香港科技大學
THE HONG KONG
UNIVERSITY OF SCIENCE
AND TECHNOLOGY

Decoherence of Cosmological Perturbations from Boundary Terms and the Non-Classicality of Gravity

Chon Man Sou

cmsou@connect.ust.hk

The Hong Kong University of Science and Technology

Copernicus Webinar

18 Oct 2022

Outline

1. The quantum state of cosmological perturbation during inflation
2. Decoherence: the quantum-to-classical transition
    - Framework and results in the literature
3. Boundary term of scalar curvature perturbation ζ: $\mathcal{L}_{\rm bd} = \partial_t(-2a^3 H M_p^2 e^{3\zeta})$
    - Its role on the well-defined variational principle and the selection of hypersurface
4. Decoherence by the boundary term
    - Affect the wavefunctional of ζ, much larger decoherence effect than the previous result by bulk interactions
5. Probe the non-classicality of gravity by decoherence
    - Motivated by the table-top experiments in quantum information: gravity-induced entanglement and non-Gaussianity of quantum states

The quantum state of cosmological perturbation during inflation

The two-mode squeezed state of cosmological perturbation

The inflation makes the vacuum becomes squeezed (Grishchuk & Sidorov, PRD 42, 3413, 1990) (Albrecht, Ferreira, Joyce & Prokopec, astro-ph/9303001)

$$\mathcal{H}_{\mathbf{k}} = \frac{k}{2}\underbrace{\left(a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} + a_{-\mathbf{k}}^{\dagger}a_{-\mathbf{k}} + 1\right)}_{\text{Harmonic oscillator}} - i\underset{\substack{\uparrow \\ \text{Time-dependent}}}{\lambda_k}\underbrace{\left(a_{\mathbf{k}}a_{-\mathbf{k}} - \text{h.c.}\right)}_{\text{squeezing}}$$

$$|\Psi\rangle = \exp\left[\frac{r_k}{\underset{\substack{\uparrow \\ \text{Squeezing parameter}}}{2}}\underbrace{\left(e^{-2i\Phi_k}a_{-\mathbf{k}}a_{\mathbf{k}} - \text{h.c.}\right)}_{\text{Two-mode squeeze}}\right]\exp\left[\underbrace{-i\Theta_k\left(a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}} + a_{-\mathbf{k}}^{\dagger}a_{-\mathbf{k}} + 1\right)}_{\text{Two-mode rotation}}\right]\underset{\substack{\uparrow \\ \text{Bunch-Davies vacuum}}}{|BD\rangle}$$

- The squeezing parameter is proportional to the e-folds after crossing the Hubble horizon

$$r_k \sim 2\log(aH/q)$$

Wavefunctional of the inflationary perturbation

The wavefunctional of the scalar curvature perturbation $\zeta^S_{\mathbf{k}} \propto a_{\mathbf{k}} + a^\dagger_{-\mathbf{k}}$

$$\Psi_G(\zeta) = N_G(\tau) \exp\left(-\frac{1}{2}\int_{\mathbf{p}} A_p(\tau)\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\right)$$

If there are interaction terms, the non-Gaussian wavefunctional has the form

$$\Psi(\zeta) \propto \exp\left(\sum_{n=2}^{\infty}\frac{1}{n!}\int_{\mathbf{p}_1,\dots,\mathbf{p}_n} A^{(n)}_{\mathbf{p}_1,\dots,\mathbf{p}_n}\zeta_{\mathbf{p}_1}\cdots\zeta_{\mathbf{p}_n}\right)$$

$$\int \frac{d^3p_1}{(2\pi)^3}\cdots\frac{d^3p_n}{(2\pi)^3}(2\pi)^3\delta^3\left(\sum \mathbf{p}_i\right)$$

- The Schrodinger wavefunctional can easily make some ansatzes
- Correlators are given by $\langle\zeta\dots\zeta\rangle \propto \int D\zeta\, \zeta\dots\zeta|\Psi(\zeta)|^2$

Decoherence by tracing out unobserved modes

Decoherence by tracing out unobserved modes

Interactions make observable modes (system $|\xi\rangle$) coupling with unobserved modes (environment $|\mathcal{E}\rangle$).

- Through time evolution, this creates entanglement between the system and environment

$$|\mathcal{E}(t_0)\rangle \otimes \left(\sum_i c_i|\xi_i(t_0)\rangle\right) \to \sum_i c_i|\mathcal{E}_i(t)\rangle \otimes |\xi_i(t)\rangle$$

- Since the environment acts like “measuring” the system, different outcomes should be orthogonal

$$\lim_{t\to+\infty}\langle\mathcal{E}_i(t)|\mathcal{E}_j(t)\rangle = \delta_{ij}$$

- The reduced density matrix defined by tracing out the environment becomes diagonal

$$\rho_R = \mathrm{Tr}_{\mathcal{E}}(|\Psi\rangle\langle\Psi|) \to |c_i|^2|\xi_i(t)\rangle\langle\xi_i(t)|$$

Decoherence of the cosmological perturbation

Decoherence by gravitational nonlinearities (Nelson, 1601.03734)

- Consider the simplest single-field inflation with trivial speed of sound, the leading bulk interaction term contributing to decoherence
  - Terms involving $\dot{\zeta} \propto \frac{\Pi}{a^3}$ is sub-dominated as the order of scale factor is smaller

$$\mathcal{L}_{\text{bulk},\zeta} = -\frac{M_p^2}{2}\epsilon(\epsilon+\eta)a\zeta^2\partial_i^2\zeta$$

- This introduces couplings between observed modes ($\mathbf{q}$) and unobserved modes ($\mathbf{k}$)

$$\Psi(\xi,\mathcal{E}) = \Psi_G(\xi,\mathcal{E})\exp\left(\int_{\mathbf{k},\mathbf{k}',\mathbf{q}}\mathcal{F}_{\mathbf{k},\mathbf{k}',\mathbf{q}}\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}'}\xi_{\mathbf{q}}\right)$$

Decoherence of the cosmological perturbation (Cont.)

The reduced density matrix obtained by the functional integral

$$\rho_R(\zeta,\tilde{\zeta}) = \int D\mathcal{E}\Psi(\xi,\mathcal{E})\Psi^*(\tilde{\xi},\mathcal{E})$$

The decoherence factor describing the suppression of off-diagonal terms

$$D(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}}) = \left|\frac{\rho_R(\xi_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}})}{\sqrt{\rho_R(\xi_{\mathbf{q}},\xi_{\mathbf{q}})\rho_R(\tilde{\xi}_{\mathbf{q}},\tilde{\xi}_{\mathbf{q}})}}\right| \sim e^{-\Gamma_{\rm deco}}$$

The 1-loop result (Nelson, 1601.03734)

$$\Gamma_{\rm deco} = \underbrace{\left(\frac{\overset{\text{Slow-roll suppressed}}{\overset{\downarrow}{\epsilon+\eta}}}{12}\right)^2 \overset{\text{Power spectrum}}{\overset{\downarrow}{\Delta_\zeta^2}} \left(\frac{aH}{q}\right)^3}_{\text{Sub-horizon environment}} + \underbrace{\frac{(\epsilon+\eta)^2\Delta_\zeta^2}{9\pi}\left(\frac{aH}{q}\right)^2\left[\log\frac{q}{k_{\rm min}}-\frac{19}{48}\right]}_{\text{Super-horizon environment}} + \dots$$

The Boundary term of ζ: well-defined variational principle

Consider the boundary term of ζ

We try to improve the estimation of decoherence by considering the non-slow-roll suppressed boundary term (Sou, Tran & Wang, 2207.04435)

$$\mathcal{L}_{\mathrm{bd}} = \partial_t(-2a^3 H M_p^2 e^{3\zeta})$$

- Already exists in the action of single-field inflation
- Usually neglected in the literature, as it has no effect on $\langle\zeta^n\rangle$, but it can contribute to $\langle\dot{\zeta}^n\rangle$ or $\langle\Pi^n\rangle$ (related to decoherence)
- Discuss why it must exist for making the variational principle well-defined
- Realize this term in the δϕ-gauge

Revisit the boundary term of ζ

The dynamics of ζ dominates by a total time derivative outside the horizon (Maldacena, astro-ph/0210603)

$$\mathcal{L} = \partial_t(-2a^3 H M_p^2 e^{3\zeta}) + \dots$$

- Proves ζ is constant outside the horizon to all orders
- This acts as a boundary term on the temporal boundary
- However, it does not contribute to correlators $\langle\zeta^n\rangle$, easily shown by the in-in formalism

$$\langle 0|\bar{T}e^{i\int_{t_0(1+i\epsilon)}^{t}\partial_{t'}K(\zeta_I,t')dt'}\zeta_I^n(t)Te^{-i\int_{t_0(1-i\epsilon)}^{t}\partial_{t'}K(\zeta_I,t')dt'}|0\rangle$$

No contribution from far past

$$= \langle 0|e^{iK(\zeta_I,t)}\zeta_I^n(t)e^{-iK(\zeta_I,t)}|0\rangle = \langle 0|\zeta_I^n|0\rangle$$

  - If the operator depends on $\dot{\zeta}$, the red terms become not commutative, leading to corrections

The boundary term of ζ must exist

Can the boundary term be removed by field redefinition?

- No, field redefinition removes boundary terms involving $\dot{\zeta}$ (Burrage, Ribeiro & Serry, 1103.4126) (Arroja & Tanaka, 1103.1102)

$$\partial_t[f(\zeta,\dot{\zeta})\dot{\zeta}] \implies \zeta \to \zeta + f(\zeta,\dot{\zeta}) \ , \ \mathcal{L}_{\text{bulk}} \to \mathcal{L}_{\text{bulk}} + f\frac{\delta S_2}{\delta\zeta}$$

The well-defined variational principle supports its existence

- With the ADM decomposition, the Ricci scalar can be decomposed 2 parts

Extrinsic curvature

Normal vector of hypersurface

$$R = {}^{(3)}R - \underset{\downarrow}{K}^2 + K^\mu_\nu K^\nu_\mu - 2\nabla_\mu(-K\overset{\downarrow}{n}^\mu + n^\nu\nabla_\nu n^\mu)$$

- Only the first part R_{ADM} is applied to do variation, whereas the covariance derivative is dropped

The boundary term of ζ must exist (Cont.)

Dropping the covariance derivative is equivalent to adding the Gibbons-Hawking-York (GHY) boundary term

$$M_p^2 \int_{\mathcal{M}} d^4x\sqrt{-g}\nabla_\mu(-Kn^\mu + n^\nu\nabla_\nu n^\mu) = -M_p^2 \int_{\partial\mathcal{M}} d^3y\sqrt{h}K$$

- If the induced metric h^{ij} on the boundary is fixed, the GHY term is the only option to make the variation well-defined (Chakraborty, 1607.05986)
- This uniquely determines the action's dominated part outside the horizon

$$\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R_{\rm ADM} + \mathcal{L}_\phi\right) = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R + \mathcal{L}_\phi\right) + S_{\rm GHY}$$

$$= \int dtd^3x\partial_t(-2a^3HM_p^2e^{3\zeta}) + \ldots$$

The boundary term and variation in the δϕ-gauge

Different choices of gauges also lead to different hypersurfaces (Rigopoulos, 1104.0292) (Prokopec & Weenink, 1209.1701)

The $t_\zeta = \mathrm{const}$ hypersurface is parameterized as follows in the δϕ-gauge, implying the value of δϕ fixed on the boundary

$$t_\varphi(\mathbf{y}) = t + \frac{\zeta(t,\mathbf{y})}{H}\ ,\ \mathbf{x} = \mathbf{y} \implies \delta\phi|_{\partial\mathcal{M}} = -\frac{\dot\phi}{H}\zeta(t,\mathbf{y}) + \mathcal{O}(\zeta^2)$$

- This requires an additional boundary term for varying the inflaton's action

$$\delta S_\phi = -\int d^3x dt_\varphi\ EOM_\phi\ \delta\phi + \int_{\partial\mathcal{M}} d^3y\sqrt{h}\left(\dot\phi + \frac{\partial\mathcal{L}_{\mathrm{bd},\phi}}{\partial\dot\phi}\right)\delta\phi|_{\partial\mathcal{M}}$$

- Adding this additional term to S_{GHY} reproduces the non-slow-roll suppressed boundary term in the δϕ-gauge

Decoherence by the boundary term

Wavefunctional with the boundary term

The wavefunctional of ζ acquires a non-Gaussian phase from the boundary term

$$H_{\mathrm{bd}}(\zeta,t)=\partial_t K(\zeta,t)$$

$$U(t,t_0)=\exp\left[-iK(\zeta,t)\right]U_0(t,t_0)$$

$$\langle\zeta|\Psi(t)\rangle=\exp\left[-iK(\zeta,t)\right]\Psi_G(\zeta,t)$$

$$\Psi(\zeta)=N_G\exp\left(-\frac{1}{2}\int\frac{d^3p}{(2\pi)^3}A_p\zeta_{\mathbf{p}}\zeta_{-\mathbf{p}}\right)\exp\left(\frac{2\mathcal{F}}{27}\int d^3x\ e^{3\zeta}\right)$$

$\uparrow$

$iS_{\mathrm{bd}}(\zeta)$

where the non-slow-roll suppressed

$$\mathcal{F}=-27ia^3HM_p^2$$

Decoherence factor

Separating the environment and system by comoving momenta

$$\zeta(\mathbf{x},t)=\int\frac{d^3k}{(2\pi)^3}\mathcal{E}_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}}+\int\frac{d^3q}{(2\pi)^3}\xi_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{x}}$$

- $\mathbf{q}$ and $\mathbf{k}$ for observed and unobserved modes respectively

For different field configurations $\zeta=\mathcal{E}+\xi\ ,\ \tilde{\zeta}=\mathcal{E}+\tilde{\xi}$

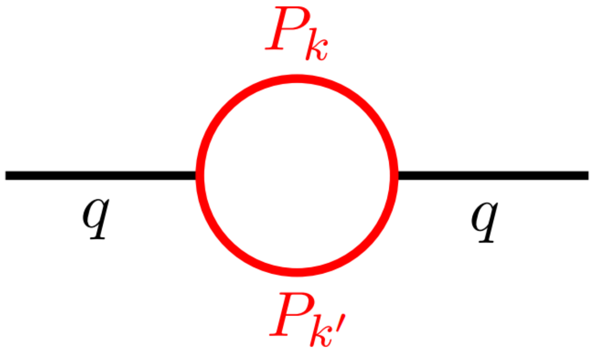
- the ratio of off- and diagonal terms of $\rho_R(\xi,\tilde{\xi})$ is the expectation value of the action difference $\Delta S_{\rm bd}(\zeta,\tilde{\zeta})=S_{\rm bd}(\mathcal{E}+\xi)-S_{\rm bd}(\mathcal{E}+\tilde{\xi})$

$$D(\xi,\tilde{\xi})=\left|\frac{\rho_R(\xi,\tilde{\xi})}{\sqrt{\rho_R(\xi,\xi)\rho_R(\tilde{\xi},\tilde{\xi})}}\right|=\left|\int D\mathcal{E}\,|\Psi_G(\mathcal{E})|^2\,e^{i\Delta S_{\rm bd}(\zeta,\tilde{\zeta})}\right|$$

One loop result

Using the Saddle-point approximation or expanding it order by order

$$D(\xi,\tilde{\xi}) \approx \exp\left[-|\mathcal{F}|^2\int_{\mathbf{k},\mathbf{k}',\mathbf{q}} \overset{\langle\mathcal{E}_{\mathbf{k}}\mathcal{E}_{\mathbf{k}}\rangle'}{\overset{\downarrow}{P_k}} P_{k'}|\Delta\xi_{\mathbf{q}}|^2\right]$$



- Leading order is a 1 loop joining 2 environment modes and 1 system mode
- Compare with the $\mathcal{O}(\epsilon^2)$ bulk interactions, the non-slow-roll suppressed $\mathcal{F}$ has larger decoherence by a few orders of magnitude
- Related to $\langle\dot{\zeta}\dot{\zeta}\rangle$ by the cubic boundary term

$$\langle 0|e^{-\int d^3x\frac{\mathcal{F}}{3}\zeta_I^3}\dot{\zeta}_I(\mathbf{x})\dot{\zeta}_I(\mathbf{y})e^{\int d^3x\frac{\mathcal{F}}{3}\zeta_I^3}|0\rangle = \langle\dot{\zeta}\dot{\zeta}\rangle_{\rm free}+\left(\frac{1}{2\epsilon M_p^2a^3}\right)^2|\mathcal{F}|^2\langle 0|\zeta_I^2(\mathbf{x})\zeta_I^2(\mathbf{y})|0\rangle$$

Probe the non-classical gravity by comparing the decoherence in semi-classical gravity

Proposals to test the non-classicality of gravity

Recently in quantum information, some proposals of table-top experiments to test quantum nature of gravity in labs

- Using gravity mediated entanglement (Marletto & Vedral, 1707.06036) (Bose et. al, 1707.06050)
  - If there is gravitational entanglement, gravity cannot be local operations and classical communication (LOCC)
- Using the non-Gaussianity by gravitational interaction of a Bose-Einstein condensate (Howl, Vedral, Naik, Christodoulou, Rovelli & Iyer, 2004.01189)
  - If the gravitational potential is classical (not an operator), the interaction Hamiltonian is at most quadratic

$$\hat{H}_{\mathrm{QG}} = \frac{m}{2}\int d^3x : \hat{\Psi}^\dagger\hat{\Psi}\underset{\text{operator}}{\hat{\Phi}}_{\mathrm{grav}} : \,, \quad \hat{H}_{\mathrm{CG}} = m\int d^3x \hat{\Psi}^\dagger\hat{\Psi}\underset{\text{C-number function}}{\Phi}_{\mathrm{grav}}$$

Cosmological perturbation in the semi-classical gravity

Consider the Schrödinger-Newton semi-classical gravity

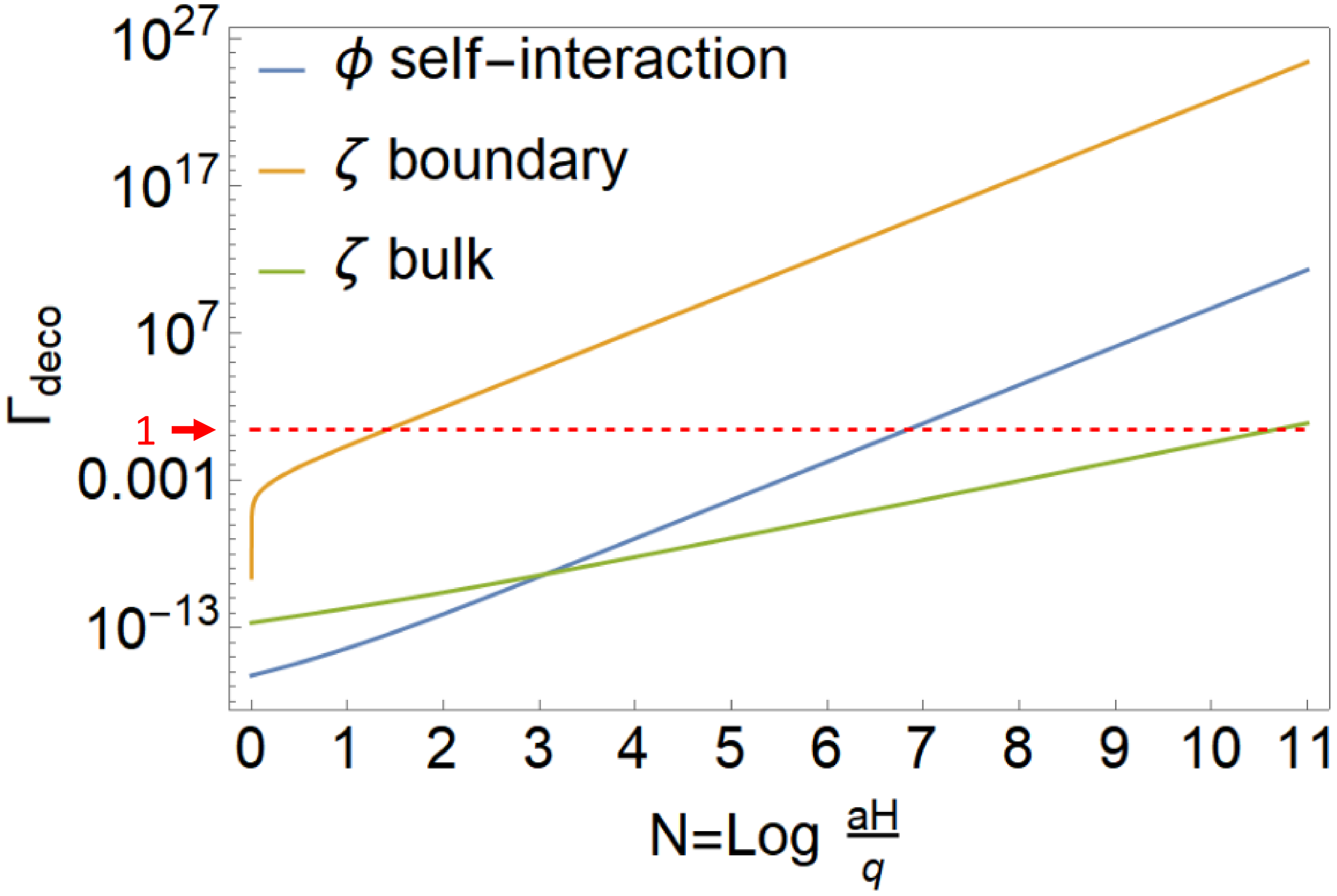
- The curvature perturbation ζ cannot describe the quantum degree of freedom, whereas the quantum variable in the δϕ-gauge still works well
  - The ζ-gauge is not well-defined, and the boundary term of its constant-time hypersurface is also not defined
- The ADM constraints becomes semi-classical equations

$$N-1=\left\langle\frac{\dot{\phi}}{2H}\varphi\right\rangle=0\ ,\ \ N^i=\left\langle\frac{\dot{\phi}^2}{2H^2}\partial_i\partial^{-2}\frac{d}{dt}\left(-\frac{H}{\dot{\phi}}\varphi\right)\right\rangle=0$$

  - The survived leading interaction is the $\mathcal{O}(\epsilon^3)$ inflaton’s self-interaction

$$\mathcal{L}_{\text{int}}\approx-\frac{a^3}{6}V'''\varphi^3$$

Compare the decoherence in different theories



$$\left|\frac{\rho_R(\xi,\tilde{\xi})}{\sqrt{\rho_R(\xi,\xi)\rho_R(\tilde{\xi},\tilde{\xi})}}\right| \sim e^{-\Gamma_{\text{deco}}}$$

$$\mathcal{L}_{\text{int},\phi} = -\frac{a^3}{6}V'''\varphi^3$$

$$\mathcal{L}_{\text{bd},\zeta} = \partial_t\left(-2a^3HM_p^2e^{3\zeta}\right)$$

$$\mathcal{L}_{\text{bulk},\zeta} = -\frac{M_p^2}{2}\epsilon(\epsilon+\eta)a\zeta^2\partial_i^2\zeta$$

The degree of decoherence and the cosmological Bell test

The Bell violation $\langle\hat{\mathcal{B}}\rangle > 2$ depends on the squeezing (r_k) and decoherence ($\Gamma_{\rm deco}$) (Martin & Vennin, 1706.05001)

- The cosmological Bell test relies on the pseudo spin on the entangled ($\mathbf{k}, -\mathbf{k}$) modes, e.g. the "x component" of the pseudo spin operator

$$\hat{S}_x(\mathbf{k}) = \int dq_{\mathbf{k}}(|q_{\mathbf{k}}\rangle\langle q_{\mathbf{k}}| - |-q_{\mathbf{k}}\rangle\langle -q_{\mathbf{k}}|) \qquad \hat{q}_{\mathbf{k}} \propto a_{\mathbf{k}} + a^{\dagger}_{\mathbf{k}}$$

- The squeezing parameter grows after crossing the horizon $r_k \sim 2\log(aH/q)$
- The decoherence $\Gamma_{\rm deco} \sim \mathcal{O}(1)$ at N e-folds after the horizon crossing
  - N=1-2 for the boundary term of ζ (be avoided to choose the hypersurface in δϕ-gauge, i.e. using density fluctuation δρ)
  - N=6-7 for the inflaton's self-interaction; N=10-11 for the bulk term of ζ

Conclusion

1. The boundary term, naturally exists in the single-field inflation, can contribute decoherene effect by trancing out unobserved modes
2. Such a boundary term is related to the well-defined variational principle and the choice of hypersurface
3. Possible realization of the table-top experiments probing the non-classicality of gravity: compare the decoherence effect and the consequence in the cosmological Bell test