Lensing of Gravitational Waves as a probe of dark matter halos

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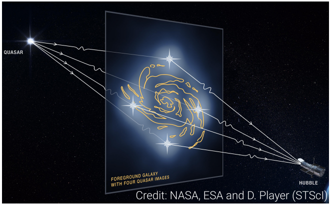
Outline

- Introduction on Gravitational lensing
- Lensing signals: methods and lens models
- Forecasts for gravitational wave (GW) detectors
- Applications to Dark Matter (DM) models
- Future directions: weak lensing
- Conclusions and outlooks

Gravitational lensing

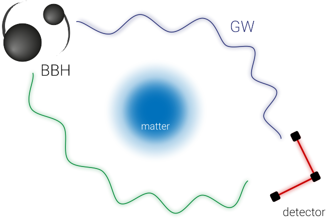
Lensing of EM waves

- Established probe at very different scales
- Powerful insights on matter distribution



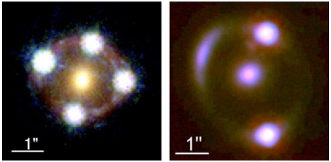
Lensing of GWs can soon become reality

- Sensitivity to $1/r$ instead of $1/r^2$
- No absorption: probe of dense DM regions

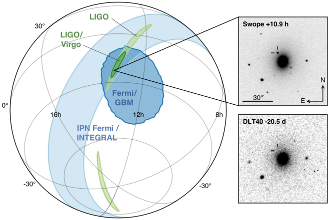


EM vs GW lensing signals

- Poor sky localization for GWs: images are not spatially resolved
- However, we can measure *time delays* and *relative magnification* of the images
- GW sources: coherent emission. *Frequency-dependent effects* (wave-diffraction effects) from the lens are detectable



[NASA - ESA - Hubble - S.H. Suyu et al.]



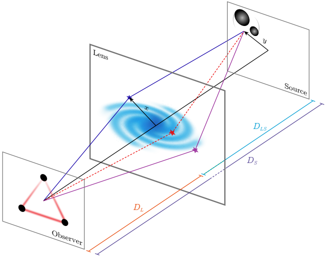
[B.P. Abbott et al., '17]

Lensing of GWs

- $g_{\mu\nu} = g_{\mu\nu}^{\mathrm{FRW}} + h_{\mu\nu}, \quad \Box h_{\mu\nu} = 0$
- Amplification factor:

$$F(w) \equiv h^L(f)/h^0(f)$$

$$= \frac{w}{2\pi i} \int \mathrm{d}^2 x \; e^{iw\phi(\boldsymbol{x},\boldsymbol{y})}$$



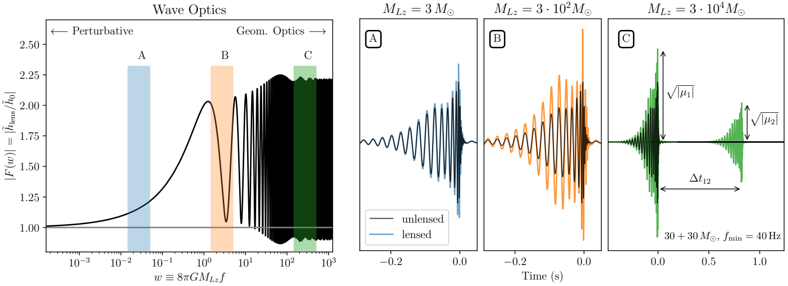
[Schneider, Gravitational Lenses '92]

- $\boldsymbol{x}$, $\boldsymbol{y}$ dimensionless distances in units of the Einstein's radius $R_{\mathrm{E}} \sim \sqrt{4GM_{Lz}D_L}$
- Fermat potential: $\phi(\boldsymbol{x},\boldsymbol{y}) \propto$ time delay

$$\phi(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2}|\boldsymbol{x}-\boldsymbol{y}|^2 - \psi(\boldsymbol{x})$$

- Lensing potential: $\psi(\boldsymbol{x})$, sourced by the projected mass distribution
- Dimensionless frequency: $w \equiv 8\pi G M_{Lz} f \simeq \frac{M_{Lz}}{10^7\, M_\odot} \cdot \frac{f}{\mathrm{mHz}}$, $M_{Lz} \equiv$ redshifted lens mass

Lensing regimes



- $w \ll 1$: wave does not feel the lens $F(w) \simeq 1 + Aw^\alpha$
- Intermediate regime $w \sim 1$: no analytic expansion for $F(w)$.
- Geometric optics (GO) $w \gg 1$: stationary-phase approx. (lens equation)

$$\boldsymbol{\nabla}_{\boldsymbol{x}}\phi(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{x} - \boldsymbol{y} - \boldsymbol{\nabla}_{\boldsymbol{x}}\psi(\boldsymbol{x}) = 0$$

solutions: images J with magnification μ_J, time delay ϕ_J, beyond GO correction Δ_J and Morse phase $n_J = 0, 1/2, 1$

$$F(w) \simeq \sum_J |\mu_J|^{1/2}\left(1 + \frac{i\Delta_J}{w}\right) e^{iw\phi_J - i\pi n_J}$$

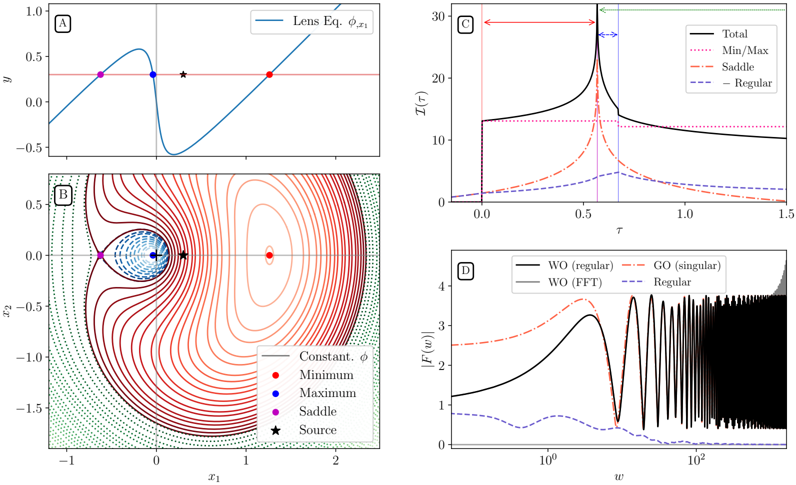
Computing $F(w)$: contour method

- Standard numerical integration is troublesome: highly oscillatory integral
- We implemented a "contour method": [A. Ulmer, J. Goodman, '94]
  evaluate the time-domain signal $\mathcal{I}(\tau)$, then use inverse Fourier transform

$$\mathcal{I}(\tau) = \int \mathrm{d}w\, e^{-iw\tau}\,\frac{F(w)}{(-iw)} = \int \frac{\mathrm{d}w}{2\pi} \int \mathrm{d}^2x\, e^{iw(\phi(\boldsymbol{x},\boldsymbol{y})-\tau)}$$
$$= \int \mathrm{d}^2x\,\delta\left(\phi(\boldsymbol{x},\boldsymbol{y})-\tau\right) = \sum_k \oint_{\gamma_k} \frac{\mathrm{d}s}{|\boldsymbol{\nabla}\phi(\boldsymbol{x}(\tau,s),\boldsymbol{y})|}$$

- Reduced to a 1D integral over contours γ_k of constant $\phi(\boldsymbol{x},\boldsymbol{y}) = \tau$.
  The sum $\sum_k$ is over stationary points (images), where the contours end.

Computing $F(w)$: contour method



Computing $F(w)$: complex-deformation method

Alternative method: exploit Cauchy's theorem [J. Feldbrugge, U.-L. Pen, N. Turok, '19+]

- Simplified example: 1D integral

$$F(w) = \int_{-\infty}^{+\infty} \mathrm{d}x\, g(x) e^{iwf(x)}$$

- Idea: deform the integration contour into the complex plane

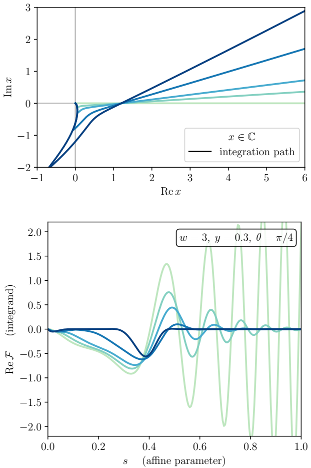
- The flow equation evolves a point $x \in \mathbb{R}$ to $z(\lambda) \in \mathbb{C}$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} z(\lambda) = i \frac{\partial f^*}{\partial z^*}$$

- The flow select the optimal integration contour

$$i \frac{\mathrm{d}}{\mathrm{d}\lambda} f = i \frac{\partial f}{\partial z} \frac{\mathrm{d}z}{\mathrm{d}\lambda} = - \left| \frac{\partial f}{\partial z} \right|^2$$

- Elegant, but numerically slower that the contour method



Lens models and lensing features

- We focus on spherically-symmetric density profiles modelling DM halos
- DM halos roughly described by the *Singular Isothermal Sphere* (SIS)

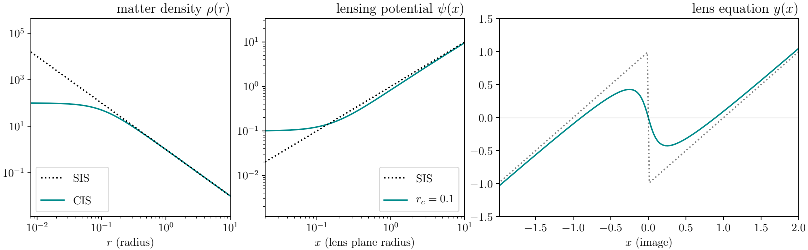
$$\rho = \frac{\sigma_v^2}{2\pi G r^2}\,, \quad \psi(x) = x$$

- We study deformations from the SIS, motivated by DM models.
  Can we distinguish different lens features?
- The presence of a core modelled by the *Cored Isothermal Sphere* (CIS)

$$\rho = \rho_0 \frac{r_c^2}{r^2 + r_c^2}\,, \quad \psi(x) = \sqrt{x^2 + x_c^2} + x_c \log\left(\frac{2x_c}{x_c + \sqrt{x^2 + x_c^2}}\right)$$

- Specific DM models (e.g. Fuzzy DM, Self-interacting DM) predict cores
  [L. Hiu+, '16; M. Kaplinghat, S. Tulin, H-B. Yu, '15]

Lens models and lensing features

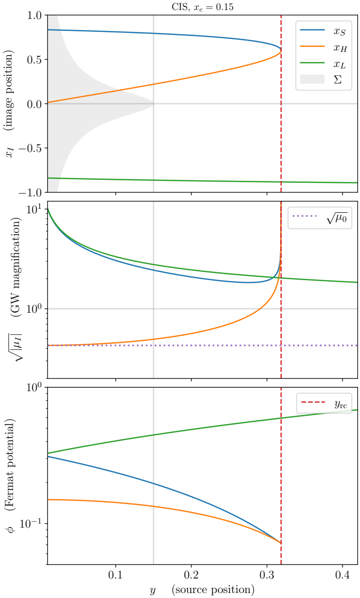


- SIS: two GO images in strong lensing (SL), one image in weak lensing (WL). Center of the lens is cuspy
- CIS with core radius r_c: three images in SL, one in WL
  $\Longrightarrow$ New central image from the core

Lens models and lensing features

Let us consider the GO regime first: $F(w)$ is described by the images only

- Central image has a finite minimum magnification $\mu_H > \mu_0 = 4x_c^2/\left(1-2x_c\right)^2$
- Potential for GW observations: for $x_c \neq 0$ an additional GW signal can be detected
- Third image allows to extract info about the core
- Time delays between images can be of order of days $\Delta T \simeq (1\,\mathrm{day})\left(M_{\mathrm{vir}}/10^{11}M_\odot\right)^{4/3}\Delta\phi$



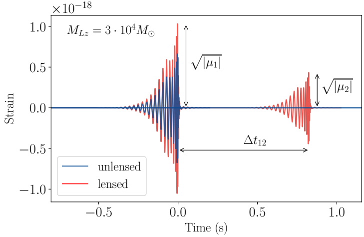
Lensing of GW: results and forecasts

- Lensing features are investigated in current detectors

  [L.Dai+, ’20; LIGO, Virgo, ’21]

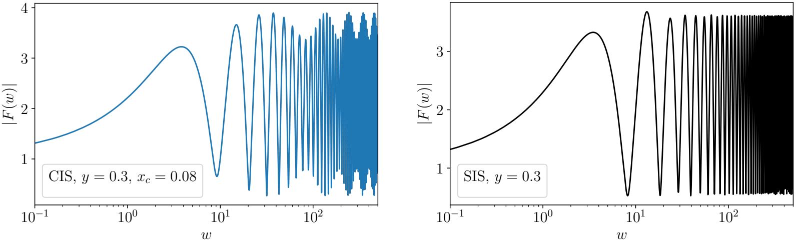
- Previous analyses mostly focused on singular lenses

  [R. Takahashi+ ’03; P. Cremonese+, ’21; H. G. Choi+, ’21; ...]



- We focus on distinguishing between different lens features: cored vs. singular DM distribution
  Evaluate sensitivities on lens parameters (core size x_c for LISA)

Probing r_c with GWs

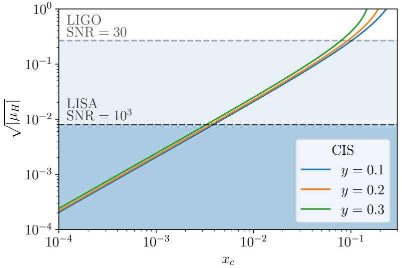


- Third image appears in frequency domain as a modulation of the signal
- This feature is not present for SIS

Probing r_c with GWs

- Central image: $\mu_H > \mu_0 = 4x_c^2/(1-2x_c)^2$
- Can be detected for large enough SNR

$$\tilde{h}_L(f) = F(f)\tilde{h}_0(f)$$

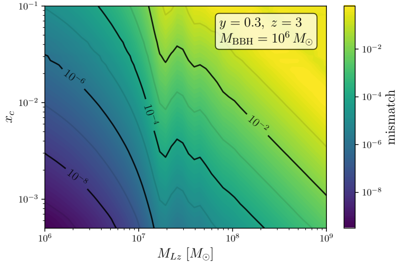


- More quantitative: mismatch $\mathcal{M}$ with SIS lens

$$\mathcal{M} \equiv 1 - \frac{(h_1|h_2)}{\sqrt{(h_1|h_1)(h_2|h_2)}}$$

- If $\mathcal{M} > \mathrm{SNR}^2$, we expect to distinguish the two signals

$$(h|g) \equiv 4\,\mathrm{Re}\int_0^{+\infty} \frac{\tilde{h}(f)^*\tilde{g}(f)}{S_n(f)}\mathrm{d}f$$



Lensing of GW: addressing degeneracies

- We perform a *Fisher matrix forecast* on source and lens parameters for LISA

  [M. Vallisneri, '07]

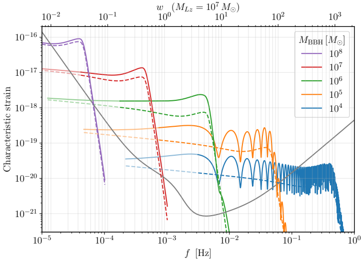
  $$\theta_i = \{D_L,\ \phi_0,\ M_{Lz},\ y,\ x_c\}$$
  $$\mathcal{F}_{ij} \equiv (\partial_i h_L | \partial_j h_L), \quad \partial_i \equiv \partial/\partial\theta_i$$
  $$\sigma_i^2 = (\mathcal{F}^{-1})_{ii}, \quad \text{marginalized posteriors}$$

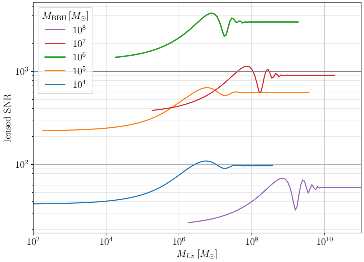
- GW sources with equal mass, non spinning and fixed orientation, using **PhenomD** waveforms

  [S. Husa+, '15, S. Khan, '15]

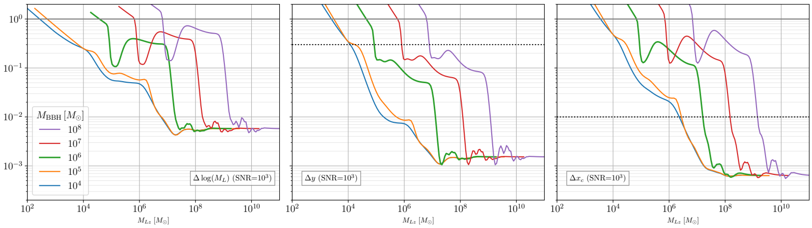
- Focus on strong-lensing regime (multiple images)

- Fiducial lens parameters:
  $M_{Lz} = 10^7\, M_\odot$, $y = 0.3$, $x_c = 10^{-2}$



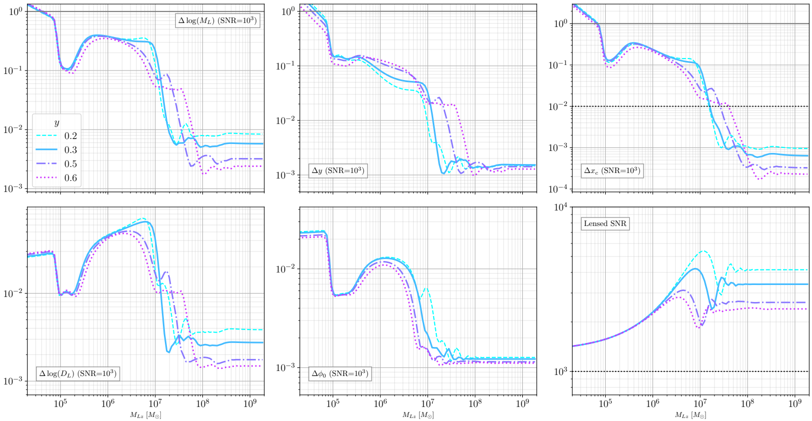


Results and forecasts: dependence on source mass



- High M_{Lz} dominated by GO regime (results saturate). Low M_{Lz} gives no lensing (lens parameters cannot be reconstructed)
- SNR typically peaked at the Innermost Stable Circular Orbit (ISCO), with $f_{\rm ISCO} \sim 1/M_{\rm BBH}$
- Lighter BBH give better constraints at small M_{Lz}: easier to have larger w at ISCO
  $w_{\rm ISCO} \sim M_{Lz}/M_{\rm BBH}$

Results and forecasts: dependence on y



Larger y improves the constraints

- M_{Lz} is probed in GO through the time delays, that increase for large y
- x_c: magnification of the third image increases with y

Application: Ultra-light DM (preliminary)

- Forecast results on lens parameters have implications for constraints on DM models

- Models of Ultra-light DM predict cores with a minimum size and mass

$$r_{1/2} \geq 0.33\,\mathrm{kpc}\frac{10^9\,M_\odot}{M_c}\left(\frac{10^{-22}\mathrm{eV}}{m_\phi}\right)^2$$

$$M_{\mathrm{vir}}^{\mathrm{min}} \simeq 1.5\cdot 10^7\,M_\odot\left(\frac{10^{-22}\mathrm{eV}}{m_\phi}\right)^{3/2}$$

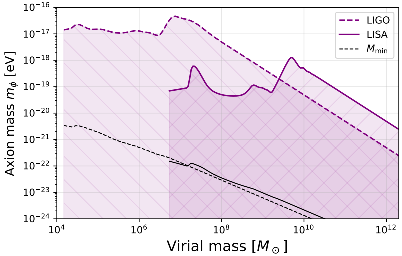
[L. Hiu+, ’16]

- A non detection of core features or of small M_{Lz} would imply bounds on DM mass, assuming halos can be described by the CIS lens

- $x_c \sim 10^{-3}$

- $R_\mathrm{E} \simeq \sqrt{GM_{Lz}d_\mathrm{eff}} \simeq 1\,\mathrm{kpc}$ for $d_\mathrm{eff} \sim 1\,\mathrm{Gpc}$

- $r_c \simeq x_c R_\mathrm{E} \sim x_c\,\mathrm{kpc}$



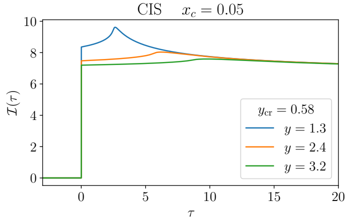
Future directions: Weak Lensing regime

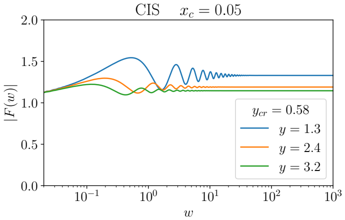
Weak lensing regime: large y and only one image

with S. Savastano, H. Villarrubia Rojo and M. Zumalacárregui

- Lensing effects decrease as $\sim 1/y$
- Instead, the lensing probability increases as $p \propto y^2$
- At intermediate w we have "remnant" oscillations, associated to the would-be images
- For $y \gg 1$, we obtain

$$\mathcal{I}(\tau) \simeq \theta(\tau)\left[2\pi + 4\frac{\mathrm{d}}{\mathrm{d}\tau}\int_0^{\pi/2}\mathrm{d}\varphi\,\psi(x)\right]$$





Conclusions and outlooks

- GW lensing is a very promising tool for DM characterization
- We implemented fast, accurate and flexible methods to evaluate lensing signals in the WO regime
- Lensed LISA ans LIGO events could test DM-halos features, such as the presence of cores

Future directions

- Investigation of the weak-lensing regime (single image): WO effects give more information about the lens model
- Include more GW parameters (e.g. LIGO/LISA antenna pattern, spins ecc..) to provide more robust lensing forecasts
- Study of more complicated lens models and configurations

Backup slides

Lensing of GWs: definitions

- Amplification factor:

$$F(f) \equiv \frac{-if}{d_{\rm eff}} \int \mathrm{d}^2\xi \, \exp\left[2\pi i f \, t_d(\boldsymbol{\xi}, \boldsymbol{\eta})\right] \,, \qquad d_{\rm eff} \equiv \frac{D_L D_{LS}}{(1+z_L) D_S} \,.$$

- Time delay:

$$t_d(\boldsymbol{\xi}, \boldsymbol{\eta}) = \frac{1}{2 d_{\rm eff}} \left| \boldsymbol{\xi} - \frac{D_L}{D_S} \boldsymbol{\eta} \right|^2 - \hat{\psi}(\boldsymbol{\xi}) + \hat{\phi}_m(\boldsymbol{\eta}) \,.$$

- Lensing potential:

$$\nabla^2_{\boldsymbol{\xi}} \hat{\psi}(\boldsymbol{\xi}) = 8\pi G \, \Sigma(\boldsymbol{\xi}) \,.$$

- Rescaling: make quantities dimensionless by rescaling by ξ_0

$$\phi(\boldsymbol{x}, \boldsymbol{y}) = \frac{d_{\rm eff}}{\xi_0^2} t_d(\boldsymbol{x}, \boldsymbol{y}) \,, \qquad \psi(\boldsymbol{x}, \boldsymbol{y}) = \frac{(1+z_L) d_{\rm eff}}{\xi_0^2} \hat{\psi}(\boldsymbol{x}, \boldsymbol{y}) \,.$$

$$\boldsymbol{x} \equiv \frac{\boldsymbol{\xi}}{\xi_0} \,, \quad \boldsymbol{y} \equiv \frac{\boldsymbol{\eta}}{\eta_0} \,.$$

- Dimensionless frequency:

$$w \equiv 8\pi G M_{Lz} f \,, \qquad M_{Lz} = \frac{\xi_0^2}{2 d_{\rm eff}}$$

Results and forecasts: correlations

- For high M_{Lz}, precision on lens parameters saturates
- In this limit, we are sensitive to linear combinations of the parameters: their accuracy increases and the parameters become almost degenerate
- Precision could drastically improve if some parameters are independently measured (e.g. EM counterparts)