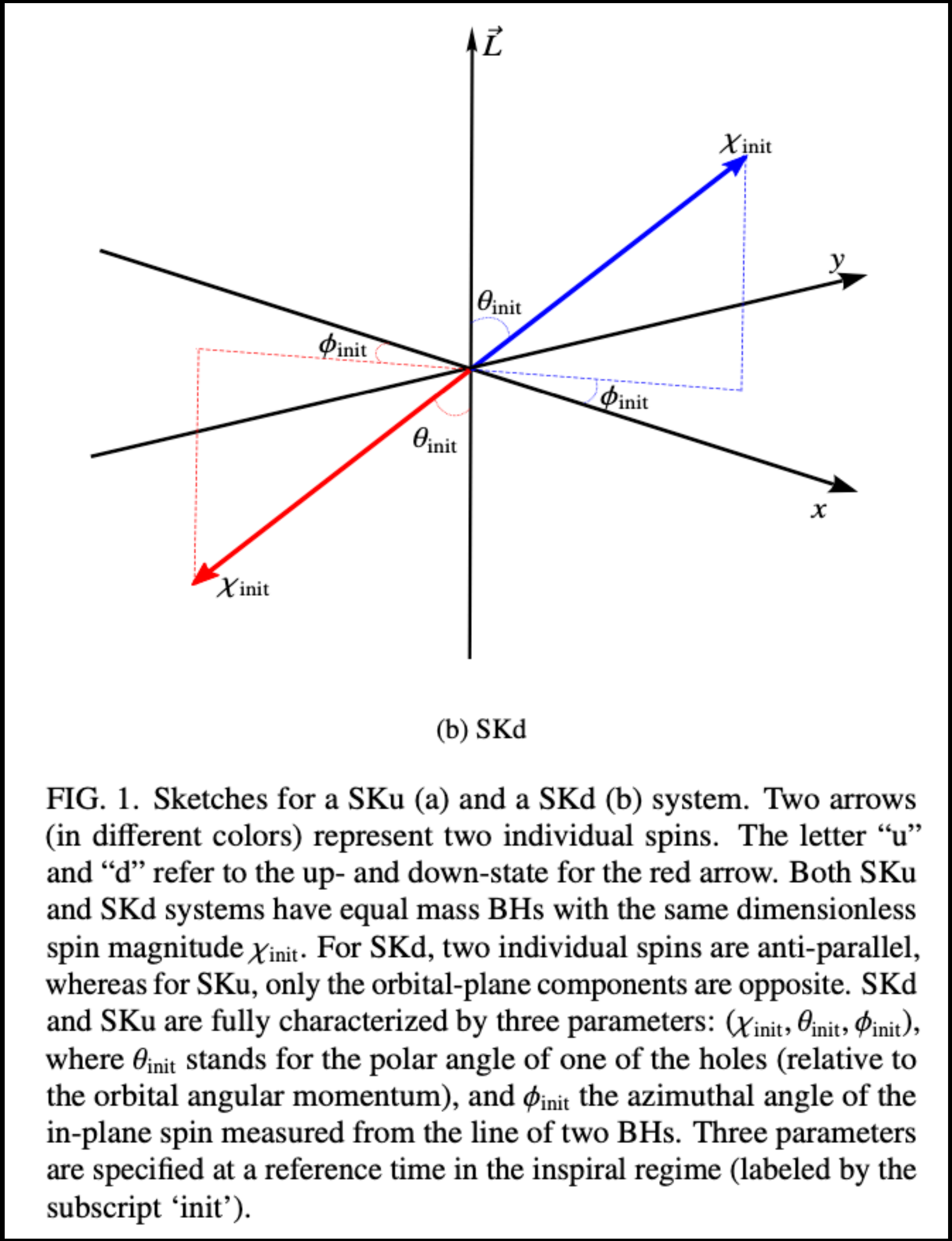
Fundamental mode ⟷ Higher overtones



N	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	$t_{\rm fit} - t_{\rm peak}$
0	0.971	-	-	-	-	-	-	-	47.00
1	0.974	3.89	-	-	-	-	-	-	18.48
2	0.973	4.14	8.1	-	-	-	-	-	11.85
3	0.972	4.19	9.9	11.4	-	-	-	-	8.05
4	0.972	4.20	10.6	16.6	11.6	-	-	-	5.04
5	0.972	4.21	11.0	19.8	21.4	10.1	-	-	3.01
6	0.971	4.22	11.2	21.8	28	21	6.6	-	1.50
7	0.971	4.22	11.3	23.0	33	29	14	2.9	0.00

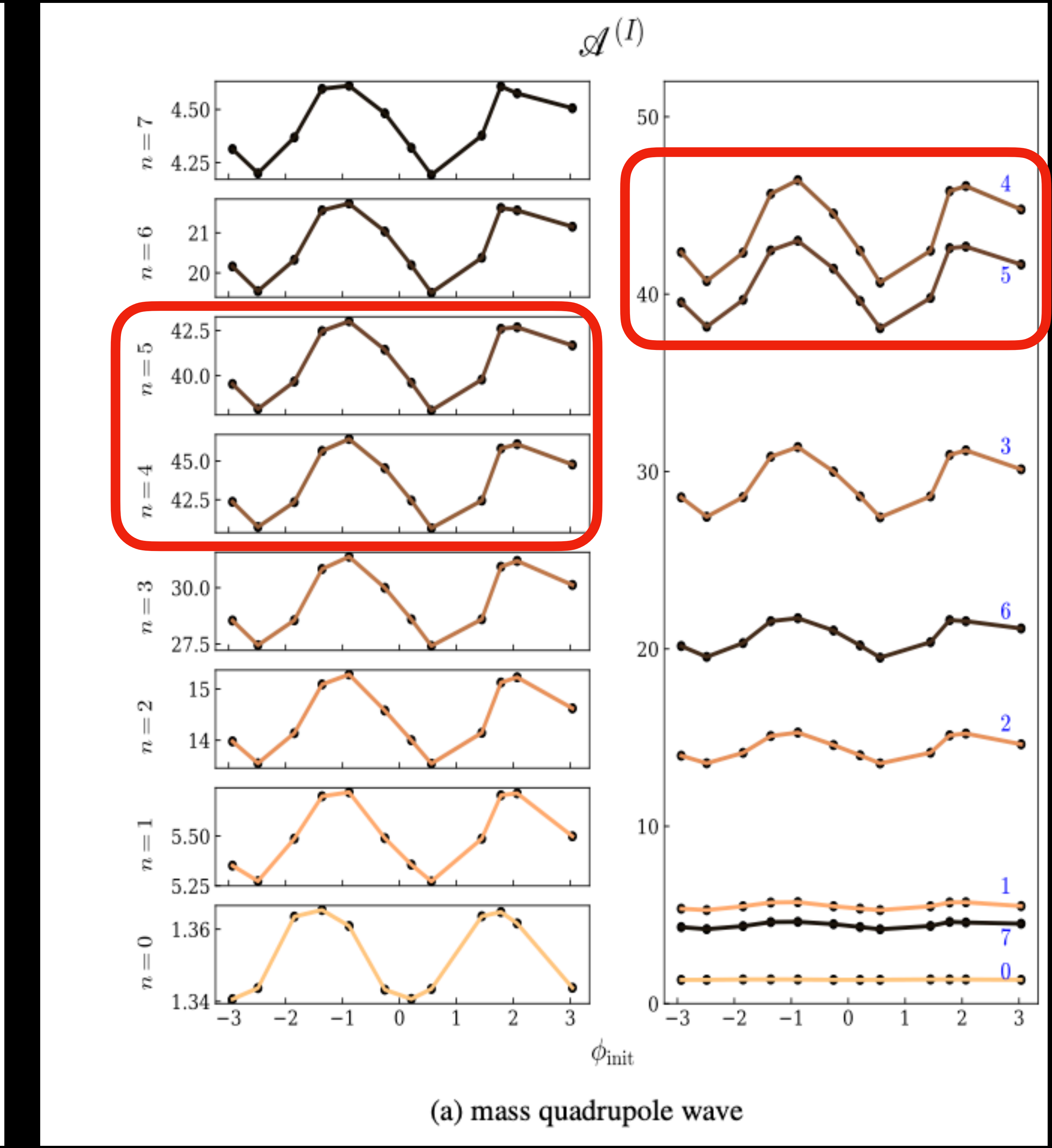
Universality of the importance of overtones

Ma, Giesler, Varma, Scheel, and Chen (2021)



(b) SKd

FIG. 1. Sketches for a SKu (a) and a SKd (b) system. Two arrows (in different colors) represent two individual spins. The letter "u" and "d" refer to the up- and down-state for the red arrow. Both SKu and SKd systems have equal mass BHs with the same dimensionless spin magnitude $\chi_{\rm init}$. For SKd, two individual spins are anti-parallel, whereas for SKu, only the orbital-plane components are opposite. SKd and SKu are fully characterized by three parameters: $(\chi_{\rm init}, \theta_{\rm init}, \phi_{\rm init})$, where $\theta_{\rm init}$ stands for the polar angle of one of the holes (relative to the orbital angular momentum), and $\phi_{\rm init}$ the azimuthal angle of the in-plane spin measured from the line of two BHs. Three parameters are specified at a reference time in the inspiral regime (labeled by the subscript 'init').

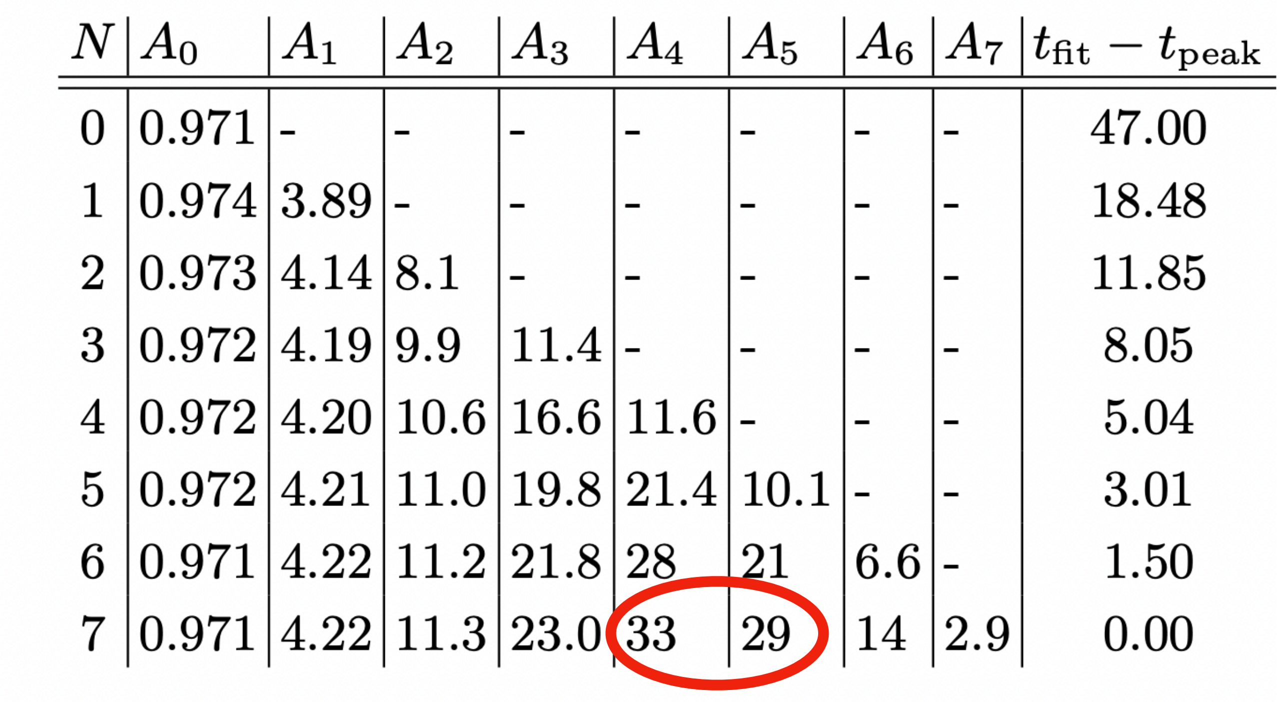


(a) mass quadrupole wave

Initial configuration of BBH

overtones

Why are the 4th and 5th overtones dominant?? (Based on fitting analysis)



N	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	$t_{\rm fit} - t_{\rm peak}$
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1	0.974	3.89	-	-	-	-	-	-	18.48
2	0.973	4.14	8.1	-	-	-	-	-	11.85
3	0.972	4.19	9.9	11.4	-	-	-	-	8.05
4	0.972	4.20	10.6	16.6	11.6	-	-	-	5.04
5	0.972	4.21	11.0	19.8	21.4	10.1	-	-	3.01
6	0.971	4.22	11.2	21.8	28	21	6.6	-	1.50
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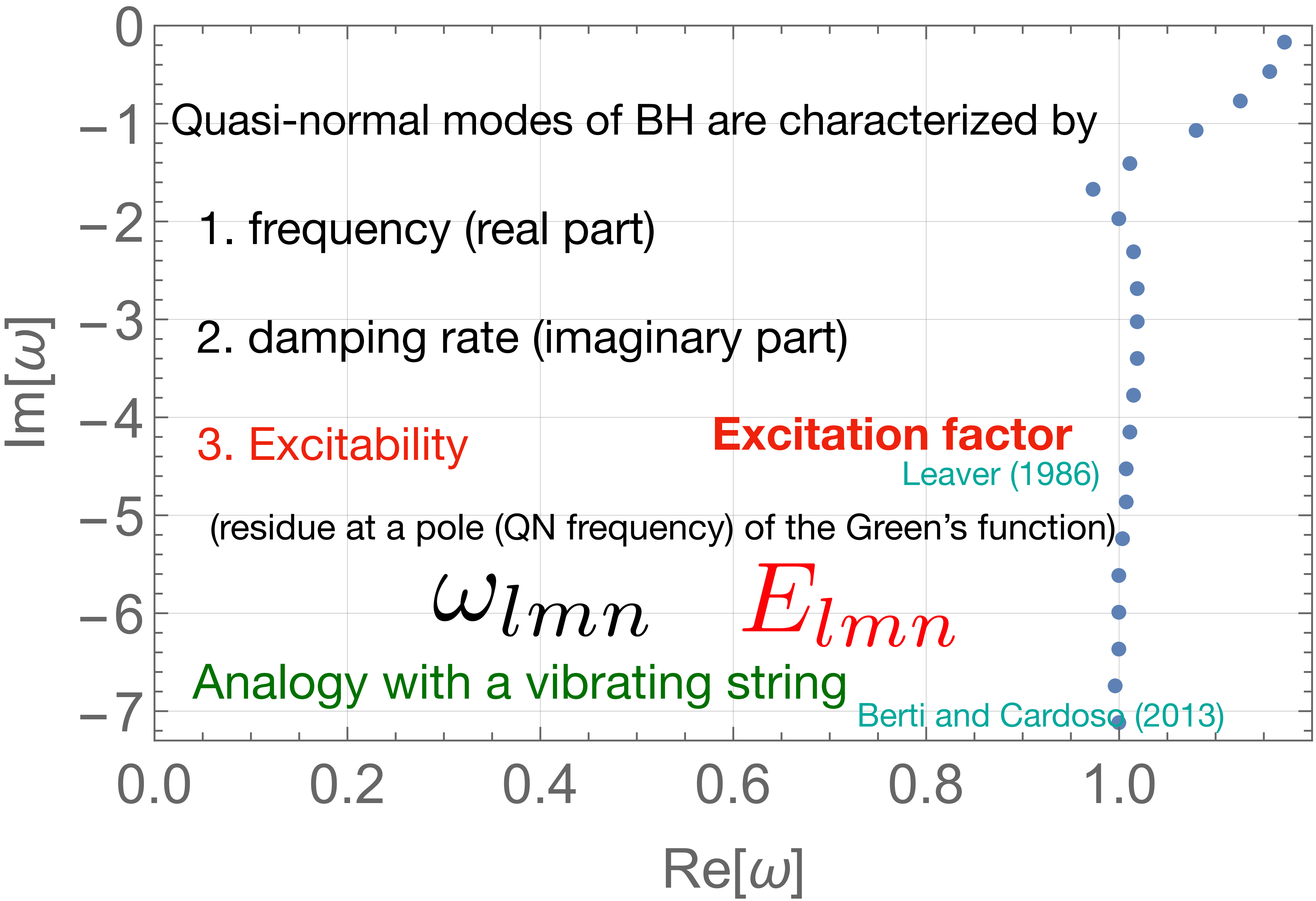
Giesler, Isi, Scheel, Teukolsky (2019)

(a) mass quadrupole wave

Ma, Giesler, Varma, Scheel, and Chen (2021)

Is there any theoretical reason or evidence??



Quasi-normal modes of BH are characterized by

1. frequency (real part)
2. damping rate (imaginary part)
3. Excitability

Excitation factor

Leaver (1986)

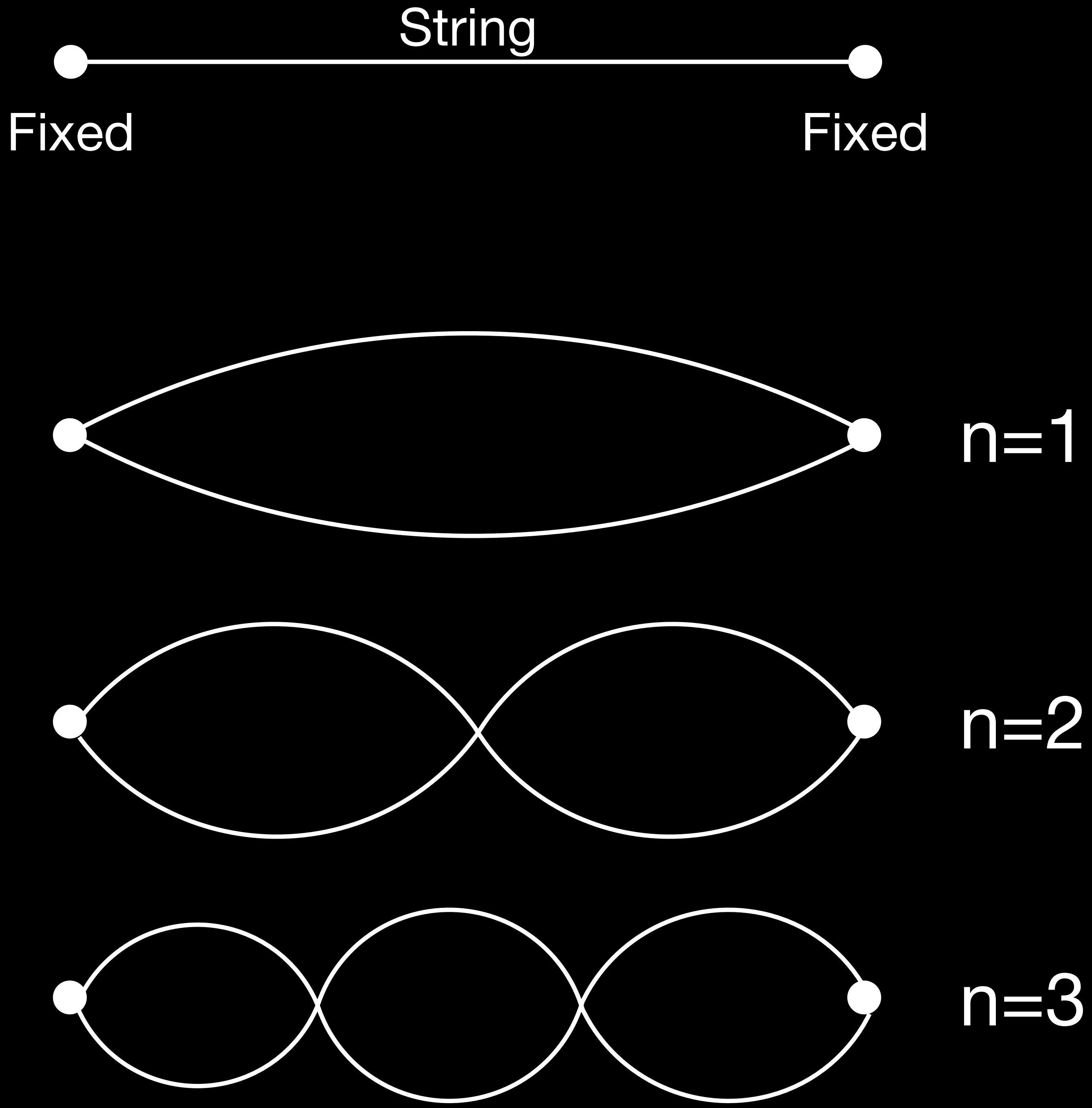
(residue at a pole (QN frequency) of the Green's function)

ω_{lmn} E_{lmn}

Analogy with a vibrating string

Berti and Cardoso (2013)

Vibrating string



String

Fixed

Fixed

n=1

n=2

n=3

Which pattern is dominant?

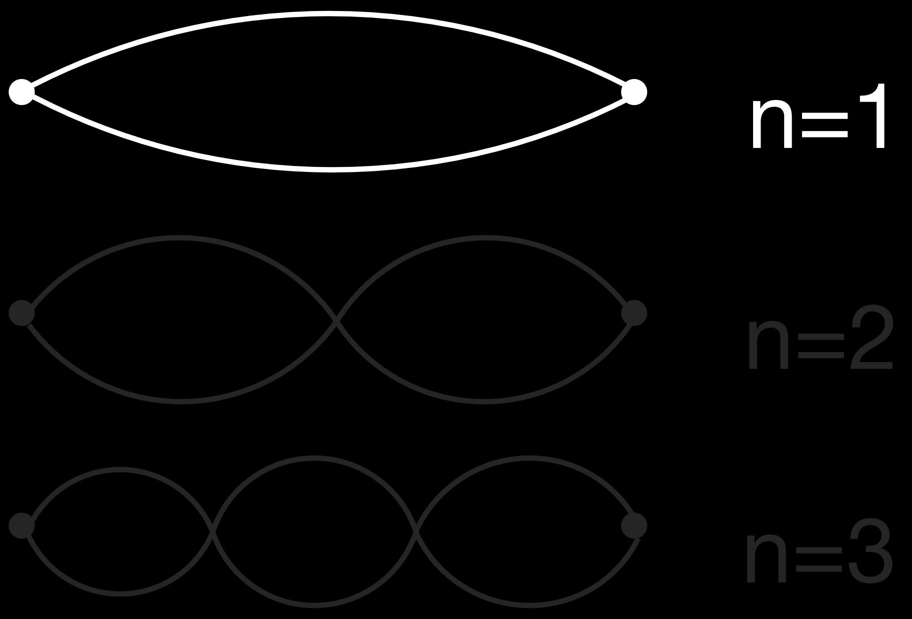
Vibrating string

$$(\partial_t^2 - \partial_x^2)u(t,x) = 0 \qquad \underbrace{u(t,x)}_{\text{waveform}} = \frac{1}{2\pi}\int d\omega dx' \underbrace{G^{(\text{string})}(x,x')}_{\text{Green's function}} \underbrace{\tilde{T}(\omega,x')}_{\text{source term}}$$

$$\boxed{G(x,x') \equiv \frac{\sin\omega x' \sin\omega(x-\pi)}{\underbrace{\omega\sin\omega\pi}_{\equiv W(\omega)}} \qquad \tilde{T}(\omega,x') \equiv e^{i\omega t_0}\left(i\omega u - \frac{\partial u}{\partial t}\right)_{t=t_0}}$$

$$= \sum_n \underbrace{E_n}_{\text{excitation factor}} \underbrace{T_n}_{\text{source factor}} \sin nx e^{-int}$$

$$\boxed{\begin{aligned} E_n &\equiv \frac{i}{\partial_\omega W|_{\omega=n}} = (-1)^n \frac{i}{\pi n} \propto \frac{1}{n} \\ T_n &\equiv (-1)^n \int dx' (inu(t_0,x') - \partial_t u(t_0,x')) \sin nx' \end{aligned}}$$



Excitation factor of the string is proportional to 1/n

n=1 is the easiest mode to excite!

Excitation of QNMs

$$h = \frac{e^{im\phi}}{r}\int d\omega dr' \sum_{lm} e^{i\omega(r^*-t+t_0)}\underline{{}_{-2}S_{lm}}(\omega,\theta)\underline{G^{(\mathrm{BH})}_{lm}}(r,r')\underline{\tilde{T}_{lm}}(r',\omega)$$

(spin-weighted) spheroidal harmonic function

Green's function

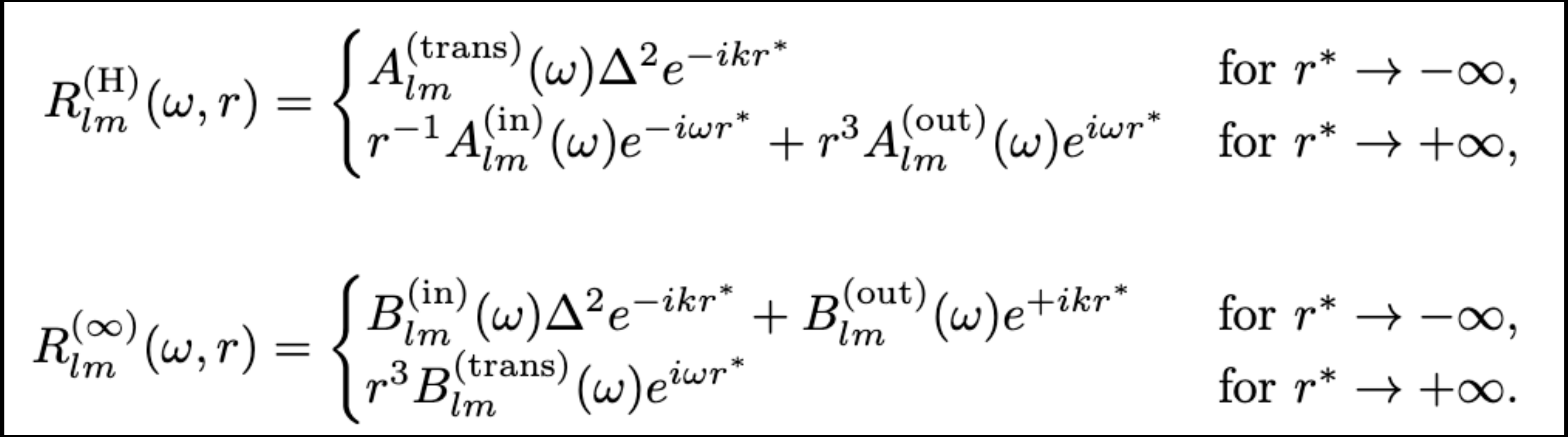
source term

$$= \frac{1}{r}\sum_{lmn}\underline{E_{lmn}}\,\underline{T_{lmn}}\,S_{lmn}e^{-i\omega_{lmn}(t-r^*)} \qquad S_{lmn} \equiv {}_{-2}S_{lm}(\omega_{lmn},\theta)$$

Source factor:
Initial data of a distorted BH

Excitation factor:
Intrinsic quantity of BHs
Quantify the “ease-of-excitaton” of QNMs
Residues of Green’s function

$$\boxed{E_{lmn} \equiv \frac{A^{(\mathrm{out})}_{lm}(\omega_{lmn})}{2i\omega^3_{lmn}}\left(\frac{dA^{(\mathrm{in})}_{lm}}{d\omega}\right)^{-1}_{\omega=\omega_{lmn}}}$$



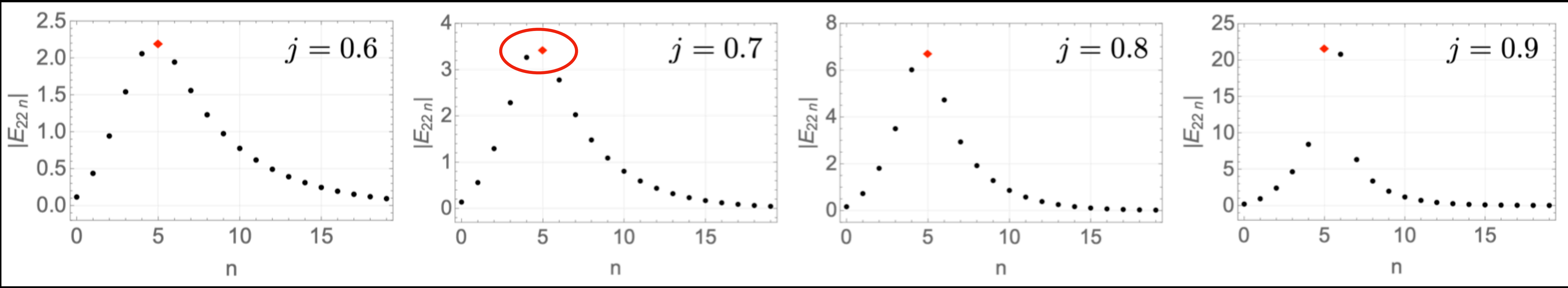
$$R^{(\mathrm{H})}_{lm}(\omega,r) = \begin{cases} A^{(\mathrm{trans})}_{lm}(\omega)\Delta^2 e^{-ikr^*} & \text{for } r^* \to -\infty, \\ r^{-1}A^{(\mathrm{in})}_{lm}(\omega)e^{-i\omega r^*} + r^3 A^{(\mathrm{out})}_{lm}(\omega)e^{i\omega r^*} & \text{for } r^* \to +\infty, \end{cases}$$

$$R^{(\infty)}_{lm}(\omega,r) = \begin{cases} B^{(\mathrm{in})}_{lm}(\omega)\Delta^2 e^{-ikr^*} + B^{(\mathrm{out})}_{lm}(\omega)e^{+ikr^*} & \text{for } r^* \to -\infty, \\ r^3 B^{(\mathrm{trans})}_{lm}(\omega)e^{i\omega r^*} & \text{for } r^* \to +\infty. \end{cases}$$

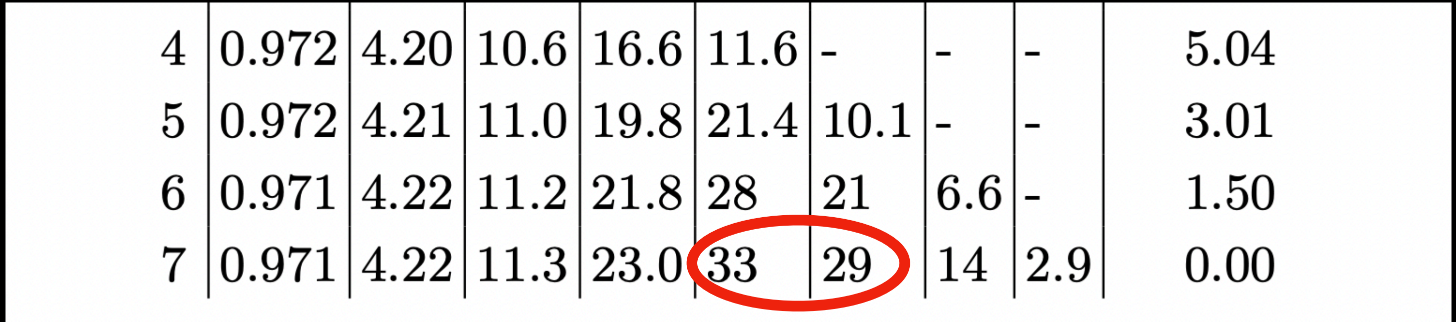
Excitation factor independent of the source of perturbation (universal quantity!!)

$$h_{22} = \frac{1}{r}\sum_{n} E_{22n} T_{22n} e^{-i\omega_{22n}(t-r^*)}$$

N.O. arXiv: 2109.09757



If the source factors have strong dependence on the overtone number “n”, the behaviour of the excitation factor is NOT meaningful…



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6	0.971	4.22	11.2	21.8	28	21	6.6	-	1.50
7	0.971	4.22	11.3	23.0	33	29	14	2.9	0.00

Giesler, Isi, Scheel, Teukolsky (2019)

$$h_{22} = \frac{1}{r}\sum_n \underline{C_{22n}}\,{}_{-2}S_{22n}\,e^{-i\omega_{22n}(t-r^*)}$$

~~challenging to compute!~~

easy to estimate!!

$$C_{22n} = E_{22n} T_{22n}$$

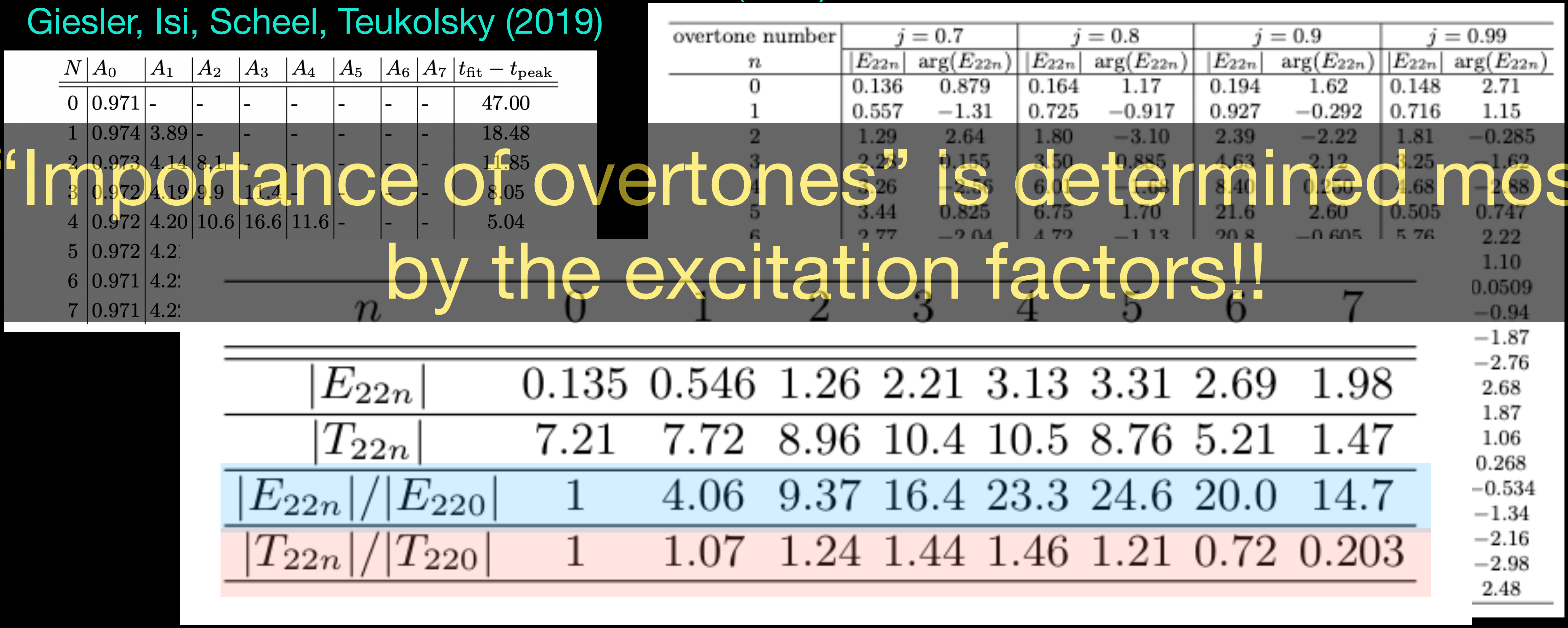
excitation coefficients

excitation factor

source factor

Giesler, Isi, Scheel, Teukolsky (2019)

N.O. (2021)



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4	0.972	4.20	10.6	16.6	11.6	-	-	-	5.04
5	0.972	4.2							
6	0.971	4.2							
7	0.971	4.2							

overtone number	$j=0.7$		$j=0.8$		$j=0.9$		$j=0.99$	
n	$\lvert E_{22n}\rvert$	$\arg(E_{22n})$	$\lvert E_{22n}\rvert$	$\arg(E_{22n})$	$\lvert E_{22n}\rvert$	$\arg(E_{22n})$	$\lvert E_{22n}\rvert$	$\arg(E_{22n})$
0	0.136	0.879	0.164	1.17	0.194	1.62	0.148	2.71
1	0.557	−1.31	0.725	−0.917	0.927	−0.292	0.716	1.15
2	1.29	2.64	1.80	−3.10	2.39	−2.22	1.81	−0.285
3	[illegible]	0.155	3.50	0.885	4.63	2.12	3.25	−1.62
4	[illegible]	[illegible]	[illegible]	[illegible]	8.40	[illegible]	4.68	−2.88
5	3.44	0.825	6.75	1.70	21.6	2.60	0.505	0.747
6	2.77	−2.04	4.72	−1.13	20.8	−0.605	5.76	2.22
								1.10
								0.0509
								−0.94
								−1.87
								−2.76
								2.68
								1.87
								1.06
								0.268
								−0.534
								−1.34
								−2.16
								−2.98
								2.48

n	0	1	2	3	4	5	6	7
$\lvert E_{22n}\rvert$	0.135	0.546	1.26	2.21	3.13	3.31	2.69	1.98
$\lvert T_{22n}\rvert$	7.21	7.72	8.96	10.4	10.5	8.76	5.21	1.47
$\lvert E_{22n}\rvert/\lvert E_{220}\rvert$	1	4.06	9.37	16.4	23.3	24.6	20.0	14.7
$\lvert T_{22n}\rvert/\lvert T_{220}\rvert$	1	1.07	1.24	1.44	1.46	1.21	0.72	0.203

"Importance of overtones" is determined mostly by the excitation factors!!

Excitation factor is SENSITIVE to "n"

Source factor is INSENSITIVE to "n"

1 Ringdown of comparable mass ratio BBH mergers

NO arXiv: 2109.09757

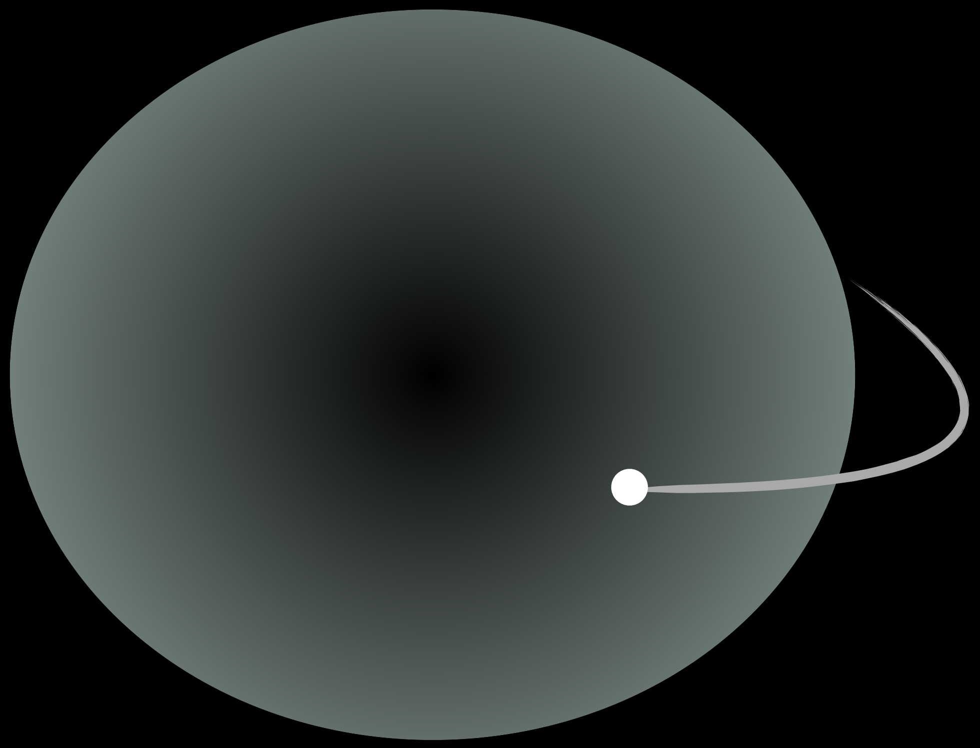
2 Ringdown of extreme mass ratio mergers

NO arXiv: 2208.02923

3 Alternative modeling of ringdown for extreme mass ratio mergers

NO arXiv: 2208.02923

Are BH overtones well excited even for an extreme-mass-ratio merger?



GW waveform induced by a particle plunging into a BH

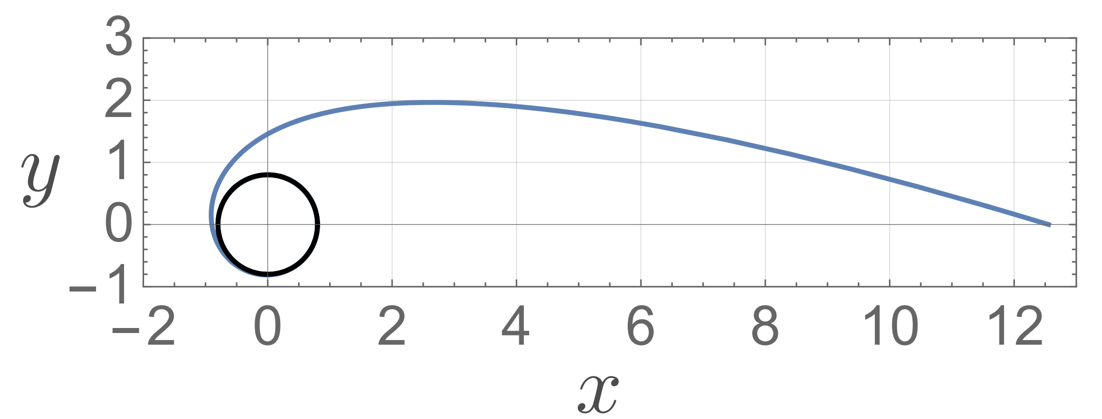
Extreme-Mass-Ratio Merger

Y. Kojima and T. Nakamura (1984)

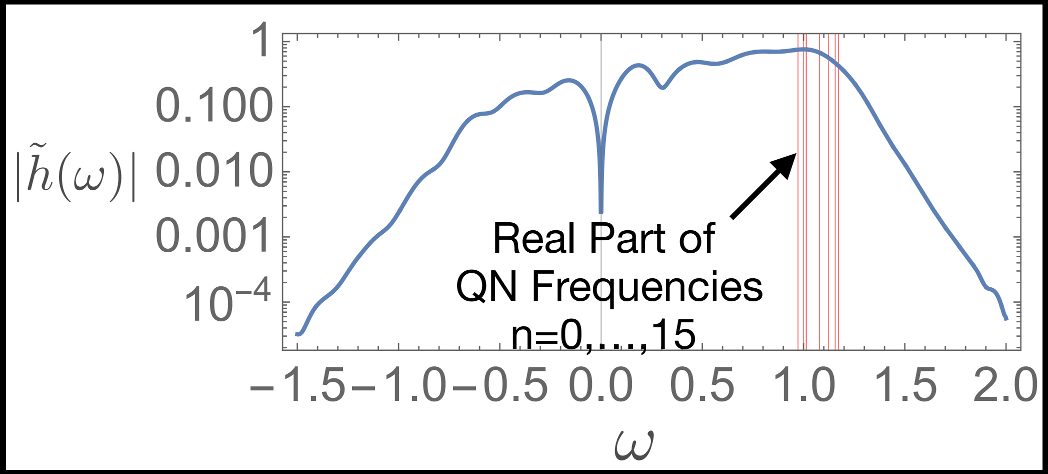
Sasaki Nakamura equation

$$\left(\frac{d^2}{dr^{*2}} - F_{lm}\frac{d}{dr^*} - U_{lm}\right) X_{lm} = T_{lm}$$

Trajectory
(solving the geodesic equation and *NO self-force*)



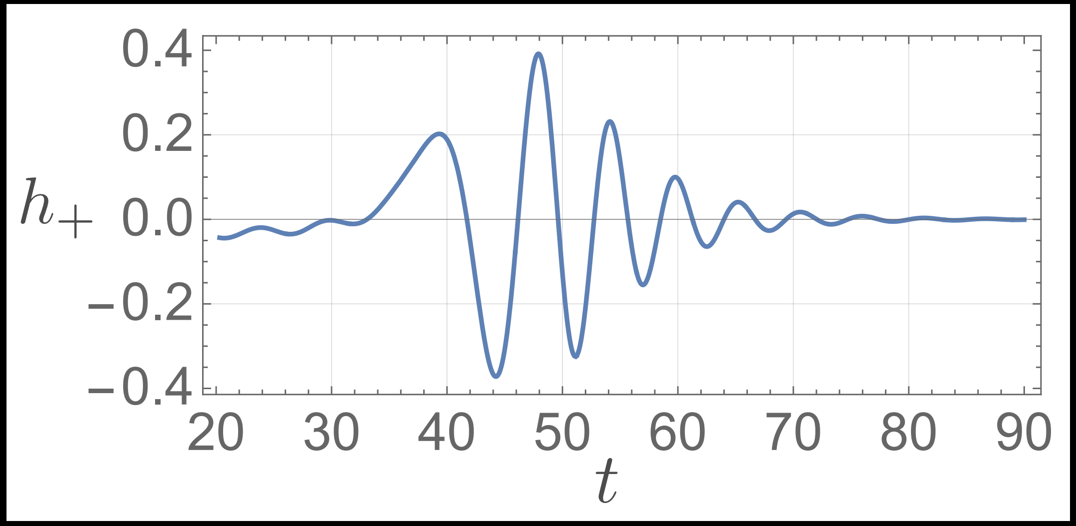
Signal (spectrum)



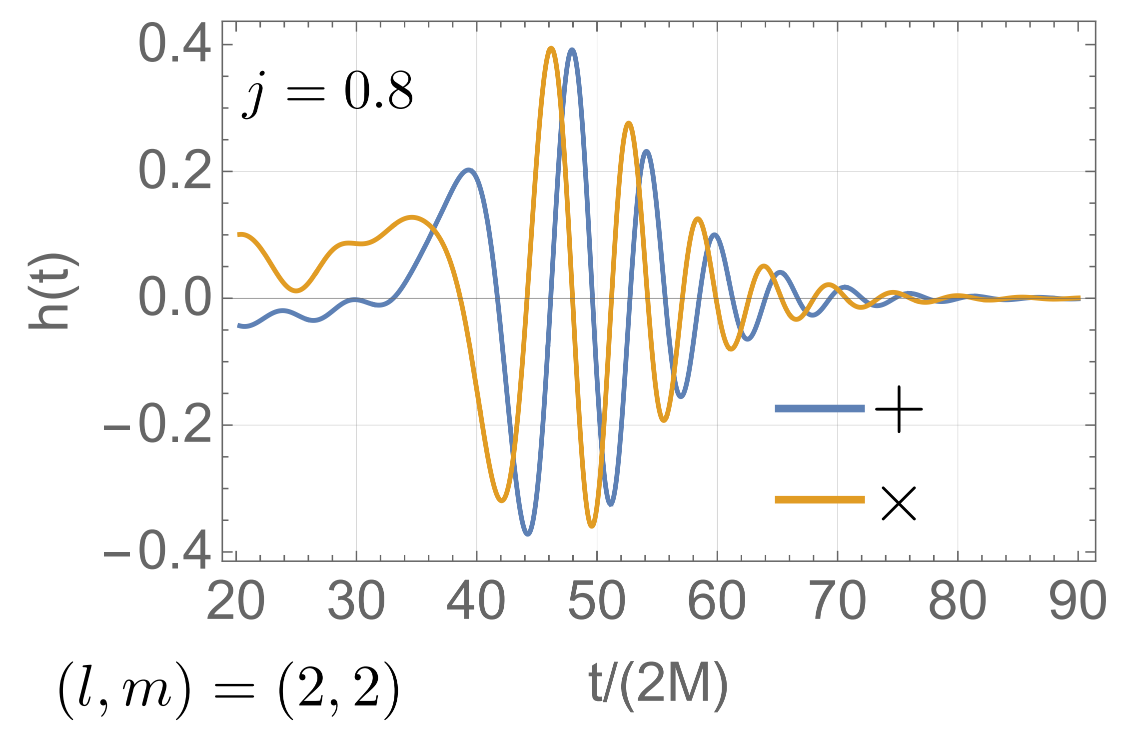
Real Part of
QN Frequencies
n=0,…,15

Inverse
Fourier Transform

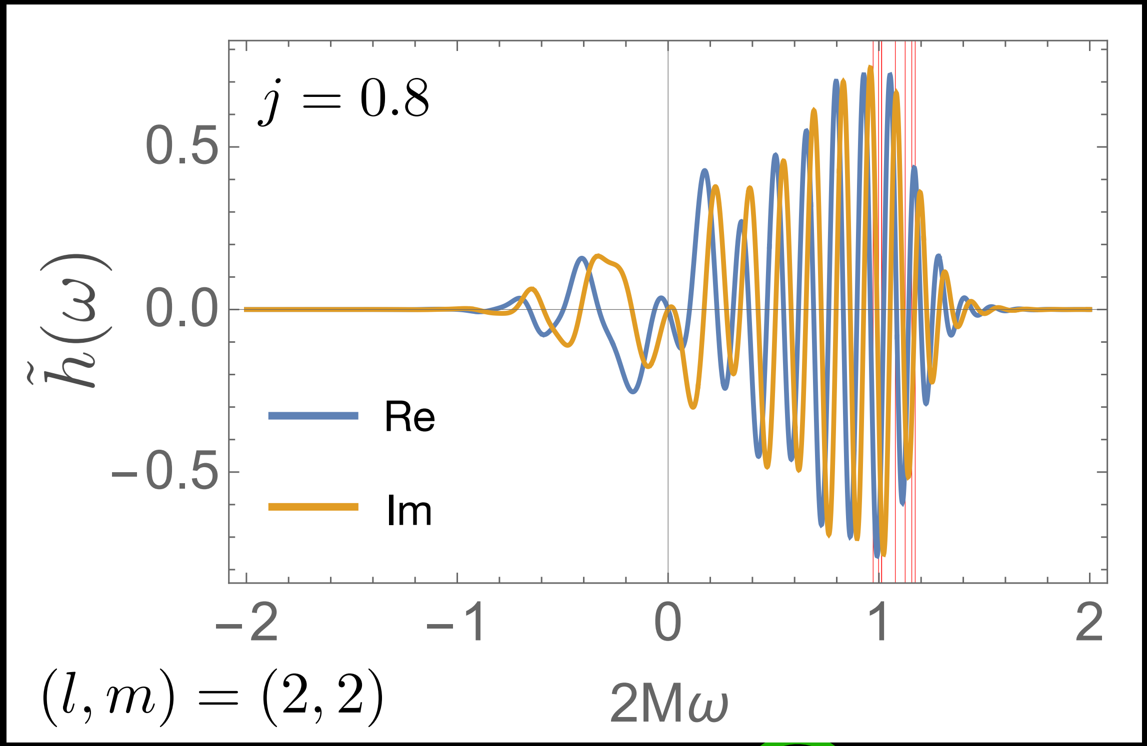
Signal (time domain)



Fitting analysis in frequency domain



Fourier transform



$$h_{\mathrm{QNM},lm}(t) = \sum_n \boxed{C_n} e^{-i\omega_{lmn}t}\theta(t - \boxed{t^*})$$

$$\tilde{h}_{\mathrm{QNM},lm}(\omega) = \frac{i}{2\pi}\sum_n \frac{\boxed{\tilde{C}_n}}{\omega - \omega_{lmn}} e^{i\omega \boxed{t^*}}$$

$$\tilde{C}_n = C_n e^{-i\omega_{lmn}t^*}$$

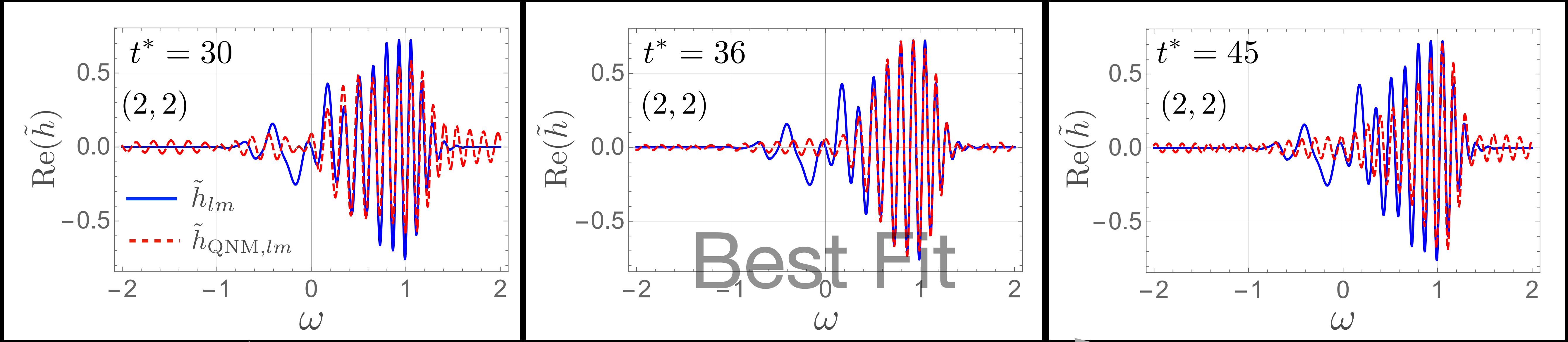
QNM fitting in time-domain waveform

see e.g. Giesler, et al. (2019)
Mourier et al. (2020)
Ma, et al. (2021)

e.g. Finch, et al. (2021)
Ma, et al. (2022) & (2023)

QNM fitting in frequency domain

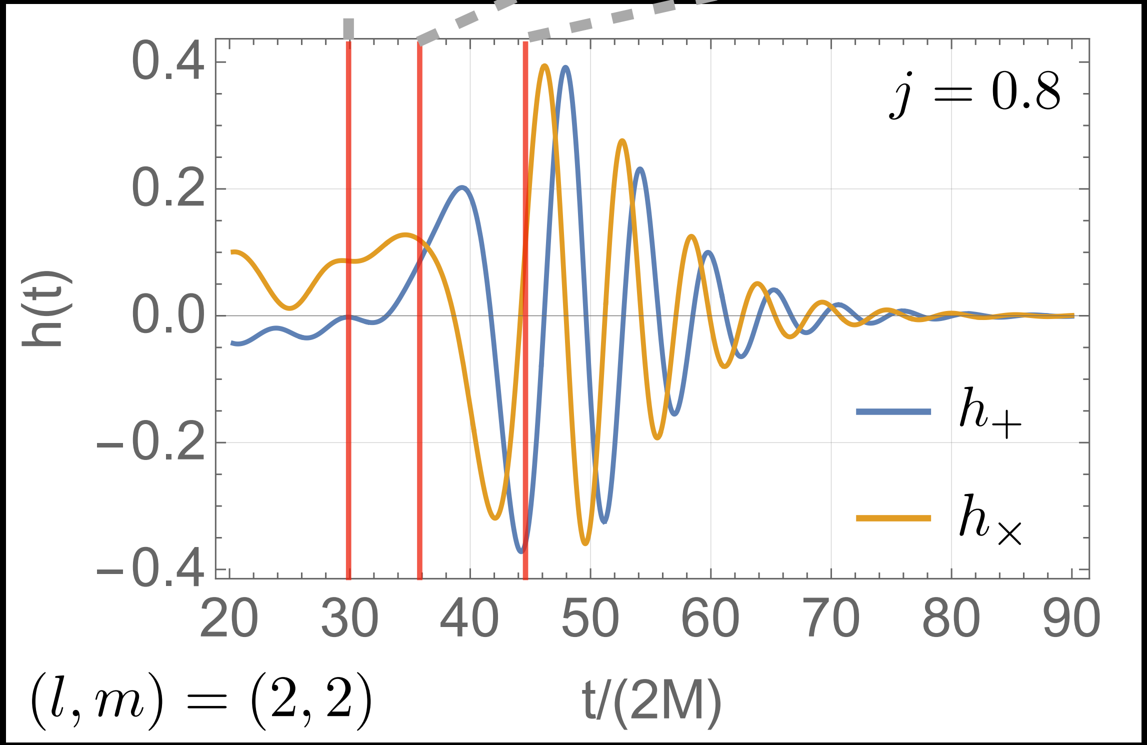
Start time of ringdown

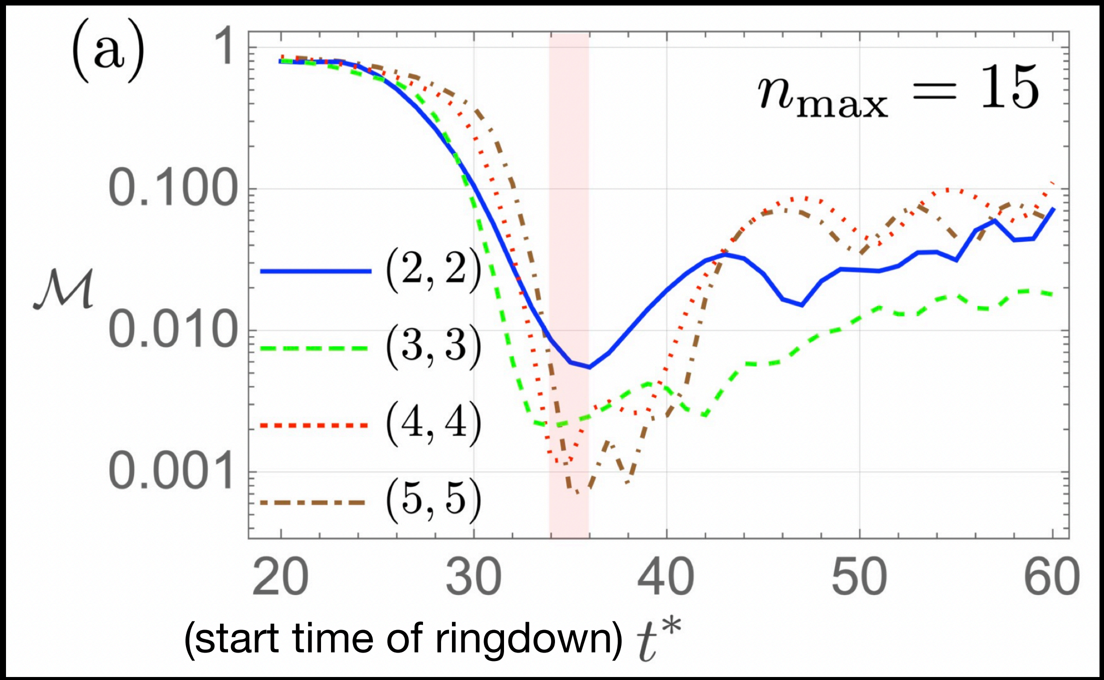


Best Fit

N. O. arXiv:2208.02923

$n_{\max} = 15$





$$\mathcal{M} \equiv \left| 1 - \frac{\langle h | h_Q \rangle}{\sqrt{\langle h | h \rangle \langle h_Q | h_Q \rangle}} \right|$$

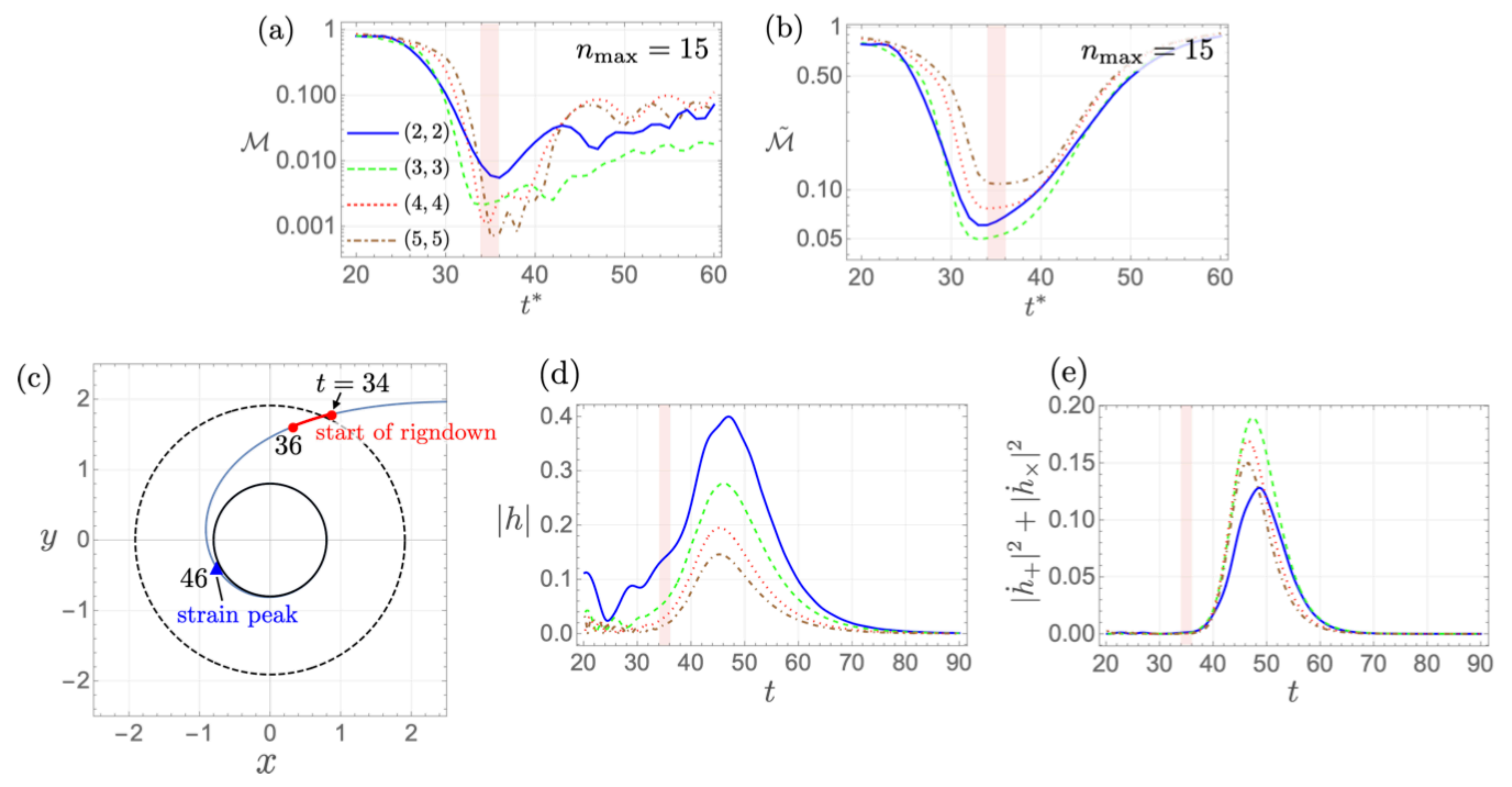
h : original

h_Q : QNM model

Start Time of Ringdown

Mismatch computed in time-domain

Mismatch computed in frequency-domain

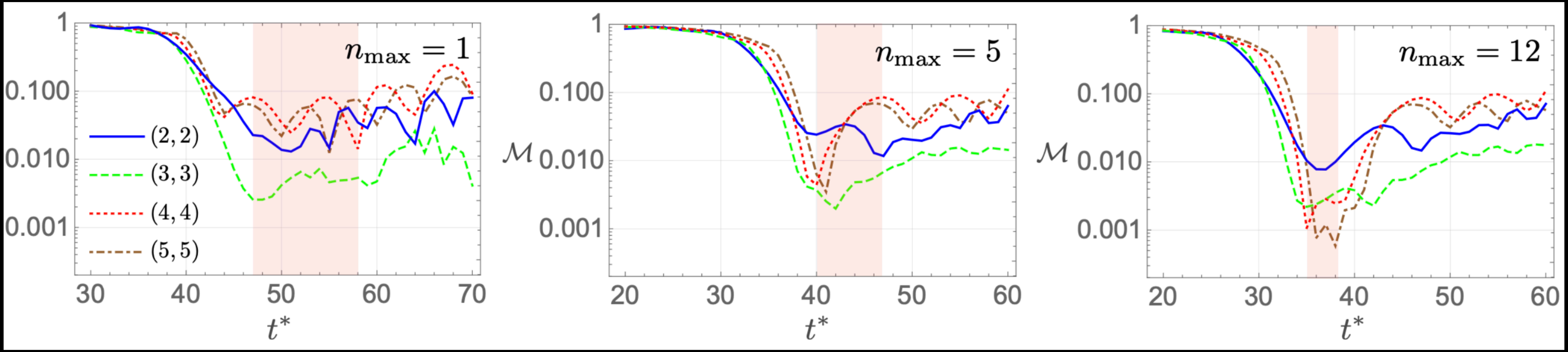


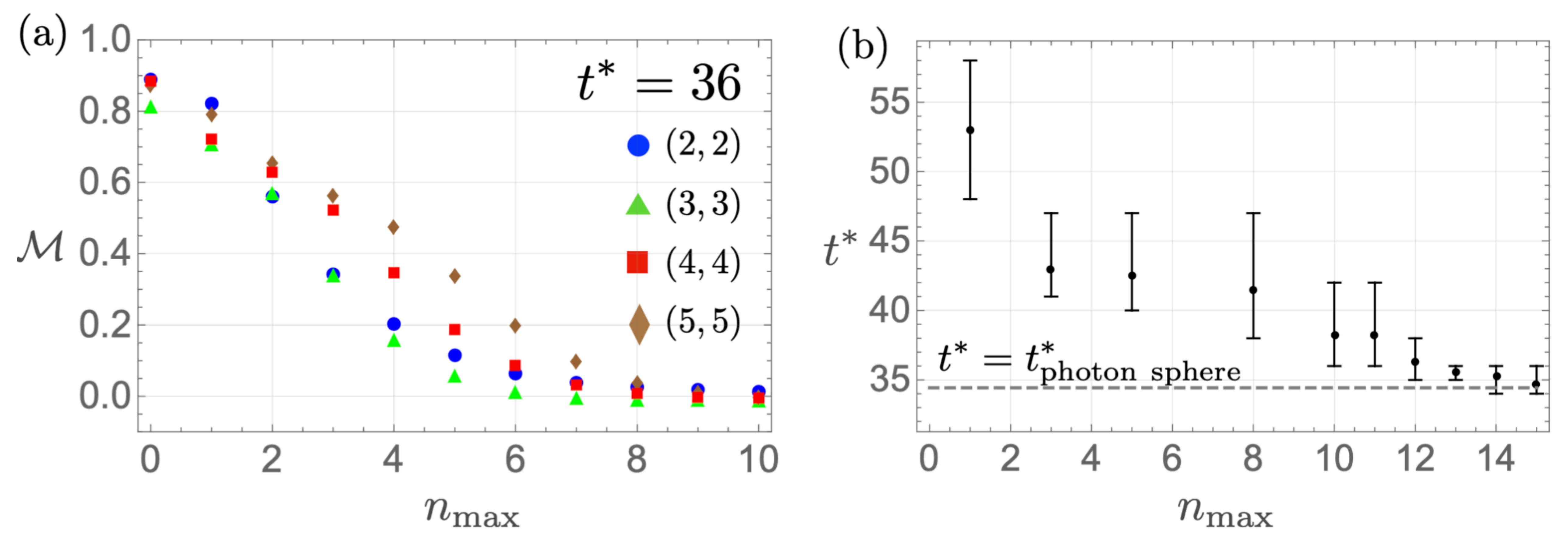
trajectory

strain

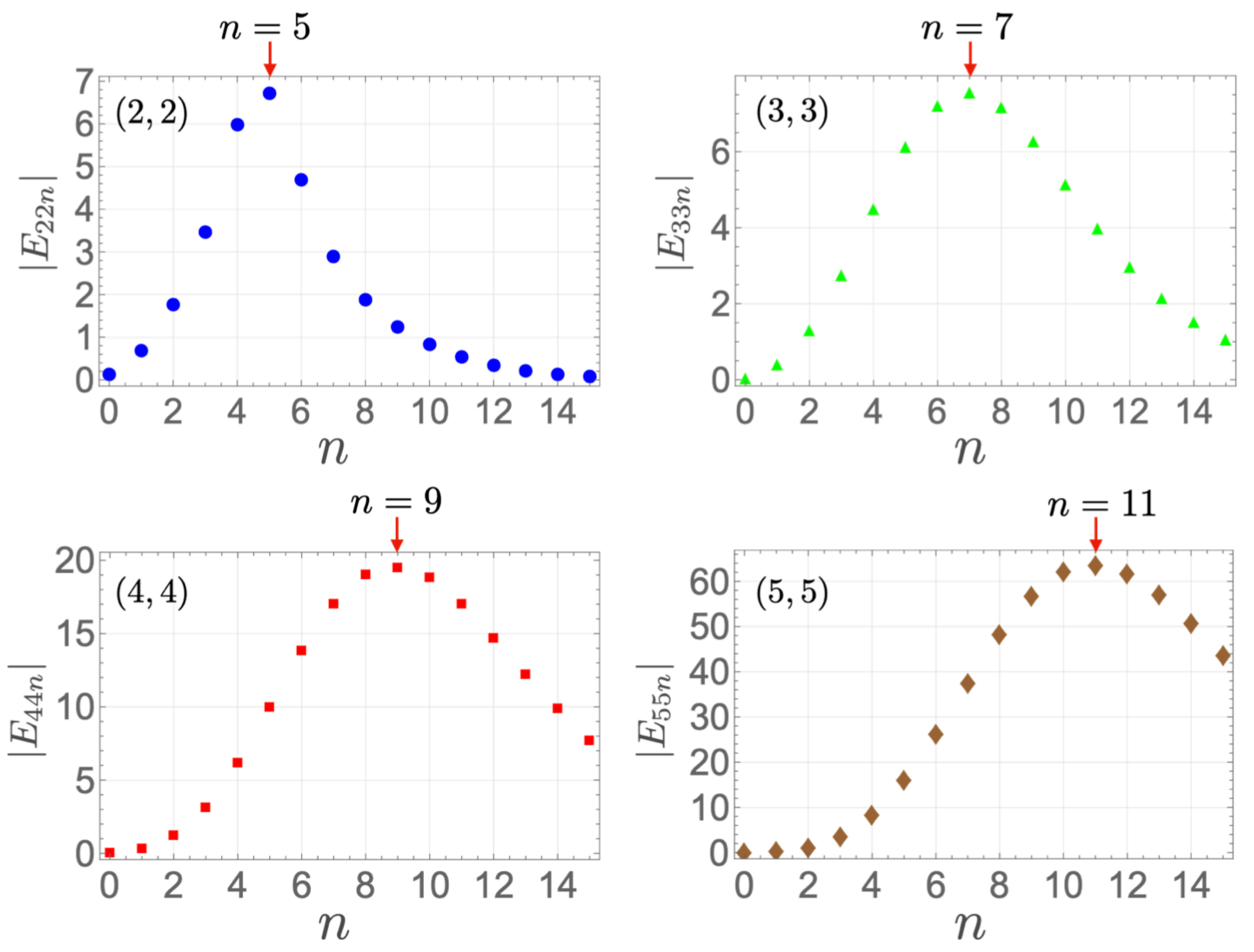
flux

Convergence of Analysis

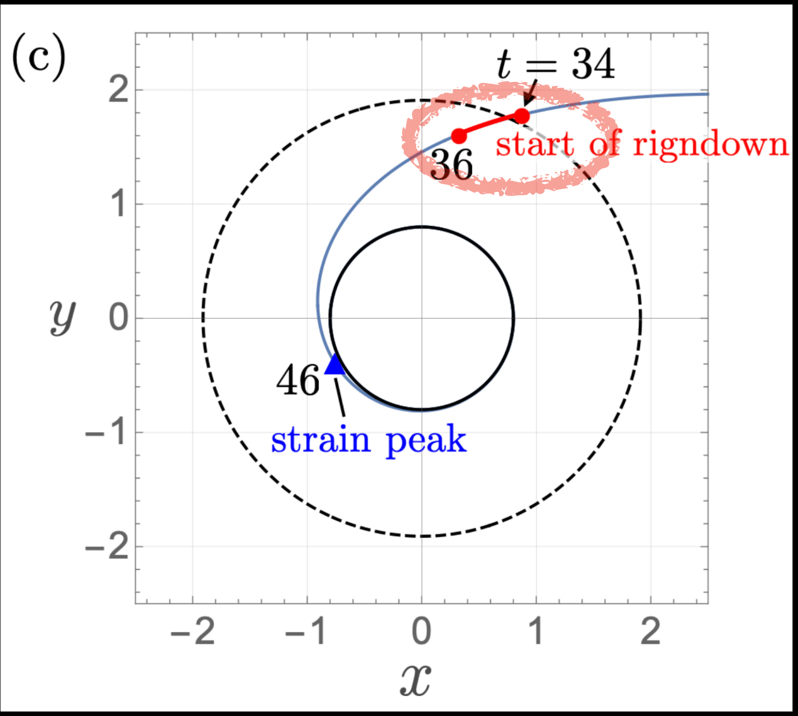


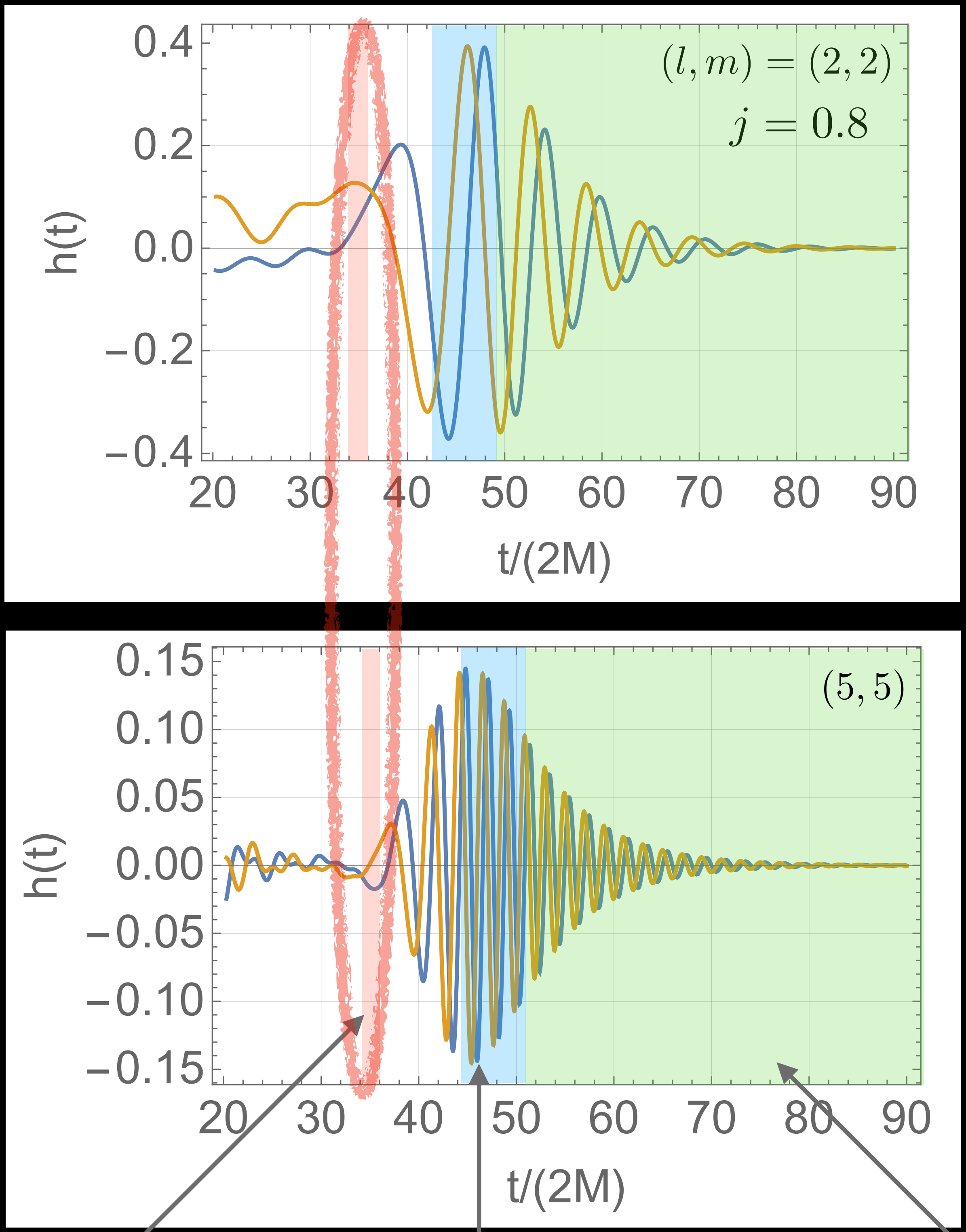


Excitation Factors



Destructive Interference of Overtones in the extreme-mass-ratio merger





Beginning of Ringdown
(Destructive Interference)

Strain Peak
(Overtones getting suppressed)

Exponential Damping
(Fundamental Mode)

1 Ringdown of comparable mass ratio BBH mergers

NO arXiv: 2109.09757

2 Ringdown of extreme mass ratio mergers

NO arXiv: 2208.02923

3 Alternative modeling of ringdown for extreme mass ratio mergers

NO arXiv: 2208.02923

Testing GR with QN modes

- QN modes are exponentially damped in time.
- Extracting QN modes from GW data involves many fitting parameters.
- Sensitive to the choice of the start time of ringdown.

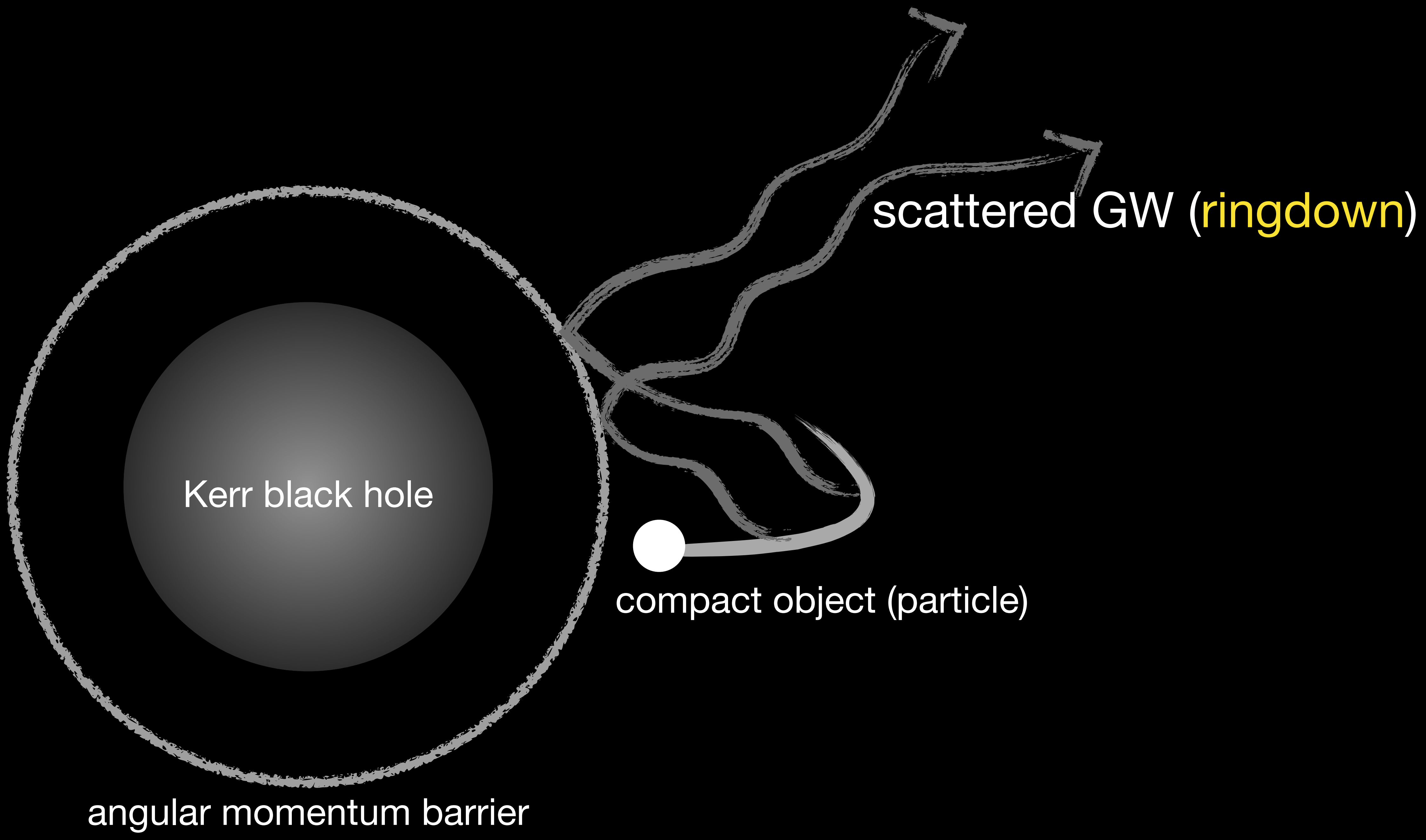
Are QN modes unique quantities to test GR in strong-gravity regimes?

Is there any other no-hair quantity to test GR?

→ greybody factor

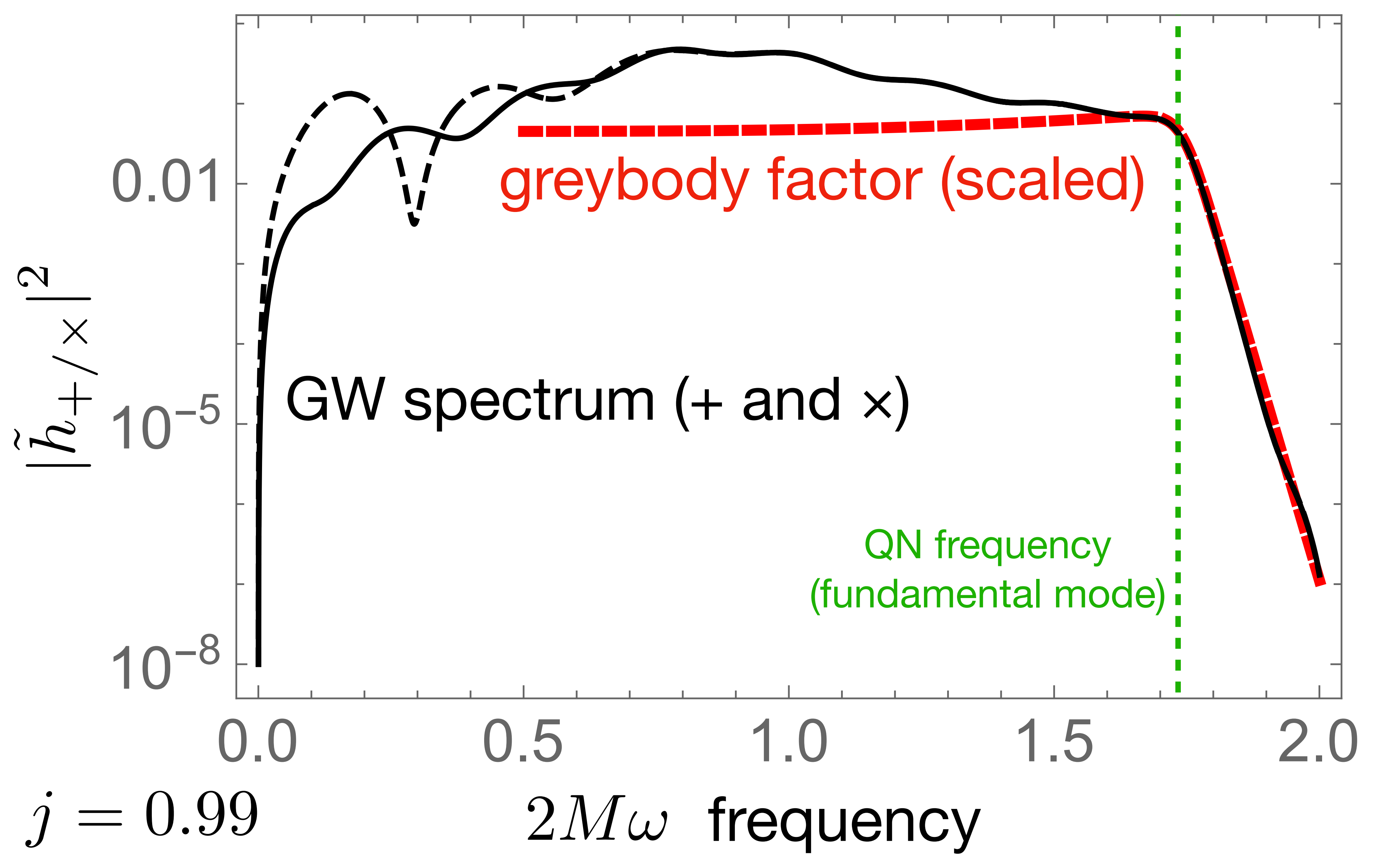
(transmissivity/reflectivity of a light ring)

Greybody Factor Imprinted on Ringdown??



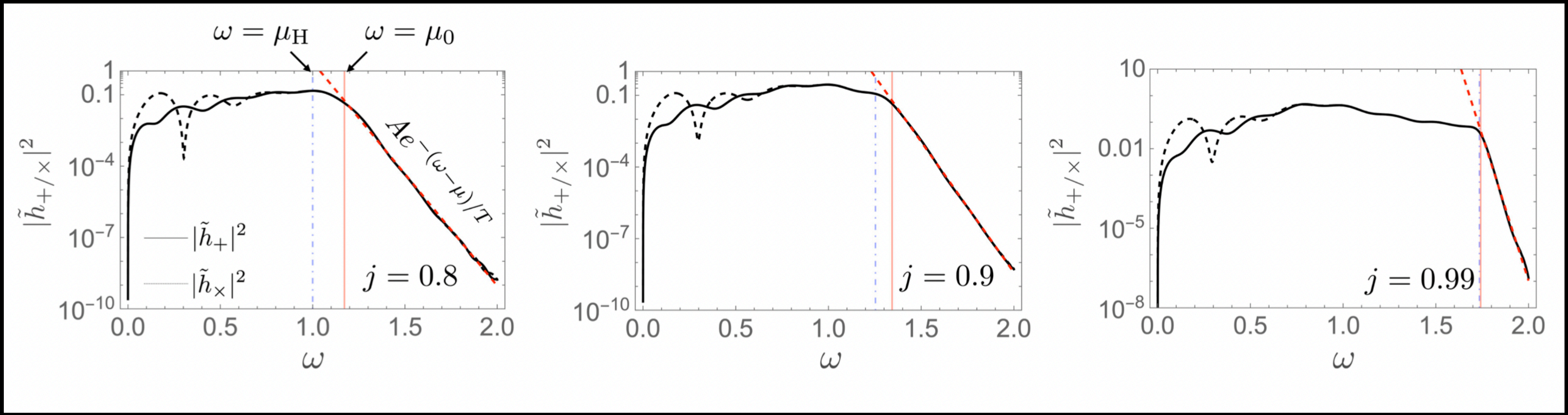
Greybody Factor Imprinted on Ringdown

NO arXiv: 2208.02923

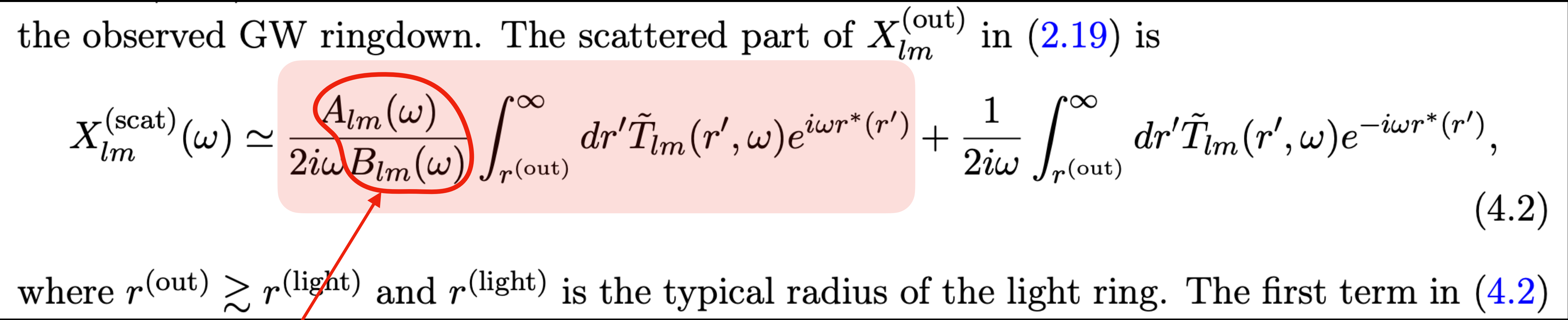


Exponential cut-off in ringdown spectrum

NO arXiv: 2208.02923



$$2M = 1 \quad (l, m) = (2, 2)$$

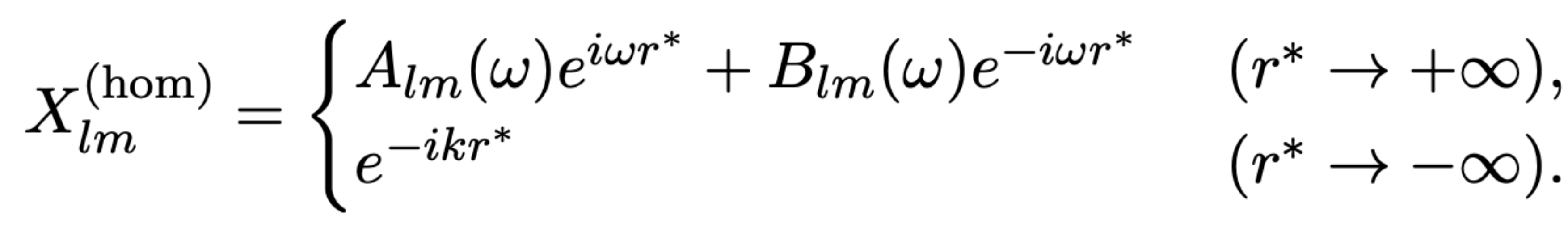


the observed GW ringdown. The scattered part of $X_{lm}^{\text{(out)}}$ in (2.19) is

$$X_{lm}^{\text{(scat)}}(\omega) \simeq \frac{A_{lm}(\omega)}{2i\omega B_{lm}(\omega)} \int_{r^{\text{(out)}}}^{\infty} dr' \tilde{T}_{lm}(r', \omega) e^{i\omega r^*(r')} + \frac{1}{2i\omega} \int_{r^{\text{(out)}}}^{\infty} dr' \tilde{T}_{lm}(r', \omega) e^{-i\omega r^*(r')}, \tag{4.2}$$

where $r^{\text{(out)}} \gtrsim r^{\text{(light)}}$ and $r^{\text{(light)}}$ is the typical radius of the light ring. The first term in (4.2)

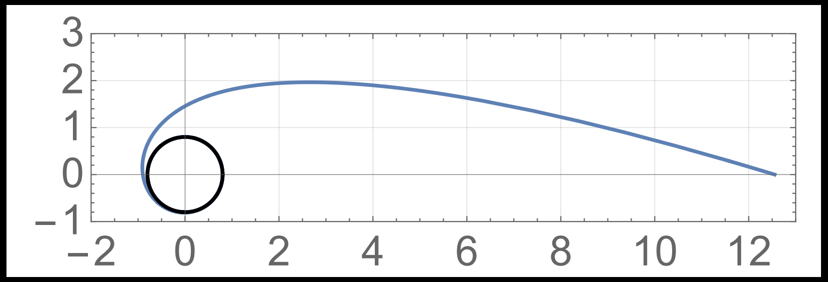
Leading to Reflectivity (greybody factor)

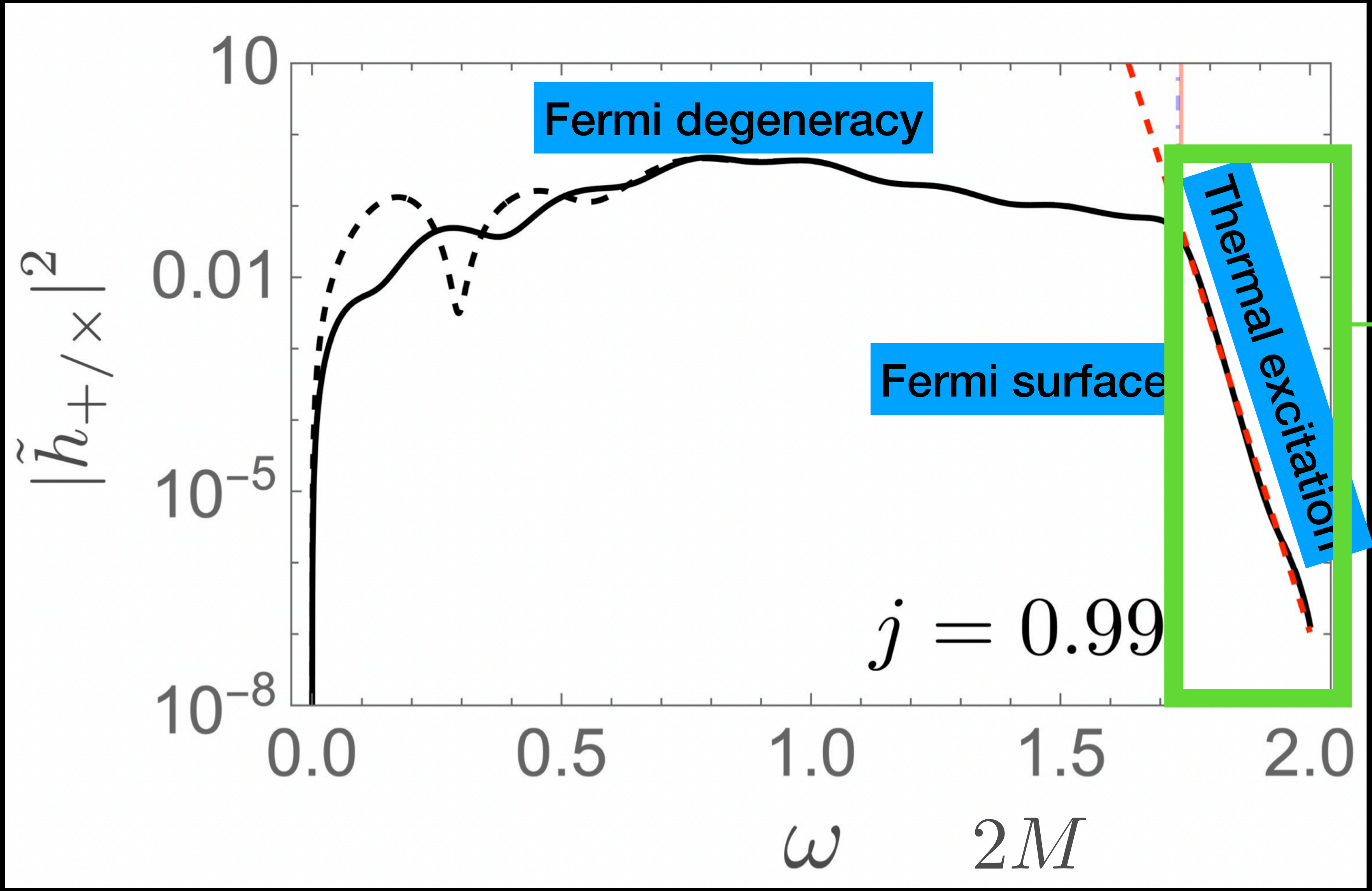


$$X_{lm}^{\text{(hom)}} = \begin{cases} A_{lm}(\omega) e^{i\omega r^*} + B_{lm}(\omega) e^{-i\omega r^*} & (r^* \to +\infty), \\ e^{-ikr^*} & (r^* \to -\infty). \end{cases}$$

Fermi degeneracy of Kerr ringdown

j=0.99 (low Hawking temperature) $T_{\rm H} = \dfrac{\sqrt{1-j^2}}{4\pi r_+}$ $\Omega_{\rm H} = \dfrac{j}{2r_+}$



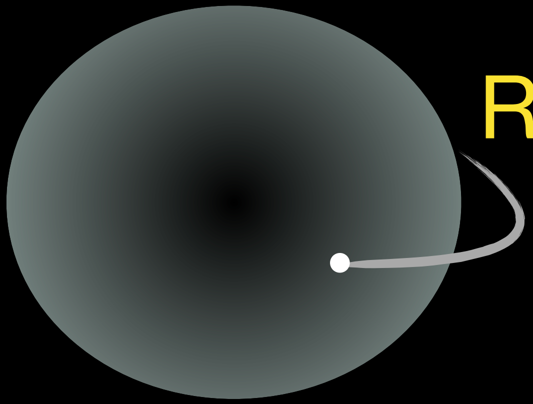


Hawking frequency (M=0.5)

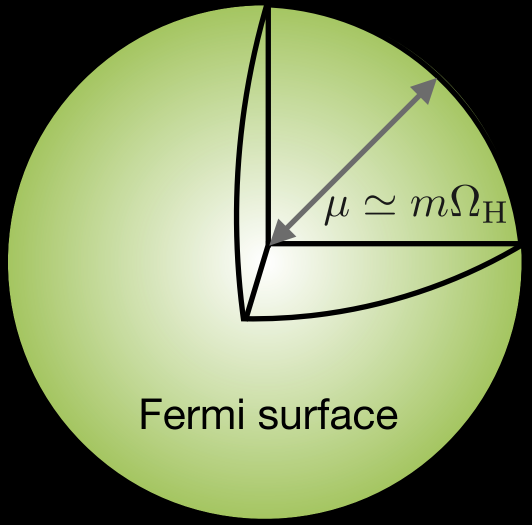
$$T_{\rm H} = 0.0197$$

Exponential cut-off in GW spectrum

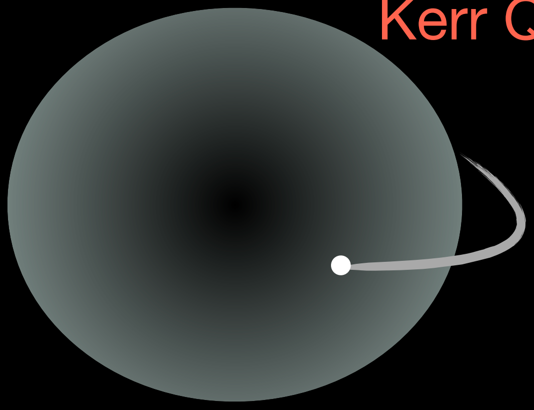
$$T_{\rm ringdown} = 0.0198(2)$$



Ringdown of a near-extremal BH
→ Thermal Fermi system??



QN modes ~ Matsubara modes



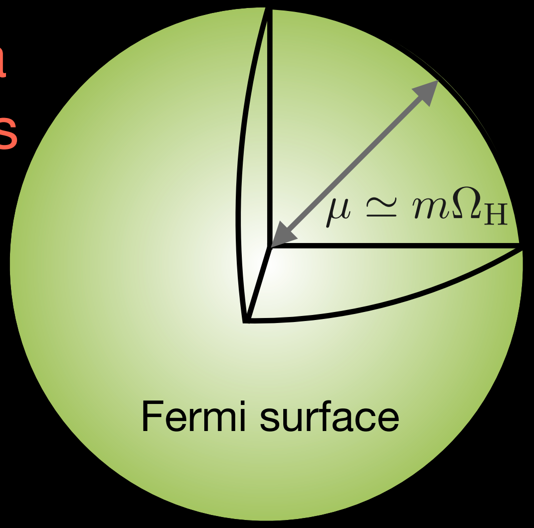
S. Hod, (2008), H. Yang, et al. (2013)

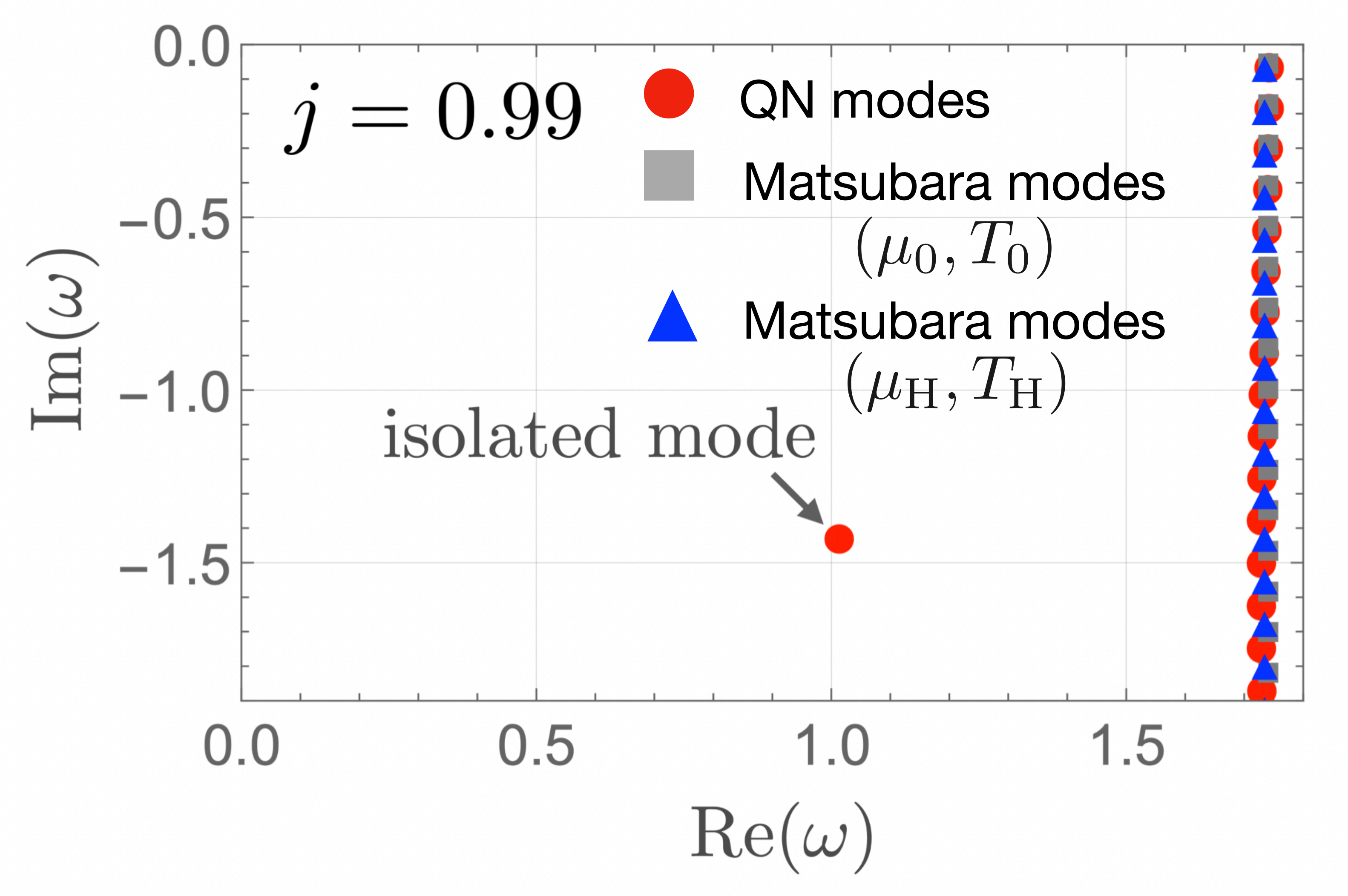
Kerr QN frequencies
for j~1

$$\omega \simeq \mu_{\rm H} - i\left(\frac{1}{2}+n\right)2\pi T_{\rm H}$$

Matsubara
frequencies

$$\frac{1}{e^{(\omega-\mu_{\rm H})/T_{\rm H}}+1}$$





Summary

Excitation of overtones

Excitability of QN modes is quantified by the “excitation factor”

It has the peak around at n=5.

↑

Consistent with the result of numerical relativity!!

ringdown sourced by an extreme mass ratio merger

Beginning of Ringdown	Strain Peak	Exponential Damping
(Destructive Interference)	(Constructive Interference)	(Fundamental Mode)

An alternative model of ringdown → greybody factors

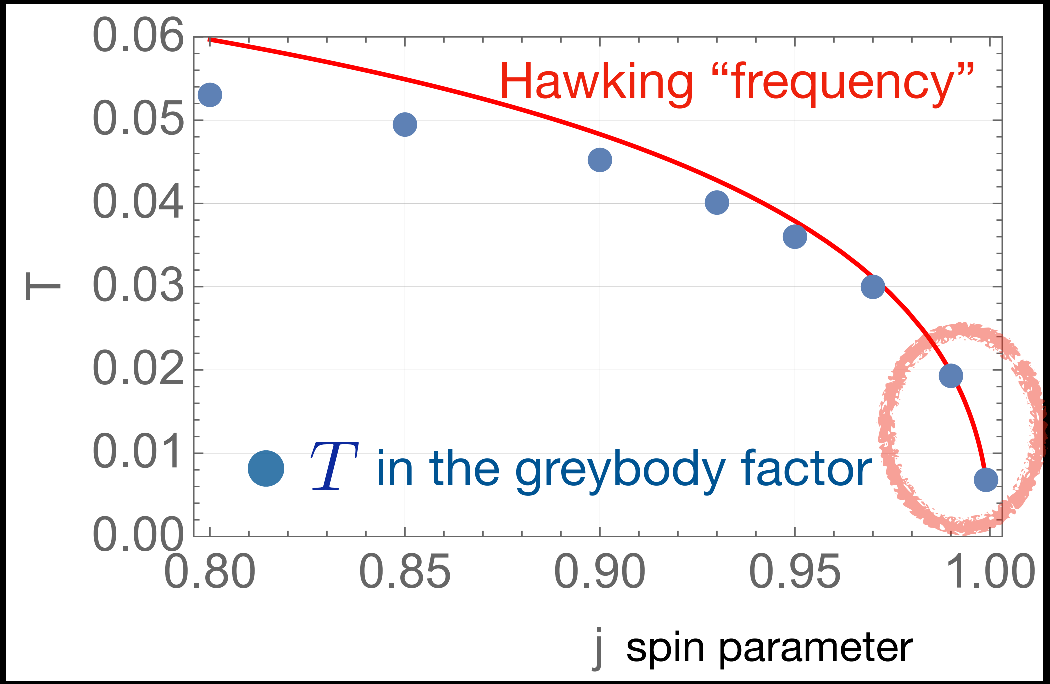
reflectivity (transmissivity) of the light ring

Greybody factor has an exponential cut-off at higher frequency region.
QN fundamental mode has a power-law tail.
Excitation of multiple QN modes lead to the exponential cut-off of ringdown spectrum?

reflectivity of a BH $\simeq$ Fermi-Dirac distribution

(WKB approximation) e.g. S. Iyer et al. (1987), R. A. Konoplya et al. (2019)

$$1-\Gamma_{lm} \simeq \frac{1}{1+e^{(\omega - m\Omega_{\mathrm{H}})/T}}$$



NO arXiv: 2208.02923

$$(\text{Hawking temperature}) = (\hbar/k_{\mathrm{B}})(\text{Hawking frequency})$$

Quantum *Classical*

apparent thermal ringdown
from an extreme-mass-ratio merger