# Where my DAEMON hides

## Power-law mass density models from fundamental principles

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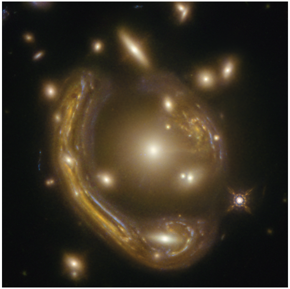
# What are suitable mass density profiles of cosmic structures?



**power-law mass densities from galaxy to galaxy-cluster scale**

$$\rho_{\mathrm{Moore}}(r) = \frac{\rho_{\mathrm{s}}}{(r/r_{\mathrm{s}})^{3/2}(1+(r/r_{\mathrm{s}})^{3/2})}$$

$$\rho_{\mathrm{PIEMD}}(r) = \frac{\rho_0}{(1+r^2/r_{\mathrm{core}}^2)(1+r^2/r_{\mathrm{cut}}^2)}$$

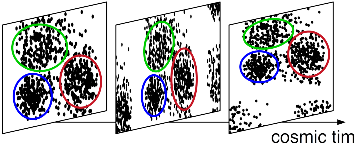
$$\rho_{\mathrm{SIS}}(r) = \frac{\sigma^2}{2\pi G r^2}$$

$$\rho_{\mathrm{NFW}}(r) = \frac{\rho_{\mathrm{s}}}{(r/r_{\mathrm{s}})(1+r/r_{\mathrm{s}})^2}$$

$$\rho_{\mathrm{Jaffe}}(r) = \frac{\rho_{\mathrm{s}}}{(r/r_{\mathrm{s}})^2(1+r/r_{\mathrm{s}})^2}$$

Image credits: ESA/Hubble, NASA, S. Suyu et al.; ESA/Hubble & NASA, S. Jha, L. Shatz

**structure = description of a `static` spatial configuration of particles**




$$\rho(r) = m_\mathrm{p}\, n(r) \propto r^{-\gamma}$$

**almost universal mass density profiles**

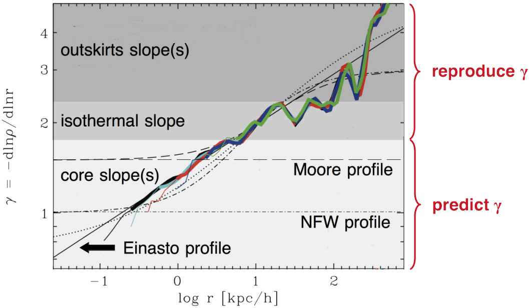
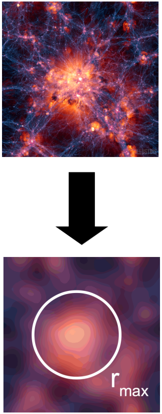



Image credits: Navarro et al. (2010)

# Prerequisites to set up a simplified, abstract model




**“microscopic” ensemble of particles**

- $n_\mathrm{p}$ identical, identically distributed point particles
- collisionless (= independent) particles
- Newtonian gravity as the only interaction
- linear superposition of grav. interactions

**“macroscopic” (dark matter) halo model**

- halo = sphere of radius $r_\mathrm{max}$
- $\rho(r) = m_\mathrm{p}\, n(r) = m_\mathrm{p}\, n_\mathrm{p}\, p(r)$
- scale-free spatial power-law $p(r)$

$$p(r_j) = N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min}) \left(\frac{r_j}{r_\sigma}\right)^{-\gamma}$$

Image credits: Aquarius A1 simulation MPA Garching

**joint PDF**

$$p_\mathrm{E} = \prod_{j=1}^{n_\mathrm{p}} p(r_j)$$

$\rightarrow$

**log-likelihood**

$$\mathcal{L}(p_\mathrm{E}) = \log\left(p_\mathrm{E}\right)$$

$\rightarrow$

**extremum config.**

$$\partial_\gamma \mathcal{L}(p_\mathrm{E}) \overset{!}{=} 0$$

$$\frac{\partial_\gamma N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min})}{N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min})} - \frac{1}{n_\mathrm{p}} \sum_{j=1}^{n_\mathrm{p}} \ln\left(\frac{r_j}{r_\sigma}\right) \overset{!}{=} 0$$

# DAEMON = DArk Emergent Matter halO explanatioN

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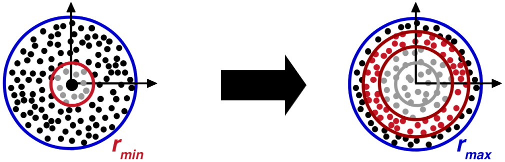


**density of a dark matter halo from simulated particle set**

$$\rho(r) = m_\mathrm{p}\, n(r) = m_\mathrm{p}\, n_\mathrm{p}\, p(r) = m_\mathrm{p}\, n_\mathrm{p}\, N(\gamma, r_\sigma, r_\mathrm{max}, r_\mathrm{min}) \left(\frac{r}{r_\sigma}\right)^{-\gamma}$$

# DAEMON = DArk Emergent Matter halO explanatioN

$$\gamma = 3 + \frac{n_\mathrm{p}}{\sum_{j=1}^{n_\mathrm{p}} \ln\left(\frac{r_j}{r_\mathrm{max}}\right)}$$


$0 \leq \gamma \leq 2$

uniform: $\gamma = 1$

variable centre

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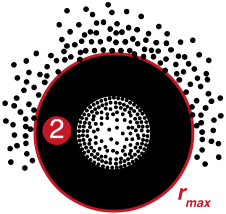
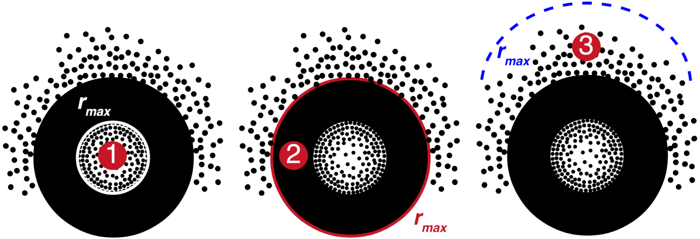



$\gamma = 2$

$n_\mathrm{p} \to \infty$

isothermal fluid

$$\gamma = 3 + \frac{n_p}{\sum_{j=1}^{n_p} \ln\left(\frac{r_j}{r_{max}}\right)}$$




| | | |
|---|---|---|
| $0 \leq \gamma \leq 2$ | $\gamma = 2$ | $\gamma = 3$ |
| uniform: $\gamma = 1$ | $n_p \to \infty$ | $r_{max} \to \infty$ |
| variable centre | isothermal fluid | isolated obj. |

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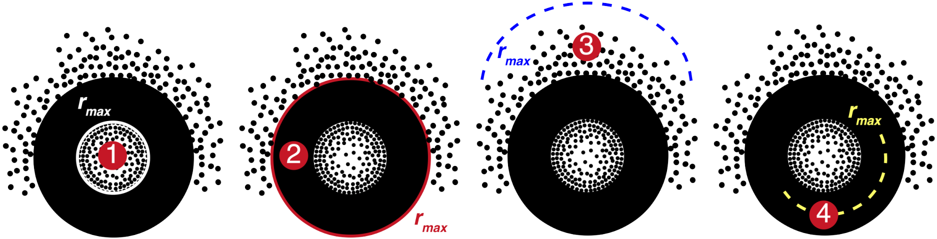
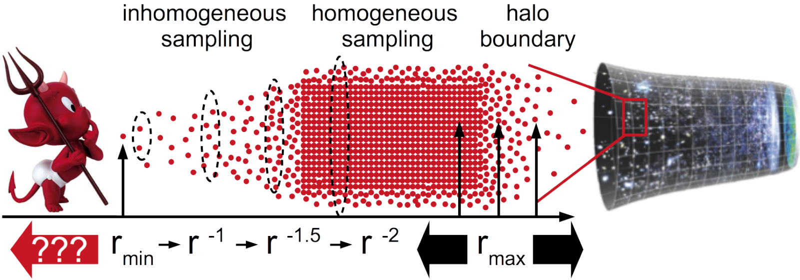



| | | | |
|---|---|---|---|
| $0 \leq \gamma \leq 2$ | $\gamma = 2$ | $\gamma = 3$ | $\gamma = 4$ |
| uniform: $\gamma = 1$ | $n_\mathrm{p} \to \infty$ | $r_\mathrm{max} \to \infty$ | $\langle \delta r \rangle \to r_\mathrm{max}$ |
| variable centre | isothermal fluid | isolated obj. | embedded obj. |

# A fundamental reason for power-law mass densities

**scale-free Newtonian gravity causes self-similar structures**

$$\phi(r) \propto r^{-(\gamma-2)} \quad \rightarrow \quad \Delta\phi(r) \propto \rho(r) \quad \rightarrow \quad \rho(r) \propto r^{-\gamma}\,, \quad \gamma > 0$$




**mass density from sample of collisionless self-gravitating particles**

Image credits: Harvey Comics; NASA/WMAP Science Team

# Comparison to standard derivation attempts

**Microcanonical ensemble for an isolated system:**

- phase space $\Omega = \left\{ \, (x_1, ..., x_{n_\mathrm{p}}; v_1, ..., v_{n_\mathrm{p}}) \,\middle|\, x_i, v_i \in \mathbb{R}^3, i = 1, ..., n_\mathrm{p} \, \right\}$
  $\rightarrow$ energy conservation implies $x$ and $v$ to carry <u>same</u> information
  $\rightarrow$ partitioning of $\Omega$ into elements arbitrary due to <u>scale-free</u> gravity
  $\rightarrow$ probability for occupying each element $p_i$: $S = -k_\mathrm{B} \sum_{i=1}^{n_\mathrm{e}} p_i \log(p_i)$
- maximum entropy $S = k_\mathrm{B} \, \log(n_\mathrm{e})$ for $p_i = 1/n_\mathrm{e}$
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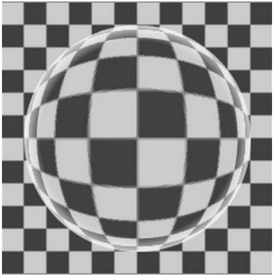
**standard statistical mechanics is NOT applicable.**

**Consequences:**

- self-similarity yields the same description on all scales
- mean "universal" mass density profile does not exist
- boundary conditions need to be specified by observed states

# Implications for applications: self-similar structure description



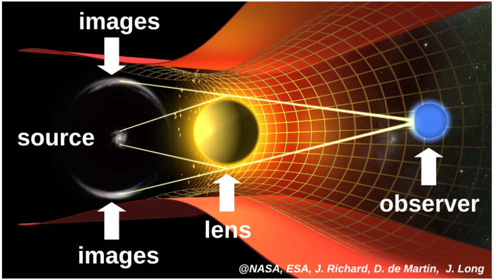
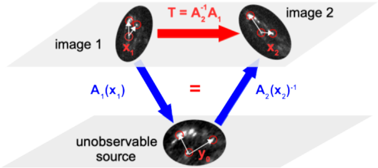


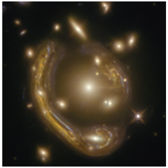

Image credits: NASA, ESA, J. Richard, D. de Martin, J. Long; S. Rodney and the FrontierSN team; T. Treu, P. Kelly and the GLASS team; J. Lotz and the Frontier Fields team; M. Postman and the CLASH team; and Z. Levay

# Implications for applications: self-similar structure description

## Strong lensing in galaxies, galaxy clusters, and plasma

- scale-free *spatial* description of structural properties:



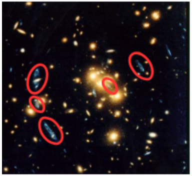
- transfer to *temporal* description of plasma properties:
  → gravitational and plasma lensing follow the same formalism
  → plasma lensing wavelength-dependent
  → infer gradients of electron number densities from time delay differences

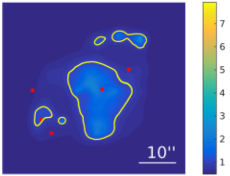
**one formalism to rule all local lens properties**

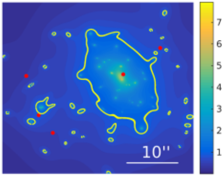
Details: Wagner, Universe 5(7), 2019

# Implications for applications: problematic mass models

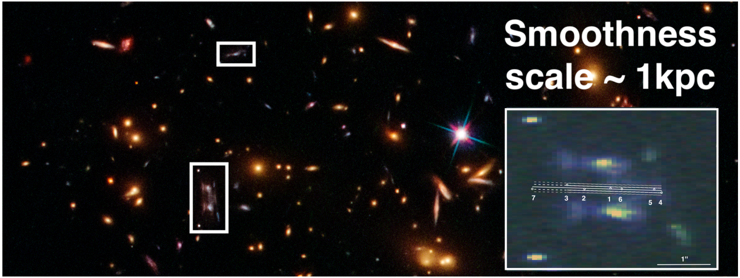
- **mass models are not unique:**






- **no mass models from sparse data:**

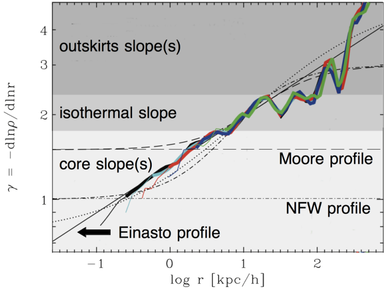


# Implications for applications: reduction to observable boundaries

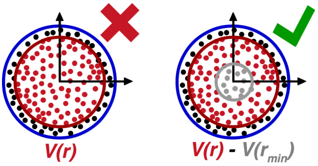
- **cusp or core at $r = 0$ in a simulated structure?**
  **(Navarro et al. MNRAS 402, 2010)**

> the number of bins or the radial range are made, this is stated explicitly in the analysis below.) These concentric shells are centred at the location of the particle identified by the SUBFIND algorithm (Springel et al. 2001) as having the minimum gravitational potential. Extensive tests show that this procedure identifies the region where the local density of the main subsystem of each halo peaks,

- **$\gamma$-value at halo centre?**




$$\rho = M/V$$




$$\rightarrow \gamma_{\mathbf{min}} = \gamma(r_{\mathbf{min}}) = 0$$

# Conclusion

- **Scale-free gravity causes self-similar structures**
  - separation of structure & dynamics
    $\longrightarrow$ **shape description indep. of cosmic assembly history**
  - effective mean field model
    $\longrightarrow$ **halo shapes are emergent structures**
  - new statistical approach
    $\longrightarrow$ **respect the nature of gravity & finite infinities**

- **Rethink data evaluation strategies**
  - mass models dependent on gauge and prerequisites
    $\longrightarrow$ **local, gauge-independent, data-based properties**
  - mass models with predictive power for hypothesis tests
    $\longrightarrow$ **clear separation of different model assumptions**

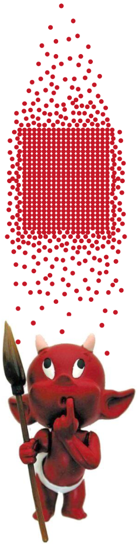


Image credits: Harvey Comics

# Thank you for your attention

**Further reading:**

- Essay (hon. mention of the GRF 2020)
  → **arXiv:2005.08975**
- Full paper (published in GREG)
  → **arXiv:2002.00960**

**More about my research:**
`thegravitygrinch.blogspot.com`

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Image credits: Harvey Comics