Trans-Planckian Censorship, Breakdown of Effective Field Theory and Emergent Cosmology

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Motivation

- Inflationary Scenario is the **current paradigm** of **early Universe cosmology.**
- Inflation is usually analyzed using an **effective field theory** (EFT) framework.
- **Fundamental conceptual problems** for an **EFT** description of a rapidly expanding universe.
- **Unitarity problem**, **Inconsistency with the 2nd law of thermodynamics.**
- We need to look beyond an EFT description of the early universe!
- Matrix Theory Cosmology: Construction of an early universe scenario based on the BFSS matrix model.

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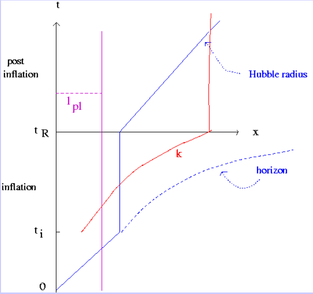
Outline

Plan

Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- → breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\boxed{\frac{a(t_R)}{a(t_i)} l_{pl} < H(t_R)^{-1}}$$

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R.B. arXiv:1911.06056

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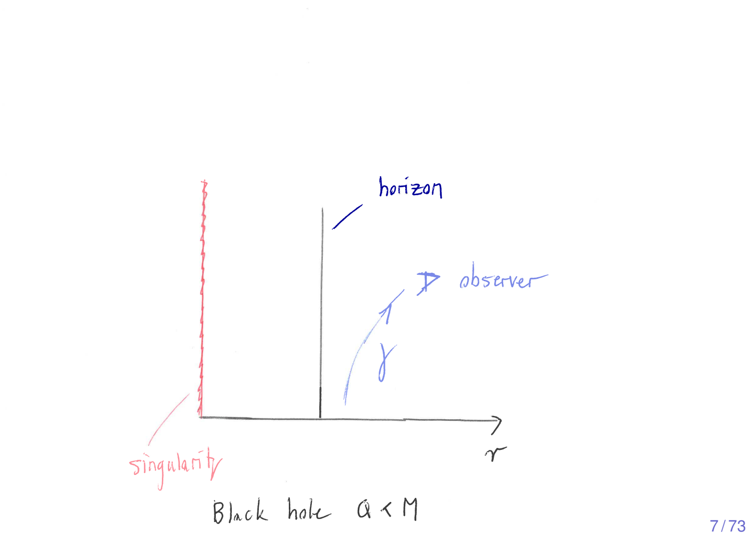

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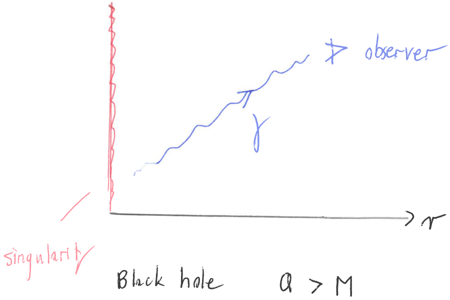

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- → Cauchy problem not well defined for observer external to black holes.
- Evolution non-unitary for external observer.
- Conjecture: ultraviolet physics → **external observer** shielded from the singularity and non-unitarity by horizon.

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Cosmological Version of the Censorship Conjecture

Translation

- Position space → momentum space.
- Singularity → trans-Planckian modes.
- Black Hole horizon → Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from the trans-Planckian modes.

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- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
- Demand: **classical region be insensitive to trans-Planckian region.**
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- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of a harmonic oscillator Hilbert space for all **comoving** wave numbers k
- IR cutoff: fixed k_{min} (comoving).
- UV cutoff: time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation → non-unitarity.
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Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy $S_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion.**
- $S_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand**: $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- $\rightarrow$ duration of inflation is bounded from above, consistent with the TCC.

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Application to EFT Description of Inflation

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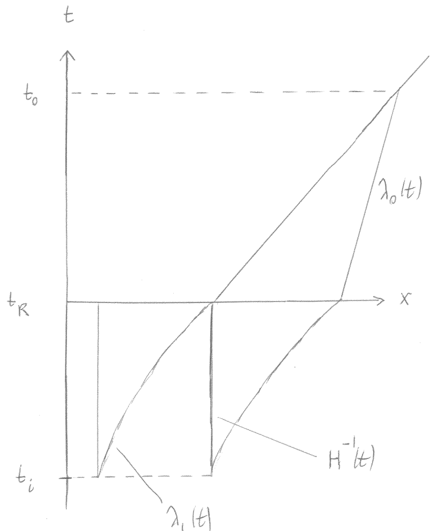

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TCC implies:

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Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

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Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

→ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

Note: Secondary tensors will be larger than the primary ones.

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Implications for Dark Energy

Dark Energy cannot be a bare cosmological constant.

Constraints on Inflation from String Theory

- There is a vast landscape of **effective field theories.**
- Any space-time dimension, and number of fields, any shape of the potential, any field range.
- Superstring theory is very **constraining**.
- Only a **small subset** of all EFTs is consistent with string theory.
- The rest lie in the swampland.

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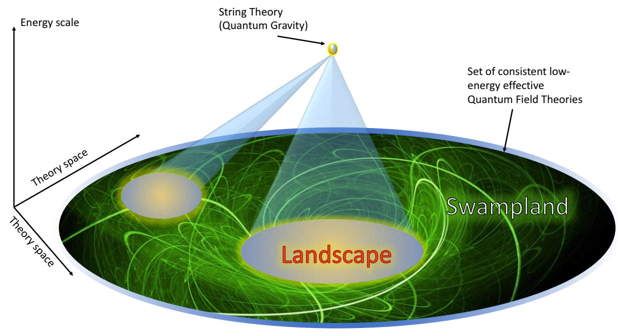
Energy scale
String Theory
(Quantum Gravity)
Set of consistent low-energy effective Quantum Field Theories
Theory space
Theory space
Swampland
Landscape

Swampland Conjectures

H. Ooguri and C. Vafa, hep-th/0605264; G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, arXiv:1806.08362; S. Garg and C. Krishnan, arXiv:1807.05193; H. Ooguri, E. Palti, G. Shiu and C. Vafa, arXiv:1810.05506.

What are conditions for habitable islands sticking out from the swamp?

- The effective field theory is only valid for $\Delta\varphi < d m_{pl}$ (field range condition).
- The potential of φ obeys (de Sitter conjecture)

$$\begin{aligned} |\frac{V'}{V}| m_{pl} &\geq c_1 \quad \text{or} \\ \frac{V''}{V} m_{pl}^2 &\leq -c_2 \end{aligned}$$

Note: d, c_1, c_2 constants of order 1.

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Consequences

- No canonical single field inflation.
- No eternal inflation.
- No bare positive Λ.
- Dark Energy is not a bare cosmological constant.
- Quintessence dark energy is constrained (L. Heisenberg et al, arXiv:1808.02877).

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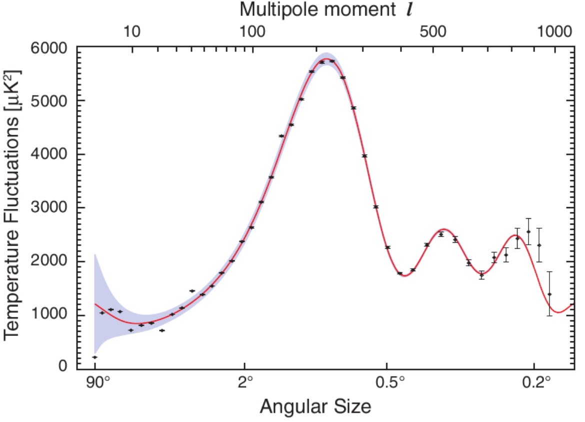
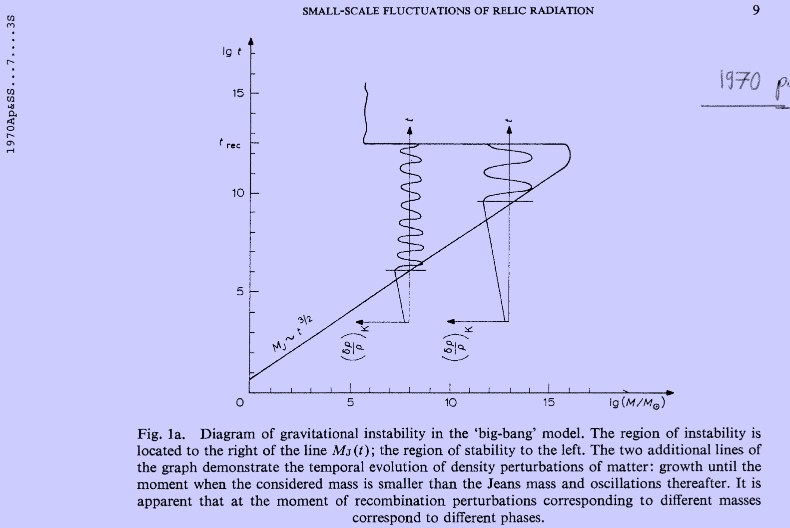
Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Early Work



SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION 9

1970Ap&SS...7....3S

1970 p

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

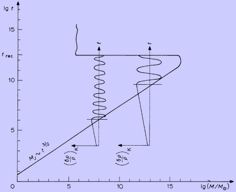
R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.

Early Work

1970Ap&SS...7....3S

1970 paper



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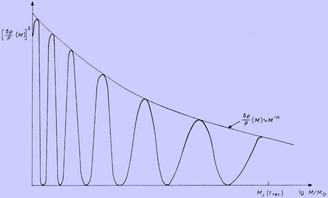

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970)

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

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- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.
- → baryon acoustic oscillations in matter power spectrum.

Hubble Radius vs. Horizon

- Horizon: Forward light cone of a point on the initial Cauchy surface.
- Horizon: region of causal contact.
- Hubble radius: $l_H(t) = H^{-1}(t)$ inverse expansion rate.
- Hubble radius: local concept, relevant for dynamics of cosmological fluctuations.
- In Standard Big Bang Cosmology: Hubble radius = horizon.
- In any theory which can provide a mechanism for the origin of structure: Hubble radius $\neq$ horizon.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the “horizon problem” of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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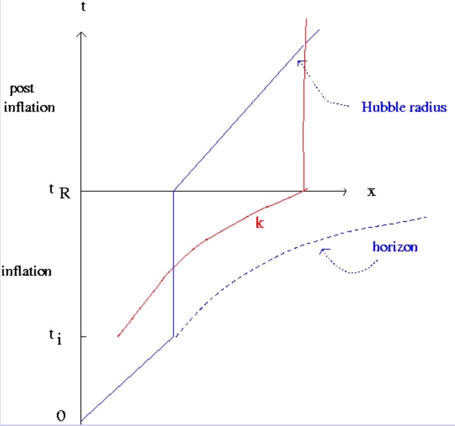
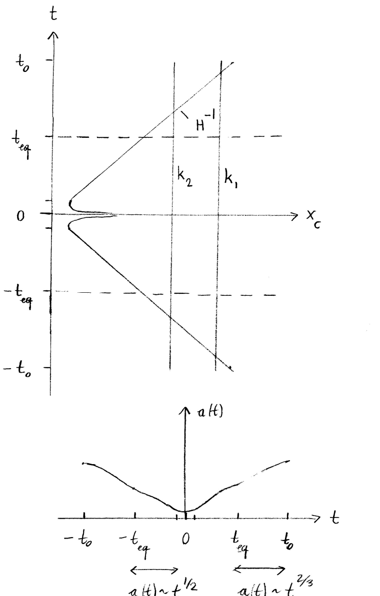
Inflation as a Solution



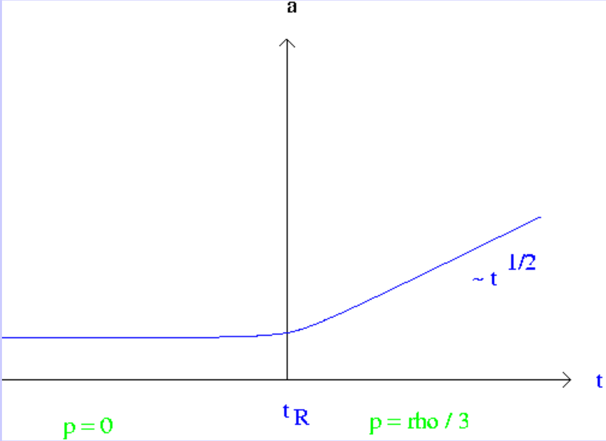
Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



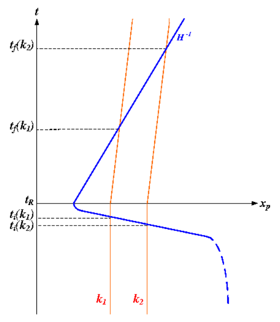
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Trans-Planckian Censorship and Cosmological Scenarios

- **Bouncing cosmologies** are consistent with the TCC provided that the energy scale at the bounce is lower than the Planck scale.
- **Emergent cosmologies** are consistent with the TCC provided that the energy scale of the emergence phase is lower than the Planck scale.
- **Inflationary cosmologies** are inconsistent with the TCC unless the energy scale of inflation is fine tuned.

All early universe scenarios require going beyond EFT.

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Obtaining Inflation

- Assumption: Space-time described by General Relativity.
- $\rightarrow$ require matter with $p < -\frac{1}{3}\rho$.
- Consider scalar field φ as matter: potential energy term has an equation of state $p = -\rho$.
- But one needs to ensure that potential energy dominates over other forms of energy!
- Require a slowly rollling scalar field:

$$\frac{V'}{V} \ll \frac{1}{m_{pl}} \, .$$

- Require rolling over large distances

$$\Delta\varphi > m_{pl} \, .$$

Challenge for Inflation

- TCC → require non-perturbative analysis of inflation.
- Constructions exist:
- G. Dvali et al: inflationary phase as a **condensate of gravitons** about Minkowski space-time (arXiv:1701.08776 [hep-th]).
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Challenge for Inflation

- TCC → require non-perturbative analysis of inflation.
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Obtaining an Ekpyrotic Bounce

J. Khoury, B. Ovrut, P. Steinhardt and N. Turok, Phys. Rev. D, 2001

- Space-time described by Einstein-Hilbert action.
- Idea: Slow contraction given by matter with equation of state $w \gg 1$.
- Obtained by assuming that matter is dominated by a scalar field φ with a negative exponential potential.
- Anisotropies diluted, creates spatial flatness
- Global attractor in initial condition space (A. Ijjas et al, arXiv:2103.00584)
- Note: **Negative exponential potentials are ubiquitous in string theory.**

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Challenge for Bouncing Cosmologies

Challenges for Ekpyrotic Cosmology:

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S-Brane and Ekpyrosis

RB and Z. Wang, arXiv:2001.00638, arXiv:2004.06437

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Adding an S-Brane to the EFT action can solve all three problems, and leads to two consistency relations for cosmological observables.

Action

Idea: At the string scale a new tower of string states becomes comparable in mass to the usual low energy degrees of freedom;
→ they must be included in the low energy effective action.

Included as an S-Brane.

$$\begin{aligned} S &= \int d^4x \sqrt{-g}\left[R + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right] \\ &- \int d^4x \kappa \delta(\tau - \tau_B)\sqrt{\gamma}\,, \end{aligned}$$

$$\kappa \equiv N\eta_s^3\,,$$

Note: The S-brane has $\rho = 0$ and $p < 0$ → can mediate the transition between contraction and expansion.

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Obtaining an Emergent Cosmology: String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
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T-Duality

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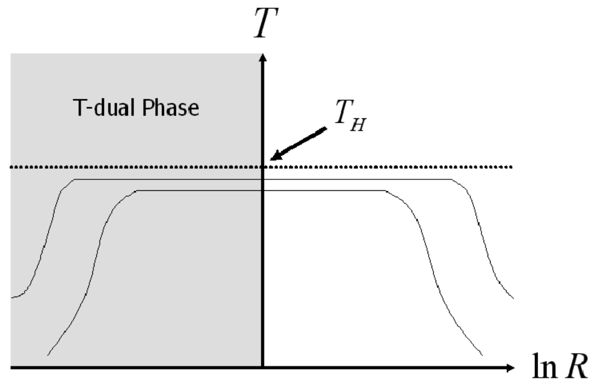
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

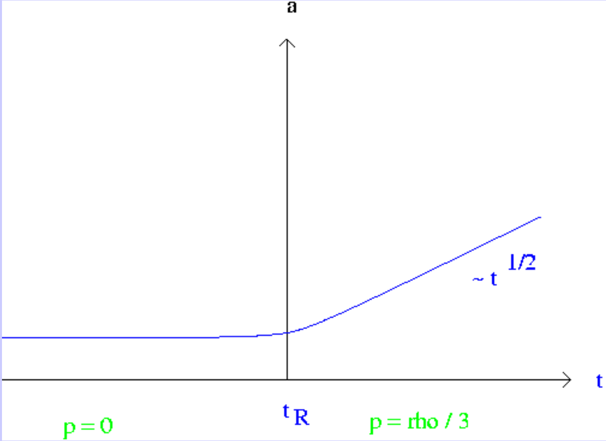
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Background for string gas cosmology



Challenge for Emergent Cosmologies

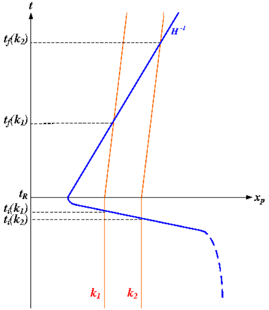
Dynamics of the emergent phase?

Require an analysis beyond EFT.

Plan

Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned} P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\ &= 8G^2 k^2 < (\delta M)^2 >_R \\ &= 8G^2 k^{-4} < (\delta\rho)^2 >_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \end{aligned}$$

Key features:

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Where do we stand?

- String Gas Cosmology appears to be an alternative to inflation for generating the structures we see in cosmological observations.
- String Gas Cosmology: nonsingular, solves the horizon problem.
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Plan

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ Hermitean matrices.
- Note: no space!
- Note: no singularities!
- Note: BFSS matrix model is a proposed non-perturbative definition of M-theory: 10 dimensional superstring theory emerges in the $N \rightarrow \infty$ limit.

BFSS Model (bosonic sector)

T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997)

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right]$$

- $X_i, i = 1, ...9$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \rightarrow \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is equivalent to the bosonic sector of the (Euclidean) IKKT matrix model.

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S = -\frac{1}{g^2}\mathrm{Tr}\Big(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\Big) ,$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Relationship between IKKT Model and Type IIB String Theory

Consider action of the Type IIB string theory in Schild gauge

$$S_{\text{Schild}} = \int d^2\sigma \alpha \left[\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\}\right) + \beta\sqrt{g}\right].$$

$$\text{Partition function}: \; Z = \int \mathcal{D}\sqrt{g}\mathcal{D}X\mathcal{D}\psi e^{-S_{\text{Schild}}}.$$

$$\begin{aligned} \text{Correspondence}: \; \{,\} \quad &\rightarrow \quad -i[,] \\ \int d^2\sigma\sqrt{g} \quad &\rightarrow \quad \text{Tr} \end{aligned}$$

Obtain grand canonical partition function of IKKT model.

Matrix Theory Cosmology

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: A_i matrices elements decay when going away from the diagonal.
- Pick $n \times n$ blocks $\tilde{X}_i(t)$ about the diagonal (n/N fixed in $N \to \infty$ limit).

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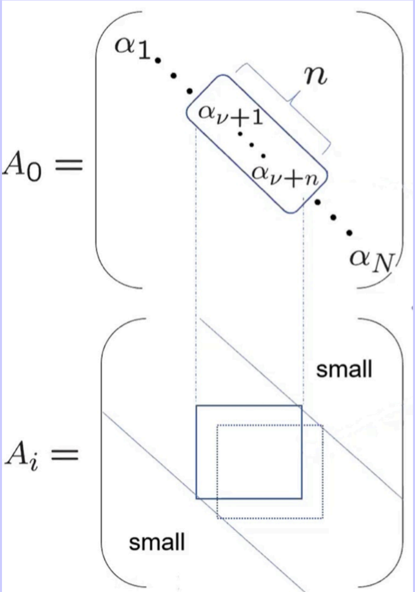
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α_1
n
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J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
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$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{X}_i)(t))^2 \right\rangle ,$$

- Space continuous in $N \to \infty$ limit.
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- Local Lorentz invariance emerges in $N \to \infty$ limit.

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$$x_i(t)^2 \equiv \left\langle \frac{1}{n} \mathrm{Tr}(\bar{X}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

- We assume that the spontaneous symmetry breaking observed in the IKKT model also holds in the BFSS model. extends to the full model (not just the bosonic sector).
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Matrix Theory Cosmology: Thermal Fluctuations

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Method:

- Consider BFSS finite temperature partition function
- Take partial derivatives with respect to T and R_i
- Obtain energy density and pressure fluctuations.

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Obtain the same results as in **String Gas Cosmology**.

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Internal energy

$$E = -\frac{d}{d\beta}\ln Z(\beta)$$

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Matsubara expansion of the action:

$$S_{BFSS} = S_0 + S_{kin} + S_{int}$$

At high temperature: S_{kin} and S_{int} suppressed compared to S_0.

To next to leading order:

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where $\chi_1 \simeq R^2\lambda^{4/3}T^{-1/2}$.

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- Derivative w.r.t. $T \rightarrow$ density fluctuations: both terms contribute.
- Derivative w.r.t. $R \rightarrow$ pressure fluctuations: only second term contributes.

Power spectrum $P(k)$ of density fluctuations: $(k = R^{-1})$

- First term dominates in the UV: Poisson spectrum.
- Second term dominated in the IR: Scale-invariant spectrum.

$$P(k) = 16\pi^2 G^2 \lambda^{4/3} N^2 \mathcal{O}(1) \sim (l_s m_{pl})^{-4}$$

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