Signatures of Primordial Gravitational Waves on the Large-Scale Structure of the Universe

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Copernicus Webinar

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Based on

- P. Bari, A. Ricciardone, N. Bartolo, D. Bertacca, and S. Matarrese, "Signatures of primordial gravitational waves on the large-scale structure of the universe," Phys. Rev. Lett., vol. 129, p. 091 301, 9 Aug. 2022. doi: 10 . 1103 / PhysRevLett . 129 . 091301.
- P. Bari, D. Bertacca, N. Bartolo, A. Ricciardone, S. Giardiello, and S. Matarrese, "An Analytical Study of the Primordial Gravitational-Wave-Induced Contribution to the Large-Scale Structure of the Universe," Sep. 2022. arXiv: 2209.05329 [astro-ph.CO] (submitted to JCAP).

Introduction

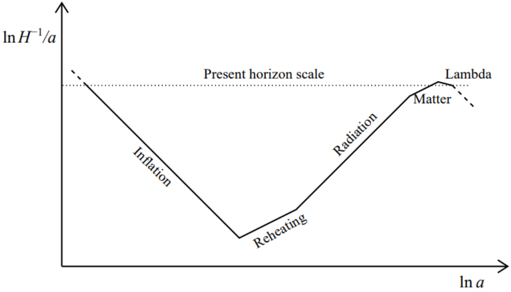


Figure: Inflation generates inhomogeneities which can explain the observed large-scale structures (LSS). (Liddle and Leach 2003)

→ almost scale-invariant, Gaussian, adiabatic power spectrum of scalar perturbations
→ Planck 2018 Results.X. ✓

Cosmological Perturbations

- Quantum Fluctuations during inflation: scalar, vector, tensor.
- Metric decomposition:

$$ds^2 = a^2(\eta)\left[-\left(1+2\sum_{r=1}^{\infty}\frac{\psi^{(r)}}{r!}\right)d\eta^2 + \sum_{r=1}^{\infty}\frac{\omega_i^{(r)}}{r!}d\eta dx^i \right.$$
$$\left. + \left\{\left(1+2\sum_{r=1}^{\infty}\frac{\phi^{(r)}}{r!}\right)\delta_{ij} + + \sum_{r=1}^{\infty}\frac{\chi_{ij}^{(r)}}{r!}\right\}dx^i dx^j\right]$$

- $\omega_i^{(r)} = \partial_i \omega^{(r)||} + \omega_i^{(r)\perp}$; $\partial^i \omega_i^{(r)\perp} = 0$
- $\chi_{ij}^{(r)} = D_{ij}\chi^{(r)||} + \partial_i \chi_j^{(r)\perp} + \partial_j \chi_i^{(r)\perp} + \chi_{ij}^{(r)T}$;
  $\partial^i \chi_i^{(r)\perp} = \partial^i \chi_{ij}^{(r)T} = \chi_i^{(r)i} = 0$

Vacuum Fluctuations to Large-Scale Structures

$$\underbrace{\delta R_{\mu\nu} - \frac{1}{2}\delta(g_{\mu\nu}R)}_{\text{perturbation in space-time}} = 8\Pi G \underbrace{\delta T_{\mu\nu}}_{\text{perturbation in energy-momentum tensor}}$$

$$\rho = \overline{\rho} + \sum_{r=1}^{\infty} \frac{\delta^{(r)}\rho}{r!}$$

Evolution equation for linear density contrast ($\delta^{(1)} = \delta^{(1)}\rho/\overline{\rho}$):

$$\delta^{(1)''} + \mathcal{H}\delta^{(1)'} - 4\pi G a^2 \overline{\rho}_m \delta^{(1)} = 0.$$

Vacuum Fluctuations to Large-Scale Structures

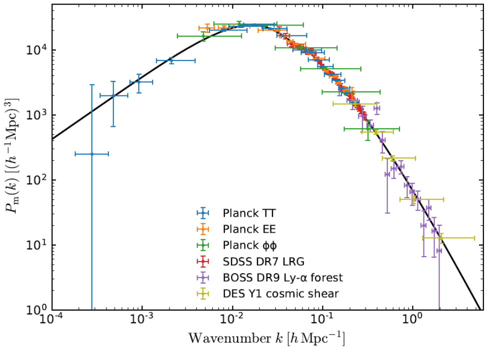


Figure: The linear matter power spectrum (at $z = 0$) inferred from different cosmological probes.(*Planck* 2018 results, Aghanim *et. al.*)

$$P_m(k) = \frac{4}{25}\frac{k^4}{\Omega_m^2 H_0^4} \overbrace{T^2(k)}^{\text{Transfer function}} \underbrace{D_+^2(a)}_{\text{Growth factor}} P_{\mathcal{R}}(k)$$

Gravitational Waves

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + \underbrace{\chi_{ij}}_{\text{GWs}})dx^i dx^j\right]$$

- Transverse, traceless, two degrees of freedom: $\chi_{\sigma=\times,+}$

$$\chi_\sigma = A_\sigma(\boldsymbol{k})\mathcal{T}(k\eta)$$

- Power spectrum defined as $(\Delta_T^2 = 4\Delta_\sigma^2)$

$$\langle A_{\sigma_1}(\boldsymbol{k}_1)A_{\sigma_2}(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2)\delta_{\sigma_1\sigma_2}\frac{2\pi^2}{k_1^3}\Delta_\sigma^2(k_1)$$

Gravitational Waves

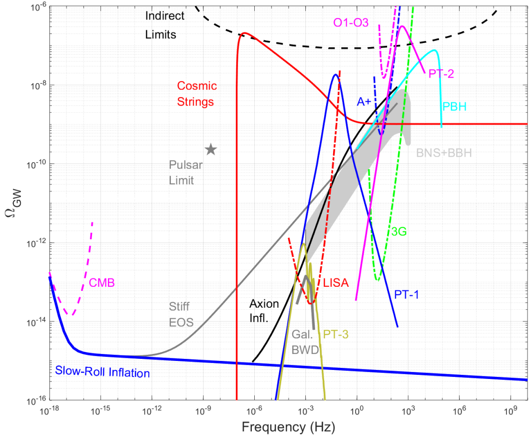


Figure: Landscape of gravitational wave cosmology (Caldwell *et. al. 2022)*

Effect of GWs on LSS: Tensor Fossils

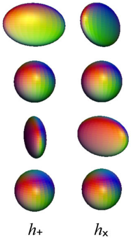


Figure: Distortions to an otherwise statistically isotropic two-point correlation function for a single Fourier mode, aimed in the $\hat{z}$ direction of the distortion pattern (Jeong *et al 2012)*

- No longer interact or very weakly interact during late-time cosmic evolution.
- Only observational effect: their imprint on the primordial curvature perturbation (can be spotted by LSS observations)

$$\langle \delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle_{h^p(\boldsymbol{K})} = (2\pi)^3 \delta^D_{\boldsymbol{k}_{1,2},\boldsymbol{K}} \underbrace{f^p_h(k_1, k_2, K)}_{\text{coupling amplitude}} \times \epsilon^p_{ab}(\boldsymbol{K}) \hat{k}^a_1 \hat{k}^b_2 h^{p*}(\boldsymbol{K})$$

Effect of GWs on LSS: Clustering of Galaxies in LSS and Cosmic Shear

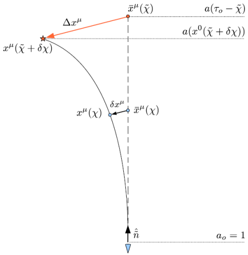


Figure: Photon geodesic is perturbed by GWs, leading to volume distortion, *z* perturbation, and magnification (Jeong *et al 2012)*

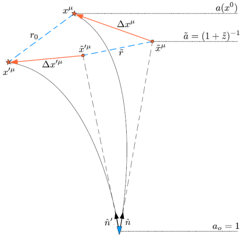


Figure: Geodesic deflection by GWs leads to cosmic shear (Schmidt & Jeong 2012)

Can we find another imprint?

Motivation

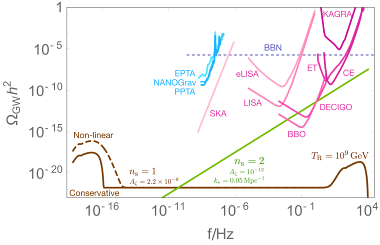
- On small and intermediate scales, second-order metric perturbations come into play
  $\longrightarrow$ mode-mixing happens
  $\longrightarrow$ perturbations can be composed of other kinds of perturbations of lower orders.

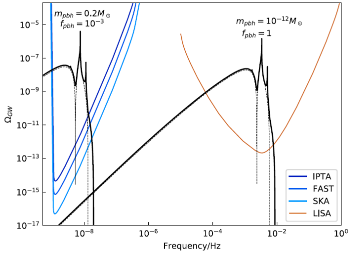
- Example: scalar induced gravitational waves are thoroughly investigated over the years (Tomita 1967, Matarrese *et. al* 1998...for a review, see Domènech 2021).

$$\chi_{ij}^{(2)''} + 2\mathcal{H}\chi_{ij}^{(2)'} - \nabla^2\chi_{ij}^{(2)} = -4\Pi_{ij}^{lm}\mathcal{S}_{lm}$$

$\mathcal{S}_{lm} \rightarrow$ quadratic in scalar perturbations

Scalar Induced Gravitational Waves





- Inevitably produced upon re-entry of curvature perturbations.
- Used to probe PBHs
- In figure: SIGW density parameter with the sensitivity curves of different observations (Above) Kohri & Terada 2018, (Below) Yuan *et. al.* 2019.

Set-up

We study the opposite effect of the scalar induced GWs, generating matter-density perturbations from the primordial tensor modes.

- Collisionless cold dark matter plus dark energy for the matter content
- No pressure gradient $\rightarrow$ co-moving, synchronous, and time-orthogonal gauge

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij}(\boldsymbol{x}, \eta)dx^i dx^j]$$

where $\gamma_{ij} = \delta_{ij} + \chi_{ij}^{(1)T} + \frac{1}{2}\left(-2\phi^{(2)}\delta_{ij} + D_{ij}\chi^{(2)\parallel}\right)$.

Second order density contrast from first order tensor modes

Evolution equation of the second order density contrast
$(\delta = \delta^{(1)} + \delta^{(2)}/2)$

$$\delta^{(2)''} + \mathcal{H}\delta^{(2)'} - 4\pi G a^2 \overline{\rho}_m \delta^{(2)} = \frac{1}{2}\chi'^{ij}\chi'_{ij} \,.$$

(Matarrese *et. al* 1998, Bruni *et. al* 2014)

⇒ no super-horizon contribution!

⇒ essentially a linear effect, sourced by the fluctuation in gravitational radiation only on subhorizon scales (See Wu *et al* 2007 for an analogous effect sourced by EM radiation):

$$\rho_{GW} = \frac{1}{32\pi G a^2}\langle\chi'^{ij}\chi'_{ij}\rangle$$

Focusing on matter domination only, we have, for the Fourier space density contrast,

$$\delta^{(2)}(\boldsymbol{k},\eta) = \sum_{\sigma,\sigma'} \int \frac{d^3\boldsymbol{k}_2}{(2\pi)^3} A_{\sigma'}(\boldsymbol{k}_2) A_\sigma(\boldsymbol{k}-\boldsymbol{k_2}) \times \epsilon_{ij}^{\sigma'}(\hat{\boldsymbol{k}}_2)\epsilon^{\sigma ij}(\widehat{\boldsymbol{k}-\boldsymbol{k_2}}) \times \left[\frac{\eta^2}{10}\int_0^\eta \frac{d\tilde{\eta}}{\tilde{\eta}} \left(\frac{3j_1(k_2\tilde{\eta})}{k_2\tilde{\eta}}\right)' \left(\frac{3j_1(|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta})}{|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta}}\right)' - \frac{1}{10\eta^3}\int_0^\eta d\tilde{\eta}\,\tilde{\eta}^4 \left(\frac{3j_1(k_2\tilde{\eta})}{k_2\tilde{\eta}}\right)' \left(\frac{3j_1(|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta})}{|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta}}\right)'\right].$$

Source GWs

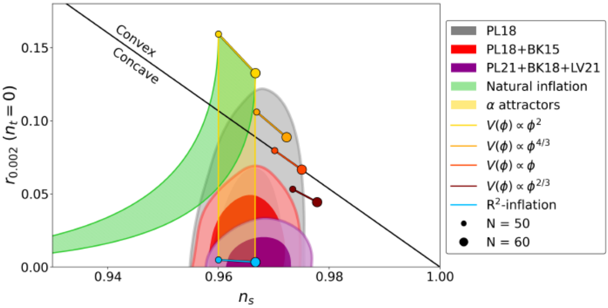


Figure: ($r_{0.002} - n_s$) plane constraining $r < 0.028$ (Galloni *et al 2022)*

On CMB scales, $r (= A_T/A_S)$ is tightly constrained.
CMB-S4, LiteBIRD $\rightarrow$ more accuracy ($r < 0.001$).

Models of GW sources: Blue-tilted spectrum

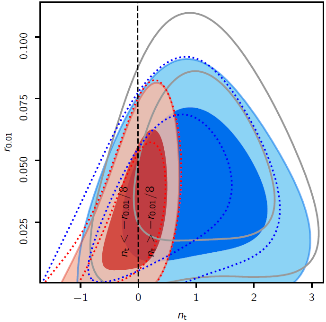


Figure: $-0.76 < n_T < 0.52$, relaxing the inflationary consistency relation, *Planck* 2018 Results. X.

- Spectator scalar fields

$$\chi_{ij}{}'' + 2\mathcal{H}\chi_{ij}{}' - \nabla^2\chi_{ij}$$

$$= -2\frac{c_s^2}{M_p^2}\hat{\Pi}_{ij}^{lm}\partial_l\delta\sigma\partial_m\delta\sigma$$

- Inflaton coupled to a gauge field

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\phi}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Power spectrum

$$\Delta_T^2(k) = A_T\left(\frac{k}{k_*}\right)^{n_T}; n_T = 0.32$$

Models of GW sources: Gaussian-bump spectrum

- Axion inflation: axion-SU(2) in the spectator sector

$$\mathcal{L} = \mathcal{L}_{inf} + \frac{1}{2}(\partial_\mu \chi)^2 - \mu^4 \left[1 + \cos\left(\chi/f\right)\right] - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\lambda}{4f} \chi F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

- Power spectrum

$$\Delta^2_T(k) = A_T e^{-\frac{1}{2\sigma^2} \ln^2\left(\frac{k}{k_p}\right)}$$

Models of GW sources: Monochromatic source

- A useful choice for analytical results
- Power spectrum

$$\Delta_T^2(k) = A_T \delta_D\left(\ln \frac{k}{k_*}\right)$$

- Result

$$\Delta_\delta^2(k) = 4 \times 10^{-5} \left(k\,\eta_0\right)^4 A_T^2 \times \left(\frac{8k_*^2}{k^2} + \frac{k^6}{16k_*^6} - \frac{k^4}{2k_*^4} + 3\frac{k^2}{k_*^2} - 8\right) \Theta(2k_* - k)$$

Comparison with linear power spectrum $P(k) = (2\pi^2/k^3)\Delta^2(k)$

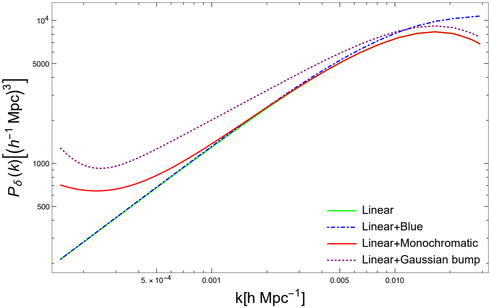


Figure: *Impact of different GW power spectra on the matter power-spectrum: i) blue-tilted* $\left(A_T = 1.26 \times 10^{-10}, n_T = 0.32, k_* = k_{\rm CMB} = 0.01{\rm Mpc}^{-1}\right)$*, ii) monochromatic* $\left(A_T = 10^{-5}, k_* = 0.008{\rm Mpc}^{-1}\right)$*, iii) Gaussian bump* $\left(A_T = 10^{-5}, \sigma = 2, k_p = 0.04{\rm Mpc}^{-1}\right)$.

Extension to radiation domination

We consider (pressure gradient ✓ one less constraint to be imposed)

- co-moving (with CDM) and time-orthogonal gauge

$$ds^2 = a^2(\eta)\left[-\left(1+2\psi\right)d\eta^2 + \gamma_{ij}\left(\boldsymbol{x},\eta\right)dx^i dx^j\right],$$

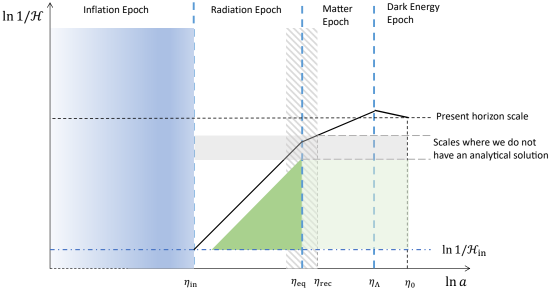
where

$$\psi = \frac{\psi^{(2)}}{2},$$
$$\gamma_{ij} = \delta_{ij} + \chi_{ij}^{(1)} - \phi^{(2)}\delta_{ij} + \frac{1}{2}D_{ij}\chi^{(2)||}$$

.

- Conservation equation → synchronous gauge regained!

Different epochs analyzed in this work



Radiation epoch

- deep radiation domination
- towards matter-radiation equality

Wang & Zhang 2019, Döring *et al 2022 discussed density perturbations from tensor-tensor couplings in RD regime, but considered only the first stage.*

Deep RD: what the linear δ does

- The potential is determined by δ_r.
- δ_m is determined by the gravitational potential, but do not themselves influence it.

$$\delta_m \sim \ln k\eta$$

Deep RD: our induced δ

- Initially $\delta^{(2)}\rho_{\rm m} \ll \delta^{(2)}\rho_{\rm r}$, hence $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} (\leftarrow \delta^{(2)}\rho_{\rm r})$
- We follow the modes with $k\eta \gg 1$.
- Defining

$$F \equiv \frac{\delta^{(2)}\rho_m}{\delta^{(2)}\rho_r} = \frac{\bar{\rho}_m}{\bar{\rho}_r}\frac{\delta_m^{(2)}}{\delta_r^{(2)}},$$

  $\delta^{(2)}\rho_m$ becomes of the same order as $\delta^{(2)}\rho_r$ at $F \sim 1$,and replaces it in the EEs at $F \sim \alpha$ at a particular time $y_\alpha (y = a/a_{\rm eq} = \bar{\rho}_m/\bar{\rho}_r)$, when

$$y_\alpha \delta_m^{(2)}(y_\alpha) = \alpha \delta_r^{(2)}(y_\alpha).$$

$\Rightarrow$ a behaviour similar to its linear counterpart!

Towards matter-radiation equality: what the linear δ does

- $\delta\rho_m \gg \delta\rho_r$ although $\overline{\rho}_m \ll \overline{\rho}_r$.
- δ_m follows Meszaros equation

$$\frac{d^2\delta_{\rm m}}{dy^2} + \frac{2+3y}{2y(y+1)}\frac{d\delta_{\rm m}}{dy} - \frac{3}{2y(y+1)}\delta_{\rm m} = 0.$$

  - $D_1(y) = y + \frac{2}{3}$,
  - $D_2(y) = D_1(y) \ln \frac{\sqrt{1+y}+1}{\sqrt{1+y}-1} - 2\sqrt{1+y}$

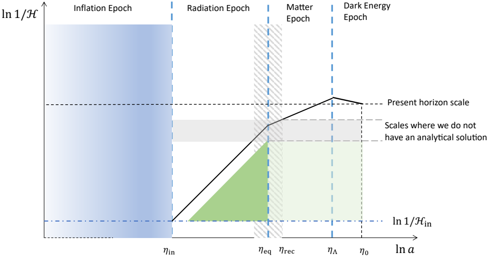
Towards matter-radiation equality: our induced δ

- $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu} (\leftarrow \delta^{(2)}\rho_{\rm m})$
- Meszaros' equation with a source term quadratic in GWs.

$$\frac{d^2\delta_{\rm m}^{(2)}}{dy^2} + \frac{2+3y}{2y(y+1)}\frac{d\delta_{\rm m}^{(2)}}{dy} - \frac{3}{2y(y+1)}\delta_{\rm m}^{(2)} = \frac{1}{2}\frac{d\chi^{ij}}{dy}\frac{d\chi_{ij}}{dy}.$$

$\rightarrow$ source term looks exactly same as what we had in MD.

Extension to CDM and Dark Energy domination



Comoving, synchronous, time-orthogonal gauge

$$\iota^2\left(1+\iota^3\right)\frac{d^2\delta_{\rm m}^{(2)}}{d\iota^2}+\frac{3}{2}\iota\left(1+2\iota^3\right)\frac{d\delta_{\rm m}^{(2)}}{d\iota}-\frac{3}{2}\delta_{\rm m}^{(2)}=\frac{\iota^2(1+\iota^3)}{2}\frac{d\chi^{ij}}{d\iota}\frac{d\chi_{ij}}{d\iota},$$

where $\iota=(\Omega_{\Lambda 0}/\Omega_{\rm m0})^{1/3}a$.

Summary and discussion

- We analyzed a new effect of 'tensor-induced scalar modes' on the present day matter power spectrum, and found that a large GW power spectrum can leave a significant imprint.
- Our effect mimics the linear perturbation in the subhorizon limit, whereas completely vanishes in the superhorizon, leaving no CMB temperature anisotropy on large scales.
- We extend the study to radiation domination and the present regime for a complete picture. We also intend to explore its high intrinsic non-Gaussianity in the future, and employ numerical methods like a Boltzmann code to quantify the effect.

Thank You