Aspects of Light Scalar Dark Matter

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Copernicus Series, February 2, 2022

QCD Axion in brief

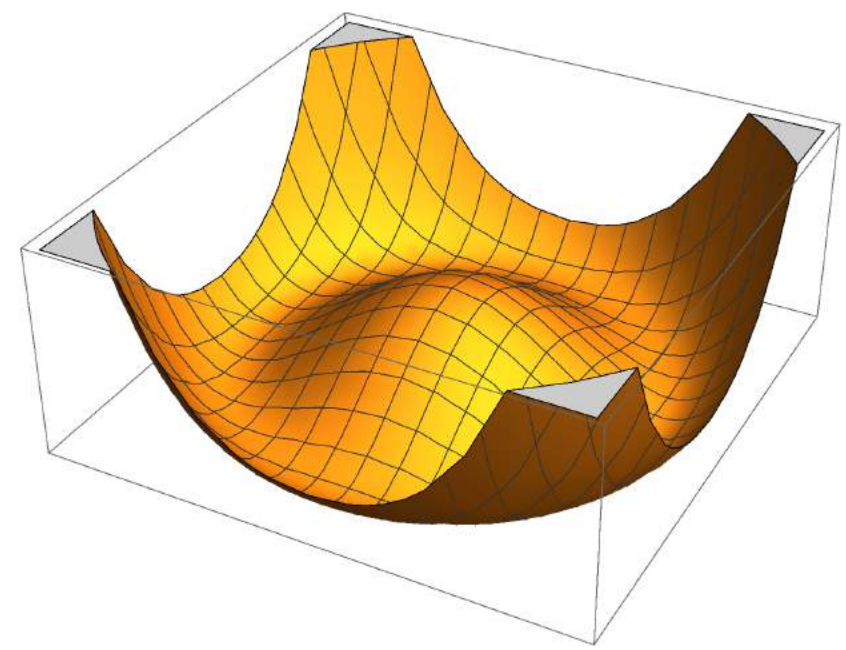
QCD-Axion

$$\Delta\mathcal{L}_{qcd} \sim \theta\, \mathbf{E}^a \cdot \mathbf{B}^a$$

$$|\theta| \lesssim 10^{-10}$$

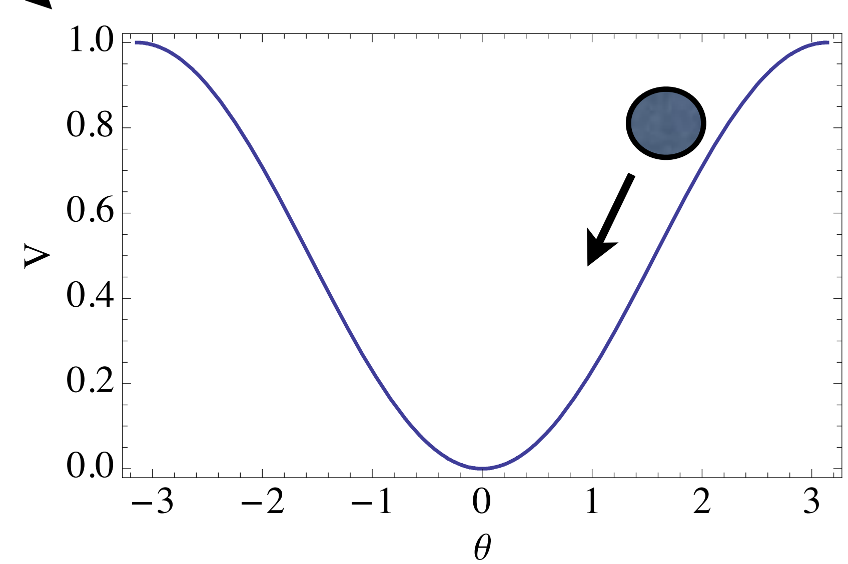
(Peccei, Quinn, Weinberg, Wilczek
Kim, Shifman, Vainshtein, Zakharov
Dine, Fischler, Srednicki, Zhitnitsky)

$$\theta \to \phi / f_a$$



$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V(\phi) = \frac{1}{2}m_a^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \dots$$



Axion mass:

$$m_a \sim \frac{\Lambda_{qcd}^2}{f_a}$$

(Attractive) Self-Coupling:

$$\lambda \sim -\frac{\Lambda_{qcd}^4}{f_a^4}$$

QCD-Axion

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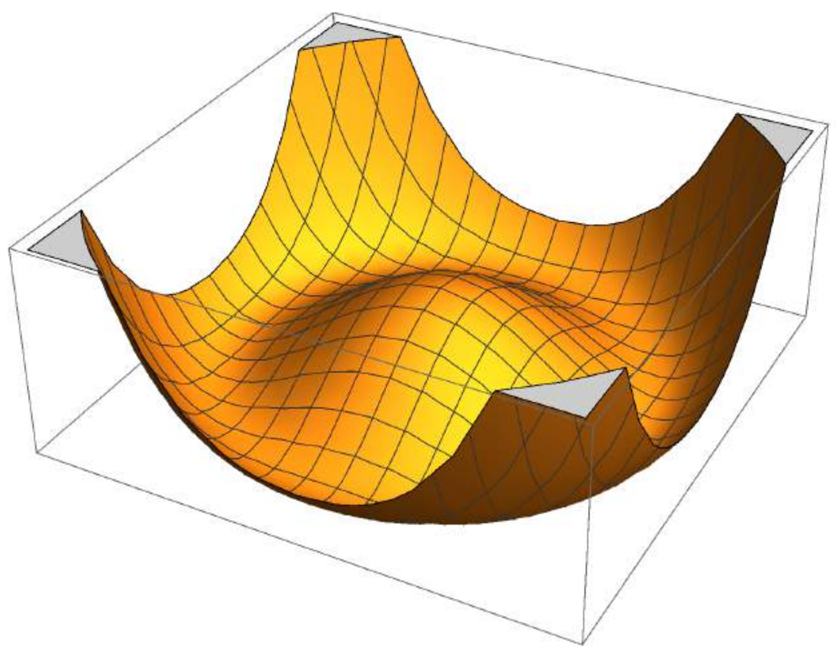
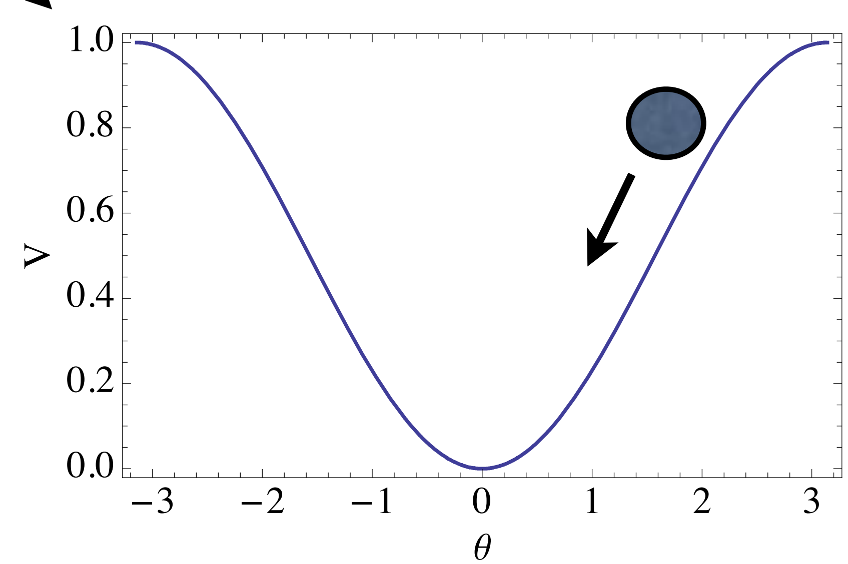


$$\Delta\mathcal{L}_a \sim \frac{\phi}{f_a}\mathbf{E}^a \cdot \mathbf{B}^a + \frac{1}{2}(\partial\phi)^2 - V(\phi)$$



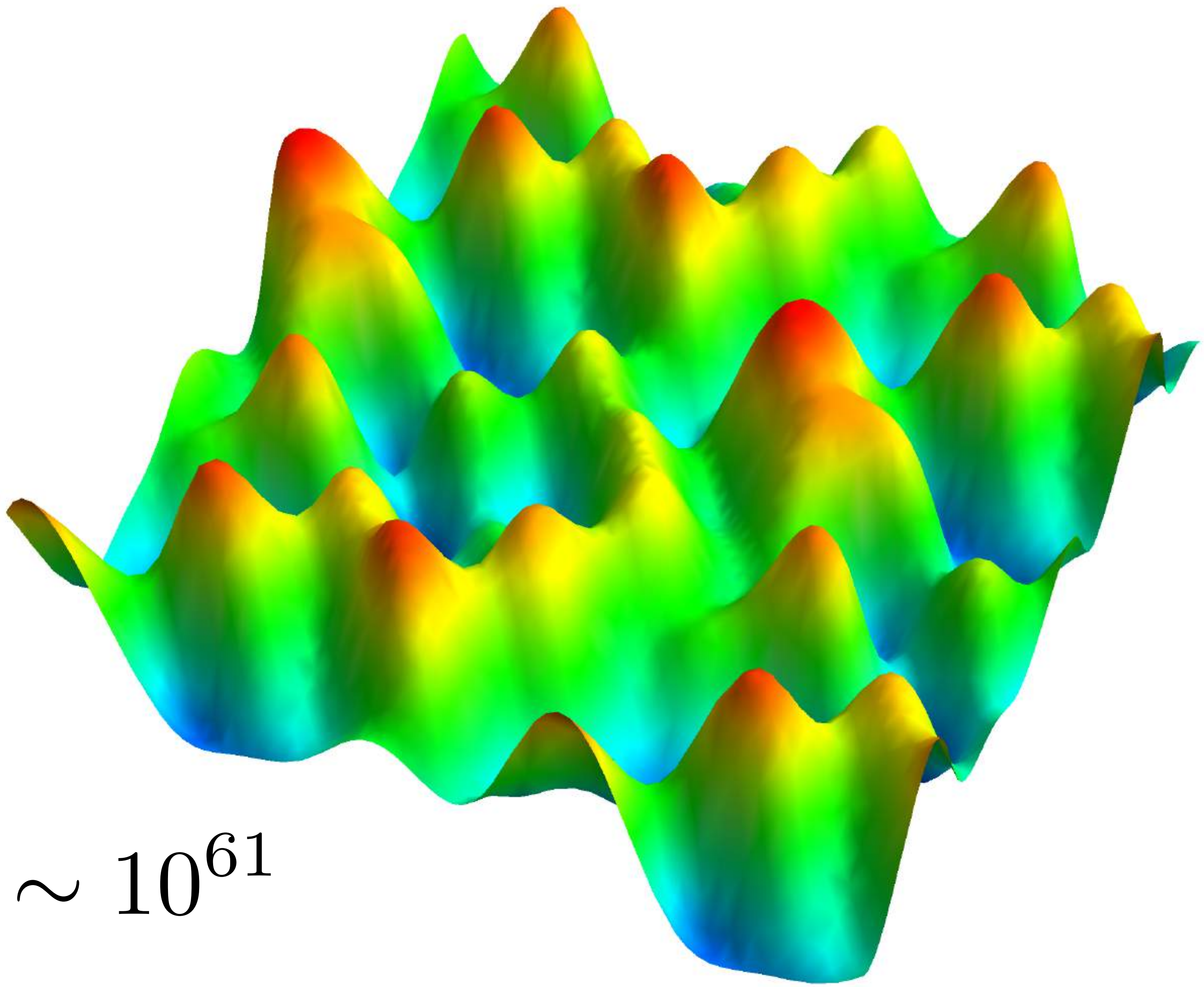
Abundance

$$\Omega_a \approx \langle\theta_i^2\rangle \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{7/6} < 0.25$$

Related issues for string-axions,
ALPs, light bosonic DM

Fluctuations and Evolution

In Post-Inflationary Scenario; Initial Distribution



$$\mathcal{N} \sim 10^{61}$$

$$(m_a \sim 10^{-5}\, eV)$$

Consider Non-Relativistic Behavior

$$\phi(\mathbf{x},t) = \frac{1}{\sqrt{2\,m}}\left(e^{-imt}\psi(\mathbf{x},t) + e^{imt}\psi^*(\mathbf{x},t)\right)$$

Hamiltonian

$$\hat{H} = \hat{H}_{\mathbf{kin}} + \hat{H}_{\mathbf{int}} + \hat{H}_{\mathbf{grav}}$$

$$\hat{H}_{\mathrm{kin}} = \int d^3x \frac{1}{2m} \nabla\hat{\psi}^\dagger \cdot \nabla\hat{\psi}$$

$$\hat{H}_{\mathrm{int}} = \int d^3x \frac{\lambda}{16m^2} \hat{\psi}^\dagger\hat{\psi}^\dagger\hat{\psi}\hat{\psi}$$

$$\hat{H}_{\mathrm{grav}} = -\frac{Gm^2}{2}\int d^3x \int d^3x' \frac{\hat{\psi}^\dagger(\mathbf{x})\hat{\psi}^\dagger(\mathbf{x}')\hat{\psi}(\mathbf{x})\hat{\psi}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}$$

Number Density

$$\hat{n}(\mathbf{x}) = \hat{\psi}^\dagger(\mathbf{x})\hat{\psi}(\mathbf{x})$$

(For corrections: Namjoo, Guth, Kaiser 2017,
Eby, Mukaida, Takimoto, Wijewardhana, Yamada 2018)

Dynamical Time Scales

Equation of Motion

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

Occupancy number change rate $\Gamma_k \equiv \frac{\dot{\mathcal{N}}_k}{\mathcal{N}_k} \sim \frac{G\,m^2\,n_{ave}}{k^2}$

(See Sikivie, Yang 2009)

Relaxation rate $\Gamma_{rel} \sim \frac{G^2\,n^2\,m^5}{k^6}$ $(\sim n\,\sigma\,v\,\mathcal{N})$

(See Levkov, Panin, Tkachev 2018)

Equilibrium with high occupancy suggests BEC

Axion BEC Literature

- Sikivie, Yang (2009)
- Erken, Sikivie, Tam, Yang (2011)
- Chavanis (2012)
- Banik, Sikivie (2013)
- Davidson, Elmer (2013)
- Saikawa, Yamaguchi (2013)
- Noumi, Saikawa, Sato, Yamaguchi (2014)
- Vega, Sanchez (2014)
- Li, Rindler-Daller, Shapiro (2014)
- Berges, Haeckel (2014)
- Banik, Christopherson, Sikivie, Todarello (2015)
- Davidson (2015)
- Eby, Suranyi, Wijewardhana (2014, 2015, 2016, 2017, 2018, 2019, 2020) [w/Leembruggen, Ma, Street, Vaz]
- ……. many more……

Part 1

Classical vs Quantum with Interactions

What About Interactions?

Fundamental claim of Sikivie, Todarello, 1607.00949

On time scales $t > \tau = 1/\Gamma$ the classical description breaks down, requiring the full quantum theory, which is the only way to see thermalization

Toy Model

Second Quantized Language

$$\hat{H} = \sum_i \omega_i \, \hat{a}_i^\dagger \hat{a}_i + \frac{1}{4} \sum_{ijkl} \Lambda_{ij}^{kl} \, \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l ,$$

Consider just 5 oscillators for simplicity

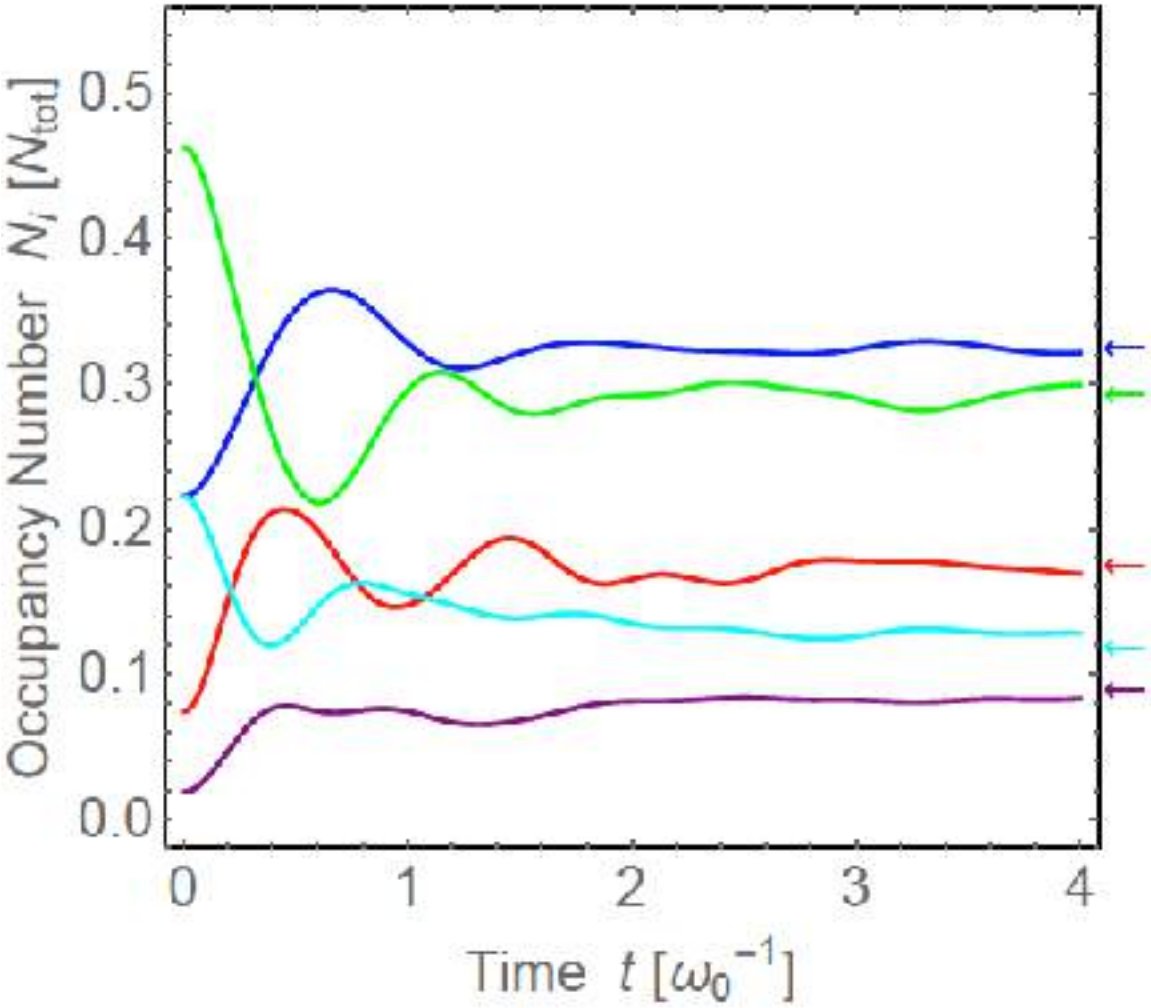
Initial quantum state $|\{N_i\}\rangle = |12, 25, 4, 12, 1\rangle$

Initial classical state $a_i = \sqrt{N_i}$

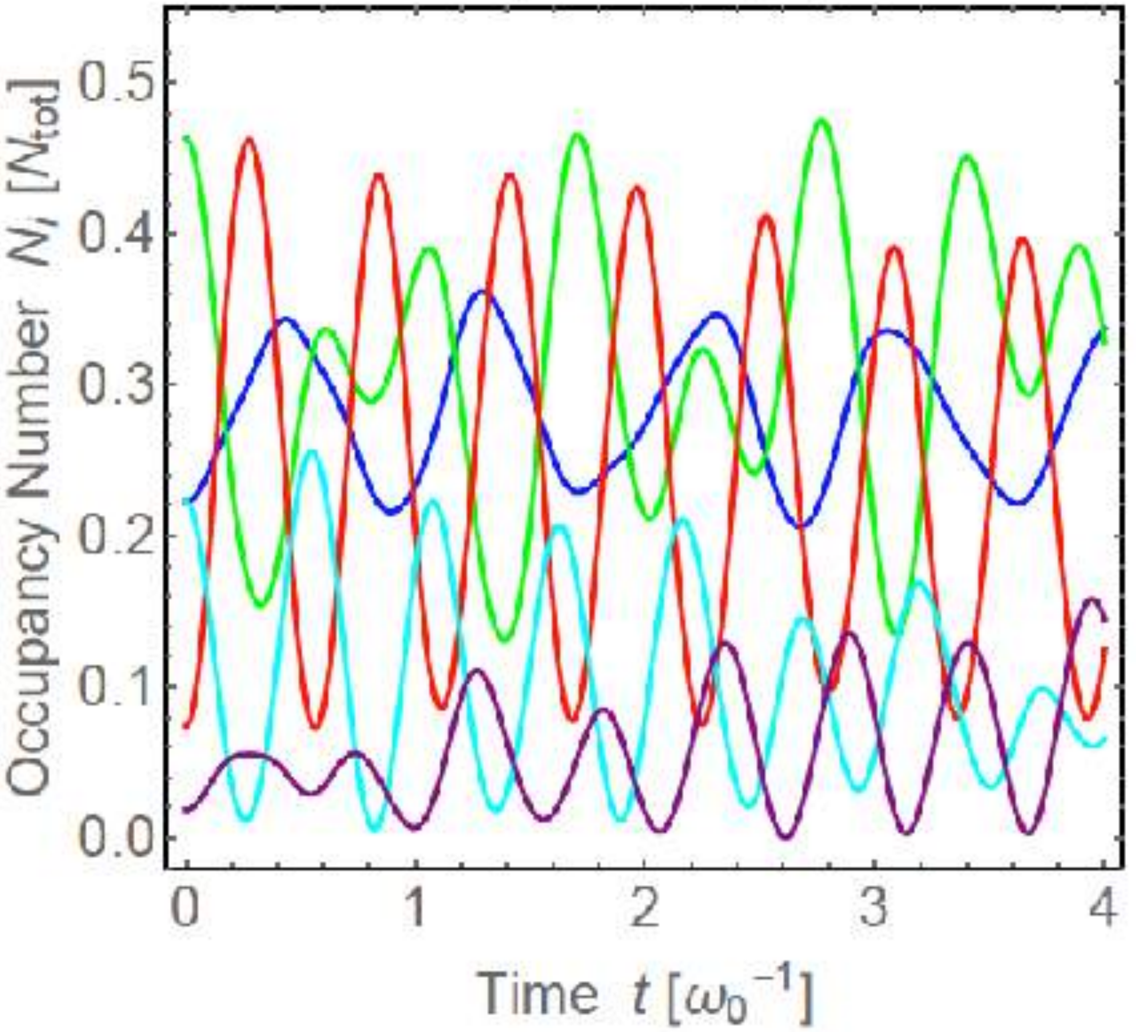
Sikivie, Todarello, 1607.00949

Quantum vs Classical??

Quantum



Classical



Sikivie, Todarello, 1607.00949

Correct Classical Treatment

Initial classical state $a_i = \sqrt{N_i}\, e^{I\theta_i}, \quad \theta_i \in [0, 2\pi)$

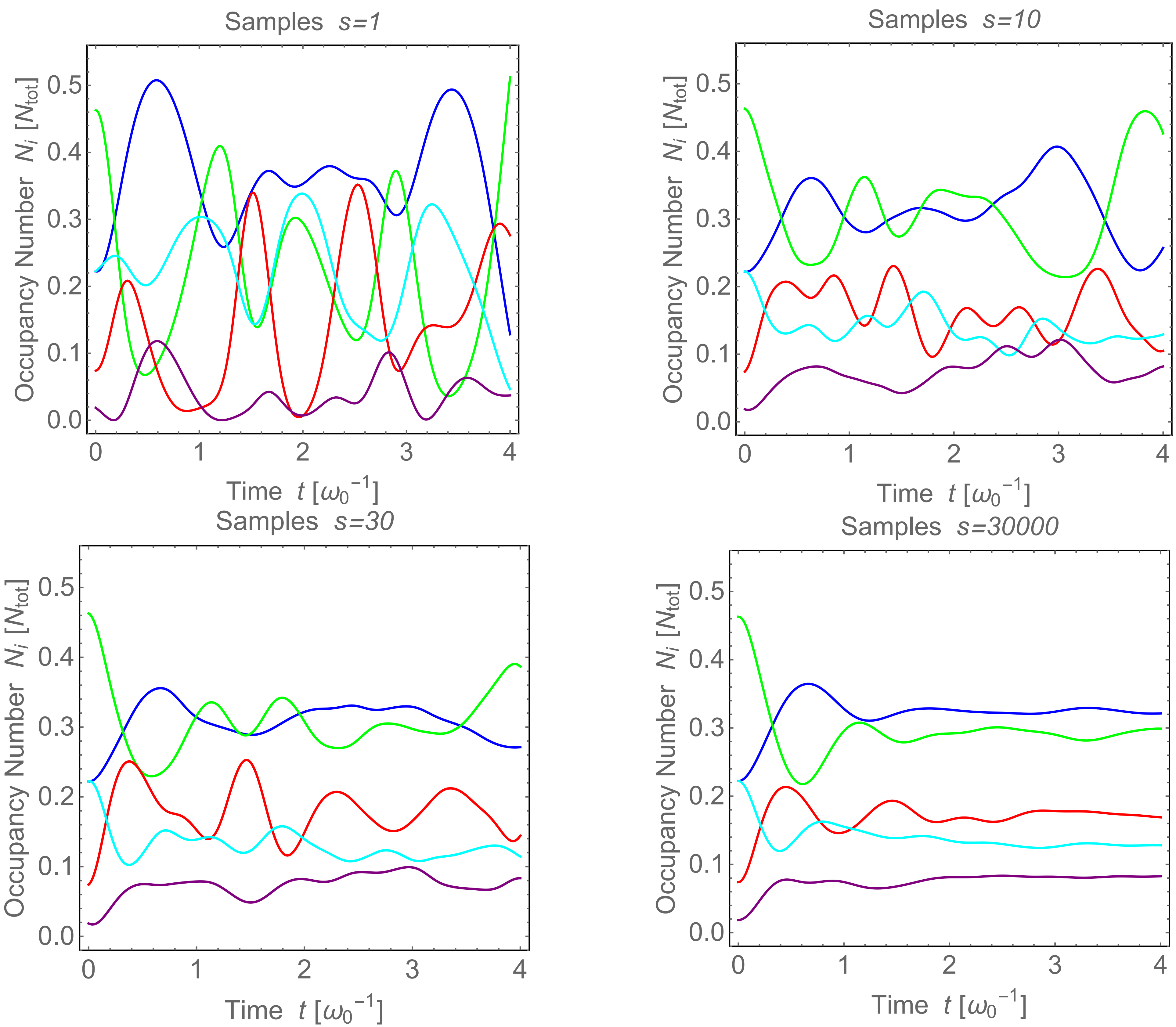
Ensemble average over random initial phases

Meaningful comparison

Connects to uncertainty in branch of wavefunction

Hertzberg 1609.01342 (JCAP)

Correct Classical Treatment



(matches quantum)

Hertzberg 1609.01342 (JCAP)

Implication for Correlation Functions

Implication for Correlation Functions

At high occupancy

$$\langle \{N_i\} | \hat{\psi}^\dagger(\mathbf{x},t)\,\hat{\psi}(\mathbf{y},t) | \{N_i\} \rangle \approx \langle \psi^*(\mathbf{x},t)\,\psi(\mathbf{y},t) \rangle_{ens}$$

Ergodic theorem

$$\langle \psi^*(\mathbf{x},t)\,\psi(\mathbf{y},t) \rangle_{ens} = \frac{1}{V}\int_V d^3z\,\psi^*_\mu(\mathbf{x}+\mathbf{z},t)\,\psi_\mu(\mathbf{y}+\mathbf{z},t)$$

Hertzberg 1609.01342 (JCAP)

Implication for Axion Simulations

Correlation functions of quantum and classical micro-states agree at high occupancy, despite the macroscopic spreading of wave-functions in these chaotic systems

Note: this is not some trivial consequence of Ehrenfest theorem…

Hertzberg 1609.01342 (JCAP)

Akin to billiard balls which exhibit chaos

Albrecht, Phillips 2012

(Return to this at end of talk)

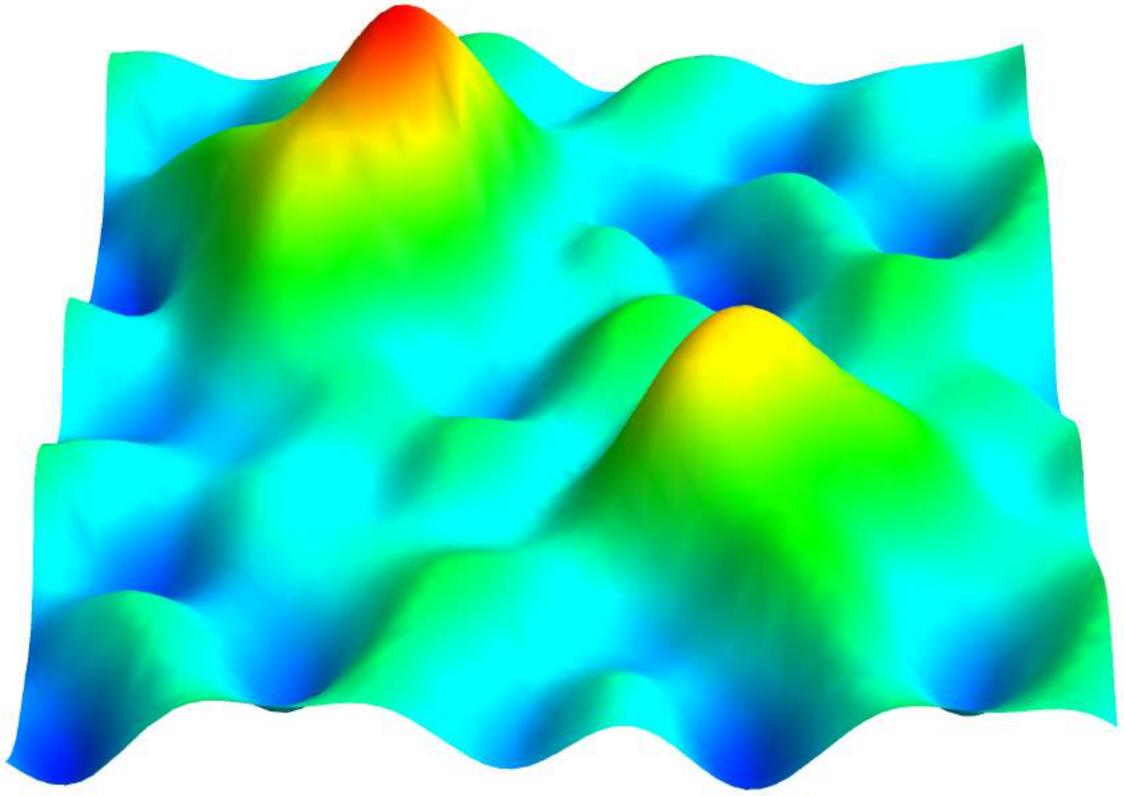
Implication for Axion Dark Matter

Statistically, axions are well described by classical field theory, after all

What is the BEC?

Miniclusters
—> Axion stars

that may exist in galaxies



Hogan, Rees 1988; Kolb, Tkachev 1993, 1994, 1995; Barranco, Bernal 2001; Guth, Hertzberg, Prescod-Weinstein 2014; Fairbairn, Marsh, Quevillon, Rozier 2017; Kitajima, Soda, Urakawa 2018; Eby, Leembruggen, Ma, Street, Suranyi, Vaz, Wijewardhana 2014 — 2020

Part 2

Light Scalar Condensates/Stars in Detail

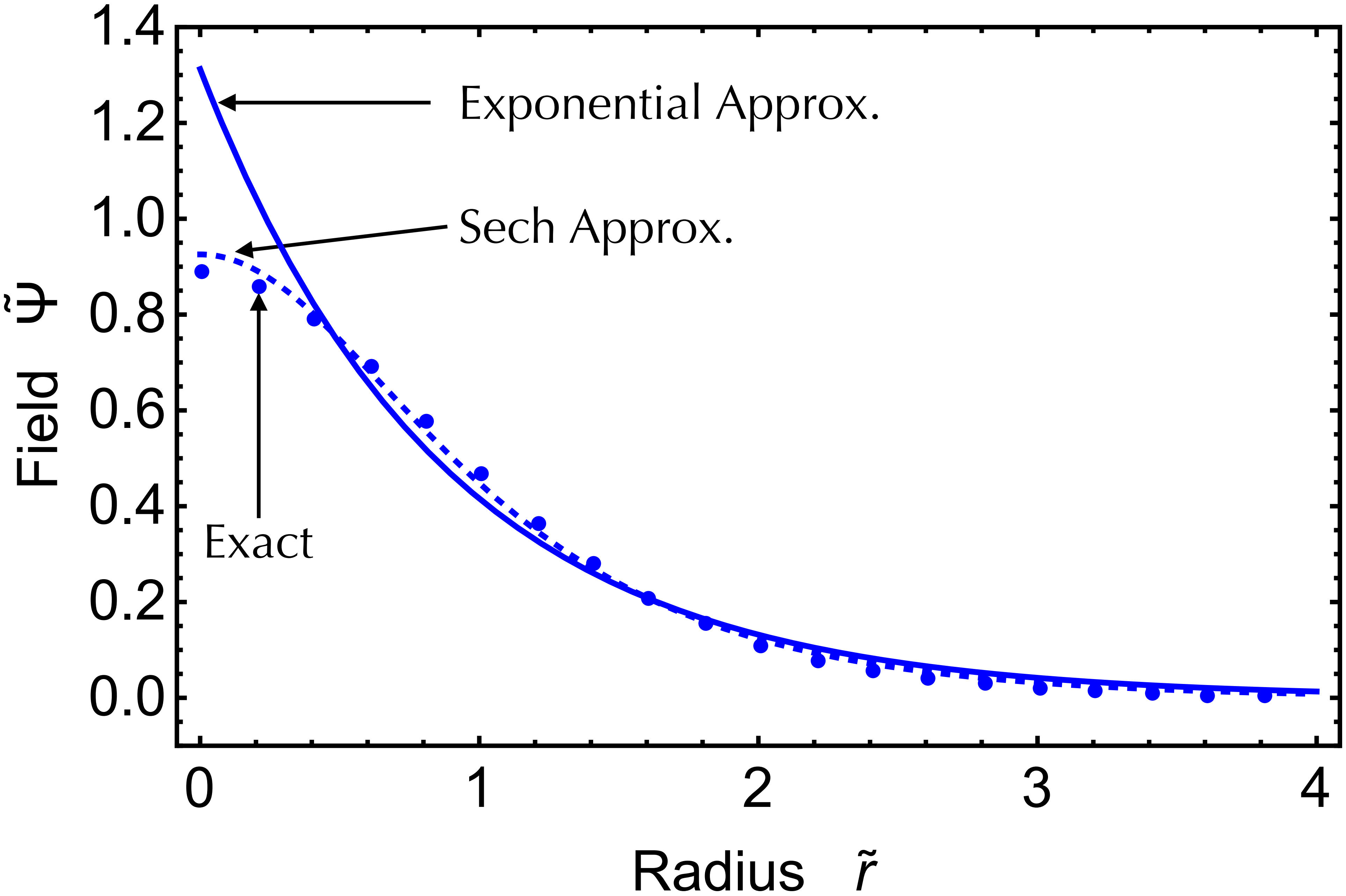
Return to Non-Relativistic Classical Field Theory

Equation of Motion

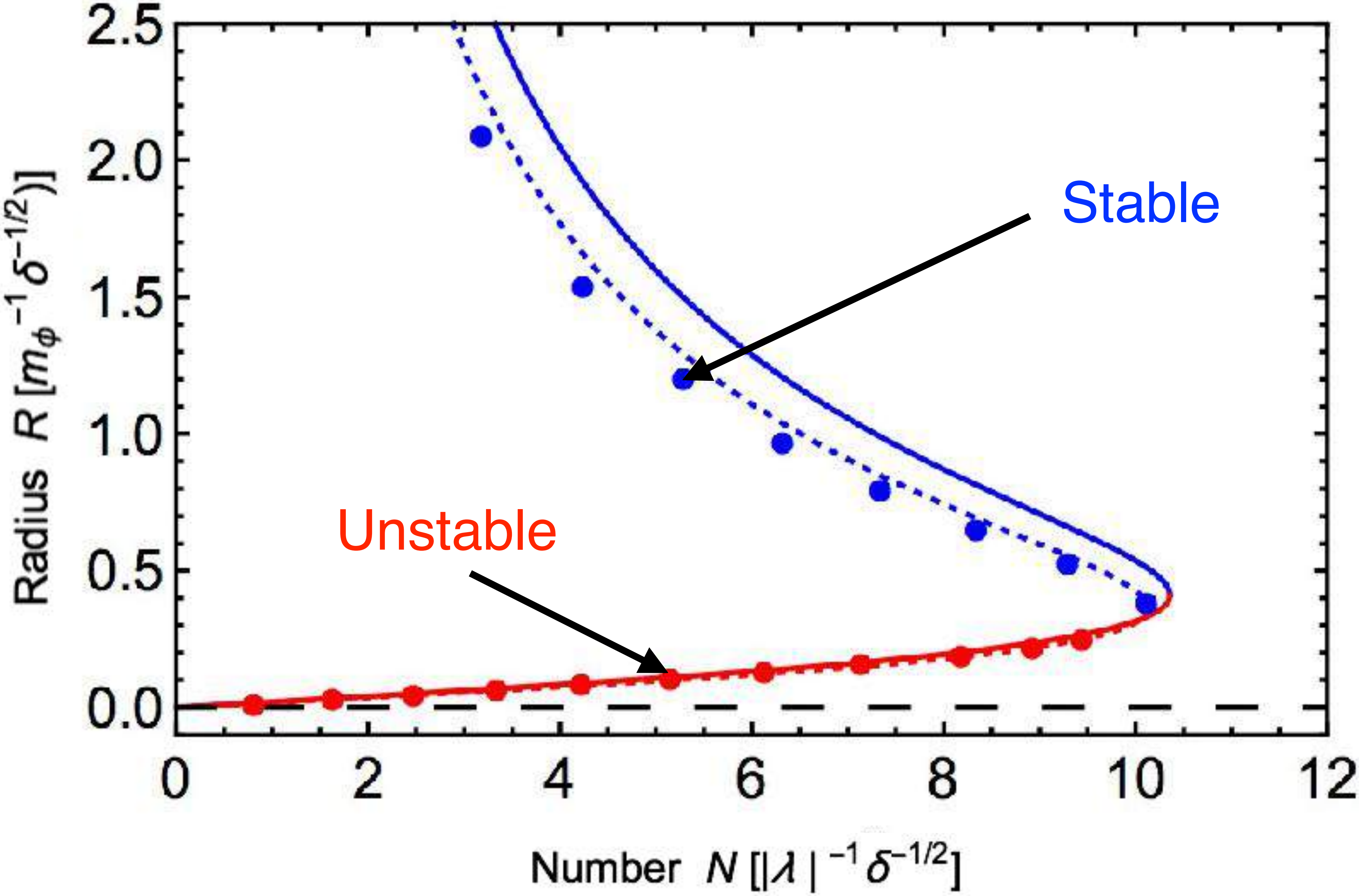
$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - Gm^2\psi\int d^3x'\,\frac{|\psi(\mathbf{x}')|^2}{|\mathbf{x}-\mathbf{x}'|}$$

$$(\lambda < 0)$$

Star Solutions (BEC) at fixed N

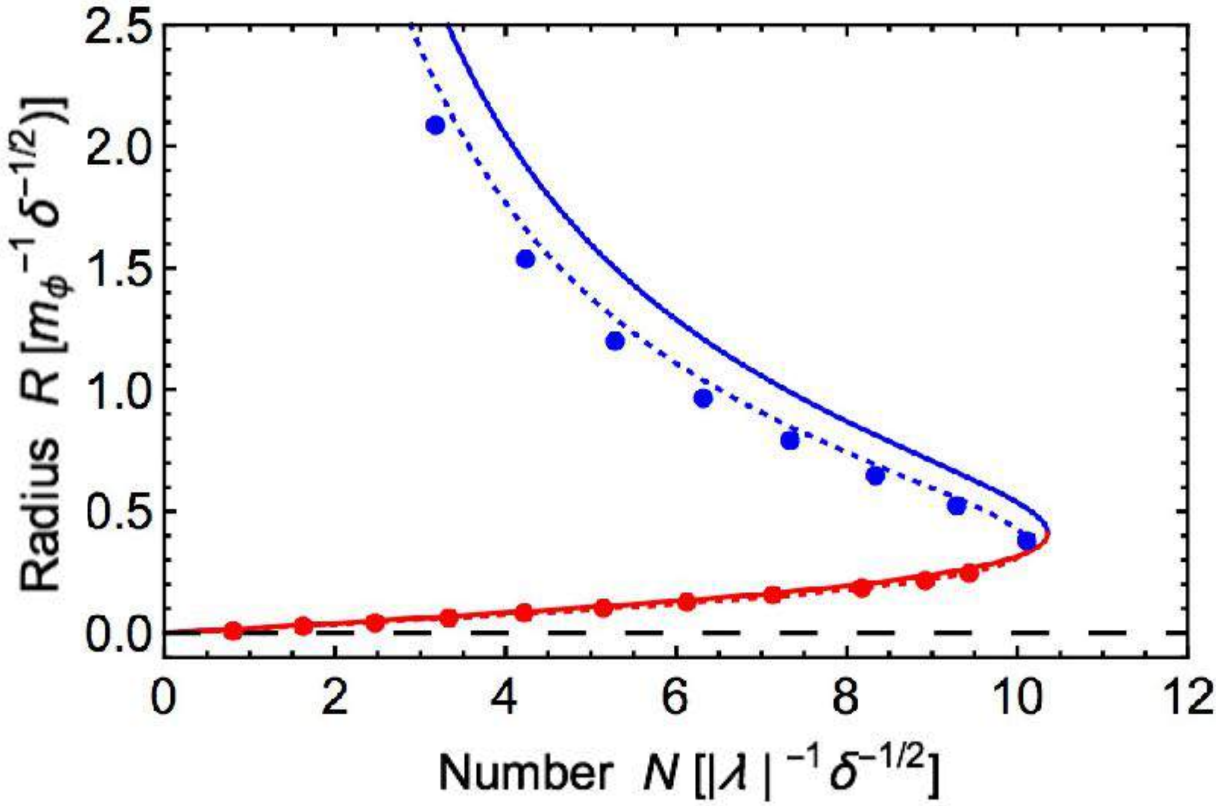


Two Branches of Solutions



See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729 (JCAP)

Two Branches of Solutions

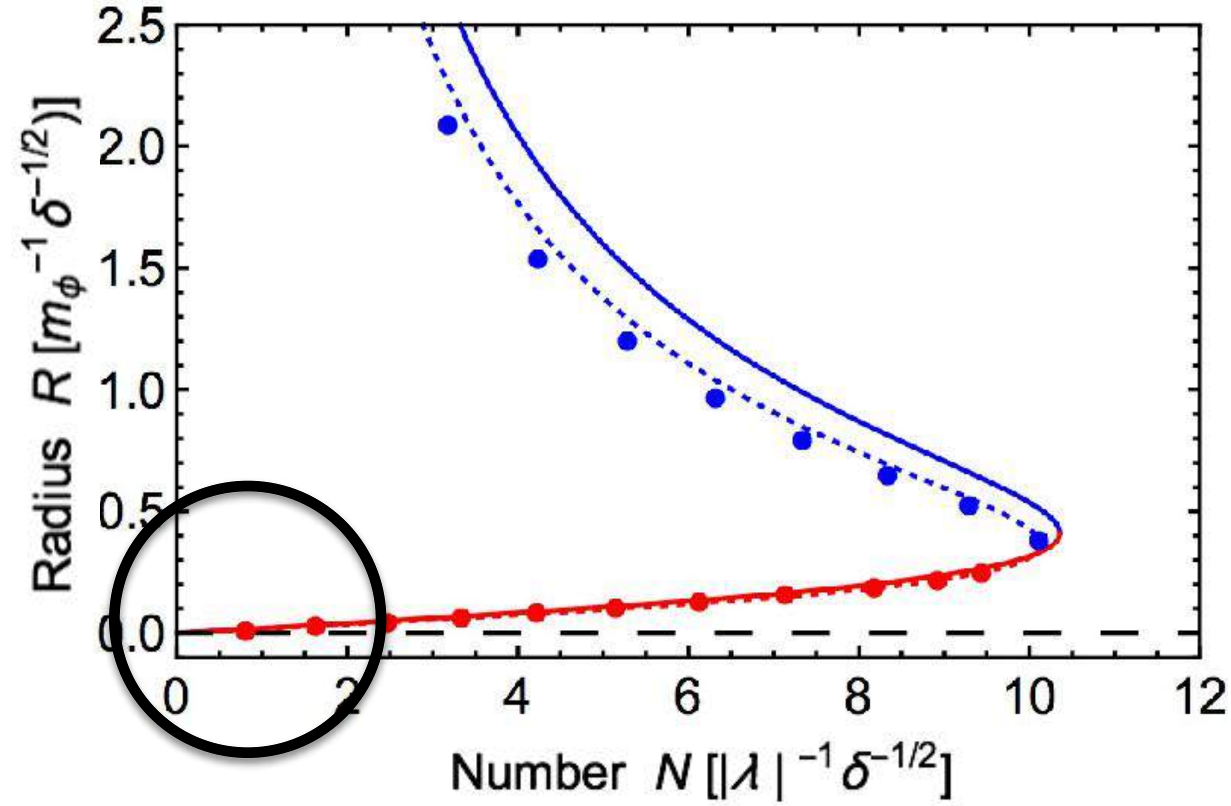


$$
\begin{aligned}
N_{max} &= \frac{f_a}{m^2\sqrt{G}}\tilde{N}_{max} \sim 8\times 10^{59}\,(\tilde{m}^{-2}\tilde{f}_a)\,,\\
M_{max} &= N_{max}\,m \qquad \sim 1.4\times 10^{19}\,\mathrm{kg}\,(\tilde{m}^{-1}\tilde{f}_a)\,,\\
R_{90,min} &= \frac{a\,(\tilde{R}_{90}/\tilde{R})}{bN_{max}Gm^3} \sim 130\,\mathrm{km}\,(\tilde{m}^{-1}\tilde{f}_a^{-1})\,,
\end{aligned}
$$

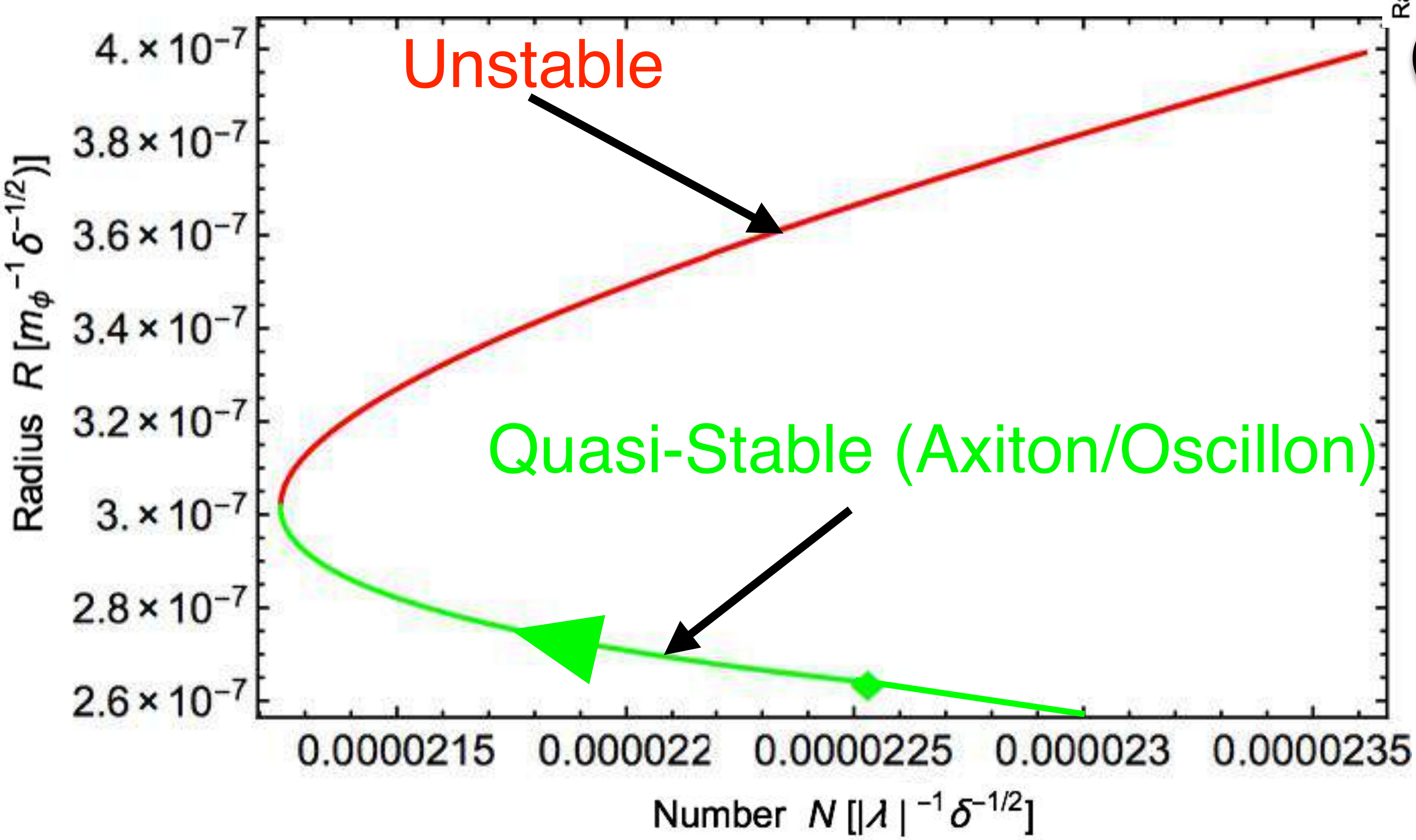
where $\tilde{f}_a \equiv f_a/(6\times 10^{11}\,\mathrm{GeV})$ and $\tilde{m} \equiv m/(10^{-5}\,\mathrm{eV})$.

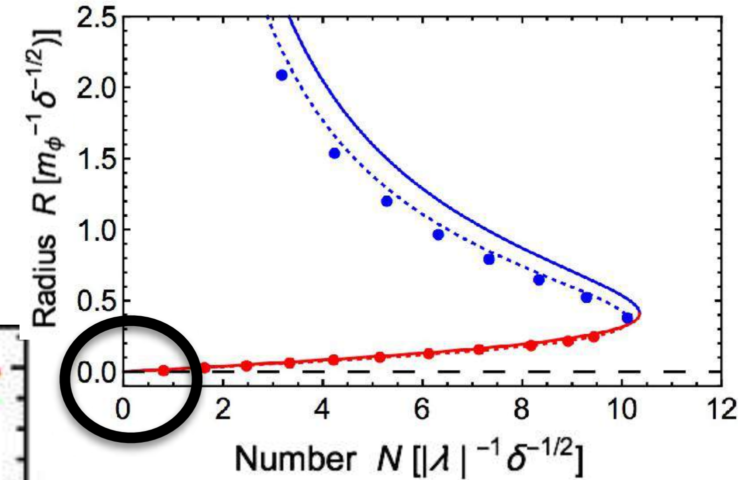
See Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729 (JCAP)

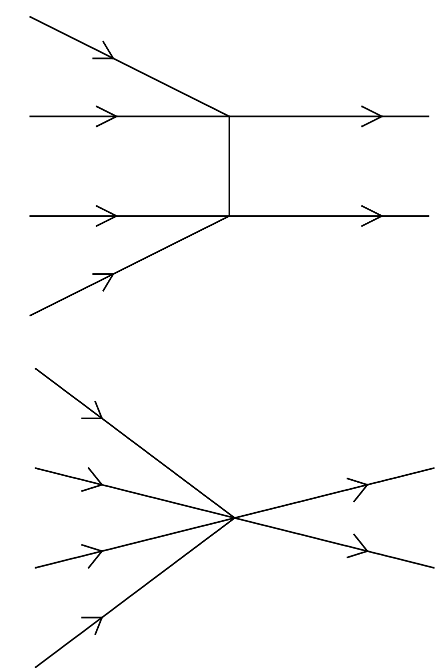
Relativistic Branch (Axiton)



Relativistic Branch (Axiton/Oscillon)





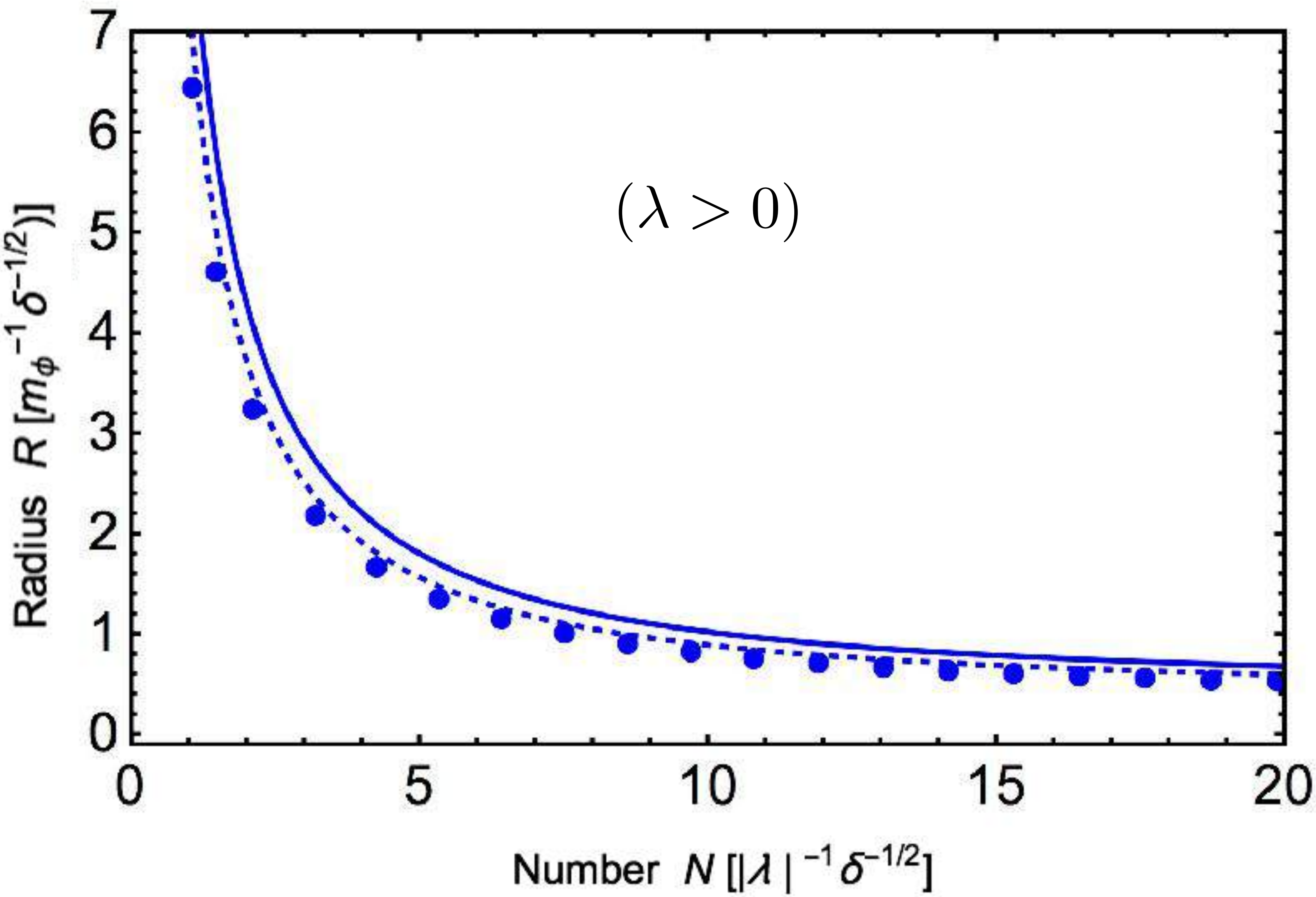


Kolb, Tkachev astro-ph/9311037; Fodor, Fogacs, Horvath, Mezei 0903.0953; Hertzberg 1003.3459; Eby, Suranyi, Wijewardhana 1512.01709; Schiappacasse, Hertzberg 1710.04729; Visinelli, Baum, Redondo, Freese, Wilczek 1710.08910; …

Repulsive Self Interactions

(see; Fan 2016)

Repulsive Self Interaction (Axion-Like Particle)



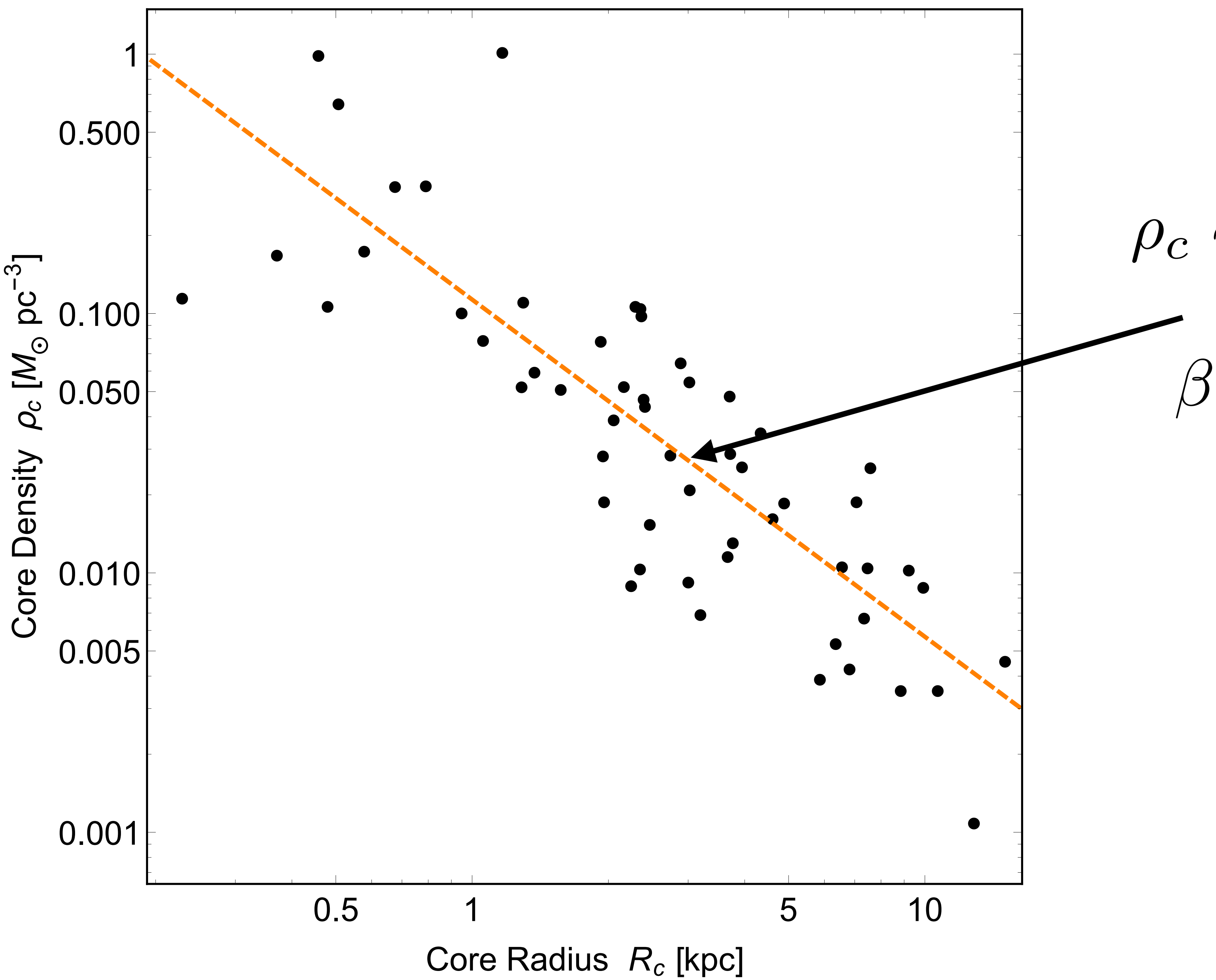
See Colpi, Shapiro, Wasserman 1986; Chavanis, Delfini 2011 and others…
Schiappacasse, Hertzberg 1710.04729; Hertzberg, Rompineve, Yang 2010.07927

Implications for Fuzzy Dark Matter

Can it explain galactic cores?

Hu, Barkana, Gruzinov 2000, ….

Core Density Vs Core Radius (Data)

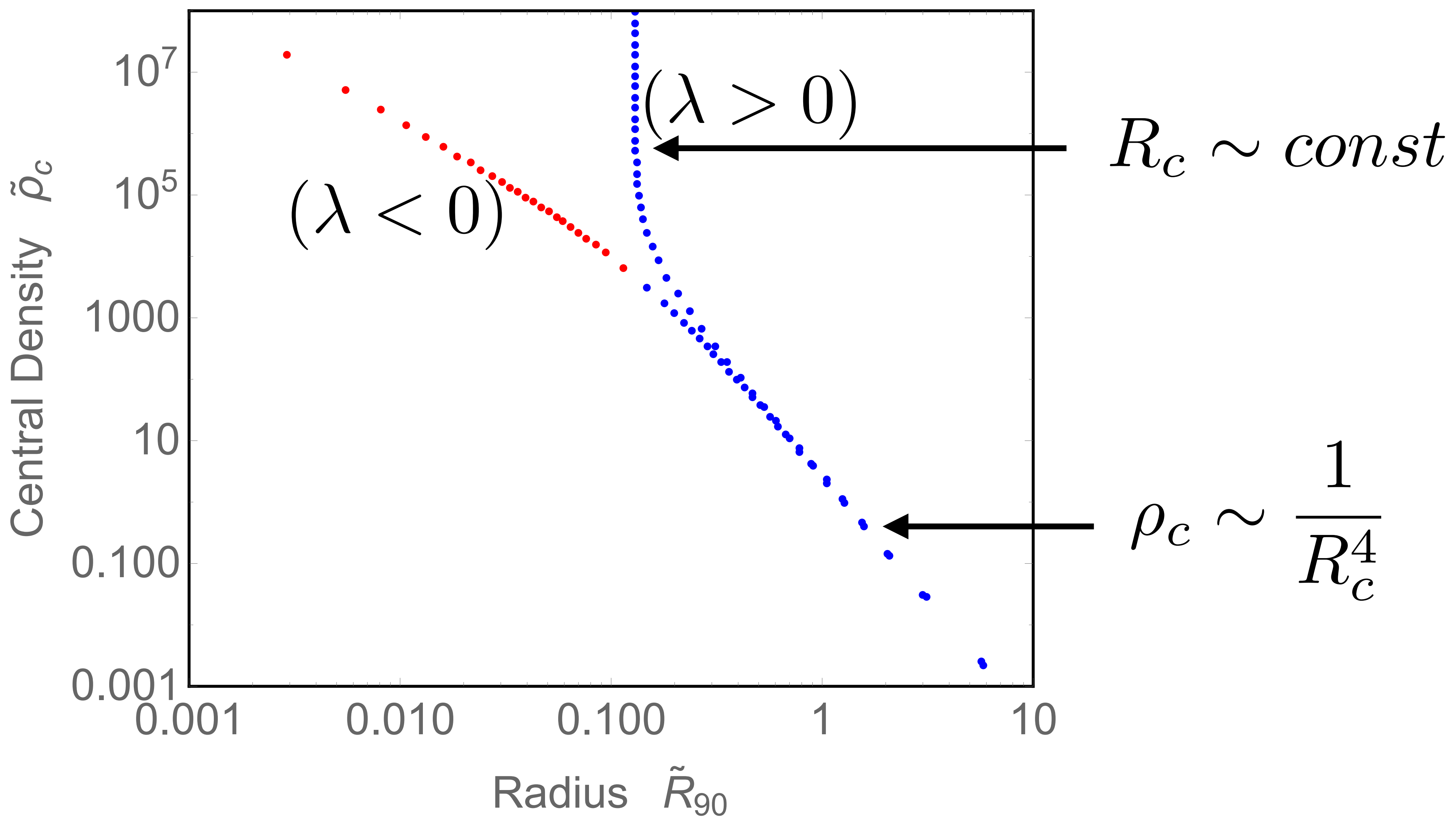


$$\rho_c \sim \frac{1}{R_c^\beta}$$

$$\beta \approx 1$$

Data from: Rodriguez, del Popolo, Marra, de Oliveira 2017

Core Density Vs Core Radius (Light Scalar in BEC)

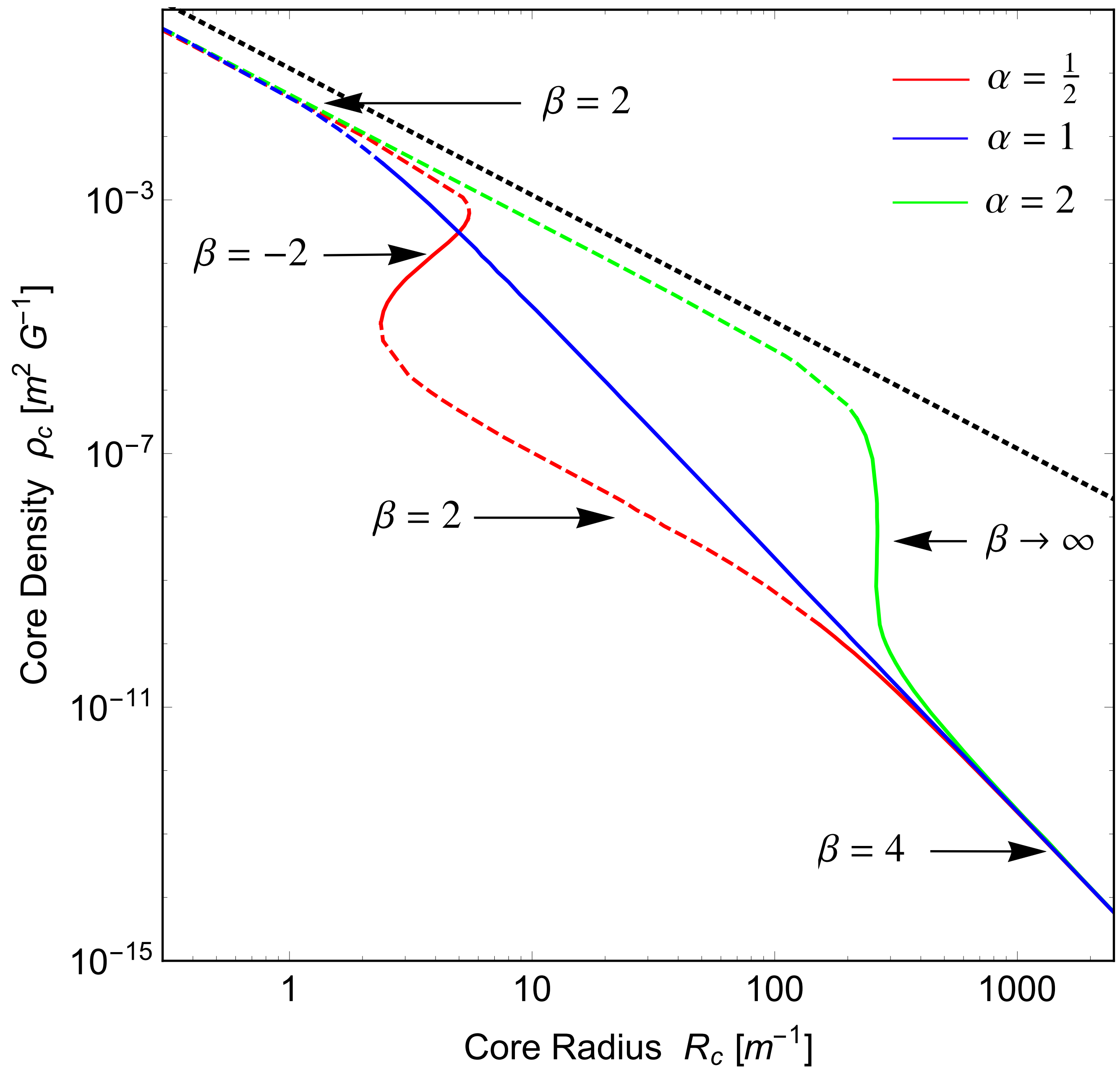


Core Density Vs Core Radius (Light Scalar in BEC)

Extension to general potentials, polytropes, full relativistic

$$V(\phi) \propto ((1+\phi^2/F^2)^\alpha - 1)$$

Solid = Stable
Dashed = Unstable

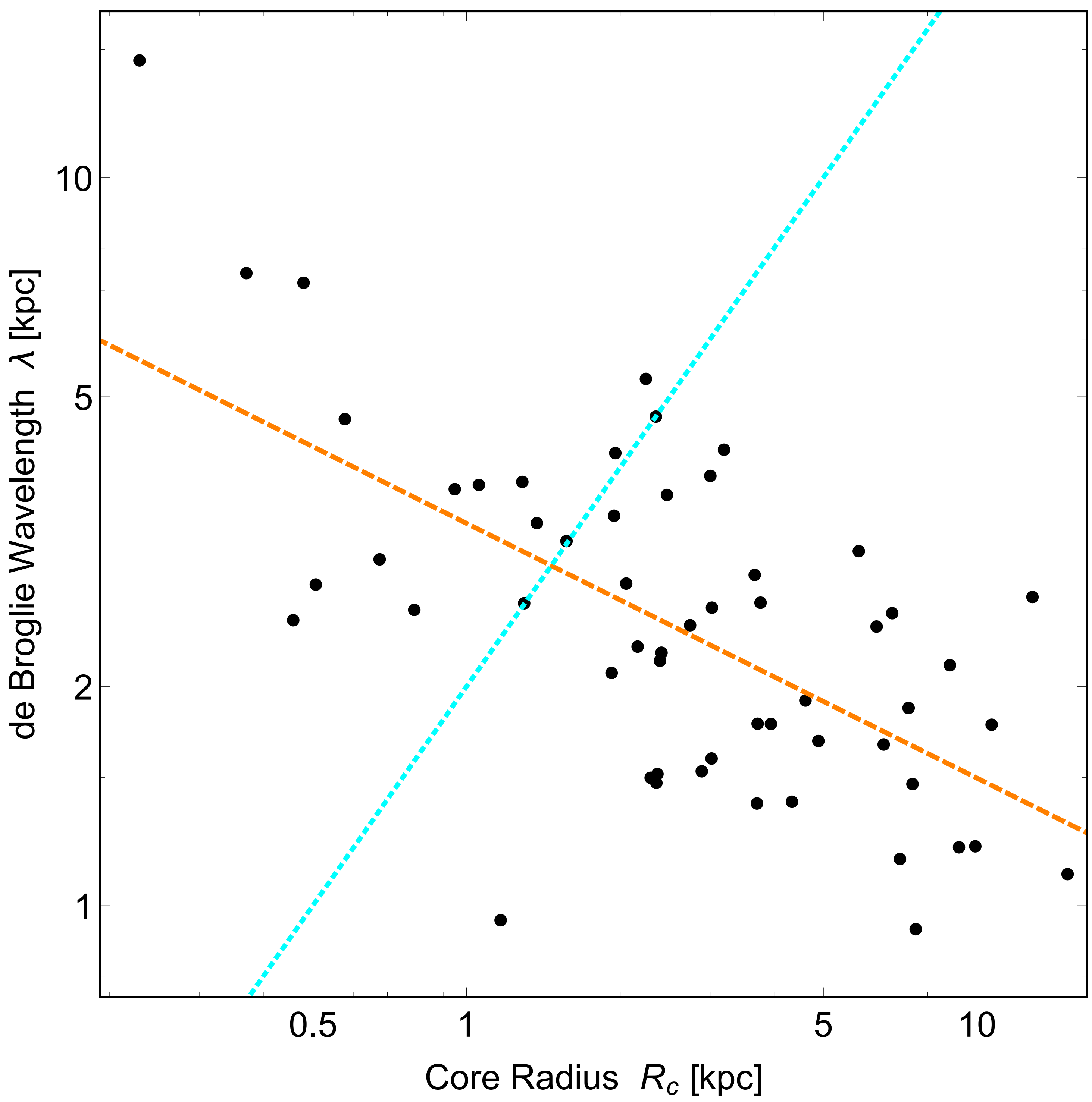


$$\rho_c \sim \frac{1}{R_c^\beta}$$

Never obtain $\beta \sim 1$ and stable

Deng, Hertzberg, Namjoo, Masoumi 1804.05921 (PRD)

Core Density Vs Core Radius (Light Scalar in BEC)



Deng, Hertzberg, Namjoo, Masoumi 1804.05921 (PRD)

Part 3

Pseudo-Scalar Resonance into Photons

Consider Axion to Photon Coupling

Photon Lagrangian

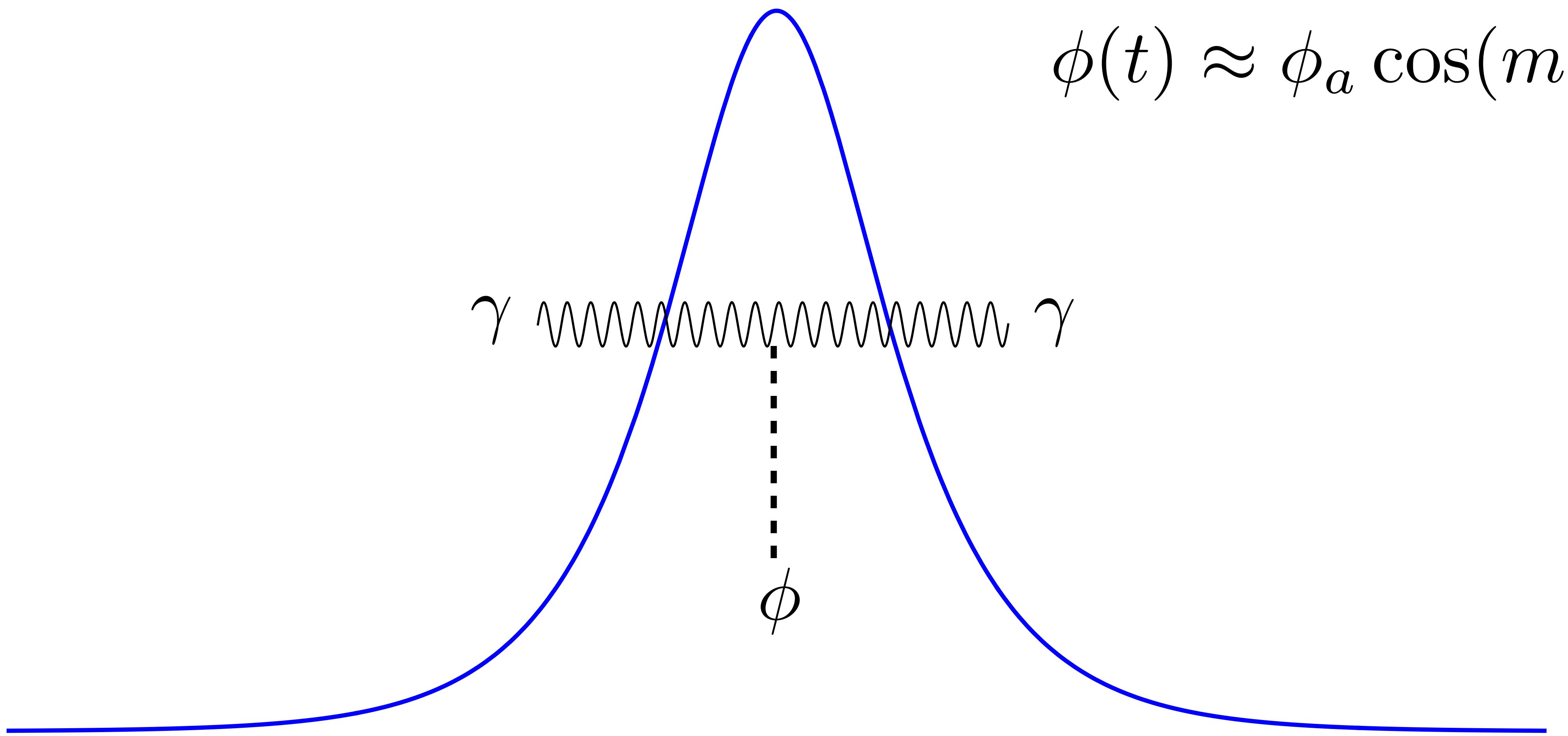
$$\mathcal{L}_\gamma = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) + g_{a\gamma}\,\phi\,\mathbf{E}\cdot\mathbf{B}$$

(Sikivie 1983; Adshead, Giblin, Scully, Sfakianakis 2015, 2016; Masaki, Aoki, Soda 2017)

Equation of motion

$$\ddot{\mathbf{A}} - \nabla^2\mathbf{A} + g_{a\gamma}\,\partial_t\phi\,\nabla\times\mathbf{A} = 0$$

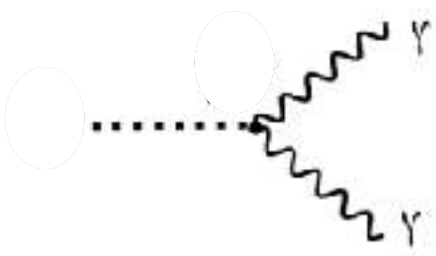
$$\phi(t) \approx \phi_a \cos(m_\phi t)$$



Homogeneous Axion Field

Mathieu Equation

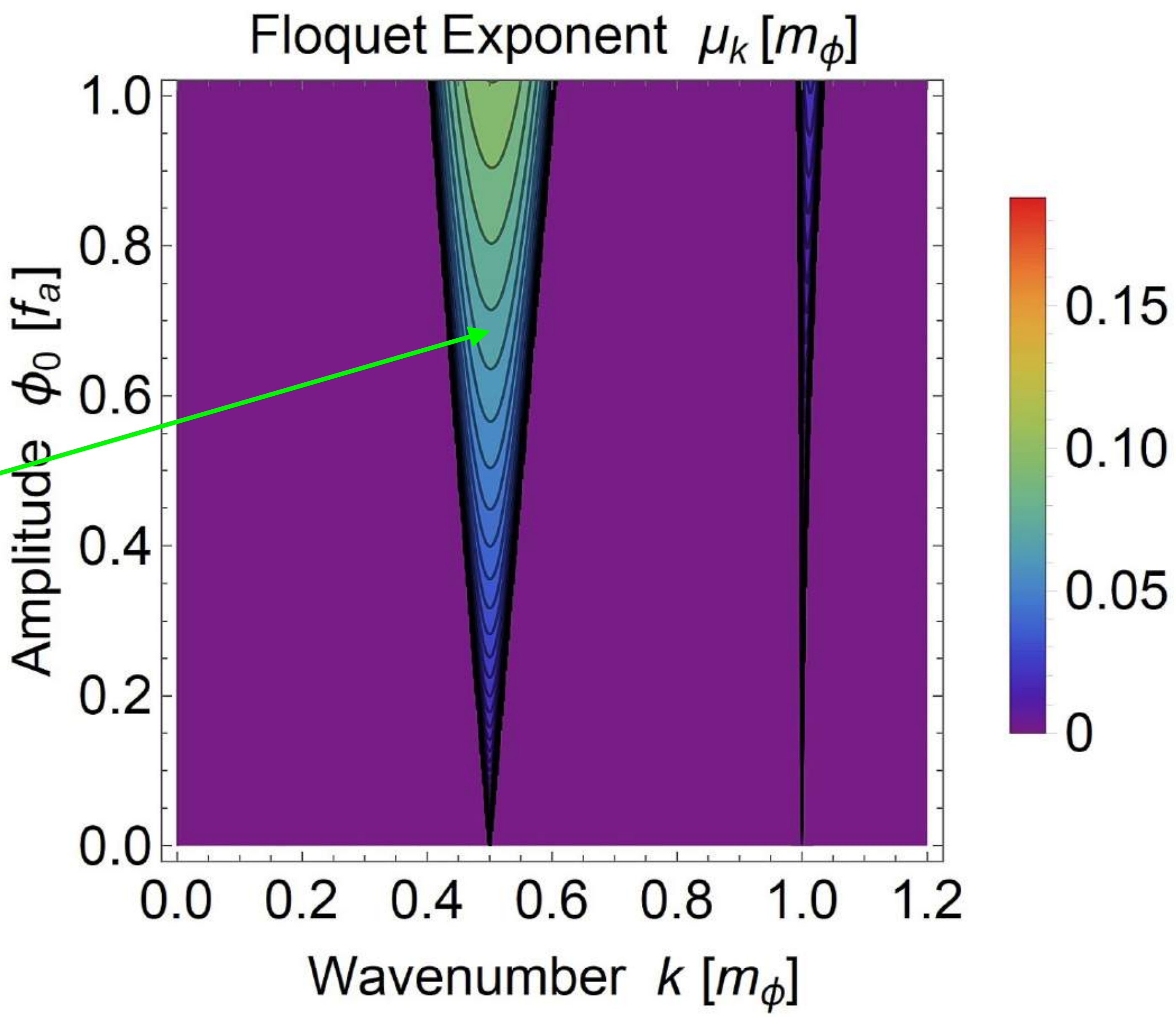
$$\ddot{\mathbf{A}}_{\mathbf{k}}^{T} + k^2 \mathbf{A}_{\mathbf{k}}^{T} + g_{a\gamma}\, k\, \partial_t \phi(t) \mathbf{A}_{\mathbf{k}}^{T} = 0$$



Parametric resonance always present

$$k \approx \frac{m_a}{2}$$

$$\mu_H^* \approx \frac{1}{4} g_{a\gamma}\, m_\phi\, \phi_a$$



e.g., Yoshimura 1996

Comment on Photon Plasma Mass

In plasma, the photon acquires an effective mass $$\omega_p^2 = \frac{4\pi\alpha\, n_e}{m_e}$$

In VERY EARLY universe, this is huge; preventing resonance

Clumps in halo: $$\omega_p^2 \approx \frac{n_e}{0.03\,\mathrm{cm}^{-3}} \left(6 \times 10^{-12}\,\mathrm{eV}\right)^2$$

Negligibly small; allowing for resonance

Inhomogeneous (Spherical) Axion Star

Decomposition into vector spherical harmonics

$$\mathbf{A}(\mathbf{r},t) = \sum_{lm} \int \frac{d^3k}{(2\pi)^3} \left[a_{lm}(k,t)\mathbf{N}_{lm}(k,\mathbf{r}) + b_{lm}(k,t)\mathbf{M}_{lm}(k,\mathbf{r})\right]$$

where

$$\mathbf{M}_{lm}(k,\mathbf{r}) = i\frac{j_l(kr)}{\sqrt{l(l+1)}}\nabla \times [Y_{lm}(\theta,\varphi)\mathbf{r}]$$

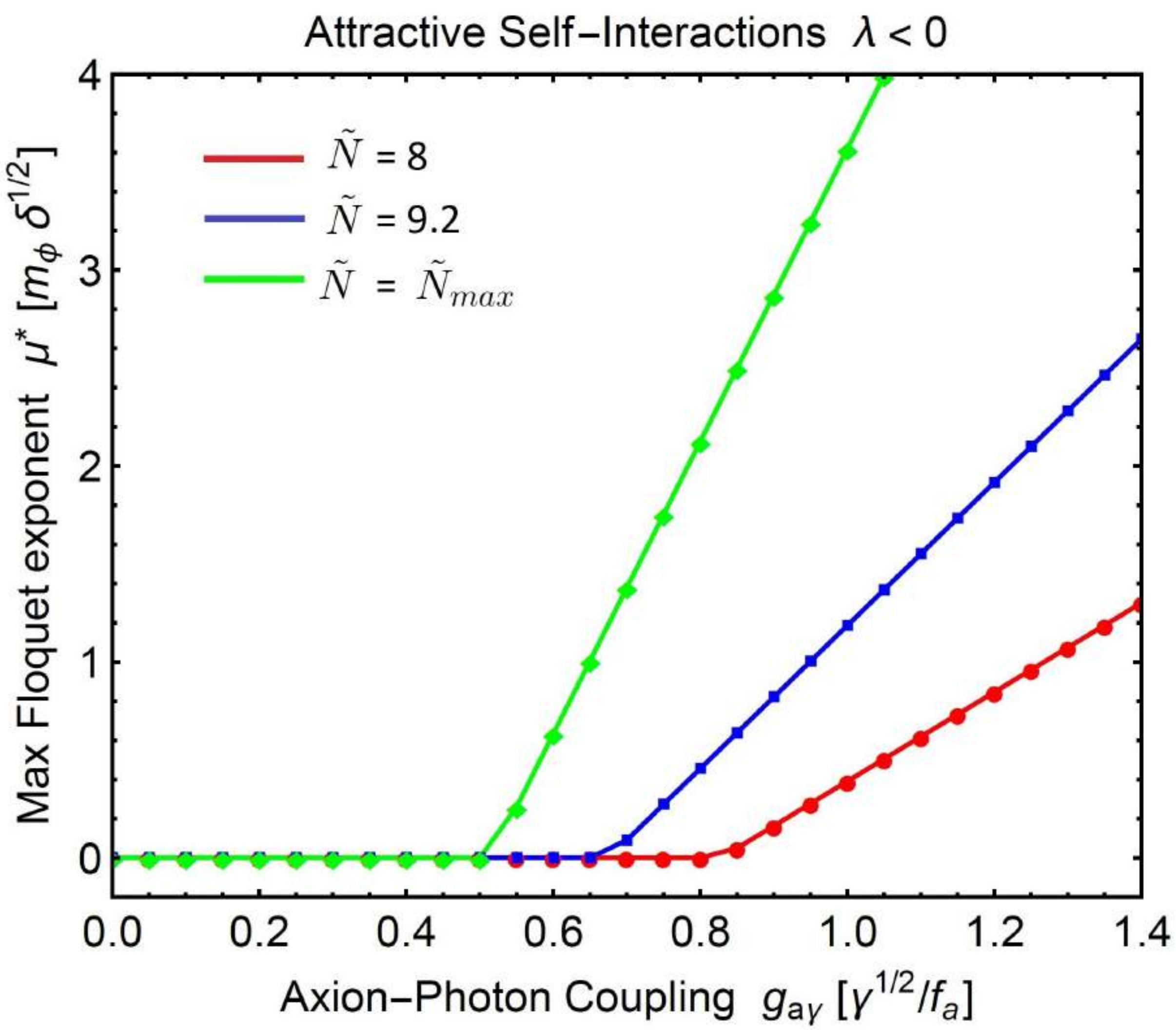
$$\mathbf{N}_{lm}(k,\mathbf{r}) = \frac{i}{k}\nabla \times \mathbf{M}_{lm}$$

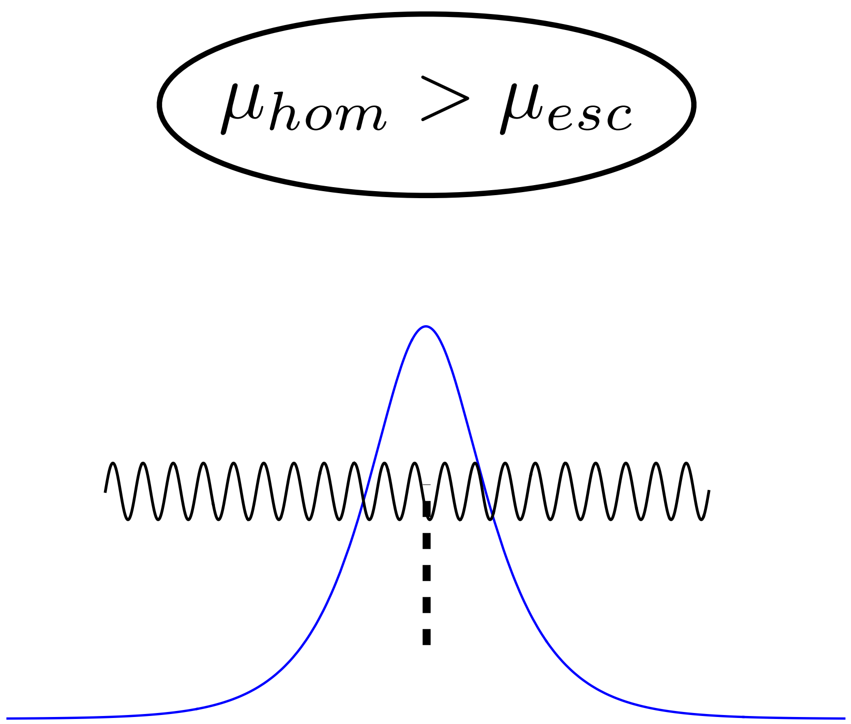
Hertzberg, Schiappacasse 1805.00430 (JCAP)

Inhomogeneous (Spherical) Axion Star

Instability channel $l = 1, m = 0, \quad b_{10} = -i\, a_{10}$

$$\ddot{a}_{10}(k,t) + k^2 a_{10}(k,t) + g_{a\gamma} k \int \frac{dk'}{(2\pi)} \partial_t \tilde{\phi}(k-k') a_{10}(k',t) = 0$$





Hertzberg, Schiappacasse 1805.00430 (JCAP)

Tkachev 1986, 1987, 2015;
Hertzberg 2010; Kawasaki, Yamada 2014

Resonance Condition (Spherical) Axion Star

(assuming attractive)

$$g_{a\gamma} > \frac{0.3}{f_a}$$

$$(\lambda < 0)$$

No resonance for standard QCD axion-photon coupling

$$g_{a\gamma} \sim \frac{\alpha}{f_a}$$

Allowed for models with enhanced couplings, decay to hidden sectors, or repulsive interactions

(e.g., Daido, Takahashi, Yokozaki 2018)

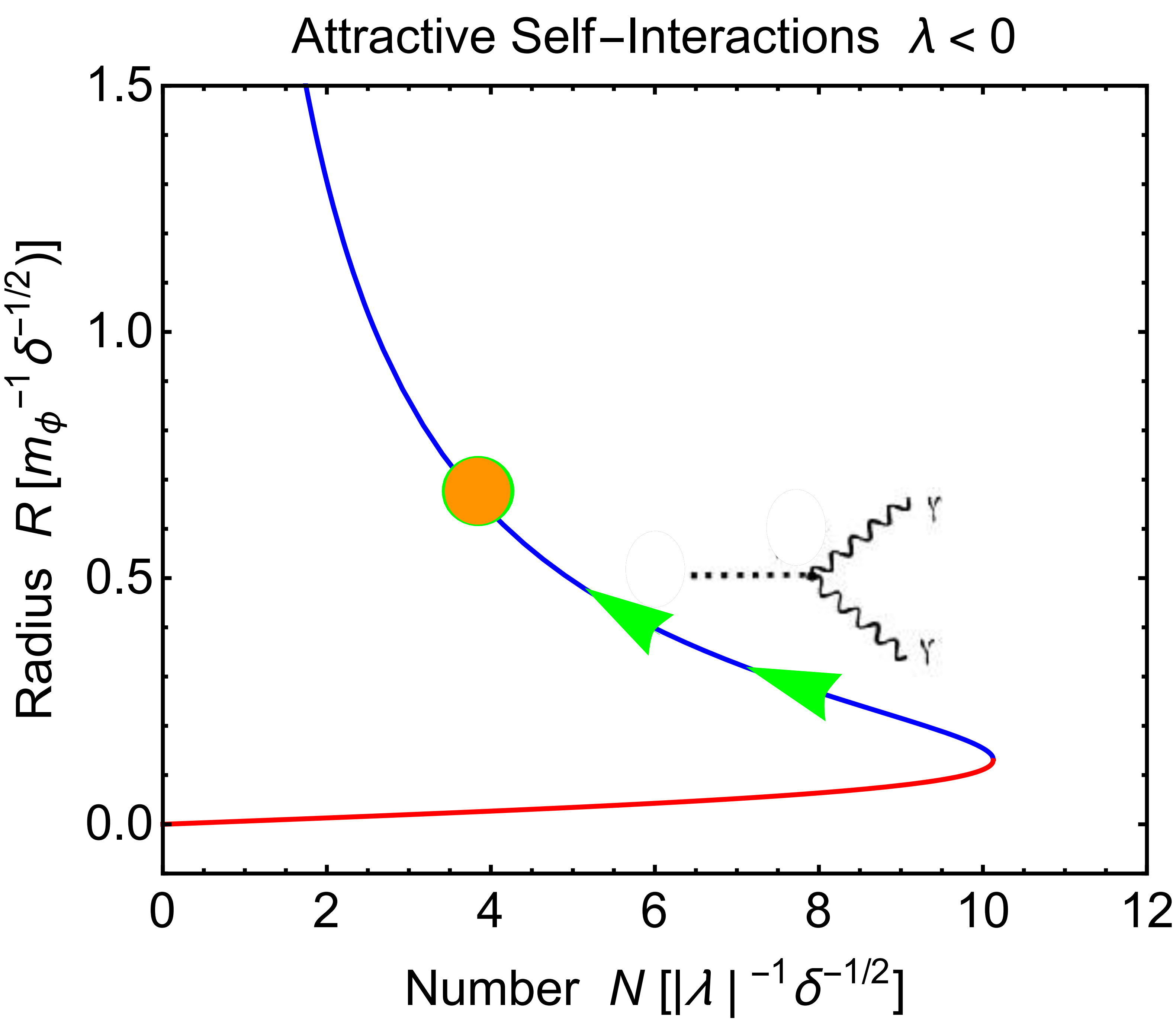
(e.g., Farina, Pappadopulo, Rompineve, Tesi 2016)

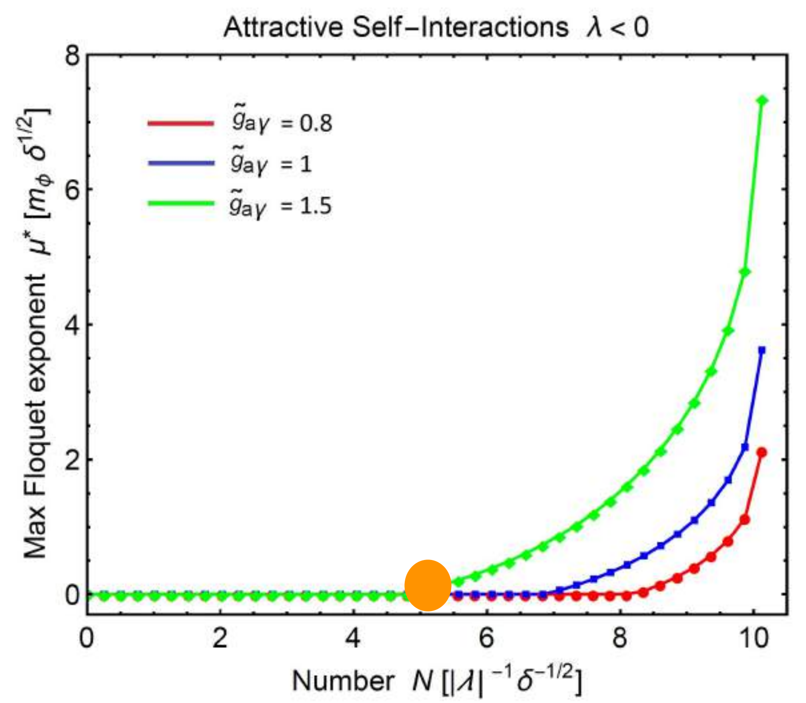
(e.g., Fan 2016)

Hertzberg, Schiappacasse 1805.00430 (JCAP)

Astrophysical Consequences

Lasing Stars Early Universe

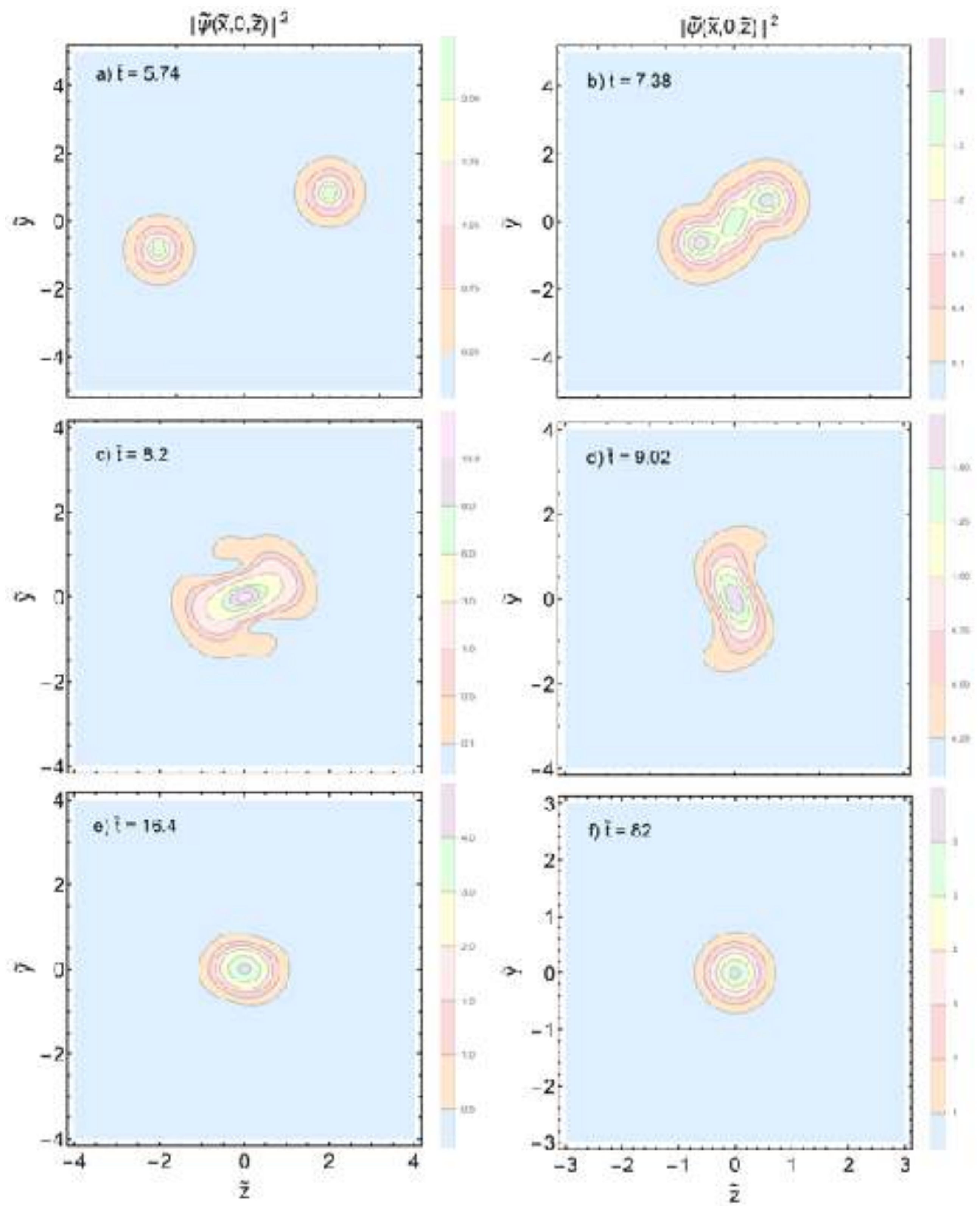




(i) Mass Pile Up

Hertzberg, Schiappacasse 1805.00430 (JCAP)

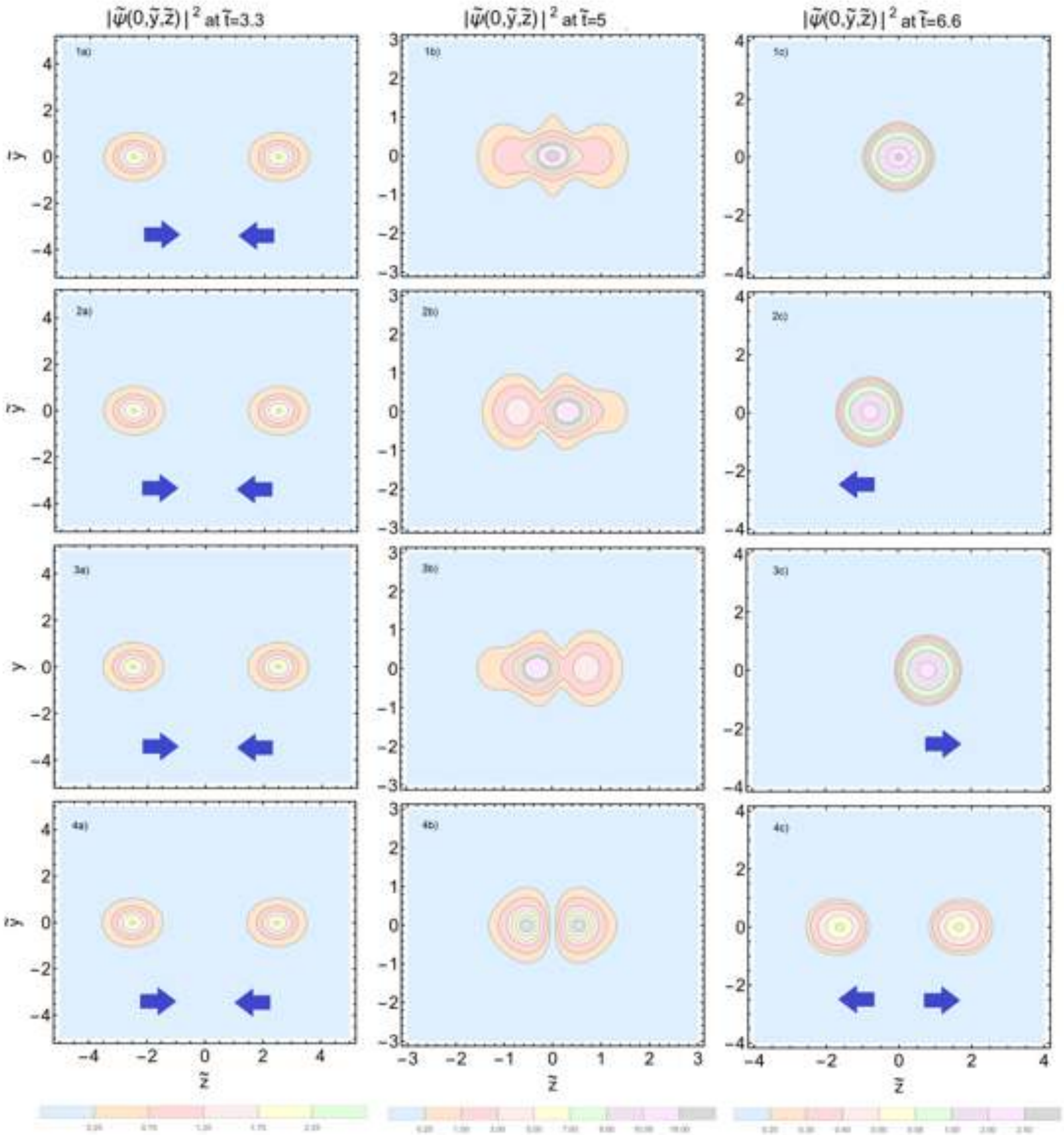
Axion Star Mergers



Hertzberg, Li, Schiappacasse 2005.02405 (JCAP)

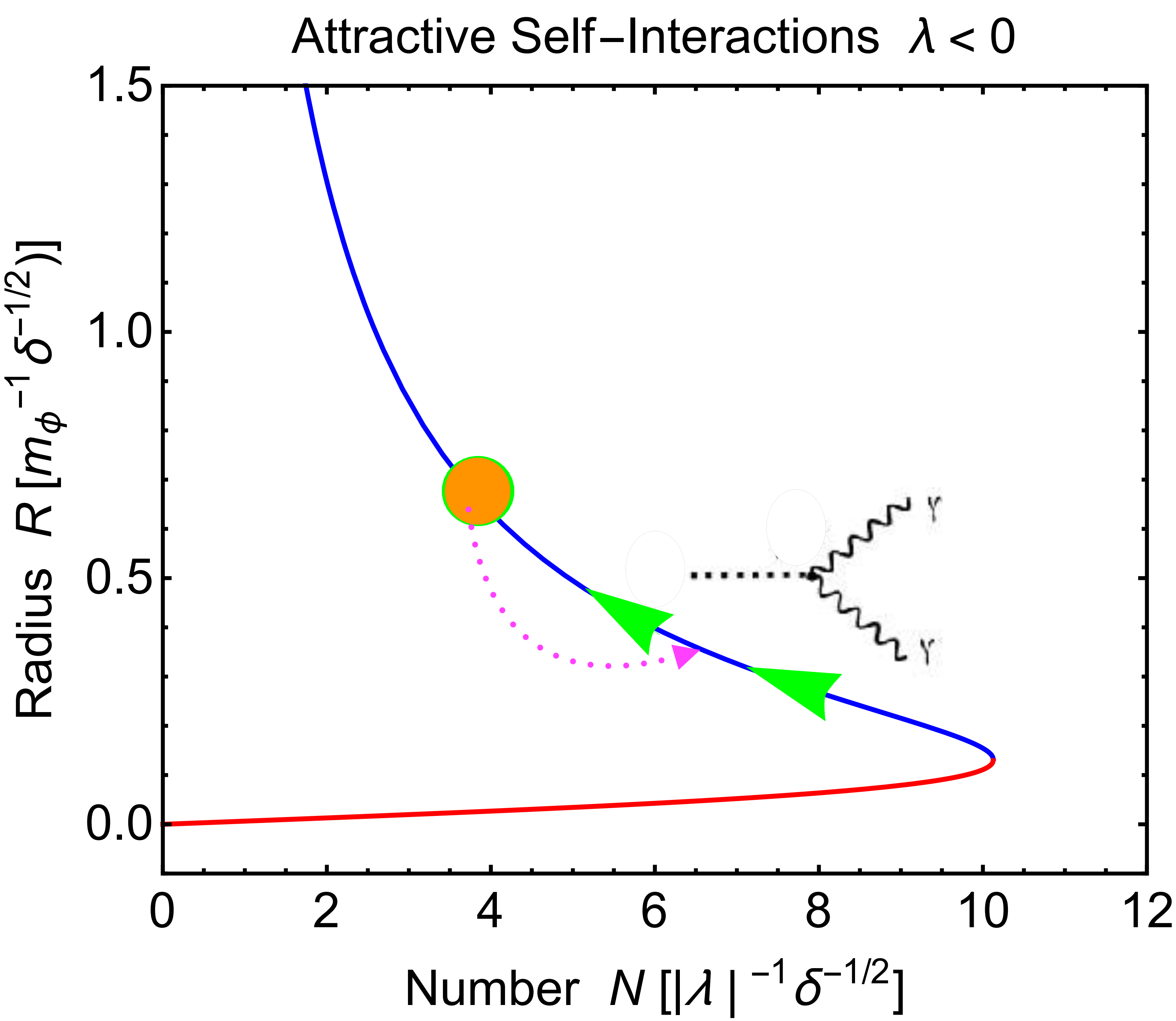
Axion Star Mergers

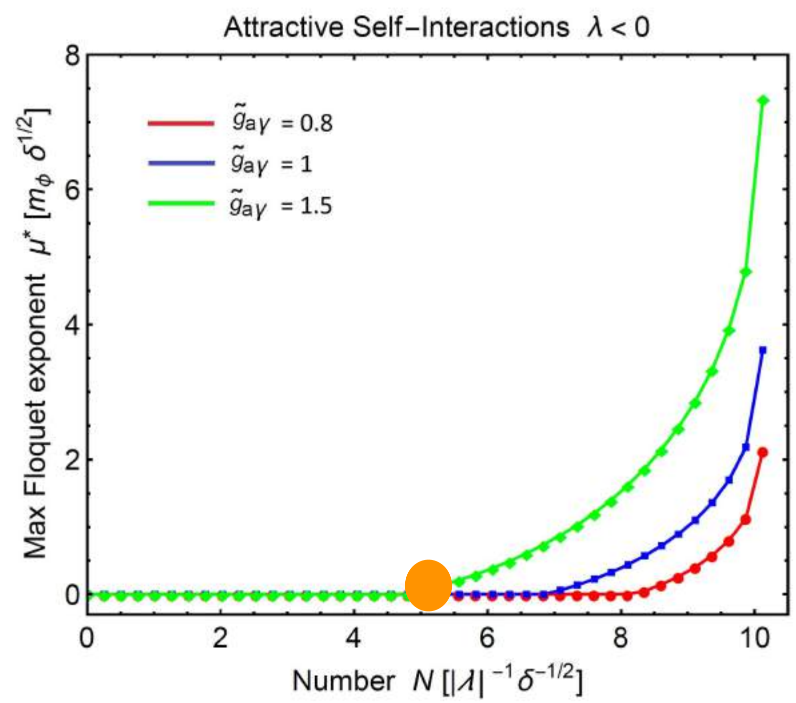
Phase dependence



Hertzberg, Li, Schiappacasse 2005.02405 (JCAP)

Lasing Stars Late Universe





(i) Mass Pile Up

(ii) Late Time Mergers;
Radio-wave Bursts

$$\lambda_{EM} = \frac{2\pi}{k} \approx \frac{4\pi}{m_a}$$
$$= \mathcal{O}(1)\, meters$$

Hertzberg, Li, Schiappacasse 2005.02405 (JCAP)

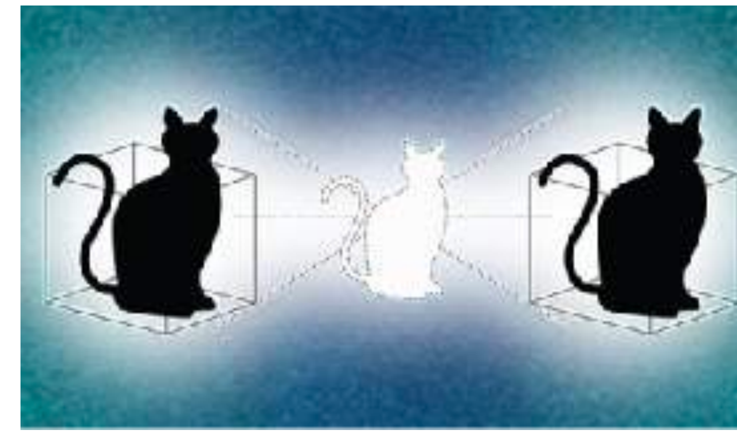
(also Tkachev 2015)

Part 4

Any Aspects of Light Scalar DM not captured by classical field theory?

Example: Quantum radiation (even at high occupancy) for appreciable self-couplings
Hertzberg 1003.3459 (PRD), Hertzberg, Rompineve, Yang 2010.07927 (PRD)

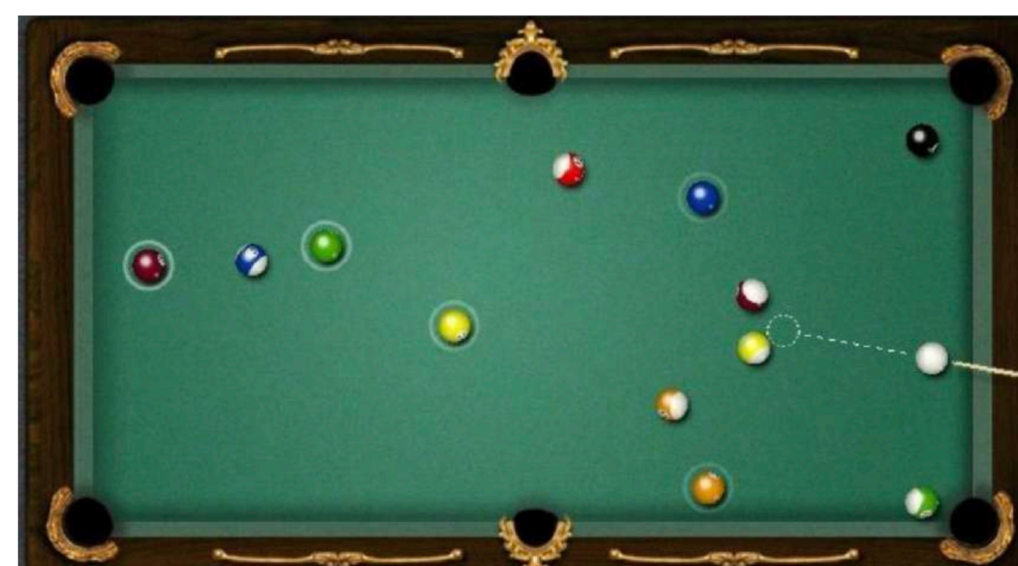
Recall; non-linear dynamics can launch states into Schrodinger cat-like states



Quantumness destroyed due to DECOHERENCE

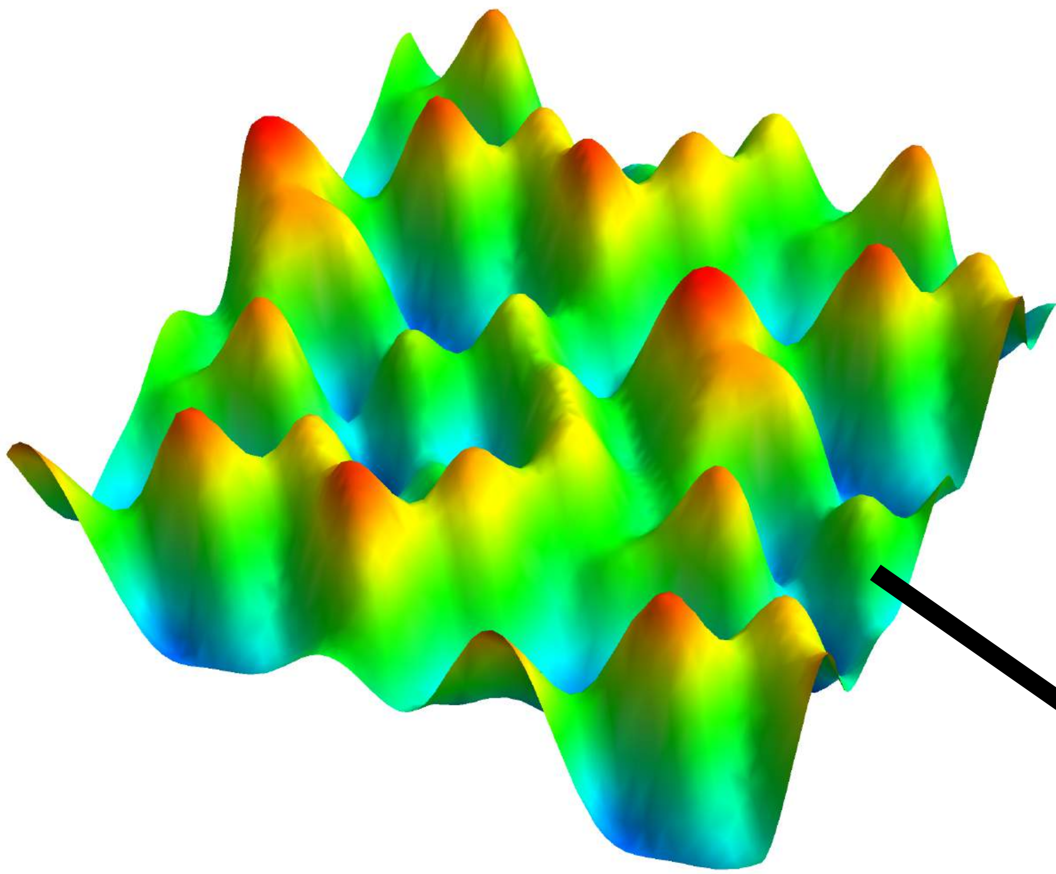
Schrodinger Cat Billiards

+

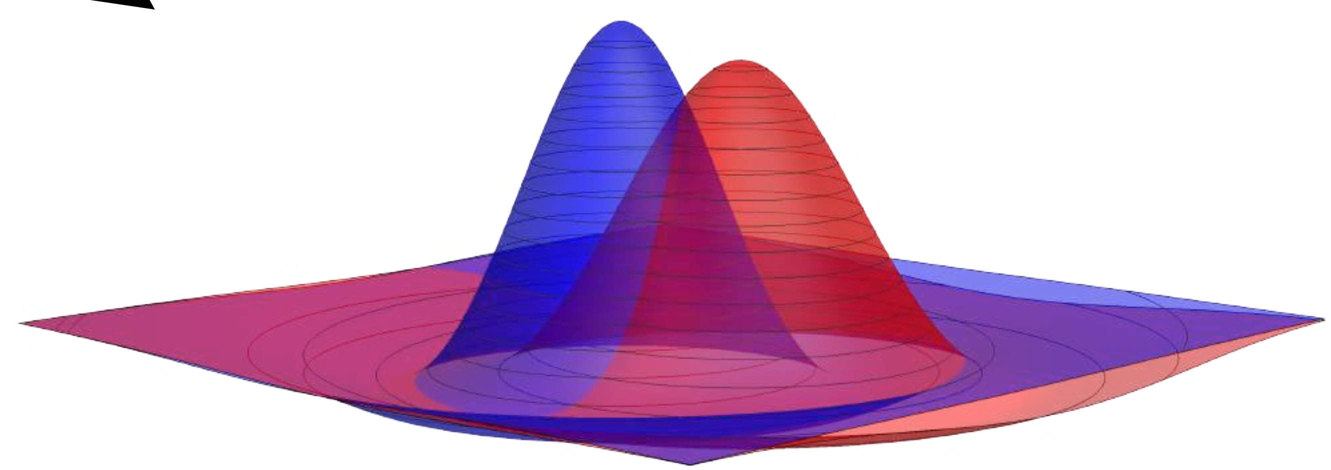


Albrecht, Phillips

Non-linear dynamics can launch states into Schrodinger cat-like states



Dark Matter Schrodinger Cat (Axions)

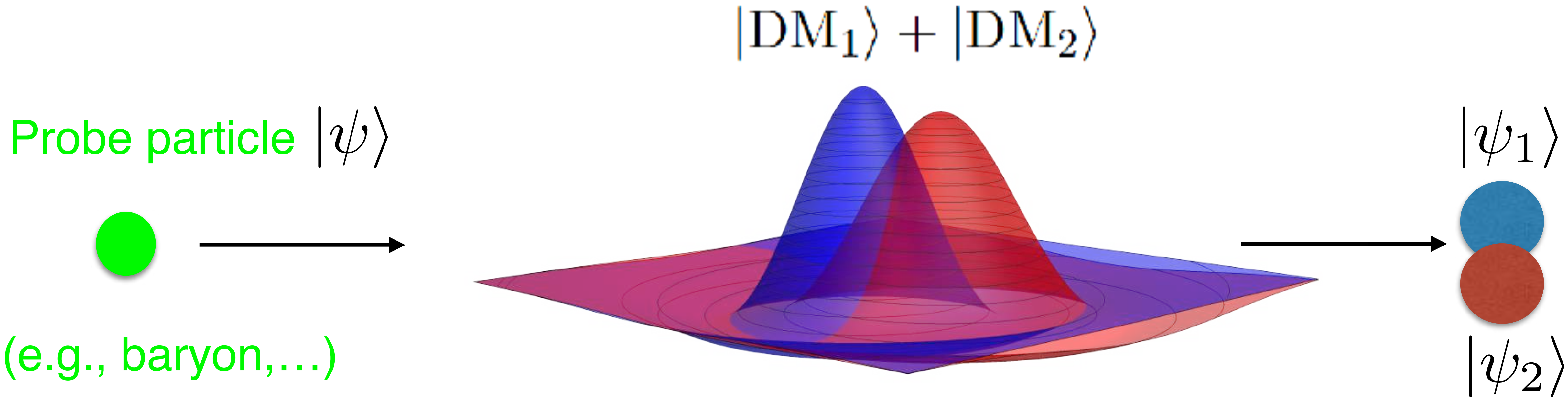


Quantumness destroyed due to DECOHERENCE???

Less clear because dark matter has tiny (non-gravitational) interactions

Could Dark Matter Schrodinger Cats Survive?

Entanglement from Gravitational Scattering



$$|\Psi_{\rm ini}\rangle = (|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle)\,|\psi\rangle \qquad |\Psi_{\rm fin}\rangle = |\mathrm{DM}_1\rangle\,|\psi_1\rangle + |\mathrm{DM}_2\rangle\,|\psi_2\rangle$$

Product State

Entangled State

Trace Out Probe Particle

$\hat{\rho} \equiv |\Psi\rangle \langle\Psi|$ Full Density Matrix

$\hat{\rho}_{\mathrm{red}} = \mathrm{Tr}_{|\psi\rangle}[\hat{\rho}]$ Reduced Density Matrix

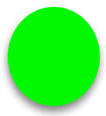
$$= |\mathrm{DM}_1\rangle \langle\mathrm{DM}_1| + \langle\psi_2|\psi_1\rangle |\mathrm{DM}_1\rangle \langle\mathrm{DM}_2| + \langle\psi_1|\psi_2\rangle |\mathrm{DM}_2\rangle \langle\mathrm{DM}_1| + |\mathrm{DM}_2\rangle \langle\mathrm{DM}_2|$$

Off diagonal elements;
controlling true quantum effects

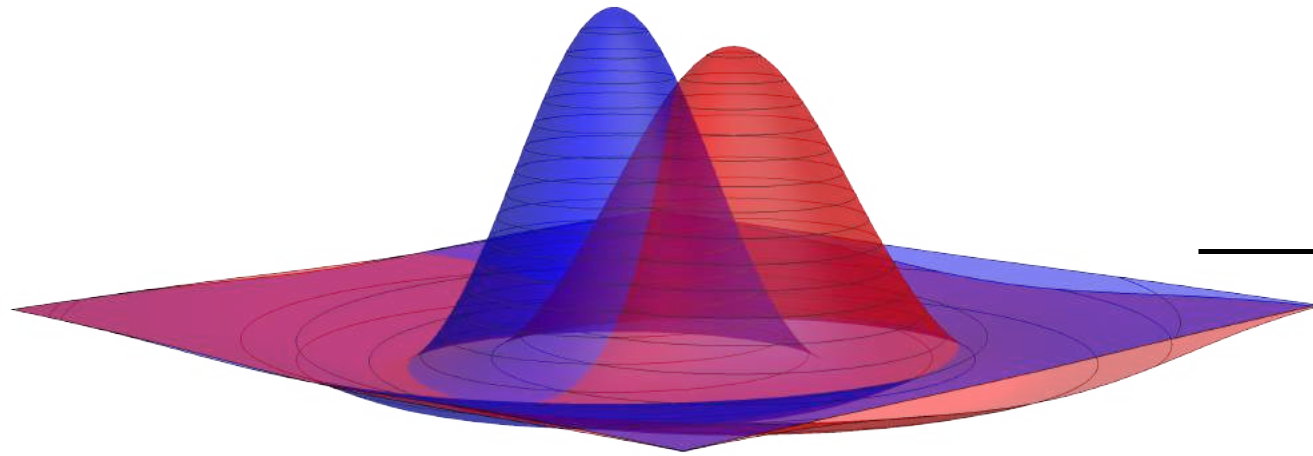
Overlap of Probe Particle States

$$|\mathrm{DM}_1\rangle + |\mathrm{DM}_2\rangle$$

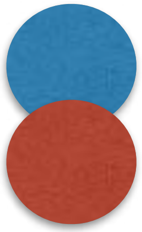
Probe particle $|\psi\rangle$



(e.g., baryon,…)



$|\psi_1\rangle$



$|\psi_2\rangle$

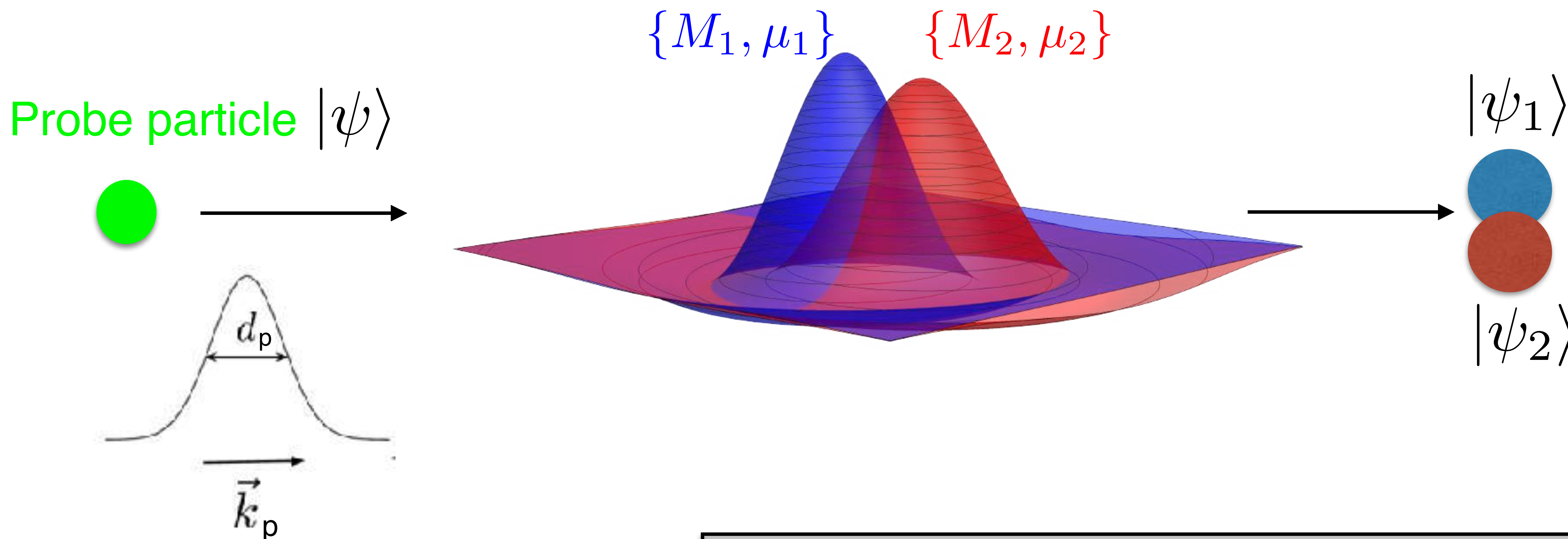
$$|\psi_i\rangle = |\mathrm{uns}\rangle + |\mathrm{sc}_i\rangle$$

$$|\langle\psi_1|\psi_2\rangle|^2 = 1 + 2\Big(\langle sc_1|sc_2\rangle_{\mathrm{R}} - \frac{1}{2}\Big\{\langle sc_1|sc_1\rangle + \langle sc_2|sc_2\rangle\Big\}\Big) + \Big(\langle sc_1|uns\rangle_{\mathrm{I}} + \langle uns|sc_2\rangle_{\mathrm{I}}\Big)^2 + O(G^3)$$

Allali, Hertzberg 2005.12287 (JCAP)

Result for Overlap of Probe Particle States

We evolve a Gaussian wave packet using perturbation theory



$$|\langle\psi_1|\psi_2\rangle|^2 = 1 - 2\Delta$$

$$\Delta_0 = \frac{2G^2m^4}{\hbar^4 k^2 d^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1 M_2}{\mu_1\mu_2}\chi_{12}\right]$$

Allali, Hertzberg 2005.12287 (JCAP)

Decoherence Rate from N-Probe Particles

Off diagonal element of density matrix

$$\prod_{n=1}^{N} |\langle\psi_1|\psi_2\rangle|_n = \prod_{n=1}^{N}(1-\Delta_b) \sim e^{-\sum_{n=1}^{N}\Delta_b}$$

Decoherence rate

$$\Gamma_{\rm dec} = n\, v \int d^2b\, \Delta_b$$

$$\Gamma_{\rm dec} = \frac{4\pi G^2 m^4 n\, v}{\hbar^4 k^2}\left[\frac{M_1^2}{\mu_1^2}\chi_{11} + \frac{M_2^2}{\mu_2^2}\chi_{22} - 2\frac{M_1 M_2}{\mu_1\mu_2}\chi_{12}\right]$$

Allali, Hertzberg 2005.12287 (JCAP)

Application to Light Diffuse DM (axions)

Diffuse scalars (axions)

$$\mu_i \sim \frac{1}{\lambda_{dB,a}} \sim \frac{2\pi}{m_a v_{vir}} \qquad M_i \sim \frac{4\pi}{3}\rho_{DM}\lambda_{dB,a}^3$$

Probe: Diffuse baryons

$$k_p \sim m_p \, v_{vir}$$

Decoherence Rate

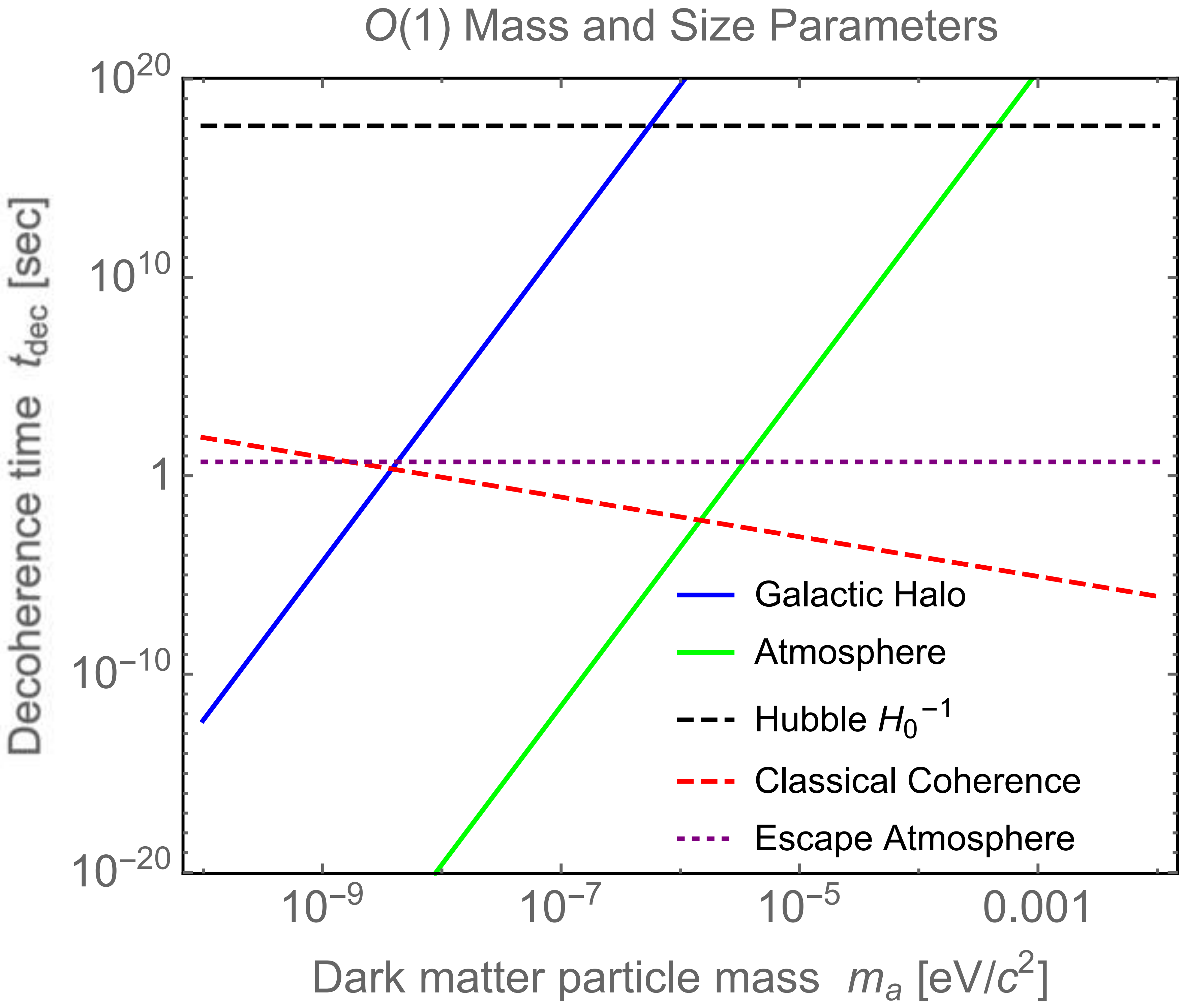
$$\boxed{\Gamma_{dec} \sim \frac{G^2\, m_p\, \rho_b\, \rho_{DM}^2}{m_a^8\, v_{vir}^9}}$$

Decoherence Time

$$t_{dec} = 1/\Gamma_{dec} \propto m_a^8$$

Allali, Hertzberg 2005.12287 (JCAP)

Application to Diffuse DM (Axions)



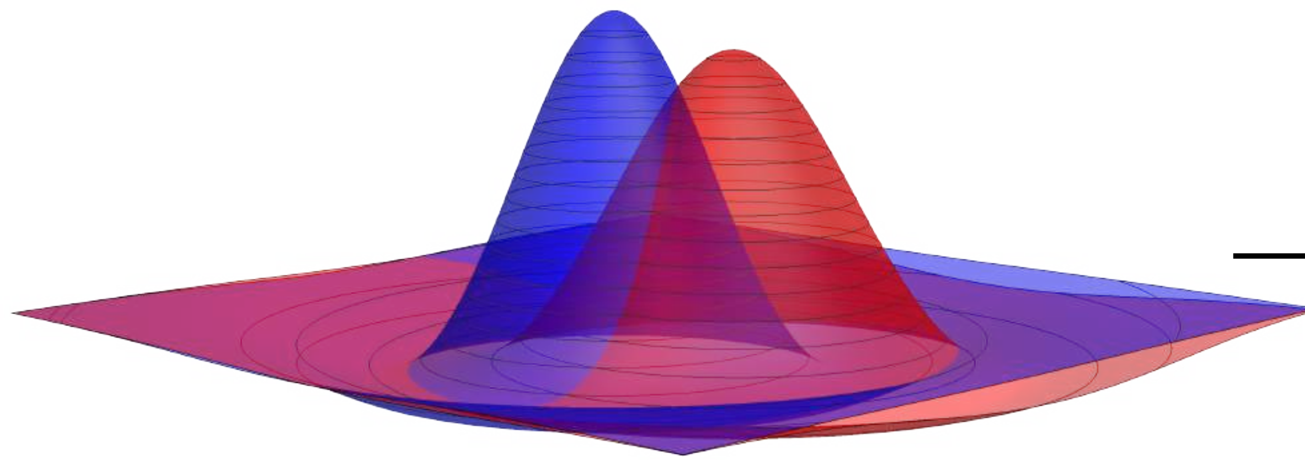
Allali, Hertzberg 2005.12287 (JCAP)

Application to Boson Stars

Boson star

Probe particle $|\psi\rangle$

(e.g., baryon,…)



$|\psi_1\rangle$

$|\psi_2\rangle$

Boson stars are much denser

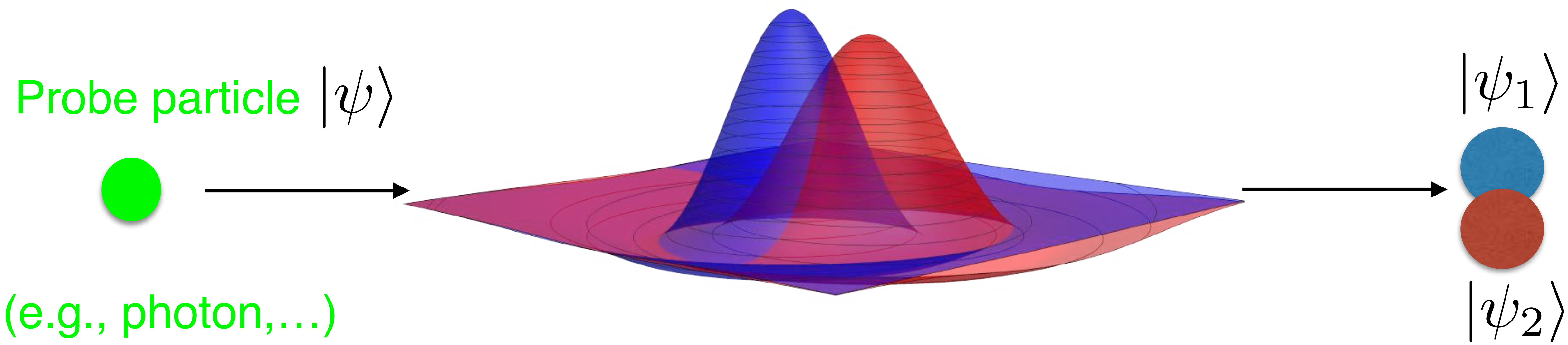
Decoherence Rate

$$\Gamma_{\rm dec} \gtrsim \frac{\hbar^2 m_p \rho_p}{v_p m_a^4} \sim 10^{21}\,\mathrm{sec}^{-1}\left(\frac{1\,\mathrm{eV}}{m_a\, c^2}\right)^4$$

Extremely rapid decoherence —> Very classical

Allali, Hertzberg 2005.12287 (JCAP)

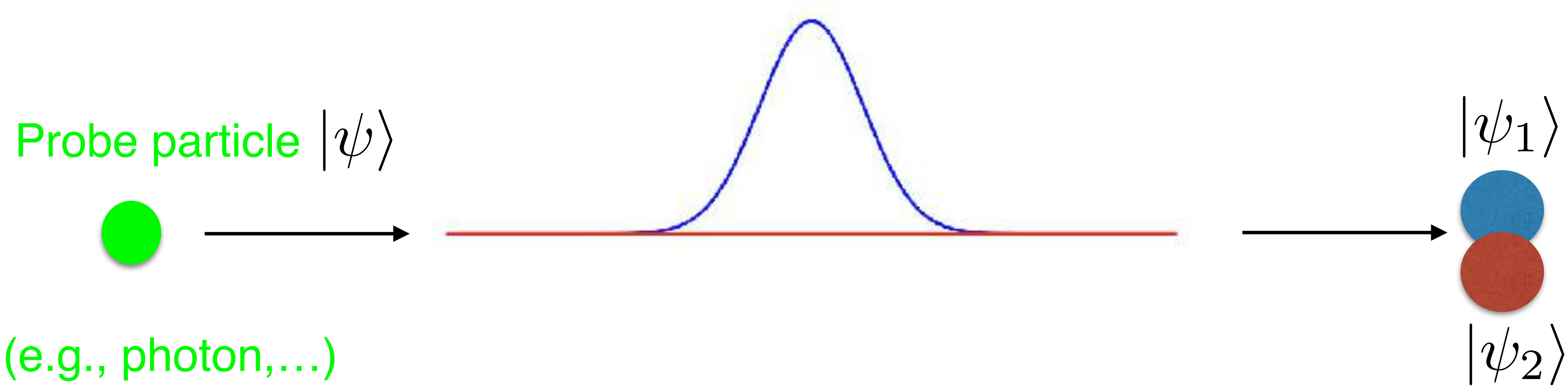
General Relativistic Extension



Rigorous quantum gravity calculation — General Relativity is a well behaved effective theory

Allali, Hertzberg 2012.12903 (PRD), 2103.15892 (PRL)

General Relativistic Extension



Rigorous quantum gravity calculation — General Relativity is a well behaved effective theory

Allali, Hertzberg 2012.12903 (PRD), 2103.15892 (PRL)

General Relativistic Extension

Starting from QFT, can derive RSE for 1-particle sub-space: (ignoring spin)

$$(i\partial_t - \sqrt{-\nabla^2 + m^2})\psi(\mathbf{x},t) = \left(\Phi(\mathbf{x},t)\sqrt{-\nabla^2+m^2} - \frac{\Psi(\mathbf{x},t)\nabla^2}{\sqrt{-\nabla^2+m^2}}\right)\psi(\mathbf{x},t)$$

Decoherence Rate
for superposition of
different phases

$$\Gamma_{dec} \propto \exp\left[-\frac{m_a^2 E_p^2}{\mu^2 k^2}\right] \sim \exp\left[-\frac{1}{v_a^2 v_p^2}\right]$$

Exponentially suppressed for non-relativistic DM or probes

So the phase is rather robust against decoherence - may be relevant to direct detection

$$|\mathrm{DM}\rangle \sim \sum_i c_i |\cos(\omega t - \mathbf{k}_a \cdot \mathbf{x} + \varphi_i)\rangle$$

(Although may decohere near black hole horizons)

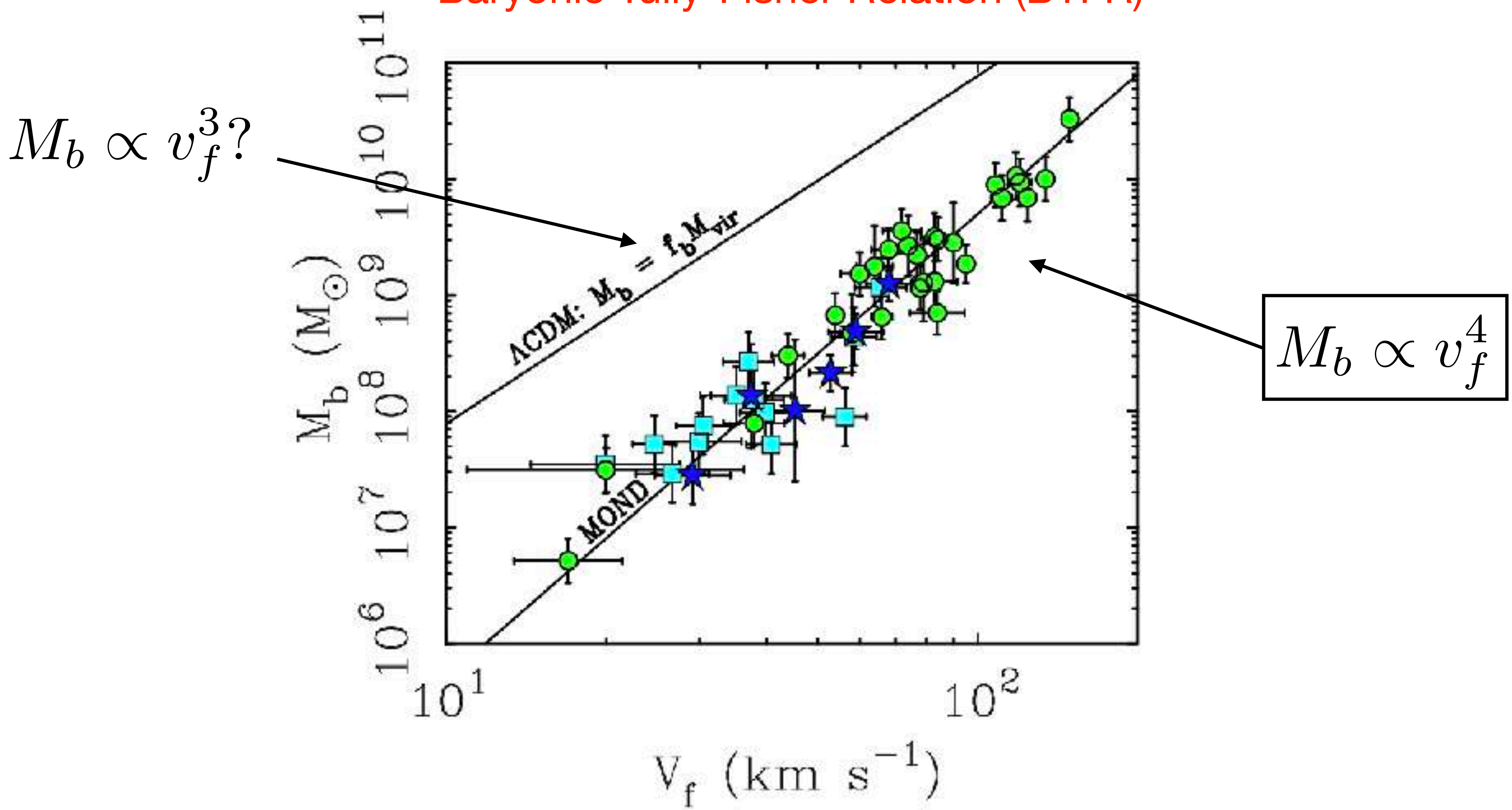
Allali, Hertzberg 2012.12903 (PRD), 2103.15892 (PRL)

Part 5

Light Scalars Producing Novel Galactic Behavior?

Possible Difficulties with CDM on Galactic Scales?

Baryonic Tully-Fisher Relation (BTFR)



(from McGaugh 2011)

Modify Gravity on Galactic Scales (MOND)?

$$a \propto \frac{M_{enc}}{R^2}$$

If instead: $\frac{v^2}{R} = a \propto \sqrt{\frac{M_{enc}}{R^2}} \Longrightarrow \boxed{M_b \propto v_f^4}$

(Milgrom)

Implementing this is very difficult:

The unique, causal, Lorentz invariant, theory of massless spin 2 particles, at large distances, is general relativity

(Feynman, Weinberg, Deser,…)

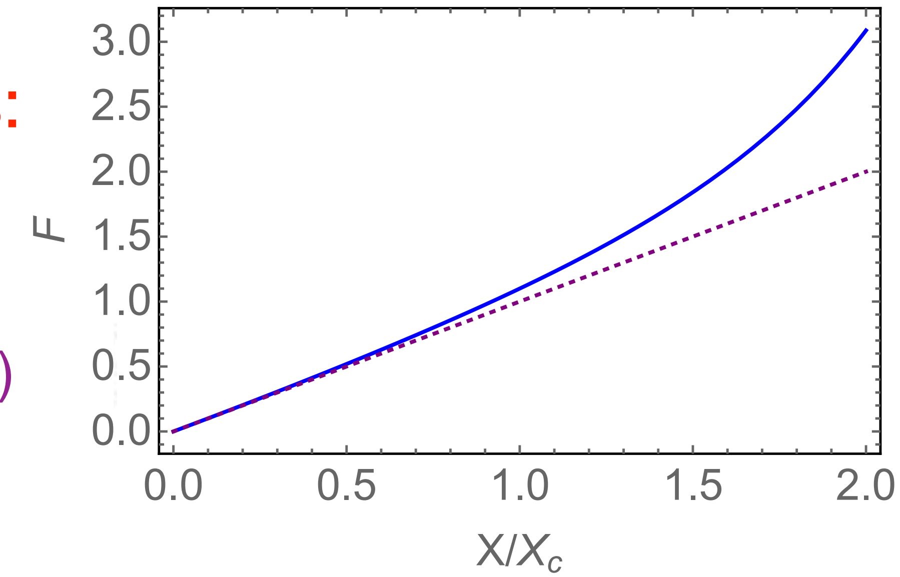
A possibility: we can add new degrees of freedom. In particular, new scalars could mediate a new long range (peculiar) interaction

Simplistic Attempt

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$S = -\int d^4x \sqrt{-g}\, [F(X,\varphi) - \tilde{\beta}\, \varphi\, T_{\rm B} + \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM}]$$

Introduce function *F* with 2 different asymptotic regimes:



Low densities/large scales $F = X$ (canonical)

(3/2 scaling)

High densities/galactic scales $\boxed{F = \tilde{\alpha}\, X \sqrt{|X|}}$ Mediates a MOND-like force

Example: $$F = X\,(1 + \tilde{\alpha}^4\, X^2)^{1/4}$$

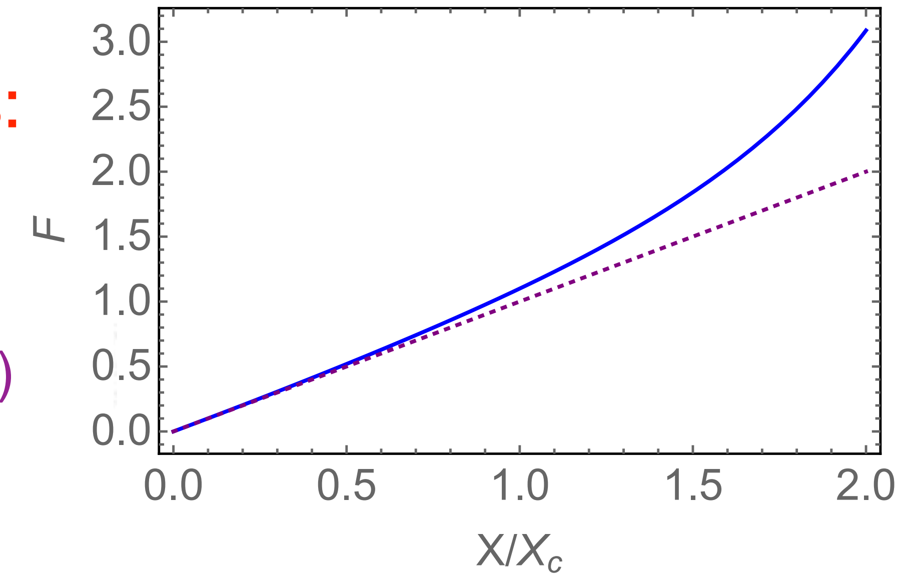
Large φ, can stay within regime of EFT

Simplistic Attempt

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$S = -\int d^4x \sqrt{-g} \left[F(X, \varphi) - \tilde{\beta}\, \varphi\, T_{\rm B} + \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM} \right]$$

Introduce function *F* with 2 different asymptotic regimes:



Low densities/large scales $F = X$ (canonical)

(3/2 scaling)

High densities/galactic scales $\boxed{F = \tilde{\alpha}\, X \sqrt{|X|}}$ Mediates a MOND-like force

$$-\frac{3\tilde{\alpha}}{2^{3/2}} \nabla \cdot (\nabla\varphi |\nabla\varphi|) = \tilde{\beta}\, T_B \qquad \mathbf{a} \propto -\mathrm{sign}(\tilde{\alpha}) \sqrt{\frac{M_{enc}}{R^2}}\, \hat{r}$$

Simplistic Attempt

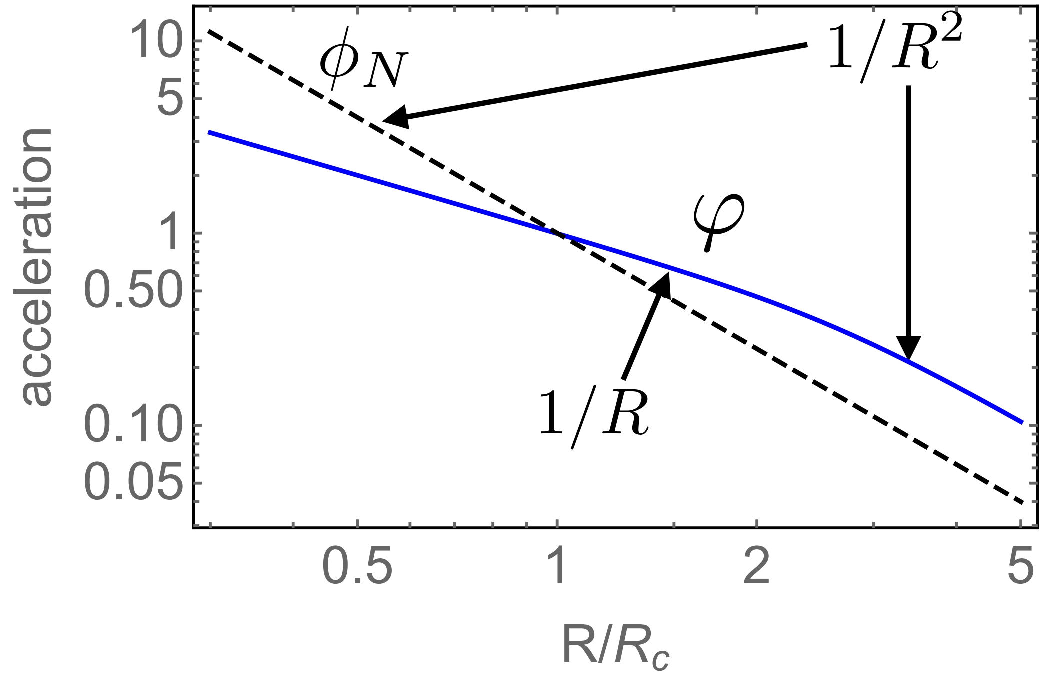
Two Problems

$$F = \tilde{\alpha}\, X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are superluminal (related details later)

$$F = X$$

Phenomenological: Although the scalar becomes canonical at large scales, it introduces another $1/r^2$ force. So it is difficult to consistently obtain the desired galactic and large scale behaviors



Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter

$U(1)$ Symmetry

Φ

$$X = g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi^*$$

Example:

$$F = \frac{1}{2}\left(X + m^2|\Phi|^2\right) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6}\left(X + m^2|\Phi|^2\right)^3$$

Reproduces CDM on large scales

(Quantum, from particle point of view,
Classical, from field point of view)

Allows for phase transition to superfluid
at galactic densities

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Goldstone θ can act as long-ranged force mediator

Berezhiani, Khoury 2015

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

$$\Phi = \rho\, e^{i(\theta + mt)}$$

Slowly varying phase θ and modulus ρ around superfluid condensate

$$X + m^2|\Phi|^2 = (\nabla\rho)^2 - 2\,m\,\rho^2\,Y \qquad \text{with} \qquad Y \equiv \dot{\theta} - m\,\phi_N - \frac{(\nabla\theta)^2}{2m}$$

At tree-level, can integrate out heavy modulus (Higgs mode)

$$\rho^2 = \Lambda\sqrt{2m|Y|}$$

(3/2 scaling)

Find low energy effective action for Goldstone is

$$F_{\text{eff}} = -\frac{2\Lambda(2m)^{3/2}}{3} Y\sqrt{|Y|}$$

By coupling to baryons, can mediate MOND-like force — reproduce BTFR, and CDM on large scales

Berezhiani, Khoury 2015

Analysis of High Energy Perturbations ε_j

Decompose into components $\Phi = (\phi_1 + i\,\phi_2)/\sqrt{2}$

Expand around superfluid $\phi_j = \phi_j^b + \varepsilon_j$ $\quad (j = 1, 2)$ $\quad \left(F' \equiv \frac{\partial F}{\partial X}\right)$

Linear equation of motion for high energy perturbations

$$\sum_{j=1}^{2} \left[F' \eta^{\mu\nu} \delta^{ij} + F'' \partial^\mu \phi_i^b \partial^\nu \phi_j^b\right] \partial_\mu \partial_\nu \varepsilon_j = 0$$

Diagonalize to obtain Higgs normal mode perturbations and associated effective metric

$$\psi = \partial^\mu \phi_1^b \partial_\mu \varepsilon_1 + \partial^\mu \phi_2^b \partial_\mu \varepsilon_2$$

$$G_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0$$

$$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

Hertzberg, Litterer, Shah 2105.02241 (JCAP)

Causal Propagation?

Obtain eigenvalues of effective metric $$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

Conditions for hyperbolicity

$$(A) \quad A \equiv F' > 0$$

$$(B) \quad B \equiv F' + 2XF'' > 0$$

(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;…)

Condition for subluminality

$$(C) \quad C \equiv -F'' \geq 0$$

Evaluate in SFDM model $A > 0$ $$B = \frac{4\,m^3\,\Lambda^4\,Y}{\rho^8} \qquad C = \frac{2\,m\,\Lambda^4\,Y}{\rho^{10}}$$

MOND regime $$Y \approx -\frac{(\nabla\theta)^2}{2m} \implies B < 0 \text{ (ghost-like)} \quad \text{and} \quad C < 0$$

Hertzberg, Litterer, Shah 2105.02241 (JCAP)

Causal Propagation? - General Analysis

Obtain eigenvalues of effective metric $$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F''(\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

Conditions for hyperbolicity

$$(A) \quad A \equiv F' > 0$$

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(Aharanov, Komar, Susskind; Wald; Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi; Bruneton;...)

Condition for subluminality

$$(C) \quad C \equiv -F'' \geq 0$$

Most general form

$$F = (X + m^2|\Phi|^2) \sum_{n=0} g_n \frac{\Lambda^{2n} \left(X + m^2|\Phi|^2\right)^n}{(\Lambda_c^2 + |\Phi|^2)^{3n}}$$

We proved that when g_n allow MOND regime $(\tilde{\alpha} > 0)$ $\Longrightarrow$ $B < 0$ (ghost-like) and $C < 0$

Hertzberg, Litterer, Shah 2105.02241 (JCAP)

Conclusions

1. Quantum correlation functions often do not follow single classical field solution. But I showed it is well modeled by classical ensemble average, or volume integral

2. Condensate are localized clumps. They have been suggested to explain the cores at centers of galaxies. But we showed this does not fit the pattern across galaxies

3. Pseudo-scalar (axion) clumps couple to photons. Parametric resonance is possible for couplings above our derived bound from occasional merger due to scalar emission

4. Macroscopic quantum states (Schrodinger cats) of light scalar dark matter might exist, and would be potentially robust against decoherence.

We found superposed spatial profiles decohere rapidly for light DM and boson stars, superposed phases live exponentially long & may launch detectors into superpositions

5. Superfluid Dark Matter is a novel way (only known?) to obtain success of CDM on large scales and success of MOND on galactic scales.

We studied a general class of models, and proved that high energy perturbations always violate hyperbolicity — ghost like behavior — in MOND regime, while intermediate regions can exhibit forms of superluminality

Thank you