Ultra-light dark matter:

the light and fuzzy side of dark matter

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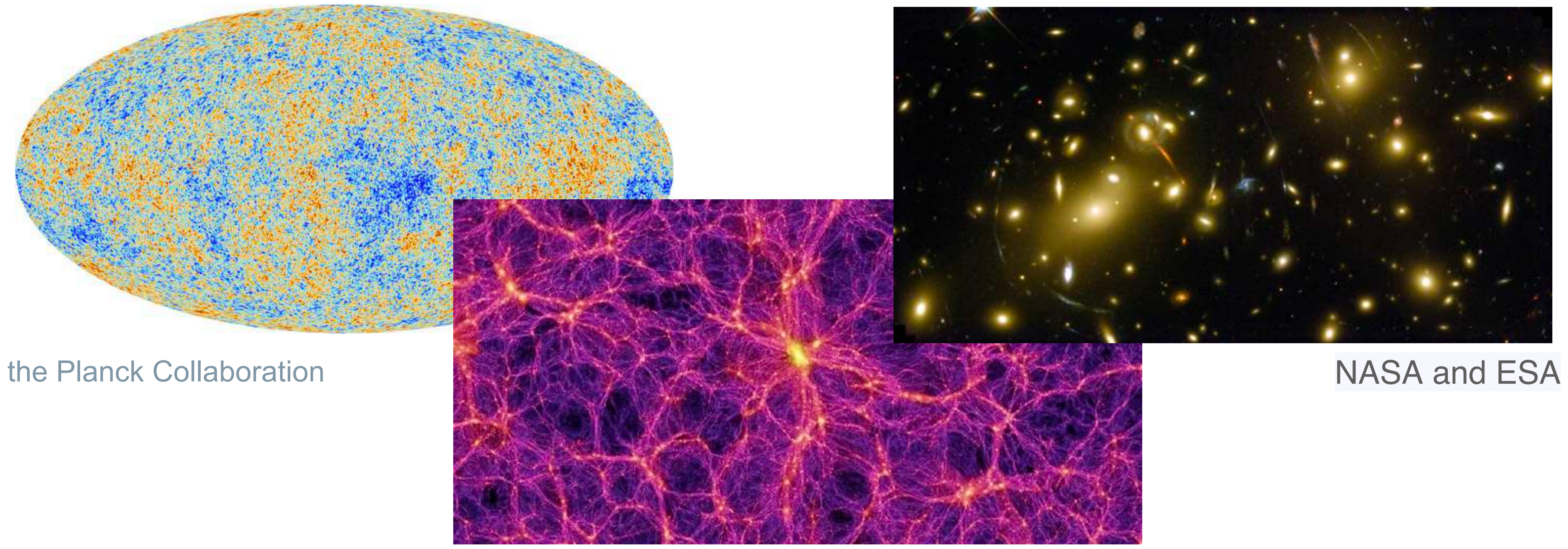
Evidences for dark matter

We can observe its effects in

Galaxies

NASA and ESA

CMB+LSS

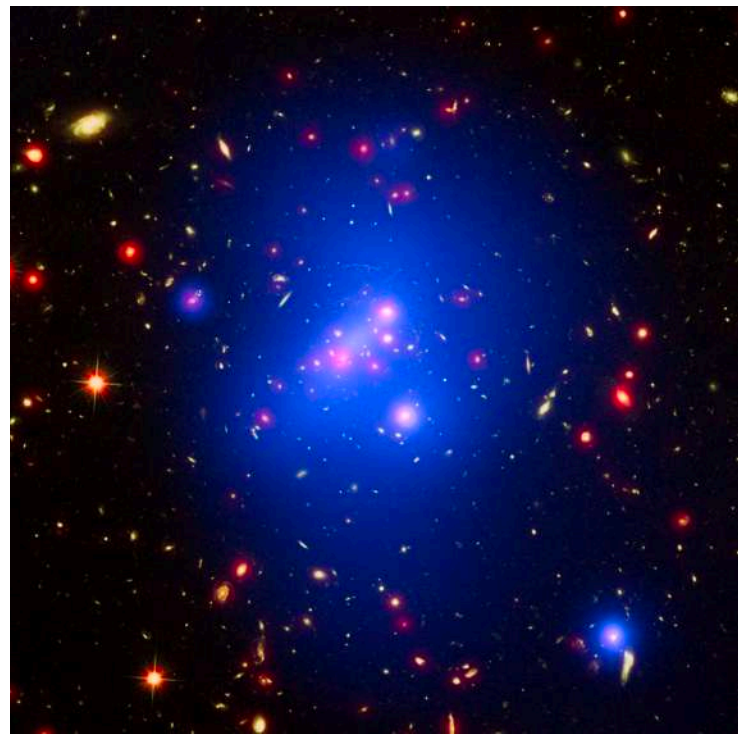


ESA and the Planck Collaboration

NASA and ESA

Springel & others / Virgo Consortium

Clusters



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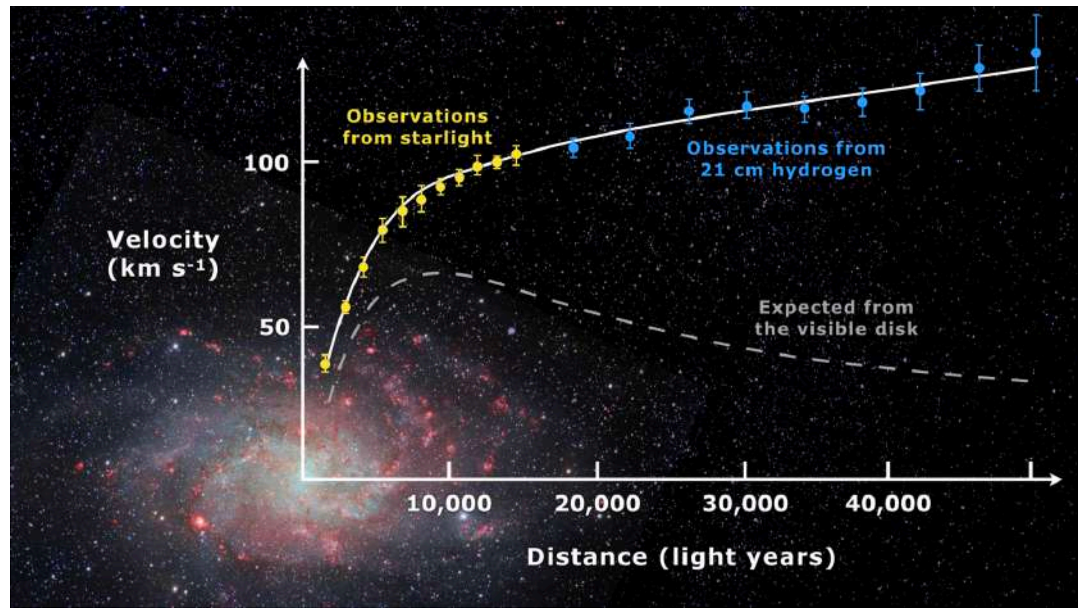
Huge amount of evidence
From all scales

Evidences for dark matter

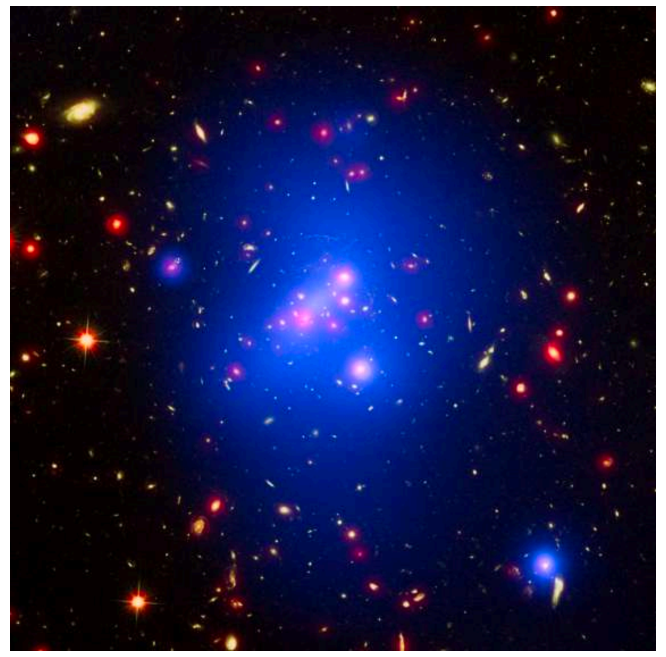
Galaxy rotation curves



Credit: Mario De Leo

- Mass fraction
- Distribution

Clusters



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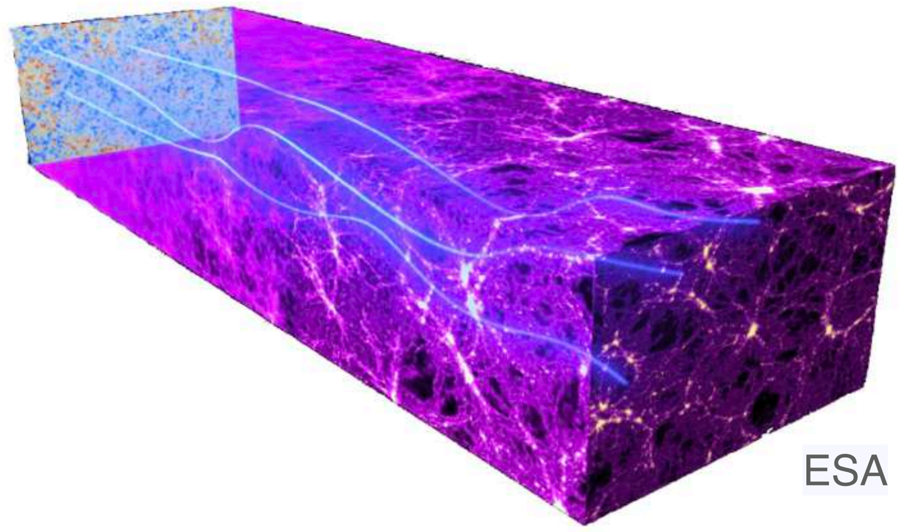
- Mass fraction
- Distribution

Lensing



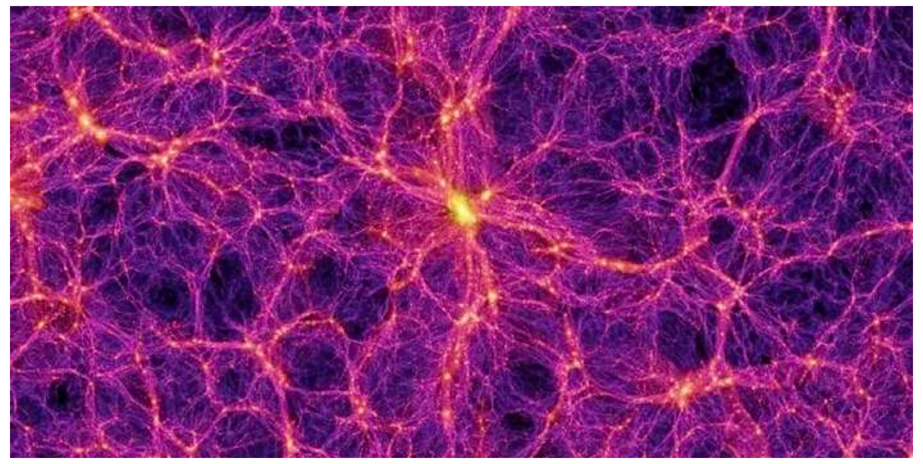
ESA

Strong lensing
- Mass fraction
- Distribution

Weak lensing
- Distribution
- Shape
- Structure

Micro lensing
- Mass fraction
- Smoothness

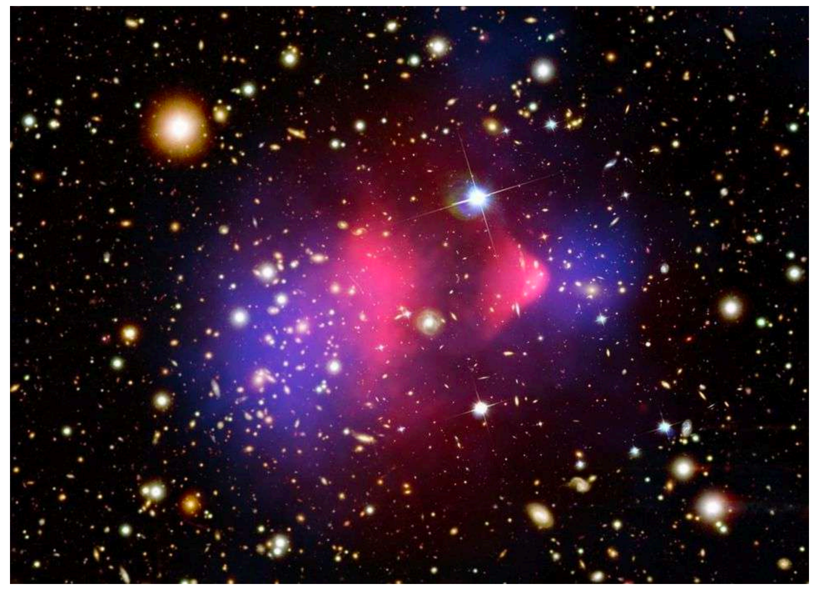
Large Scale Structure



Springel & others / Virgo Consortium

CMB/LSS
- Ratio of DM/collisional matter
- Thermal history

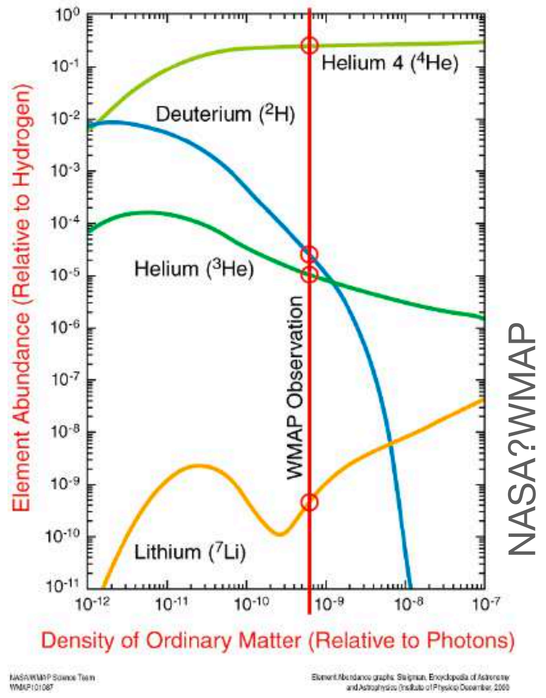
Cluster collision



NASA/CXC/CfA and NASA/STScI

- Distribution
- Separation from collisional matter
- Self-interaction

Big Bang Nucleosynthesis



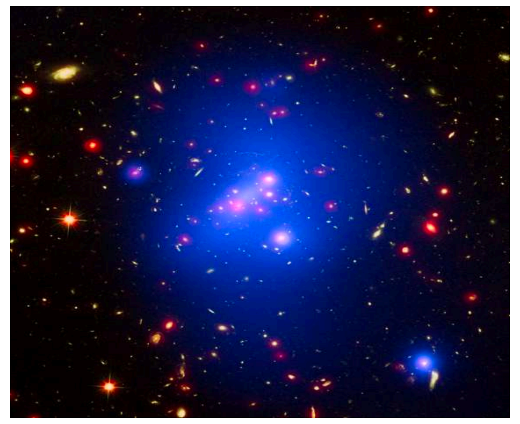
NASA?WMAP

- Amount of baryons

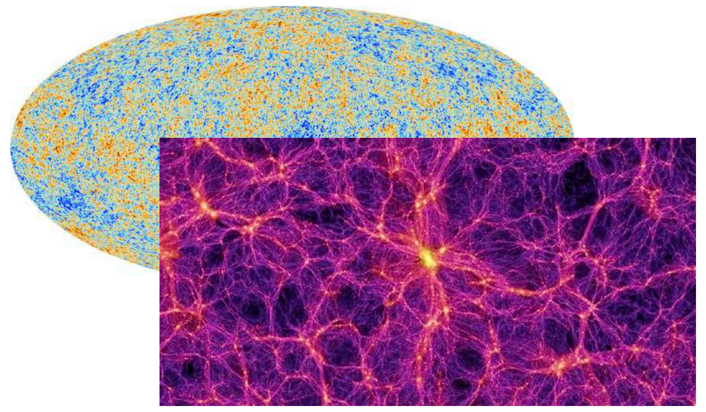
Based on K. Mack

What we know about dark matter

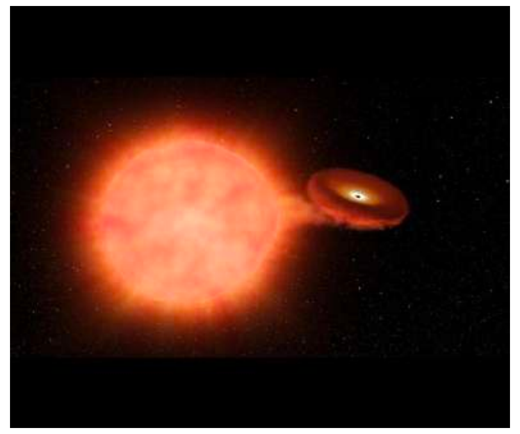
Galaxies



Clusters



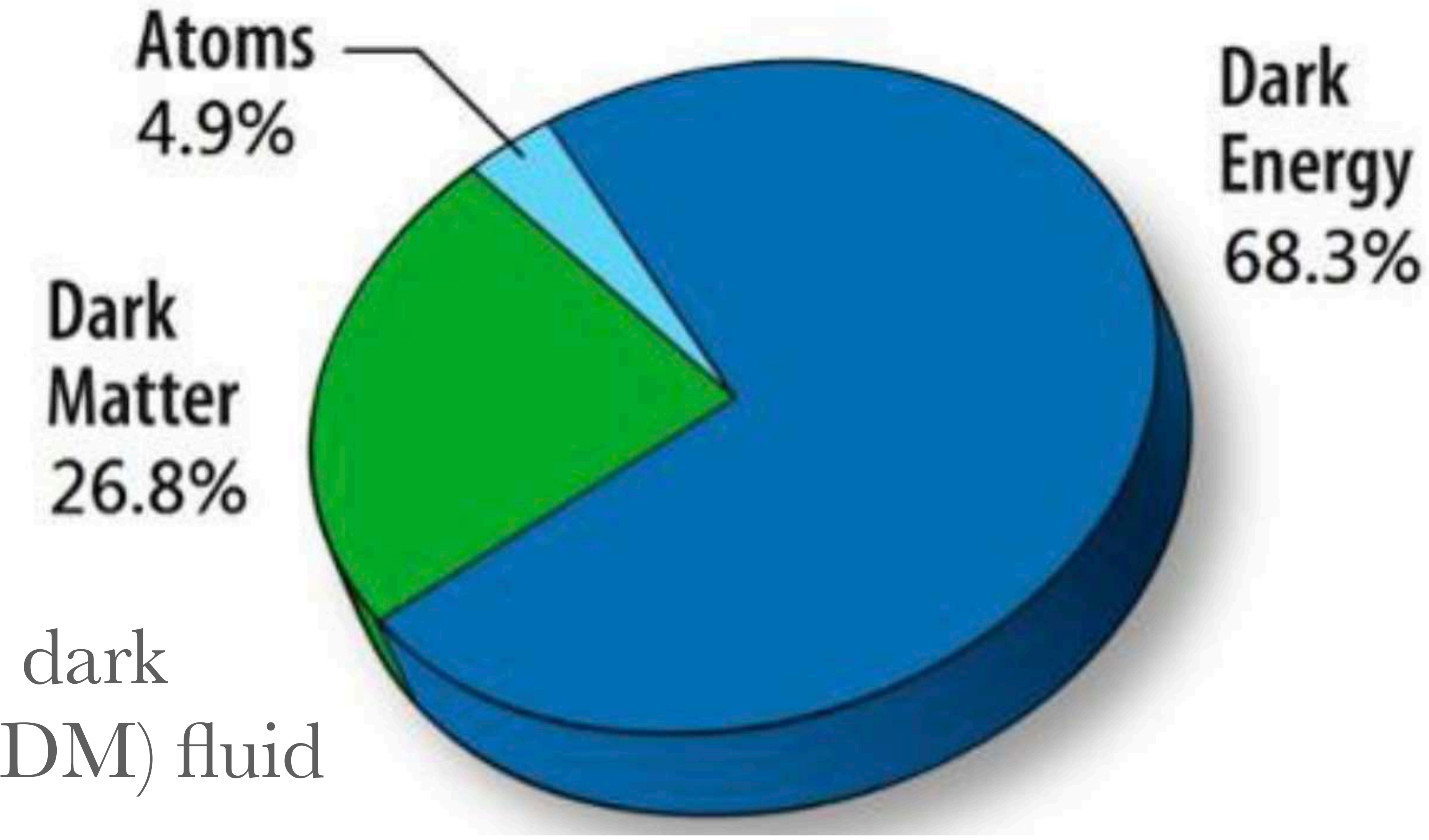
Large scales



Sn Ia

ΛCDM – the standard cosmological model

Successful description of our universe with 6 free parameters, tested to sub-percent precision.



DM: cold dark matter (CDM) fluid

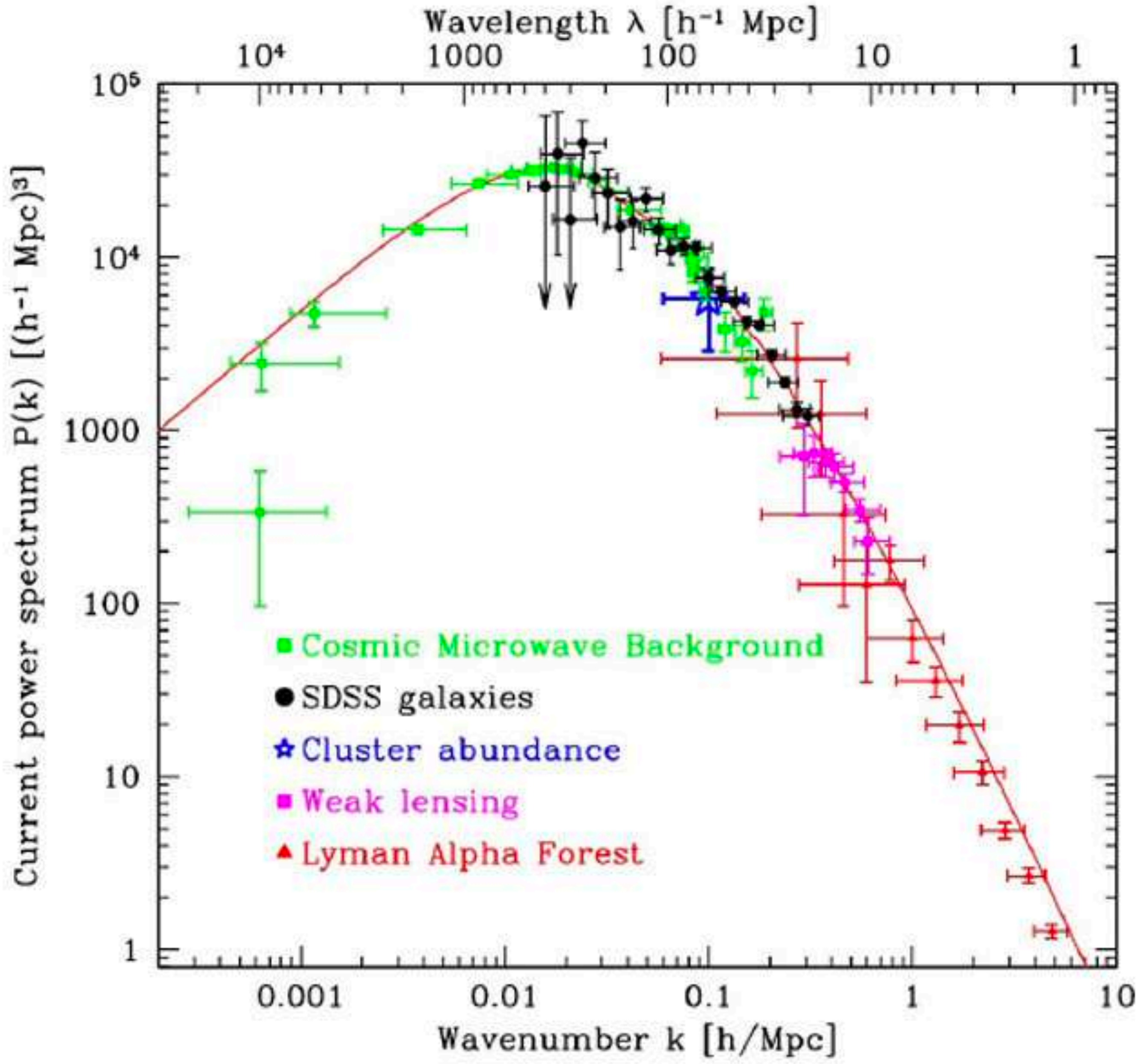
Λ

ΛCDM

simple but exotic model!

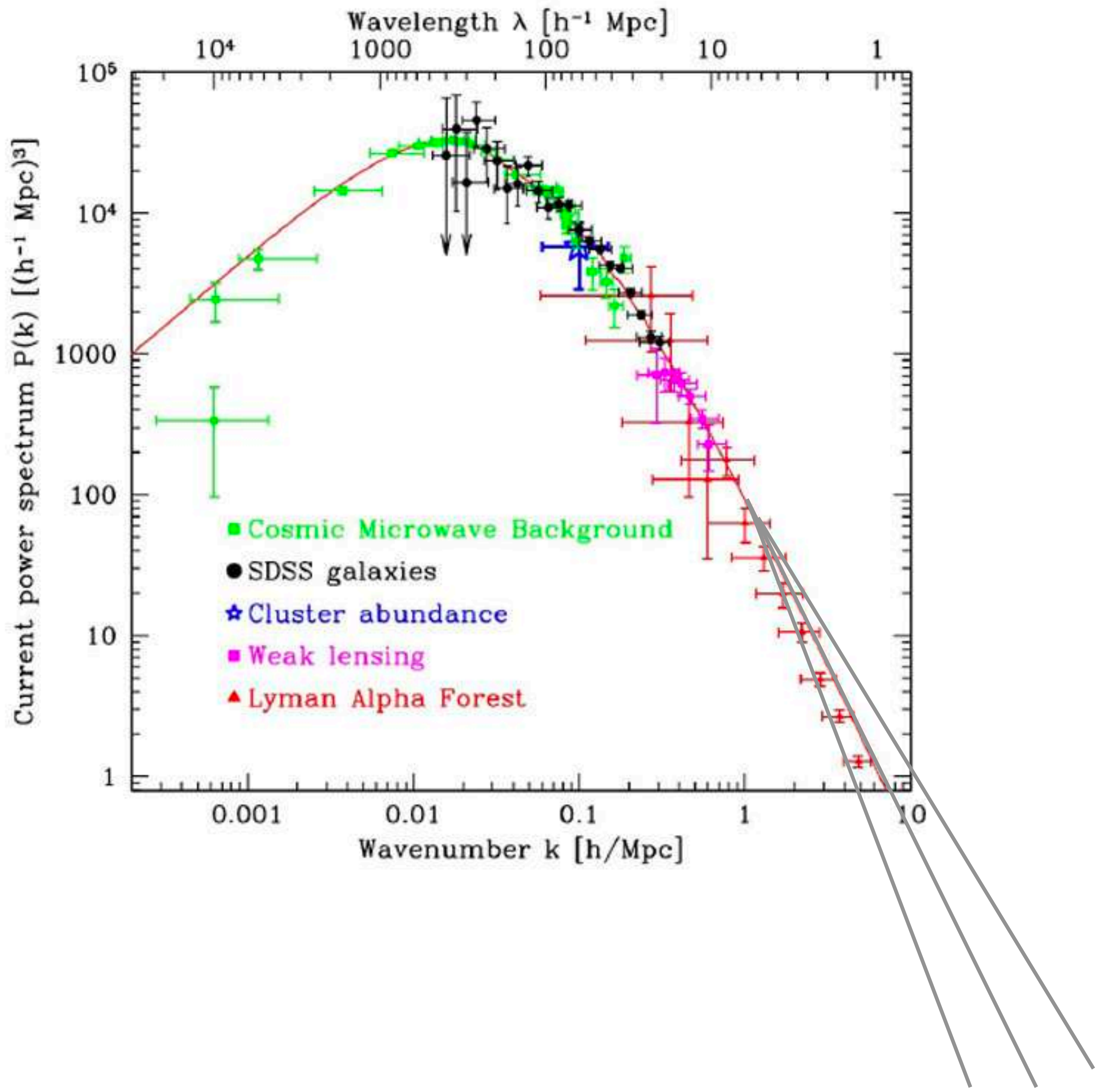
Cold dark matter

- Cold: moves much slower than *c*
- Presureless: gravitational attractive, clusters
- Dark (transparent): no/weakly electromagnetic interaction
- Collisionless: no/weakly self-interaction or interaction with baryons
- Abundance: amount of dark matter today known



What we don't know

- What is DM? Nature
- ~~Cold~~ → How cold it is? WDM
- ~~Pressureless~~ → Cluster on all scales?
- ~~Dark~~ → Non-gravitational interaction? Milicharged DM
- ~~Collisionless~~ → How small sefl-interaction? SIDM



Although still behaves like CDM on large scales

Small scale behaviour: still “weakly" constrained and small scale challenges

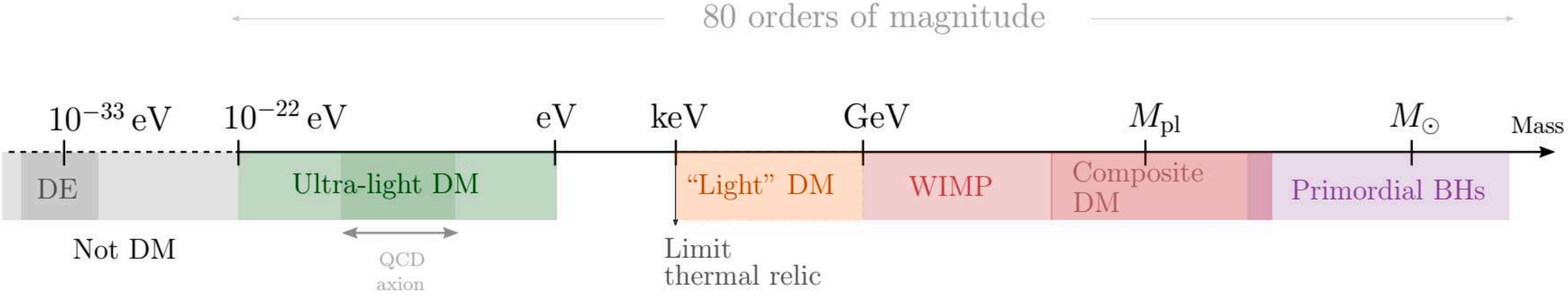
Small scale curiosities: cusp-core, missing satellites, BTFR, …

What we don't know

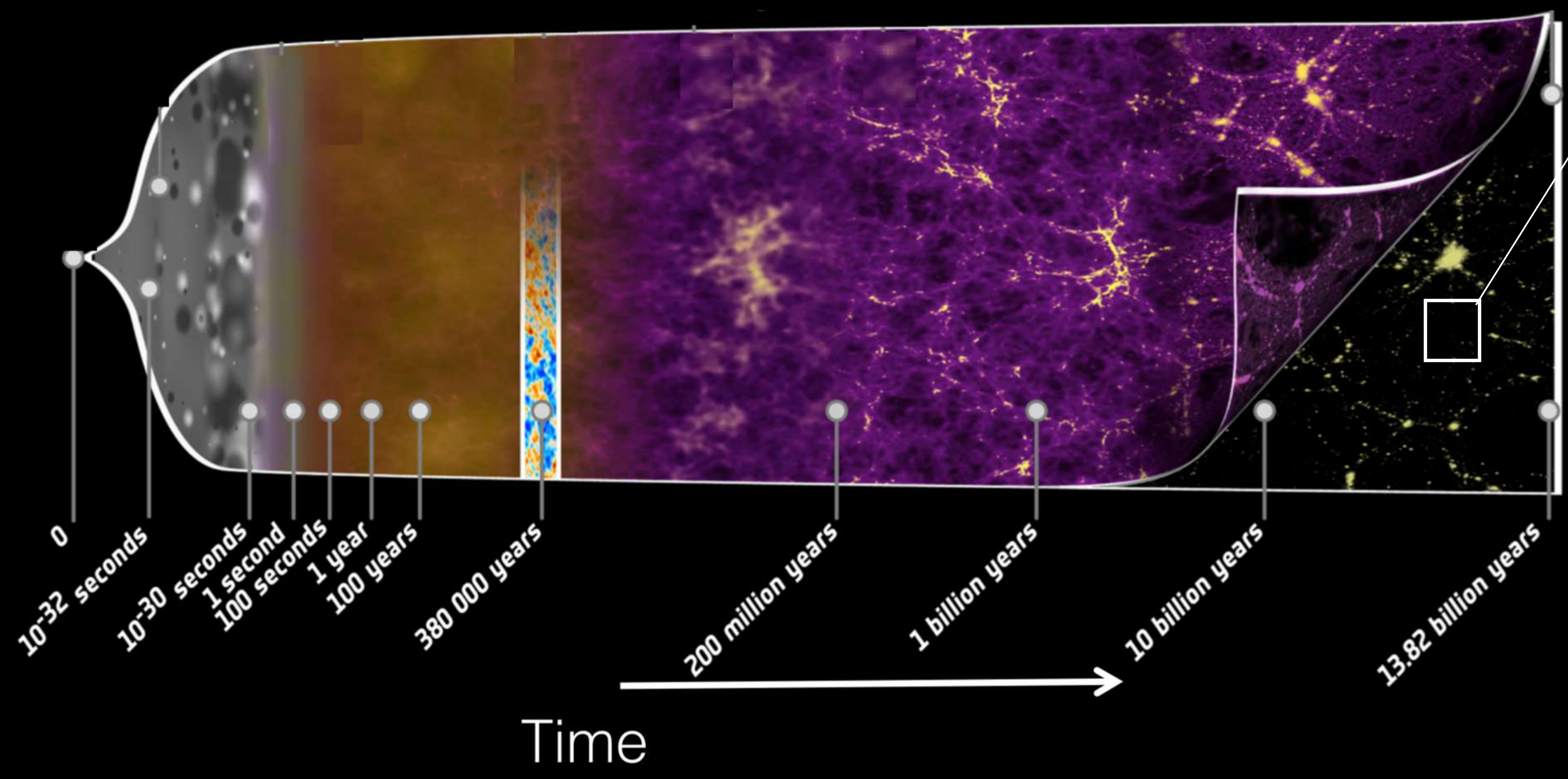
- What is DM? What is the nature of DM?

State of the "art"

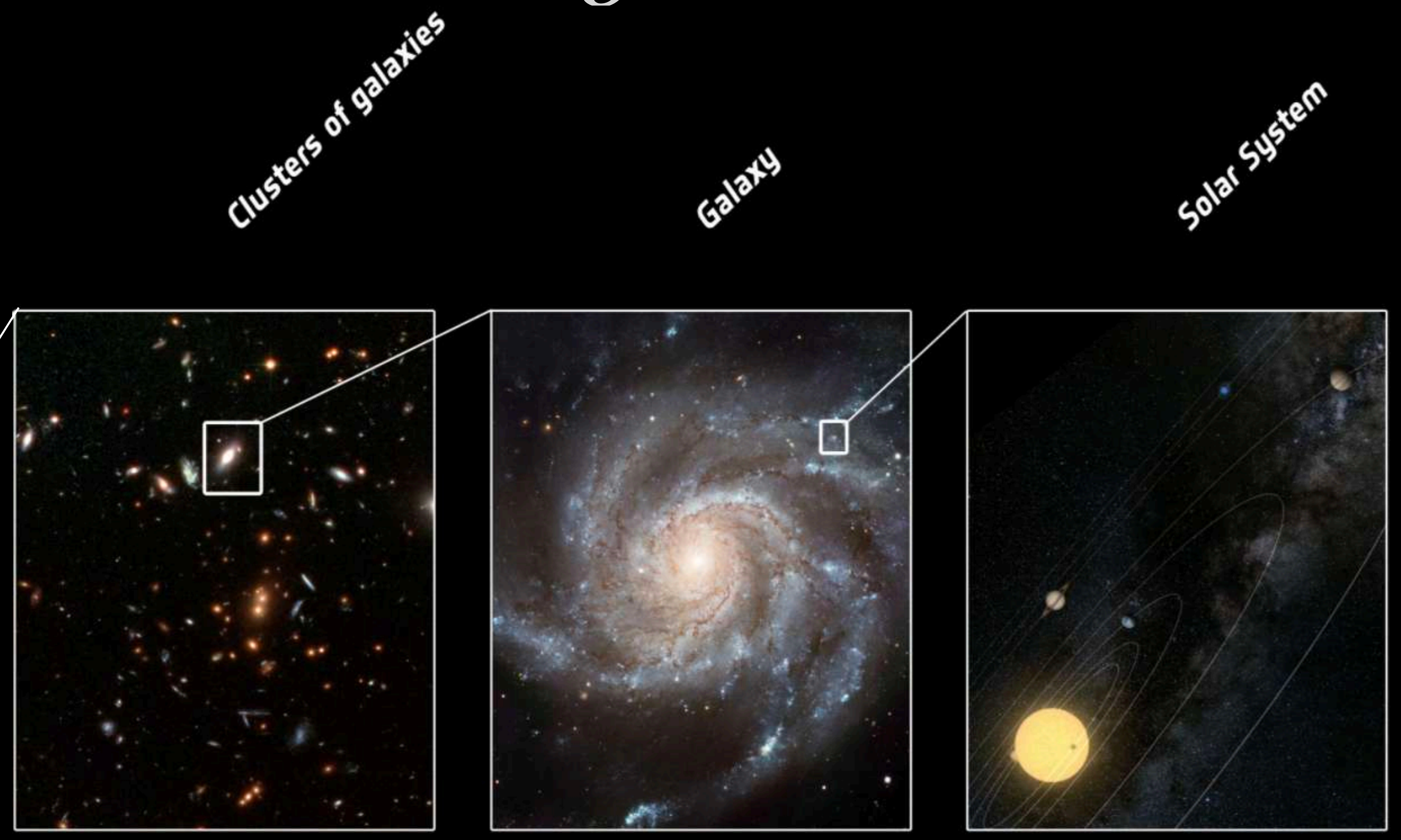
Mass scale of DM



Small scales can offer some hints of the nature of DM



Credit: Nasa

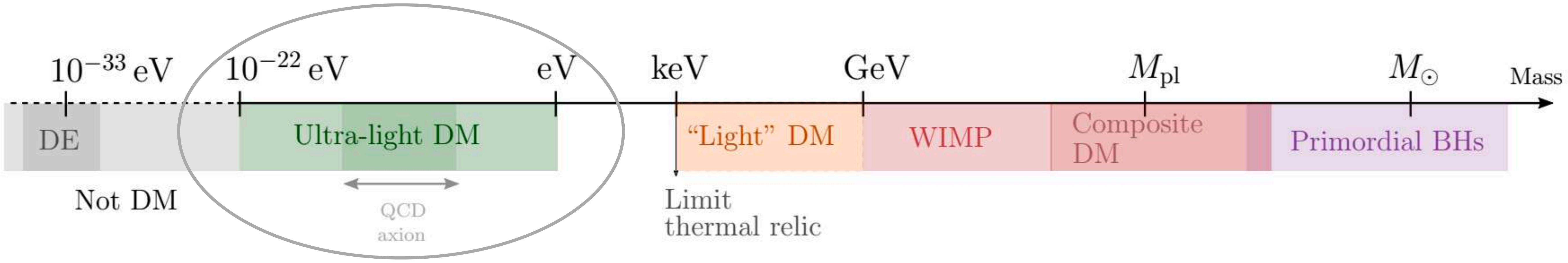


Small Scales

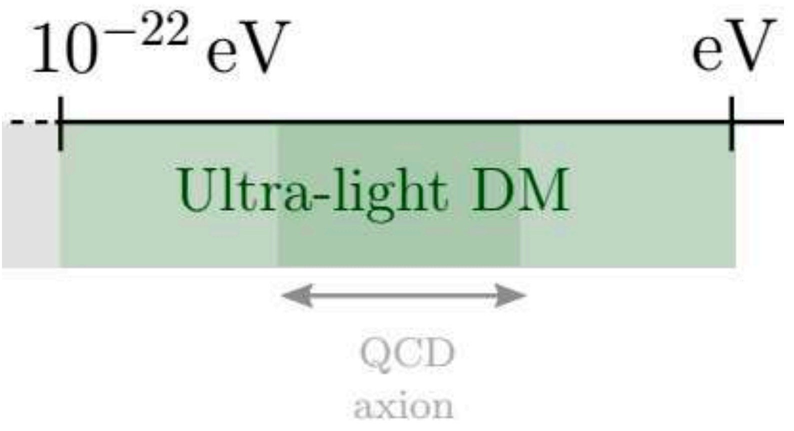
Opportunity to probe the nature of DM!

Astrophysical Observables → DM Distribution → Nature of DM Microphysics Particle physics

Ultra-light dark matter

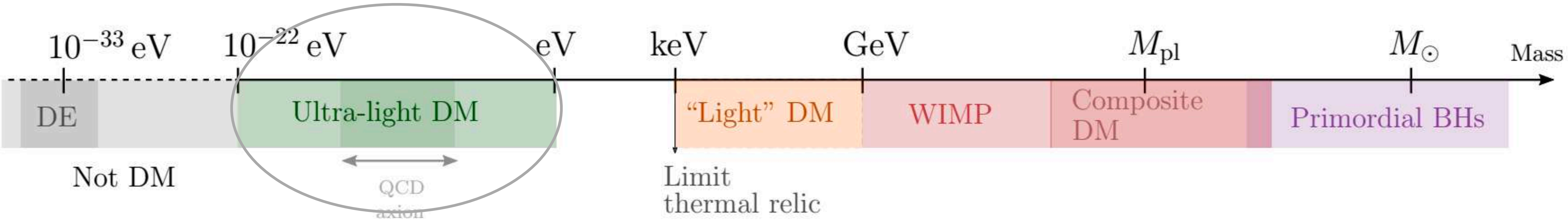


Ultra-light Dark Matter

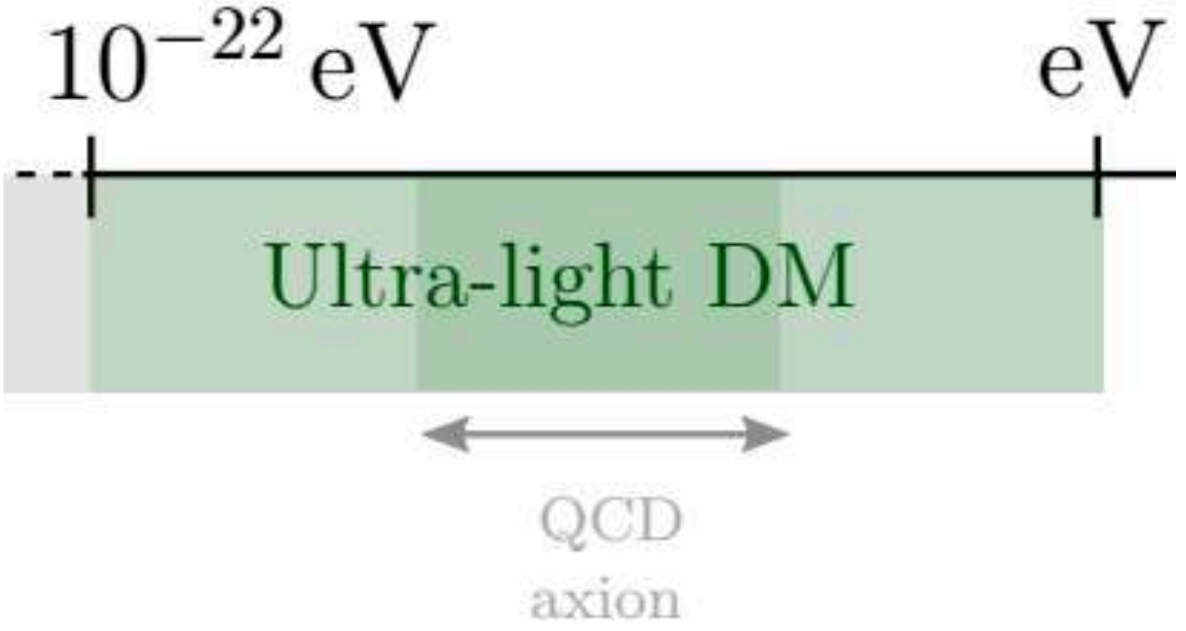


Ultra-light candidate, cold → Large $\lambda_{\mathrm{dB}} \sim 1/mv$

Lightest possible candidate for DM



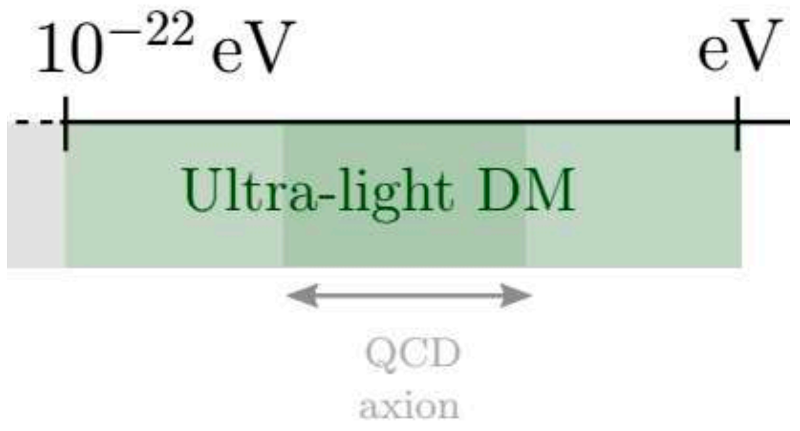
10^{-57} kg 10^{-35} kg



→ Bosons

Non-thermally produced

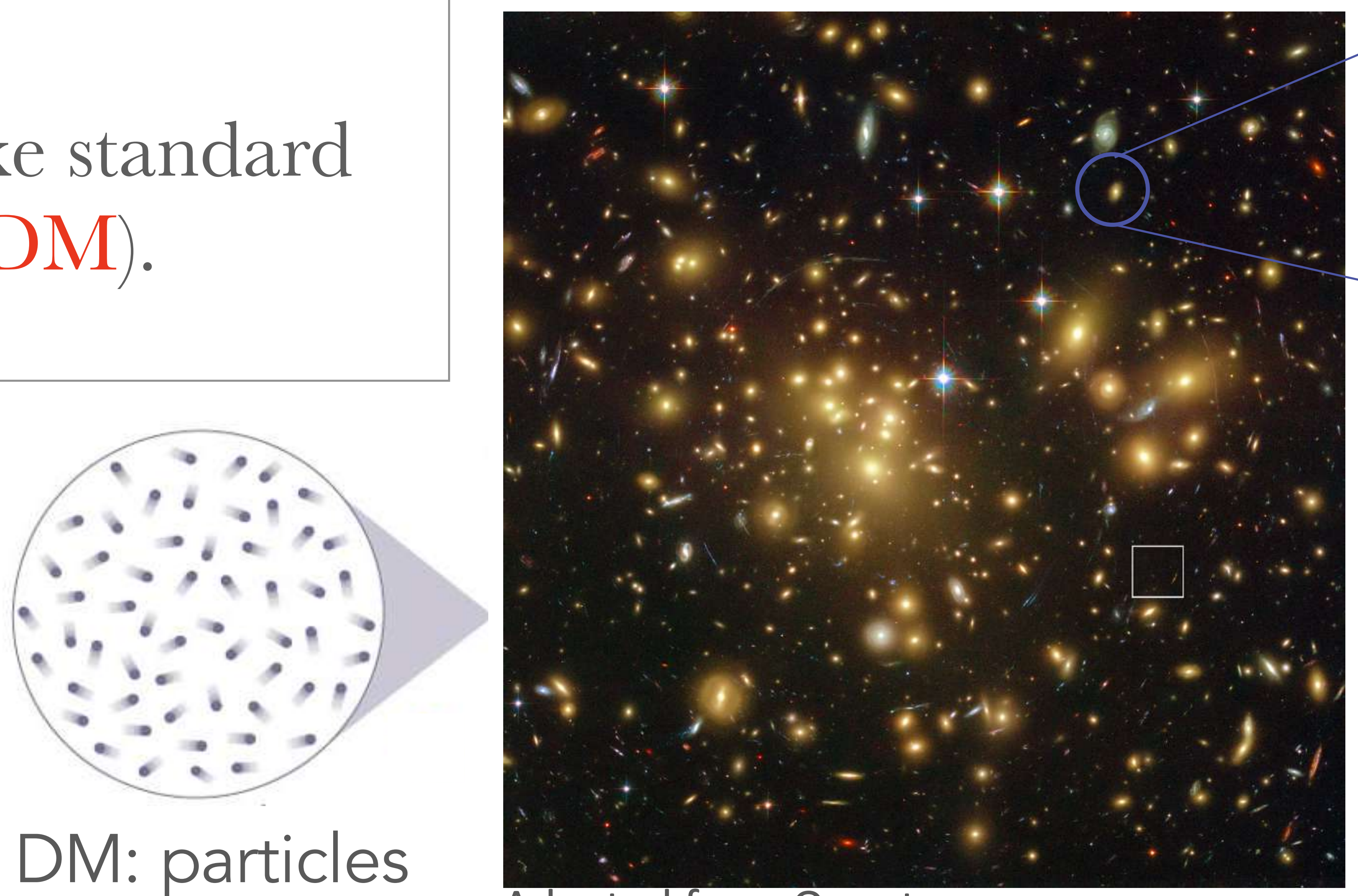
Ultra-light Dark Matter



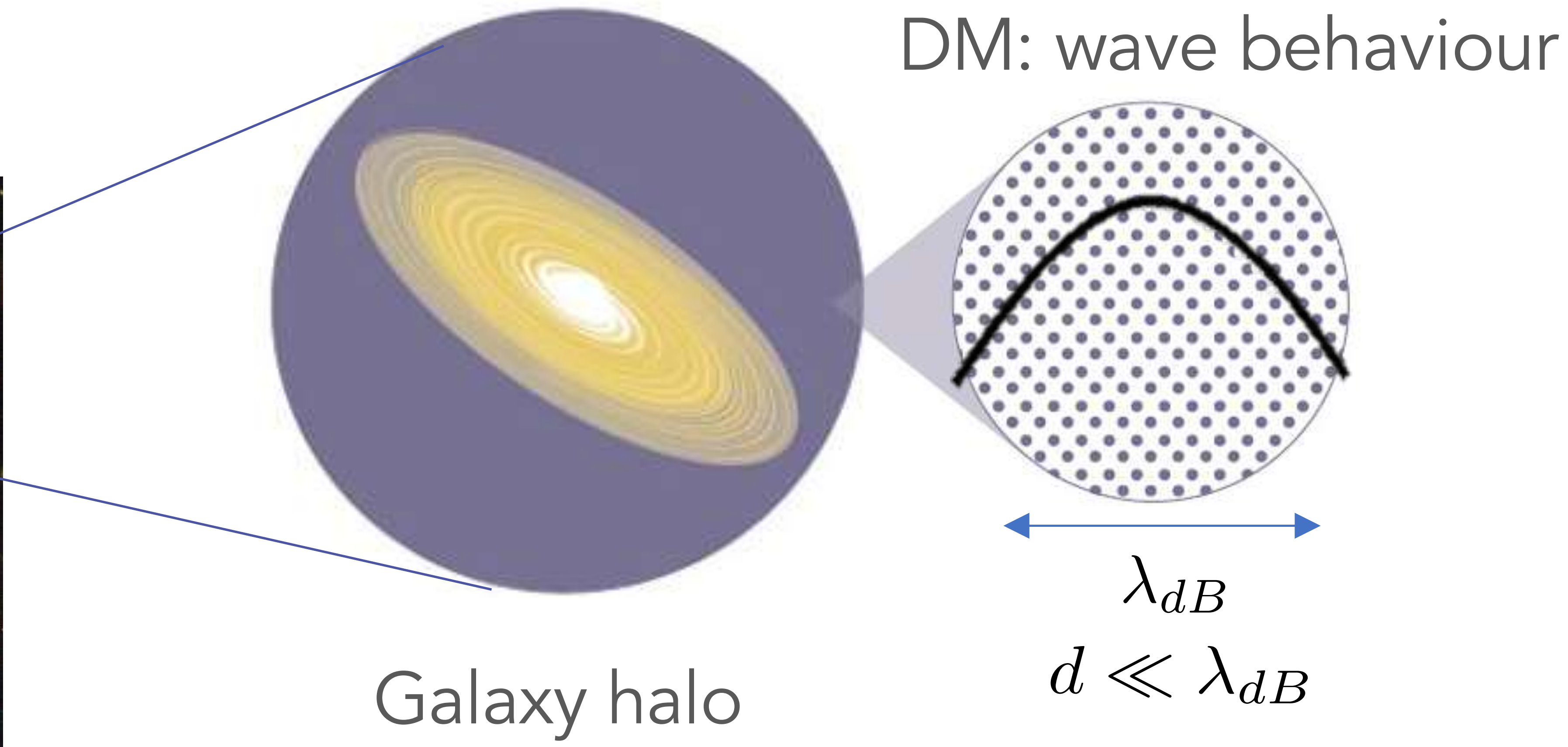
Ultra-light candidate $\longrightarrow$ Large $\lambda_{\mathrm{dB}} \sim 1/mv$

Lightest possible candidate for DM

Large scales:
DM behaves like standard particle DM (CDM).



Adapted from Quanta



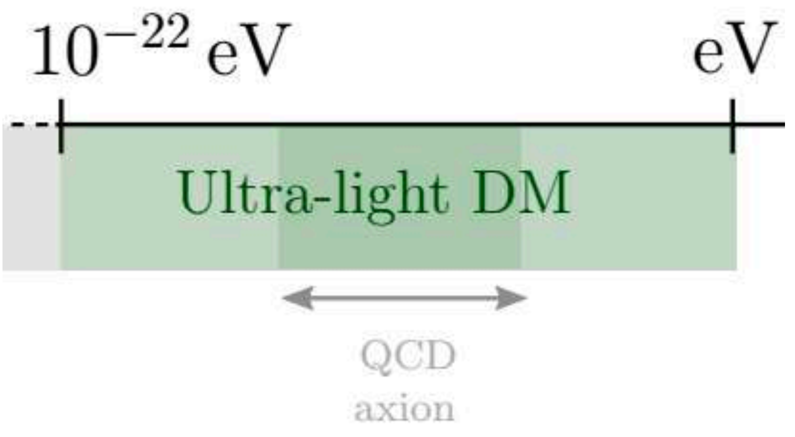
Small scales:
DM behaves like a wave

10^{-60} kg $\qquad$ 10^{-35} kg

$$\boxed{10^{-25}\,\mathrm{eV} \lesssim m \lesssim \mathrm{eV}}$$

$\lambda_{dB}^{ULDM} \sim \mathrm{pc} - \mathrm{kpc}$

Motivations of the ULDM



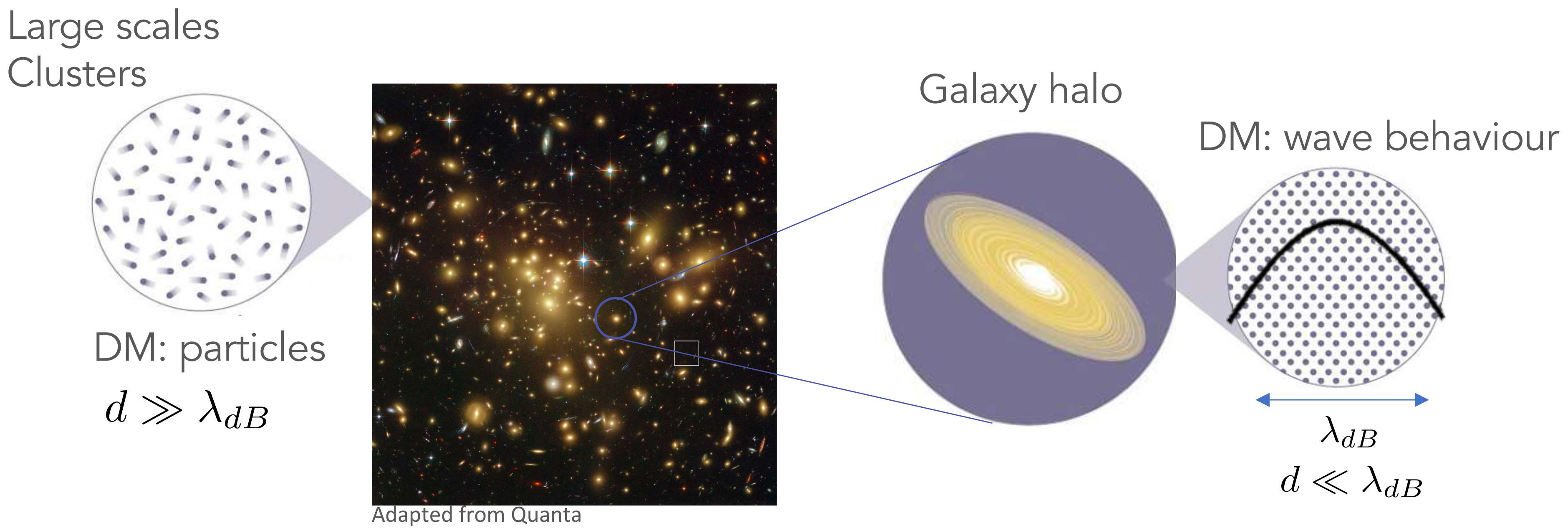
- Particle physics/HEP/condensed matter motivation

Candidates: Axions, ALPs, UL particles, …

- Might address small scales problems

- **Rich phenomenology on small scales:**

  - Wave nature manifest on galactic scales

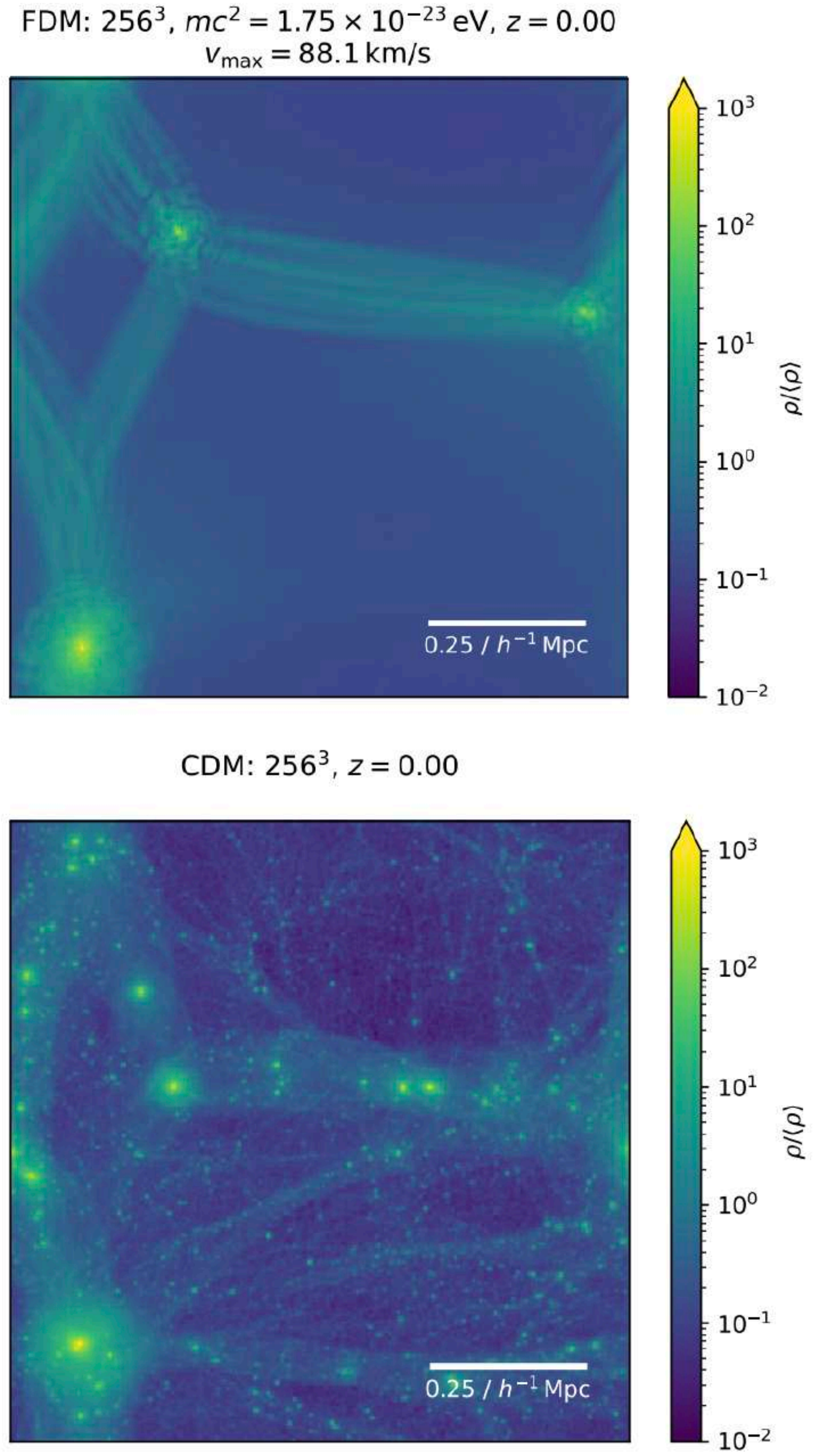


Adapted from Quanta

Phenomenology

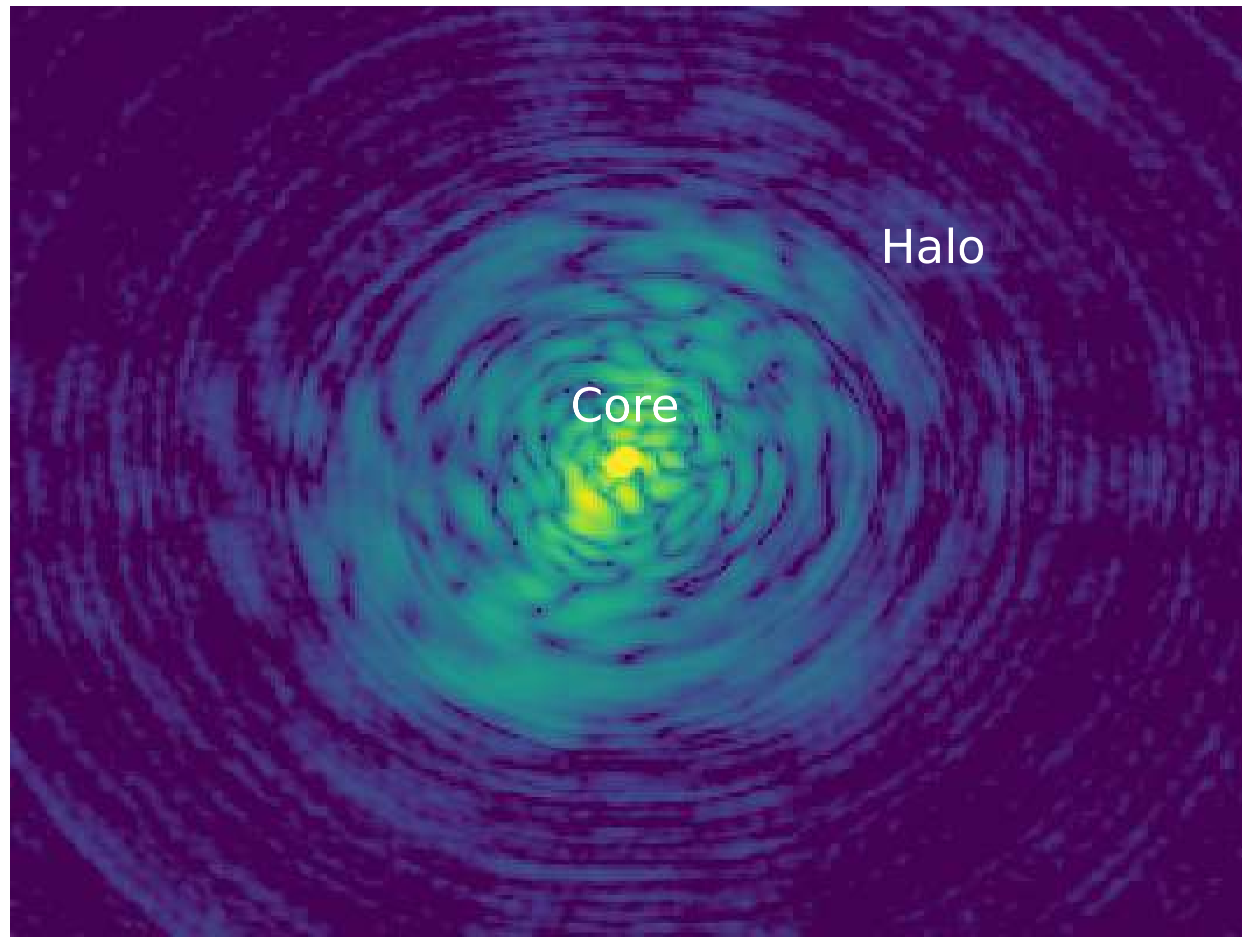
RICH PHENOMENOLOGY ON SMALL SCALES

Suppression of small structures

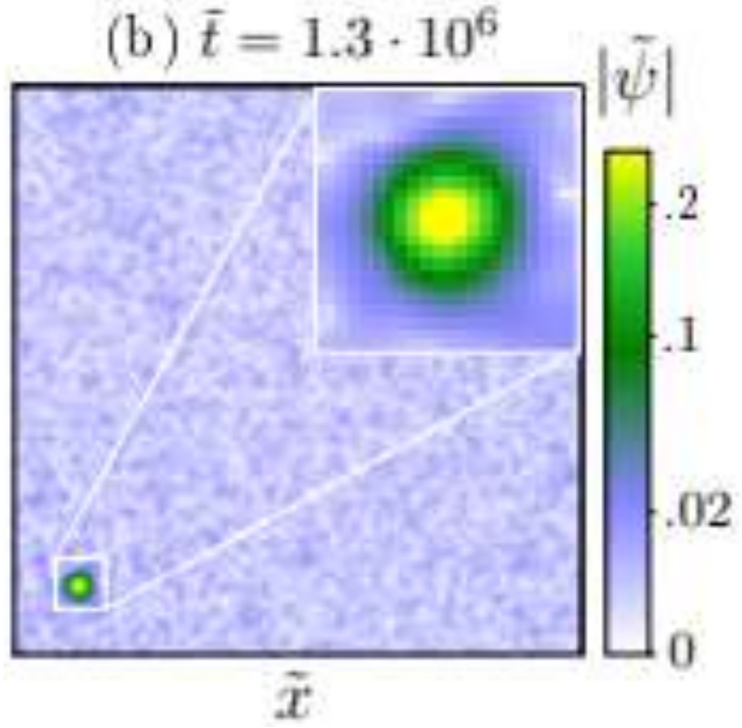


S. May et al. 2021

Formation of a solitonic core

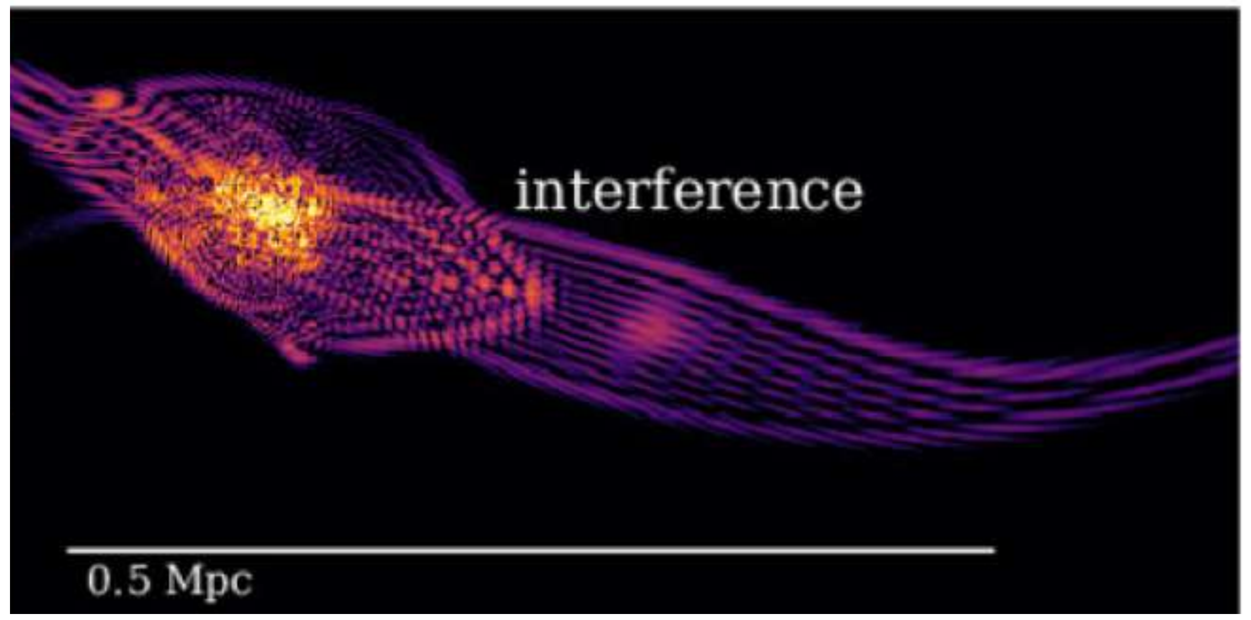


Dynamical effects



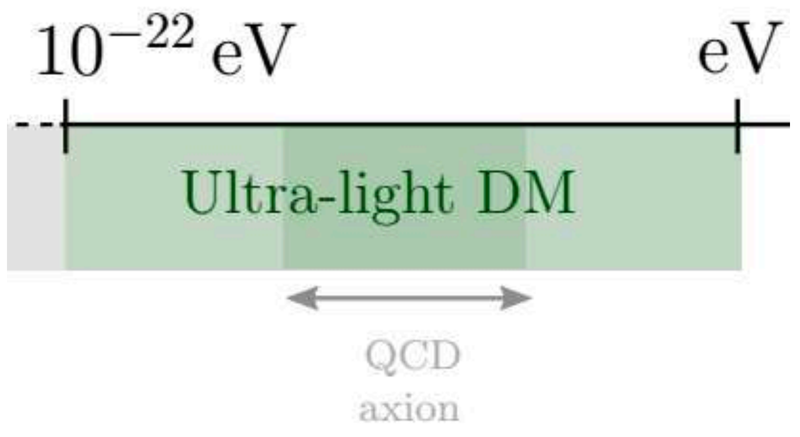
Levkov et al. 2018

Wave interference

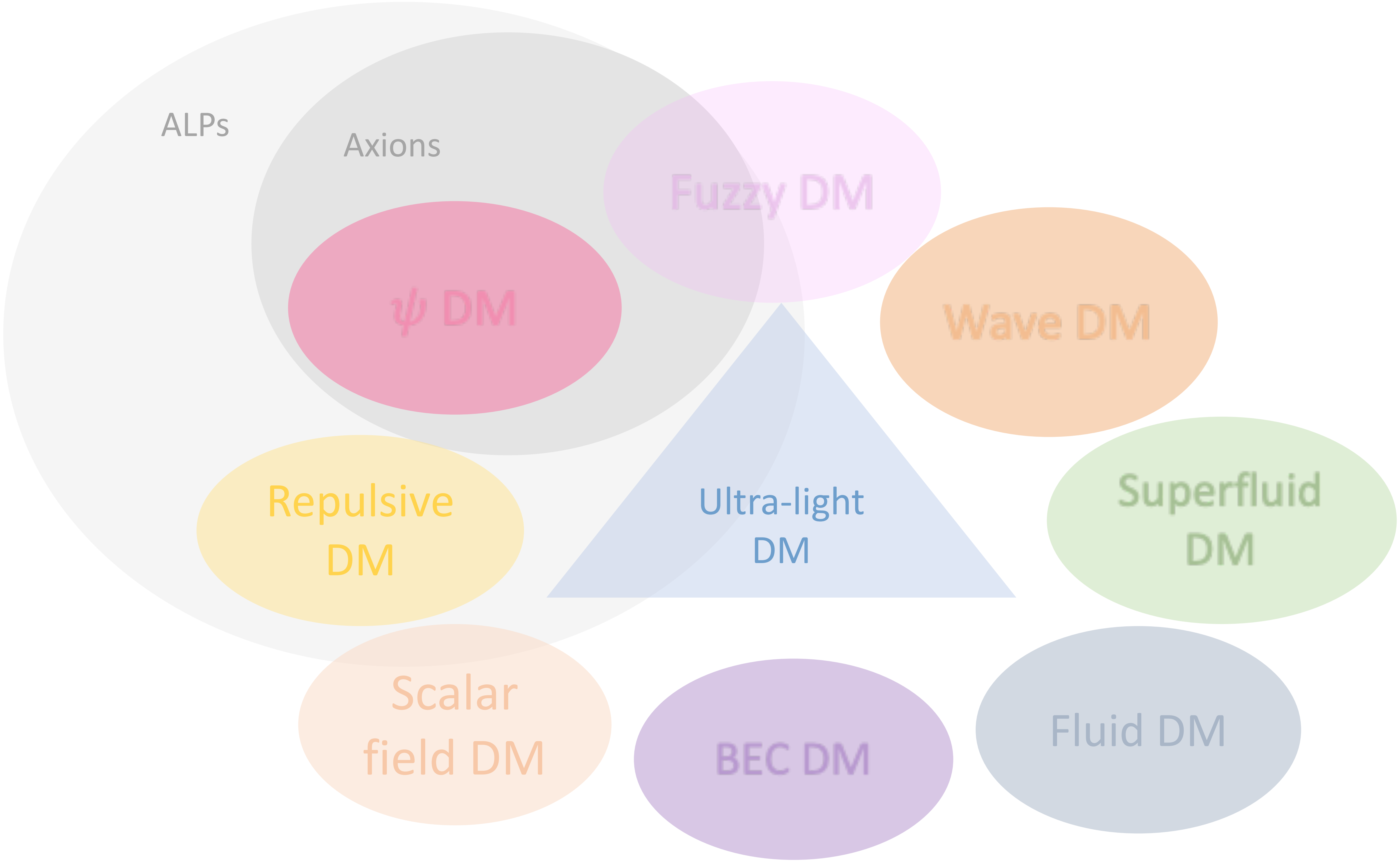


Mocz et al. 2017

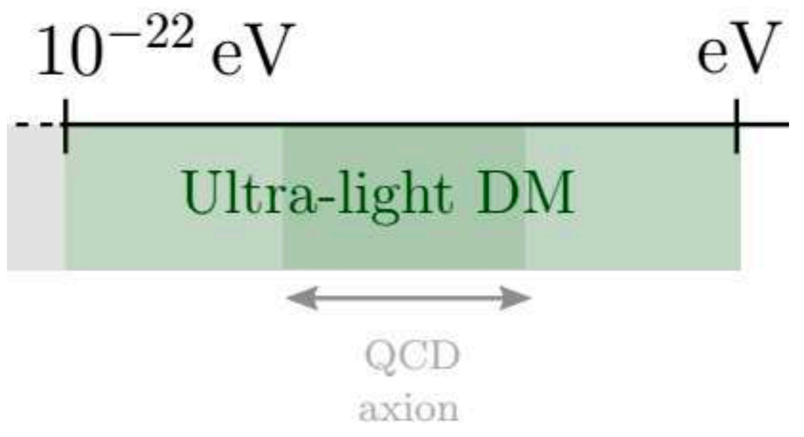
Ultra-light Dark Matter - models



There are many ways to have a DM with this property → many ULDM models in the literature
However, each of these models presents a different dynamics on small scales - different phenomenology



Ultra-light Dark Matter -classes



3 classes:

Fuzzy DM (FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

DOFs m

Self Interacting FDM (SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

m g

DM Superfluid

- Forms a superfluid in galaxies
- MOND behaviour interior of galaxies

Axion and ALP (axion like particles)

$$i\dot{\psi} = \left(-\frac{1}{2m}\nabla^2 + \frac{g}{8m^2}|\psi|^2 - m\Phi\right)\psi$$

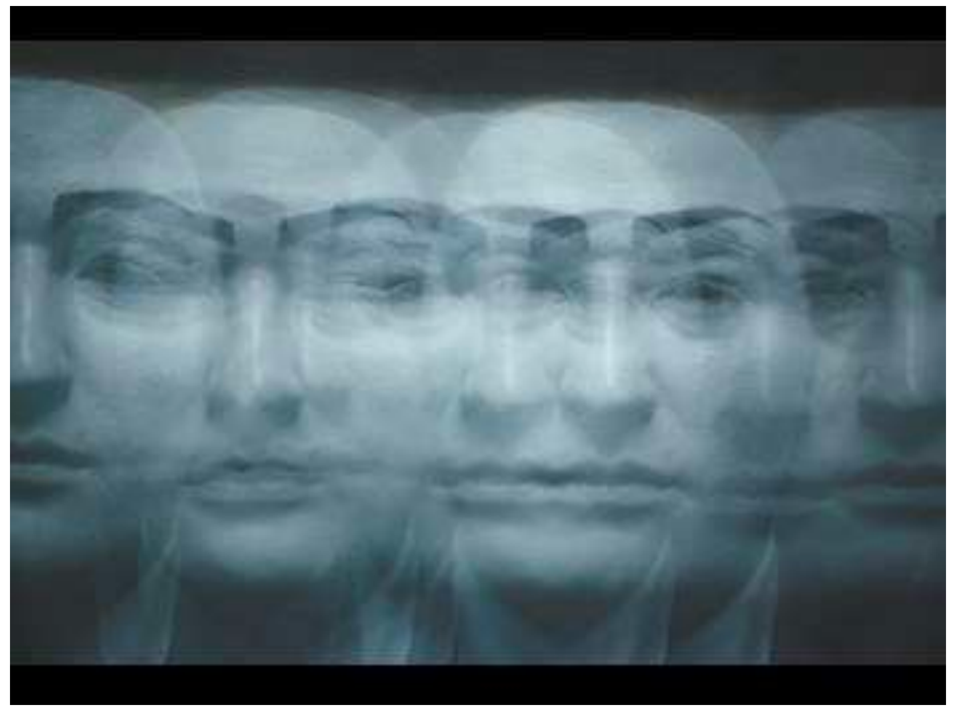
$$\mathcal{L} = P(X)$$

⟶ Connection with condensed matter and particle physics!

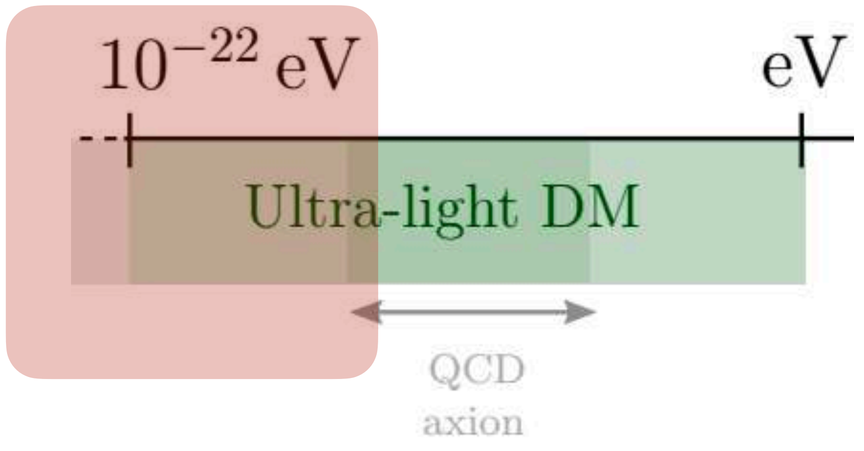
"Ultra-light dark matter", **E.Ferreira**, 2020. The Astronomy and Astrophysics Review.

Fuzzy dark matter

Self interacting fuzzy dark matter



Fuzzy dark matter



Fuzzy DM
(FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

m

Wave DM
Ultra-light axions

Self Interacting FDM
(SIFDM)

- Presence of (weakly) self-interaction
- Condensation under gravity + SI (superfluid)

m g

Hu W, Barkana R, Gruzinov A (2000 a,b)

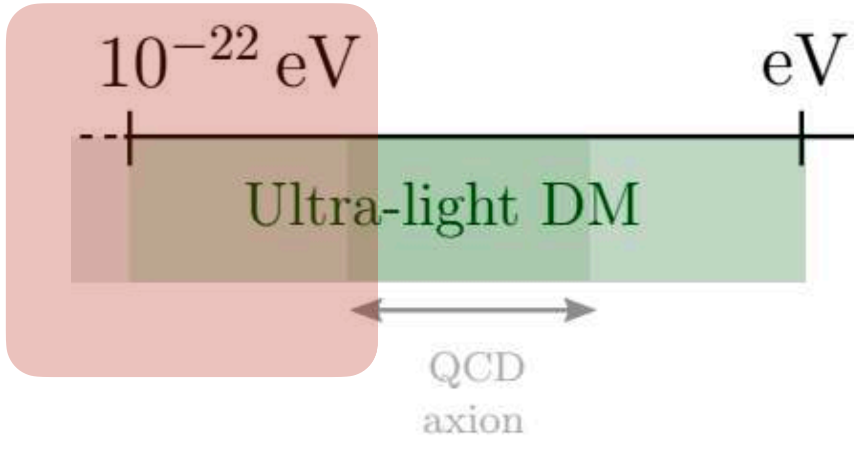
(Reviews: EF (2021), J. Niemeyer (2019), L. Hui (2021))

Idea:

$$m_{\mathrm{fdm}} \sim 10^{-22}\,\mathrm{eV}$$

address the small scale problems+ rich phenom.

Fuzzy dark matter



Fuzzy DM
(FDM)

- Gravitationally bounded ultra-light scalar field model
- Condensation under gravity (BEC)

m

Wave DM
Ultra-light axions

Focus in spin 0 particles here!

(Some of the grav. phenom. is carried for vectors, for example)

Hu W, Barkana R, Gruzinov A (2000 a,b)

(Reviews: *EF (2021), J. Niemeyer (2019), L. Hui (2021)*)

Idea:

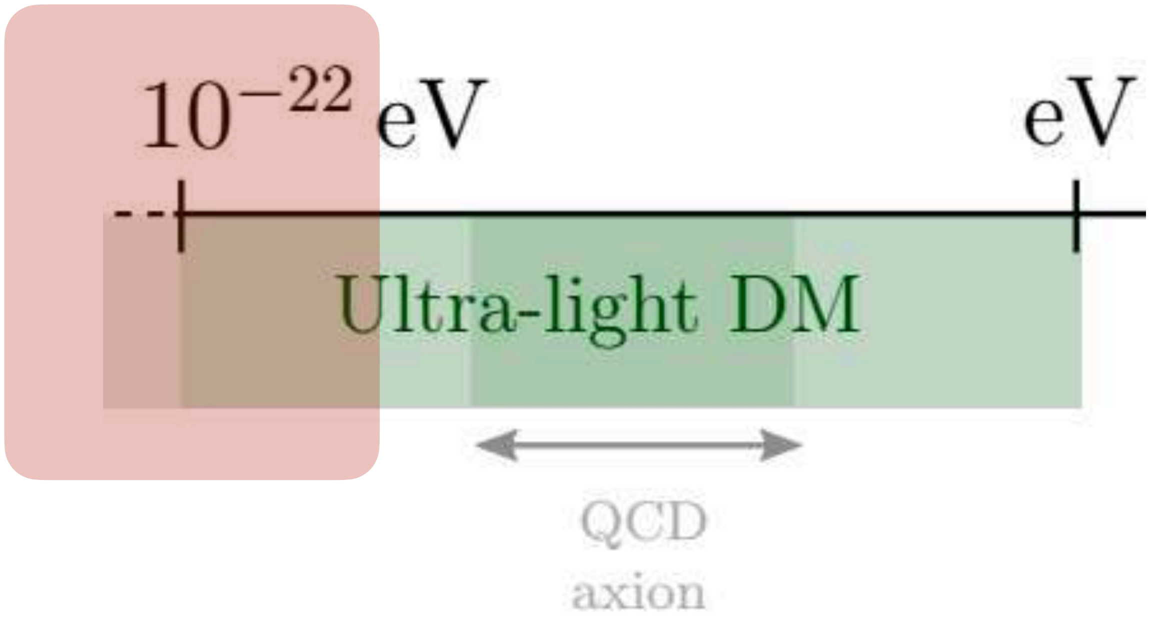
$$m_{\rm fdm} \sim 10^{-22}\,{\rm eV}$$

address the small scale problems+ rich phenom.

Motivation: particle physics

FDM candidates

- Natural candidate for a light scalar field is a pseudo-Nambu Goldstone boson (breaking of an approximate symmetry)



Known PNGB: QCD axion → Candidate for DM

(Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978)

Axion-like particles

Axions or Axion like particles (ALP)

Axions and ALPs are pseudo Nambu Goldstone bosons from the spontaneous symmetry breaking of a $U_{PQ}(1)$ ($U(1)$) symmetry, and are described by the complex field: $\Psi = v\, e^{i\phi/f_a}$

$$v_{0,ssb} = f_a/\sqrt{2} \longrightarrow \phi \to \phi + c$$

Non-perturbative effects (from string theory or instatons) induce a potential:

$$V(\phi) = \Lambda_a^4\,[1 - \cos(\phi/f_a)] \xrightarrow[\phi \ll f_a]{} \frac{1}{2}m^2\phi^2 + \frac{g}{4}\phi^4 + \cdots$$

Motivation: particle physics

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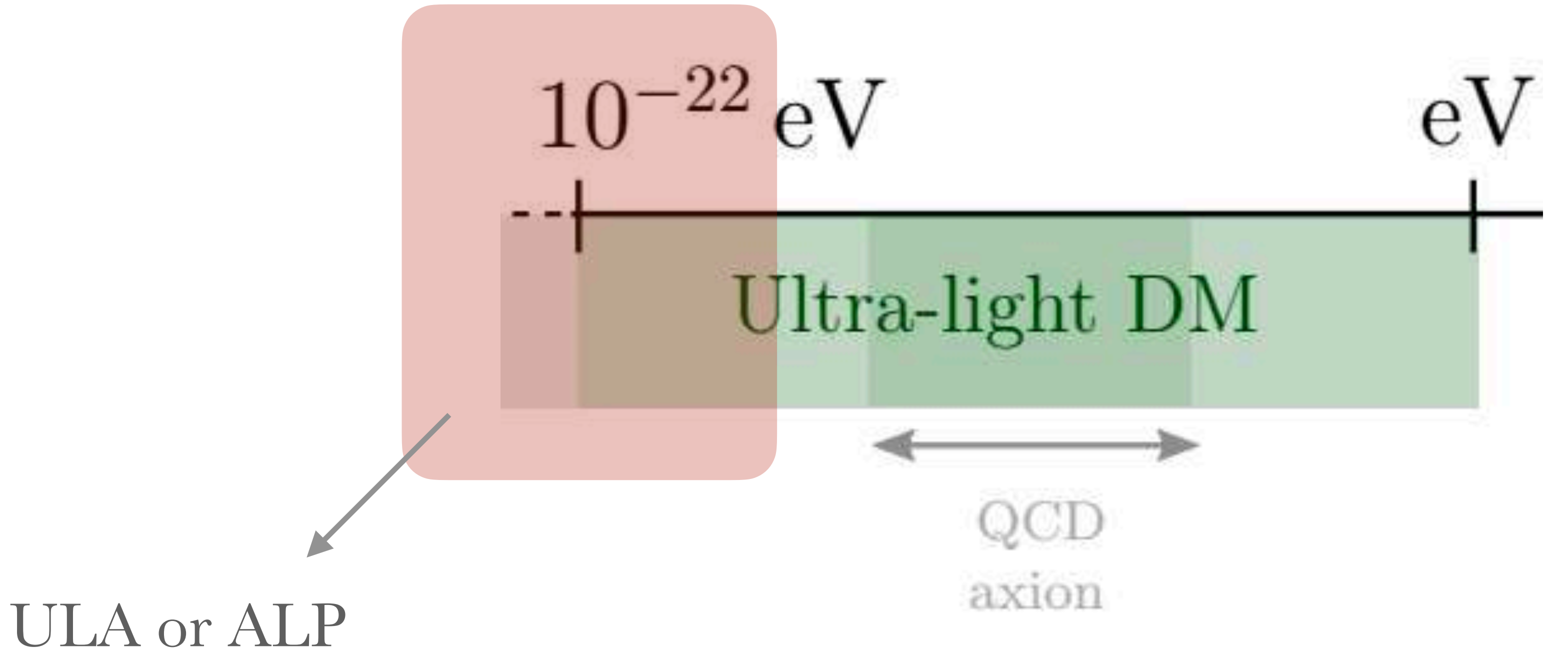
(Peccei and Quinn 1977; Weinberg 1978; Wilczek 1978)



Axion-like particles or ultra-light axions:

- ALPs expected in string theory *(Arvanitaki et al., Svrcek, Witten)*
- Can generate PNGB that are ultra-light

(Francesca Chadha-Day's talk!)

- Formation mechanism: needs to have a relic abundance that gives the correct DM abundance

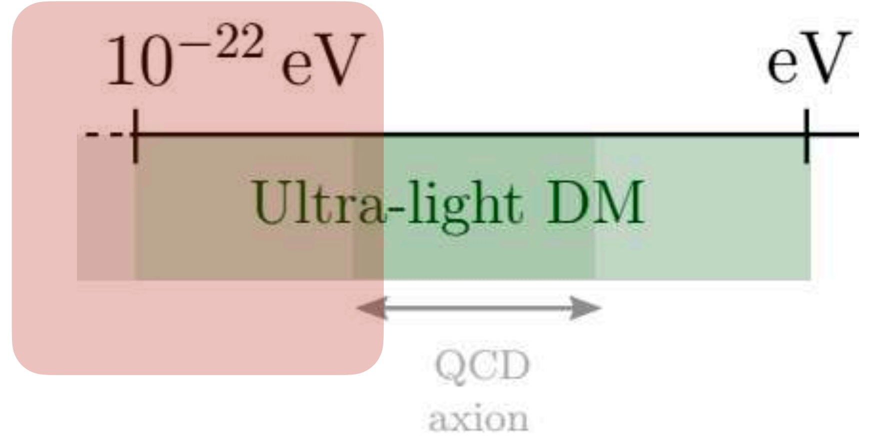
Non-thermal mechanism (e.g. mis-alignement)

$$\Omega_{\rm matter} \sim 0.1 \left(\frac{f_a}{10^{17}\,{\rm GeV}}\right)^2 \left(\frac{m}{10^{-22}\,{\rm eV}}\right)$$

L. Hui

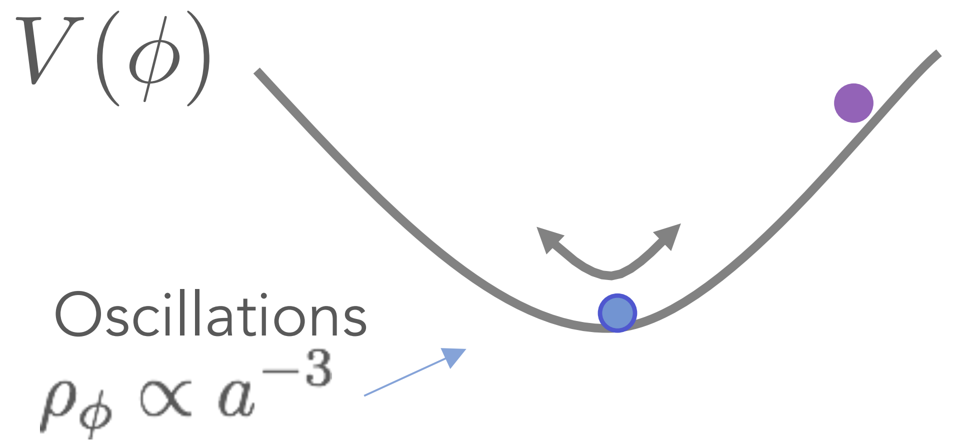
* Axion and ALP interact with photons (and neutrinos) *(Chris McCabe and Francesca Calore's talk!)*

Cosmological evolution

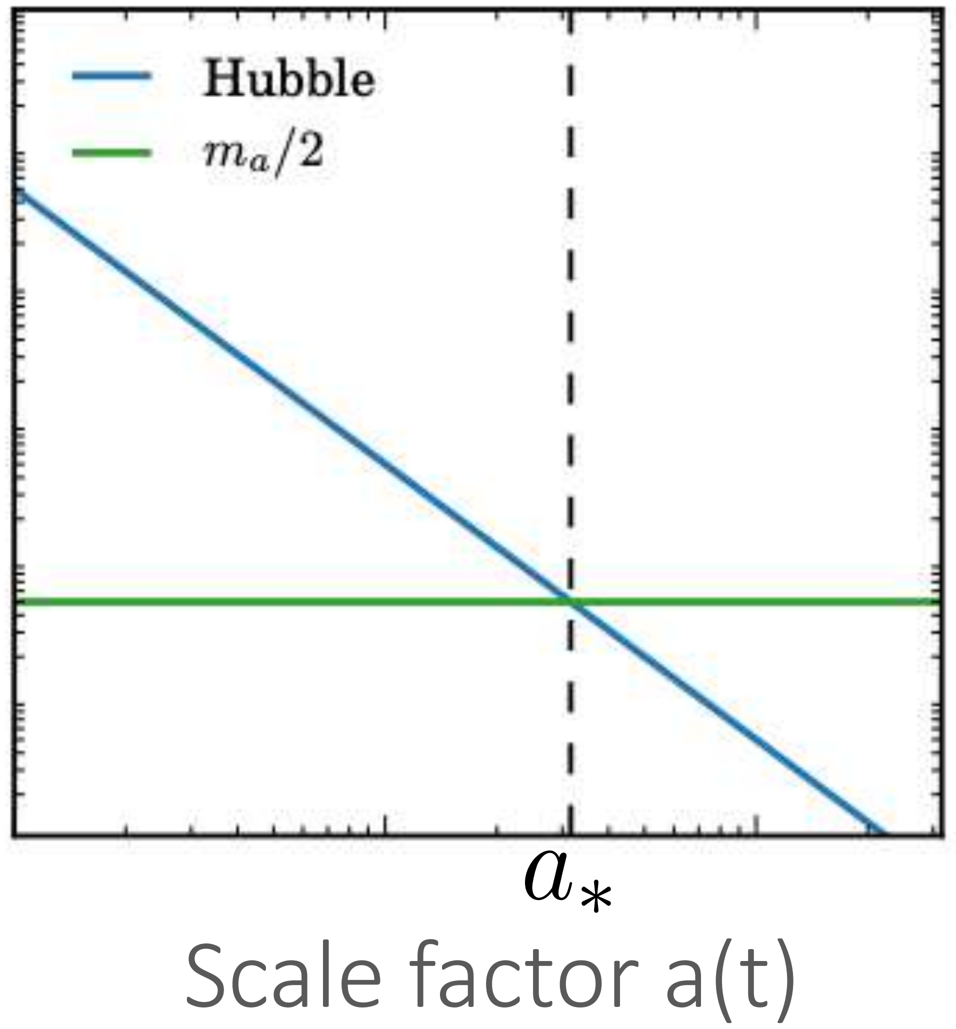


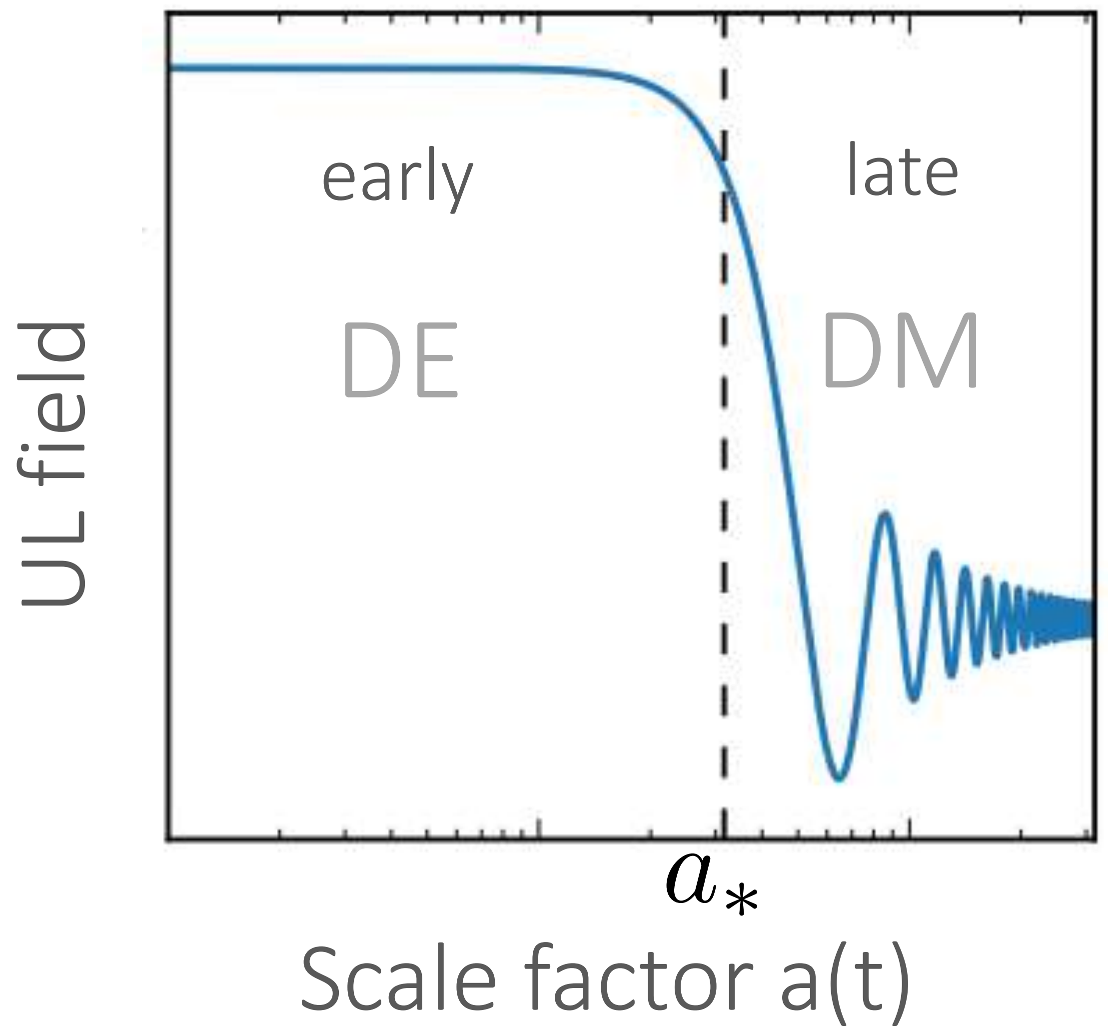
$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0$$

FDM

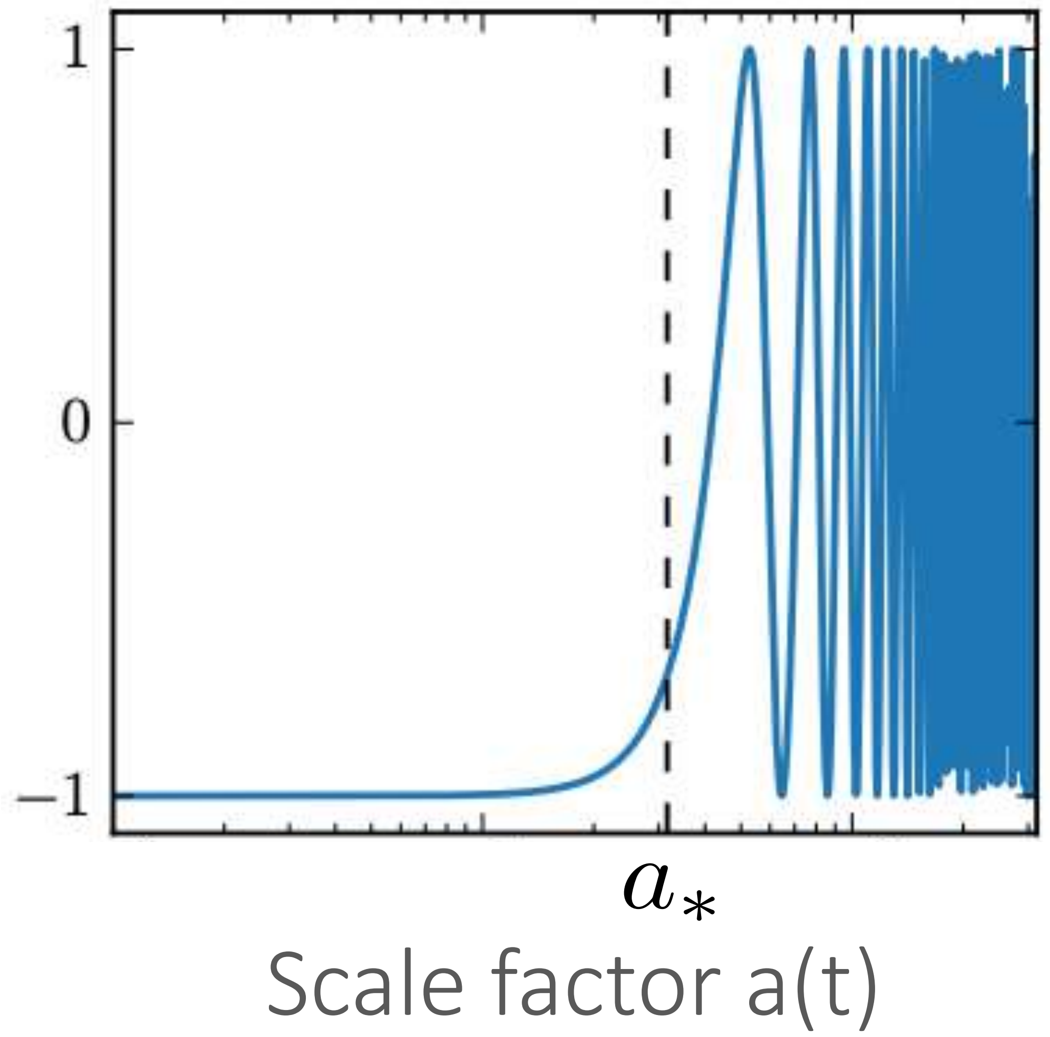


$$\left\{\begin{array}{llllll} H \gg m & \Longrightarrow & \phi_{\text{early}} = \phi(t_i) & \longrightarrow & \omega = -1 & \text{DE} \\ H \ll m & \Longrightarrow & \phi_{\text{late}} \propto e^{imt} & \longrightarrow & \langle \omega \rangle = 0 & \text{DM} \end{array}\right.$$



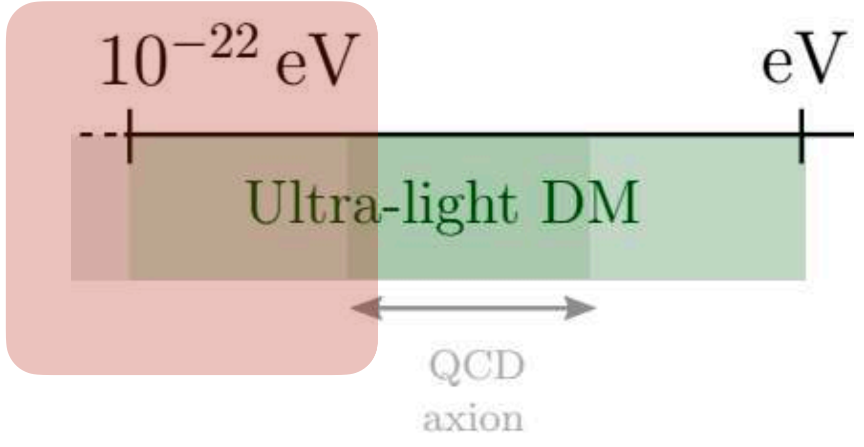


$H(t_*) = m$

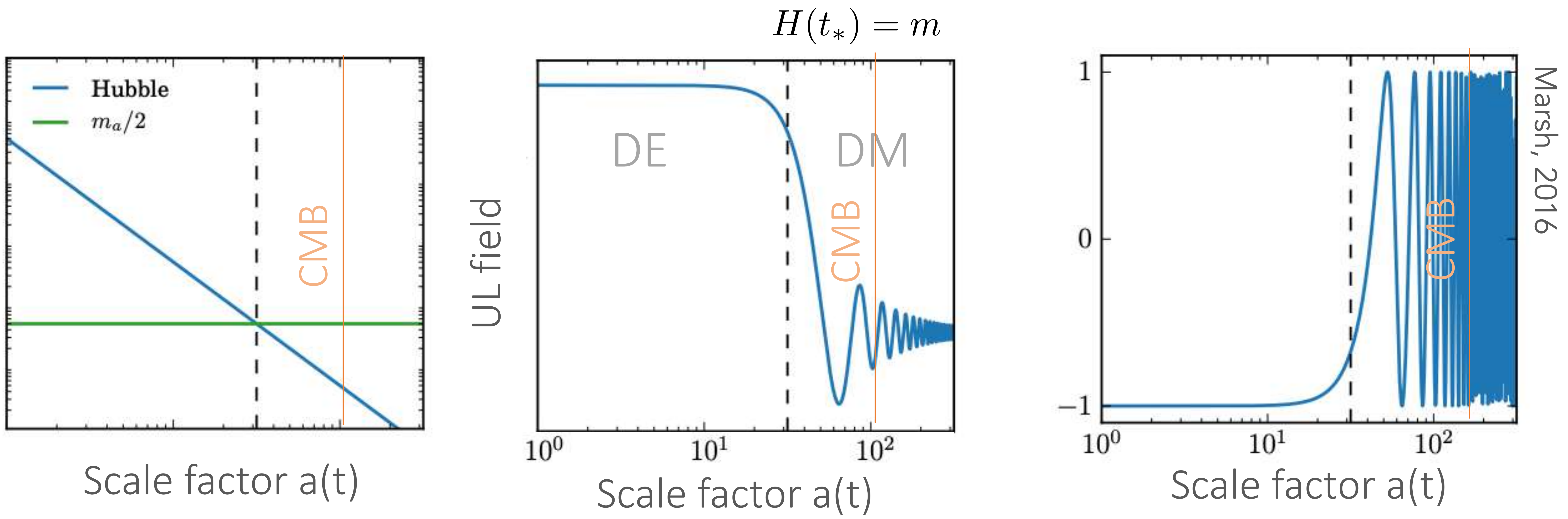


Marsh, 2016

Cosmological evolution



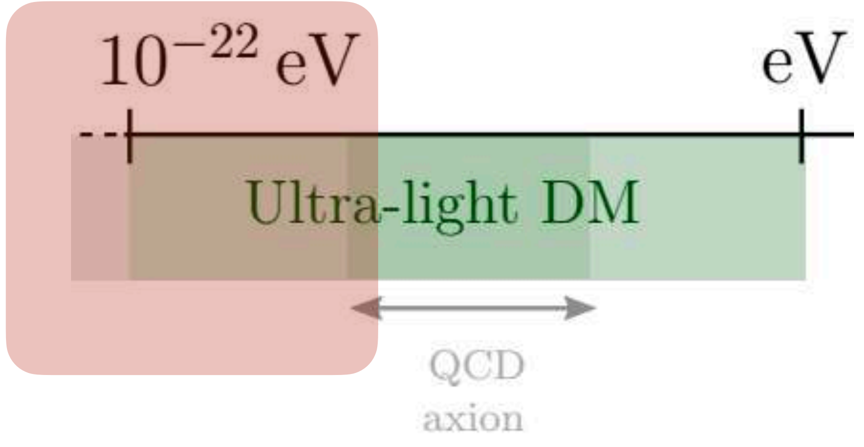
In order to behave like DM: start oscillating before matter-radiation equality



Marsh, 2016

$$m > 10^{-28}\,\mathrm{eV} \sim H(a_{\mathrm{eq}})$$

Structure formation - non-relativistic regime



Evolution on small scales: take non-relativistic regime of the theory, relevant for structure formation.

Schrödinger-Poisson system : describe the FDM and the SIFDM

$$\begin{cases} i\dot{\psi} = \left(-\frac{1}{2m}\nabla^2 + \boxed{\frac{g}{8m^2}|\psi|^2} - m\Phi \right)\psi \\ \nabla^2\Phi = 4\pi G(m|\psi|^2 - \bar{\rho}) \end{cases}$$

Schrödinger equation
(Gross-Pitaevskii)

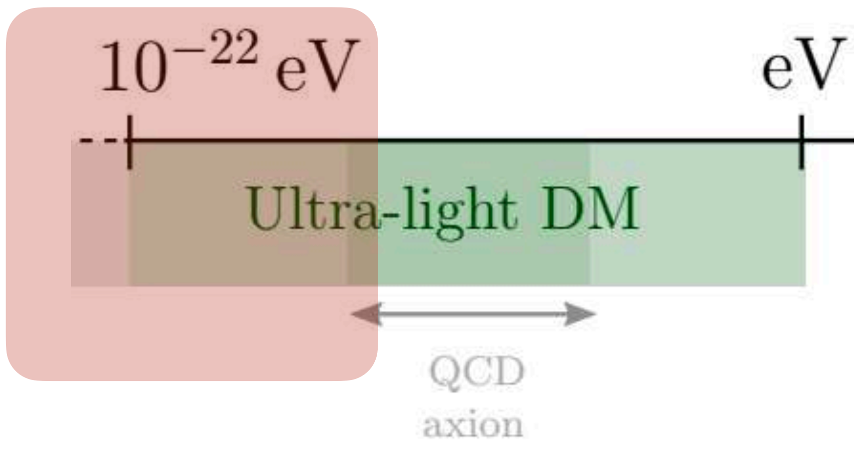
Poisson equation

$g = 0 \longrightarrow$ FDM

$g \neq 0 \longrightarrow$ SIFDM

Fundamentally different than
CDM/WDM/SIDM!

Structure formation - non-relativistic regime



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Schrödinger equation
(Gross-Pitaevskii)

Poisson equation

$g = 0$ → FDM

$g \neq 0$ → SIFDM

Fundamentally different than
CDM/WDM/SIDM!

Madelung equations $\quad (\psi \equiv \sqrt{\rho/m}\, e^{i\theta}$ and $\mathbf{v} \equiv \nabla\theta/m)$

$$P_{int} = K\rho^{(j+1)/j} = \frac{g}{2m^2}\rho^2$$

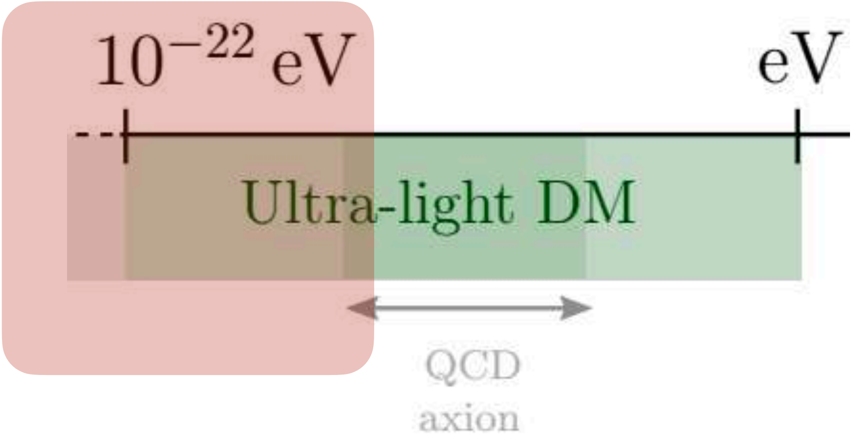
$$\dot{\rho} + \nabla\cdot(\rho\,\mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{m}\left(V_{grav} - \boxed{P_{int}} - \boxed{\frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}\right)$$

Quantum pressure

FLUID
DESCRIPTION

Structure formation - perturbation and stability



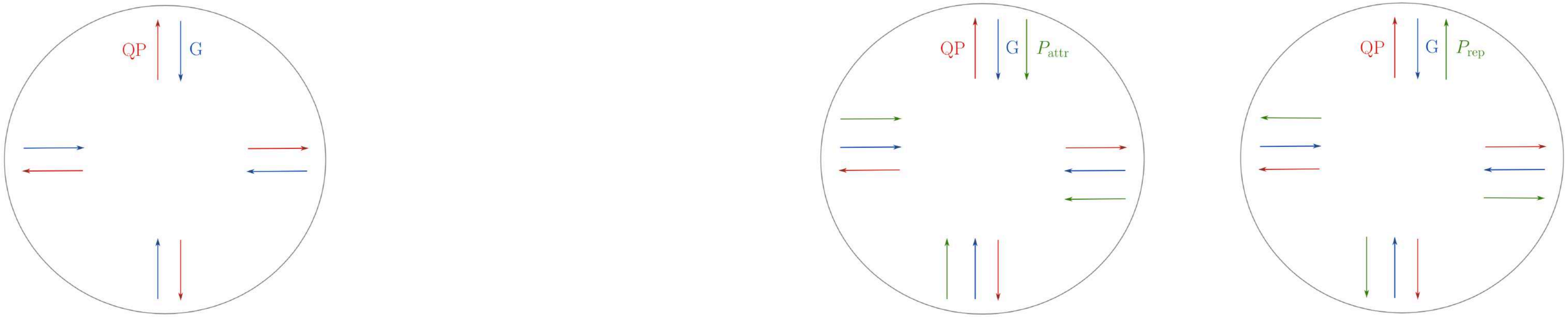
Competition between gravity and pressure (quantum pressure and interaction)

SIFDM

FDM

ATTRACTIVE

REPULSIVE



$g < 0$

$g > 0$

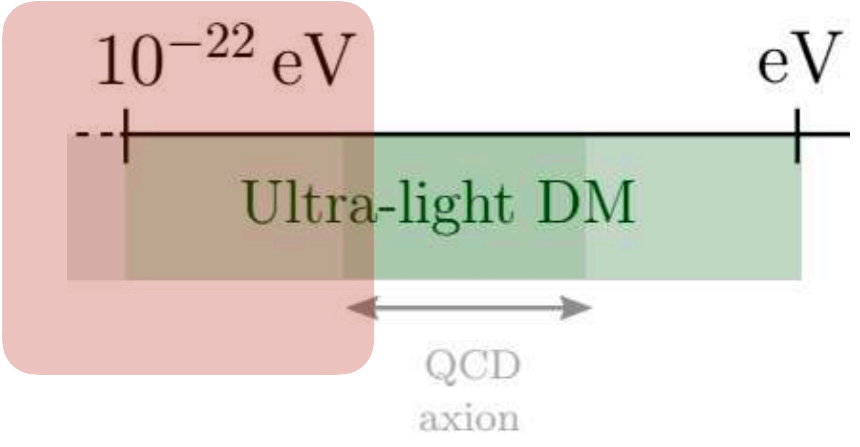
$P_{\rm int} = \frac{g}{2m^2}\rho^2$

$$\dot{\rho} + \nabla \cdot (\rho\,\mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{m}\left(V_{grav} - \boxed{P_{int}} - \boxed{\frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}}\right)$$

Quantum pressure

Structure formation - perturbation and stability



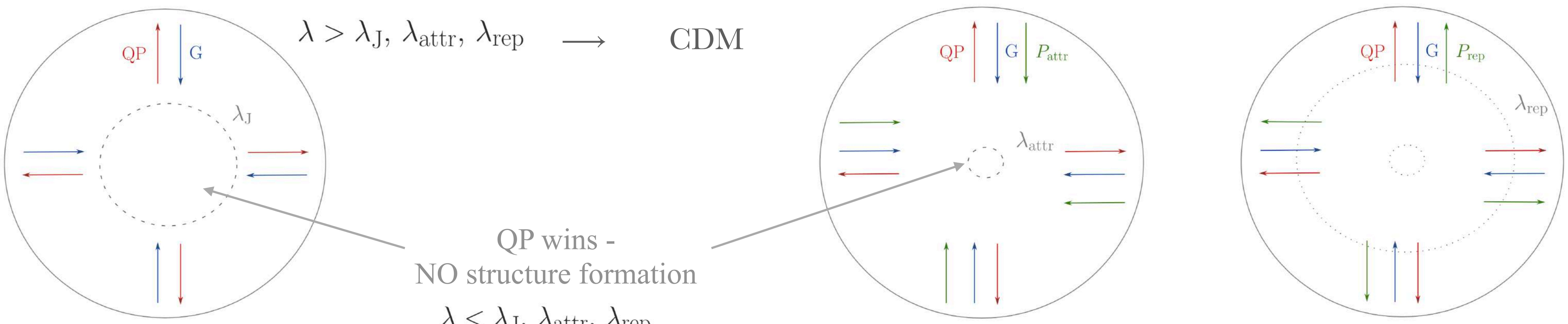
Finite clustering scale - no structure formation on small scales

SIFDM

FDM

ATTRACTIVE

REPULSIVE



$\lambda > \lambda_{\rm J},\ \lambda_{\rm attr},\ \lambda_{\rm rep} \quad \longrightarrow \quad$ CDM

QP wins -
NO structure formation

$\lambda < \lambda_{\rm J},\ \lambda_{\rm attr},\ \lambda_{\rm rep}$

$g < 0$

$g > 0$

Finite size coherent core – Bose stars

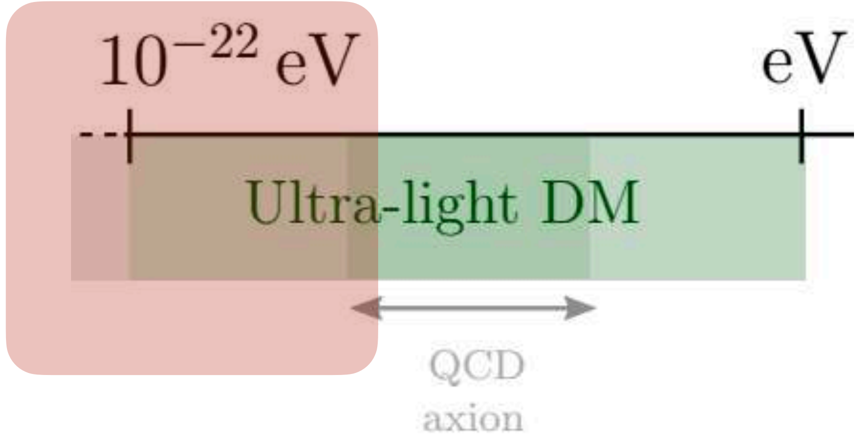
$$\lambda_J = 55 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-1/2} \left(\frac{\rho}{\bar{\rho}}\right)^{-1/4} (\Omega_m h)^{-1/4}\,\mathrm{kpc}$$

$m \leq 10^{-20}\mathrm{eV} \ \Rightarrow\ \lambda_{dB} > \mathcal{O}(\mathrm{kpc})$ Galactic scales

For attractive interactions can only form localized clumps (solitons)

QCD axion: $m \sim 10^{-5}\,\mathrm{eV}$
$\lambda_a \sim -10^{-48}$ $\longrightarrow$ $l_{soliton} \sim 10^{-5}\,\mathrm{kpc}$

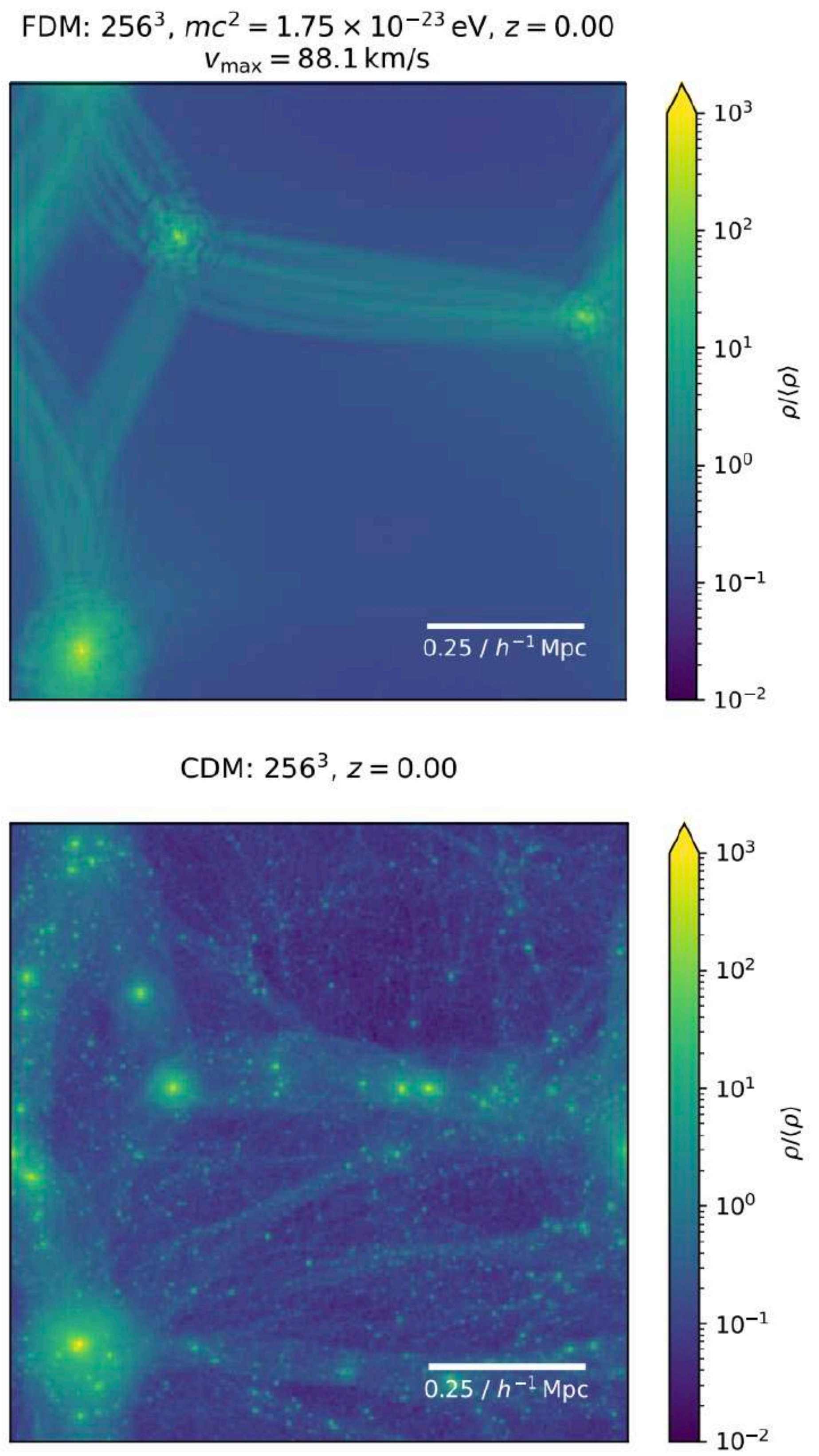
Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

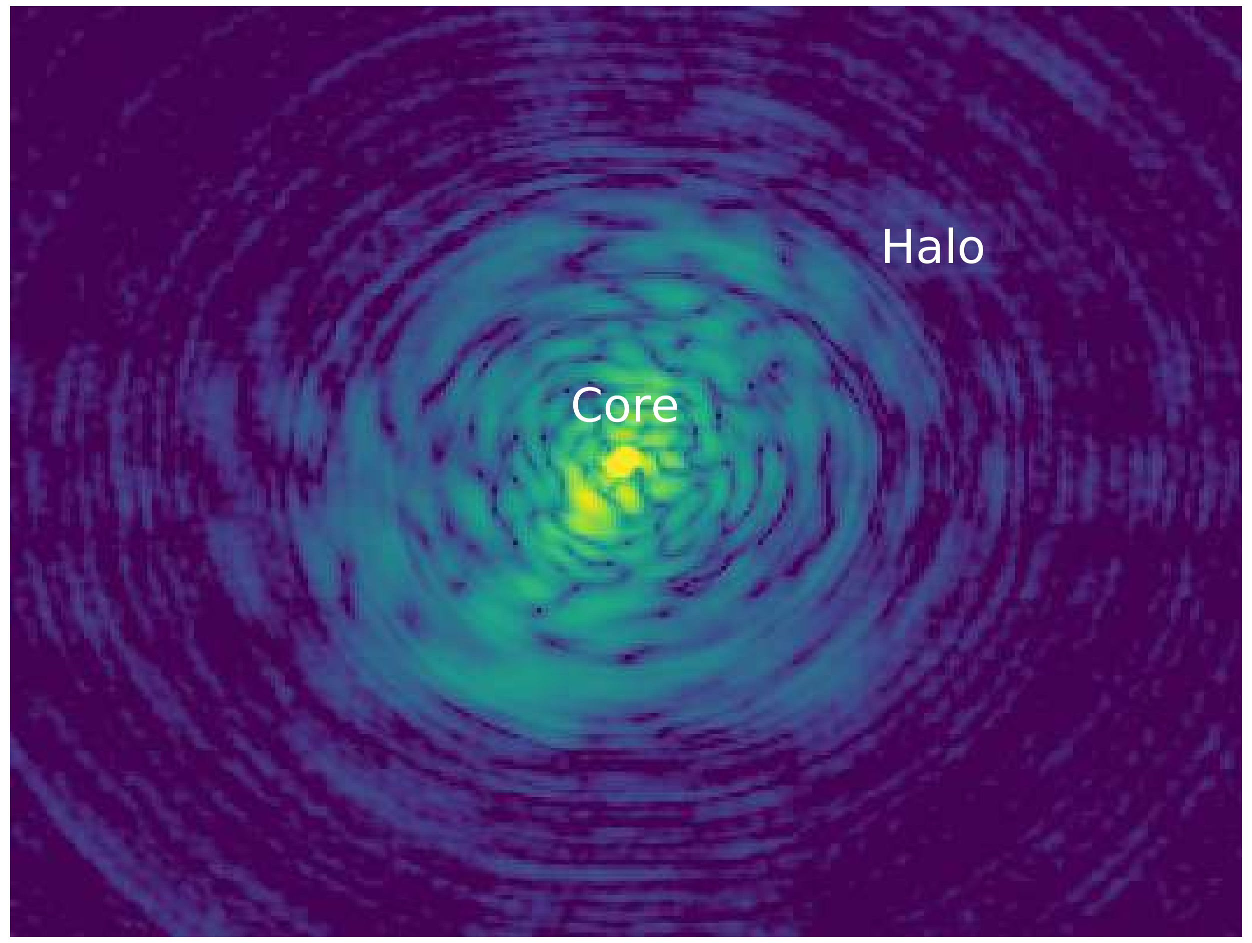
* Focus only in gravitational signatures

Suppression of small structures

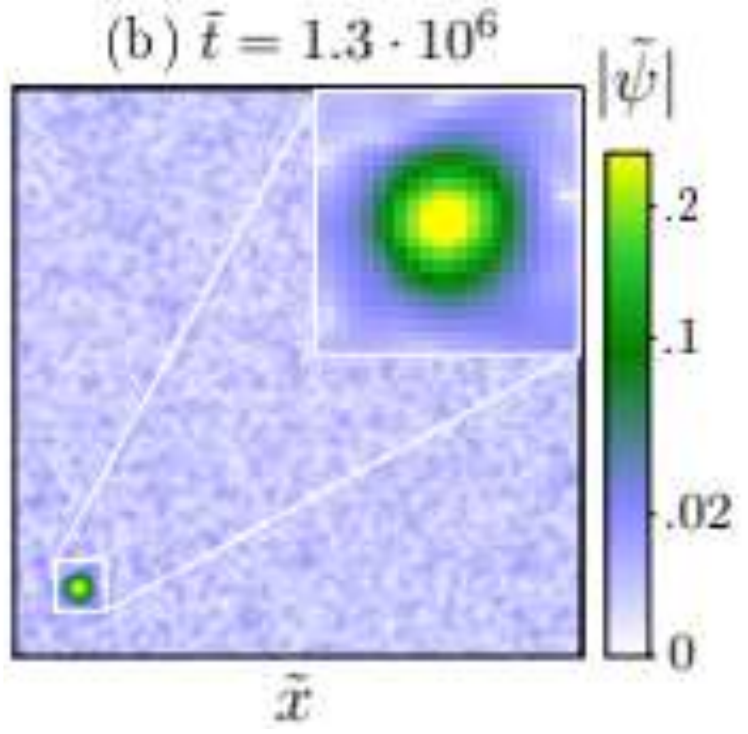


S. May et al. 2021

Formation of a solitonic core

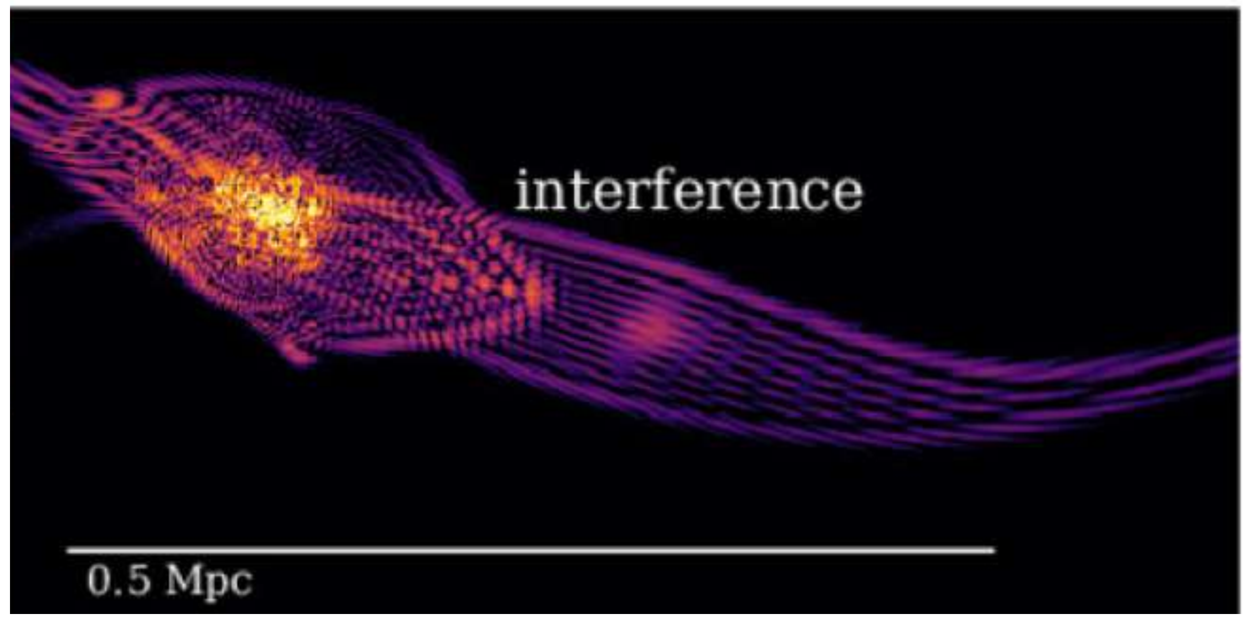


Dynamical effects



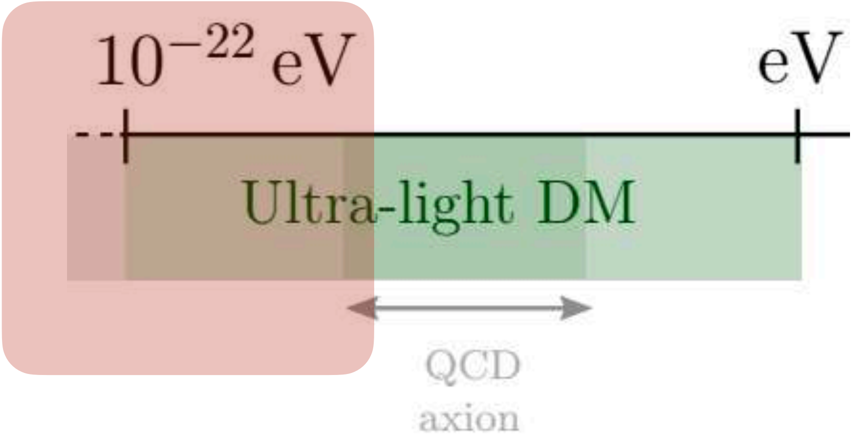
Levkov et al. 2018

Wave interference



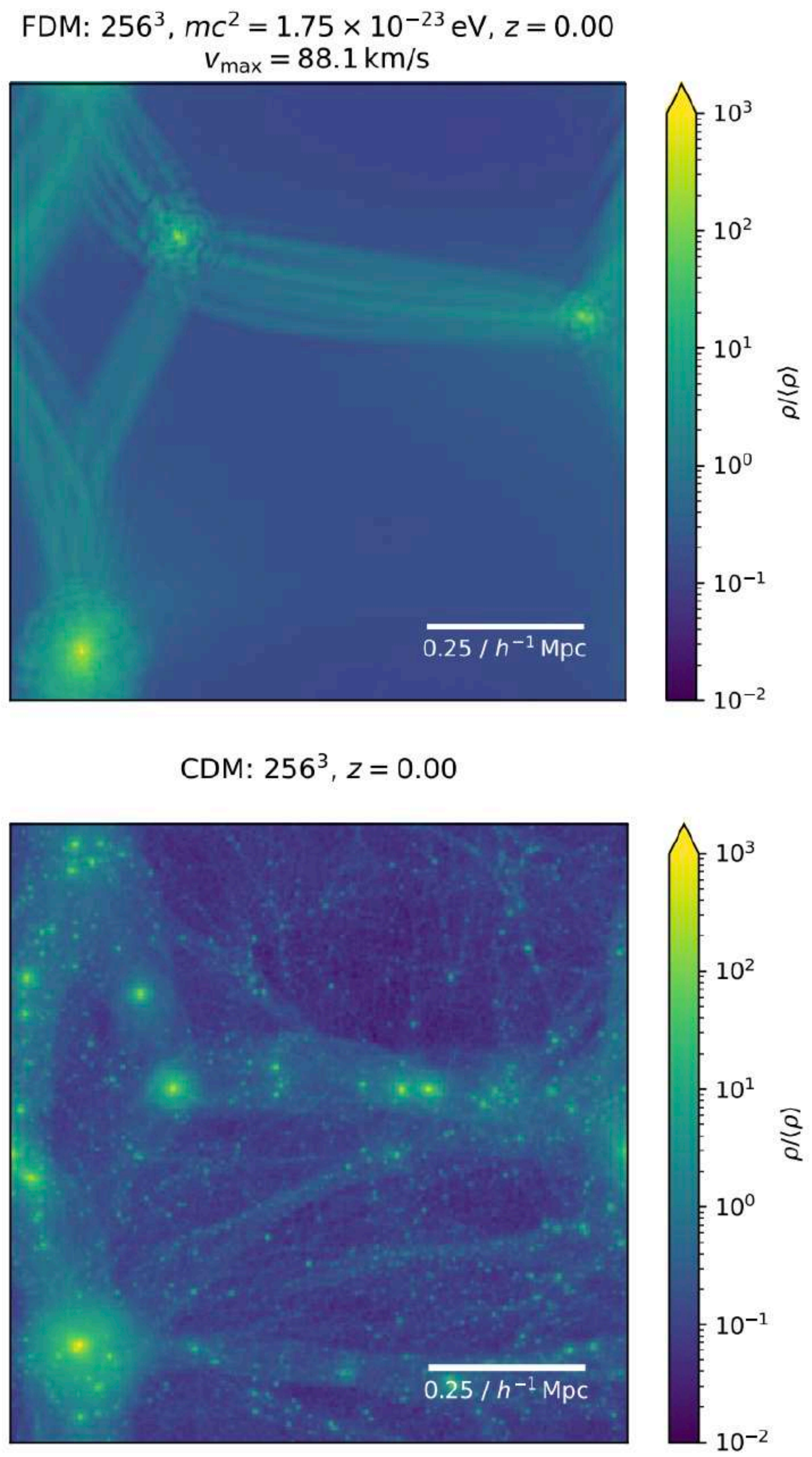
Mocz et al. 2017

Phenomenology



RICH PHENOMENOLOGY ON SMALL SCALES

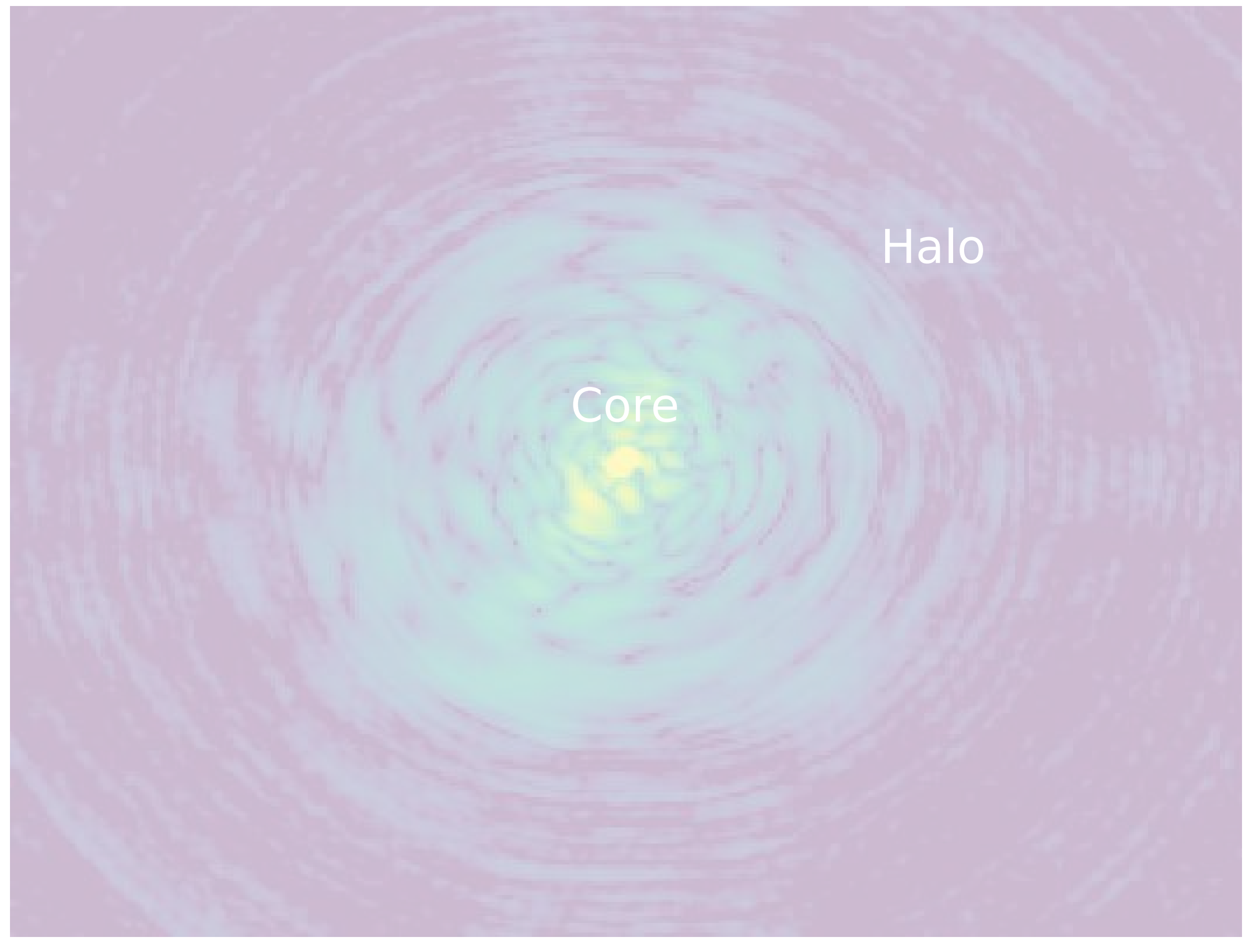
Suppression of small structures


FDM: 256^3, $mc^2 = 1.75 \times 10^{-23}$ eV, $z = 0.00$
$v_{\max} = 88.1$ km/s

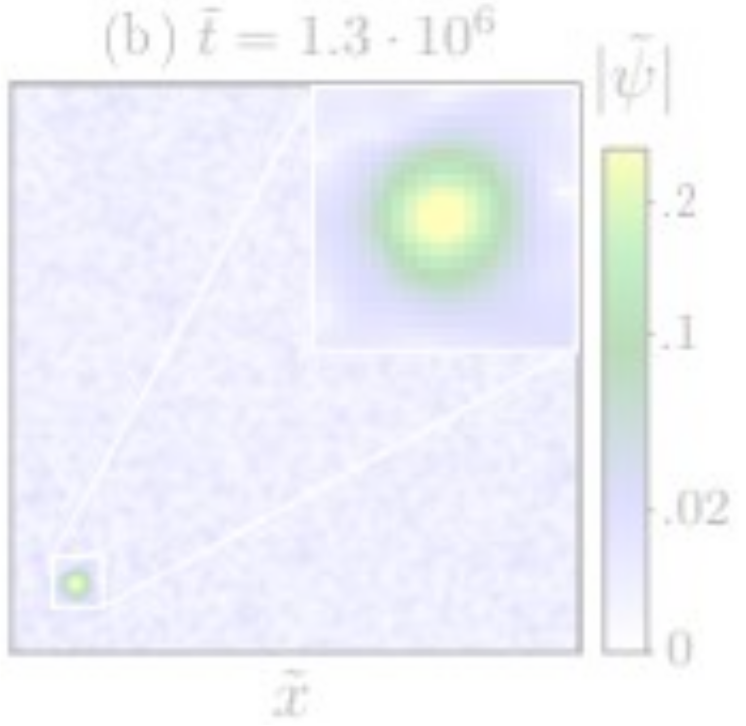
CDM: 256^3, $z = 0.00$

S. May et al. 2021

Formation of a solitonic core



Dynamical effects



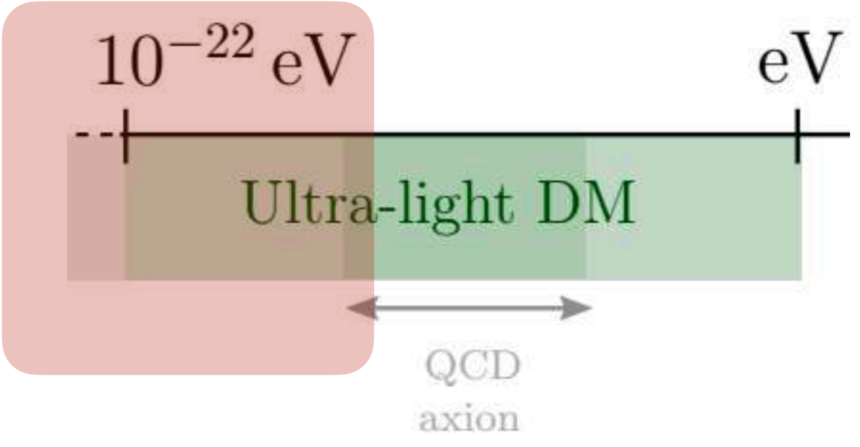
Levkov et al. 2018

Wave interference

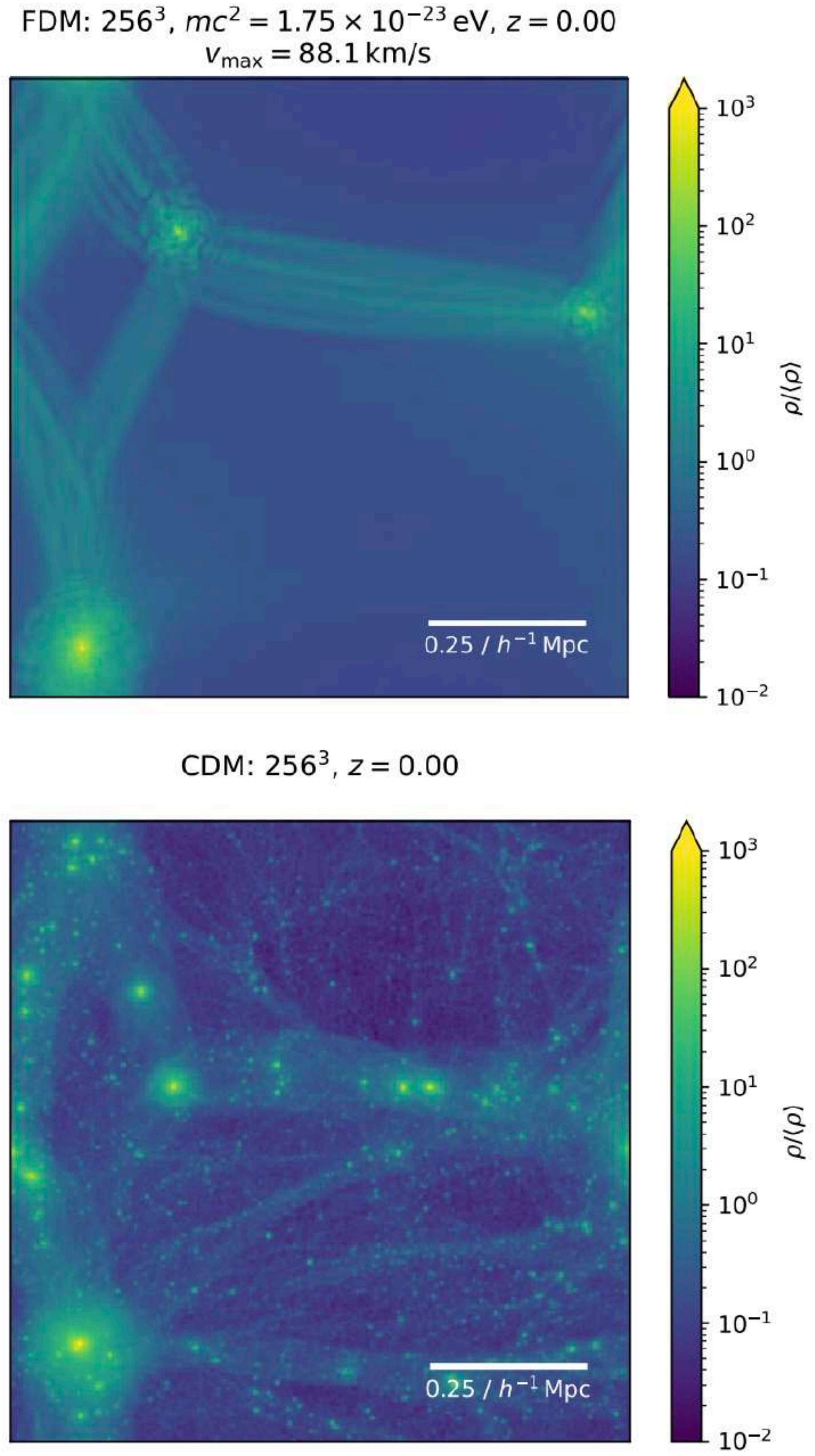
Mocz et al. 2017

Phenomenology

Suppression of small structures



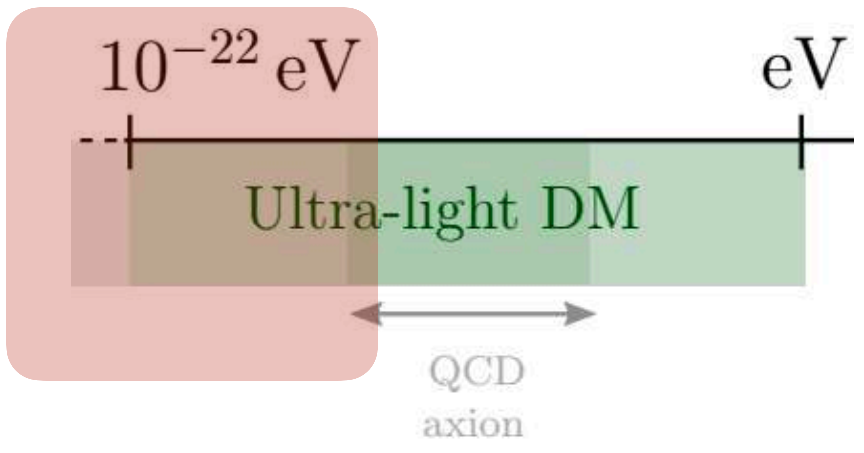
Finite Jeans length λ_J or $\lambda_{\mathrm{attr}}, \lambda_{\mathrm{rep}}$ → No small scale structure



S. May et al. 2021

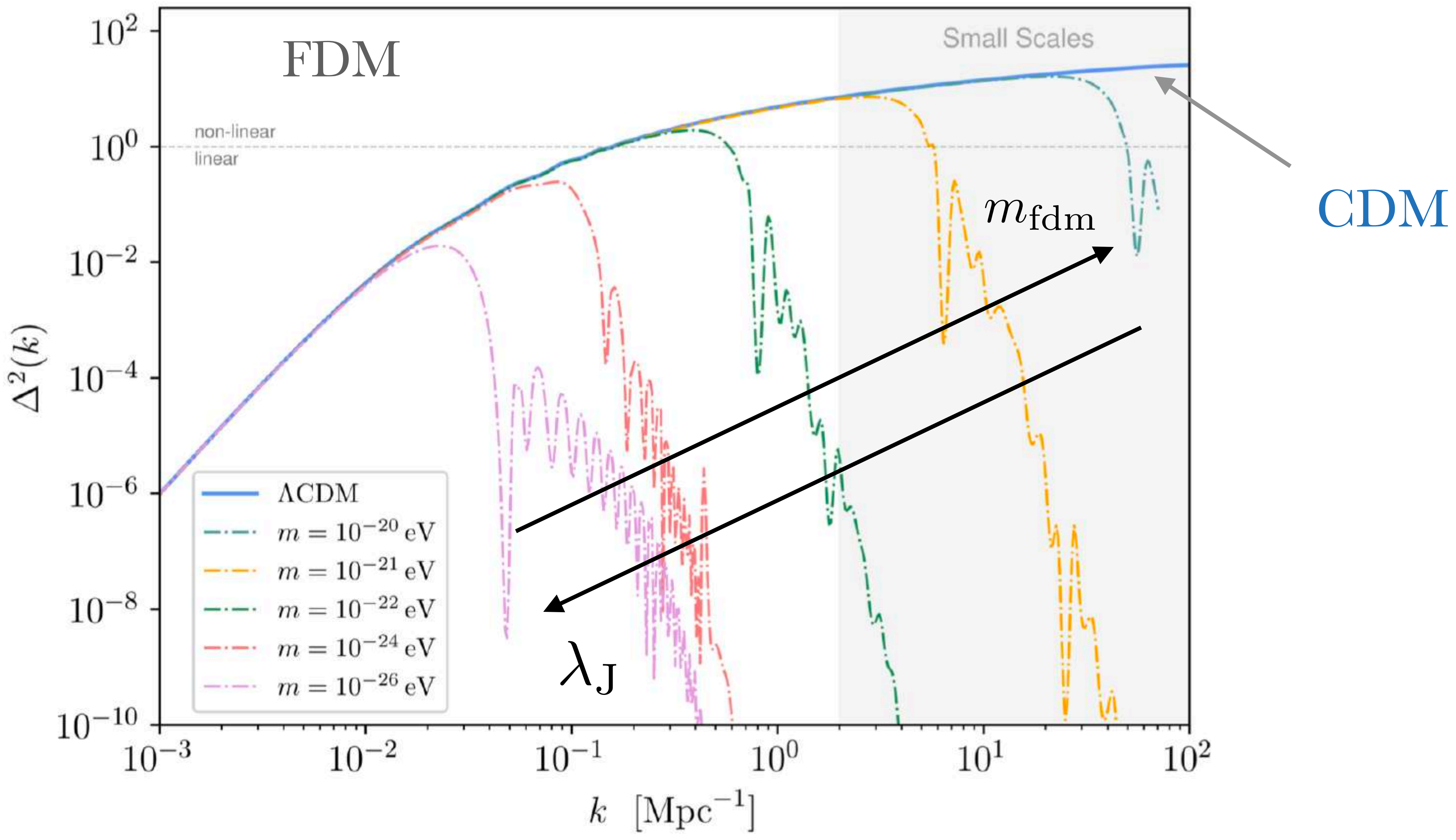
Phenomenology

Suppression of small structures

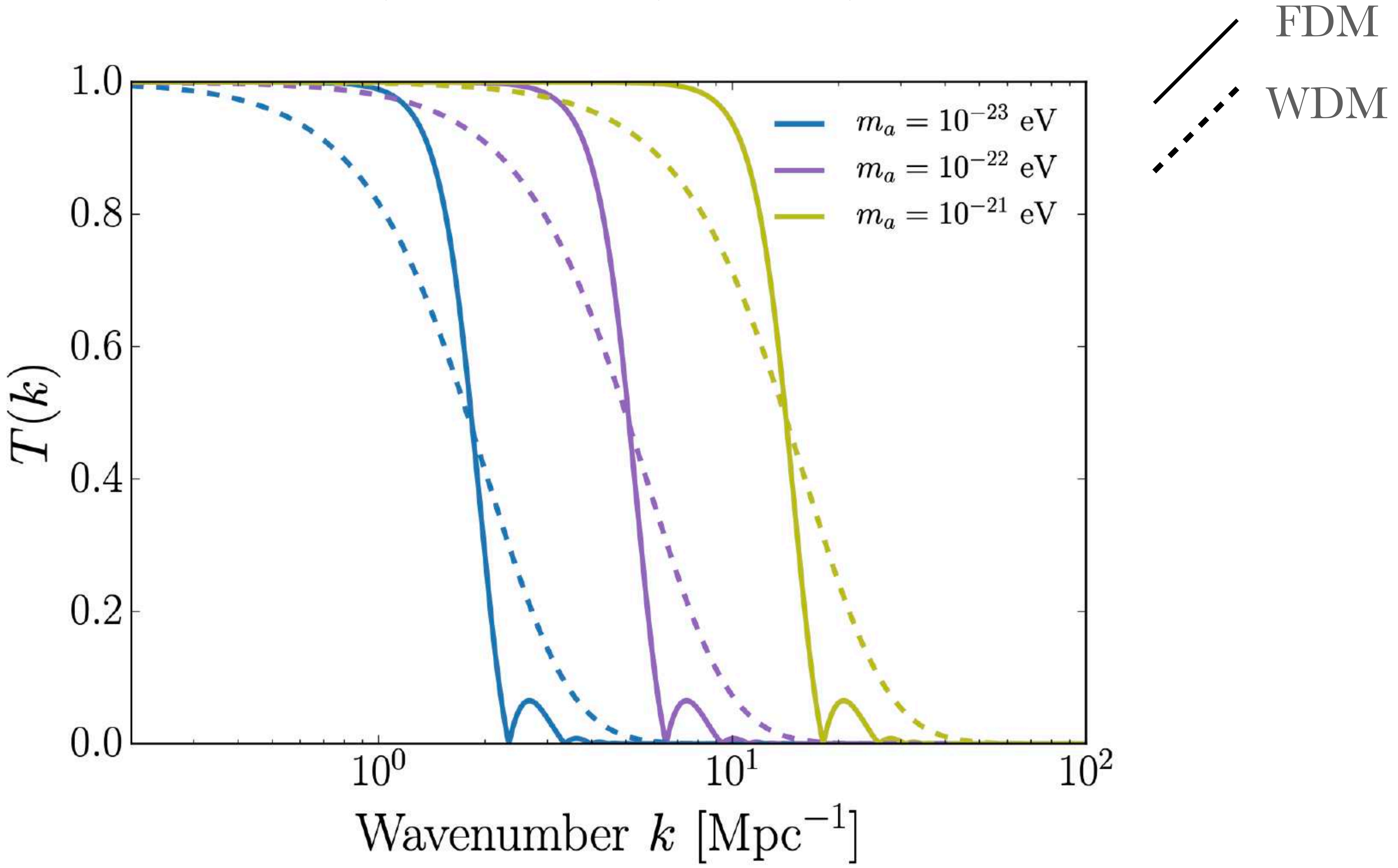


Finite Jeans length λ_J or $\lambda_{attr}, \lambda_{rep}$ → Suppresses small scale structure

POWER SPECTRUM



TRANSFER FUNCTION



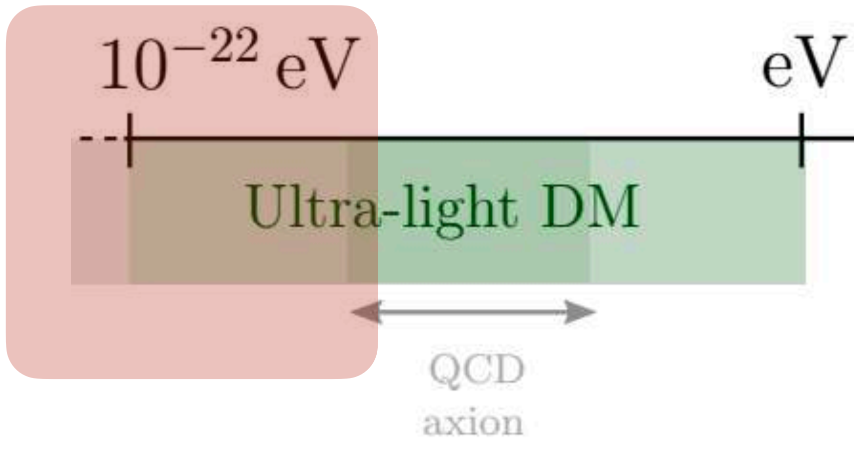
$$P_X(k,z) = T_X^2(k,z)\, P_{\Lambda CDM}(k)$$

$$\begin{cases} T_{WDM} = [1 + (\alpha k)^{2\mu}]^{-5/\mu} \\ T_{FDM} = \frac{\cos x_J^3(k)}{1+x_J^8(k)} \end{cases}$$

- Degenerate with WDM

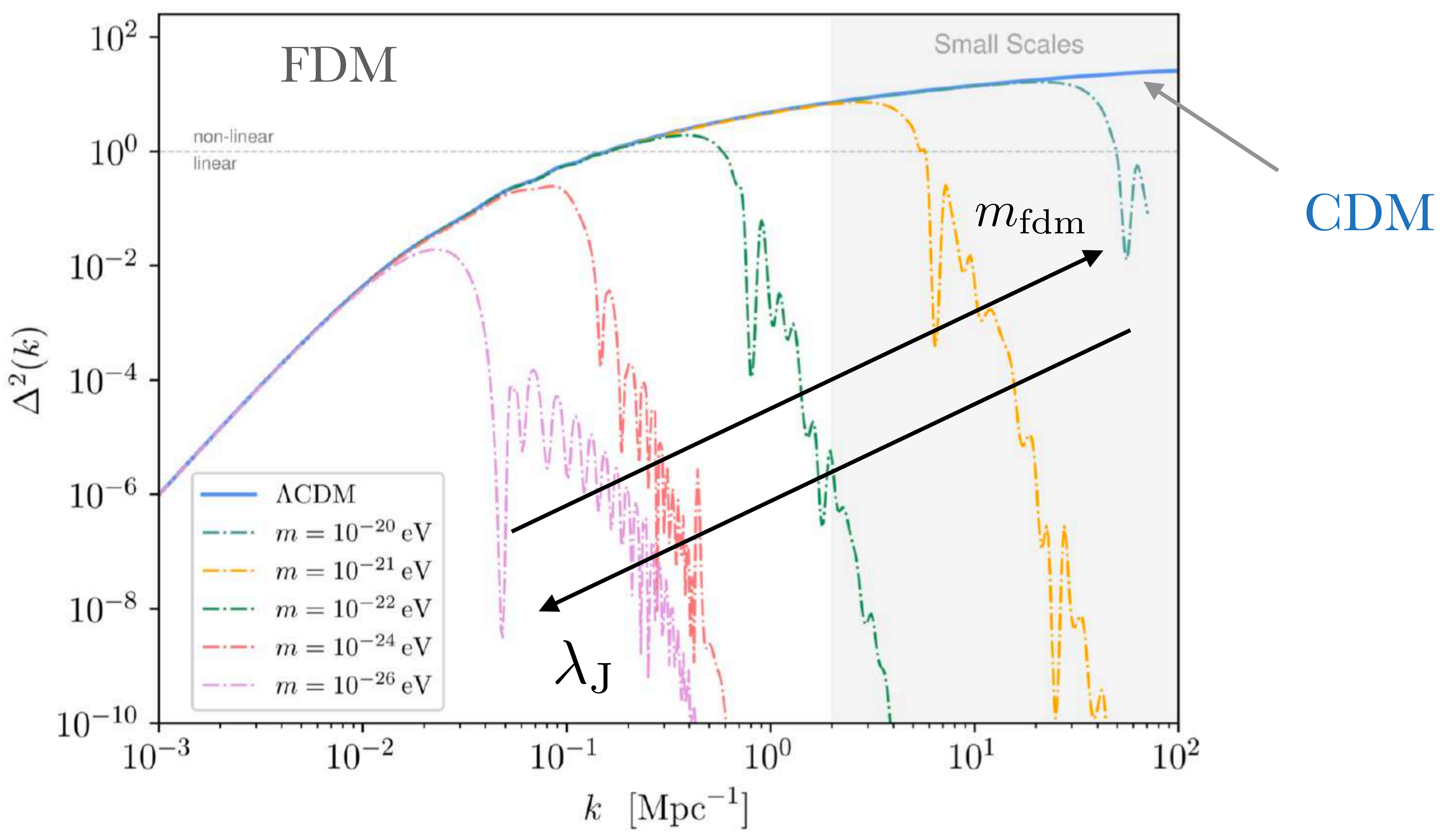
Phenomenology

Suppression of small structures

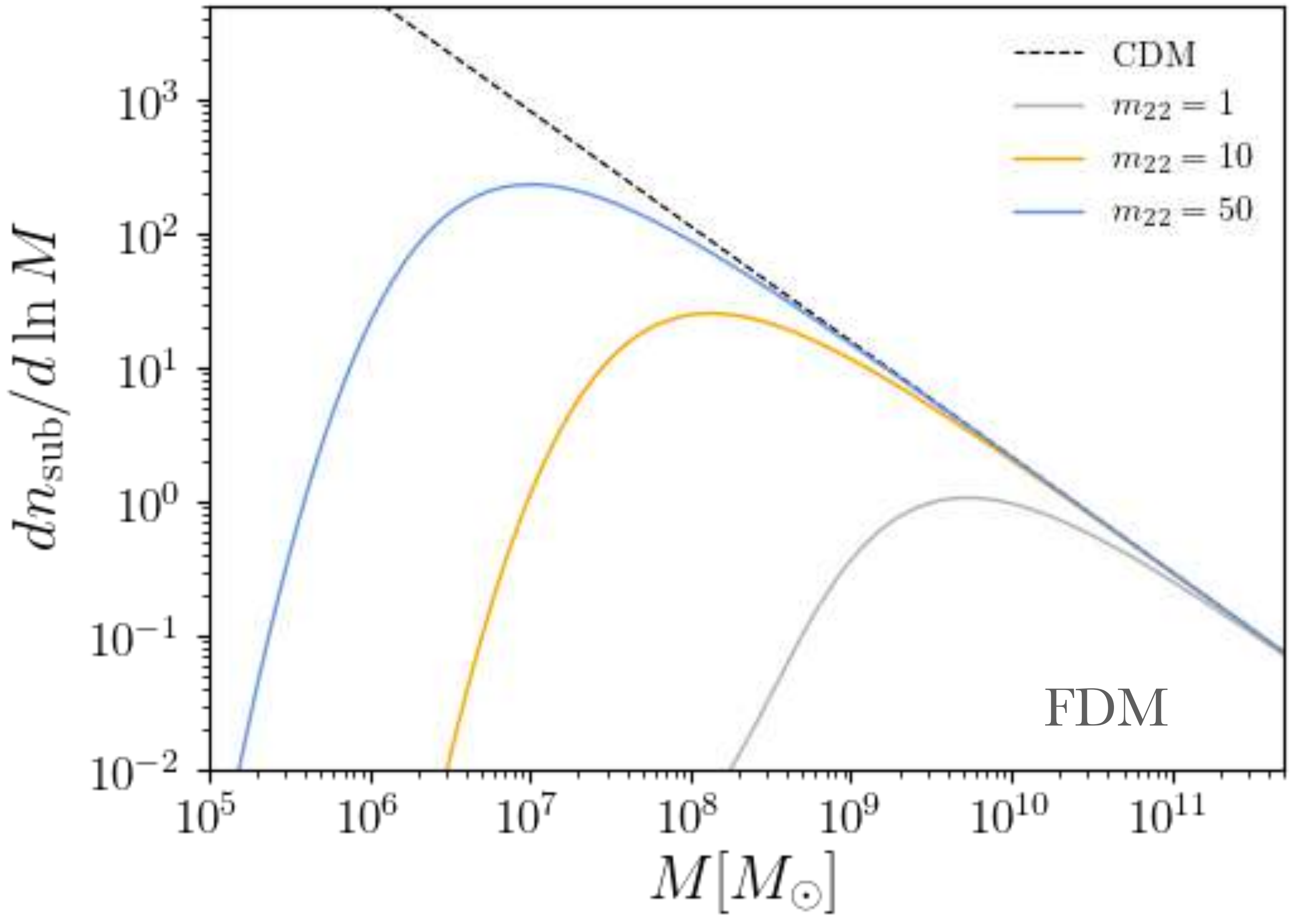


Finite Jeans length λ_J or $\lambda_{\text{attr}}, \lambda_{\text{rep}}$ → Suppresses small scale structure

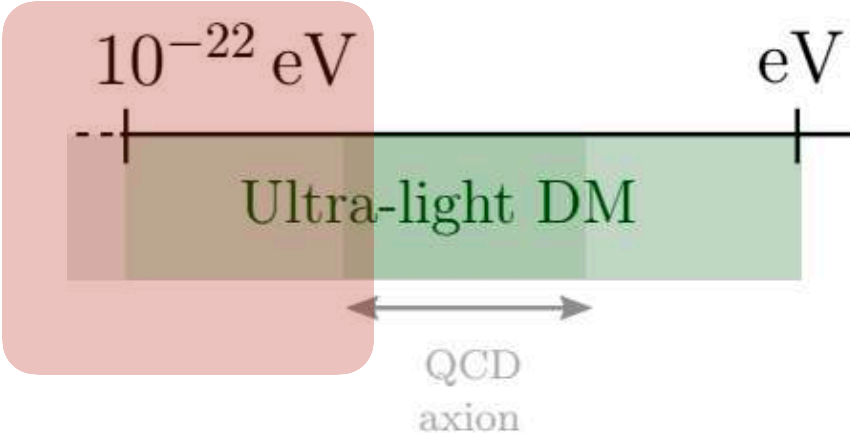
POWER SPECTRUM



(sub) HALO MASS FUNCTION

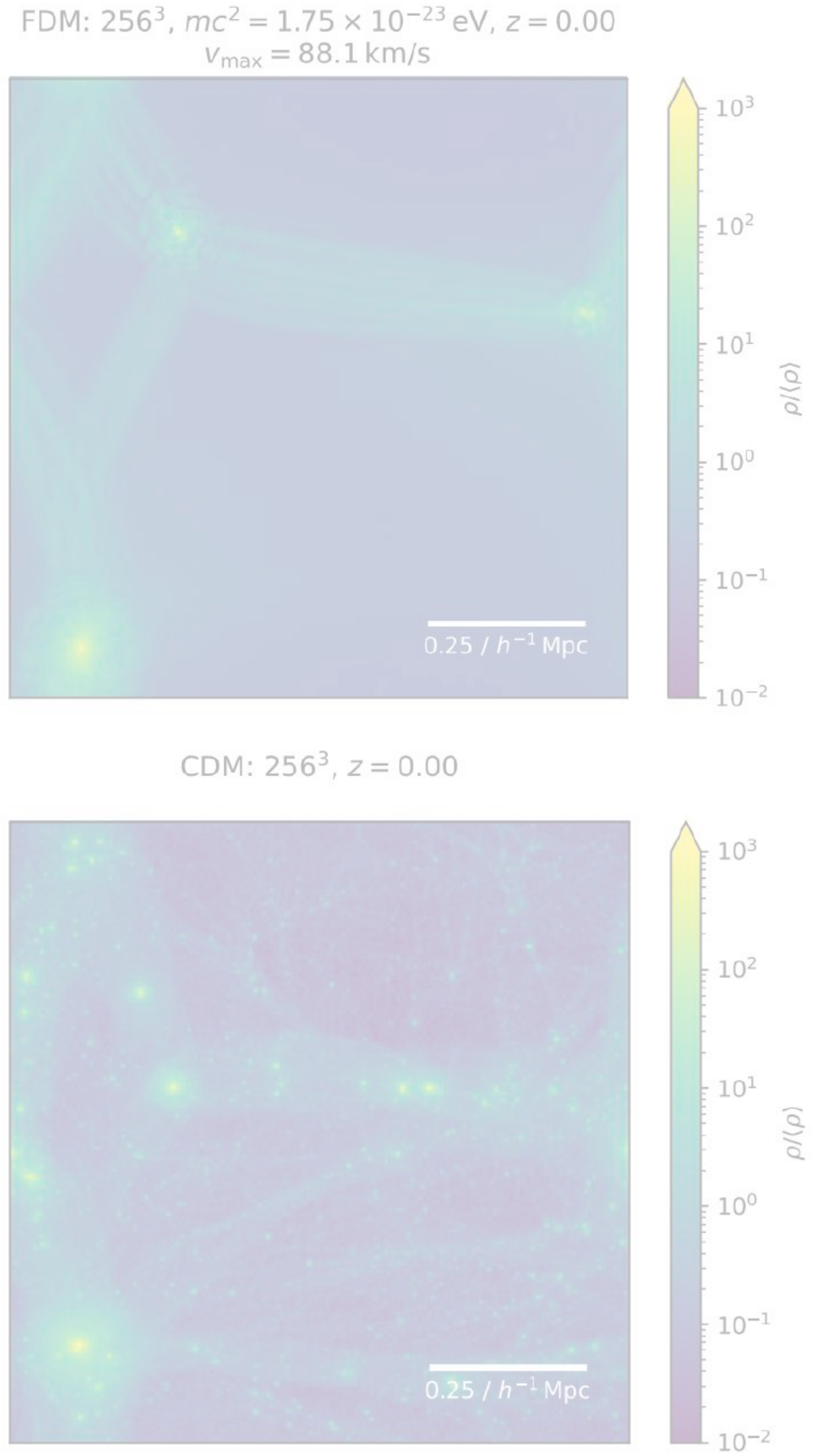


Phenomenology



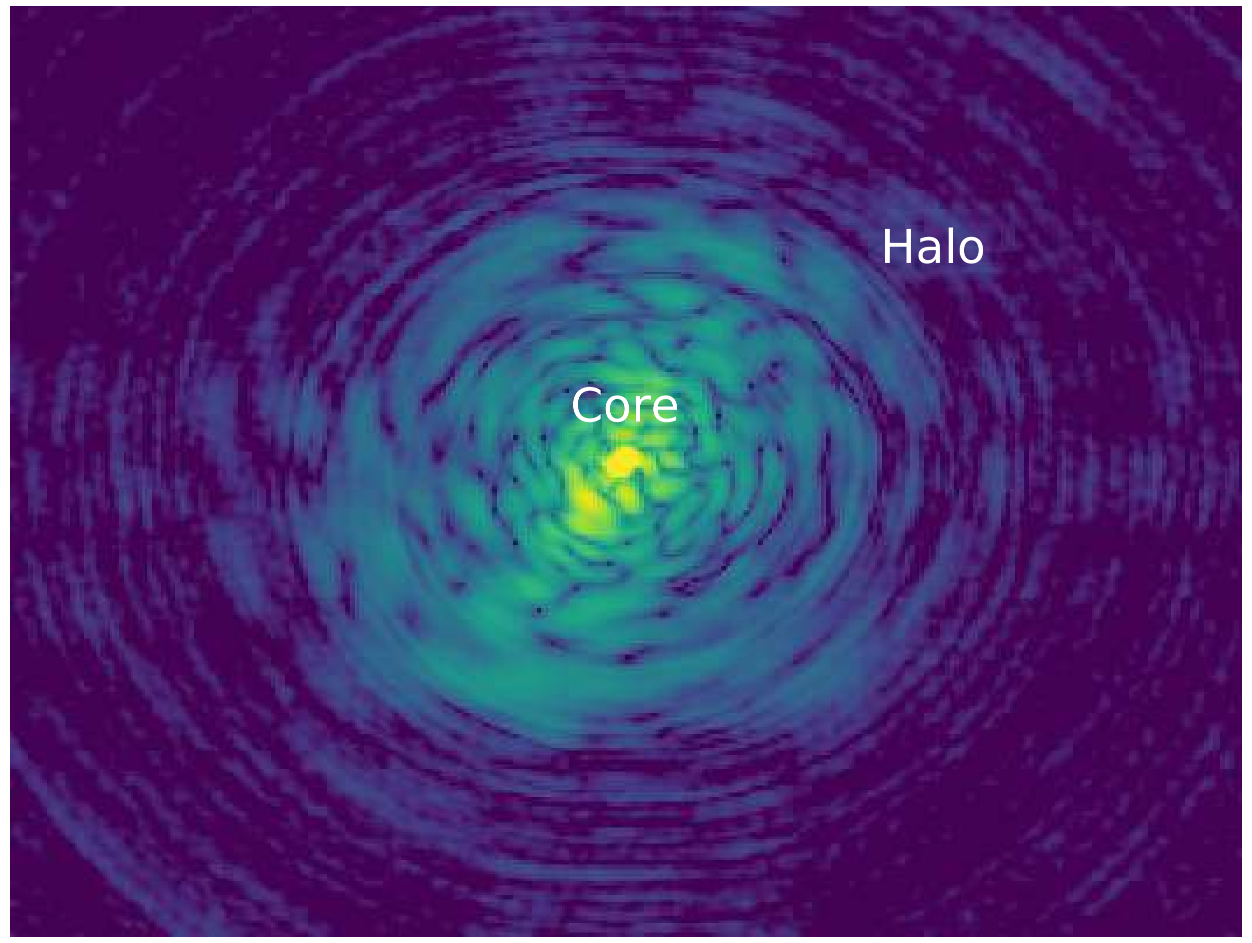
RICH PHENOMENOLOGY ON SMALL SCALES

Suppression of small structures

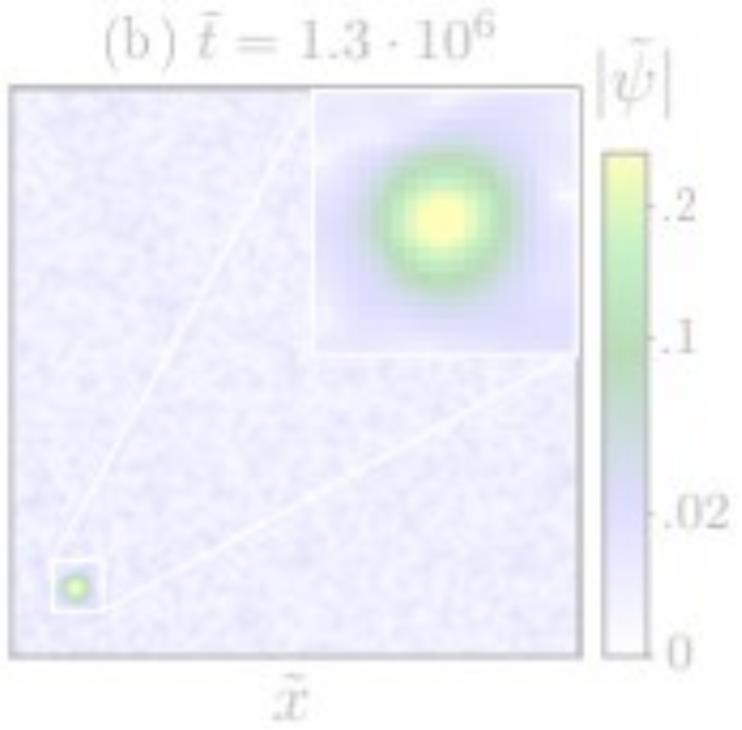


S. May et al. 2021

Formation of a solitonic core



Dynamical effects



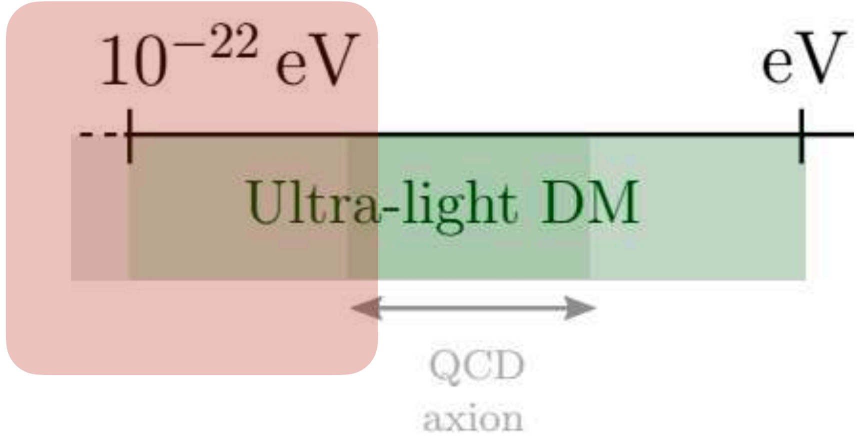
Levkov et al. 2018

Wave interference

Mocz et al. 2017

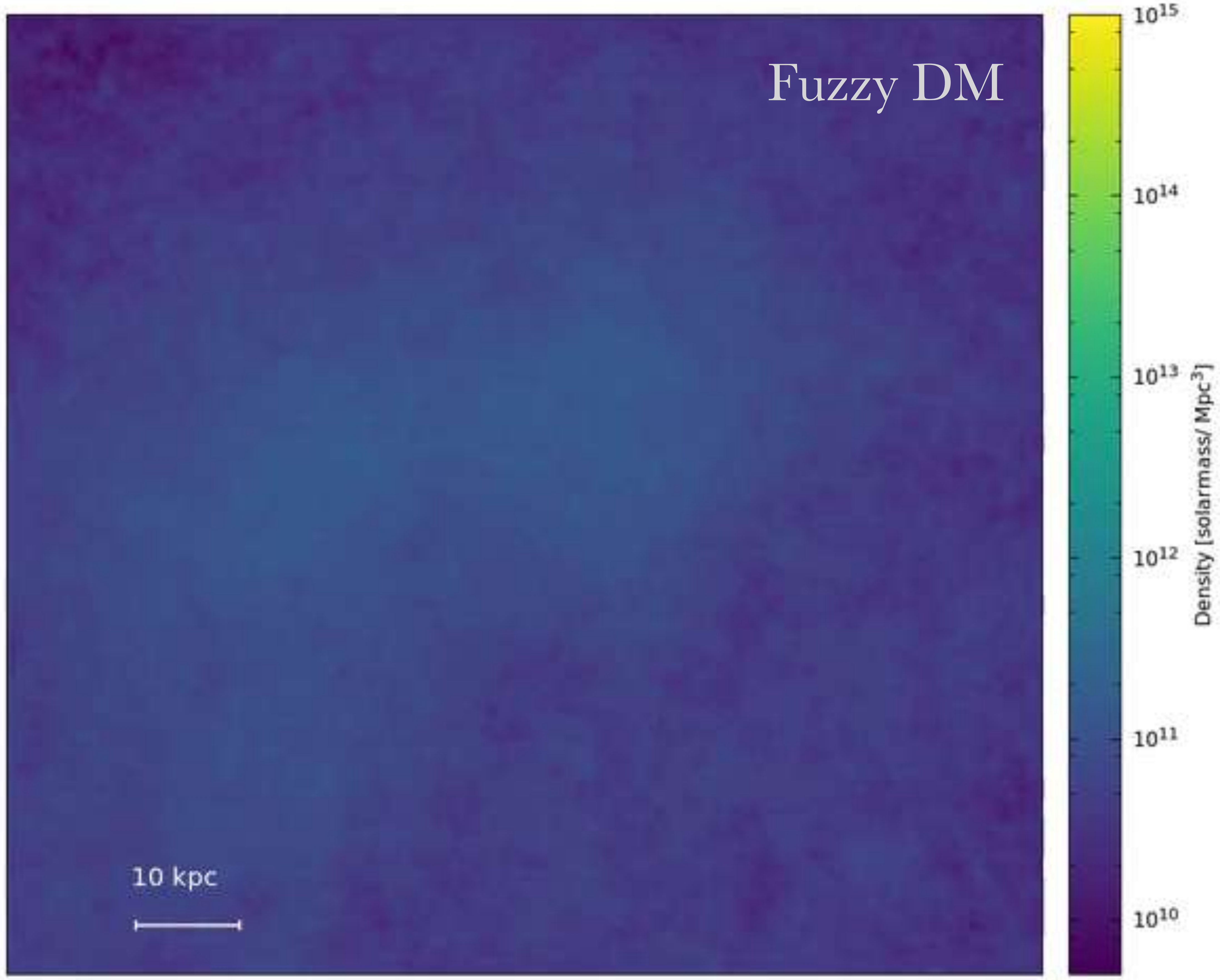
Phenomenology

Formation of cores

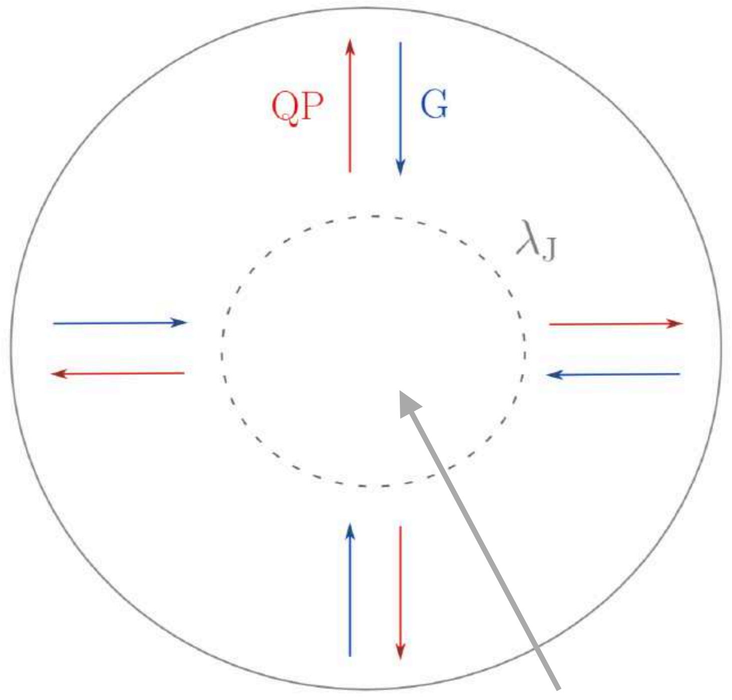


$m = 10^{-22}\,\mathrm{eV}$ $N = 512^3$ $L = 300\,\mathrm{kpc}$

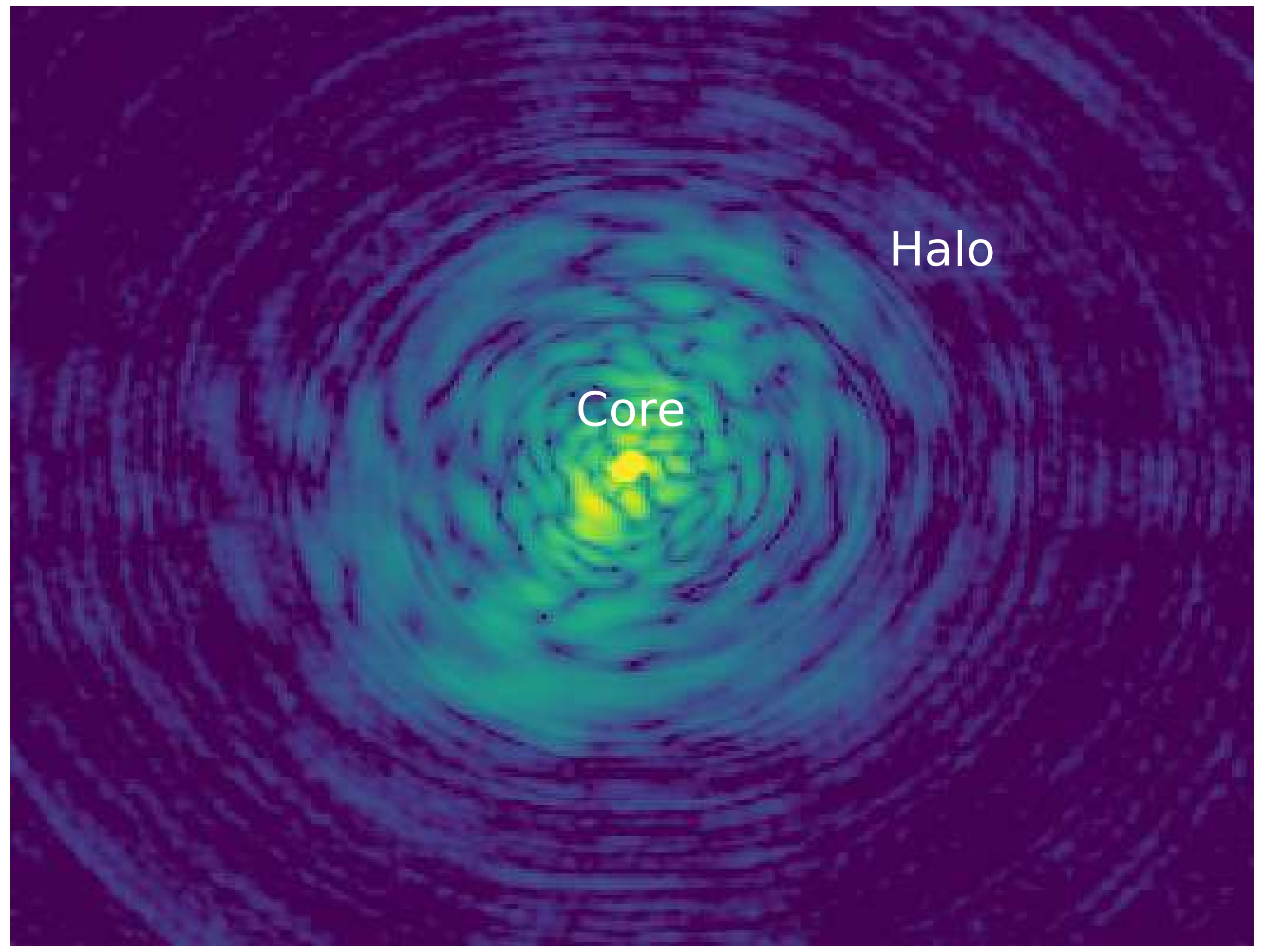
NON-LINEAR evolution: need simulations



Simulation by Jowett Chan

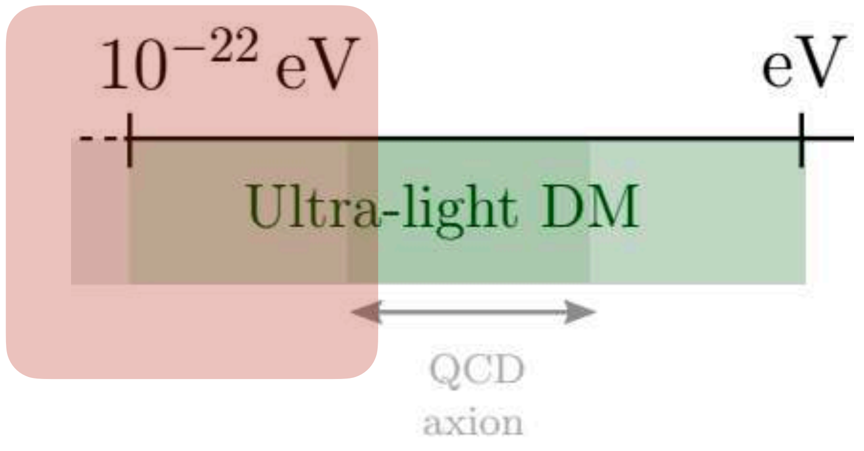


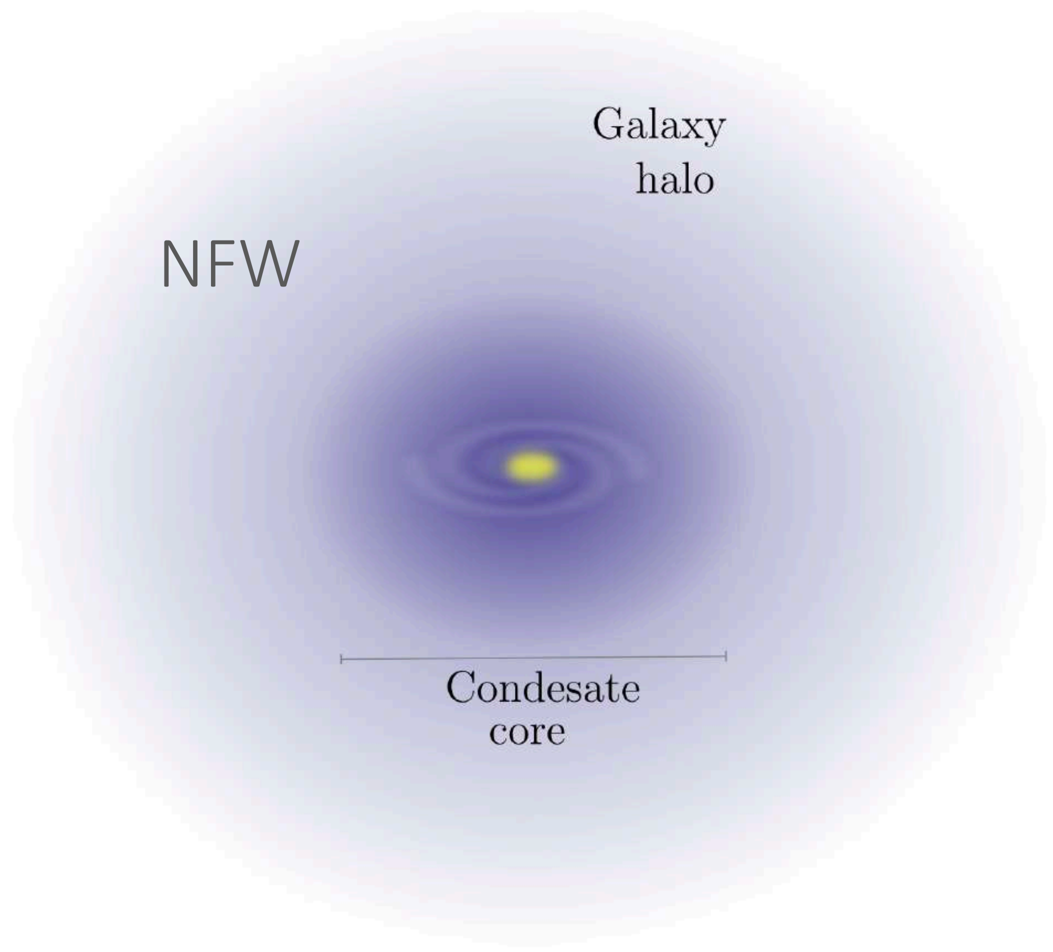
NO structure formation
Stable, oscillating solution



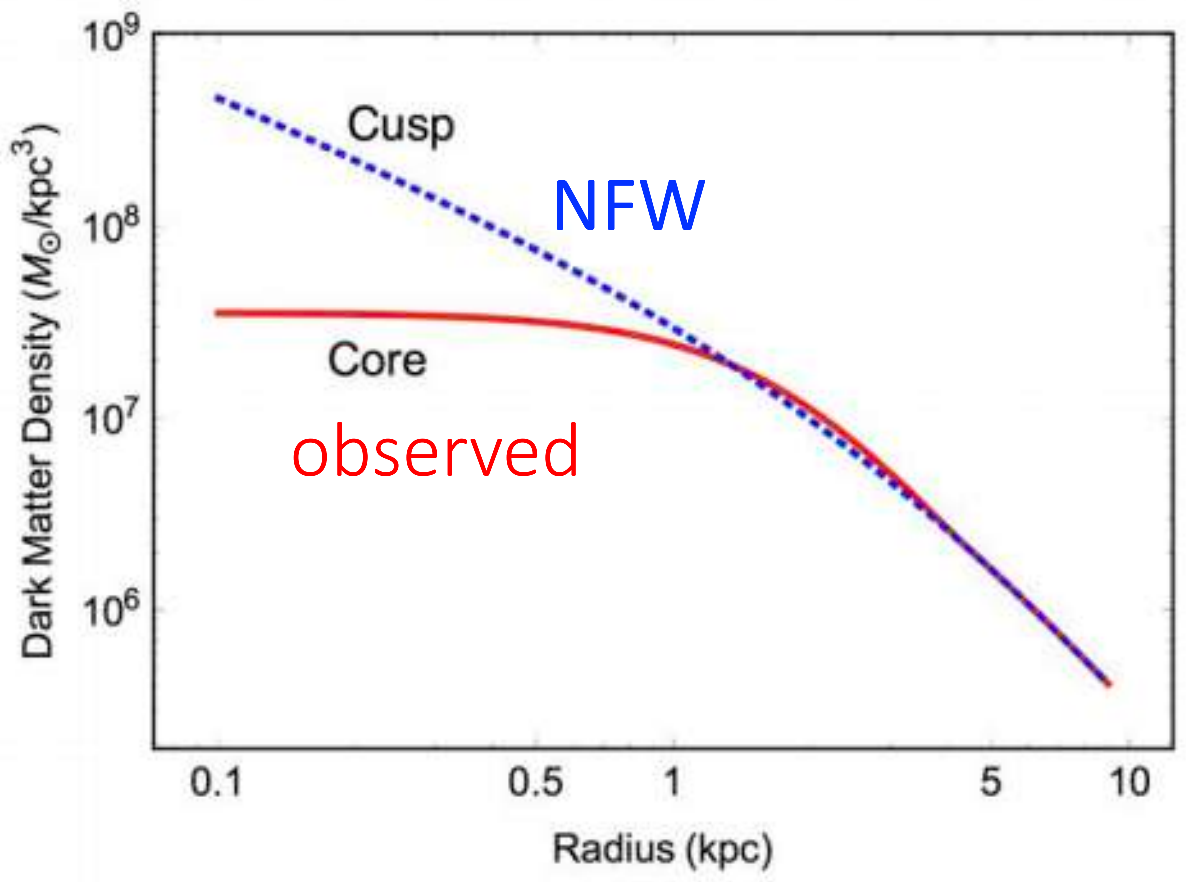
Phenomenology

Formation of cores





$$\rho(r) \simeq \begin{cases} \rho_c & \text{for } r \leq r_c \\ \rho_{\rm NFW} & \text{for } r \geq r_c \end{cases}$$



FDM From simulations Schive et al. 2014, fitting function: Stable core solution

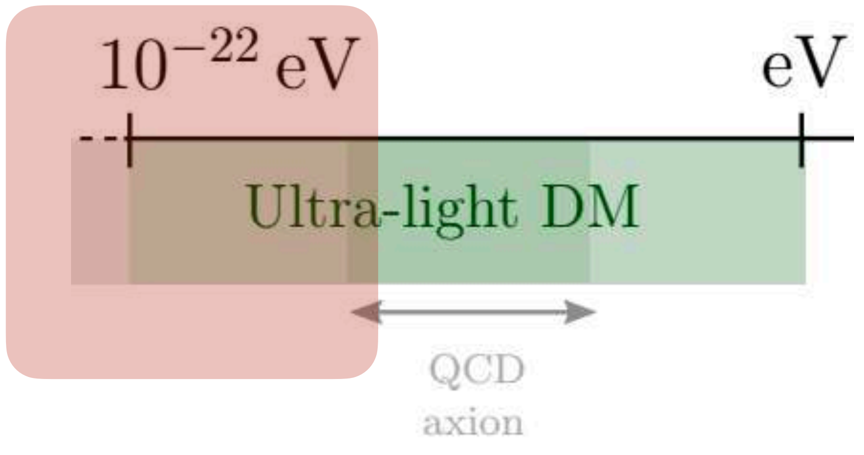
$$\rho_c \simeq \frac{1.9\times10^{-2}}{[1+0.091\,(r/R_{1/2,c})^2]^8}\left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-2}\left(\frac{r_c}{\mathrm{kpc}}\right)^{-4} M_\odot\,\mathrm{pc}^{-3}\,,$$

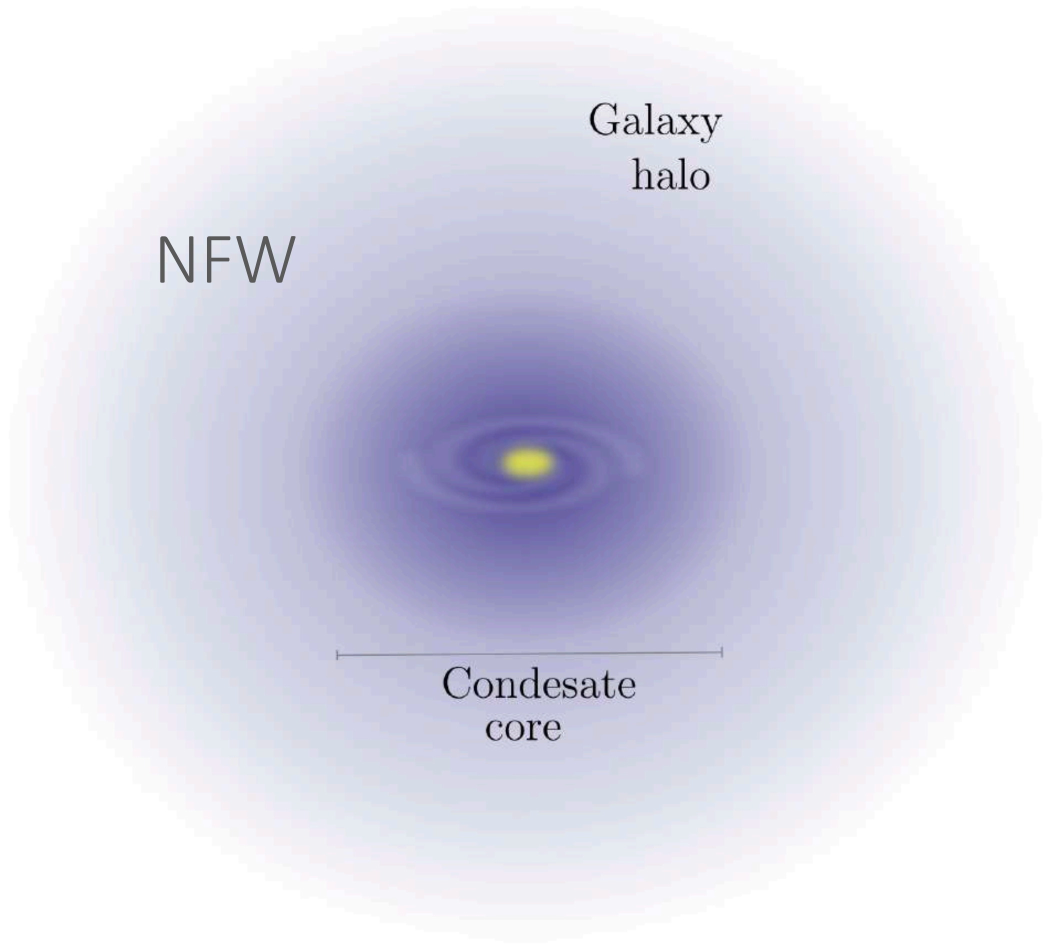
$$r_c \simeq 0.16\left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-1}\left(\frac{M}{10^{12}\,M_\odot}\right)^{-1/3}\mathrm{kpc}\,.$$

Relations used to compare
with observations

Phenomenology

Formation of cores





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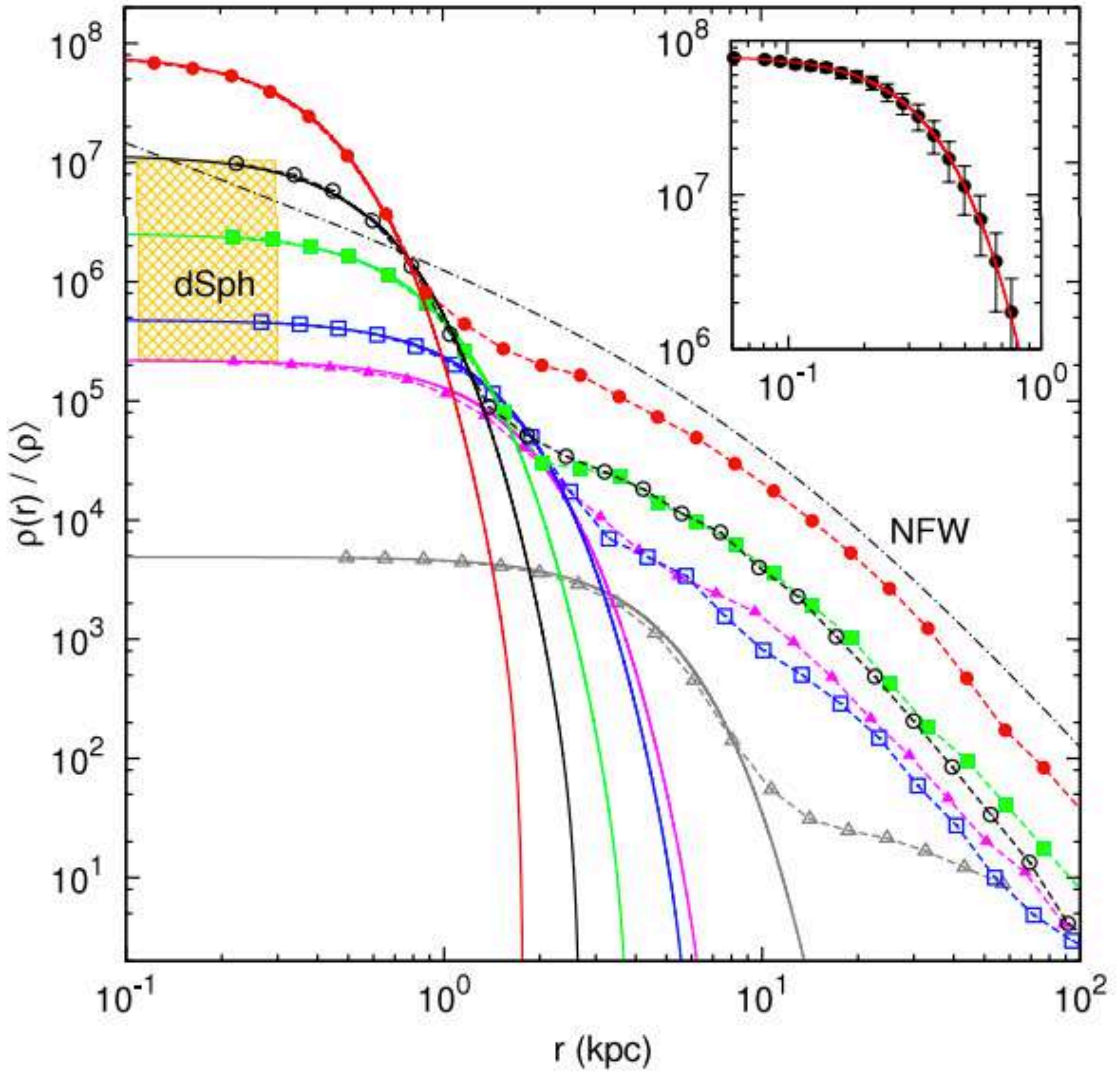
Schive et al. 2014



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Relations used to compare
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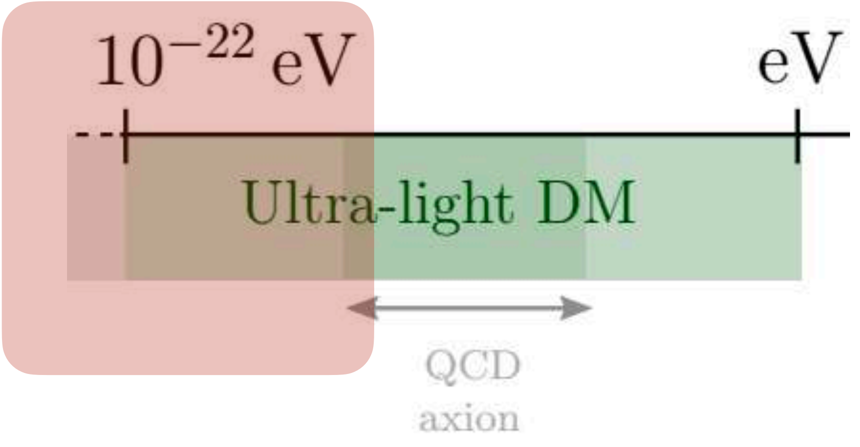
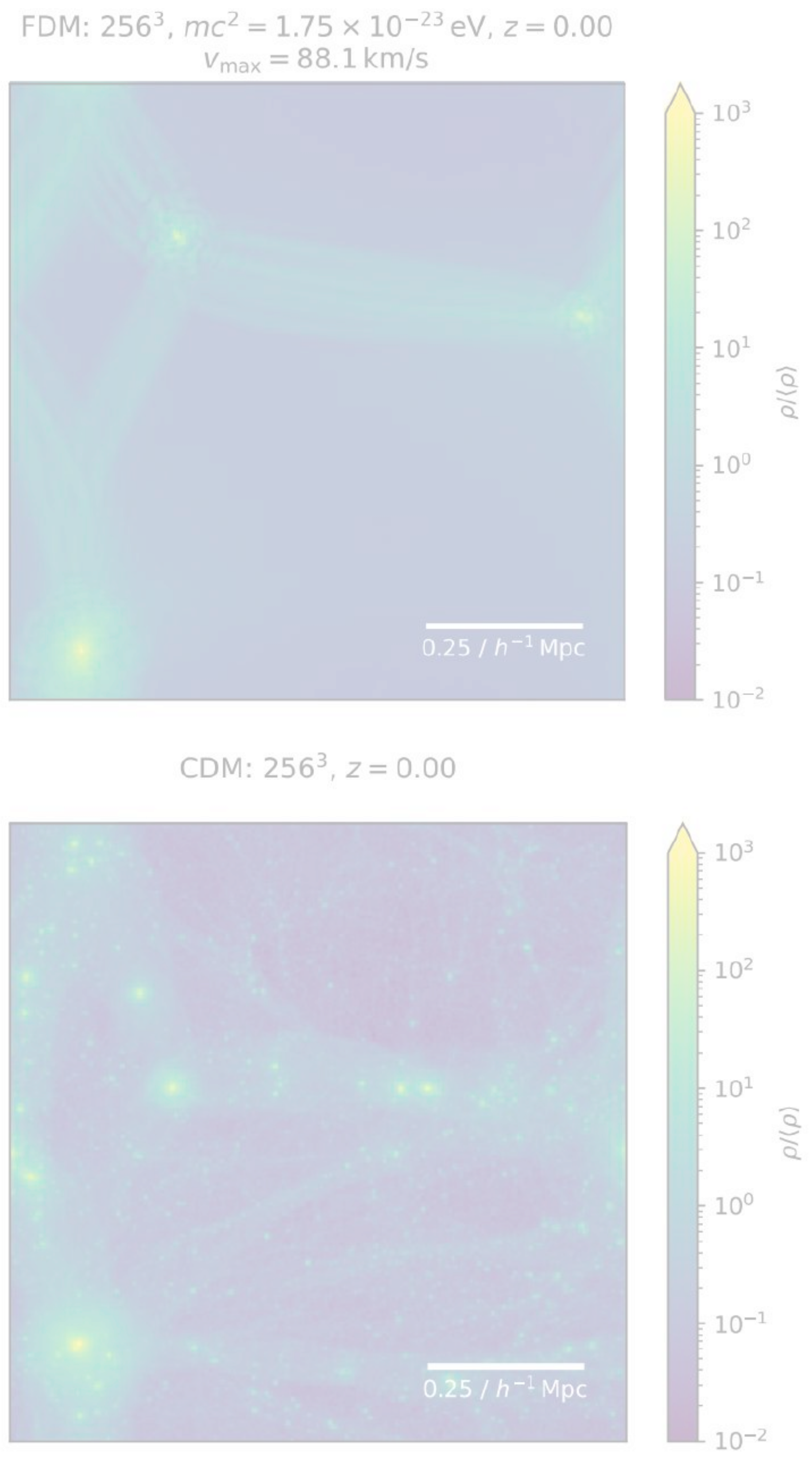
Phenomenology



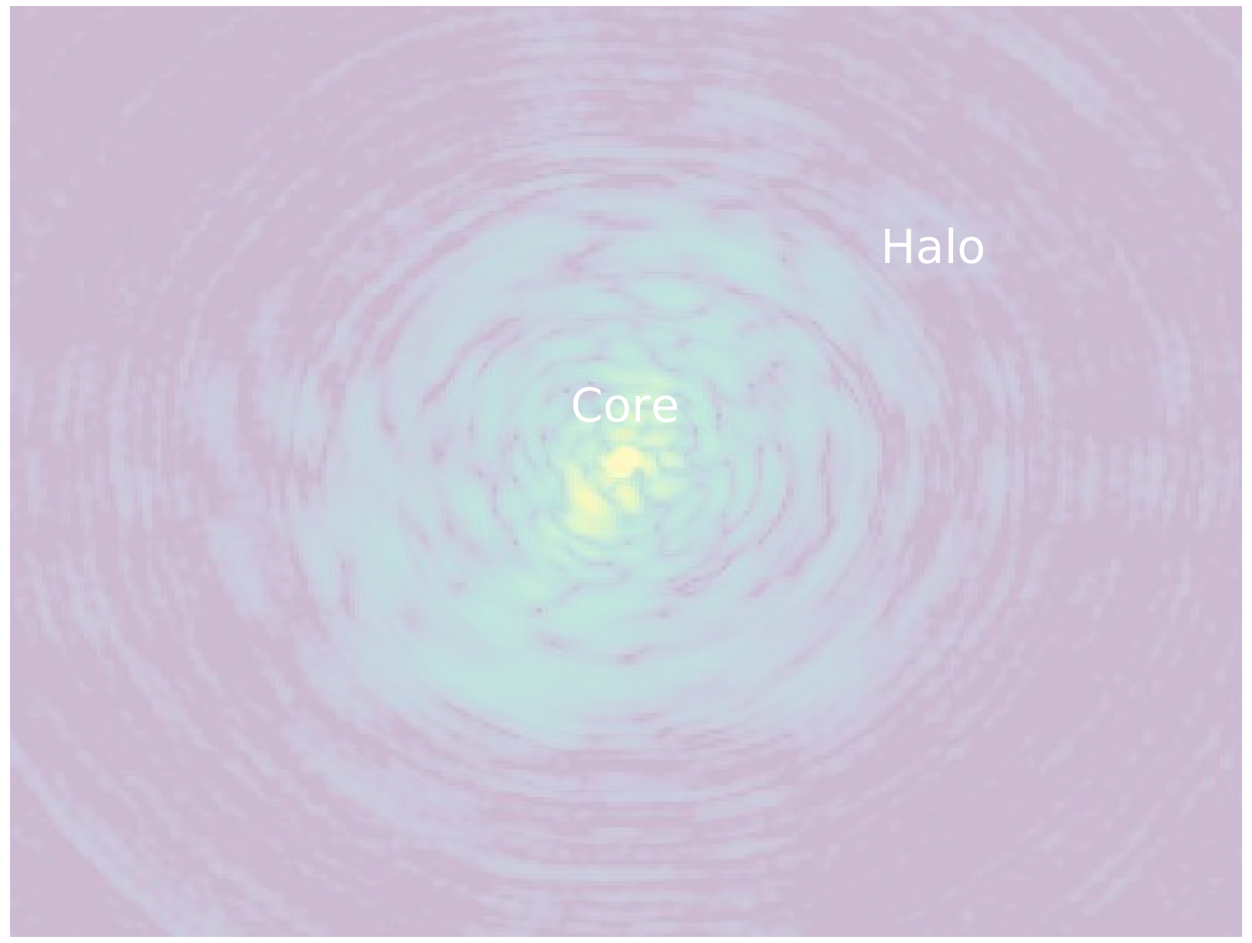
RICH PHENOMENOLOGY ON SMALL SCALES

Suppression of small structures

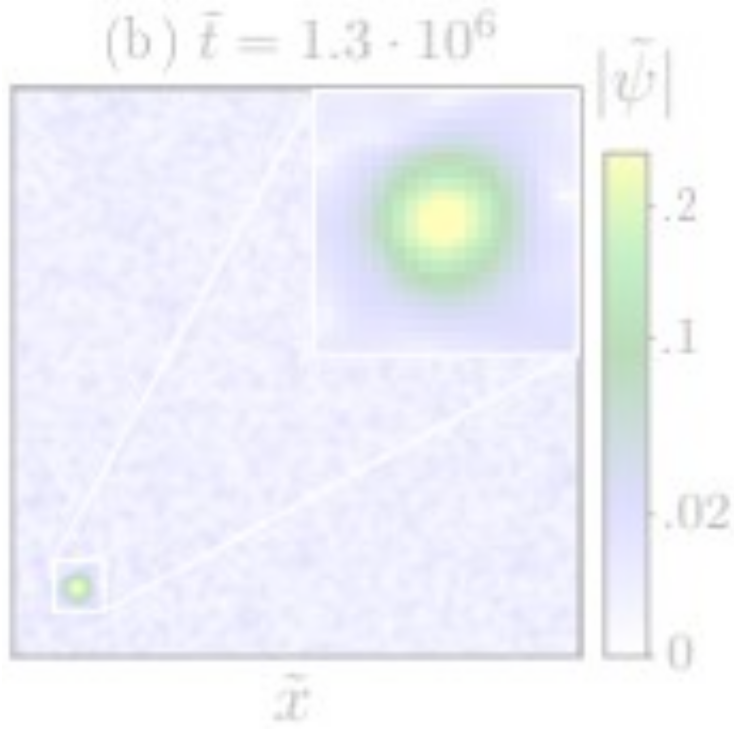


S. May et al. 2021

Formation of a solitonic core

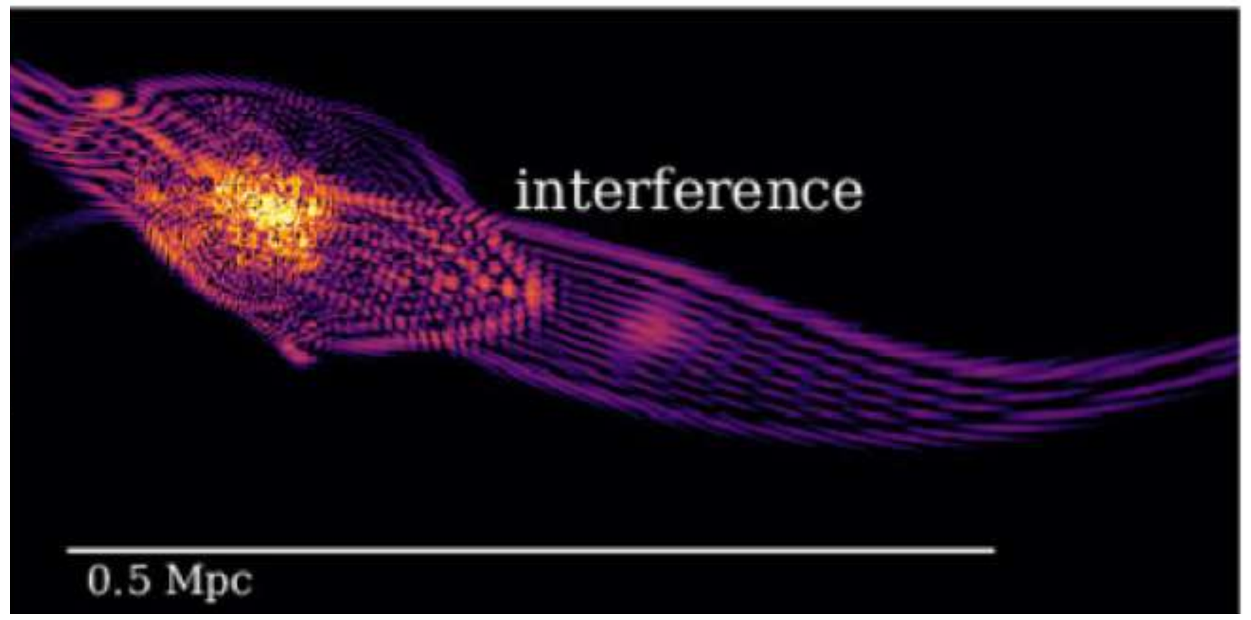


Dynamical effects



Levkov et al. 2018

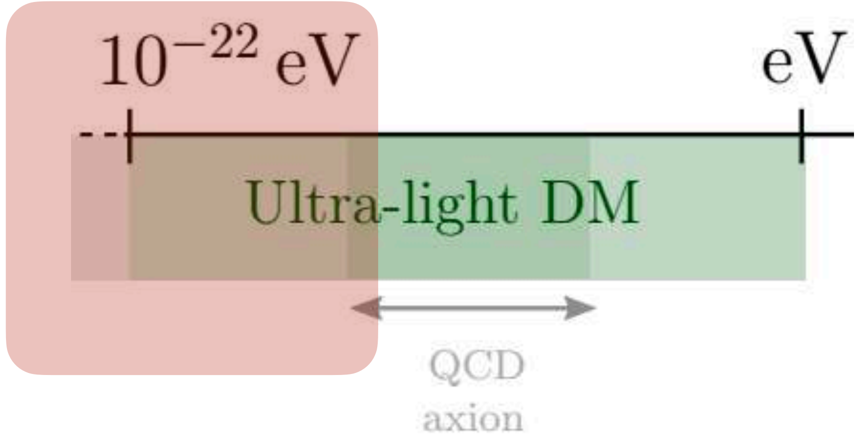
Wave interference

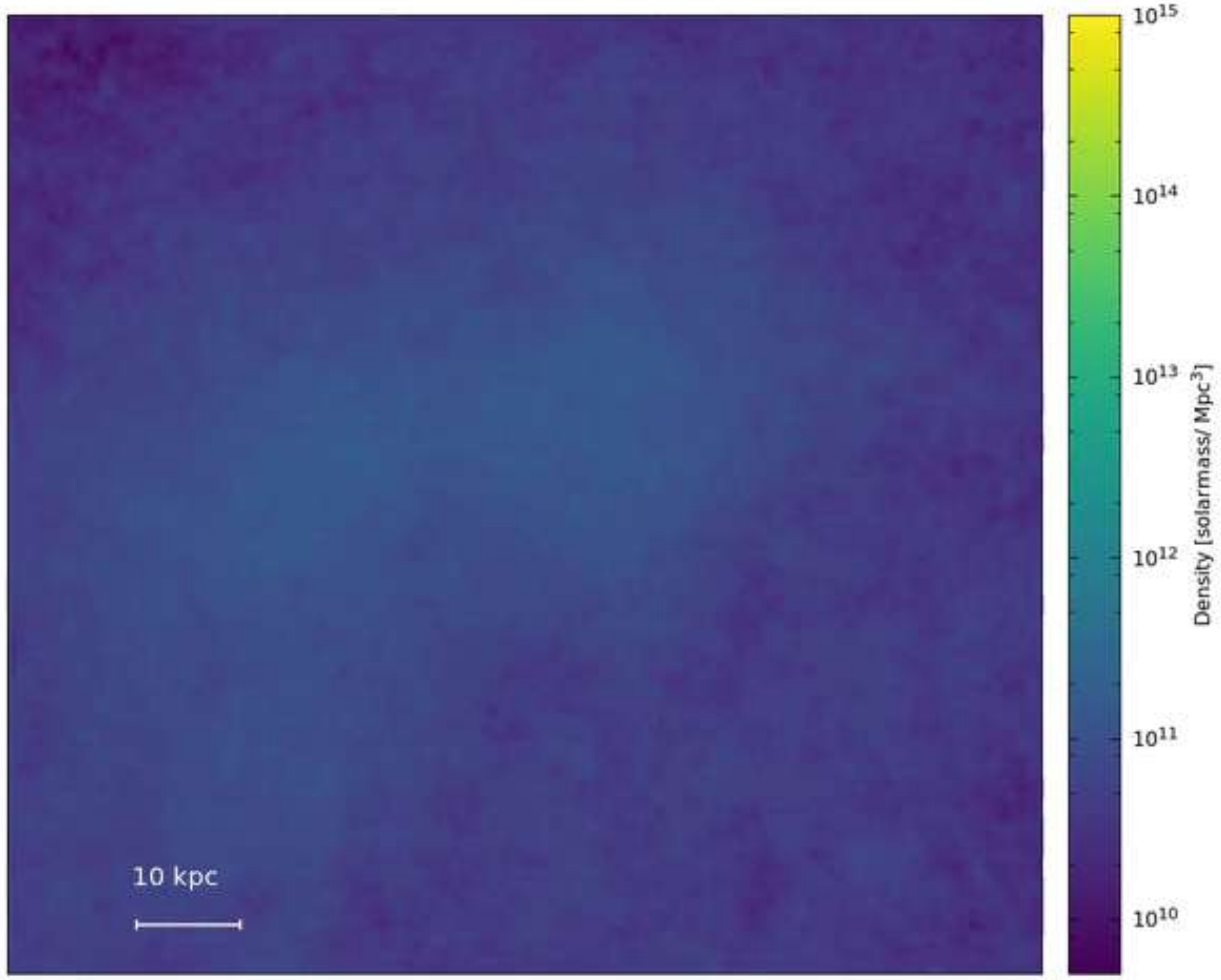


Mocz et al. 2017

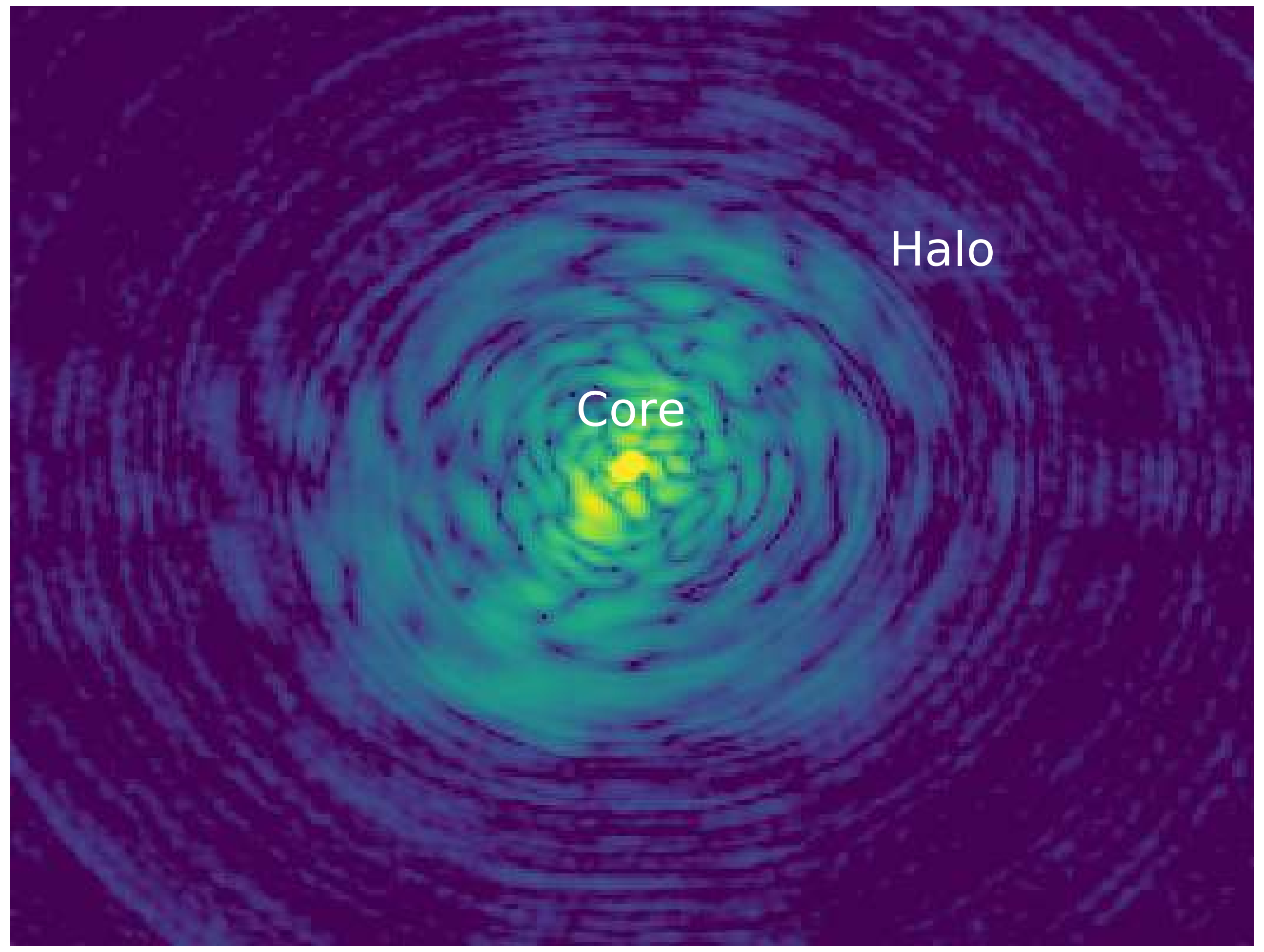
Phenomenology

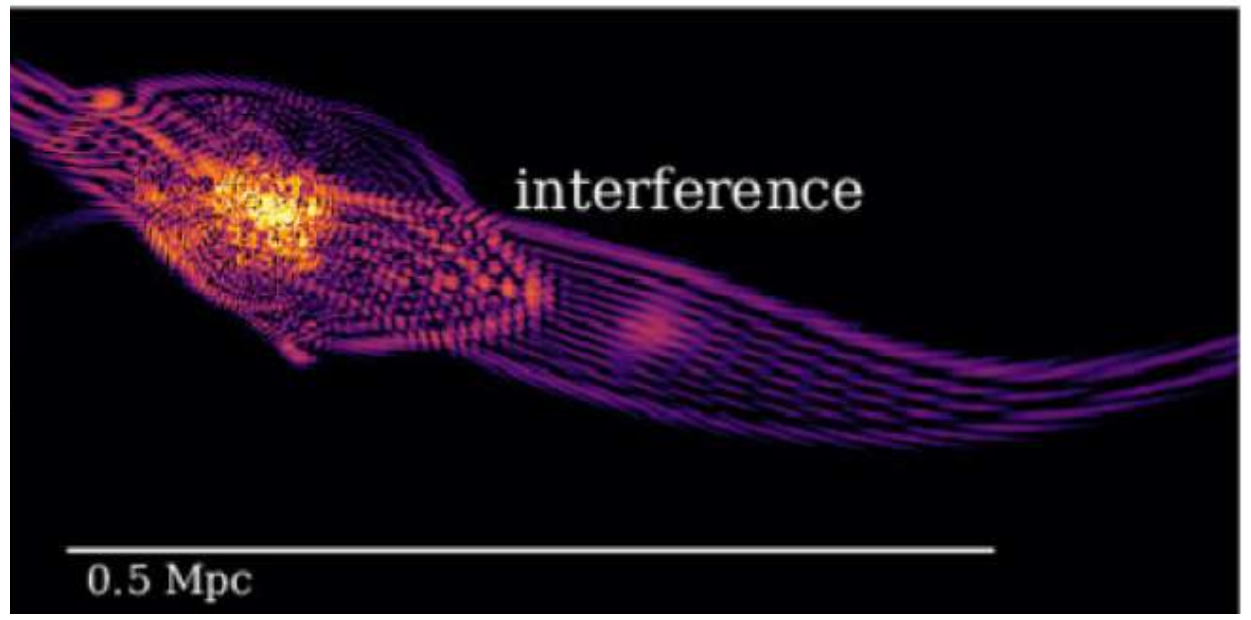
Wave interference: granules and vortices





Simulation by Jowett Chan





Mocz et al. 2017

Order one fluctuations in density ⟶ Constructive interference: granules

Destructive interference

⟶ $\sim \lambda_{\mathrm{dB}}$ Hard to observe!

Phenomenology

Vortices

Vortices are sites where the fluid velocity has a non-vanishing curl

$$(\psi \equiv \sqrt{\rho/m}\, e^{i\theta} \quad \text{and} \quad \mathbf{v} \equiv \nabla\theta/m)$$

$$\dot{\rho} + \nabla \cdot (\rho\, \mathbf{v}) = 0$$

$$\dot{\mathbf{v}} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{m}\left(V_{grav} - P_{int} - \frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

Vel. field is a gradient flow ⟶ irrotational fluid, no vorticity

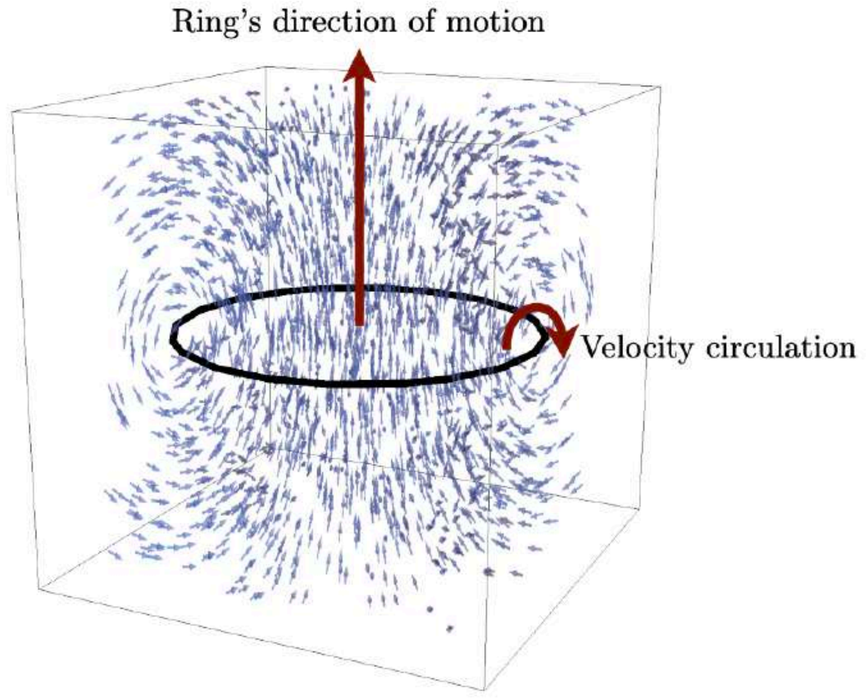
Two ways:
- regions where the density vanishes
- transfer of angular momentum (superfluids only)

Fuzzy DM

Interference of waves leads to vortices - where there is destructive interference

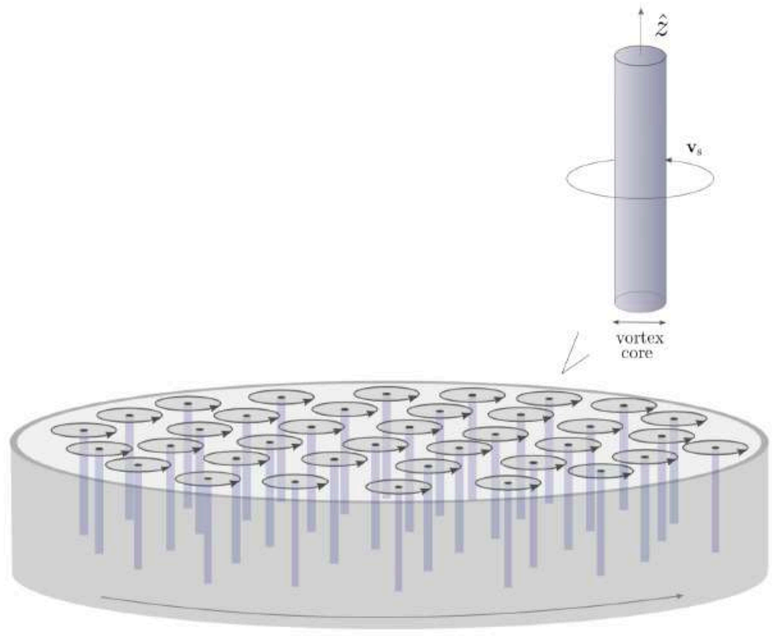
General defet in 3D

$$\mathcal{C} = \frac{1}{m}\oint_{\partial A} \mathrm{d}\theta = \frac{2\pi n}{m}$$

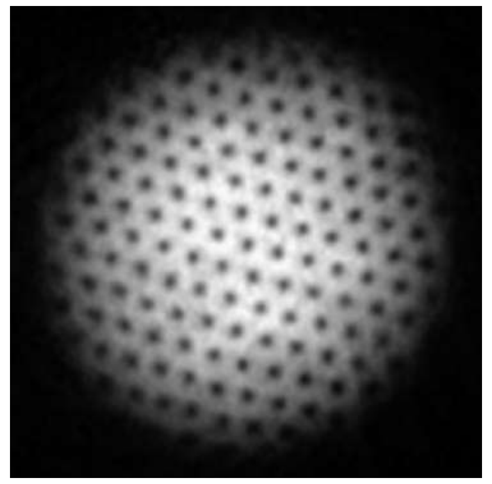


Self-interacting Fuzzy DM

Superfluid cannot rotate uniformly. If the superfluid rotates faster than the critical vel., network of vortices are formed.



EF, 2020



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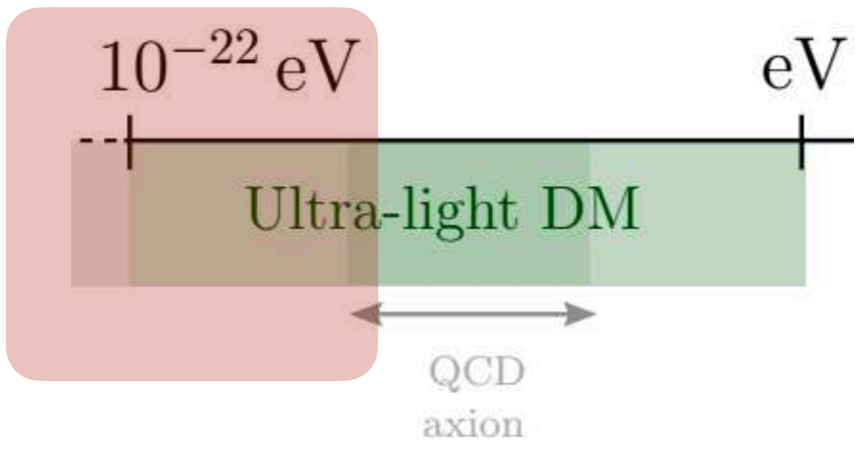
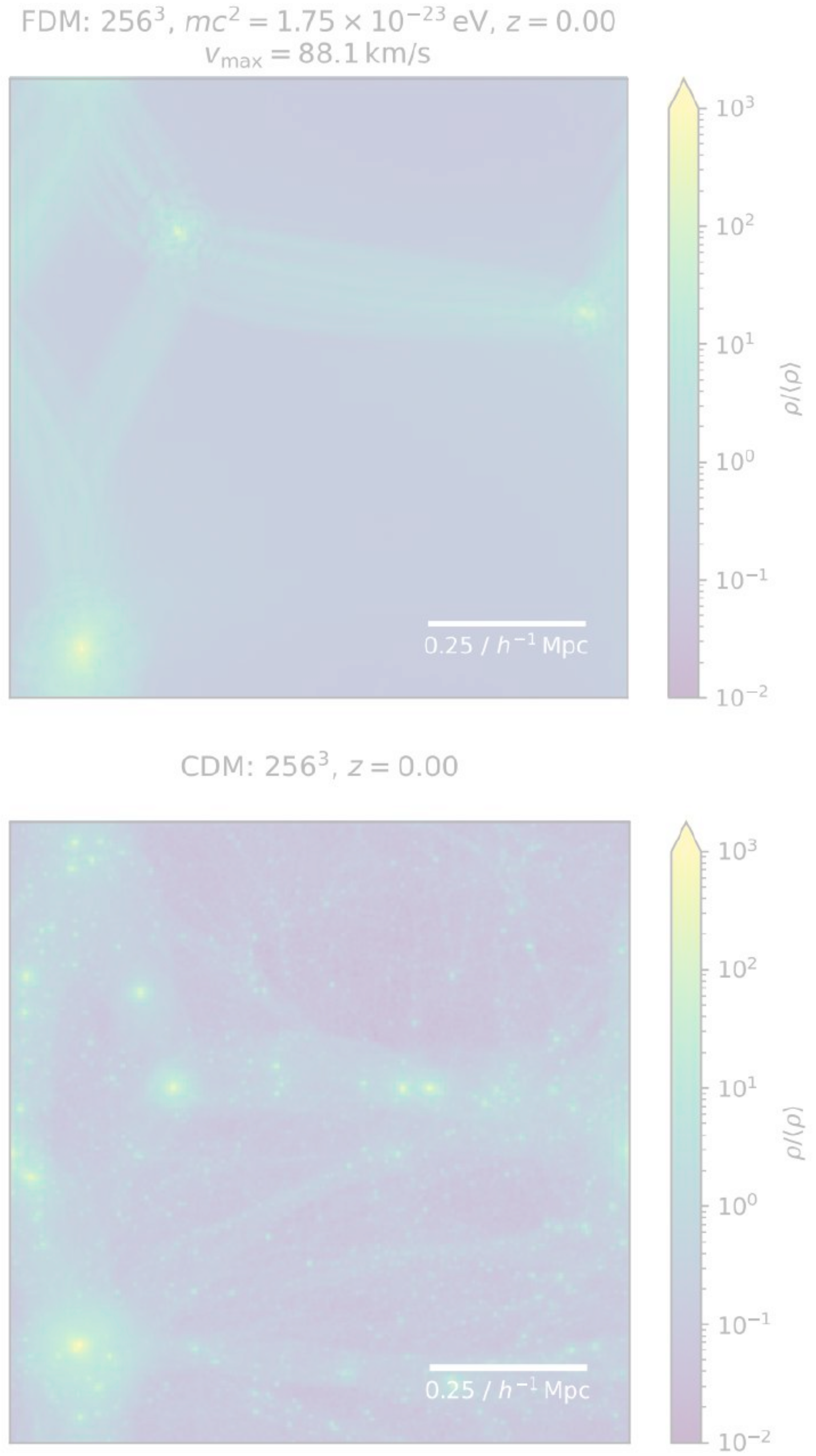
Phenomenology



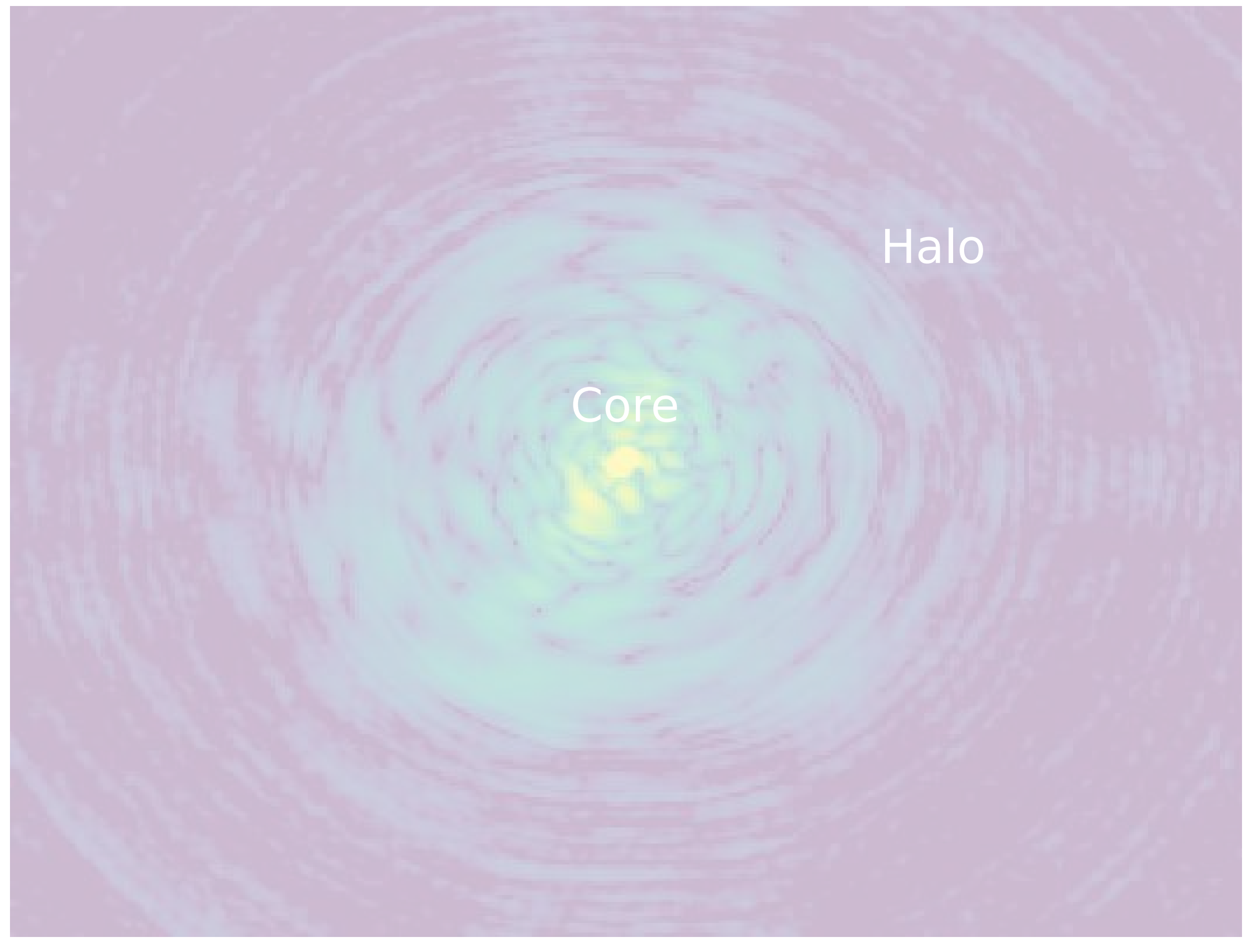
RICH PHENOMENOLOGY ON SMALL SCALES

Suppression of small structures

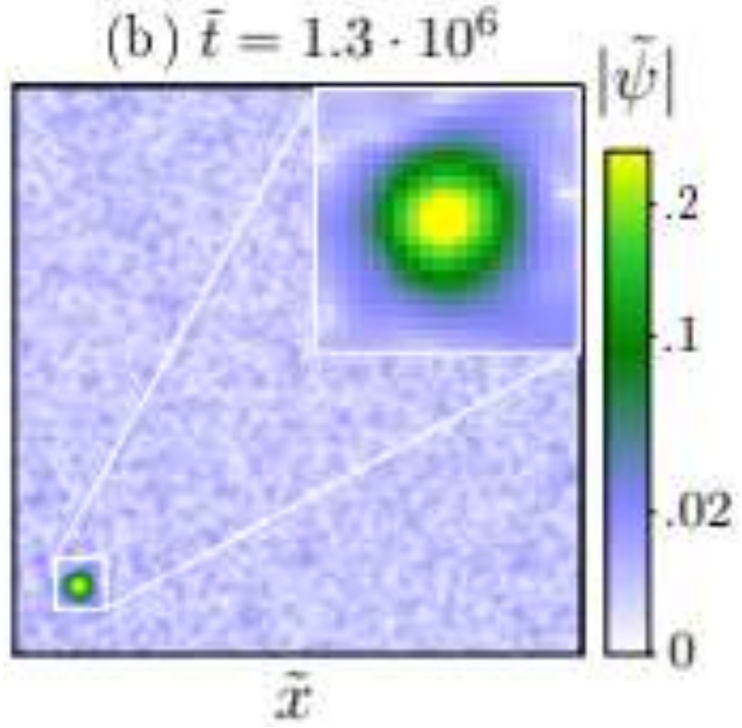


S. May et al. 2021

Formation of a solitonic core



Dynamical effects



Levkov et al. 2018

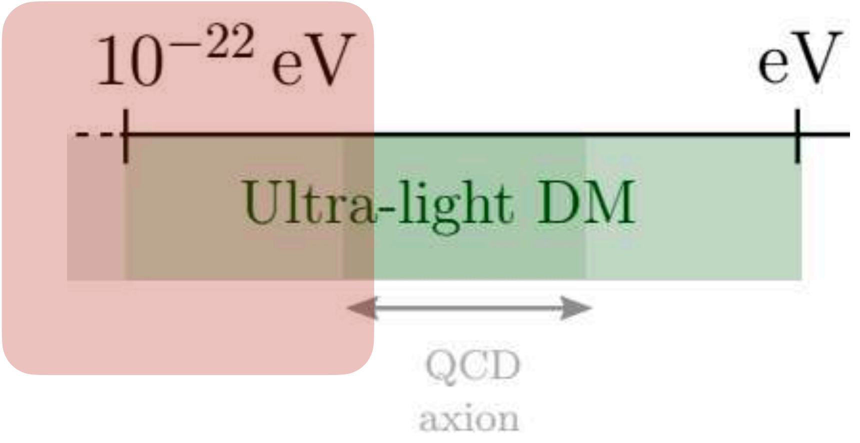
Wave interference

Mocz et al. 2017

Phenomenology

Dynamical effects

Relaxation, oscillation, friction, and heating

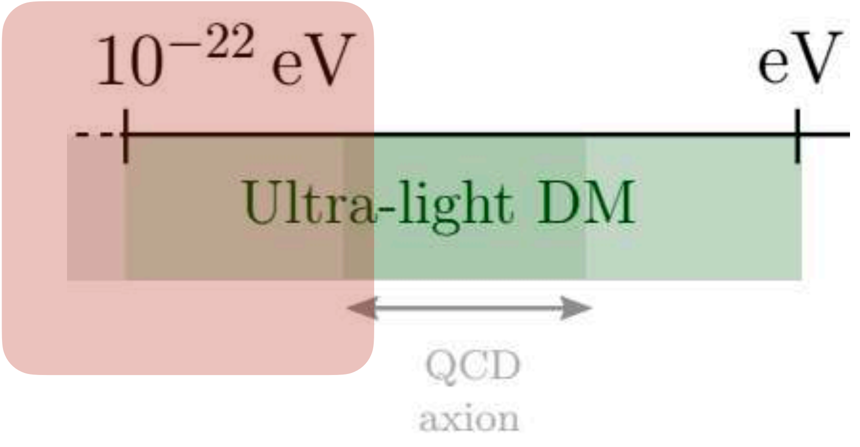


Phenomenology

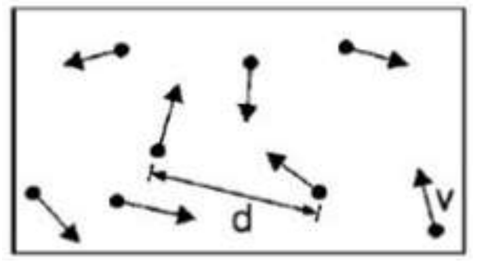
Dynamical effects

Relaxation, oscillation, friction, and heating

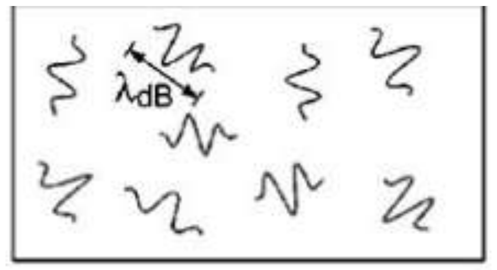
Formation of a BEC / superfluid



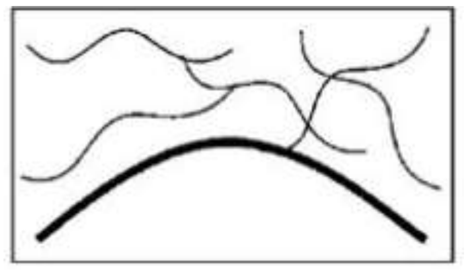
Bose Einstein Condensate



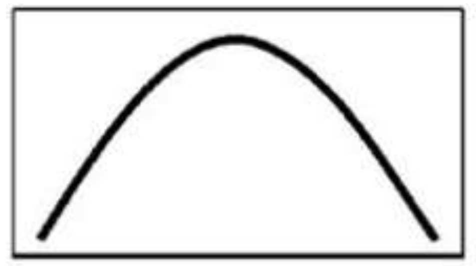
High temperature
Thermal velocities



Low temperature
$\lambda_B \sim T^{-1/2}$
"wave packets"



$T = T_c$
BEC
"matter wave overlap"
$d \sim \lambda_{dB}$



$T = 0$
Pure BEC
"giant matter wave"

- Bose Einstein condensate (BEC): macroscopic occupation of the ground state
- At low temperatures, each particle wave function overlap - single wave function describes the entire fluid.

Superfluid

- Appears at low T after the superfluid condenses into a BEC.
- Effective dynamics: fluid flows without friction

Description

Mean field approximation:
Large N, dilute

$$\hat{\Psi}(\mathbf{r},t) = \psi(\mathbf{r},t) + \delta\hat{\Psi}(\mathbf{r},t)$$

with $\psi(\mathbf{r},t) = \langle\hat{\Psi}(\mathbf{r},t)\rangle$
Fixed $n_0 = |\psi(\mathbf{r},t)|^2$

classical field
"wavefunction of the condensate"

small perturbation: describes
depletion of the condensate

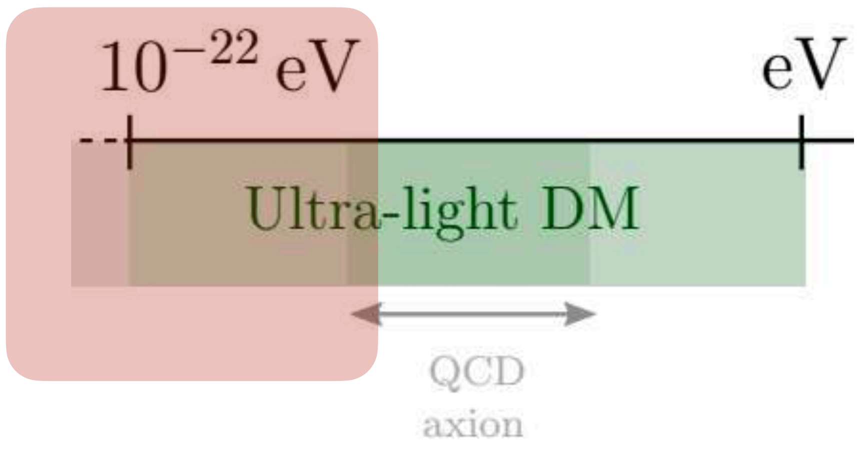
Credit: Peking University

$$\boxed{i\partial_t\psi(\mathbf{r},t) = \left(-\frac{\nabla^2}{2m} + V_{trap}(\mathbf{r}) + U_0|\psi(\mathbf{r},t)|^2\right)\psi(\mathbf{r},t)}$$

Non-linear Schrödinger equation - Gross-Pitaevskii equation

Phenomenology

Dynamical effects



Relaxation, oscillation, friction, and heating

Formation of a BEC / superfluid

- Thermalization (and condensation) *seem* to happen inside the galaxy! Formation of a soliton (ground state) or Bose star in the interior of galaxies

A. Guth M. Hertzberg, C. Prescod-Weinstein (2014)

- Formation of a condensate and a core occur from gravitational interaction.

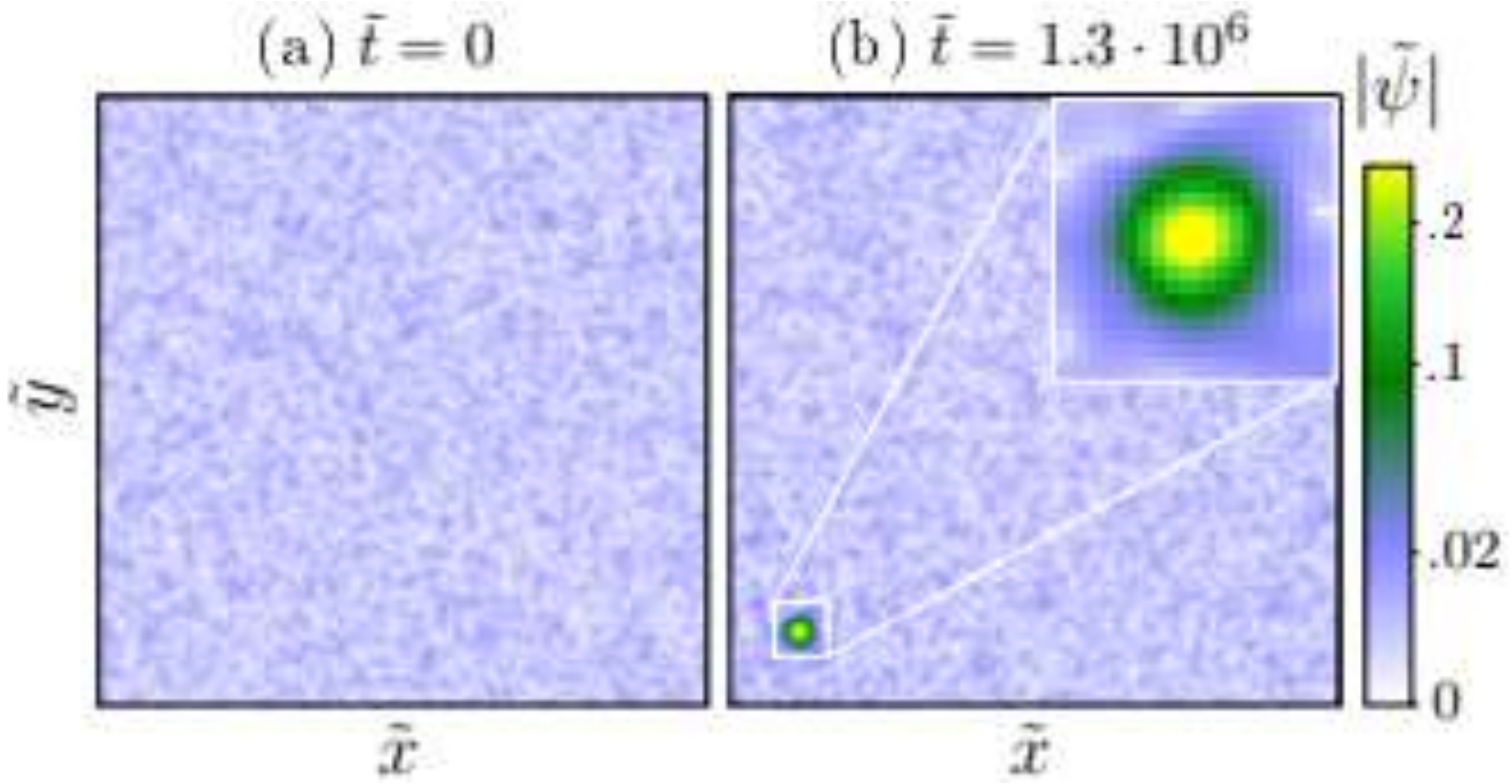
Condensation/relaxation time: $\tau_{\rm gr} \gg \tau_{\rm int}$

$$\tau_{\rm gr} \sim 10^6\,{\rm yr}\left(\frac{m}{10^{-22}\,{\rm eV}}\right)^3\left(\frac{v}{30\,{\rm km/s}}\right)^6\left(\frac{\rho}{0.1\,M_\odot/{\rm pc}^3}\right)^{-2}$$

$$\tau_{\rm int} = \frac{1}{\sqrt{8}|g|n}$$

Smaller than the age of the universe!

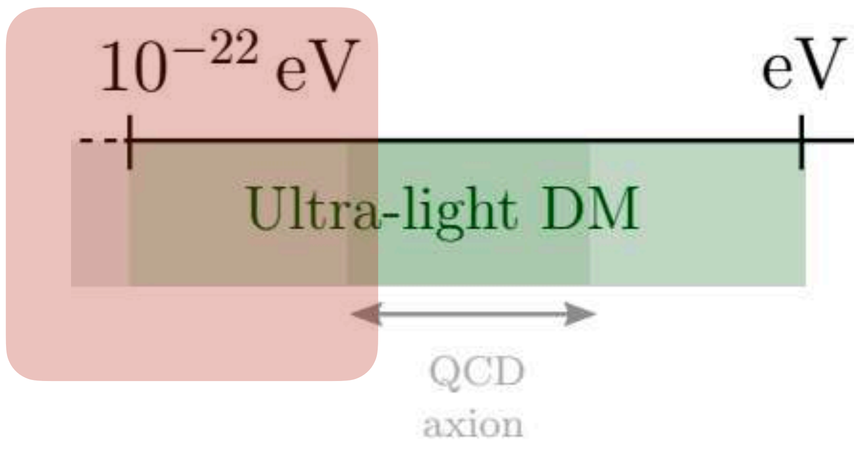
Wave turbulence theory



Levkov et al. 2018, Kirpatrick et al. 2020

Phenomenology

Dynamical effects



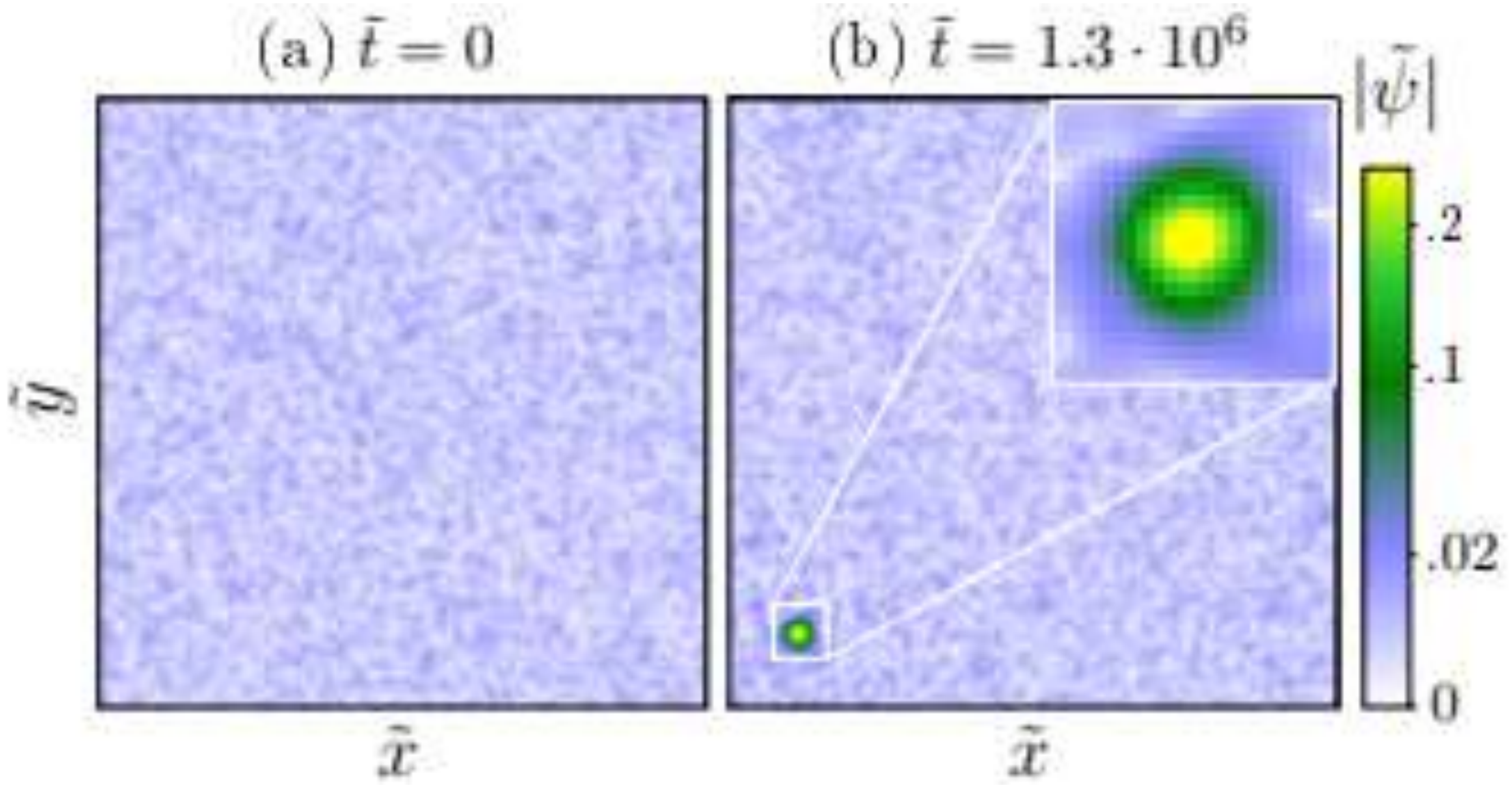
Relaxation, oscillation, friction, and heating

Formation of a BEC / superfluid

Open question!

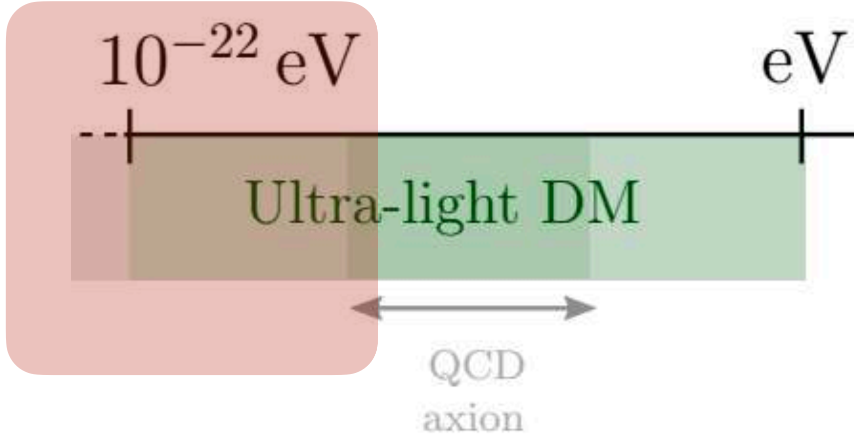
- Need theoretical work to describe *analytically* the formation of these solitons

- Cosmologically, classical or quantum field?

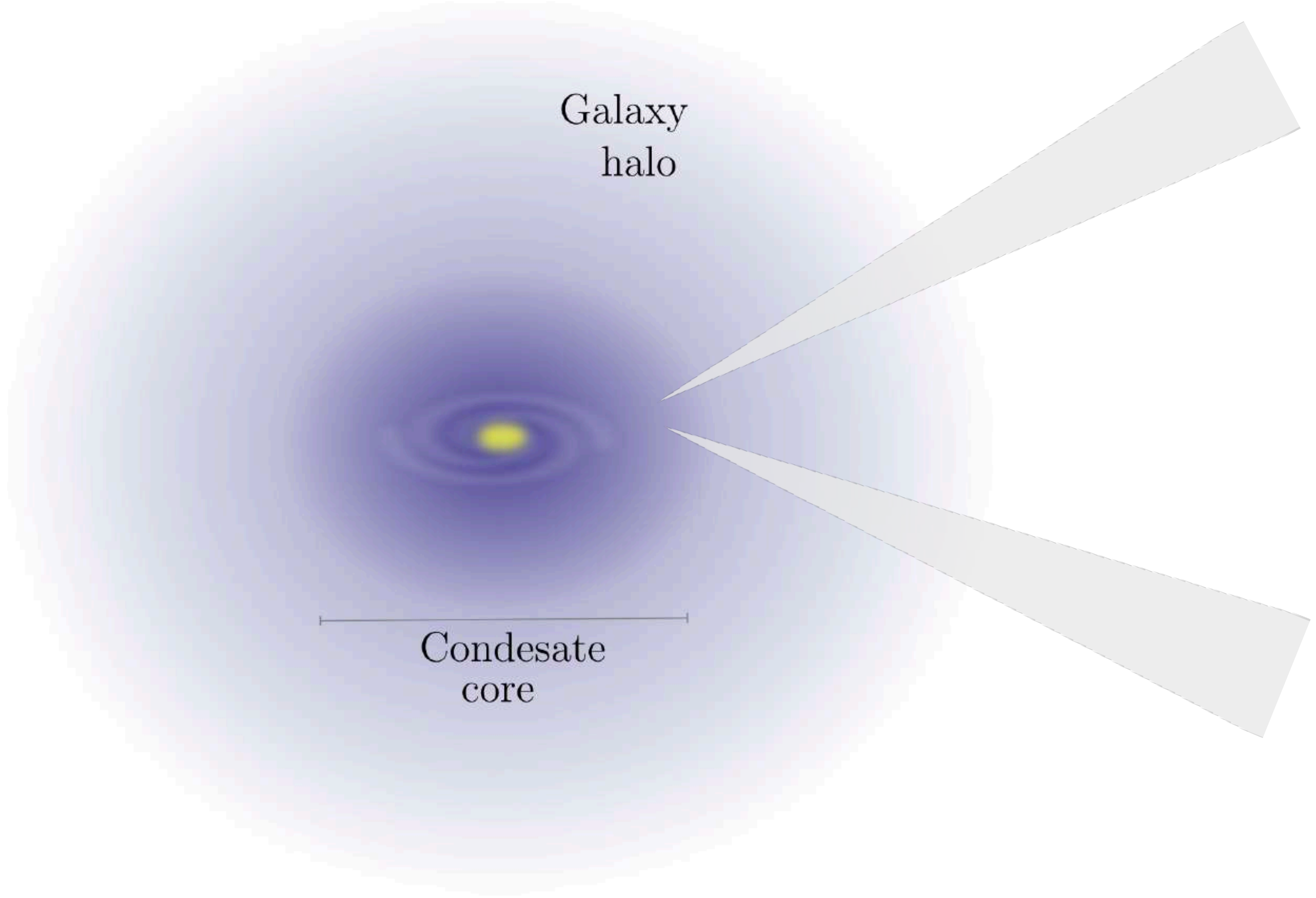


Phenomenology

Dynamical effects



Relaxation, oscillation, friction, and heating

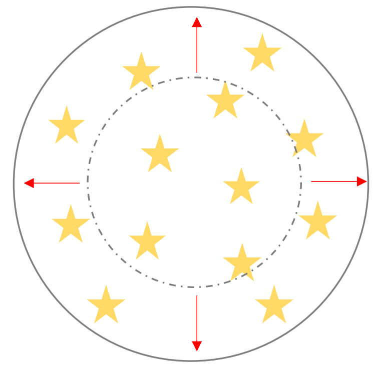


Heating

FDM granule

m_{eff}

System (star)
gains energy



Friction

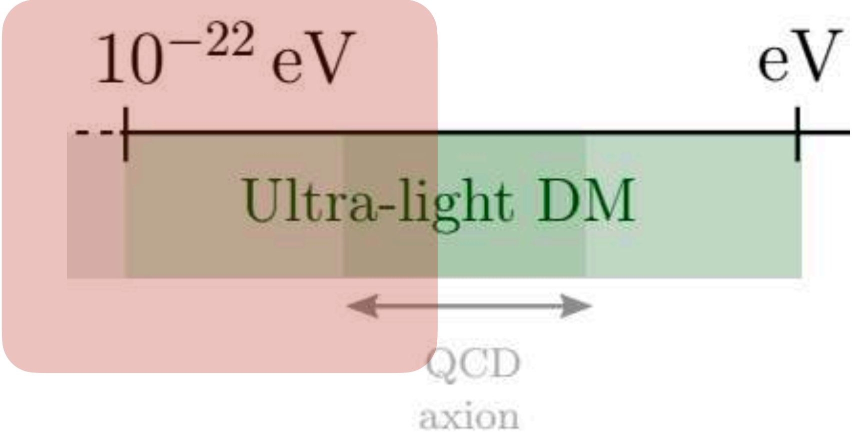
FDM granule

m_{eff}

Globular cluster

System (GC or BH)
loses energy

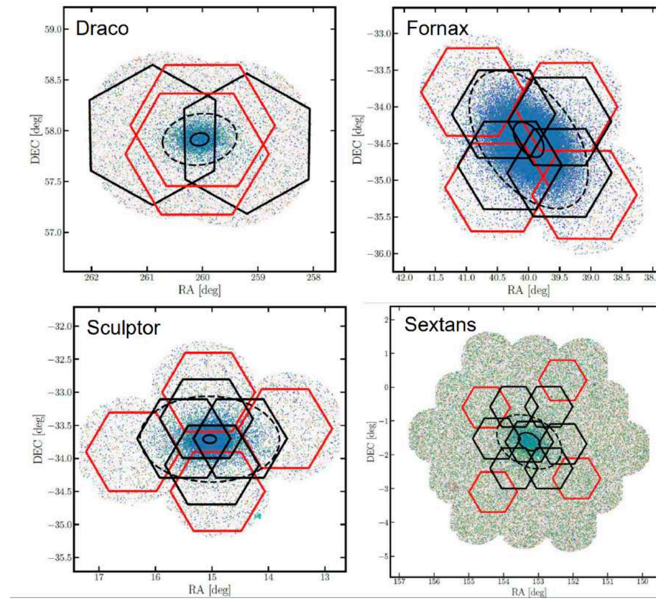
Observational implications and constraints



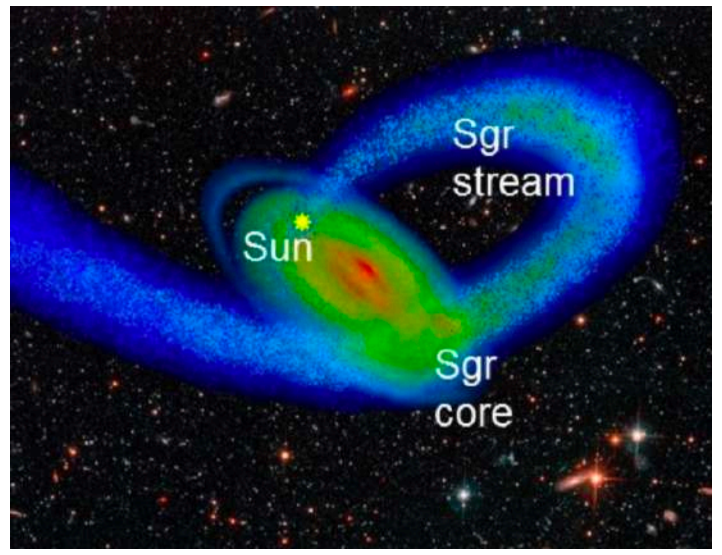
Galaxies

NASA and ESA

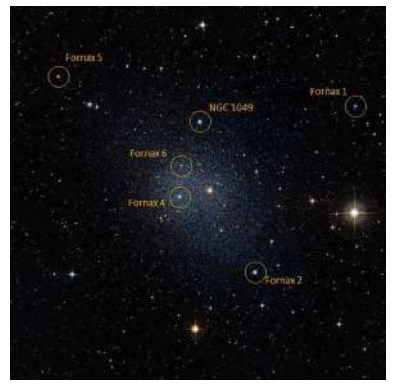
Dwarfs



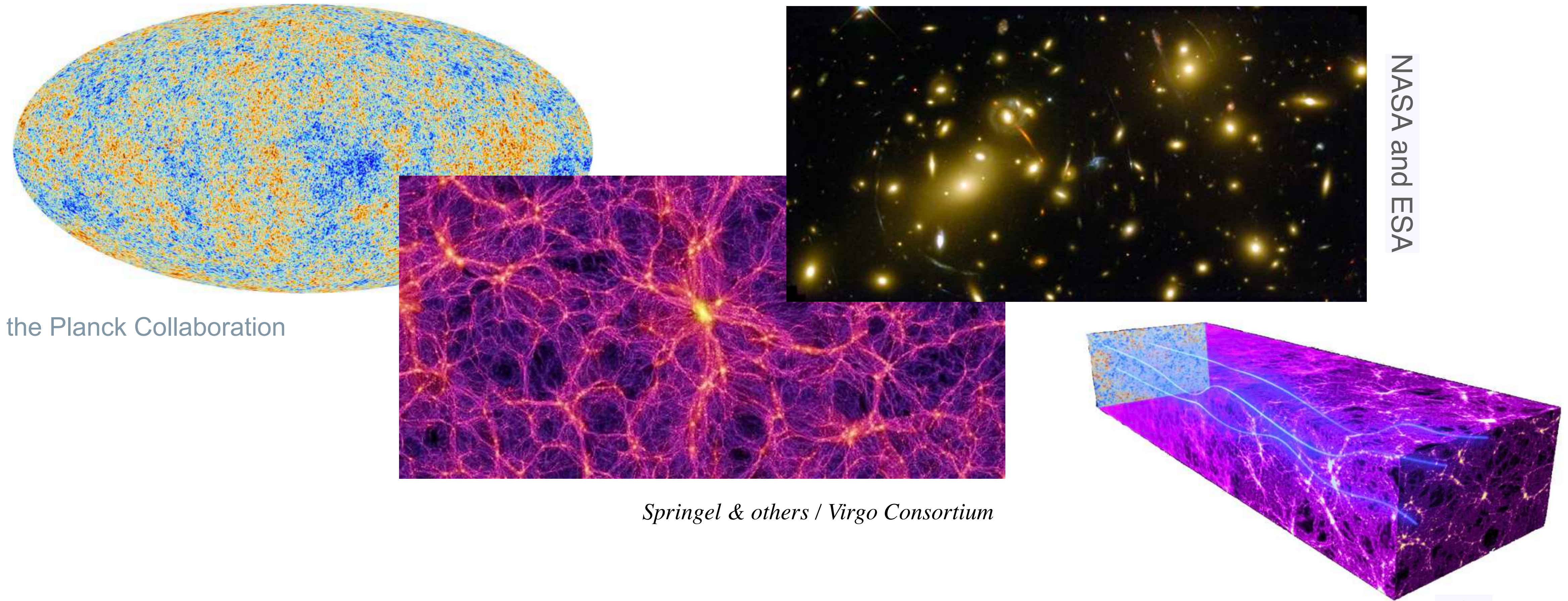
Stellar stream



Globular clusters



CMB+LSS



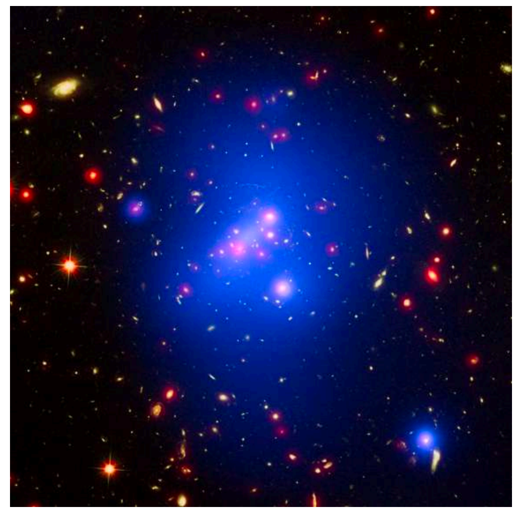
ESA and the Planck Collaboration

NASA and ESA

Springel & others / Virgo Consortium

ESA

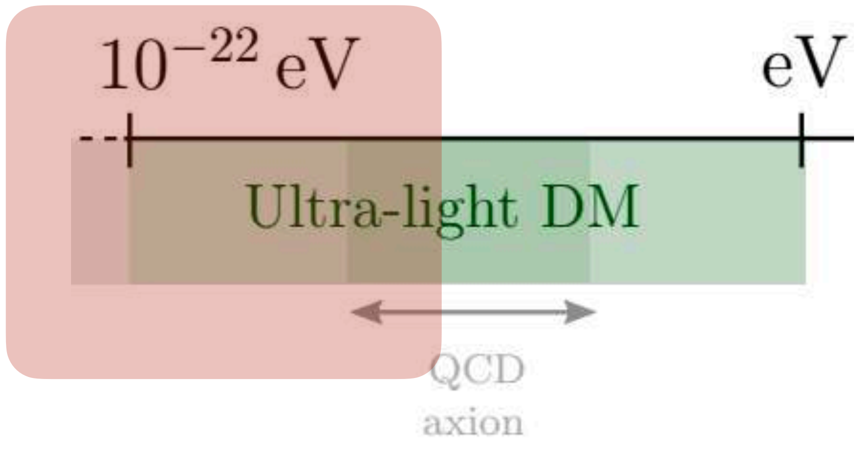
Clusters



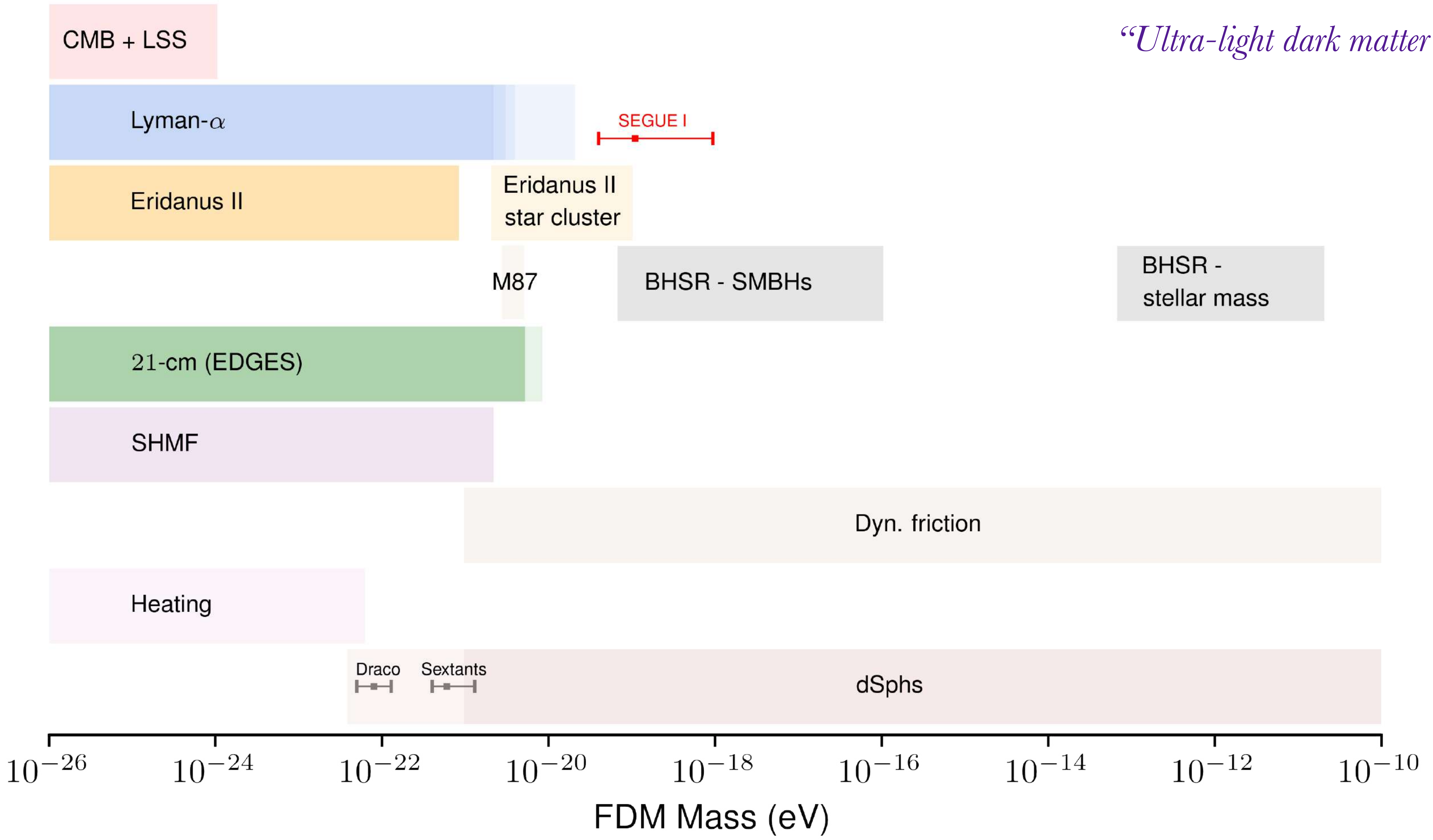
CC BY 4.0

Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass



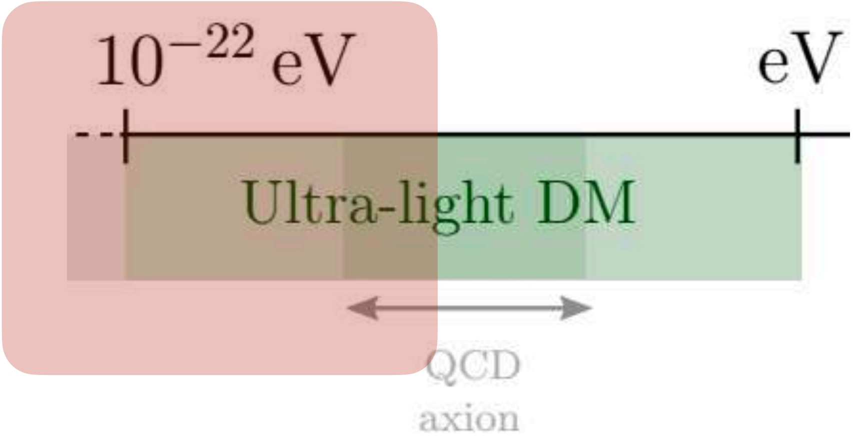
"Ultra-light dark matter", **E.F.**, 2020. The Astronomy and Astrophysics Review.

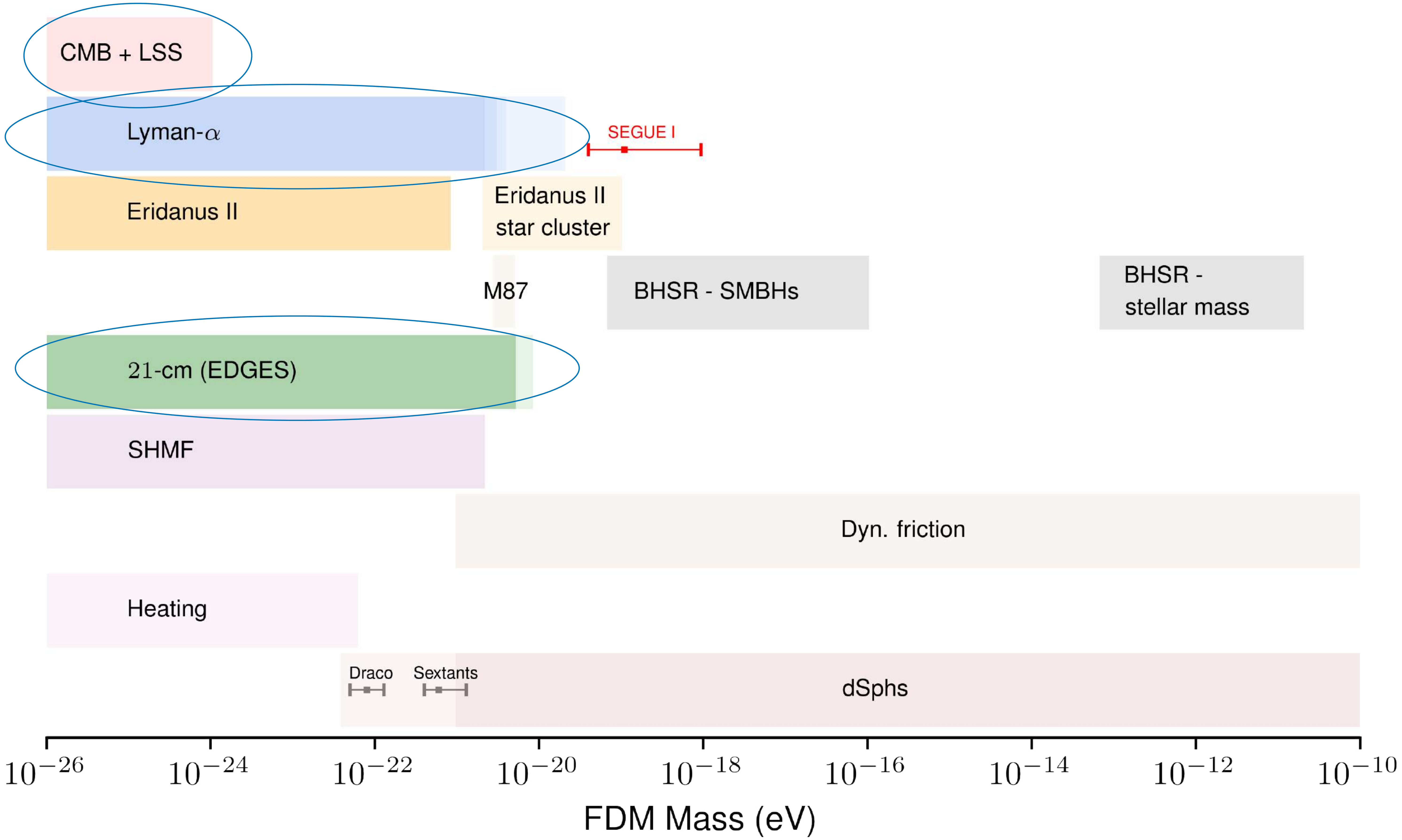


Bounds consider FDM is all DM

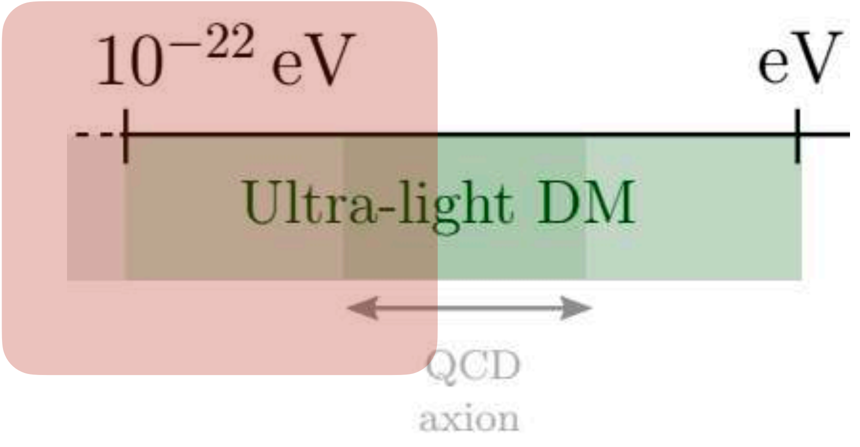
Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass





"Ultra-light dark matter", **E.F.**, 2020



Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass

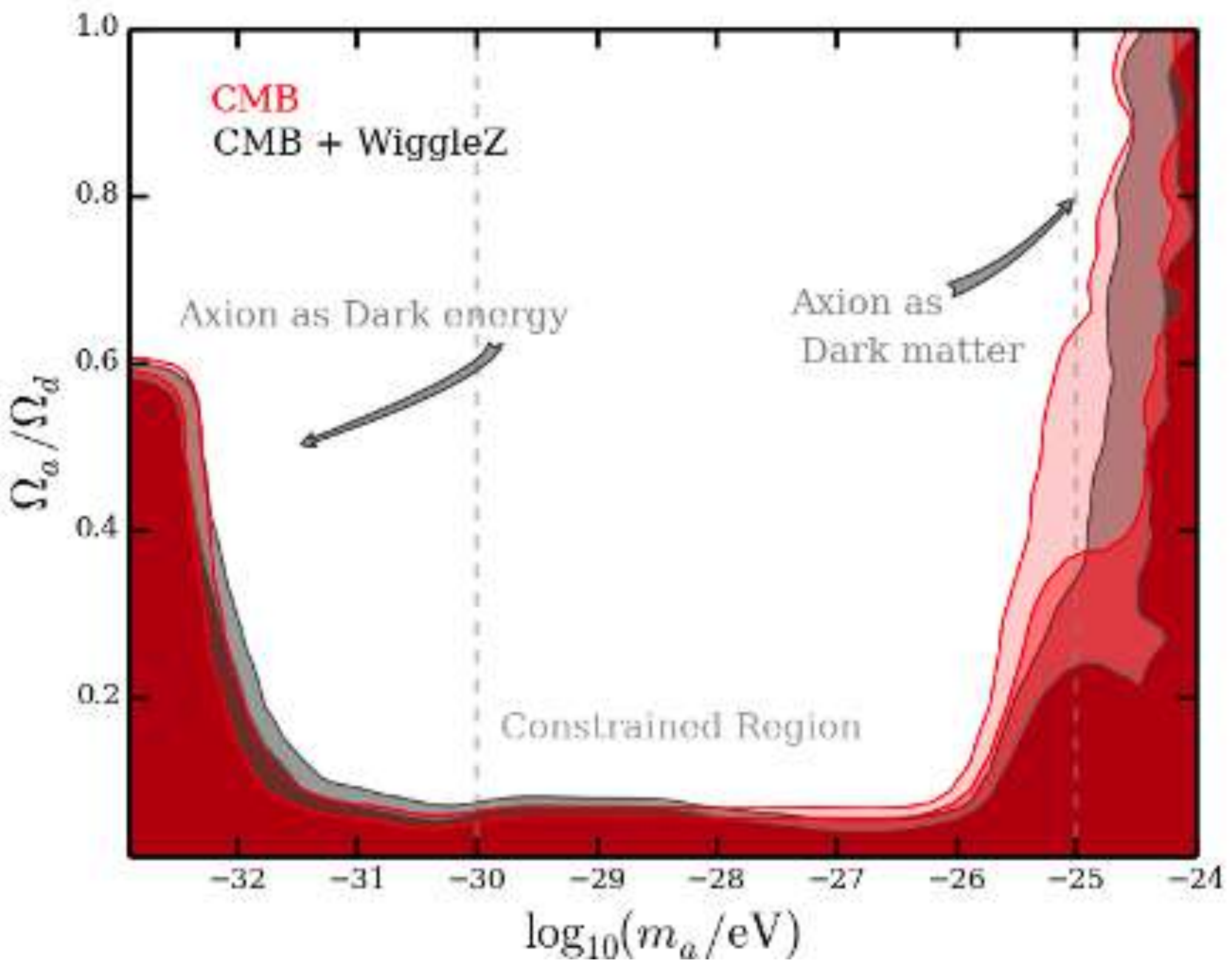
Suppression of small structures

CMB/LSS

Lyman alpha

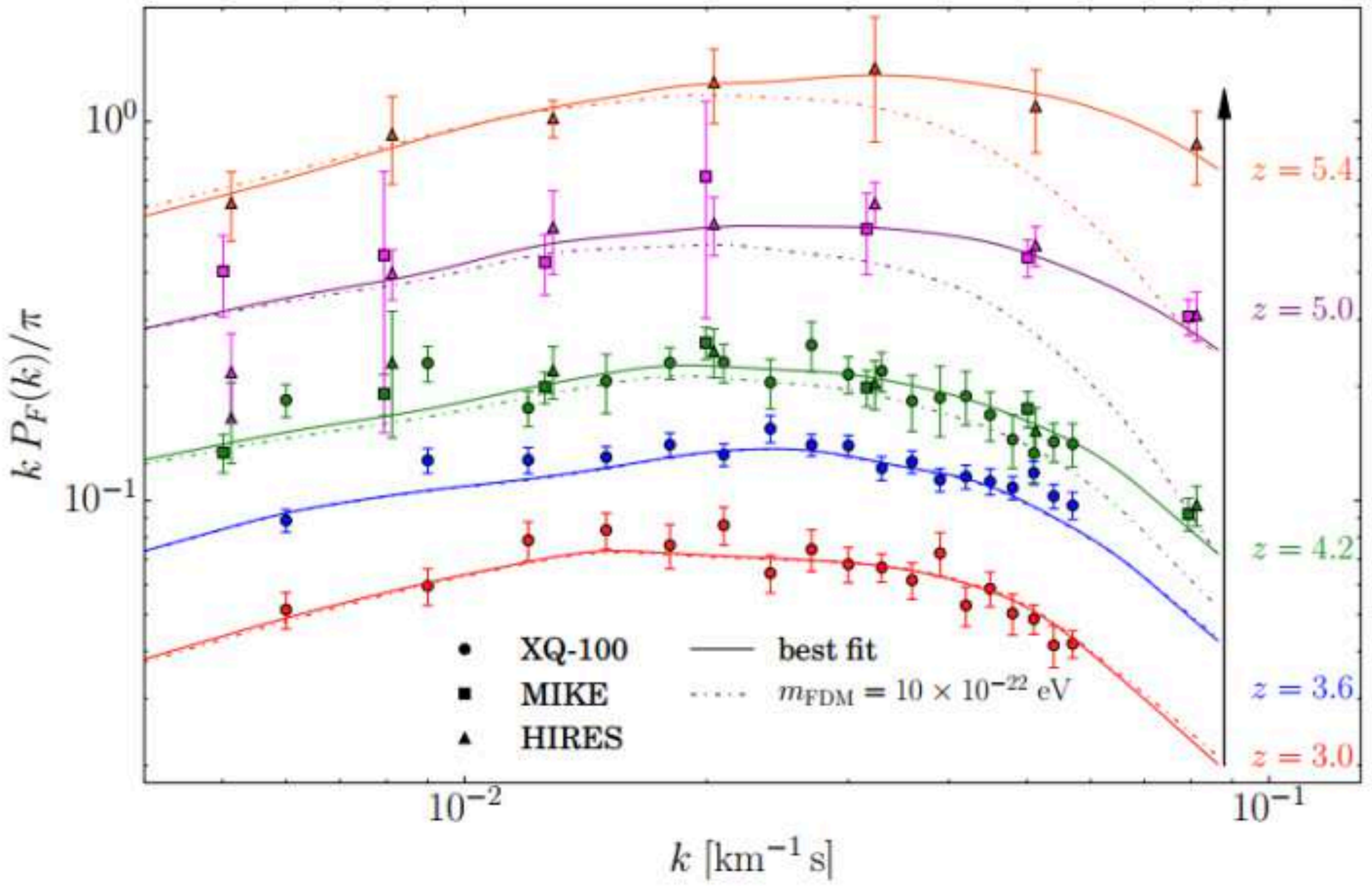
Global 21 cm

Hlozek et al, (2015, 2018)



$m \gtrsim 10^{-24}\,\mathrm{eV}$

Armengaud et al. (2017); Iršič et al. (2017);
Rogers et al. (2020)



$m \gtrsim 2 \times 10^{-20}\,\mathrm{eV}$

so enough Mpc-scale power in Ly-α forest at z = 5.

Suppressed small scale structure

↓

Postpone Ly-α coupling, heating, reionization H

↓

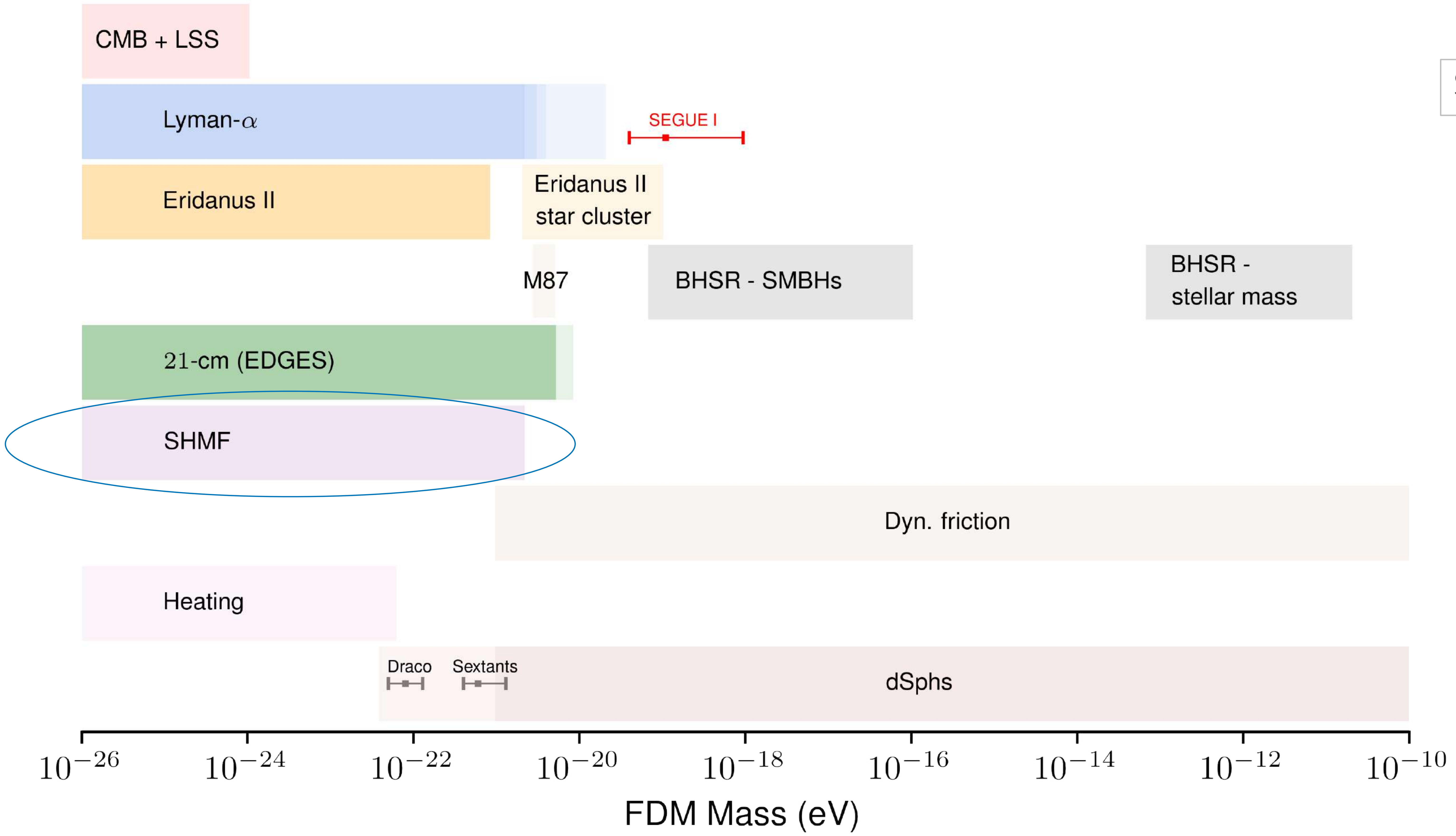
Smaller 21-cm global signal

$m \gtrsim 6 \times 10^{-22}\,\mathrm{eV}$

EDGES global 21 cm signal
Olof Nebrin et al.(2019)

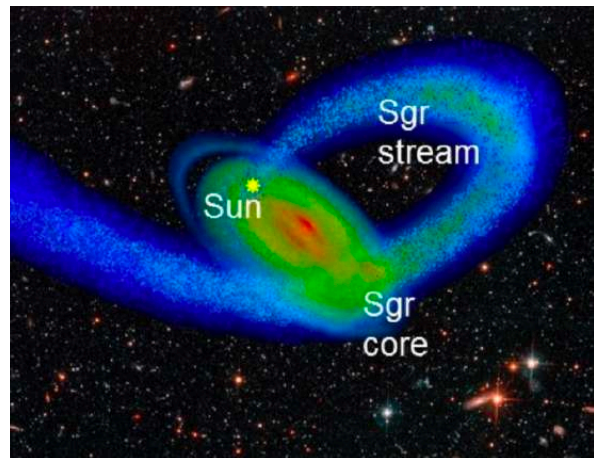
Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass



Suppression of small structures

Stellar streams

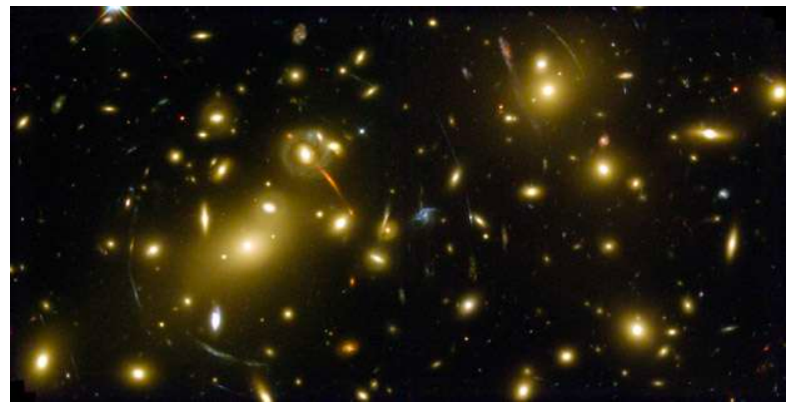


- DM properties encoded in variations density in stellar streams
- Opportunity to probe nature of DM
- GD-1 : compatible with CDM

Ibata et al. (2020): at this stage, hard to disentangle DM signal.

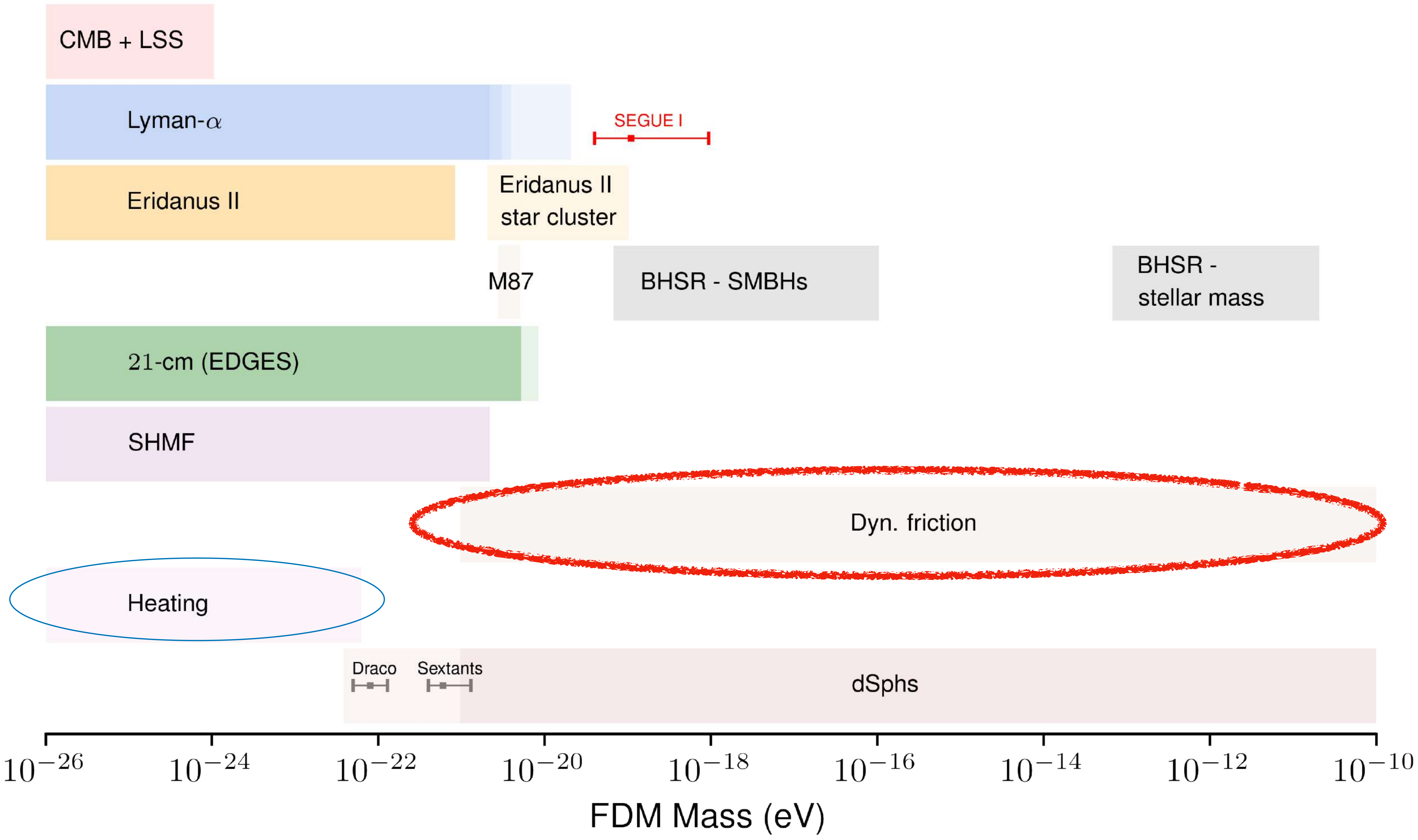
Schutz 2020: bound in the FDM sub-halo mass function using stellar streams and grav. lensing

Grav. lensing



Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass

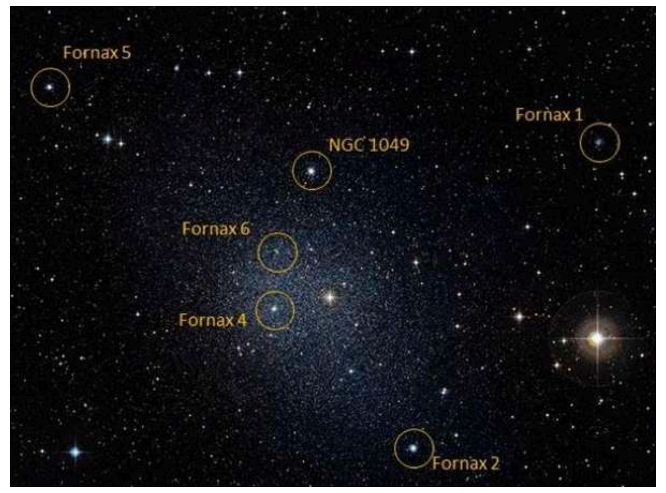


Dynamical effects

Globular clusters

$$m < 10^{-21}\,\mathrm{eV}$$

ESO/Digitized Sky Survey 2



Lancaster et al. 2020

Heating of the MW disk

Church et al. 2019

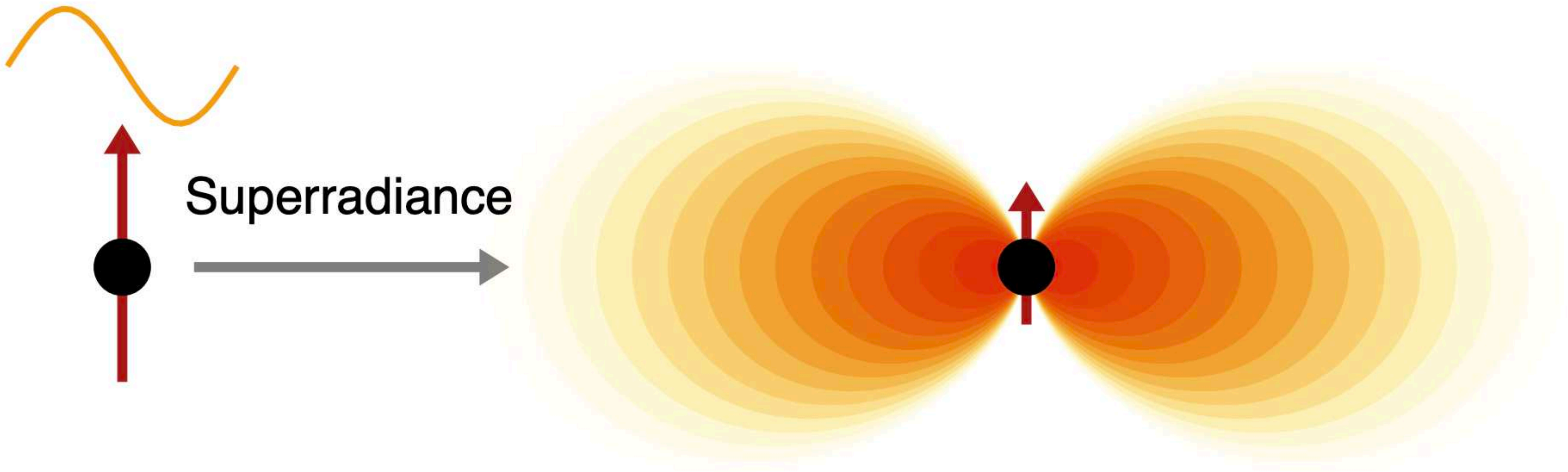
$$m > 0.6 \times 10^{-22}\,\mathrm{eV}$$

Black Hole Superradiance

Zeldovich (1972) Starobinsky (1973) Arvanitaki et al. [0905.4720]

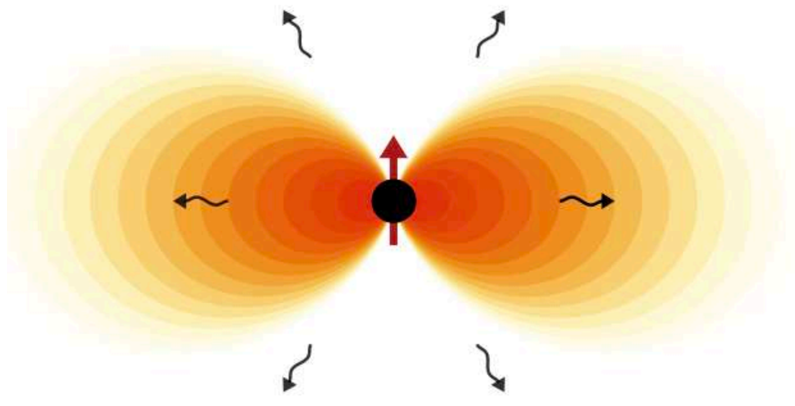
A cloud of ultra-light bosons (and vector fields) can be created around rotating black holes - if the particle Compton wavelength is of the order of the size of the BH

Structure like a "gravitational atom"

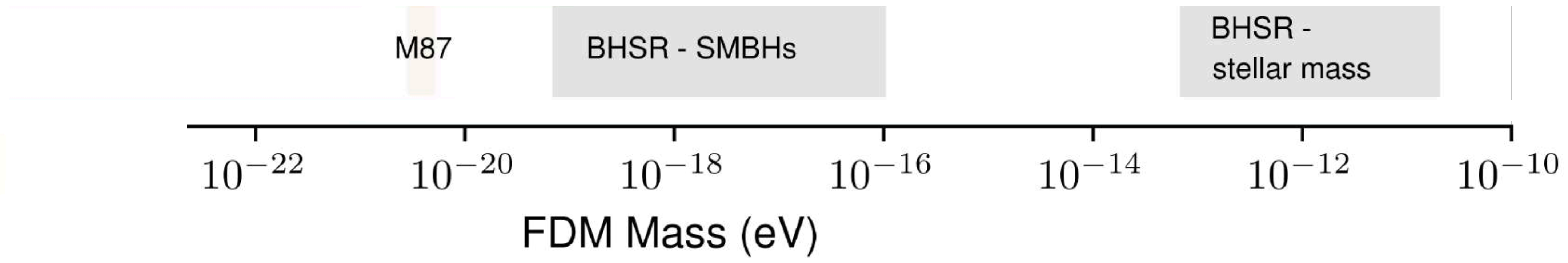


H. Chia et al, 2018

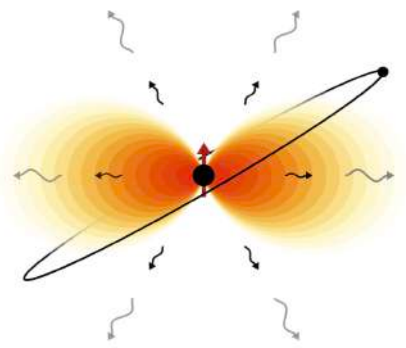
Emits gravitational waves



H. Chia et al, 2018



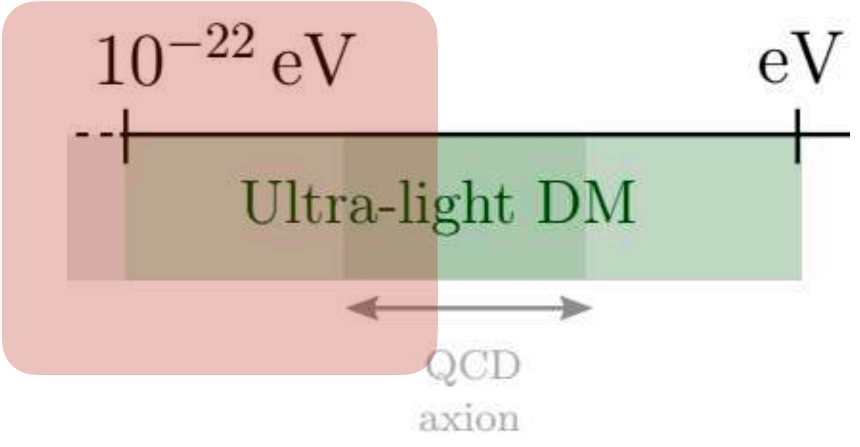
Dynamics can be altered by the presence of a companion - binary



H. Chia et al, 2018

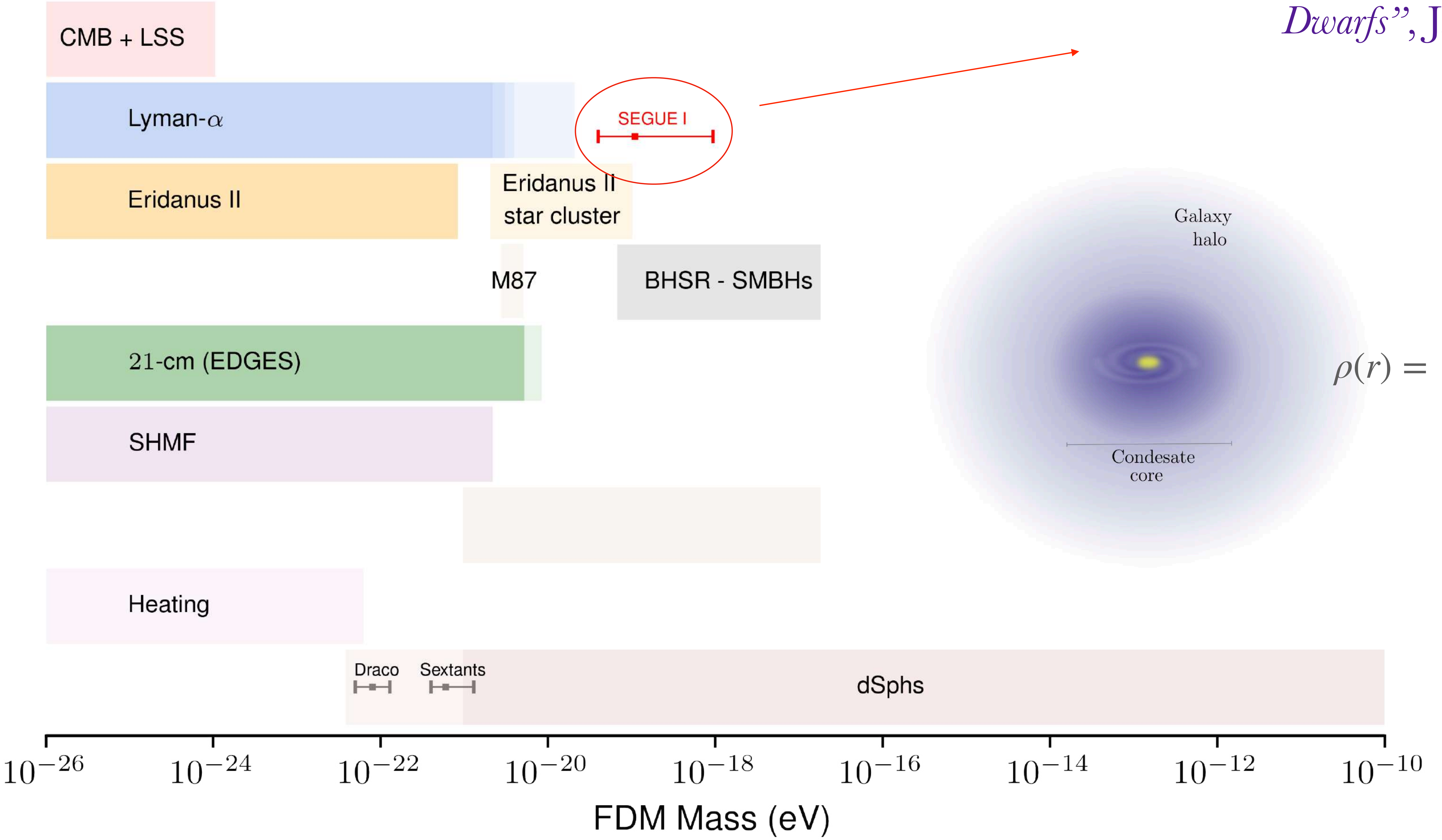
Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass



Presence of a core

"Narrowing the mass range of Fuzzy Dark Matter with Ultra-faint Dwarfs", J. Chan, **E.F.**, K. Hayashi, 2021.



FDM SIMULATIONS

$$\rho(r) = \begin{cases} \rho_{\text{soliton}} \simeq \dfrac{\rho_c}{\left[1+0.091(r/r_c)^2\right]^8}\,, & r < r_\epsilon \\ \rho_{\text{NFW}} = \dfrac{\rho_s}{(r/r_s)(1+r/r_s)^2}\,, & r > r_\epsilon \end{cases}$$

Ultra-light Dark Matter

FDM mass from Ultra-faint dwarfs

Hayashi, **E.F**,Chan, 2021.

Ultra-faint dwarfs (UFD): ideal laboratory to study DM

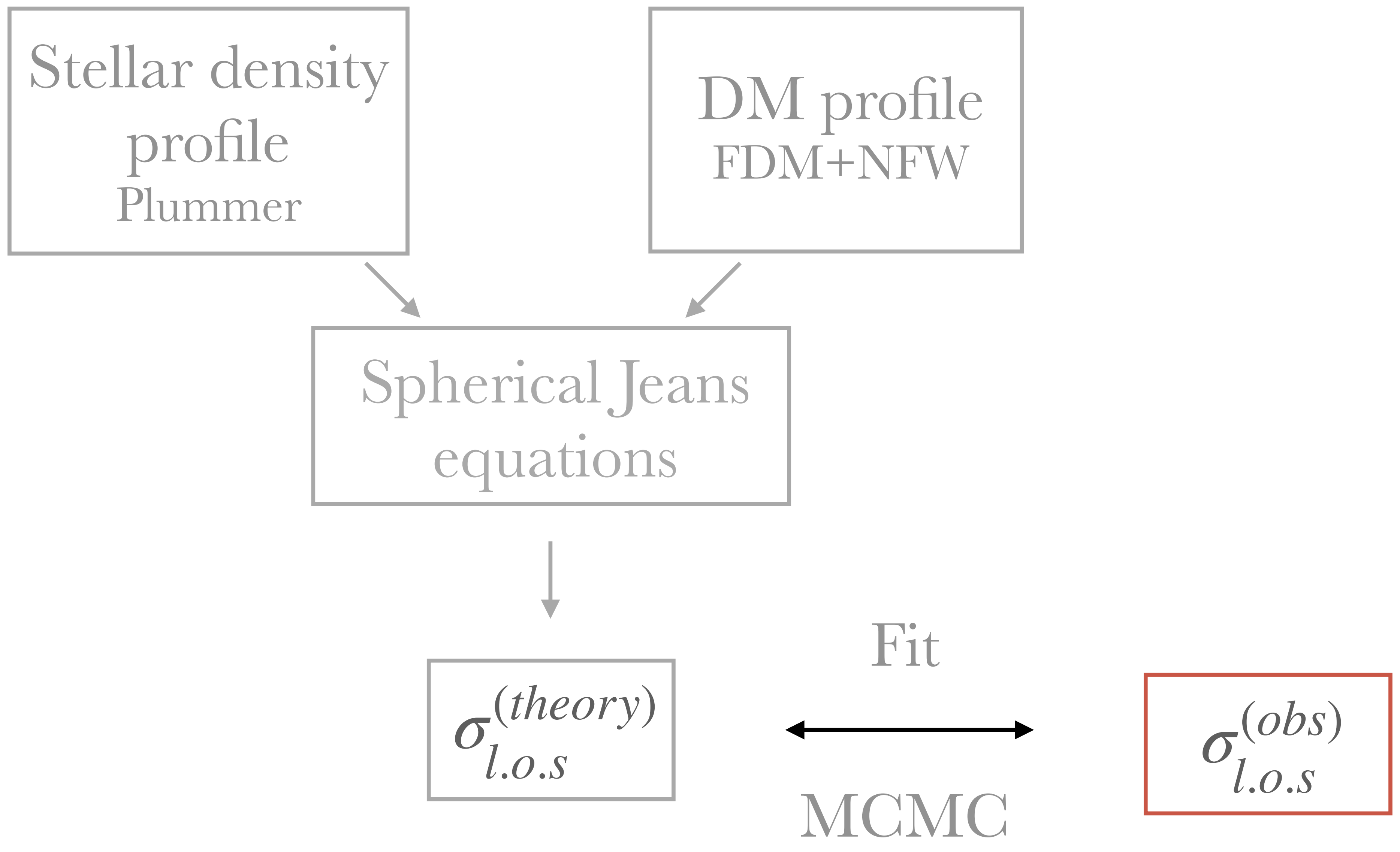
Stellar kinematic data from 18 UFDs to fit the FDM profile:

FDM SIMULATIONS

$$\rho_c(r) = 1.9 \times 10^{12} \left(\frac{m}{10^{-23}\,\mathrm{eV}}\right)^{-2} \left(\frac{r_c}{\mathrm{pc}}\right)^{-4} \quad [M_\odot\,\mathrm{pc}]$$

$$r_c \simeq 1600 \left(\frac{m}{10^{-23}\,\mathrm{eV}}\right)^{-1} \left(\frac{M_{\mathrm{halo}}}{10^{12}\,M_\odot}\right)^{-1/3} \quad [\mathrm{pc}]$$

$$\rho(r) = \begin{cases} \rho_{\mathrm{soliton}} \simeq \dfrac{\rho_c}{\left[1 + 0.091(r/r_c)^2\right]^8}, & r < r_\epsilon \\ \rho_{\mathrm{NFW}} = \dfrac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, & r > r_\epsilon \end{cases}$$



Stellar density profile
Plummer

DM profile
FDM+NFW

Spherical Jeans equations

$\sigma^{(theory)}_{l.o.s}$

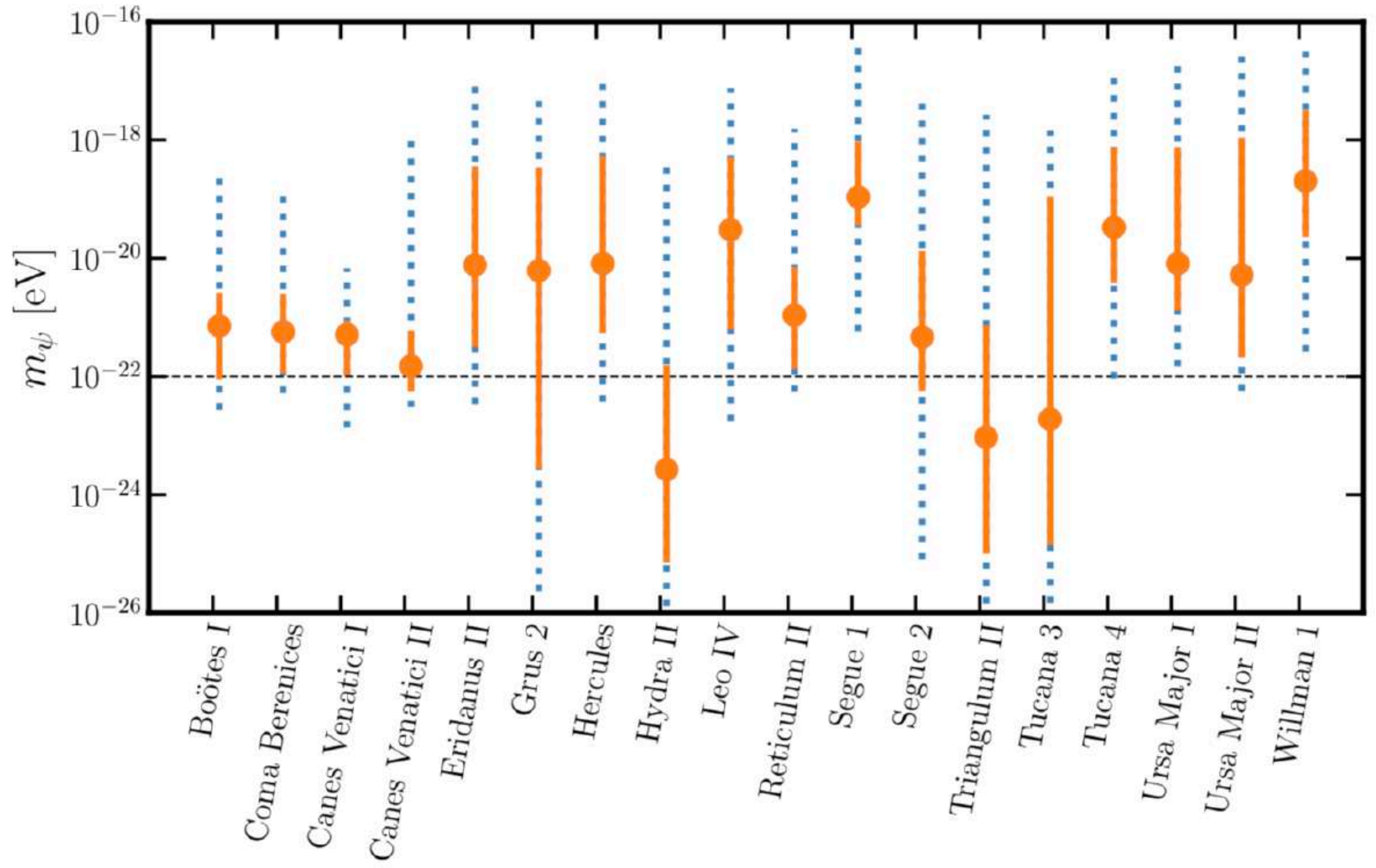
Fit

MCMC

$\sigma^{(obs)}_{l.o.s}$

Parameter space: $\left\{m, M_{\mathrm{halo}}, r_\epsilon, r_s, r_\beta, \beta_0, \beta_\infty, \eta, r_h, v_{sys}\right\}$

Velocity anisotropy

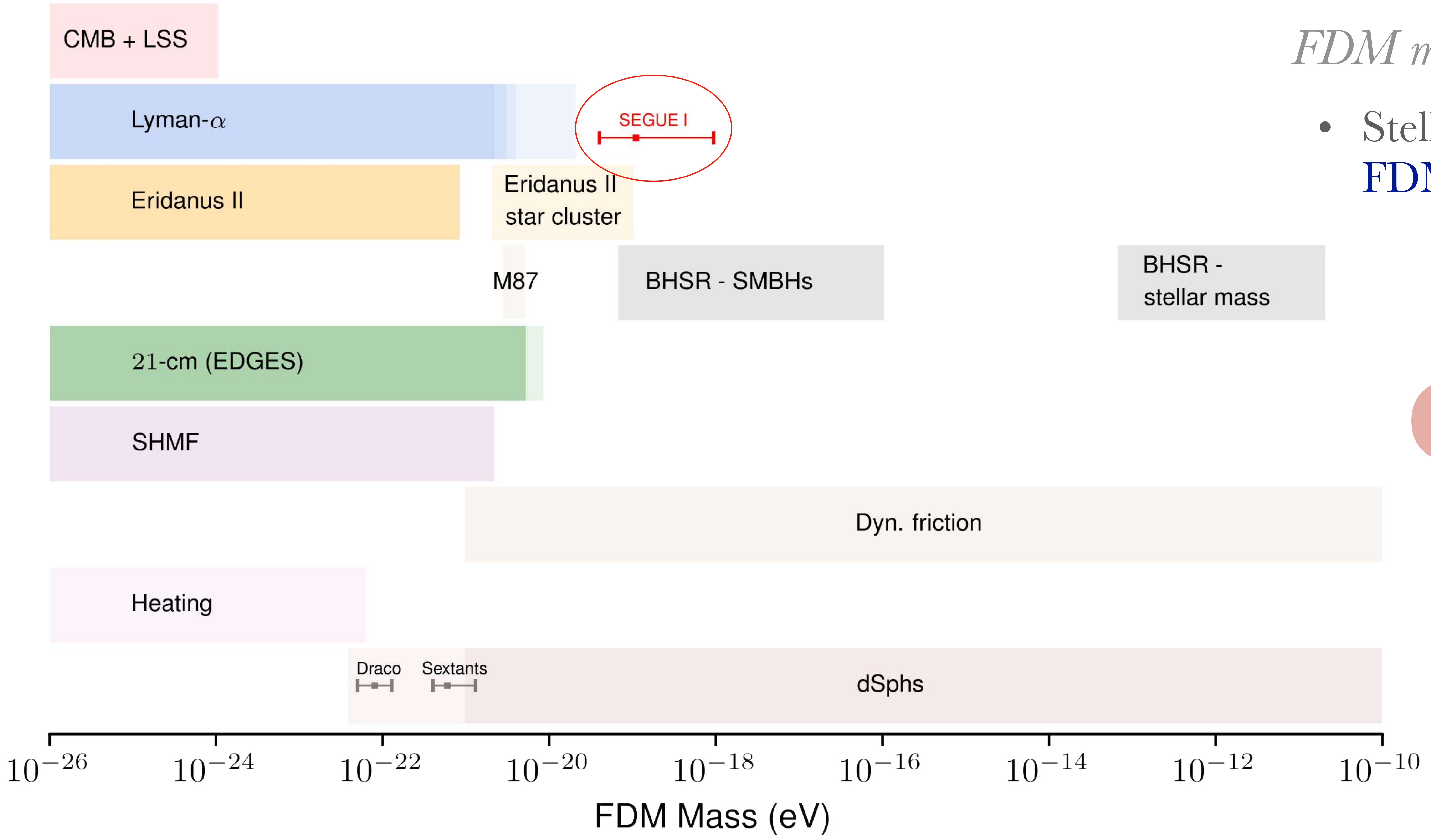


Strongest constraint on m_{FDM} to date!

Ultra-light Dark Matter

Fuzzy Dark Matter - bounds on the mass

Hayashi, **E.F**,Chan, 2021.

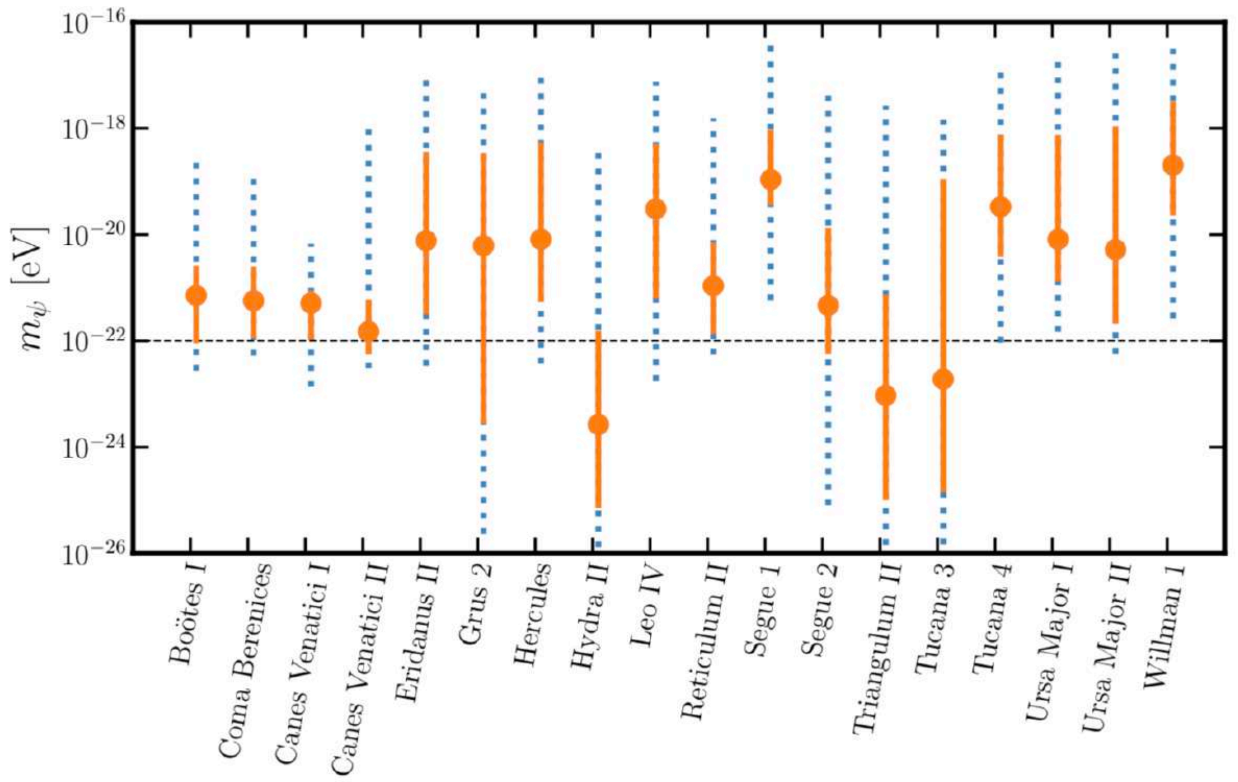


FDM mass from Ultra-faint dwarfs

- Stellar kinematic data from 18 UFDs to fit the FDM profile from simulations

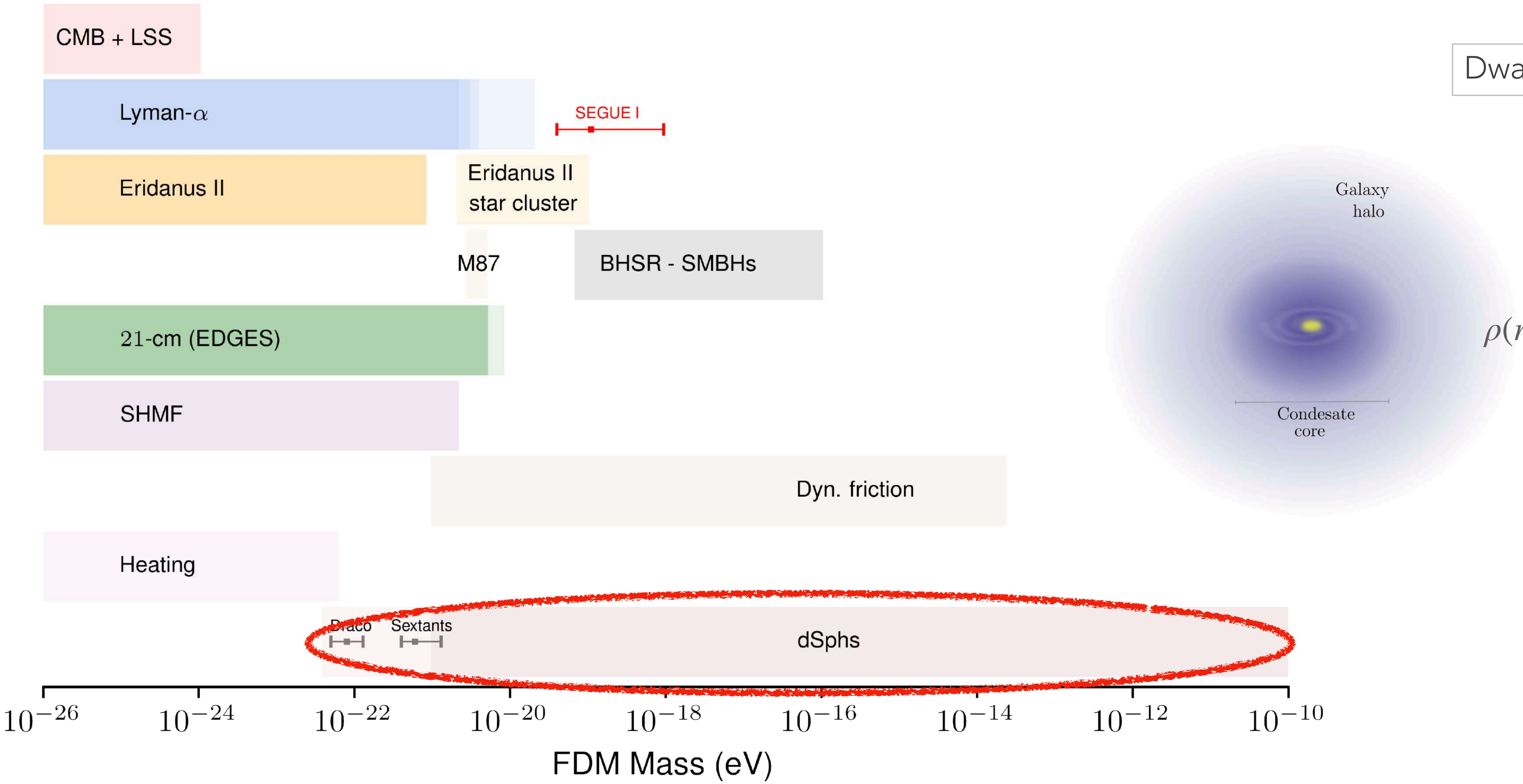
$$m_{\rm FDM}^{(Seg1)} = 1.1^{+8.3}_{-0.7} \times 10^{-19}\,{\rm eV}$$

Strongest constraint on $m_{\rm FDM}$ to date!



Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass



DWARFS

Dwarf Spheroidals (dSphs)

FDM SIMULATIONS

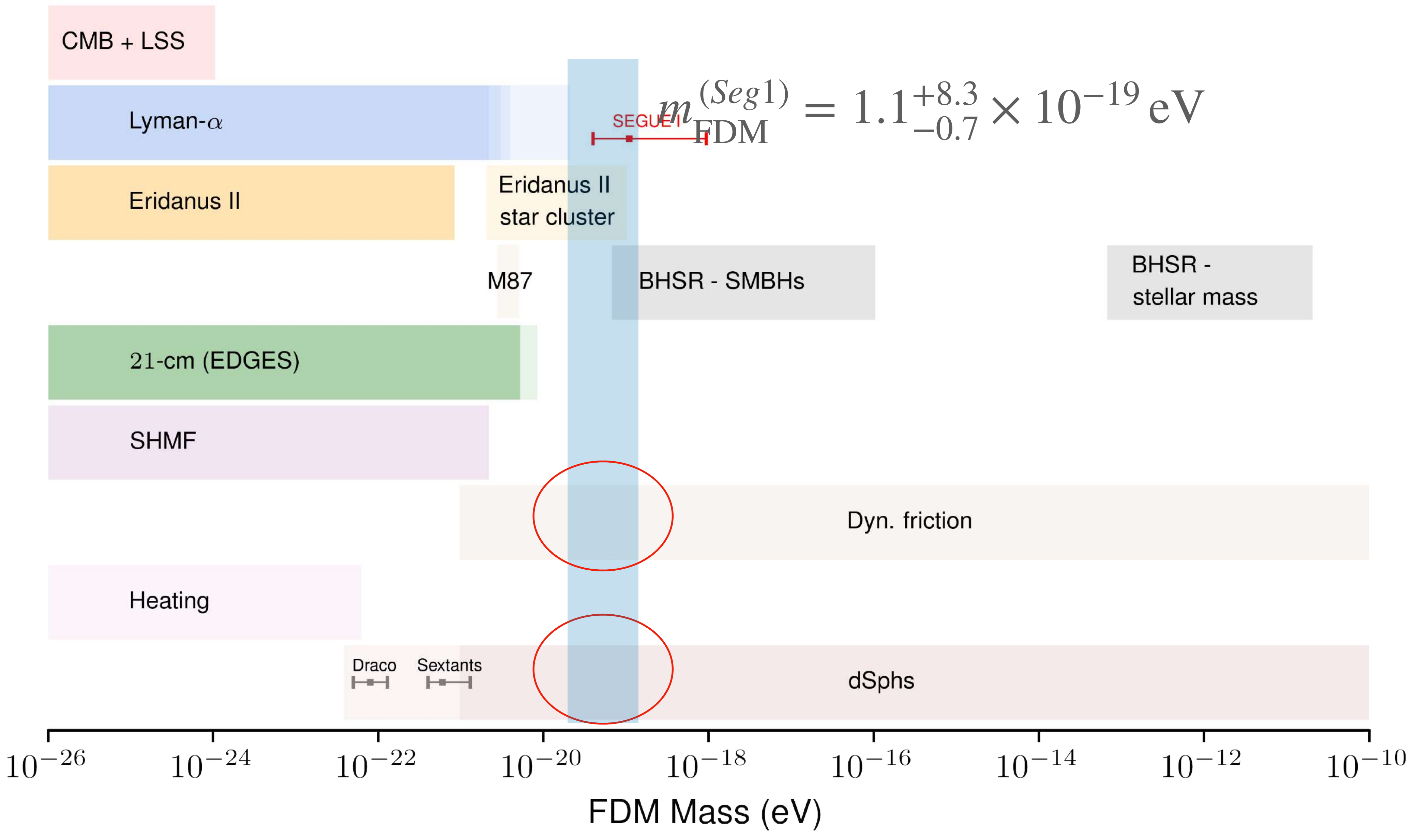
$$\rho(r) = \begin{cases} \rho_{\text{soliton}} \simeq \dfrac{\rho_c}{\left[1 + 0.091(r/r_c)^2\right]^8}, & r < r_\epsilon \\ \rho_{\text{NFW}} = \dfrac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, & r > r_\epsilon \end{cases}$$

Fornax - Sculptor

$$m < 0.8 \times 10^{-22}\ \text{eV}$$

Gonzalez-Morales et al. 2017, Safarzadeh and Spergel 2019

Constraints on the mass



$$m_{\rm FDM}^{(Seg1)} = 1.1^{+8.3}_{-0.7} \times 10^{-19}\,{\rm eV}$$

Incompatibility between all bounds and the dSphs (Fornax and Sculptor) bounds

Possible reasons for this incompatibility:

- *Influence of baryons:* baryonic processes can change the density structure of their halo - we are not probing the intrinsic DM profile.
- *Universality of the core profile*: FDM soliton profile might be too simplistic, could change for different systems (might also depend on baryons)
- *Core-mass relation:* might need to be better understood. ≠ relation in ≠ simulations
- *Challenge for the FDM model*

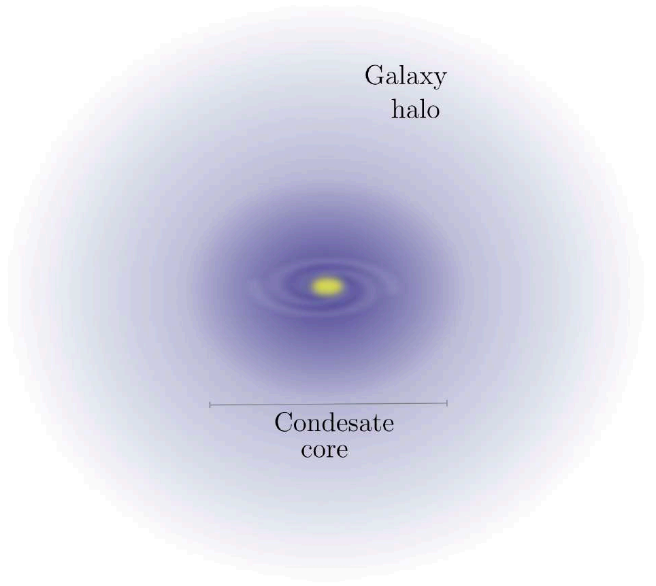
Safarzadeh and Spergel 2019; Hayashi, EF, Chan 2021

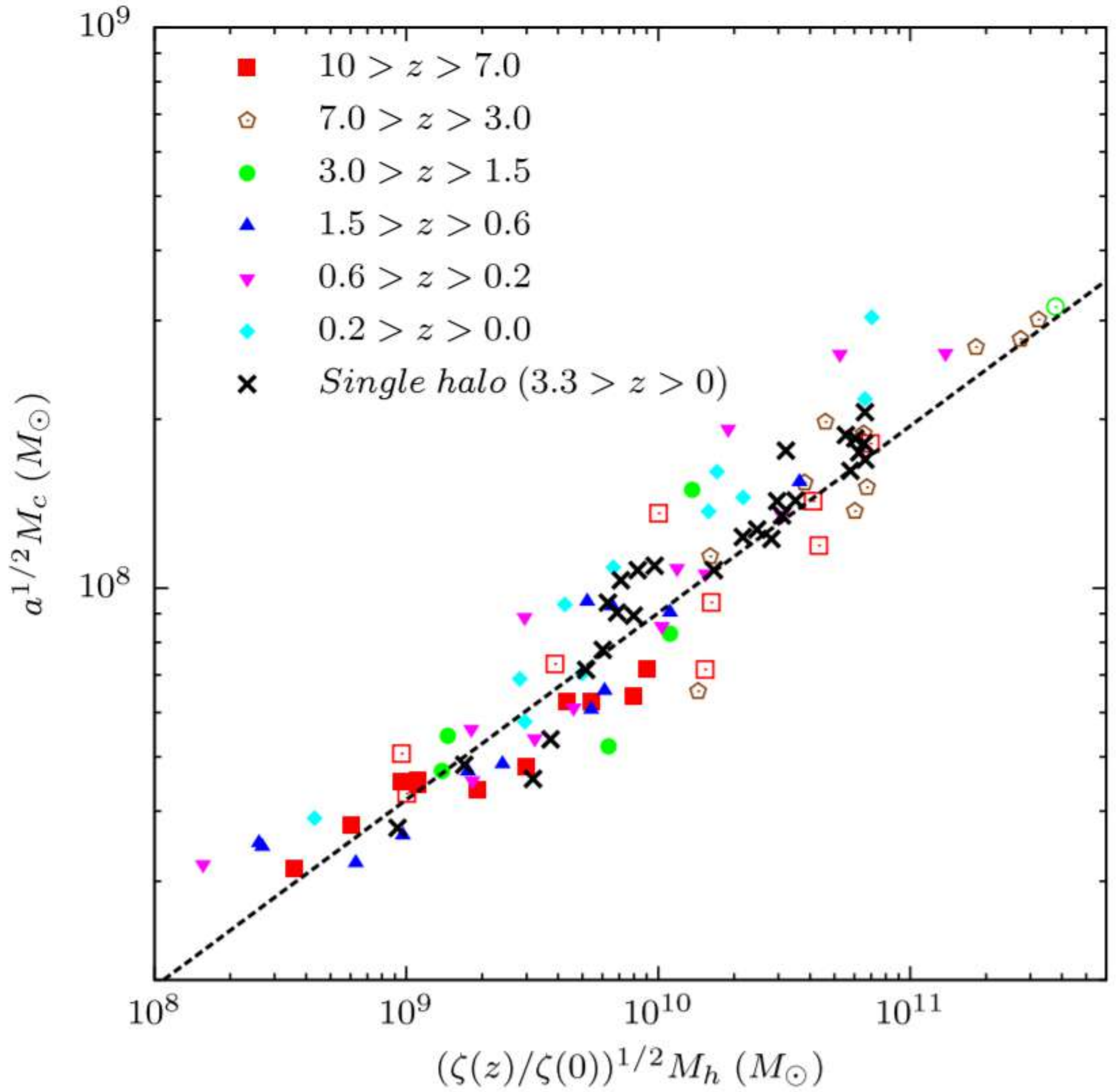
FDM - Core-halo mass relation

J. Chan et al. 2021

We want to study how the core relates to the halo mass

$$\rho_c \simeq \frac{1.9\times 10^{-2}}{[1+0.091\,(r/R_{1/2,c})^2]^8}\left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{-2}\boxed{\left(\frac{r_c}{\mathrm{kpc}}\right)}^{-4} M_\odot\,\mathrm{pc}^{-3}, \quad \xrightarrow[?]{} M_h$$





Schive et al. 2014

Schive et al 2014

$$M_c \propto M_h^{1/3}$$

Velocity dispersion tracing

$$\sigma_c \sim \sigma_h$$

Mocz et al 2017

$$M_c \propto M_h^{5/9}$$

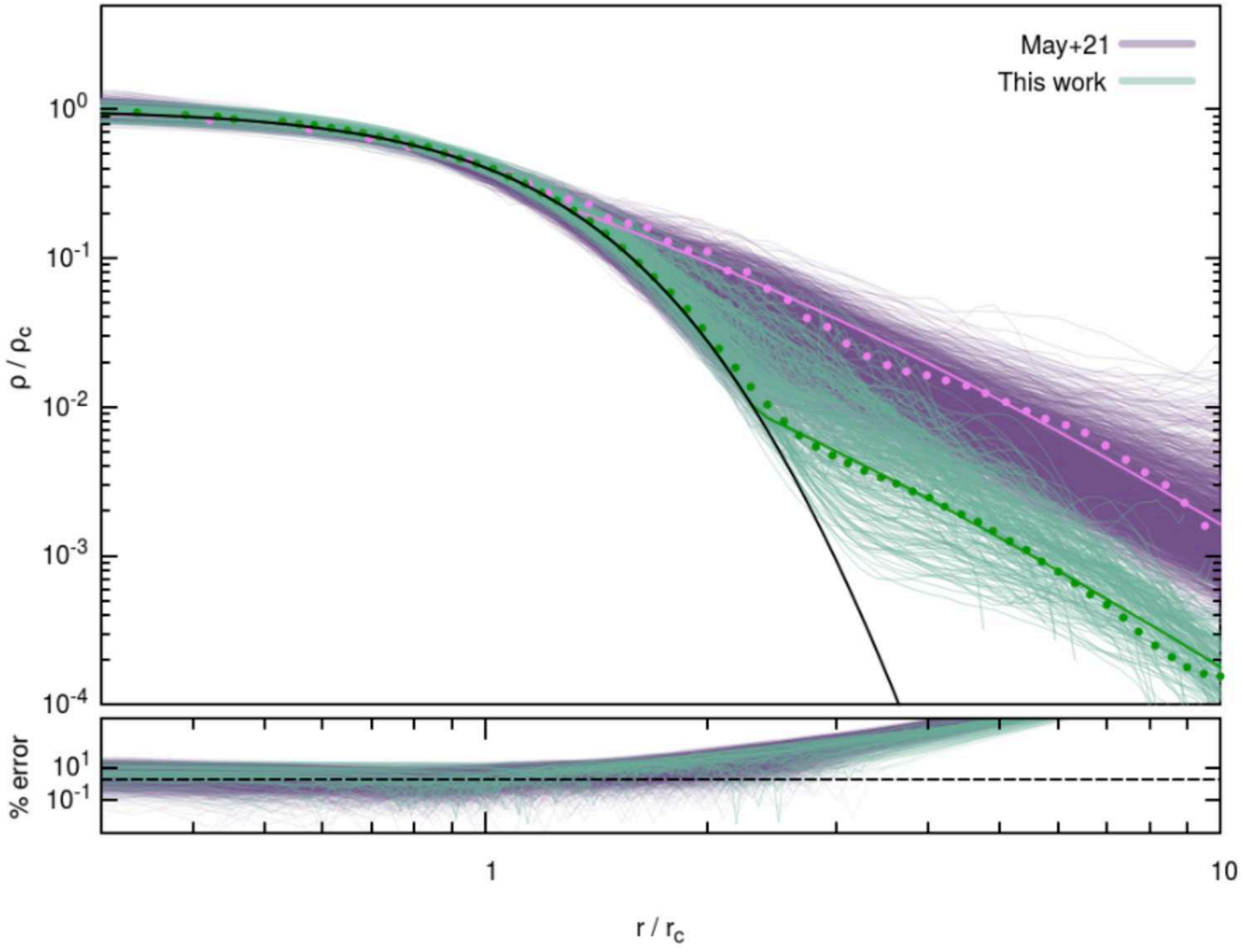
Energy tracing

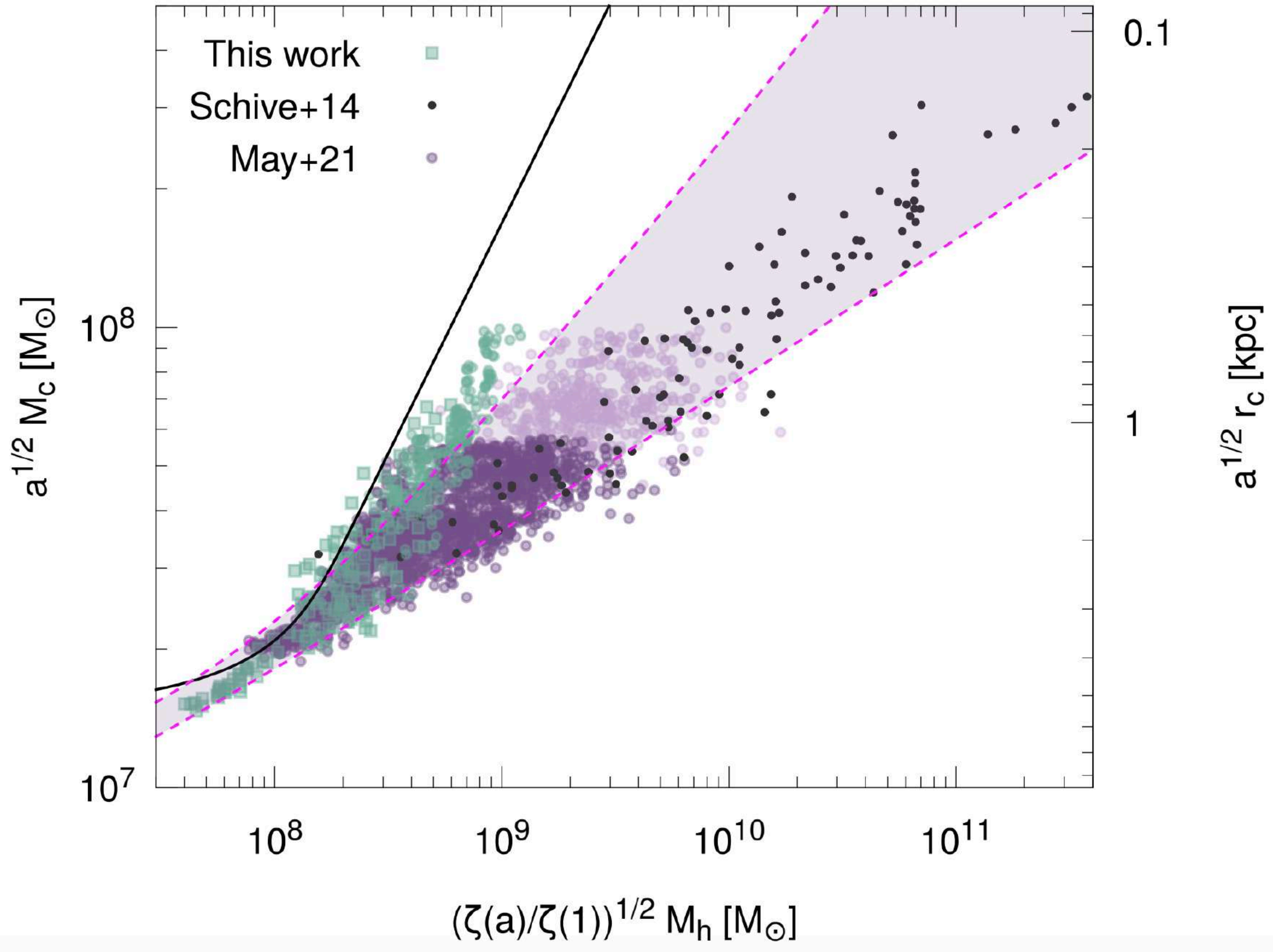
$$M_c\,\sigma_c^2 \sim M_h\,\sigma_h^2$$

$$\underbrace{\text{Velmatt et al 2018}}_{=\ \text{Schive}},\ \underbrace{\text{Nori et al 2020, Nima et al 2020}}_{\neq\ \text{Schive}}$$

FDM - Core-halo mass relation

J. Chan, EF, S. May, K. Hayashi, M. Chiba 2021

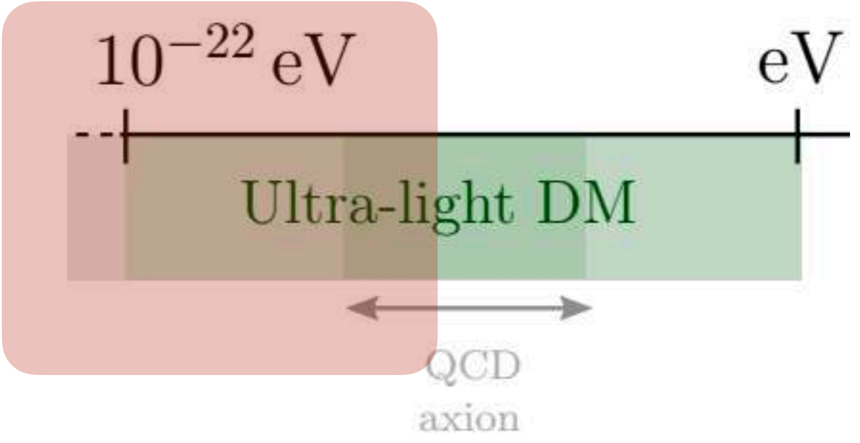




Steeper slope → Smaller core → Smaller mass

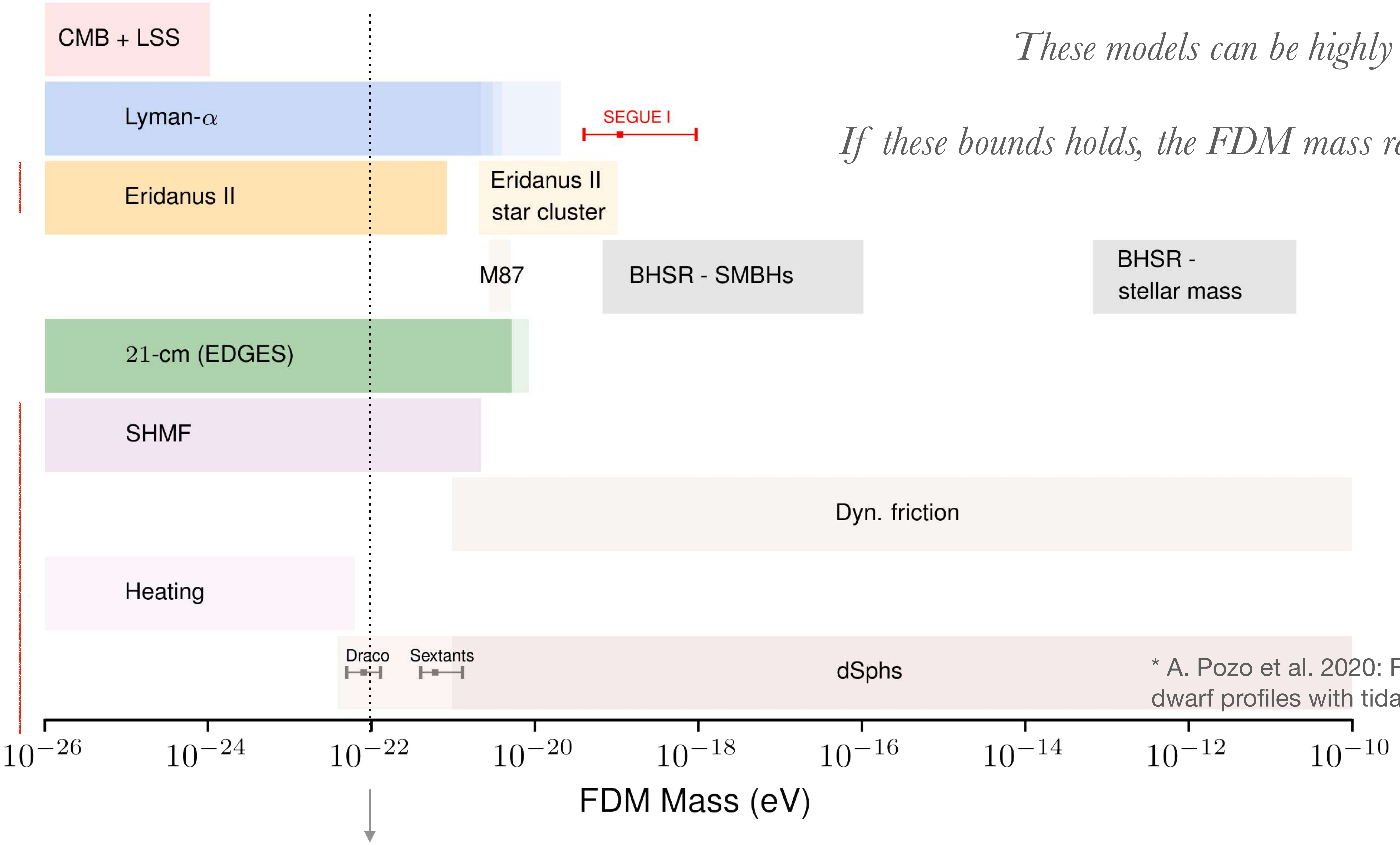
Observational implications and constraints

Fuzzy Dark Matter - bounds on the mass



These models can be highly constrained

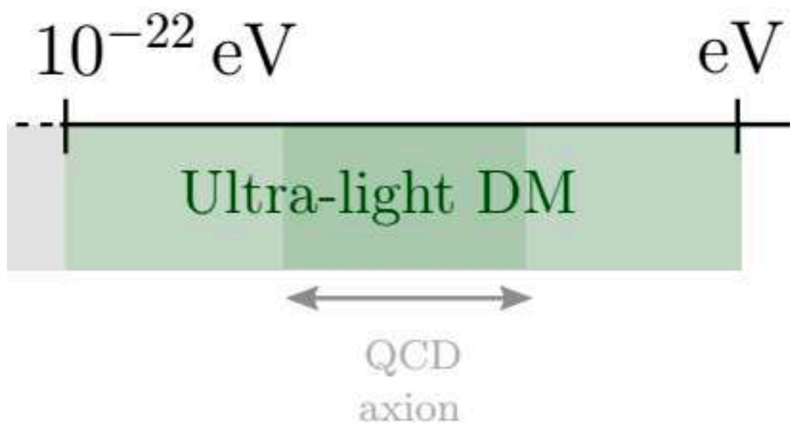
If these bounds holds, the FDM mass range is narrowing down



* A. Pozo et al. 2020: FDM with $m \sim 10^{-22}$ eV explains classical dwarf profiles with tidal stripping included

Sweet spot for solving small scale problems

Observational implications and constraints



Bounds on the mass and other parameters

Self interacting FDM

m g

* Check: Lasha Berezhiani et al (2020)

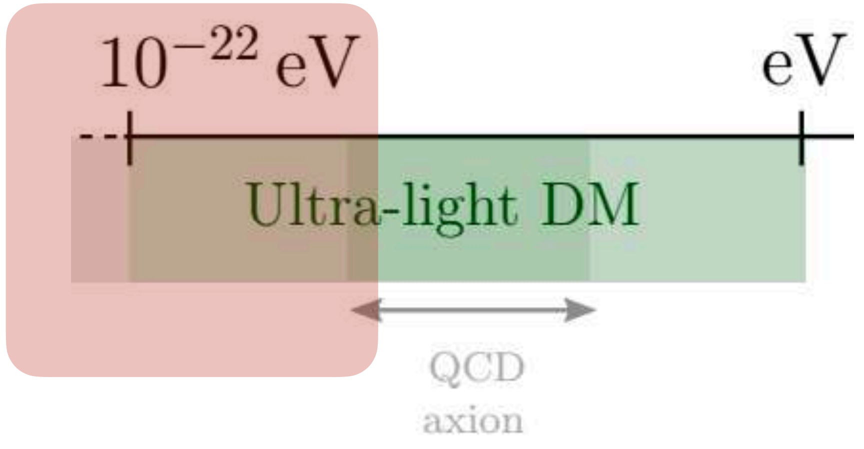
DM Superfluid

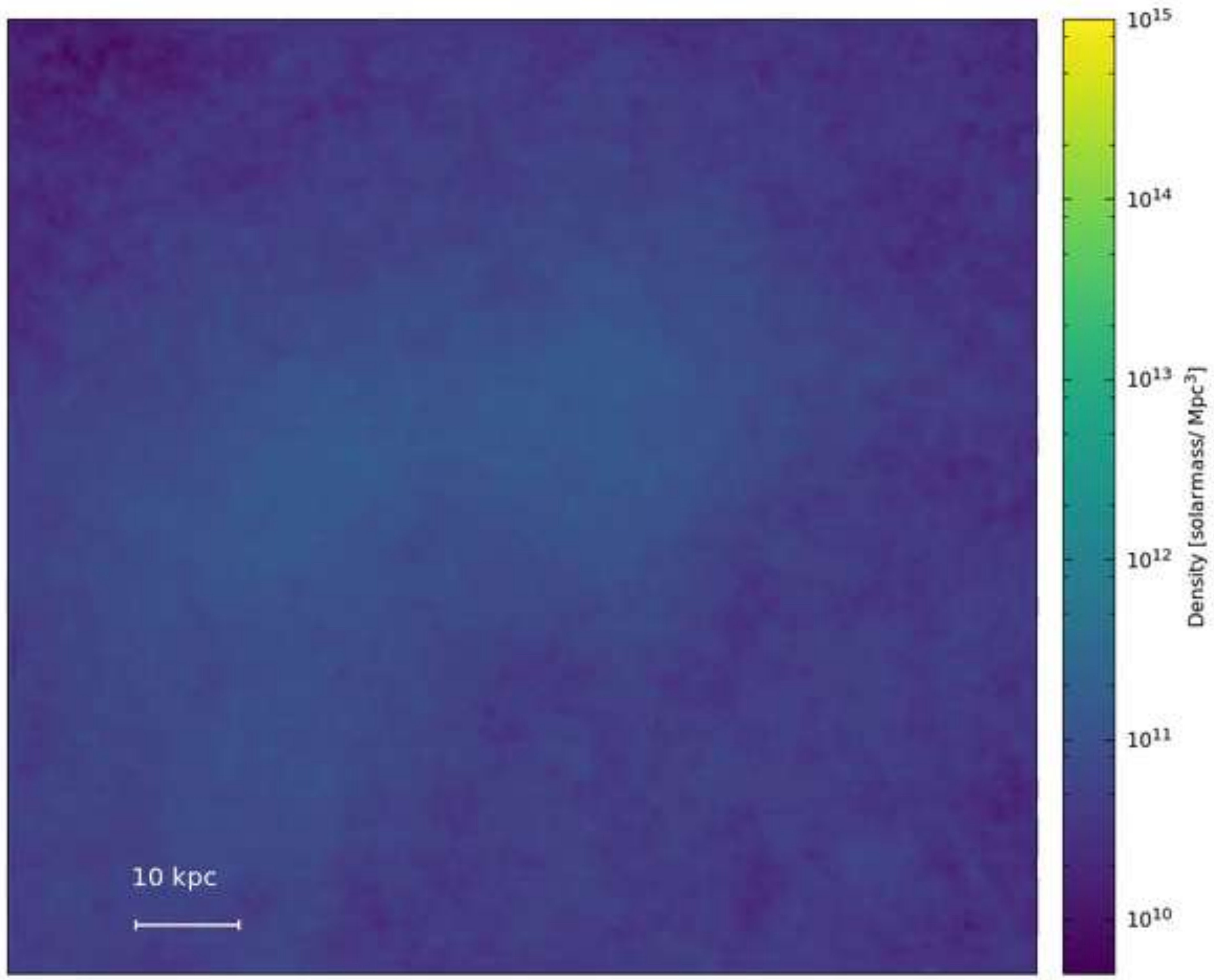
$$\mathcal{L} = P(X)$$

Still highly unconstrained

Interference pattern

Granules





Simulation by Jowett Chan

Strong lensing:
James H. H. Chan, Hsi-Yu Schive, Shing-Kwong Wong, Tzihong Chiueh, Tom Broadhurst, 2020
- Flux ratio anomalies of a few tens of percent
- Produce rare hexad and octad images

Stellar streams:
Neal Dalal, Jo Bovy Lam Hui, Xinyu Li, 2020

Sub-galactic power spectrum:
Hezaveh et al. (2016)

Order one fluctuations in density ⟶ Constructive interference: granules
Destructive interference

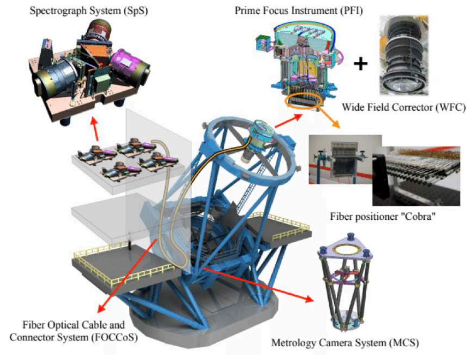
↳ $\sim \lambda_{\mathrm{dB}}$ Hard to observe!

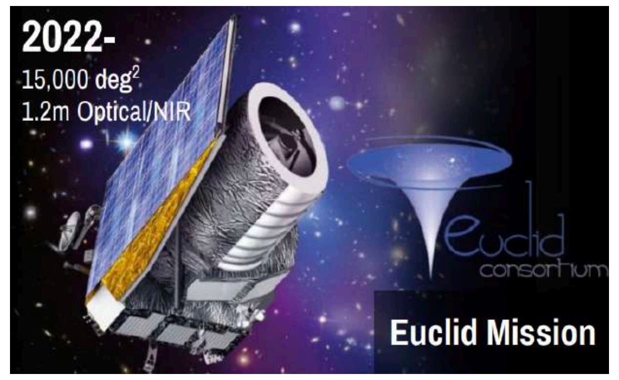
Future - signals in cosmology

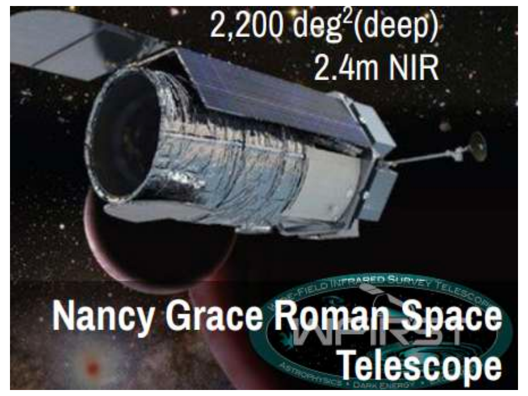
Observations

Photometric and spectroscopic surveys

Prime Focus Spectrograph (PFS)







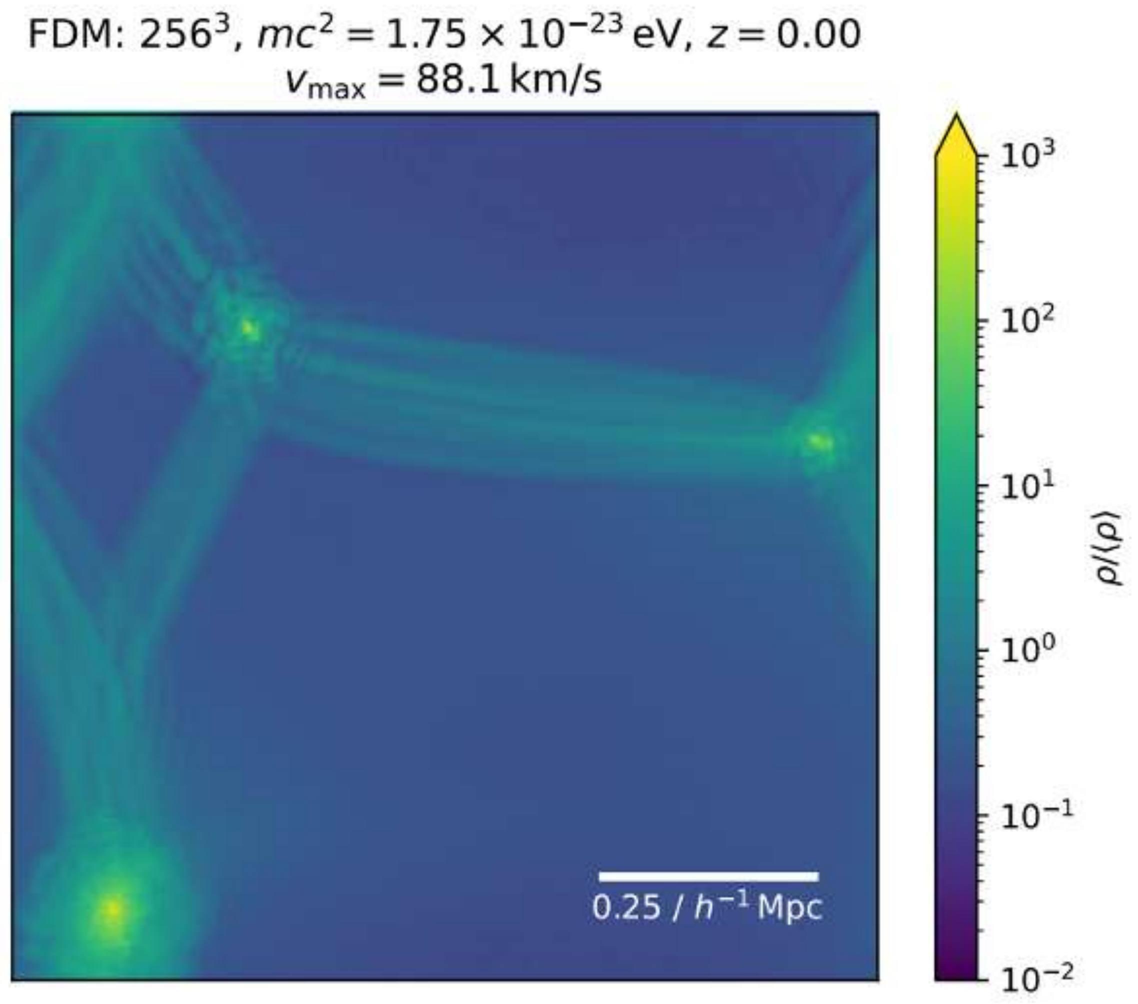
CMB

CMB-S4
Next Generation CMB Experiment

21cm

SKA
SQUARE KILOMETRE ARRAY

Simulations



New probes

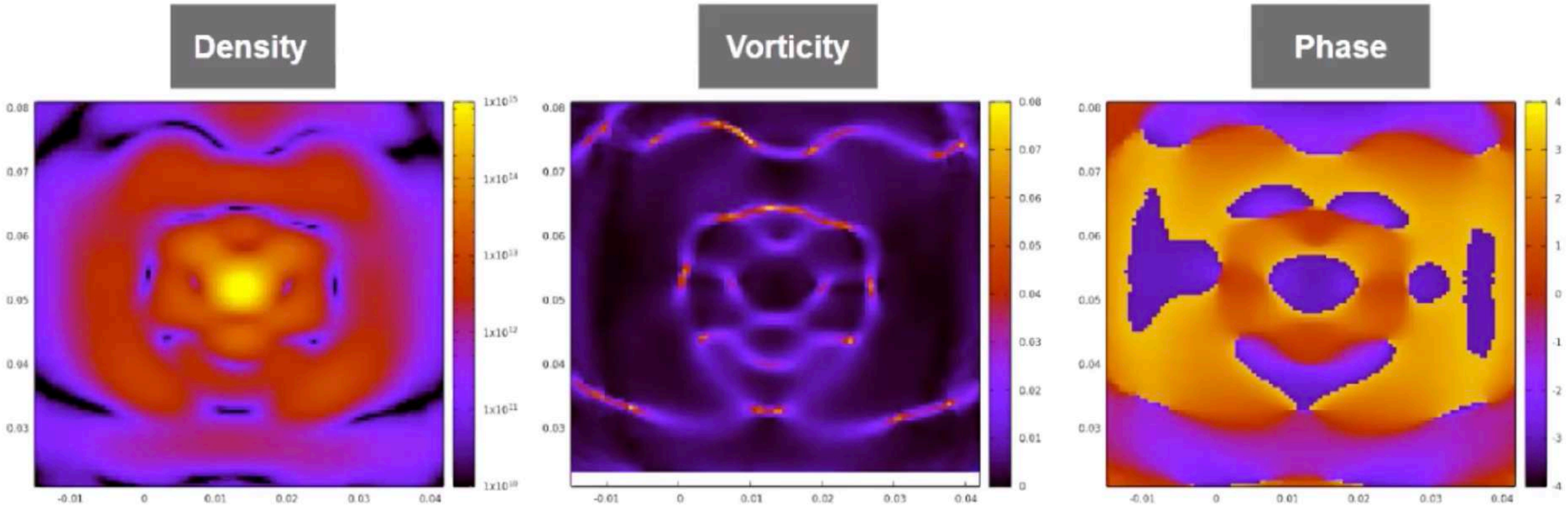
Modified from Jia Liu

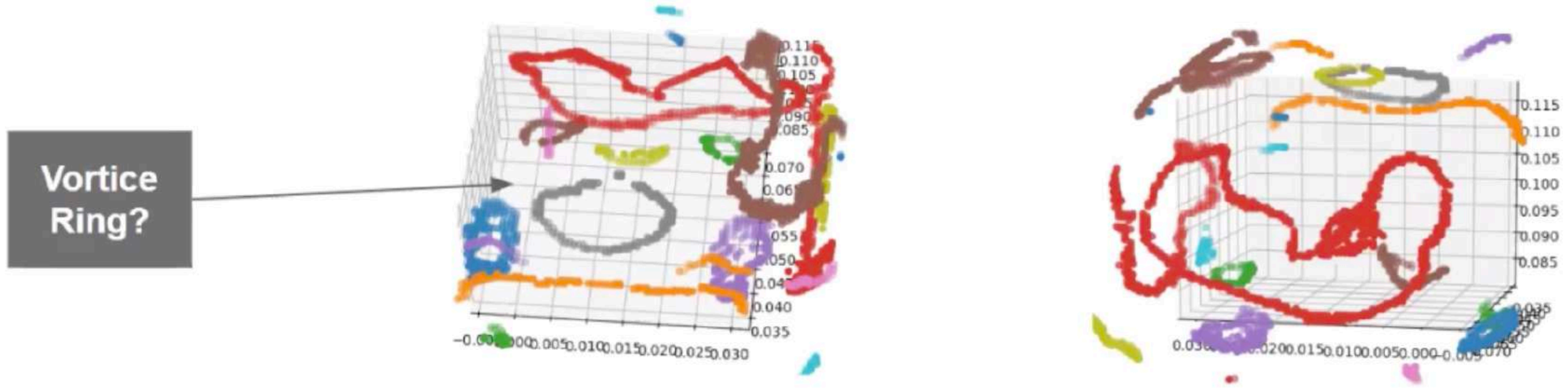
Ongoing: Vortices in FDM and SIFDM

PRELIMINARY

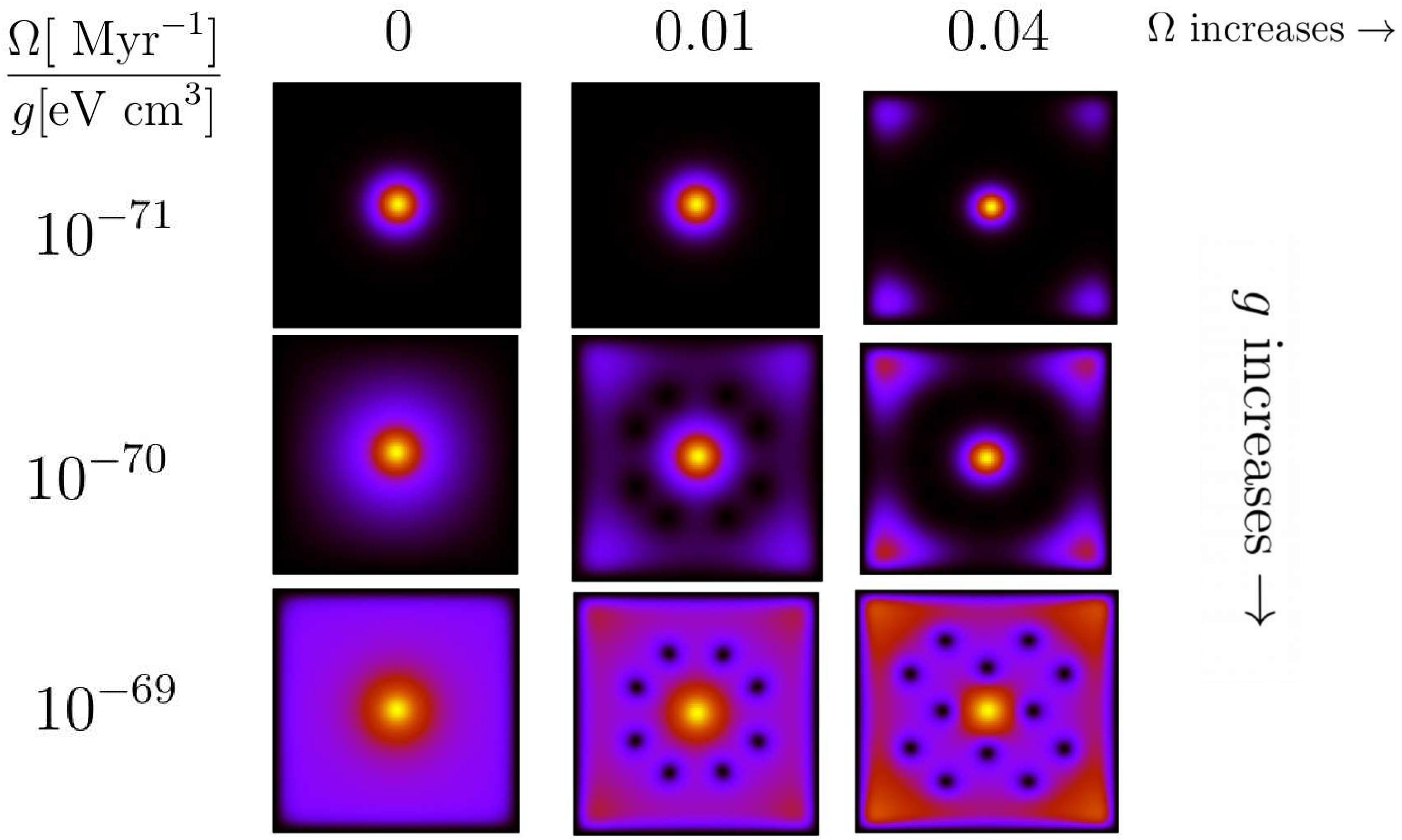
In collaboration with Jowett Chan

Fuzzy DM





Self-interacting Fuzzy DM

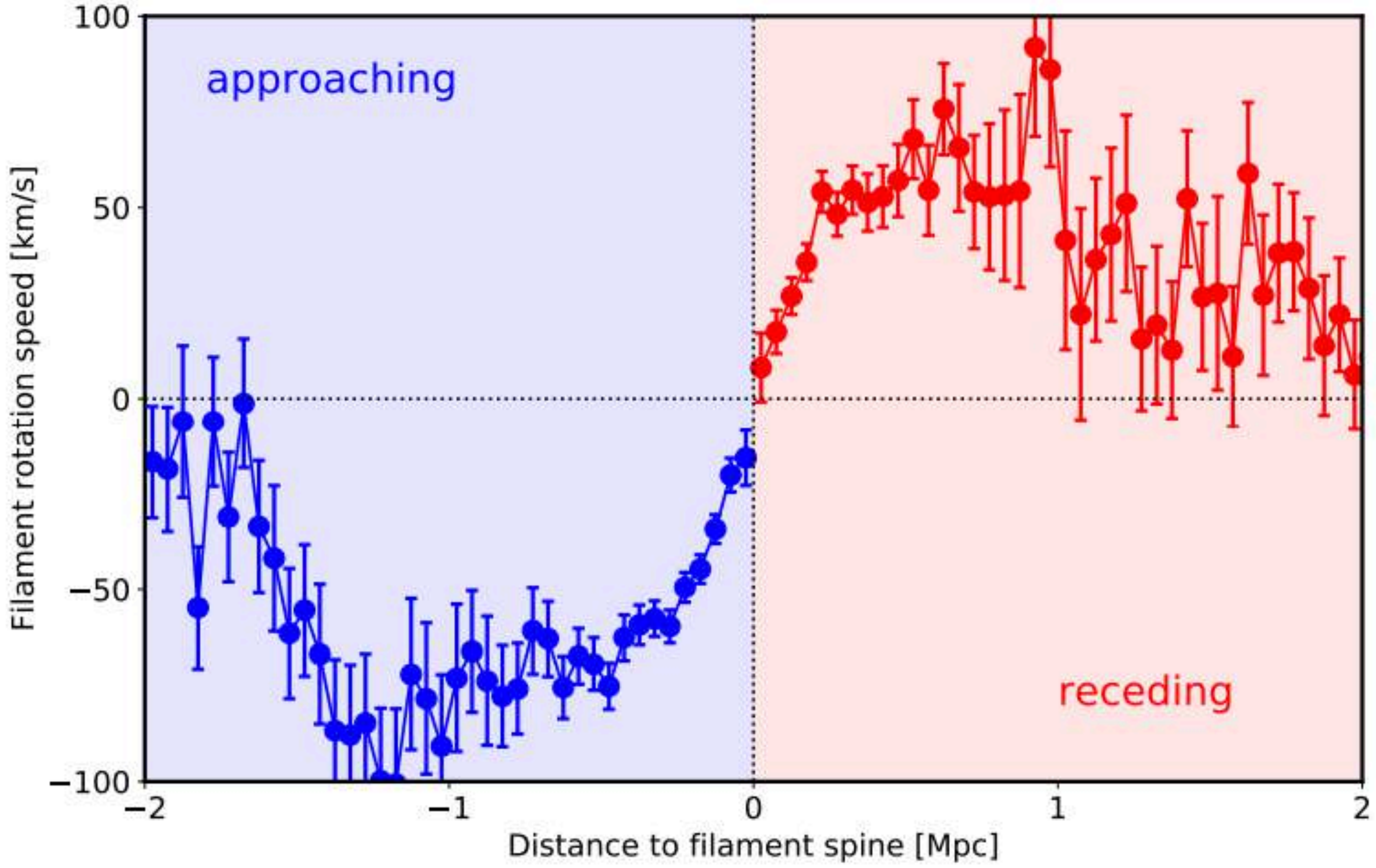


- \+ Improve theoretical understanding of these DM vortices *In collaboration with P. Bittar*

What is the predicted size and abundance of vortices in the halo?
Are they observable?
Strong lensing? Stellar streams?
Can they be formed in the filaments?

Rotation of filaments: vortices

Peng Wang, Noam I. Libeskind, Elmo Tempel, Xi Kang, Quan Guo, "*Possible observational evidence that cosmic filaments spin*" (2021)



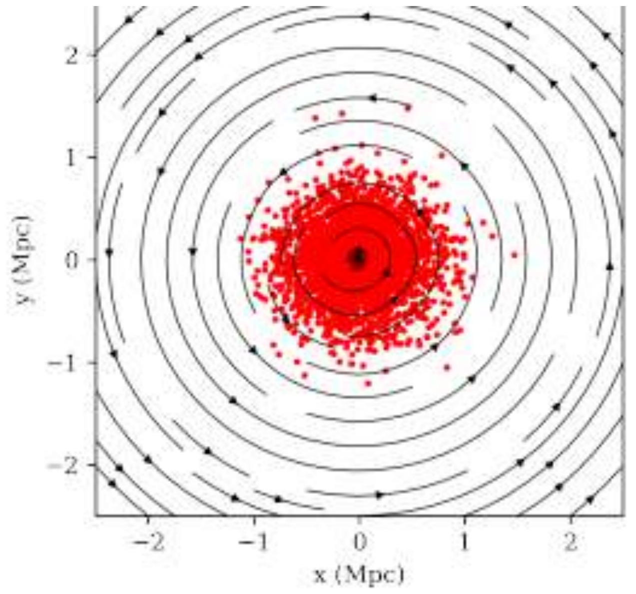
- Stacking thousands of filaments and examining the velocity of galaxies perpendicular to the filament's axis (via their red and blue shift)
- Found that filaments display motion consistent with rotation ⟶ largest objects known to have angular momentum

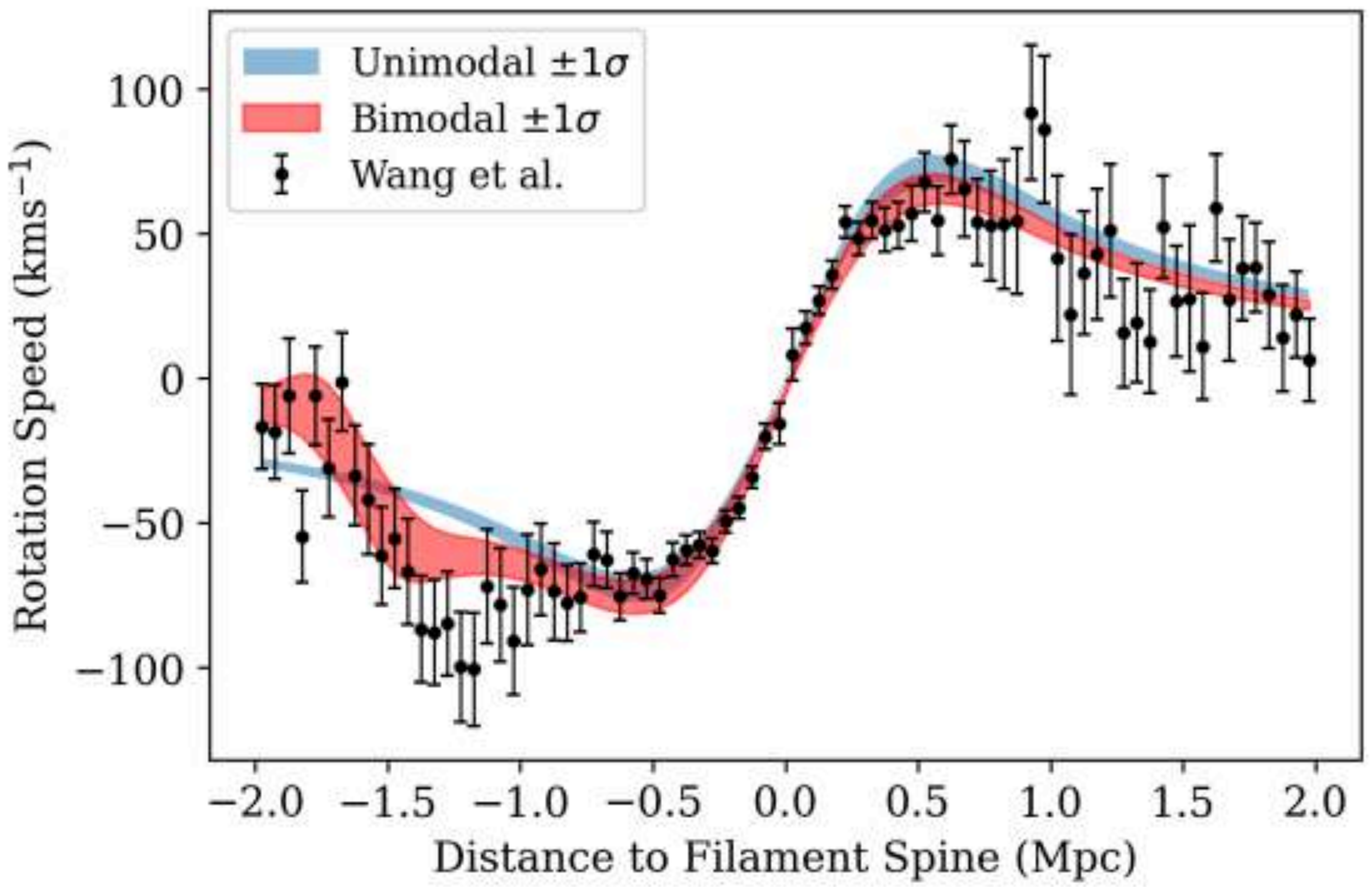
Rotation of filaments: vortices

- Not clear that we can get spinning cosmic filaments in LCDM
  - Seems to be difficult to theoretically explain the acquisition of angular momentum on megaparsec scales
  - Some simulations seem to be finding spinning cosmic filaments

Stephon Alexander, Christian Capanelli, Elisa G. M. Ferreira, and Evan McDonough, *"Cosmic Filament Spin from Dark Matter Vortices"* (2021)

- Suggest that a collection of (ULDM) vortices enclosed in a cylindrical volume aligned with the axis of a filament are able to generate rotations at the Mpc scale and reproduce the result of Wang et al (2021)





$$R = 0.51^{+0.02}_{-0.02}\ \mathrm{Mpc}$$

$$\frac{N_V}{m} = 2.9^{+0.2}_{-0.2} \times 10^{25}\ \mathrm{eV}^{-1}.$$

For example, for a $m \sim 10^{-22}\mathrm{eV} \longrightarrow N_V \sim 3000$

Compatible with:
- in regions where the density vanishes (*Hui et al 2020*, *Lague et al 2020*)
- Transfer of angular momentum (*Berezhiani, 2015*)

Ongoing: Simulation of the SIFDM

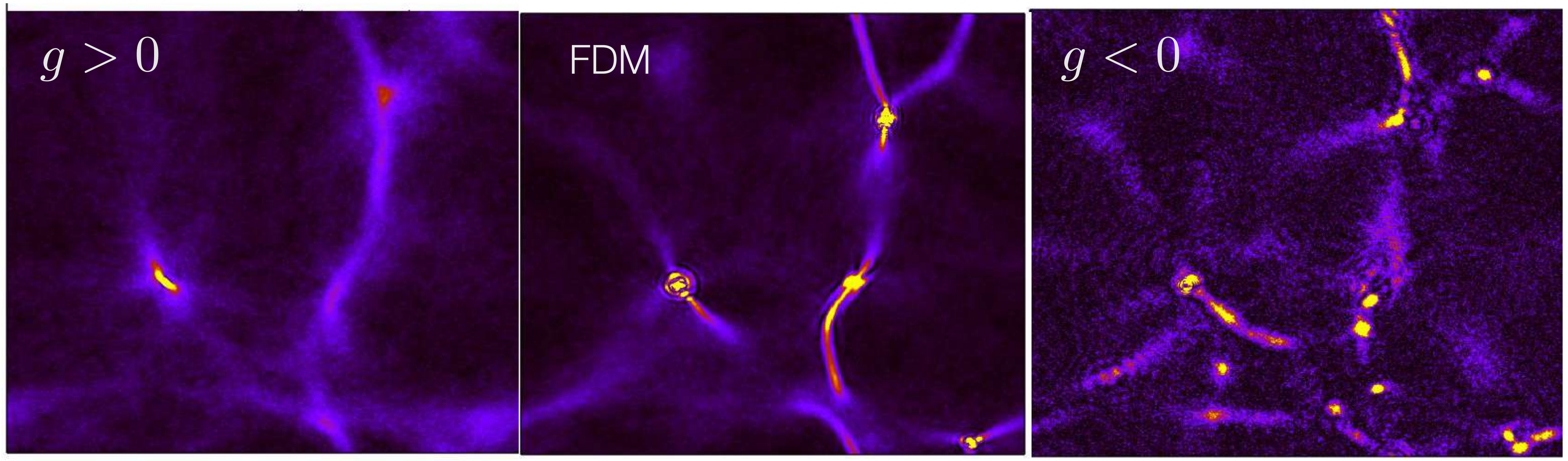
In collaboration with Jowett Chan

SIFDM can present very different phenomenology - very few simulations of this class

Solving the *Schrödinger-Poisson equations* using a splitting spectral method

L = 1 Mpc/h
N = 980^3
z_i = 50
timestep = 1000

PRELIMINARY



m = 1e-22 eV
a_s = 2e-74 cm

m = 1e-22 eV
a_s = 0 cm

m = 1e-22 eV
a_s = - 2e-74 cm

Other simulations with SIFDM: Amin et al. 2019,
Hartman et al. 2019 (2 fluid – Madelung), Glennon et al 2020 (PySIUltraLight).

Simulations of ULDM

- Hybrid simulations: large scales (hydro) + small sales (SP-sims)
- Soliton mergers
    - Soliton oscillations
- Adding baryons

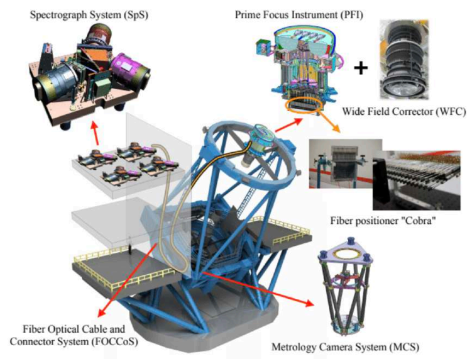
(See works from S. May & V. Springel, L. Hui, Veelmat, Niemeyer & Schwabe, Schive, Chiueh & Broadhurst, Mocz et al., …)

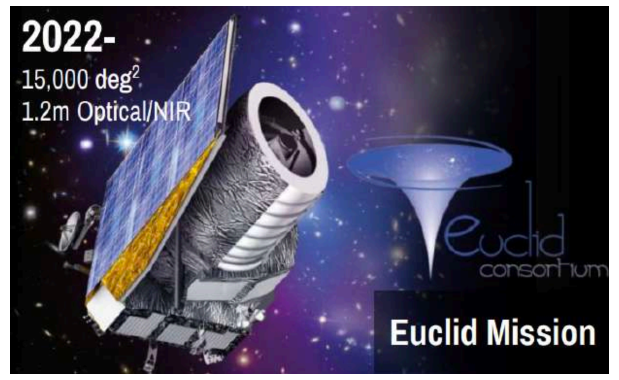
Future - signals in cosmology

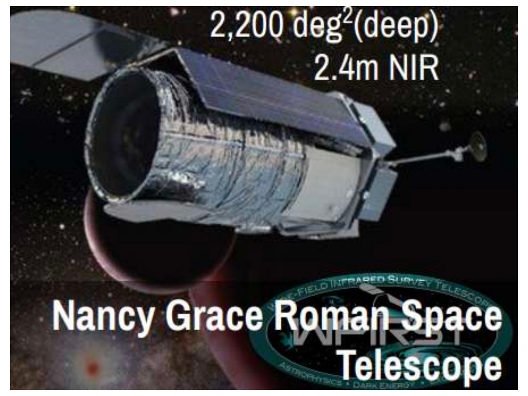
Observations

Photometric and spectroscopic surveys

Prime Focus Spectrograph (PFS)







CMB

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Next Generation CMB Experiment

21cm

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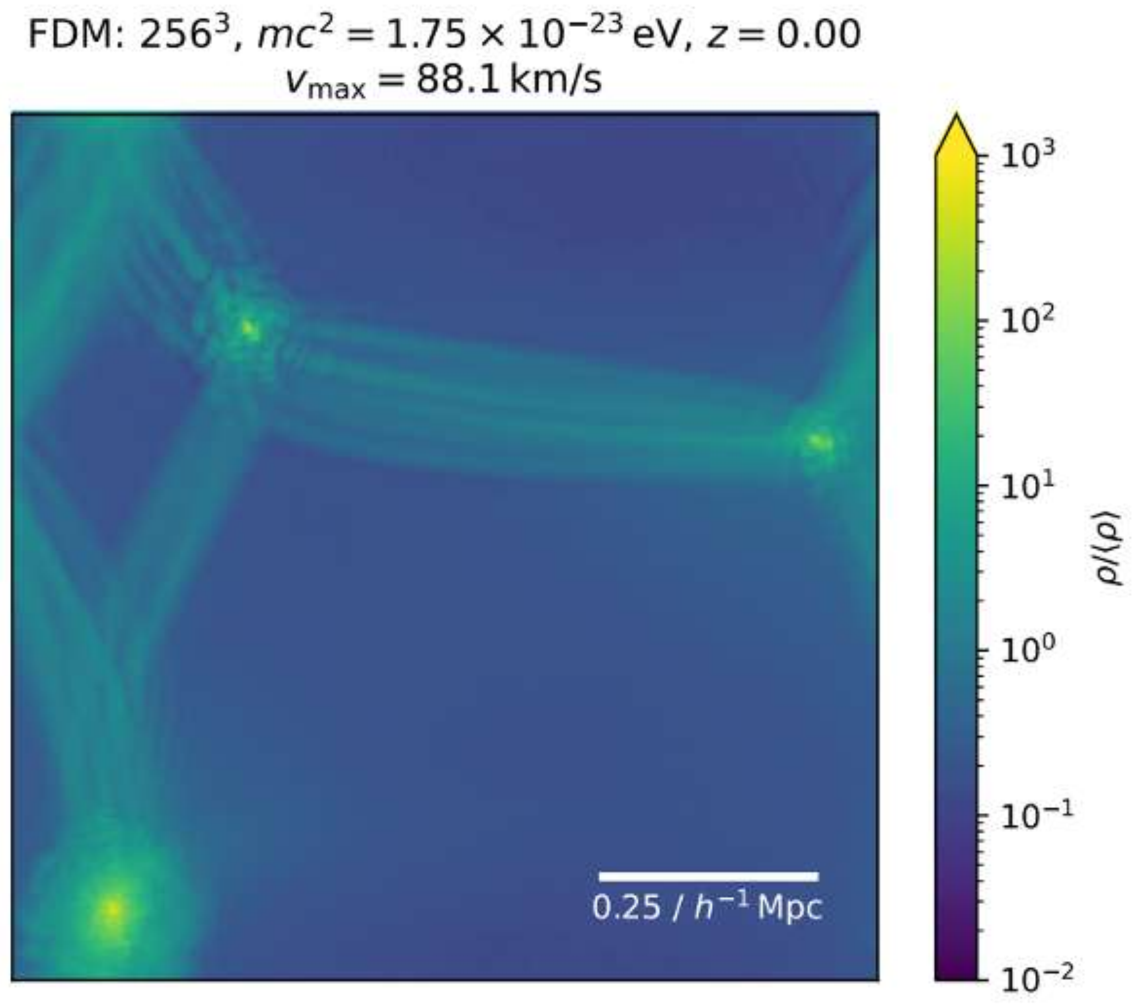
Simulations



New probes

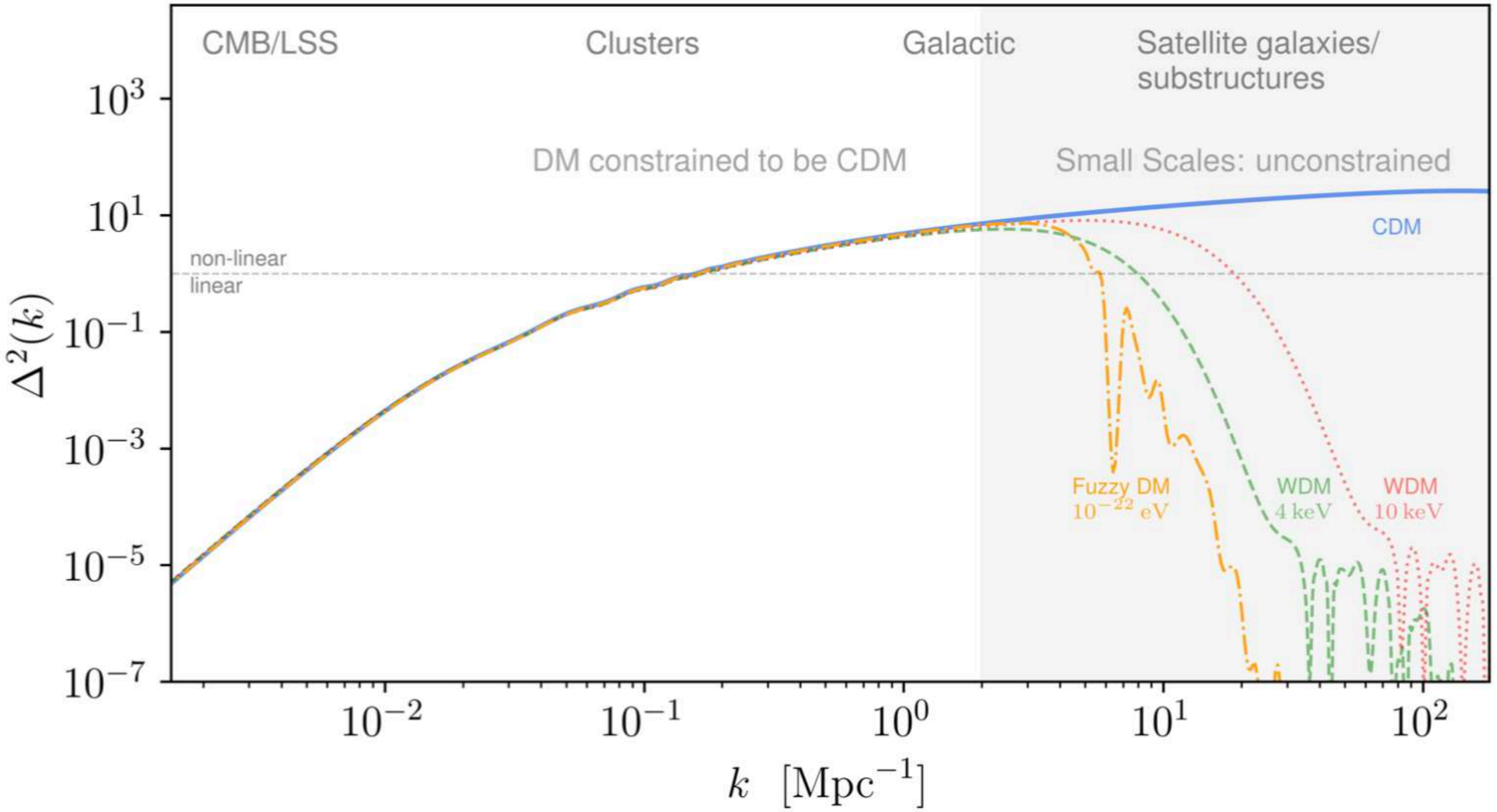
Substructures
- strong lensing
- stellar streams

Small scale information from PS
- substructure convergence PS

Modified from Jia Liu

Sub-galactic power spectrum

Using gravitational probes, strong lensing and stellar streams, to describe substructures



Sub-galactic power spectrum

Using gravitational probes, strong lensing and stellar streams, to describe substructures

A. Diaz Rivero, et al. (2017); Diaz Rivero, et al. , (2018)

Substructure convergence power spectrum

Bayer et al. (2018) ; Auger et al. 2009
FDM: *Kawai et al. (2021)*

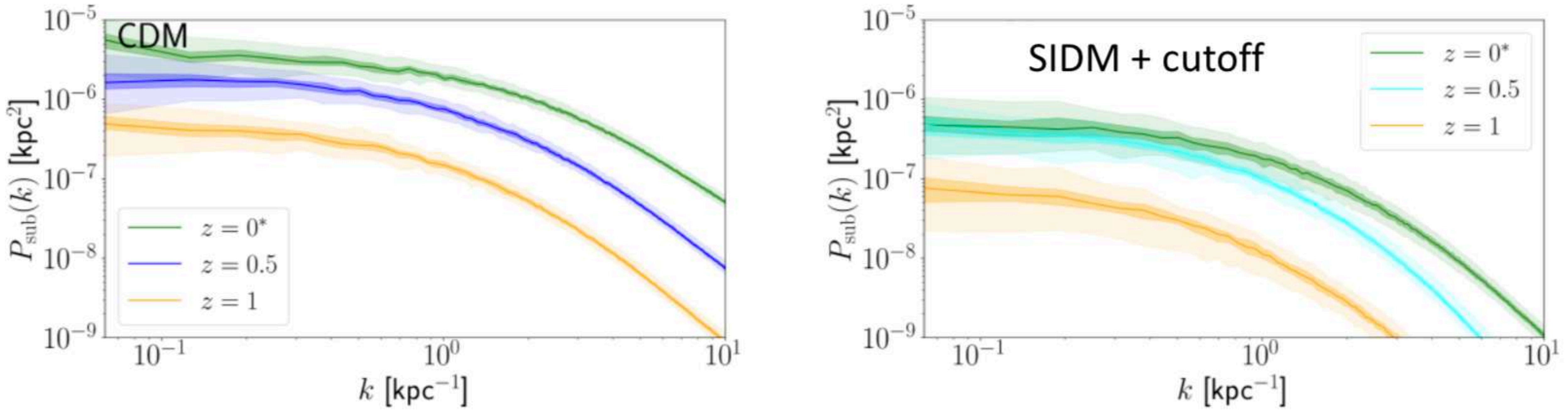
Develop a formalism to compute the substructure convergence power spectrum for different populations of dark matter subhalos.

Hezaveh et al. (2016) (projected mPS by using strong lensing)

Change of language: instead of talking about lensing perturbations in terms of individual subhalos, look at the correlation function of the projected density field.

(based on Dvorkin's slide)

$$P_{\text{sub}}(k) = P_{\text{1sh}}(k) + P_{\text{2sh}}(k)$$



Sub-galactic power spectrum

Using gravitational probes, strong lensing and stellar streams, to describe substructures

Substructure convergence power spectrum

Sten Delos and Fabian Schmidt (2021)

Stellar streams: perturbed by passing substructure. Good gravitational probe, since given their low dynamical temperature and negligible self-interaction, it retains the memory of those encounters.

THIS WORK: Fully analytical understanding of the stream perturbations!

Power spectrum of a stream's stellar density is analytically related to that of the substructure background:

$$\underset{\text{Stream power}}{\boxed{P_*(k,t)}} = \chi_*\left(k\sigma_0 t, \frac{D}{k\sigma_0^3}\right)\frac{k^2t^2}{3}P_{\Delta v}(k,t)$$

$$P_{\Delta v}(k,t) = 16\pi^4 G^2\bar{\rho}^2 k^2 t\int_k^\infty \frac{\mathrm{d}q}{q}\frac{\overset{\text{Substructure power}}{\boxed{\mathcal{P}(q)}}}{q^6}\int \mathrm{d}^3\boldsymbol{u}\,\frac{f(\boldsymbol{u})}{u}\theta_H(qu-kv)$$

Previous:
- Mostly numerical
- Perturbations ⟶ sub-halo mass function

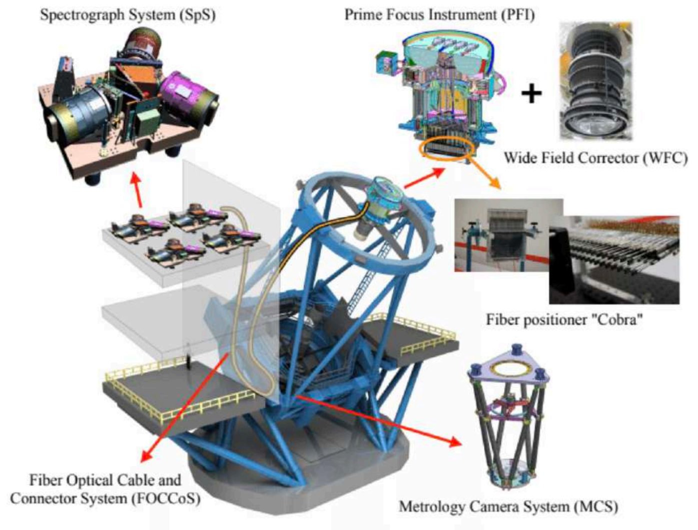
Relates the stellar stream perturbation to the surrounding matter distribution, from dark and luminous substructure

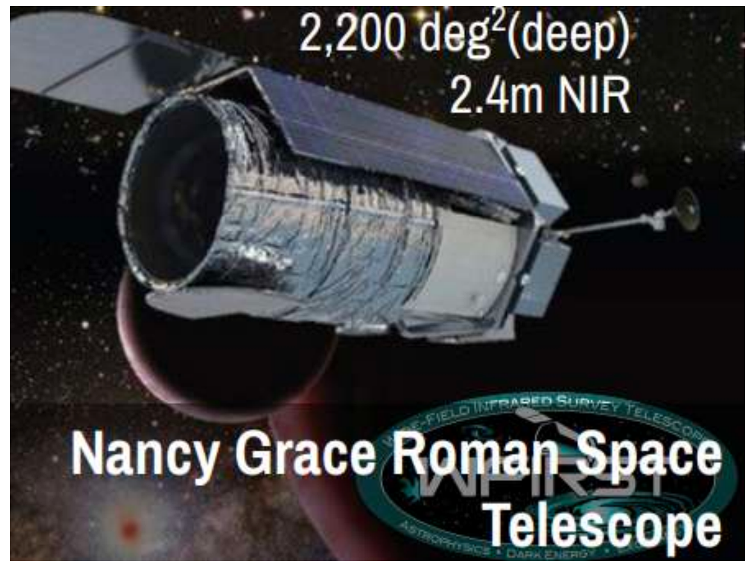
Future - signals in cosmology

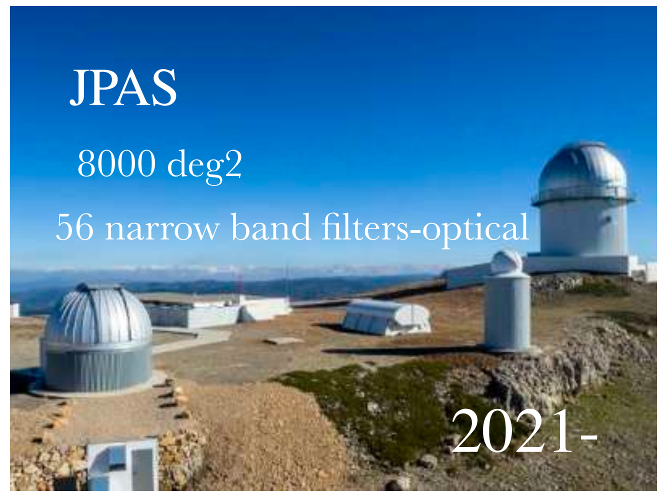
Observations

Photometric and spectroscopic surveys

Prime Focus Spectrograph (PFS)

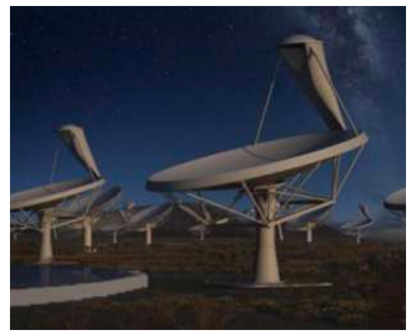






GWs

21cm



SKA
SQUARE KILOMETRE ARRAY

CMB

CMB-S4
Next Generation CMB Experiment

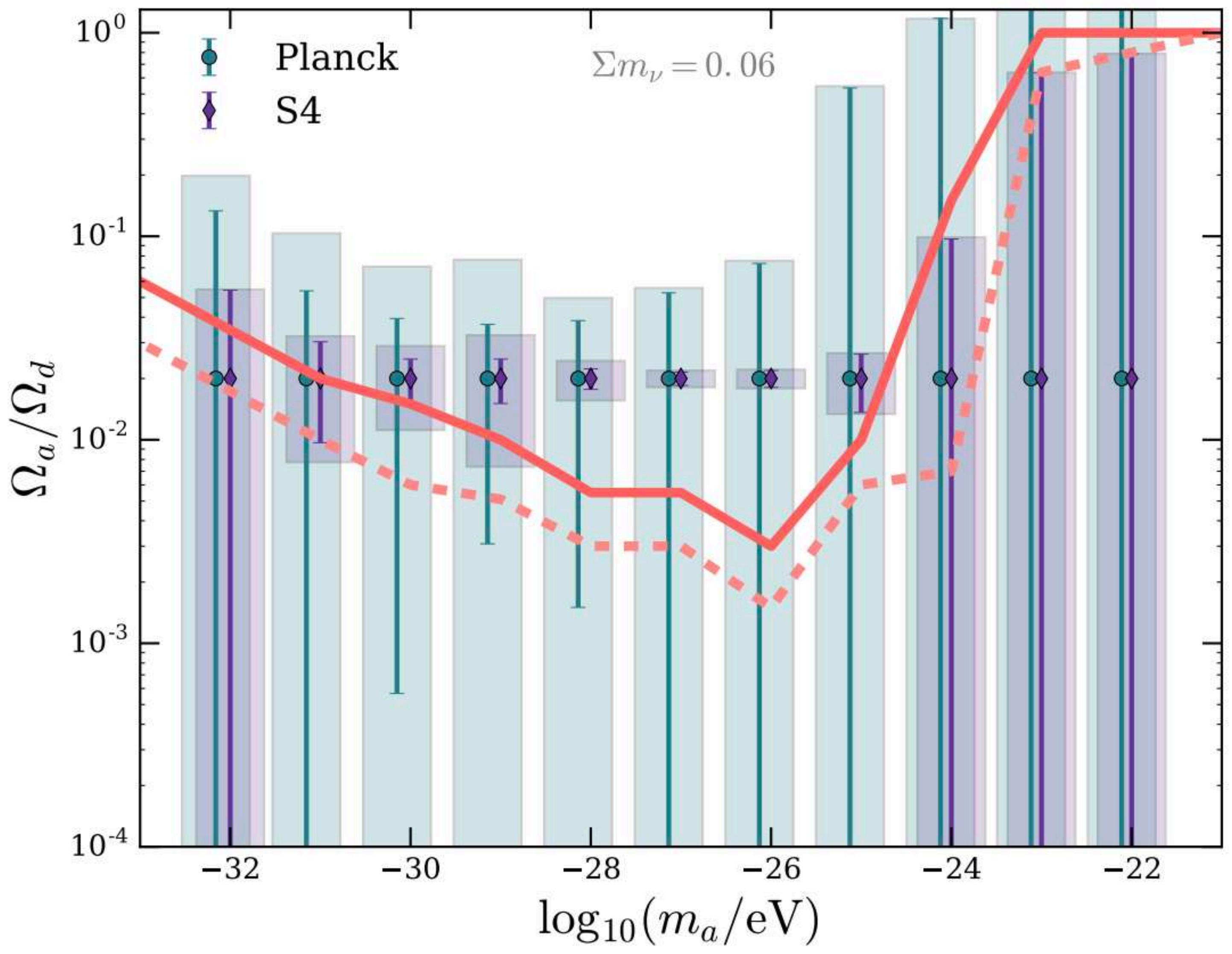
Modified from Jia Liu

Future - Cosmic Microwave Background

TESTING ULTRA LIGHT DM CMB

CMB - S4

Constraints on Ω_a/Ω_d



Hlozek et al., 2016

Significantly improve constraints on the composition of the dark sector!

Constraints on the *optical depth* $\tau(r_{\mathrm{rec}})$

Constraint the ULDM mass

Kinematic Sunyaev–Zel'dovich effect: sensitive to the duration of the reionization

- *LiteBIRD*
- *Advances ACTPol*
- *CMB-S4*

PFS (Prime Focus Spectrograph)

PFS is going to be exquisite to measure the properties of DM

GOAL

PFS: spectroscopy part of *SuMIRe project*

DM with PFS ⟶ synergy between science goals

Galaxy archeology

- Nature of DM (dSphs)
- Structure of MW dark halo
- Streams
- Stellar kinematics and chemical abundances – MW & M31

Cosmology

- Power spectrum
- HSC+PFS
- Linear growth (RSD)

Galaxy evolution

- Small-scale tests of structure growth
- Halo-galaxy connection M_*/M_{200}
- Physics of cosmic reionization via LAEs & 21cm studies
- Tomography of gas and DM

Wide & deep survey of MW dwarf galaxies w. Subaru/PFS

PFS (Prime Focus Spectrograph)

Ongoing

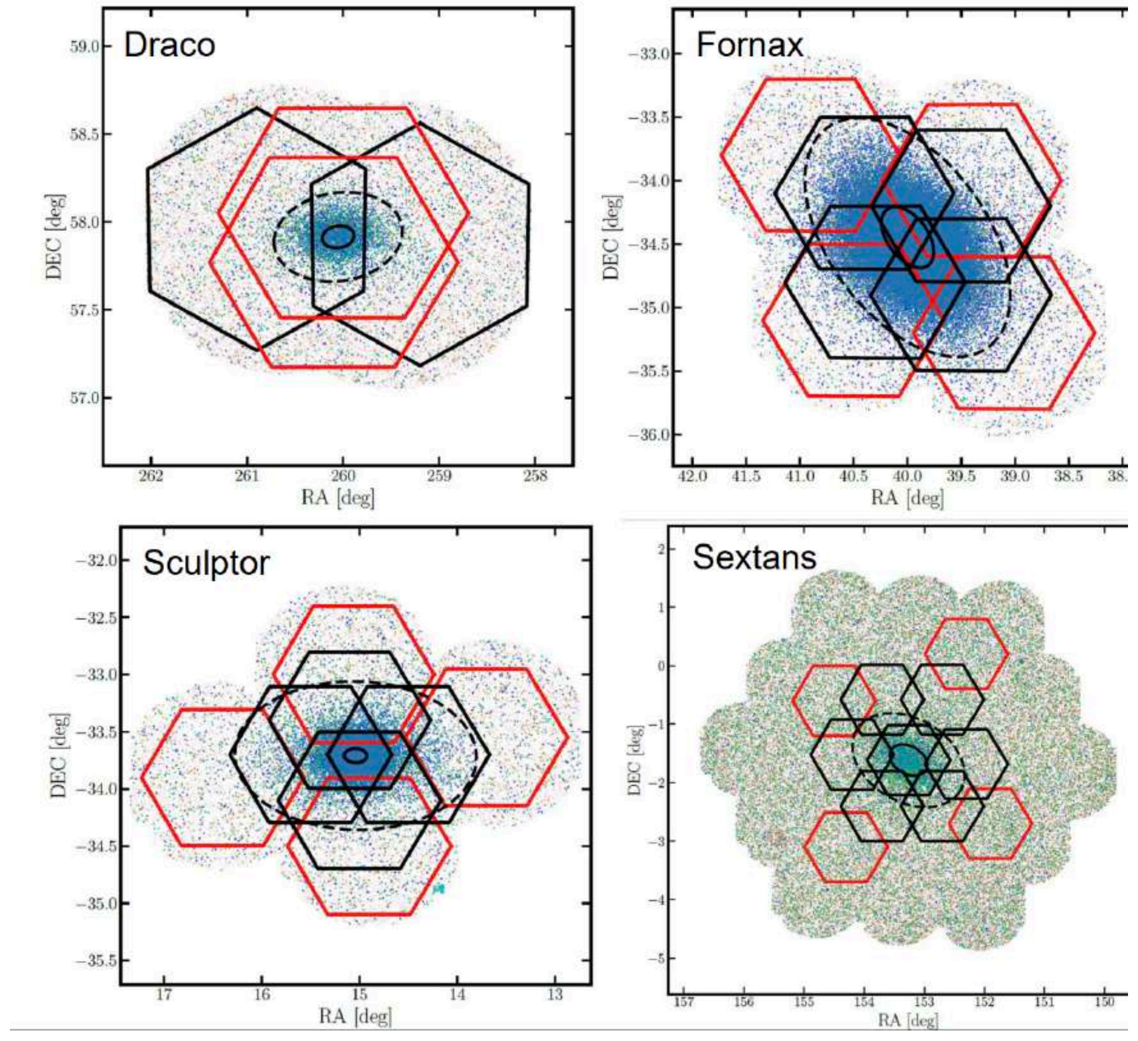
TESTING ULTRA LIGHT DM/DM with PFS

GOAL

Galaxy archeology

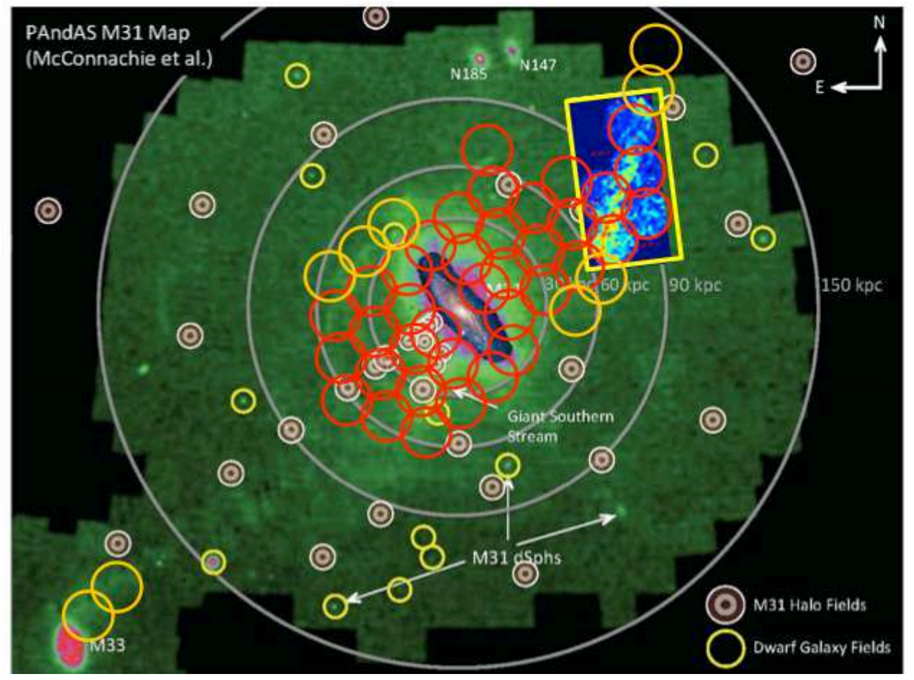
- Nature of DM (dSphs)
- Structure of MW dark halo
- Streams
- Stellar kinematics and chemical abundances – MW & M31

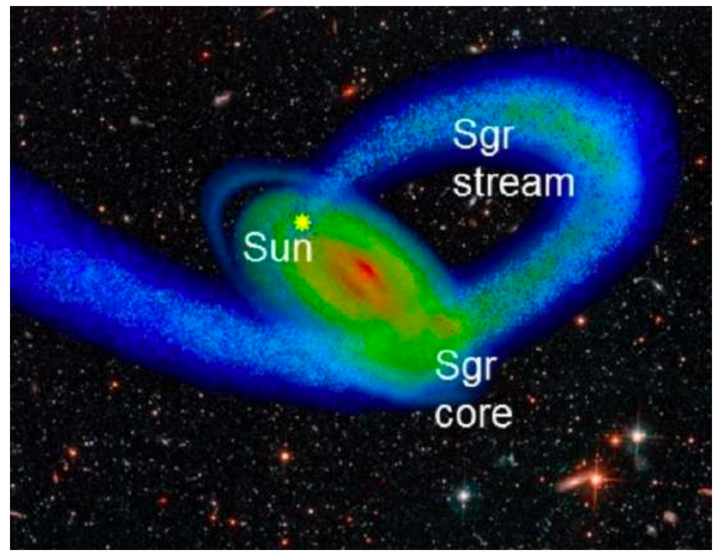
Wide & deep survey of MW dwarf galaxies w. Subaru/PFS



dSphs

M31





MW outer disk

- MW dwarf satellites - DM halo profile and [Fe/H] & [α/Fe] over largest areas ⟶ Unique & high impact
- M31 halo - DM subhalos, chemo-dynamics with spectroscopic [Fe/H] and [α/Fe]
- MW halo/streams/disks - Chemo-dynamics of the MW outer disks, halo dynamics, constraints on the Galactic potential ⟶ Unique: beyond reach of *Gaia* and VLT

GA ⟶ potential to put unprecedented constraints on ULDM. Potential for discovery!

PFS (Prime Focus Spectrograph)

DM Science with PFS

DM with PFS ⟶ synergy between science goals

Galaxy archeology

- Nature of DM (dSphs)
- Structure of MW dark halo
- Streams
- Stellar kinematics and chemical abundances – MW & M31

Cosmology

- Power spectrum
- HSC+PFS
- Linear growth (RSD)

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- Small-scale tests of structure growth
- Halo-galaxy connection M_*/M_{200}
- Physics of cosmic reionization via LAEs & 21cm studies
- Tomography of gas and DM

Wide & deep survey of MW dwarf galaxies w. Subaru/PFS

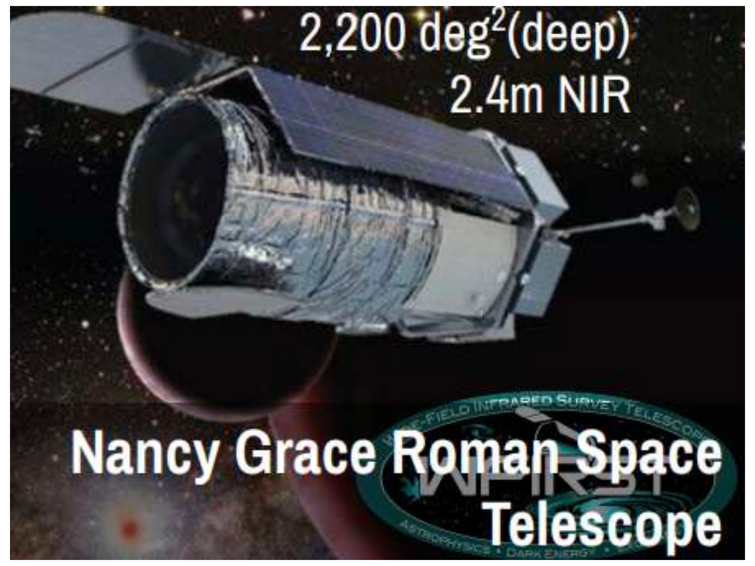
Use PFS GA, GE and cosmology to constrain the properties of DM.

LSST will discover tens of thousands of lensed galaxies.
This vast increase in sample sizes (in coordination with other facilities, e.g. HST, ALMA) will provide much stronger statistical constraints on dark matter models than what is currently possible.

LSS probes

PFS in coordination with:

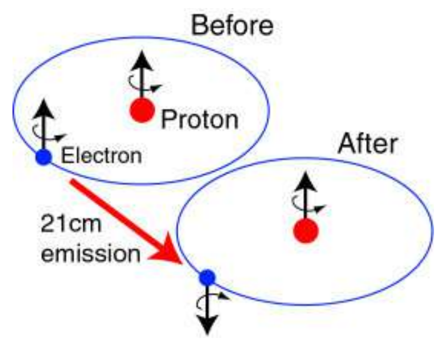
- Vera C. Rubin Legacy Survey of Space and Time (LSST)
- Atacama Large Millimeter/submillimeter Array (ALMA)
- Nancy Grace Roman Space Telescope (WFIRST)
- *Gaia*
- …



"…will provide much stronger statistical constraints on dark matter models"

1902.01055

Future - BINGO *telescope*

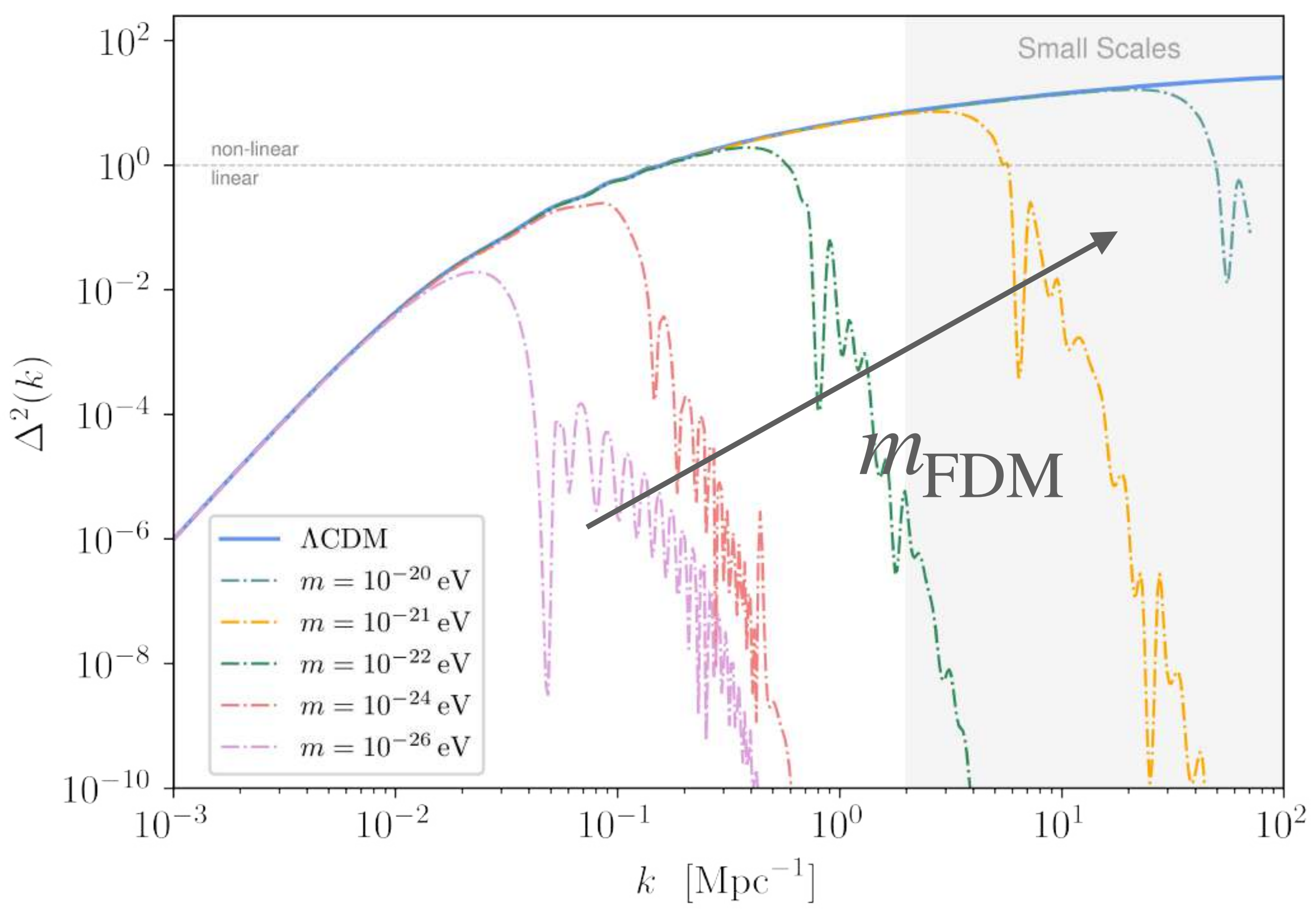


21-cm

Ultra-light DM (FDM) with 21-cm intensity mapping

- Intensity mapping (IM) - 3D tomographic map: great potential as a future cosmological probe
- Complementary to forest probes
- Capacity to probe power spectrum for *smaller scales*

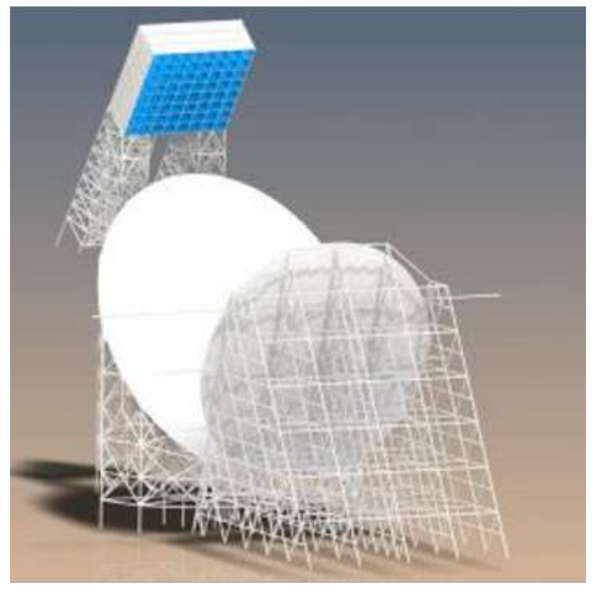
With 21-cm we can probe: m Ω_a/Ω_t



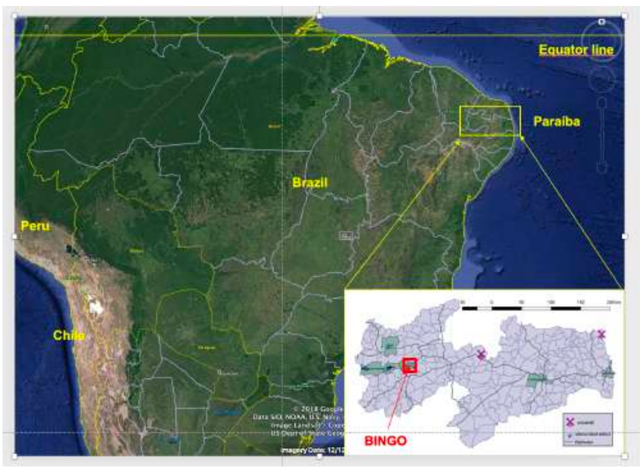
- Suppression of PS
- Increase in b_{HI}

BINGO (BAO In Neutral Gas Observations)

Intensity mapping - BAO



- Dish diameter: 40m
- Area : 15 x 200deg^2 – drift scan
- Frequency range: 960 - 1260MHz
- Redshift range: 0.12 - 0.48



- Main goals: DE, FRBs
- Constraints on DM

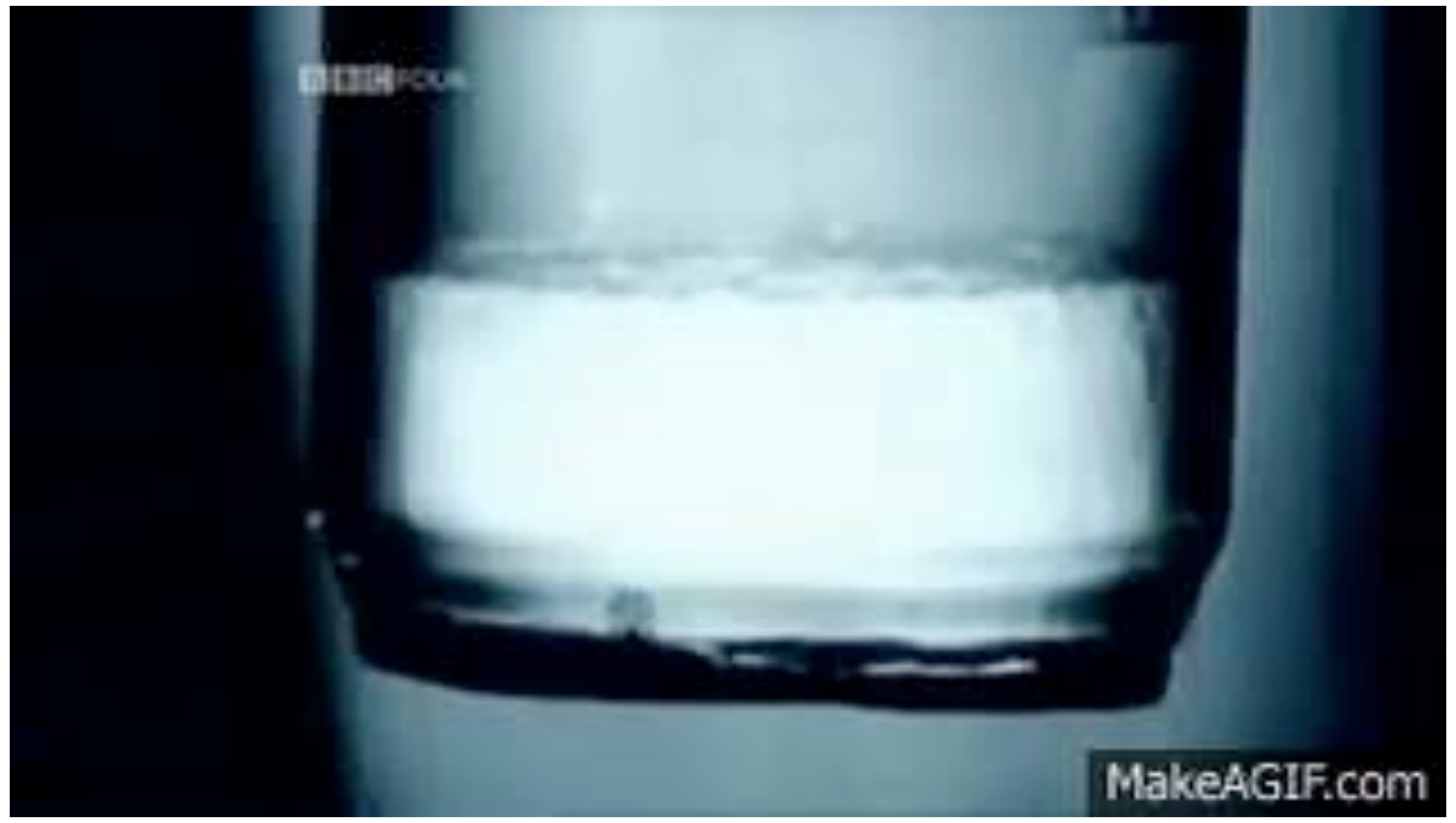
Observation start: end of 2022

FORECAST $\sigma(\Omega_a/\Omega_T)_{bingo} = 0.2$

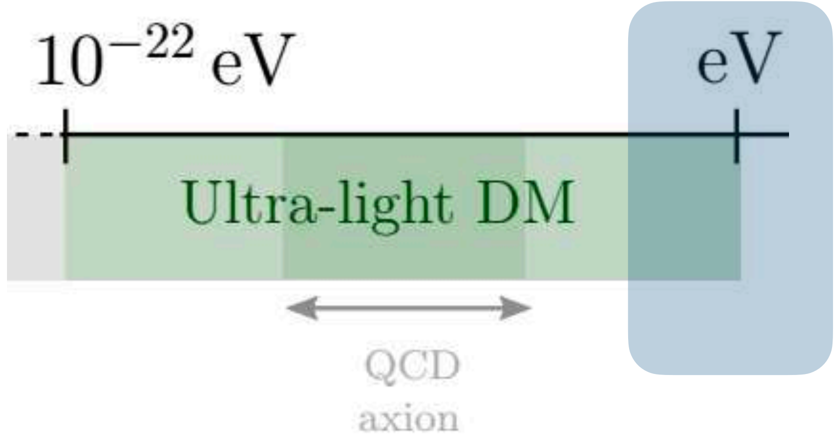
Bauer et al 2020
Carucci et al 2018

"The BINGO project I", Abdalla, **E.F.**, et al, 2021
+The BINGO project II - VII, including E.F.

Superfluid Dark Matter

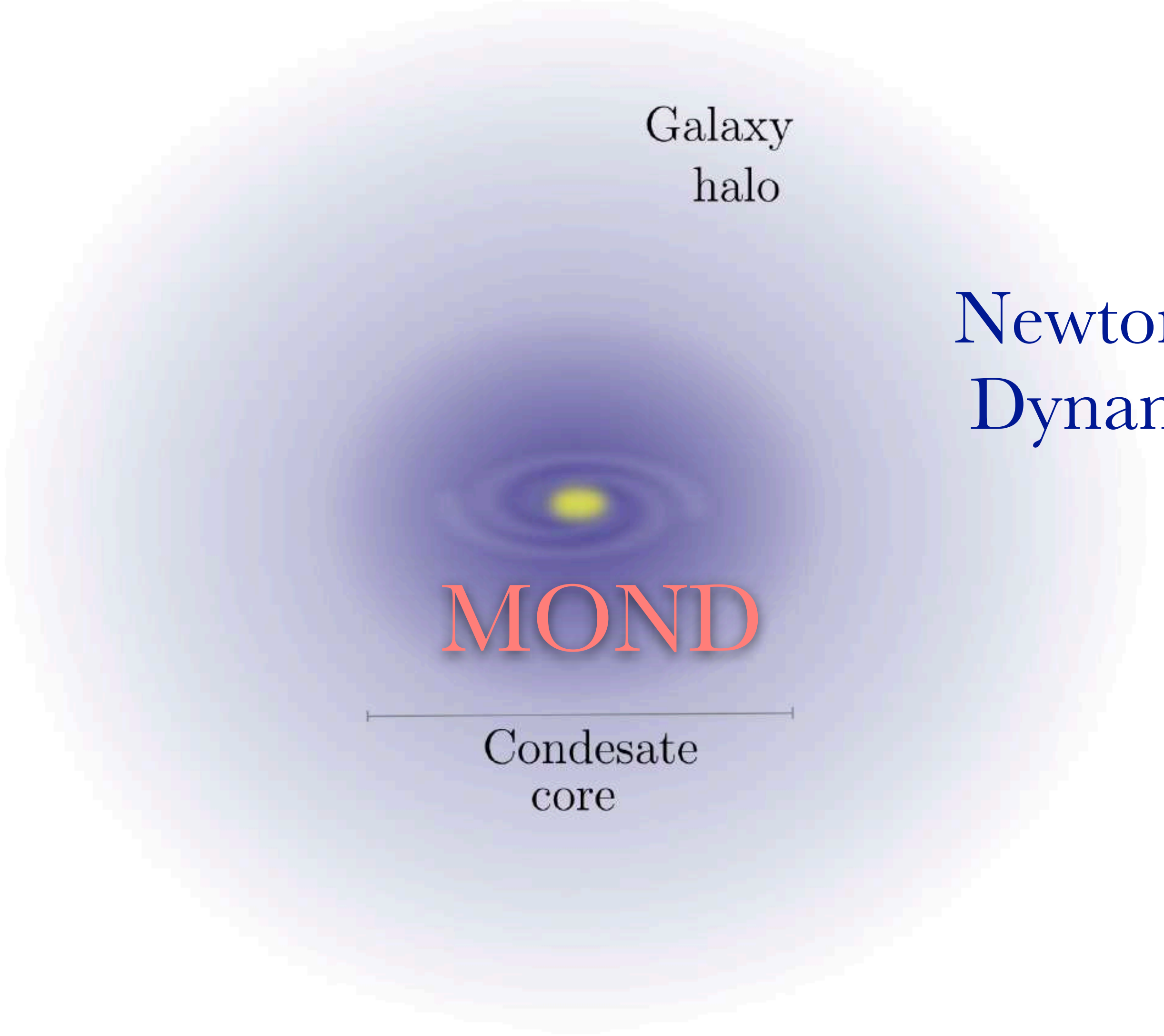


Superfluid Dark Matter



Lasha Berezhiani and Justin Khoury (2016)

Large scales:
DM behaves like standard particle DM (CDM).



Galactic scales:
DM forms a superfluid
⟶ emergent MOND dynamics in galaxies

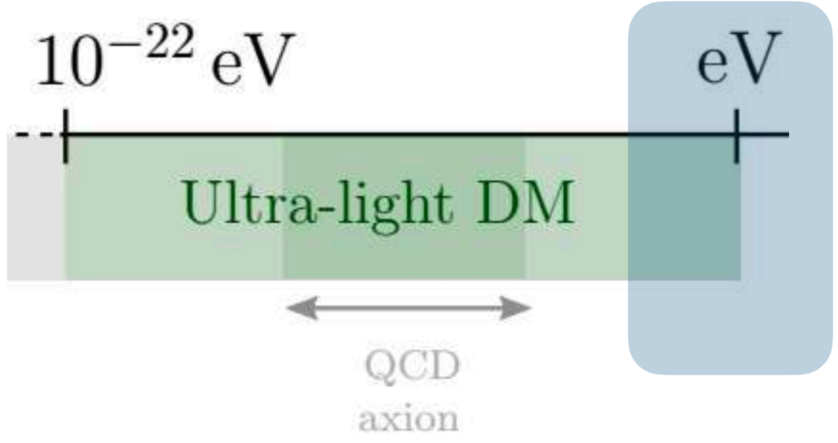
$$a = \begin{cases} a_N^b, & a_N^b \gg a_0. \\ \sqrt{a_N^b a_0}, & a_N^b \ll a_0. \end{cases}$$

Similar phenomenology than the FDM & SIFDM + explains the rotations curves and scaling relations

Suppresses small structures, dyn. effects, formation of cores

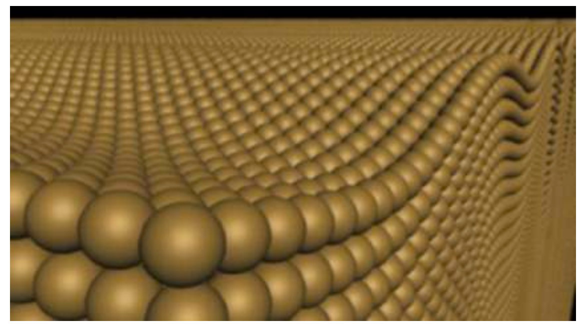
Superfluid Dark Matter

How to construct - MOND from phonons



Lasha Berezhiani and Justin Khoury (2016)

EFT of superfluids $\mathcal{L} = P(X)$ $\quad X = \dot{\theta} - m\Phi - \frac{(\vec{\nabla}\theta)^2}{2m}$



Crystal Lens

$$\Psi = (v+\rho)e^{i(\mu t+\theta)}$$

Low energy: only θ excited - phonon
Nambu Goldstone boson

Different phenomena $P(X) \propto \left(\dot{\theta}/m\right)^n$

$n = 2: \quad P \sim \rho^2$ BEC
$n = 3/2: \quad P \sim \rho^3$ "MOND"
$n = 5/2: \quad P \sim \rho^{5/3}$ Unitary Fermi gas

To describe non-relativistic MOND, it is imposed that:

⟶ Leads to an equation of state $P \sim \rho^3$ required to describe MOND

$$P(X) = \frac{2\Lambda\,(2m)^{3/2}}{3} X\sqrt{|X|}$$

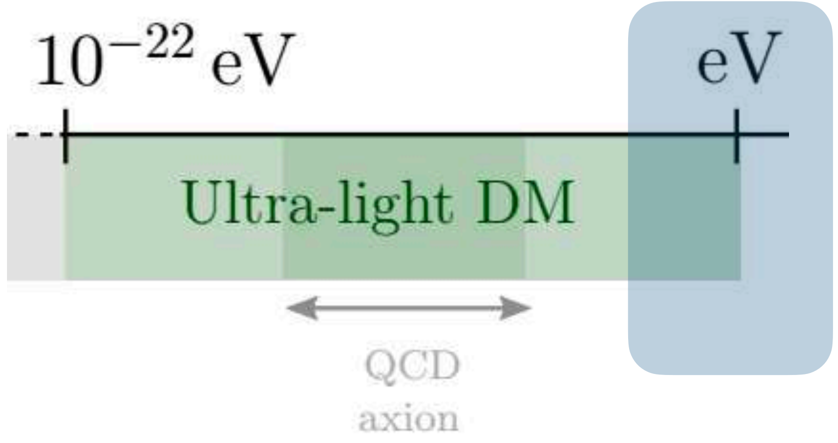
To mediate the MONDian force, couple phonons to baryons:

$$\mathcal{L}_{int} \sim \frac{\Lambda}{M_{pl}}\theta\rho_b$$

Softly breaks shift symmetry

$$\Lambda = \sqrt{a_0 M_{pl}} \sim 0.8\,\mathrm{meV}$$

Superfluid Dark Matter



- Newtonian limit: $|\vec{\nabla}\Phi| > 3\,a_0$

$$\Longrightarrow \quad \boxed{\vec{\nabla}^2\Phi = \frac{\rho_s + \rho_b}{2M_{pl}^2}}$$

- MOND limit: $|\vec{\nabla}\Phi| < 3\,a_0$

$$\Longrightarrow \quad \boxed{\vec{\nabla}\cdot\left(\frac{|\vec{\nabla}\Phi|}{a_0}\vec{\nabla}\Phi\right) = \frac{\rho_s + \rho_b}{2M_{pl}^2}}$$

Only DM

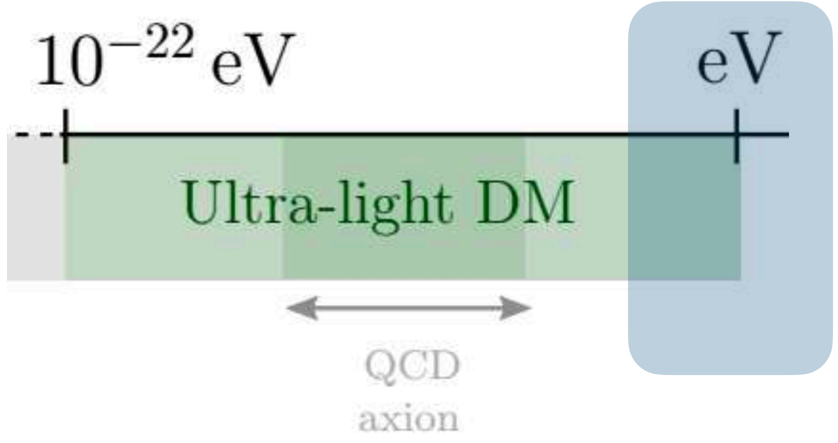
Mostly DM

"MOND"

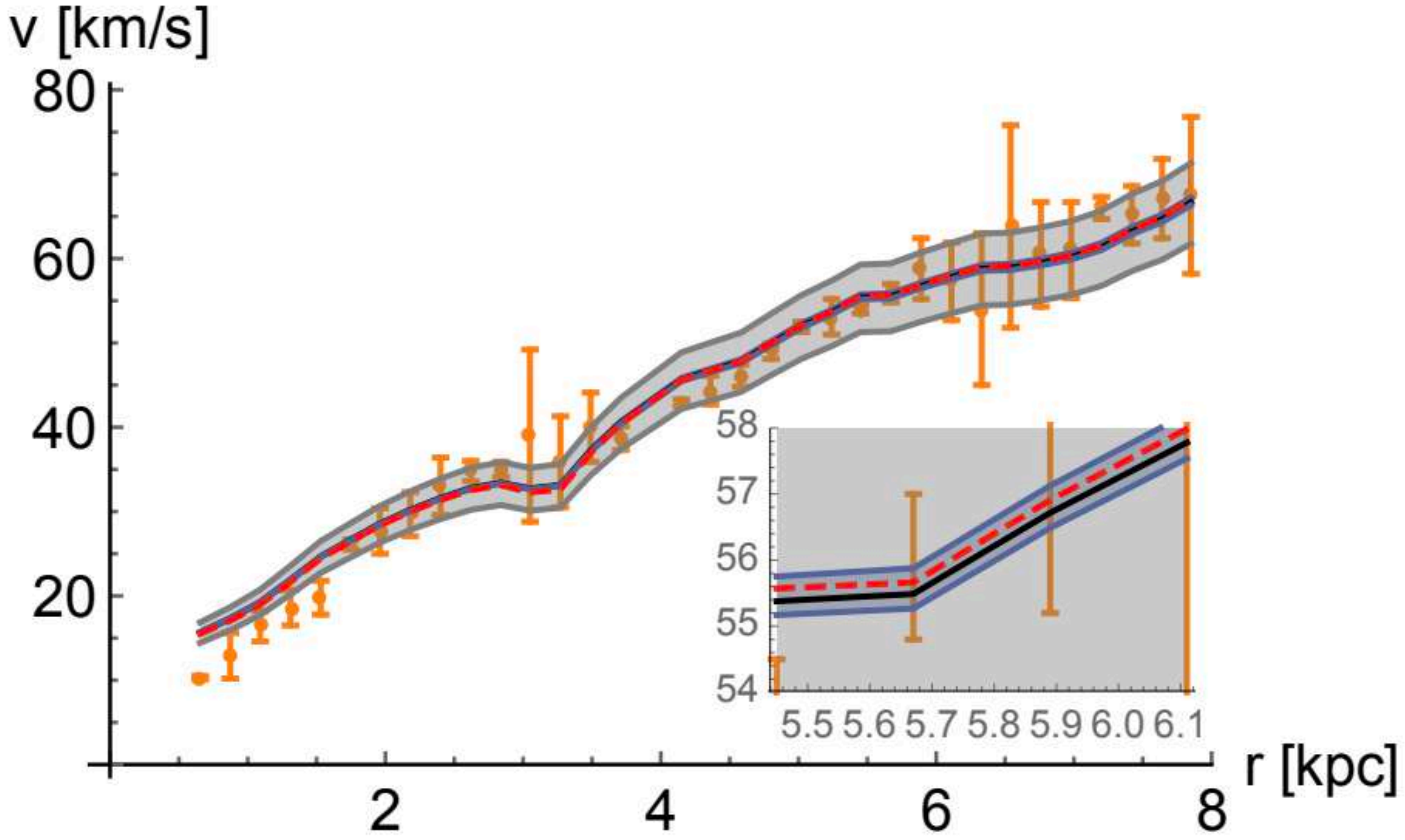
No MOND

Superfluid Dark Matter

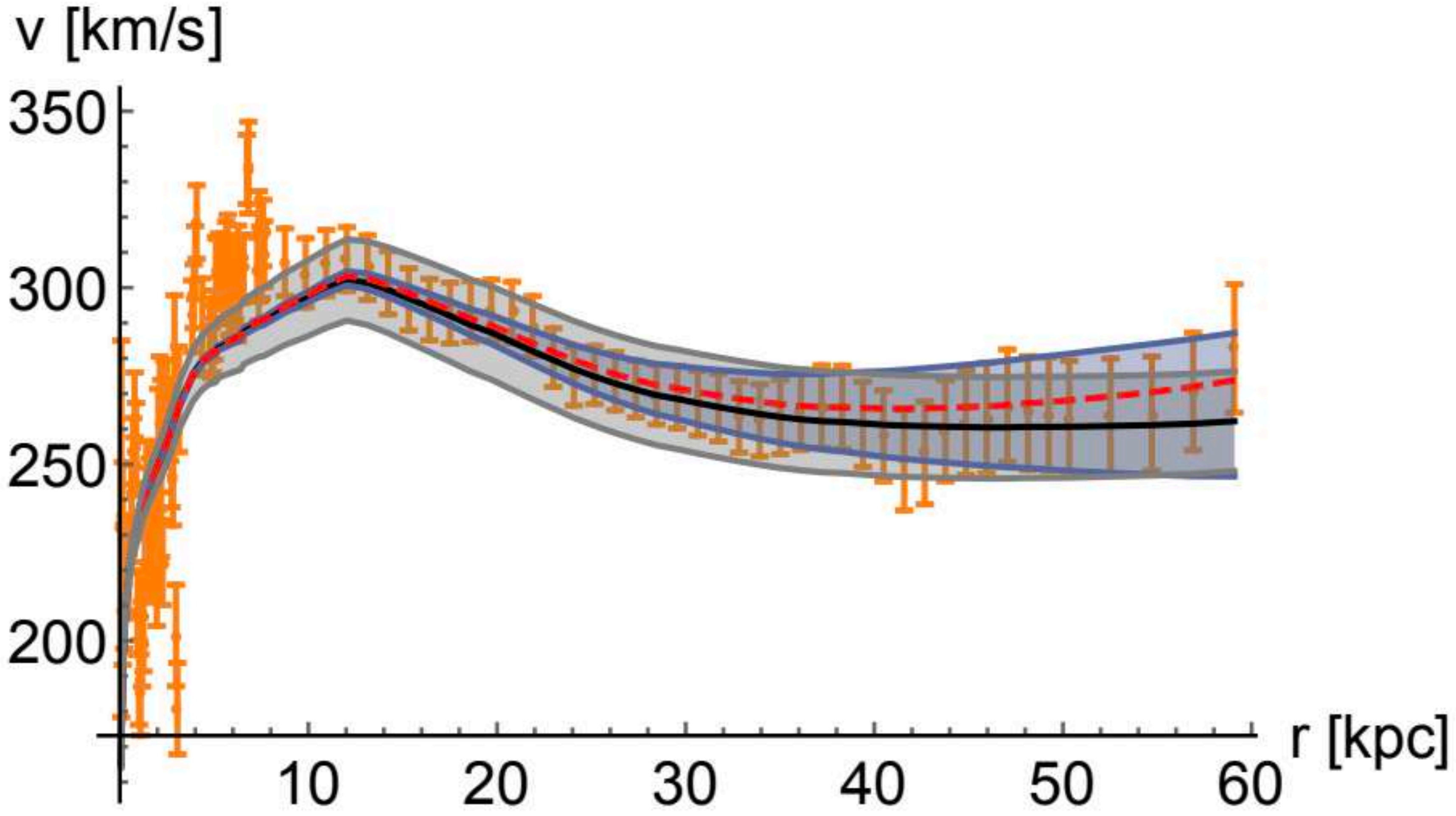
Rotation curves



Low surface brightness



High surface brightness



Superfluid core:

$$R_{halo} = 57\,\mathrm{kpc}$$

$$R_{Sf} = 40\,\mathrm{kpc}$$

58% of the total mass of the halo

$$R_{halo} = 445\,\mathrm{kpc}$$

$$R_{Sf} = 79\,\mathrm{kpc}$$

25% of the total mass of the halo

Superfluid Dark Matter

Observational consequences

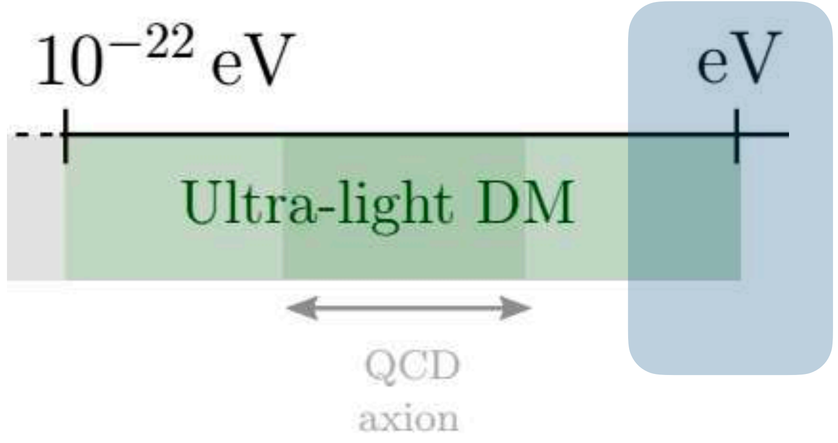


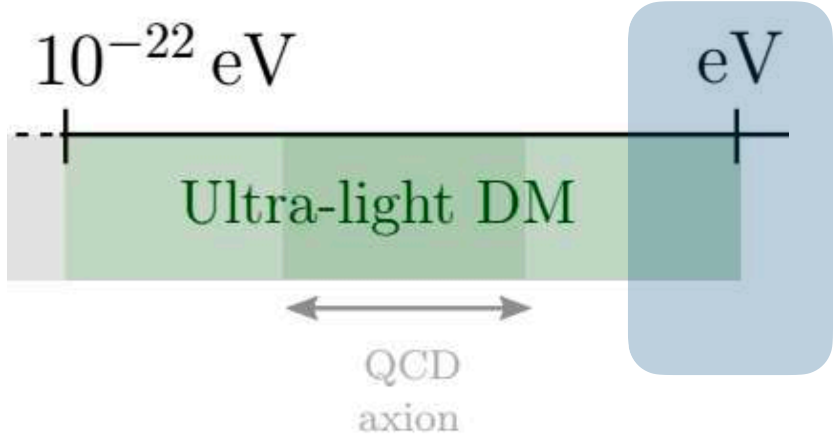
Table 2: Summary of observational consequences of superfluid DM from [124].

System	Behavior
Rotating Systems	
Solar system	Newtonian
Galaxy rotation curve shapes	MOND (+ small DM component making HSB curves rise)
Baryonic Tully–Fisher Relation	MOND for rotation curves (but particle DM for lensing)
Bars and spiral structure in galaxies	MOND
Interacting Galaxies	
Dynamical friction	Absent in superfluid core
Tidal dwarf galaxies	Newtonian when outside of superfluid core
Spheroidal Systems	
Star clusters	MOND with EFE inside galaxy host core — Newton outside of core
Dwarf Spheroidals	MOND with EFE inside galaxy host core — MOND+DM outside of core
Clusters of Galaxies	Mostly particle DM (for both dynamics and lensing)
Ultra-diffuse galaxies	MOND without EFE outside of cluster core
Galaxy-galaxy lensing	Driven by DM enveloppe $\Longrightarrow$ not MOND
Gravitational wave observations	As in General Relativity

Berezhiani et al. 2018)

Superfluid Dark Matter

Dynamical Friction

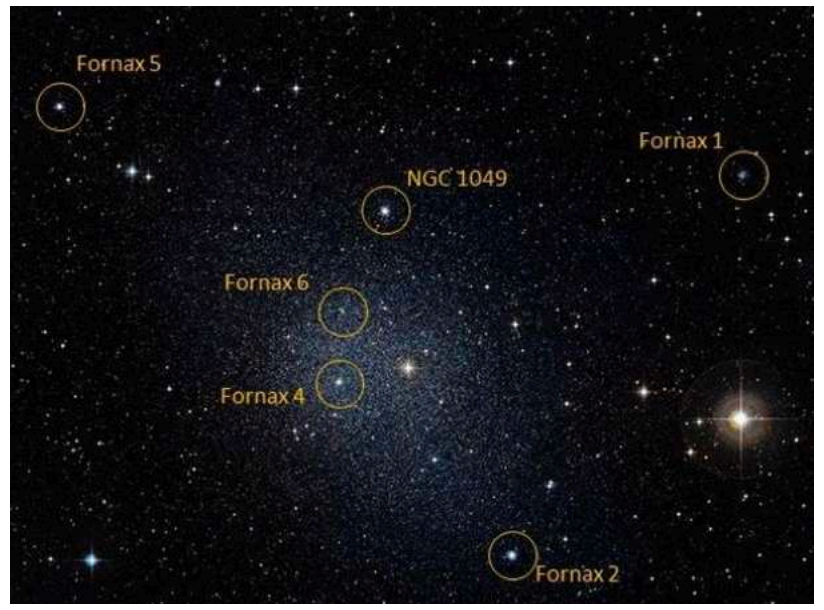


Inner region of galaxy:
Superfluid core

Superfluid flows without friction

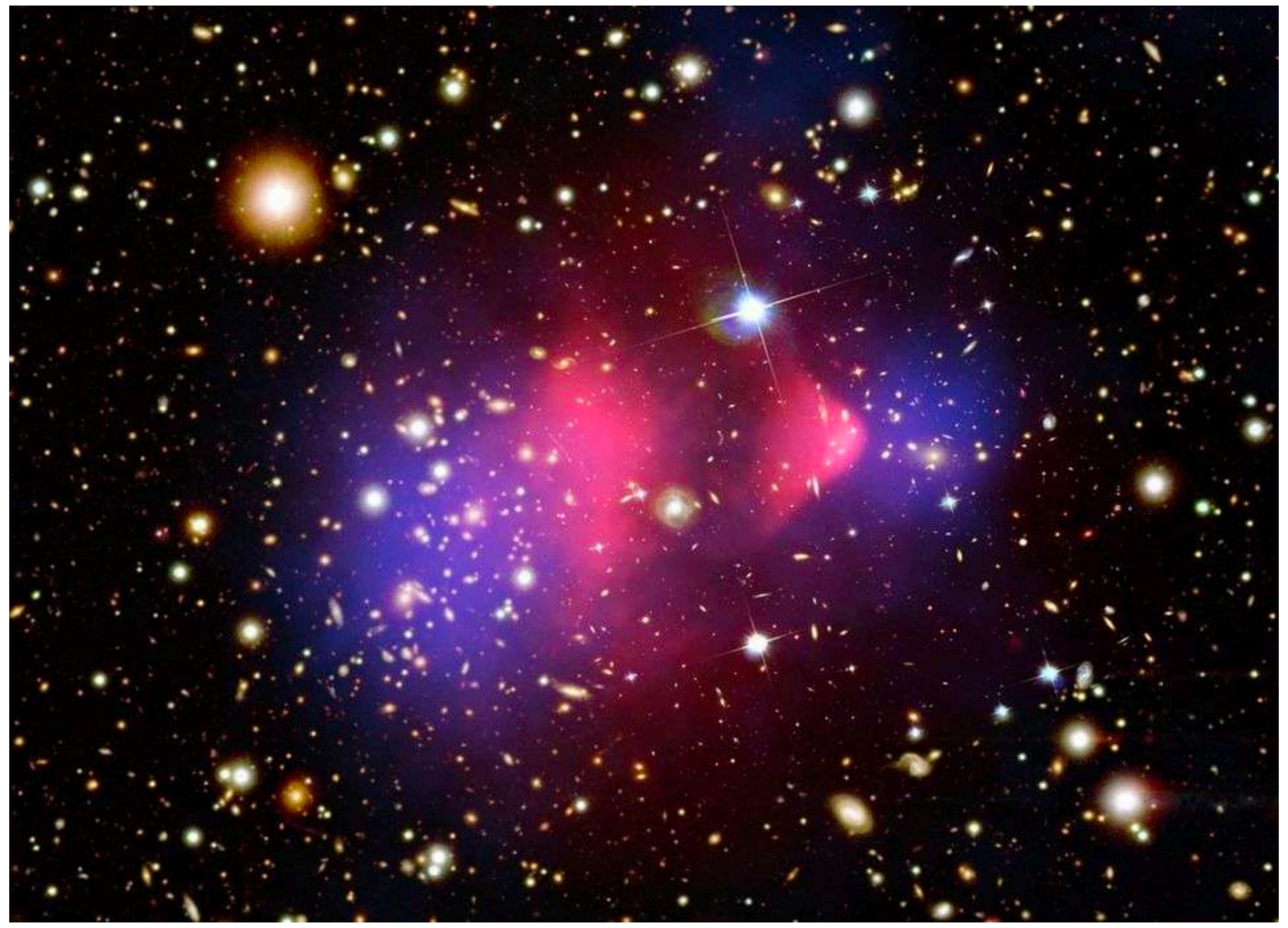
BUT: what about the Bullet Cluster?

- Fornax: globular cluster should have merged with Fornax due to dynamical friction.
  Superfluid → no friction
  Can explain these glob. Clusters



ESO/Digitized Sky Survey 2

Complete analysis in: B. Elder et al., JCAP 1910 (2019) no.10, 074

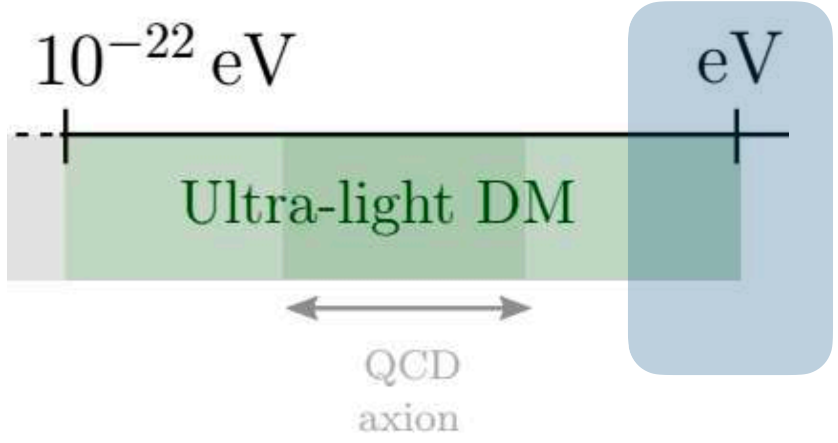


Composite Credit: X-ray: NASA/CXC/CfA/ M.Markevitch et al.; Lensing Map: NASA/STScI; ESO WFI; Magellan/U.Arizona/ D.Clowe et al. Optical: NASA/STScI; Magellan/U.Arizona/D.Clowe et al.;

Large cluster subsonic and small cluster supersonic (Sf core)
Bullet cluster as expected!

Landa criteria for superfluid
$v < v_c$

Superfluid Dark Matter



Superfluid DM model presents a very interesting behaviour in galaxies, being able to reproduce MOND from DM

- Presents only a phenomenological non-relativistic description
- Need to develop cosmology
- Does not present many constraints yet.

Presents opportunities of theoretical and observational advances!

Ultra-light fields as Dark Energy

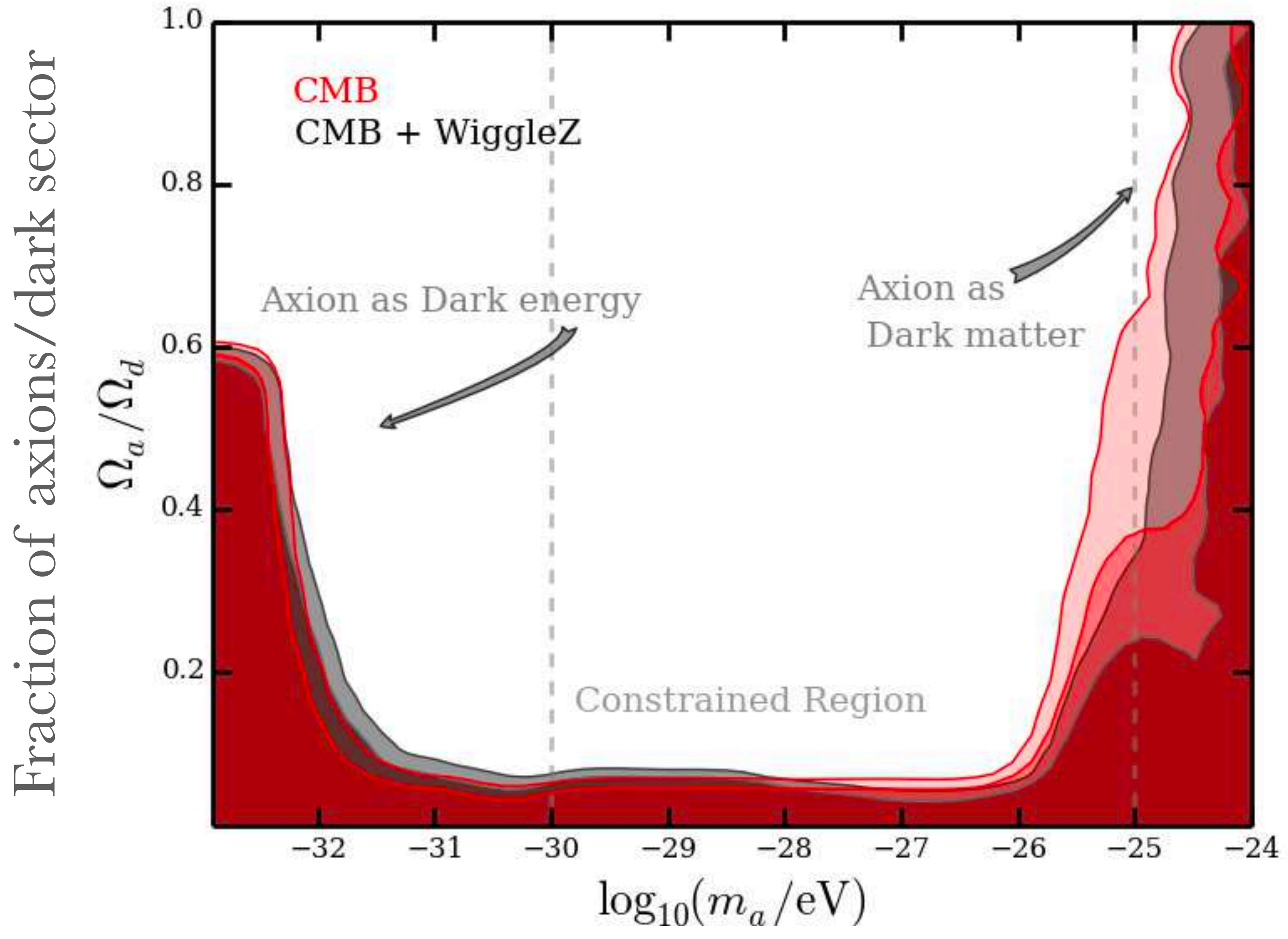
Ultra-light fields as Dark Energy

Fuzzy Dark Matter

Behave as dark energy with $w \sim -1$ for

$$m_{\rm fdm} < 10^{-32}\,{\rm eV}$$

Hlozek et al, 2015; Jiangang Kang et al. (2019)



Ultra-light fields as Dark Energy

Unified superfluid dark sector

- DM superfluid with two interacting distinguishable states.
- Phonons: propagate with different phases for each species

  $\longrightarrow$ Potential for the $(\theta_1 - \theta_2)$

- Prediction for clustering

Unified framework
w/ DM alone!

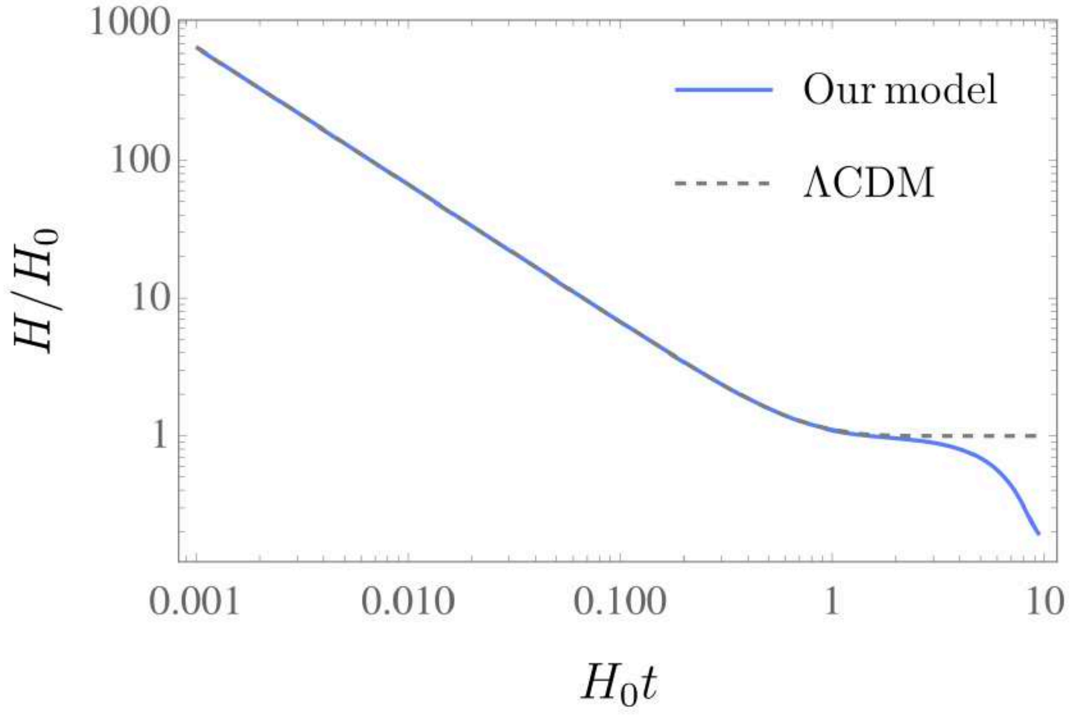
- Acceleration from interactions (no dark energy)
- Use condensed matter methods in cosmology – effective change of the dynamics, no change in the fundamental theory.

"Unified superfluid dark sector", EF, G. Franzmann, J. Khoury, R. Brandenberger, 2018

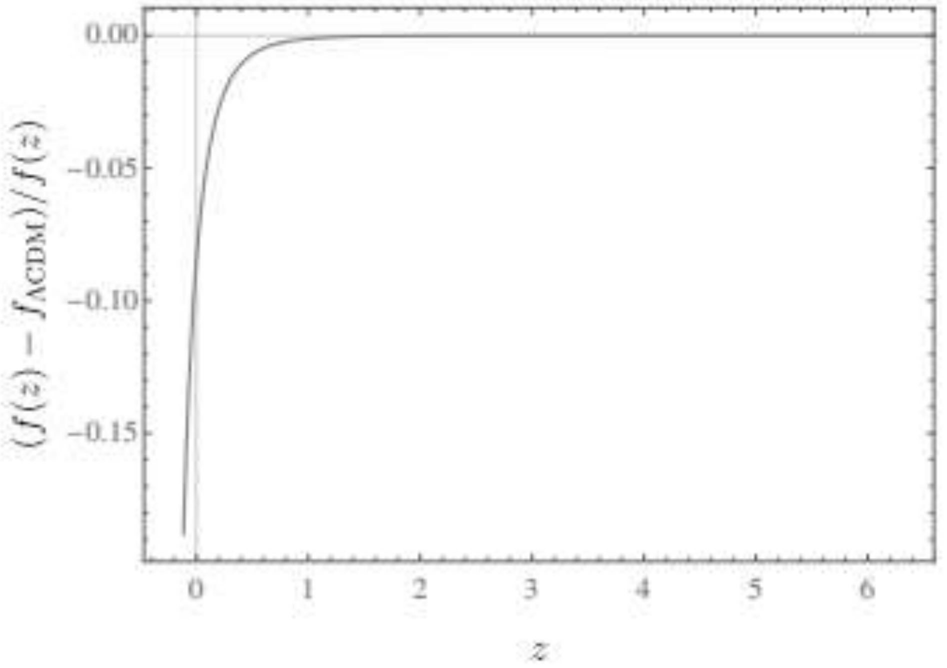
$$\mathscr{L} = P(X_1) + P(X_2) - M^4\left[1 + \cos(\theta_1 - \theta_2)/f\right]$$

Dark matter *Potential - dark energy*

Background evolution



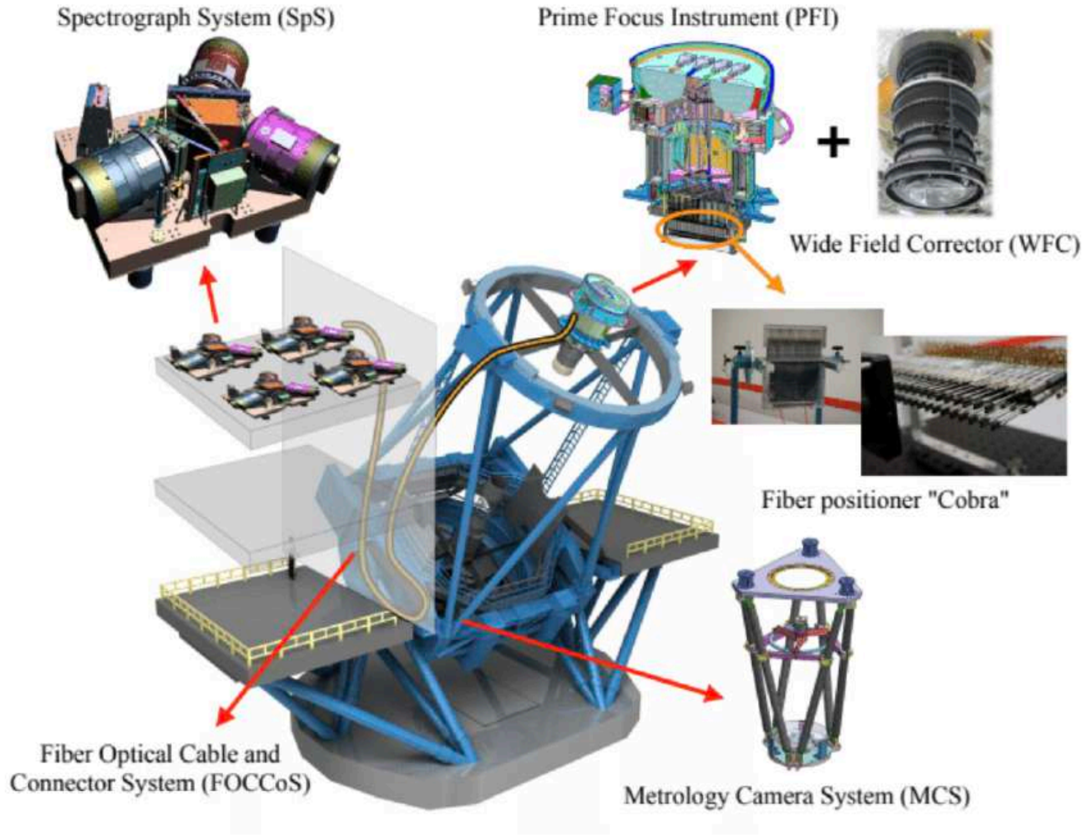
Clustering:
growth factor



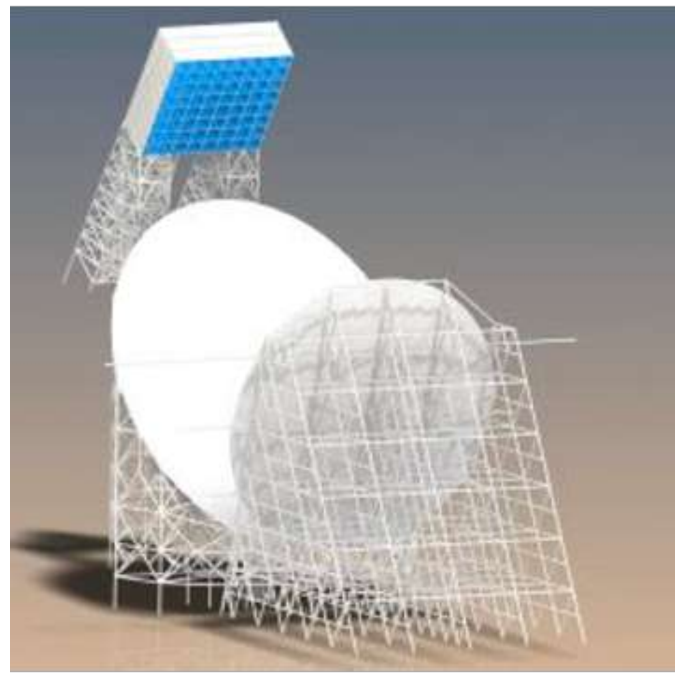
Future

Search for DE - main goal of many of these experiemnts

Prime Focus Specctrograph (PFS)



BINGO telescope

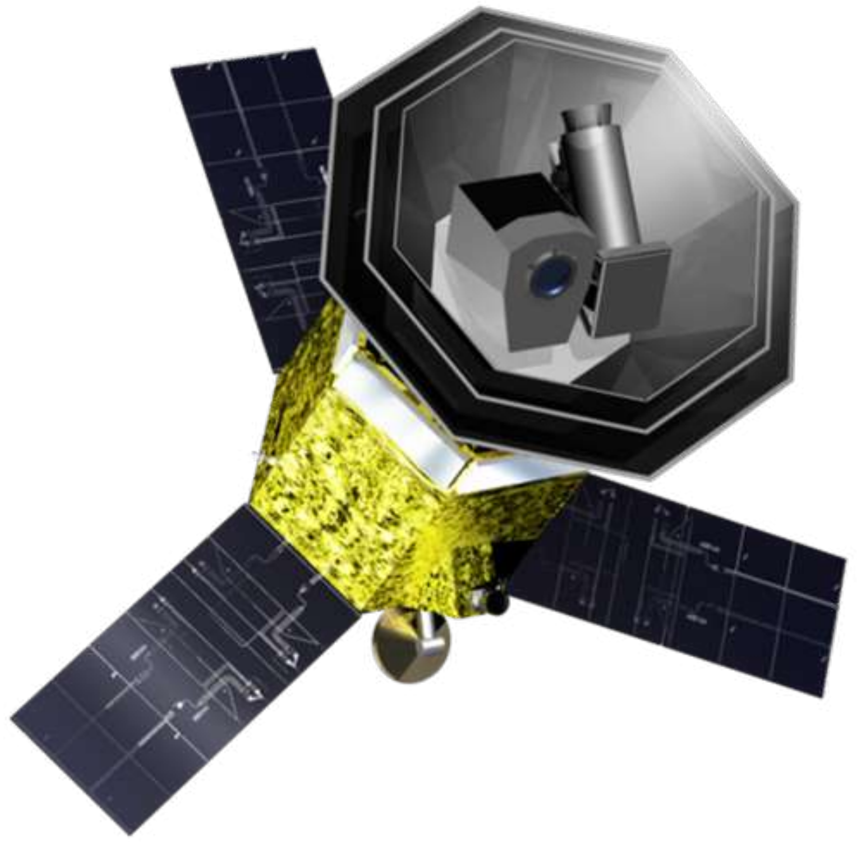


Vera Rubin observatory (LSST)

CMB-S4

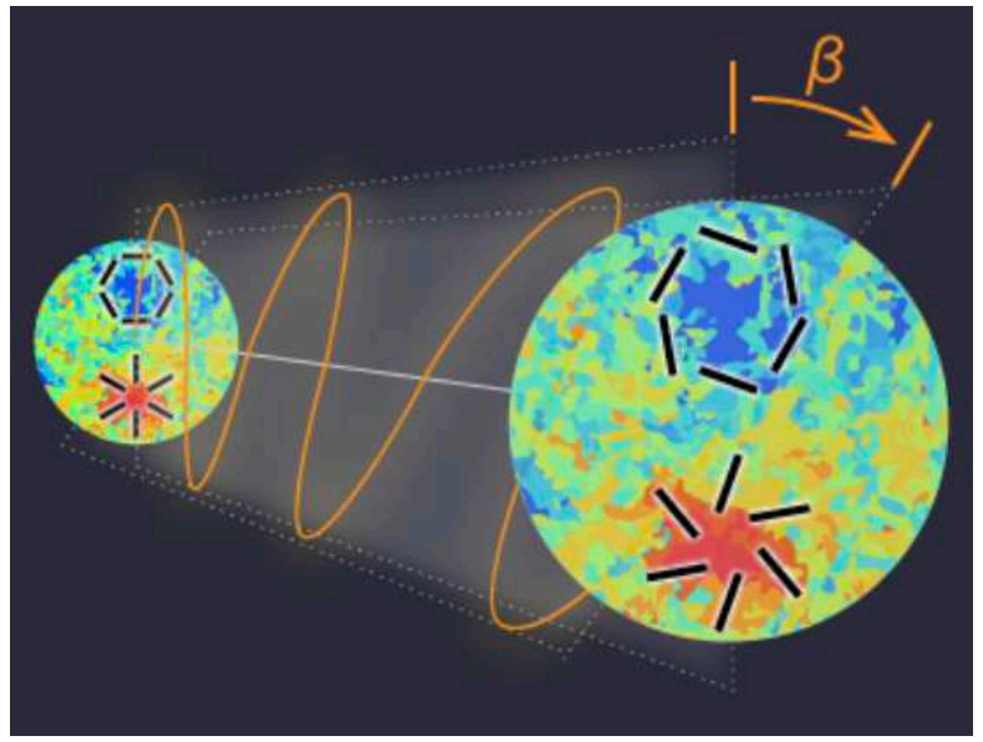
CMB-S4
Next Generation CMB Experiment

LiteBIRD



Future - Cosmic Microwave Background

Cosmic Birefringence from axions



Minami/Komatsu

Parity-violating physics in polarisation of the cosmic microwave background

↓

Rotation of the CMB polarization plane

Minami , Komatsu 2020

Could be cause by an ultra-light axion that behaves like dark energy

LiteBIRD can possibly constraint this effect

- Develop models with such axion
- Study their predictions
- Forecasts for LiteBIIRD

Summary

Ultra-Light Dark Matter

Well motivated DM models
Rich and distinct phenomenology on small scales
Testable prediction
One of the leading candidate for DM

Small Scales

Opportunity to probe the microphysics, particle physics properties of DM
Small scales provide strong constraints in these models
FDM mass being narrowed down
Incompatibility between dwarf bounds

Core-halo mass relation

Requires further investigation
Aa different relation could change the mass bounds
Simulations - relation not universal ? Large spreading?

Vortices

Vortices might exist in our simulations.
Need to improve their identification

Thank you very much!

Extra slides

FDM simulation

Spectral technique to solve the SP system

$$i\hbar\partial_t\psi_c(t,\mathbf{x}) = -\frac{\hbar^2}{2m\,a(t)^2}\nabla_c^2\psi_c(t,\mathbf{x}) + \frac{m}{a(t)}\Phi_c\,\psi_c(t,\mathbf{x})$$
$$\nabla_c^2\Phi_c(t,\mathbf{x}) = 4\pi Gm\left(|\psi_c(t,\mathbf{x})|^2 - \langle|\psi_c|^2\rangle(t)\right)$$

Time evolution of the wave function

$$\Psi\left(\boldsymbol{x},t+\Delta t\right) = T\exp\left[-\frac{\mathrm{i}\Delta t}{\hbar}\int\mathrm{d}t'\left(-\frac{\hbar^2}{2m}\nabla^2 + mV\left(\boldsymbol{x},t'\right)\right)\right]\Psi\left(\boldsymbol{x},t\right)$$

Small Δt:

$$\Psi\left(\boldsymbol{x},t+\Delta t\right) = \exp\left(\frac{\mathrm{i}\hbar\Delta t}{2m}\nabla^2 - \frac{\mathrm{i}m\Delta t}{2\hbar}V\left(\boldsymbol{x},t+\Delta t\right) - \frac{\mathrm{i}m\Delta t}{2\hbar}V\left(\boldsymbol{x},t\right)\right)\Psi\left(\boldsymbol{x},t\right),$$

Split into 3 operations (Baker-Campbell-Haussdorff formula)

$$\Psi\left(\boldsymbol{x},t+\Delta t\right) = \exp\left(-\frac{\mathrm{i}m\Delta t}{2\hbar}V\left(\boldsymbol{x},t+\Delta t\right)\right)\exp\left(\frac{\mathrm{i}\hbar\Delta t}{2m}\nabla^2\right)\exp\left(-\frac{\mathrm{i}m\Delta t}{2\hbar}V\left(\boldsymbol{x},t\right)\right)\Psi\left(\boldsymbol{x},t\right)$$

Operator Splitting Spectral Method

$$\psi_c^{n+1} \approx \underbrace{e^{i\Phi_c\,\Delta t/2}}_{3^{\mathrm{rd}}}\,\underbrace{\mathcal{F}^{-1}\Big[e^{ik^2\Delta t}}_{2^{\mathrm{nd}}}\,\underbrace{\mathcal{F}^{-1}\Big[e^{i\Phi_c\,\Delta t/2}\,\psi_c^n\Big]}_{1^{\mathrm{st}}}\Big]$$

$$\Delta t \sim \Delta x^2$$

Timestep criteria

FDM simulation

Spectral technique to solve the SP system

$$i\hbar\partial_t\psi_c(t,\mathbf{x}) = -\frac{\hbar^2}{2m\,a(t)^2}\nabla_c^2\psi_c(t,\mathbf{x}) + \frac{m}{a(t)}\Phi_c\,\psi_c(t,\mathbf{x})$$
$$\nabla_c^2\Phi_c(t,\mathbf{x}) = 4\pi Gm\left(|\psi_c(t,\mathbf{x})|^2 - \langle|\psi_c|^2\rangle(t)\right)$$

Operator Splitting Spectral Method

$$\psi_c^{n+1} \approx \underbrace{e^{i\Phi_c\,\Delta t/2}}_{3^{\mathrm{rd}}}\underbrace{\mathcal{F}^{-1}\left[e^{ik^2\Delta t}\right.}_{2^{\mathrm{nd}}}\underbrace{\left.\mathcal{F}^{-1}\left[e^{i\Phi_c\,\Delta t/2}\,\psi_c^n\right]\right]}_{1^{\mathrm{st}}}$$

$$\Delta t \sim \Delta x^2$$

Timestep criteria

FDM simulation

The fields ψ and Φ are discretised on a uniform Cartesian mesh with N^3 grid points - allow numerical computations using Fast Fourier transform. It follows the operations:

- Calculate the potential
- $\psi_c \leftarrow e^{-i \frac{m}{\hbar} \frac{1}{a} \frac{\Delta t}{2} \Phi_c} \psi_c$ (kick) (20a)
- $\psi_c \leftarrow \mathrm{FFT}^{-1}\left(e^{-i \frac{\hbar}{m} \frac{1}{a^2} \frac{\Delta t}{2} k^2} \, \mathrm{FFT}(\psi_c)\right)$ (drift) (20b)
- $\Phi_c \leftarrow \mathrm{FFT}^{-1}\left(-\frac{1}{k^2} \, \mathrm{FFT}\left(4\pi G m \left(|\psi_c|^2 - \langle |\psi_c|^2 \rangle\right)\right)\right)$

  (update potential) (20c)
- $\psi_c \leftarrow e^{-i \frac{m}{\hbar} \frac{1}{a} \frac{\Delta t}{2} \Phi_c} \psi_c$ (kick) (20d)
- Go to step (20a) (20e)

Schrödinger-Poisson system

$$i\hbar \partial_t \psi_c(t, \mathbf{x}) = -\frac{\hbar^2}{2m\, a(t)^2} \nabla_c^2 \psi_c(t, \mathbf{x}) + \frac{m}{a(t)} \Phi_c \, \psi_c(t, \mathbf{x})$$

$$\nabla_c^2 \Phi_c(t, \mathbf{x}) = 4\pi G m \left(|\psi_c(t, \mathbf{x})|^2 - \langle |\psi_c|^2 \rangle (t) \right)$$

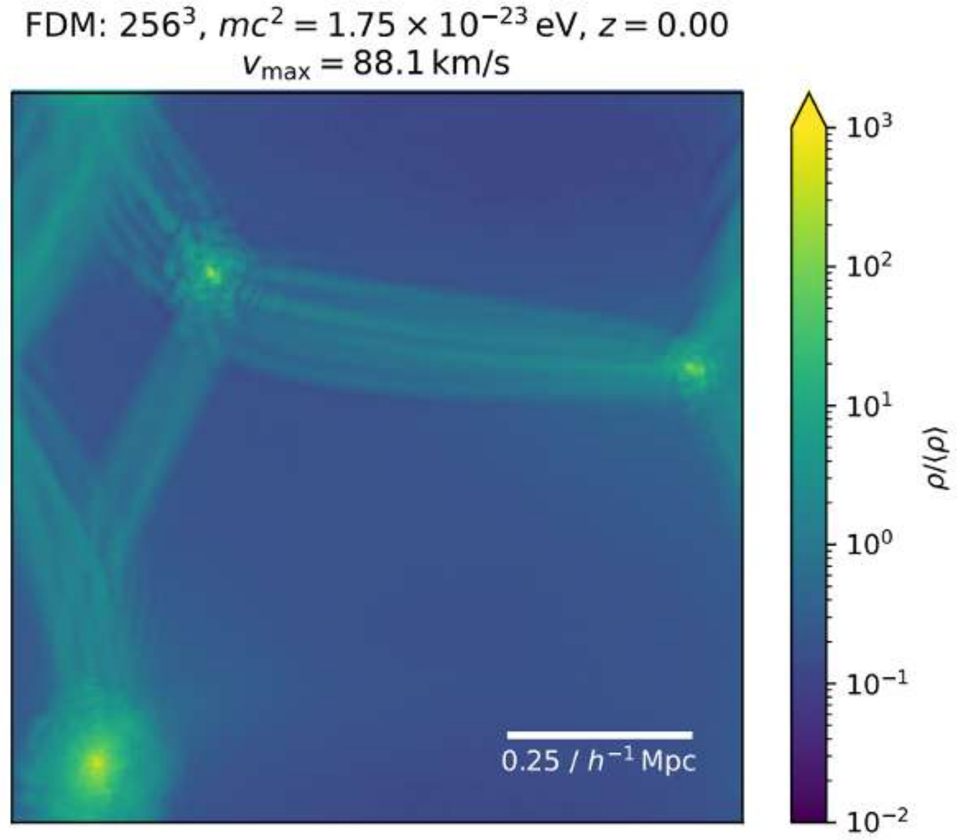
May et al. 2020
Steps (20a) to (20e) implemented
as a module in the AREPO code

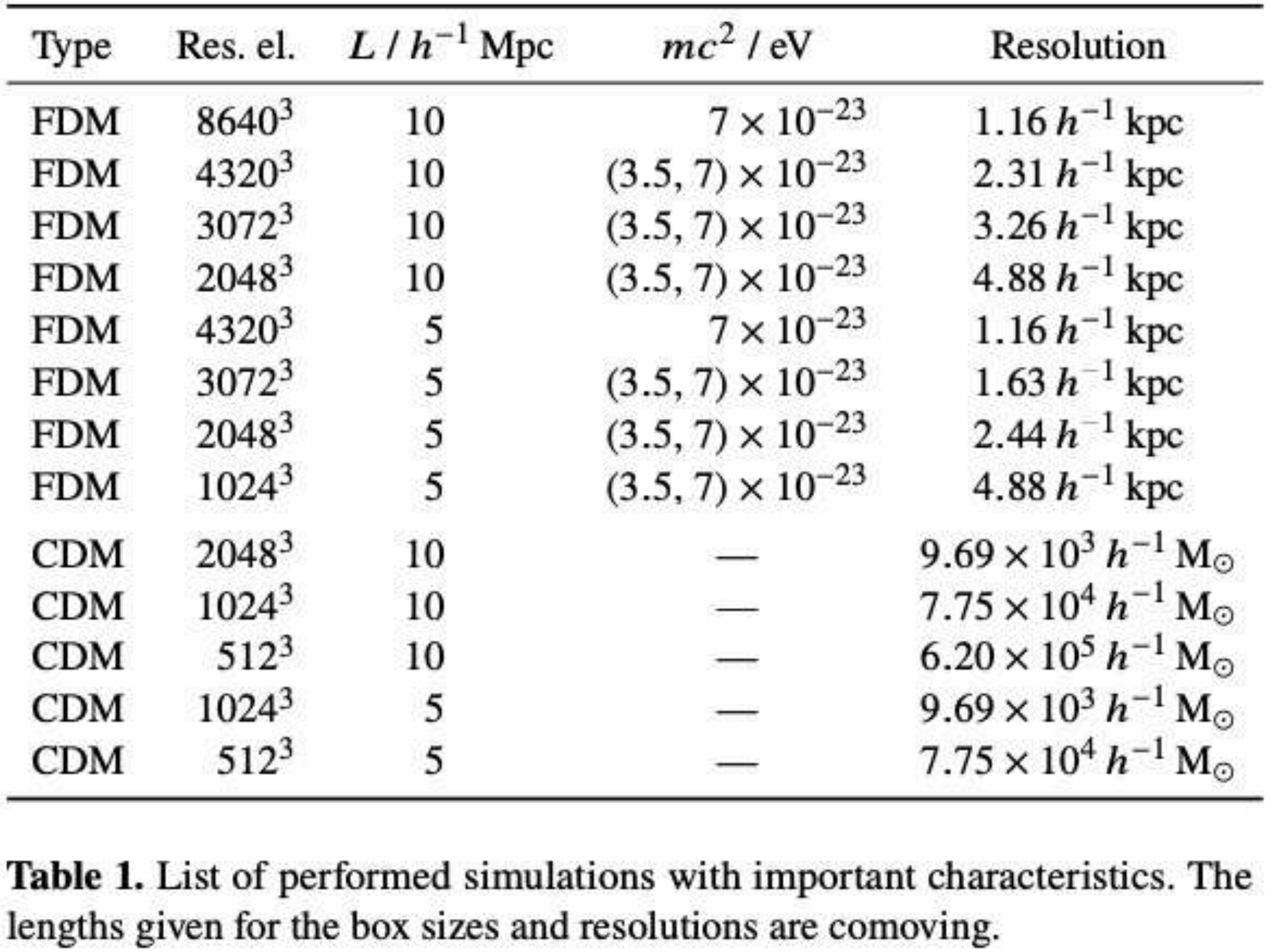
Jowett
Own implementation

FDM simulation

May et al 2020: Box size and resolution

Largest three-dimensional cosmological simulations of FDM structure formation to low redshifts





Type	Res. el.	L / h^{-1} Mpc	mc^2 / eV	Resolution
FDM	8640^3	10	7×10^{-23}	$1.16\,h^{-1}$ kpc
FDM	4320^3	10	$(3.5, 7) \times 10^{-23}$	$2.31\,h^{-1}$ kpc
FDM	3072^3	10	$(3.5, 7) \times 10^{-23}$	$3.26\,h^{-1}$ kpc
FDM	2048^3	10	$(3.5, 7) \times 10^{-23}$	$4.88\,h^{-1}$ kpc
FDM	4320^3	5	7×10^{-23}	$1.16\,h^{-1}$ kpc
FDM	3072^3	5	$(3.5, 7) \times 10^{-23}$	$1.63\,h^{-1}$ kpc
FDM	2048^3	5	$(3.5, 7) \times 10^{-23}$	$2.44\,h^{-1}$ kpc
FDM	1024^3	5	$(3.5, 7) \times 10^{-23}$	$4.88\,h^{-1}$ kpc
CDM	2048^3	10	—	$9.69 \times 10^3\,h^{-1}\,\mathrm{M}_\odot$
CDM	1024^3	10	—	$7.75 \times 10^4\,h^{-1}\,\mathrm{M}_\odot$
CDM	512^3	10	—	$6.20 \times 10^5\,h^{-1}\,\mathrm{M}_\odot$
CDM	1024^3	5	—	$9.69 \times 10^3\,h^{-1}\,\mathrm{M}_\odot$
CDM	512^3	5	—	$7.75 \times 10^4\,h^{-1}\,\mathrm{M}_\odot$

Table 1. List of performed simulations with important characteristics. The lengths given for the box sizes and resolutions are comoving.

Simulations: $\{\Omega_m = 0.3,\ \Omega_b = 0,\ \Omega_\Lambda = 0.7,\ H_0 = 70\,\mathrm{km\,s^{-1}}(h = 0.7),\ \sigma_8 = 0.9\}$

IC: $z = 127$