Wave dark matter

Lam Hui
Columbia University

Collaboration with:
Bovy, Bryan, Dalal, Joyce, Kabat, Landry, Law, Li,
Ostriker, Santoni, Sun, Tomaselli, Tremaine, Trincherini,
Witten, Wong, Yavetz - esp. 1610.08297, 2004.01188

Review article on wave dark matter 2101.11735

arXiv: 2105.01069

Ladder Symmetries of Black Holes:

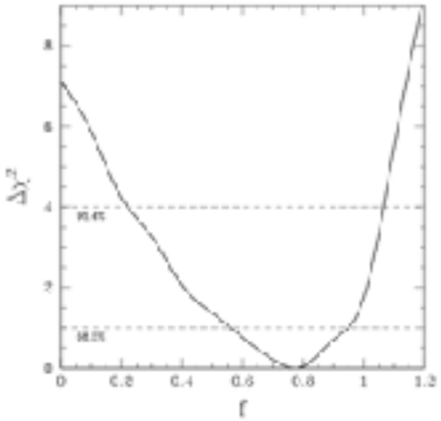
Implications for Love Numbers and No-Hair Theorems

with A. Joyce, R. Penco, L. Santoni, A. Solomon

Rich evidence for dark matter - from its gravitational effects

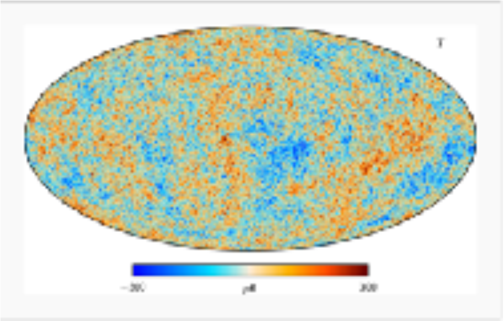
- Dynamical measurements.

- Gravitational lensing measurements.

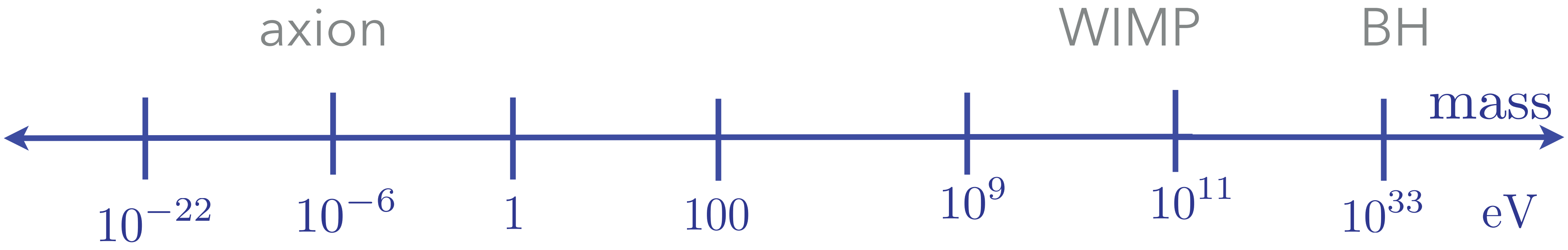


Hoekstra, Yee, Gladders

- Growth of perturbations.



We have rich evidence for the existence of DM, but remain ignorant about its basic properties e.g. mass:



axion

WIMP

BH

mass

10^{-22} 10^{-6} 1 100 10^{9} 10^{11} 10^{33} eV

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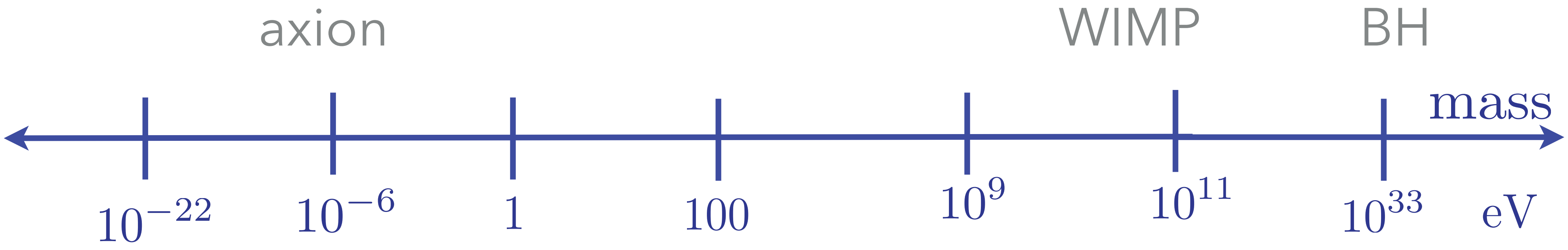

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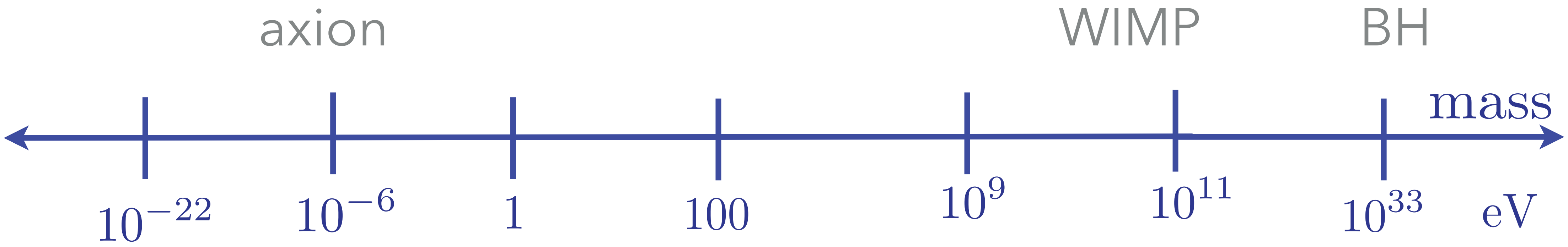

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Question: at what mass is the interparticle separation < de Broglie wavelength?
(1/mv)

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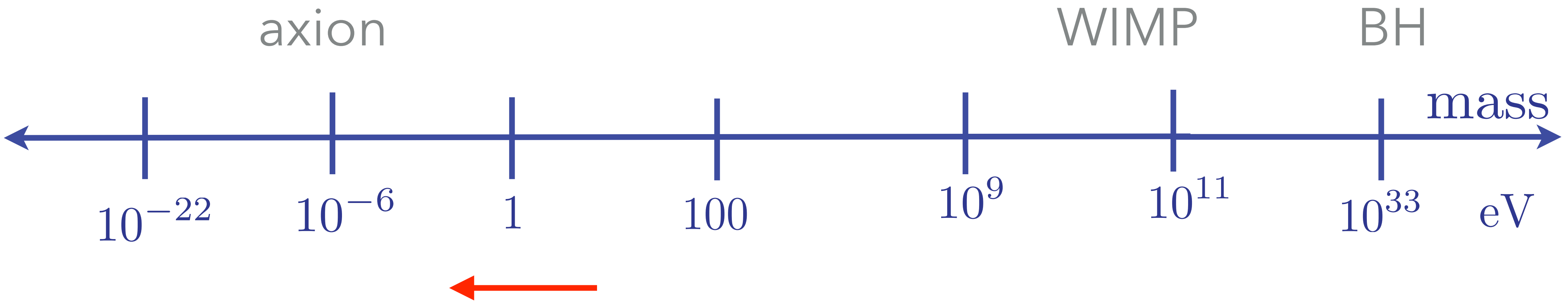

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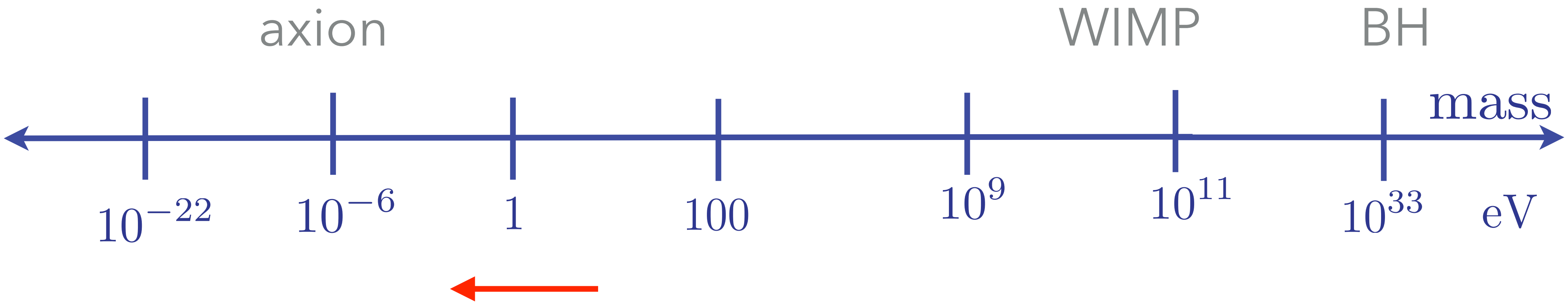

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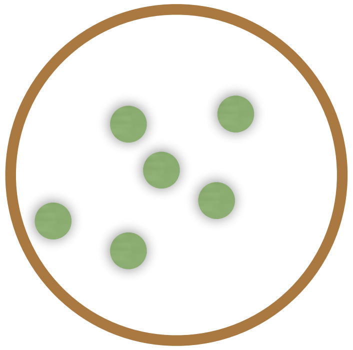
Question: at what mass is the interparticle separation < de Broglie wavelength?
(1/mv)

wave regime $m < 30\,\mathrm{eV}$

$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$

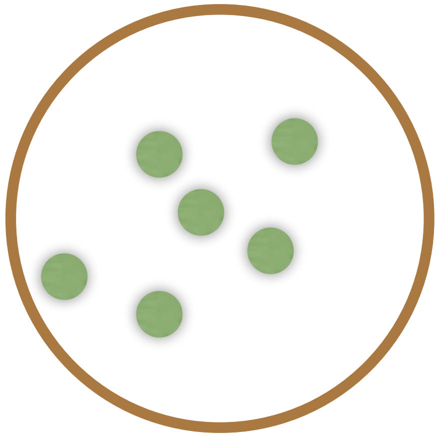
$$10^{4}\,\mathrm{cm} \quad \text{for} \quad m = 10^{-6}\,\mathrm{eV}$$

$$100\,\mathrm{pc} \quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}$$



bosonic

Let's discuss:



Particle physics motivations

Wave dynamics and phenomenology

Astrophysical implications (ultra-light DM)

Experimental implications (light DM)

$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$

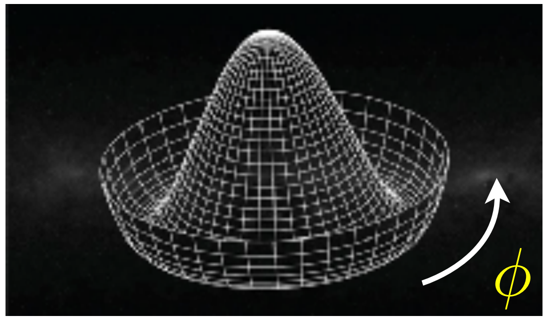
$$10^{4}\,\mathrm{cm} \quad \text{for} \quad m = 10^{-6}\,\mathrm{eV}$$ QCD axion

$$100\,\mathrm{pc} \quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}$$ Fuzzy DM (Hu, Barkana, Gruzinov)

Particle physics motivations

- A natural candidate for a light (scalar) particle is a pseudo-Nambu-Goldstone boson.

  A well known example is the QCD axion (Peccei, Quinn; Weinberg; Wilczek; Kim; Shifman, Vainshtein, Zakharov, Zhitnitsky; Dine, Fischler, Srednicki; Preskill, Wise, Wilczek; Abbott, Sikivie).

  There are also many axion-like-particles in string theory (Svrcek, Witten; Arvanitaki et al.)

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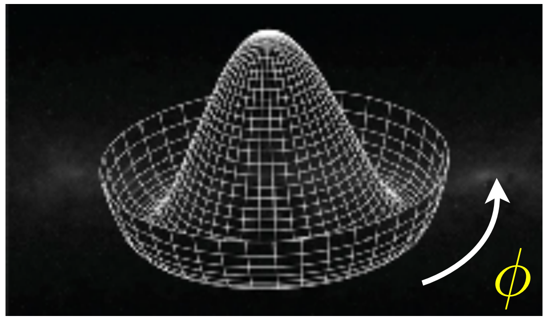

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Footnote on ultra -light version

mass m ← 10^{-22} eV →

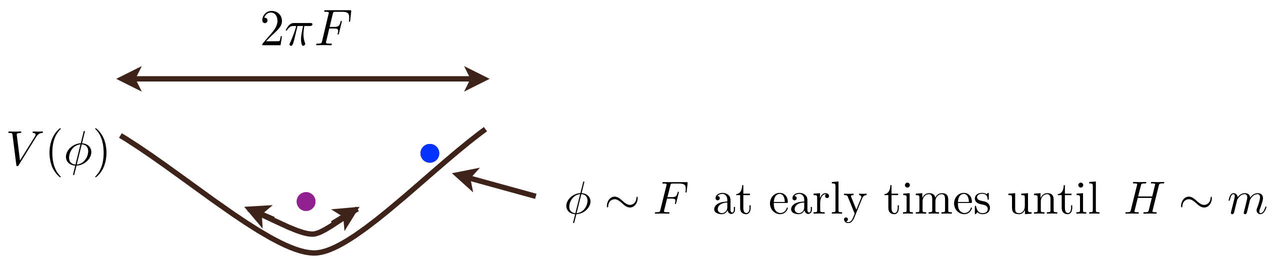
Fuzzy dark matter (FDM)
Hu, Barkana, Gruzinov
Amendola, Barbieri

- Consider an angular field (a pseudo Nambu-Goldstone) of periodicity $2\pi F$ i.e. an axion-like field with a potential from non-perturbative effects (not QCD axion).

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F]) \qquad m \sim \Lambda^2/F$$

(candidates: Arvanitaki et al.
Svrcek, Witten)

- Relic abundance matches dark matter abundance (mis-alignment mechanism).



$$\Omega_{\rm matter} \sim 0.1 \left(\frac{F}{10^{17}\,\rm GeV}\right)^2 \left(\frac{m}{10^{-22}\,\rm eV}\right)^{1/2}$$

(Preskill, Wise, Wilczek; Abbot, Sikivie; Dine, Fischler, with constant m)

Dynamics of wave dark matter:

- Ignoring self-interactions $\longrightarrow$ $-\Box\phi + m^2\phi = 0$ Klein Gordon equation

In the non-relativistic limit , useful to define: $\phi = \frac{1}{\sqrt{2m}}\left[\psi e^{-imt} + \psi^* e^{imt}\right]$

$$\ddot{\phi} \propto -m^2\psi e^{-imt} - im\dot{\psi}e^{-imt} + \cancel{\ddot{\psi}e^{-imt}} + \text{c. c.}$$

$$i\dot{\psi} = \left[-\frac{\nabla^2}{2m} + m\Phi_{\text{grav.}}\right]\psi$$

Schrodinger equation

- $\Phi_{\text{grav.}}$ is the gravitational potential, determined by:

$$\text{Poisson eq.}: \ \nabla^2\Phi_{\text{grav.}} = 4\pi G\rho = 4\pi G m|\psi|^2$$

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- An alternative viewpoint: ψ as a (classical) fluid. $\psi = \sqrt{\rho/m}\, e^{i\theta}$ i.e. $\rho = m\,|\psi|^2$

mass conservation $\dot{\rho} + \nabla\cdot\rho v = 0 \ \text{ where } \ v = \frac{1}{m}\nabla\theta$

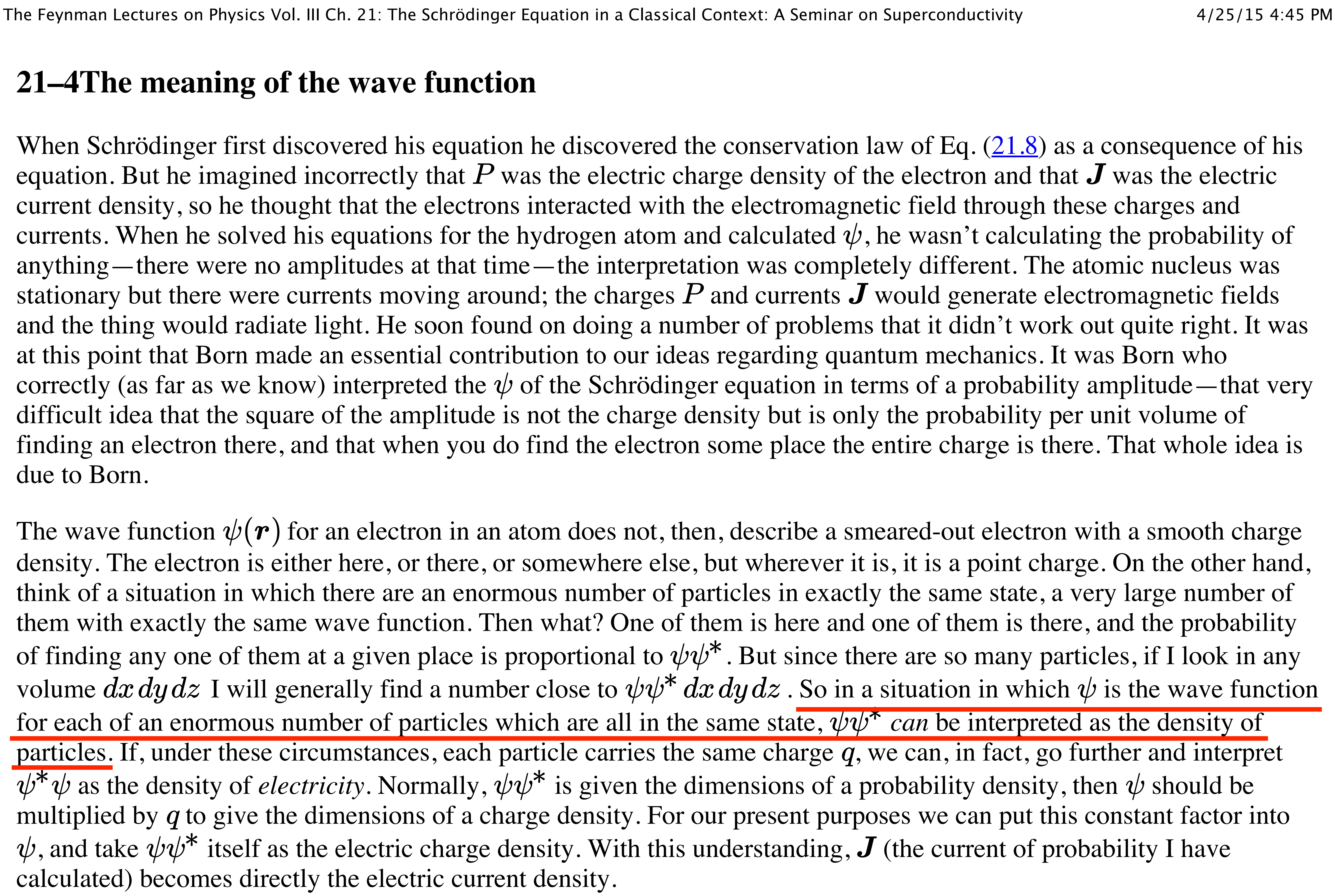
Euler equation $\dot{v} + v\cdot\nabla v = -\nabla\Phi_{\rm grav.} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$

superfluid

(see also Berezhiani, Khoury; Fan; Alexander, Cormack)

Feynman Lectures Vol. 3

Classical or quantum?
Think about number-phase commutator.



The Feynman Lectures on Physics Vol. III Ch. 21: The Schrödinger Equation in a Classical Context: A Seminar on Superconductivity 4/25/15 4:45 PM

21–4The meaning of the wave function

When Schrödinger first discovered his equation he discovered the conservation law of Eq. (21.8) as a consequence of his equation. But he imagined incorrectly that P was the electric charge density of the electron and that $\boldsymbol{J}$ was the electric current density, so he thought that the electrons interacted with the electromagnetic field through these charges and currents. When he solved his equations for the hydrogen atom and calculated ψ, he wasn't calculating the probability of anything—there were no amplitudes at that time—the interpretation was completely different. The atomic nucleus was stationary but there were currents moving around; the charges P and currents $\boldsymbol{J}$ would generate electromagnetic fields and the thing would radiate light. He soon found on doing a number of problems that it didn't work out quite right. It was at this point that Born made an essential contribution to our ideas regarding quantum mechanics. It was Born who correctly (as far as we know) interpreted the ψ of the Schrödinger equation in terms of a probability amplitude—that very difficult idea that the square of the amplitude is not the charge density but is only the probability per unit volume of finding an electron there, and that when you do find the electron some place the entire charge is there. That whole idea is due to Born.

The wave function $\psi(\boldsymbol{r})$ for an electron in an atom does not, then, describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge. On the other hand, think of a situation in which there are an enormous number of particles in exactly the same state, a very large number of them with exactly the same wave function. Then what? One of them is here and one of them is there, and the probability of finding any one of them at a given place is proportional to $\psi\psi^*$. But since there are so many particles, if I look in any volume $dx\,dy\,dz$ I will generally find a number close to $\psi\psi^*\,dx\,dy\,dz$. So in a situation in which ψ is the wave function for each of an enormous number of particles which are all in the same state, $\psi\psi^*$ *can* be interpreted as the density of particles. If, under these circumstances, each particle carries the same charge q, we can, in fact, go further and interpret $\psi^*\psi$ as the density of *electricity*. Normally, $\psi\psi^*$ is given the dimensions of a probability density, then ψ should be multiplied by q to give the dimensions of a charge density. For our present purposes we can put this constant factor into ψ, and take $\psi\psi^*$ itself as the electric charge density. With this understanding, $\boldsymbol{J}$ (the current of probability I have calculated) becomes directly the electric current density.

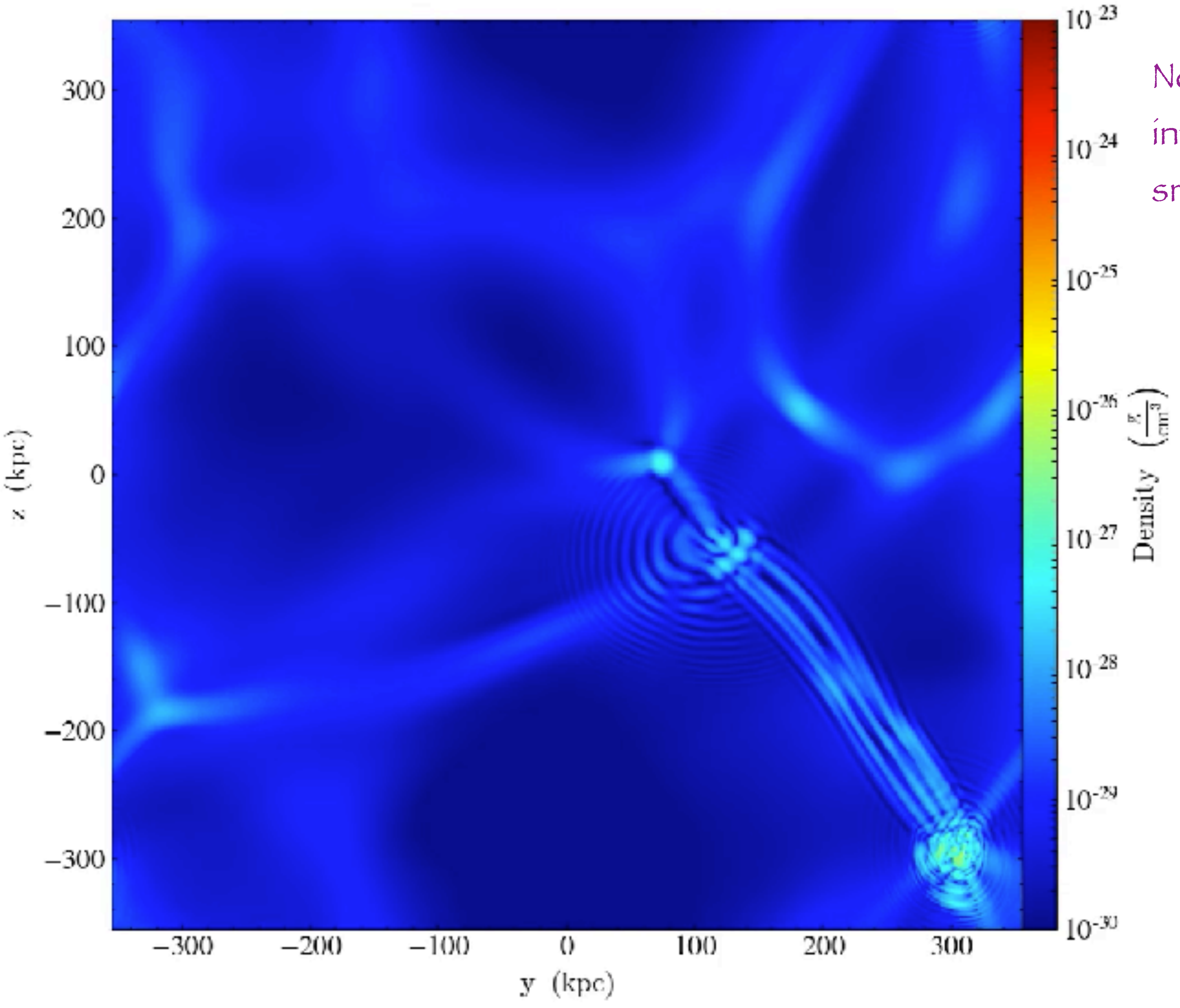
Long history of scalar field as dark matter:

Baldeschi, Ruffini, Gelmini; Turner; Press, Ryden, Spergel; Sin; Hu, Barkana, Gruzinov; Peebles; Goodman; Lesgourgues, Arbey, Salati; Amendola, Barbieri; Chavanis; Suarez, Matos; Matos, Guzman, Urena-Lopez …

Dark matter as superfluid:

Rindler-Daller, Shapiro; Berezhiani, Khoury; Fan; Alexander, Cormack; Alexander, Gleyzer, McDonough, Toomey; Ferreira, Franzmann, Khoury, Brandenberger …

Wave effects in a cosmological simulation



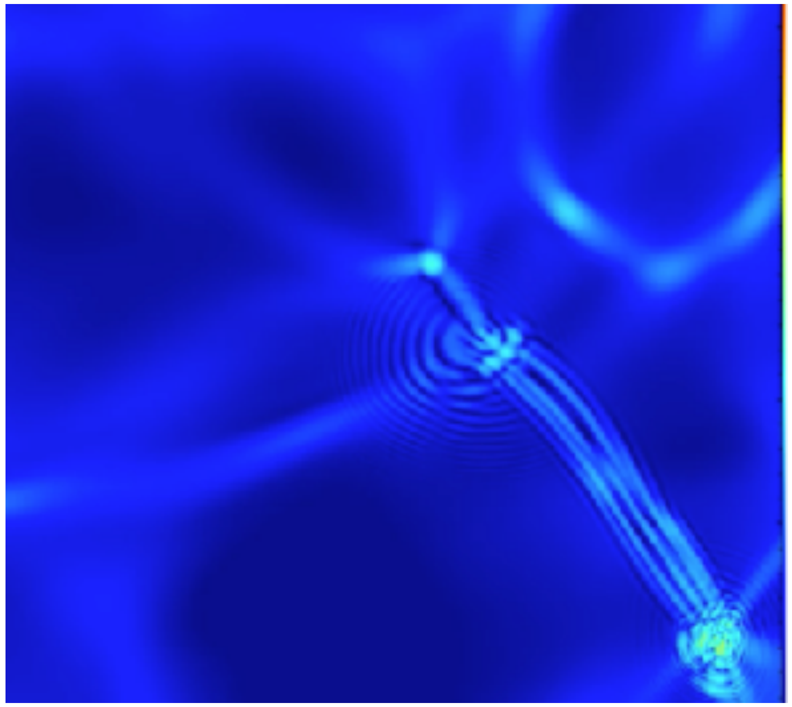
Note the irony: interference enhances small scale power.

Li, LH, Bryan

See Schive, Chiueh, Broadhurst; Veltmaat, Niemeyer; Schwabe, Niemeyer, Engels; Mocz et al.; Nori, Baldi; Kendall, Easther

Wave effects from light/ultra-light DM:

- Lyman-alpha forest
- solitonic halo core
- interference substructure
- vortices
- dynamical friction
- evaporation of sub-halos by tunneling
- direct detection
- detection by pulsar timing array
- gravitational lensing
- scattering of tidal streams
- soliton oscillations
- black hole hair



Vortices

- Consider again fluid formulation: $\psi = \sqrt{\rho/m}\, e^{i\theta}$

$$\dot{\rho} + \nabla \cdot \rho v = 0 \quad \text{where} \quad v = \frac{1}{m}\nabla\theta$$

$$\dot{v} + v \cdot \nabla v = -\nabla\Phi_{\text{grav.}} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- Naively, vorticity cannot exist, because the velocity field is a gradient flow. In addition, one might think Kelvin's theorem should hold i.e. no vorticity is generated if there's no vorticity to begin with.

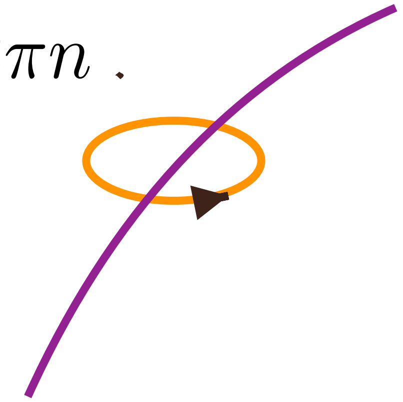
- The loophole: where $\rho = 0$. Note: such complete destructive interference can only occur in the late universe when O(1) fluctuations are present. No vortices in the early universe.

- See condensed matter literature.

Structure of a vortex

- Generically, in 3D, the set of points where both the real & the imaginary parts of the wavefunction vanish fall on a line i.e. a line/string defect.

- The phase of the wavefunction must wraps around the line by $2\pi n$.
Thus, a vortex: $\oint \vec{v} \cdot d\vec{\ell} = 2\pi n/m$

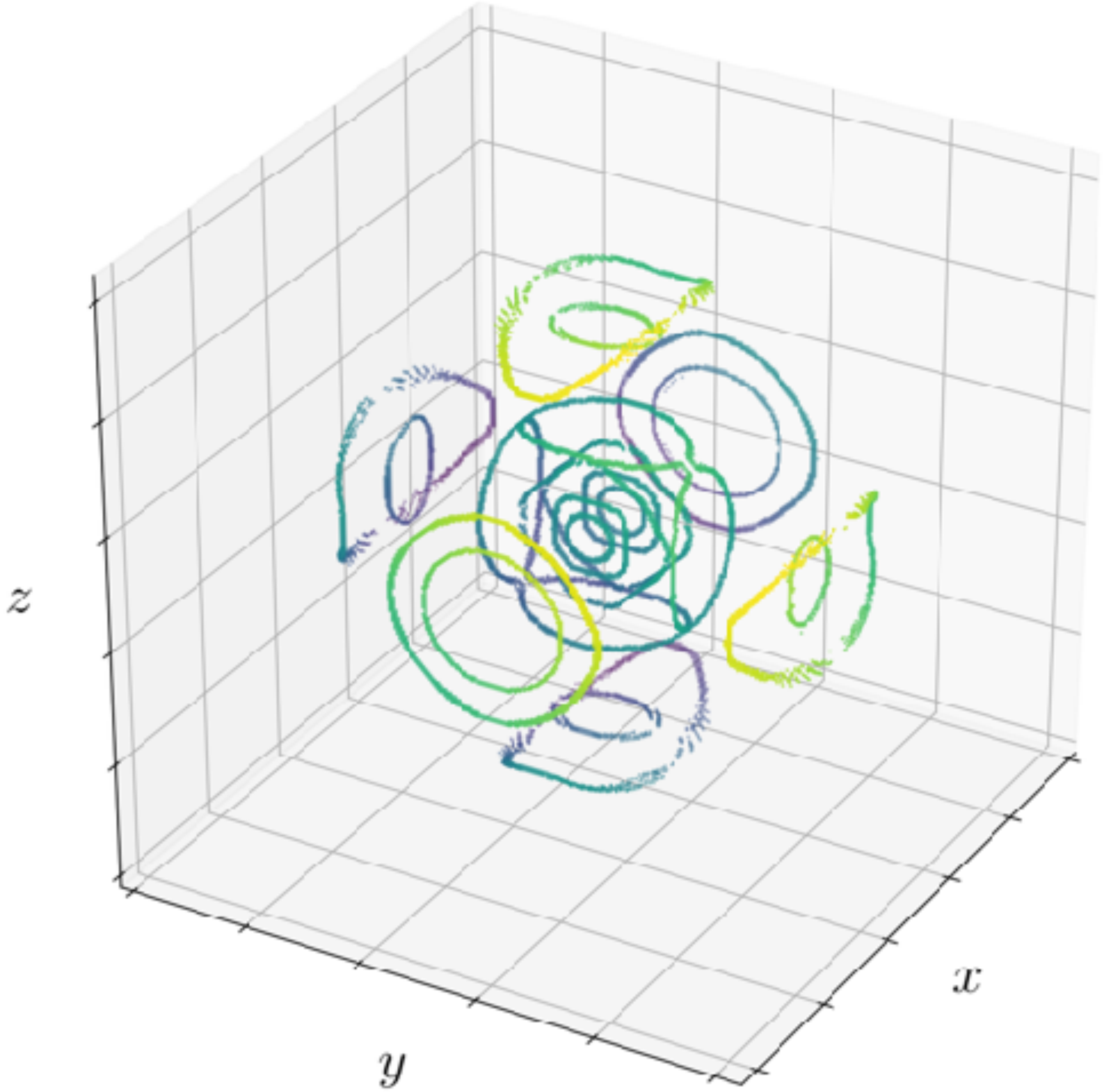


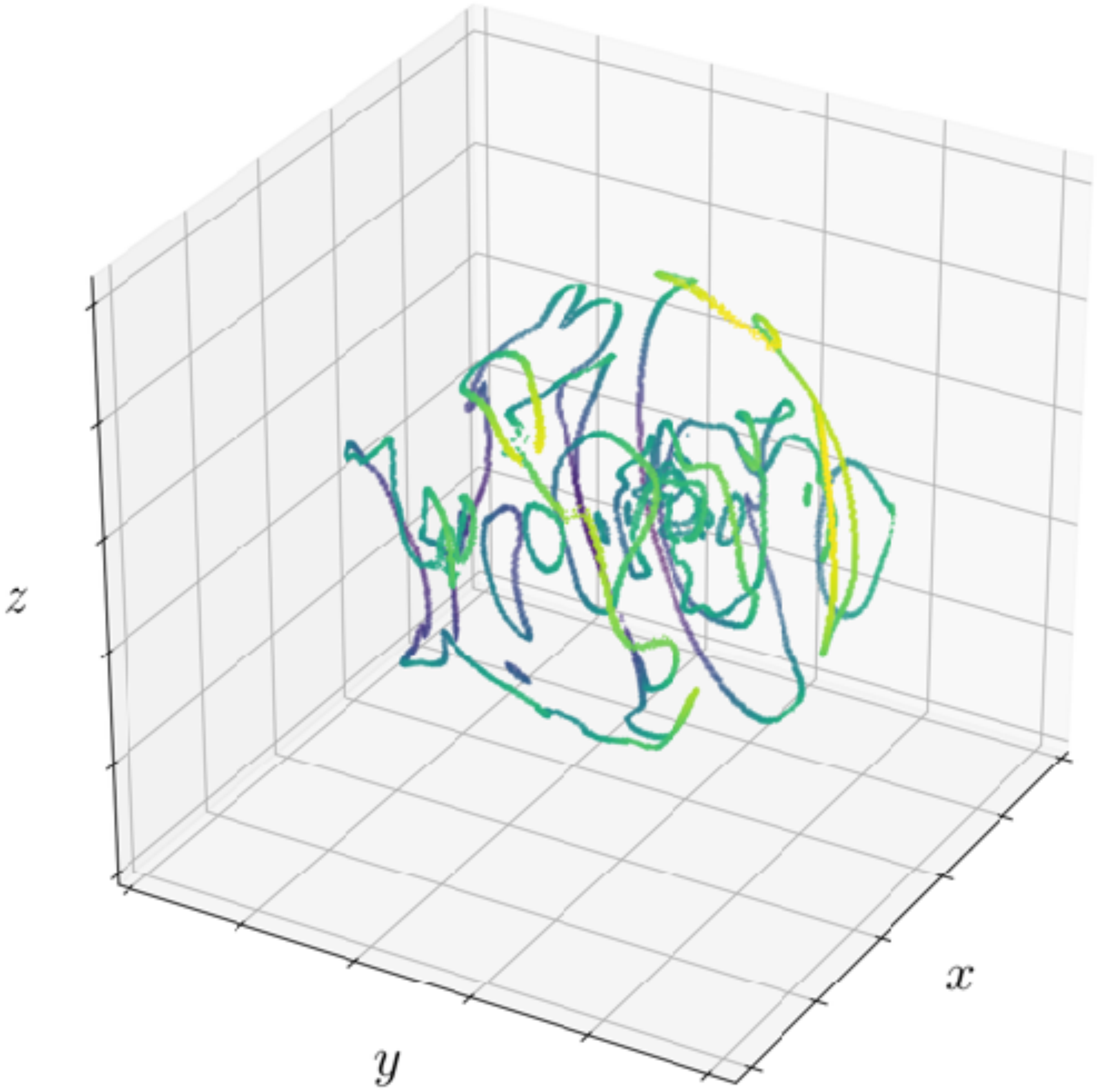
- Taylor expansion reveals further details (case of $n = 1$):

$$\psi(\vec{x}) \sim \cancel{\psi(0)} + \vec{x} \cdot \vec{\partial}\psi|_0 + ... \qquad \rho \sim r^2 \qquad (\text{also } v \sim 1/r)$$

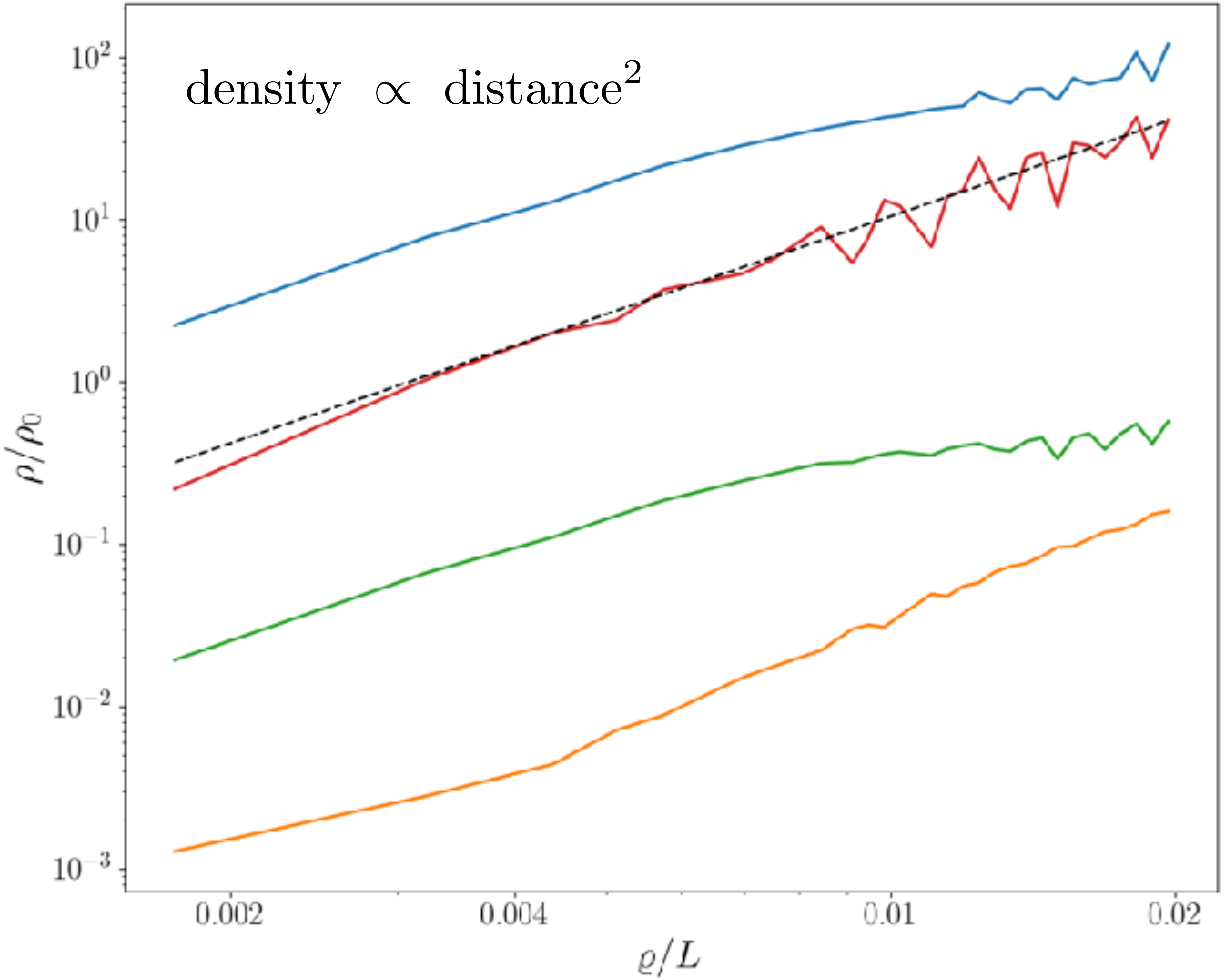
- Vortex generally takes the form of a loop i.e. vortex ring.

Note: this is not the usual axion string.


z
y
x


z
y
x

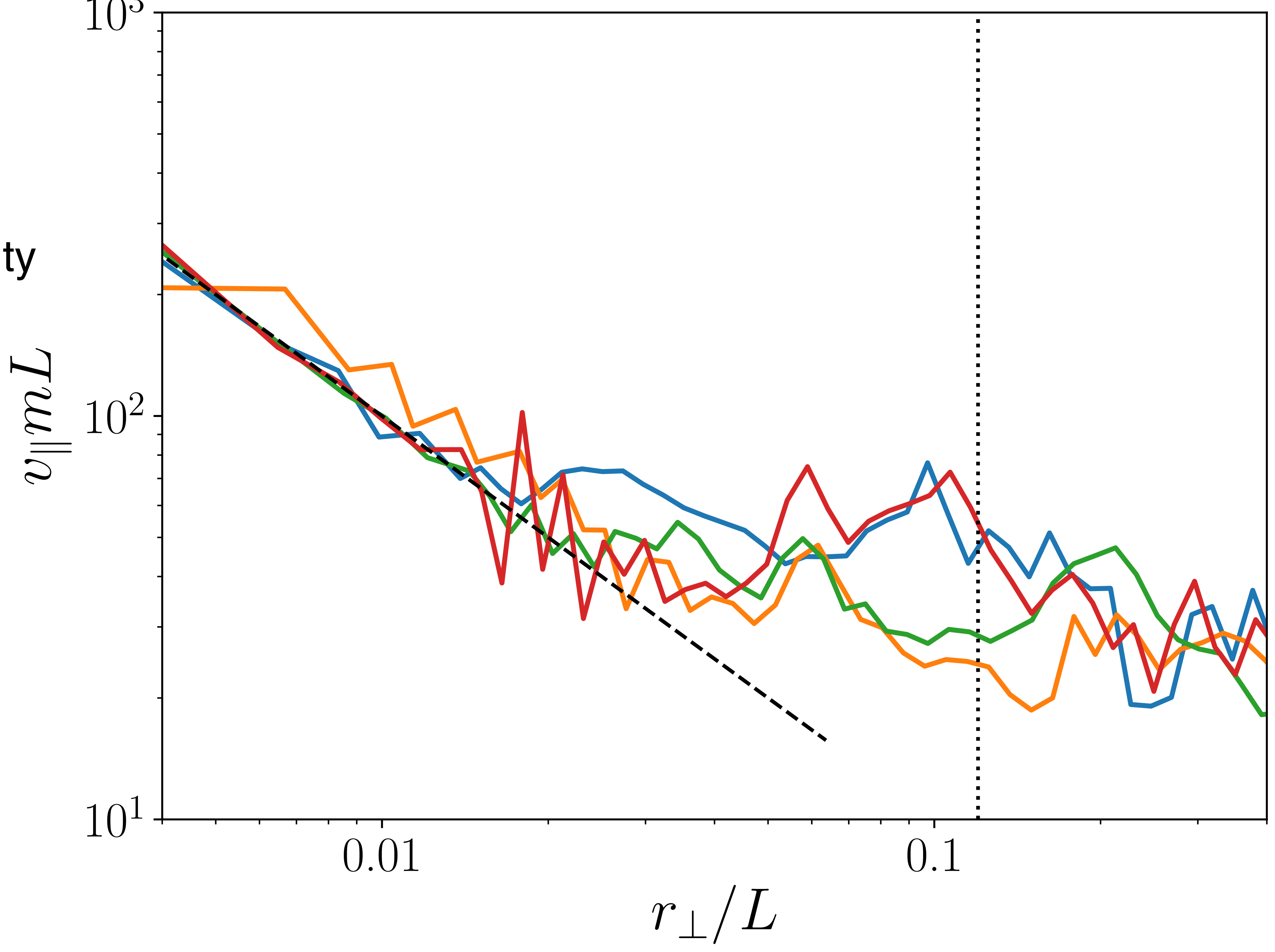
density



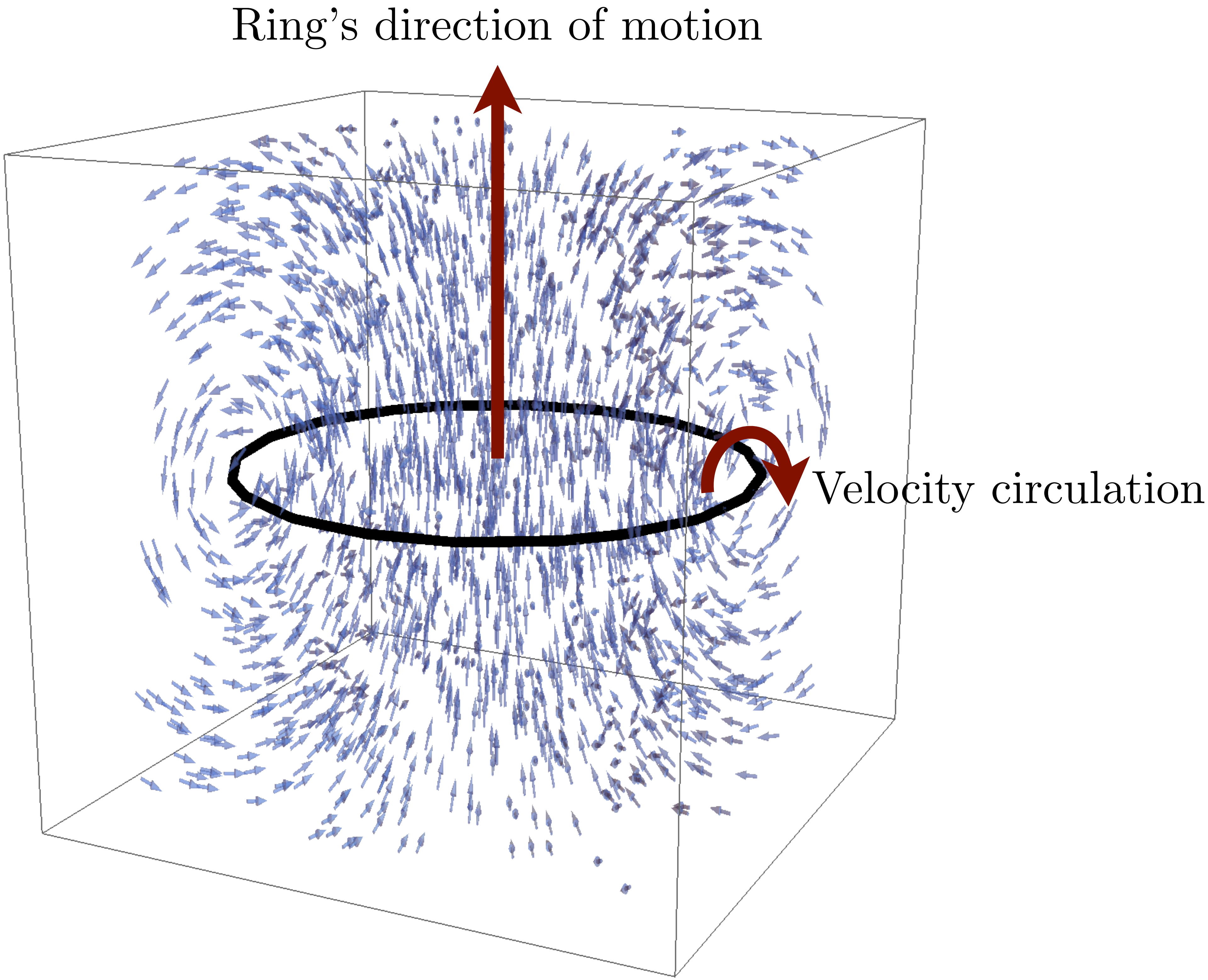
density $\propto$ distance^2

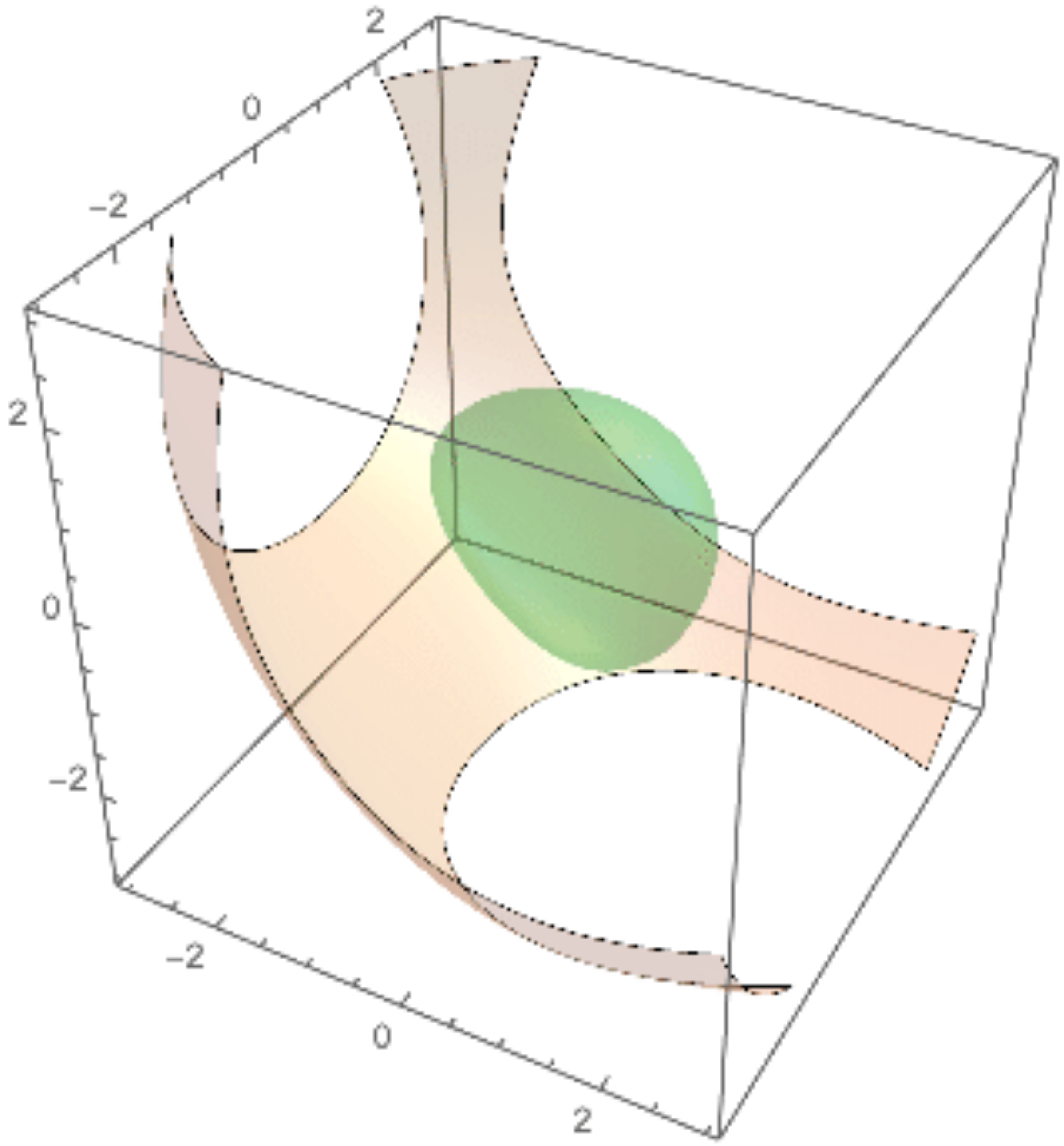
distance from vortex

velocity



distance from vortex


Ring's direction of motion
Velocity circulation


2
0
-2
2
0
-2
-2
0
2

Additional comments on vortices:

- Should defects be rare? No - roughly one vortex ring per de Broglie volume. Can compute this analytically for a model halo composing of a superposition of waves with random phases: essentially looking for zero-crossing.

  Note: this holds even if the halo has no net angular momentum.

- Smaller rings move faster: $v \sim \dfrac{1}{mR}$. Curved segments also move faster.

- Vortices (and interference substructures) are transient phenomena. Coherent time scale is de Broglie time $1/mv^2$ (million years for ultra-light). Vortices can't arbitrarily appear or disappear - Kelvin's theorem.

- Angular momentum eigenstates have vortices, though angular momentum does not require vortices (e.g. can always add s-wave with large amplitude).

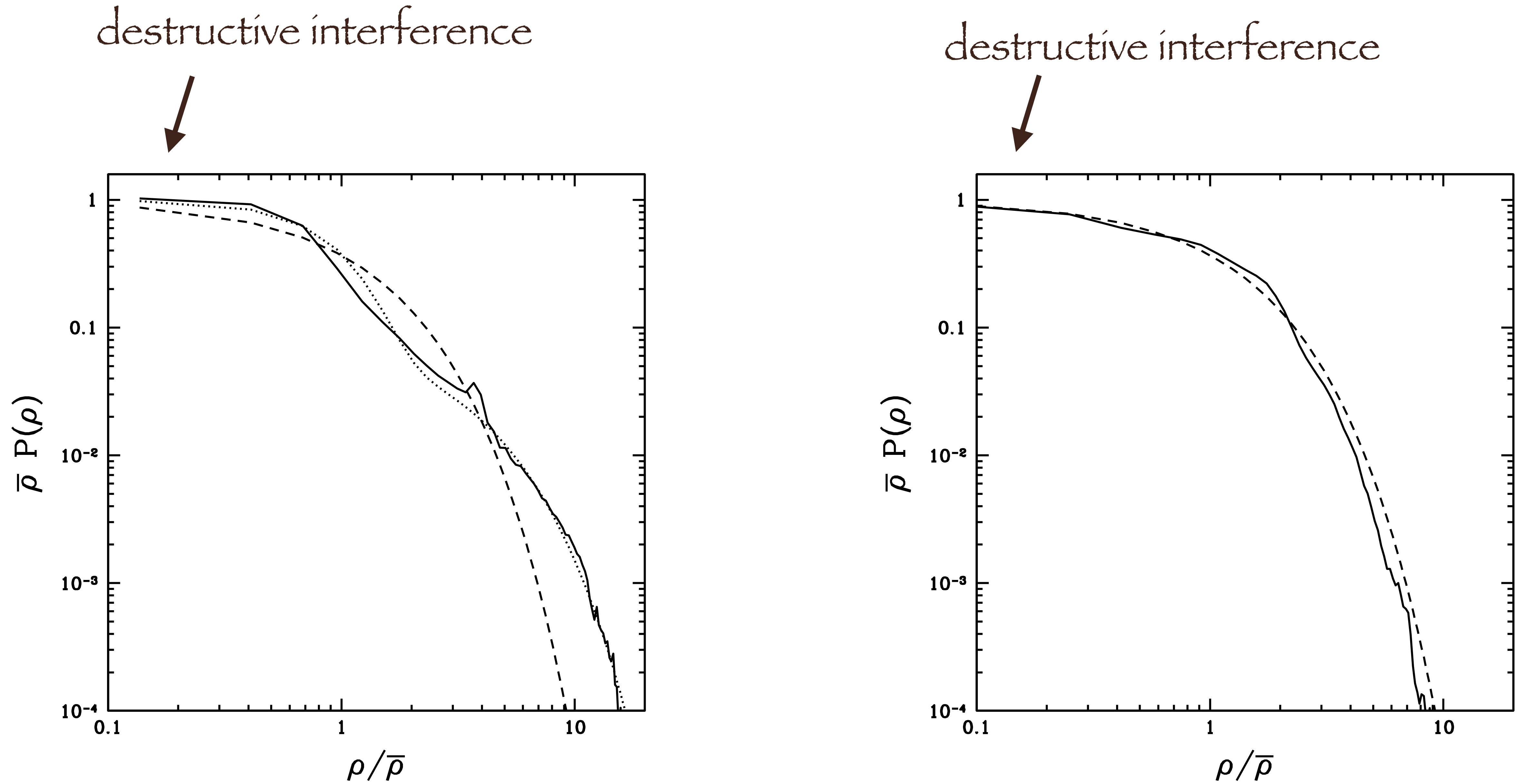


Figure 5: The one-point probability distribution of density: $P(\rho)d\rho$ gives the probability that the density ρ takes the values within the interval $d\rho$. The solid lines are measured from numerical wave simulations of two halos that form from mergers of smaller seed halos and gravitational collapse. The left panel is from a simulation where the initial seed halos are distributed uniformly, and the right panel is from a simulation where the initial seed halos are distributed randomly. The dashed line in each panel shows the analytic prediction from the random phase halo model: $\bar{\rho}P(\rho) = e^{-\rho/\bar{\rho}}$. The dotted line on the left panel is $\bar{\rho}P(\rho) = 0.9\, e^{-1.06(\rho/\bar{\rho})^2} + 0.1\, e^{-0.42(\rho/\bar{\rho})}$. See [72] for details.

Observational signatures (for ultralight DM)

- Strong gravitational lensing (flux anomaly)

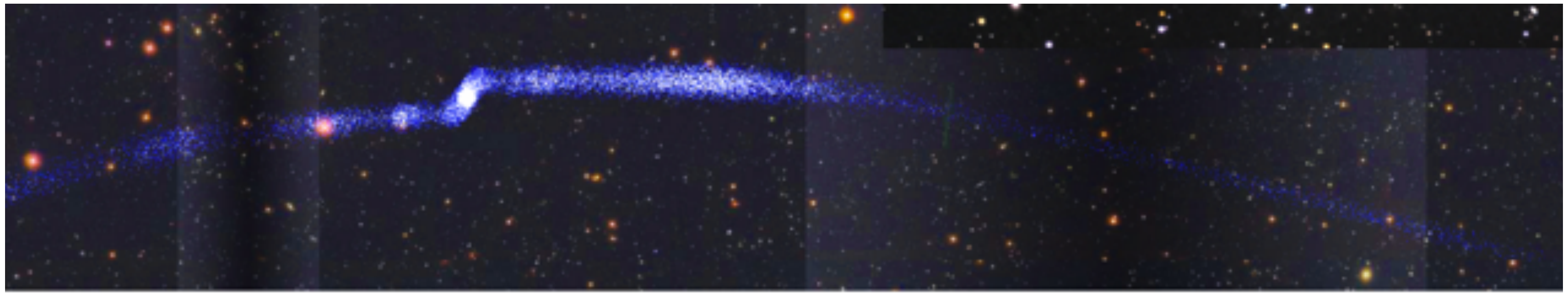
e.g. Two very close images should have the same magnification, absent substructure (Dalal, Kochanek).

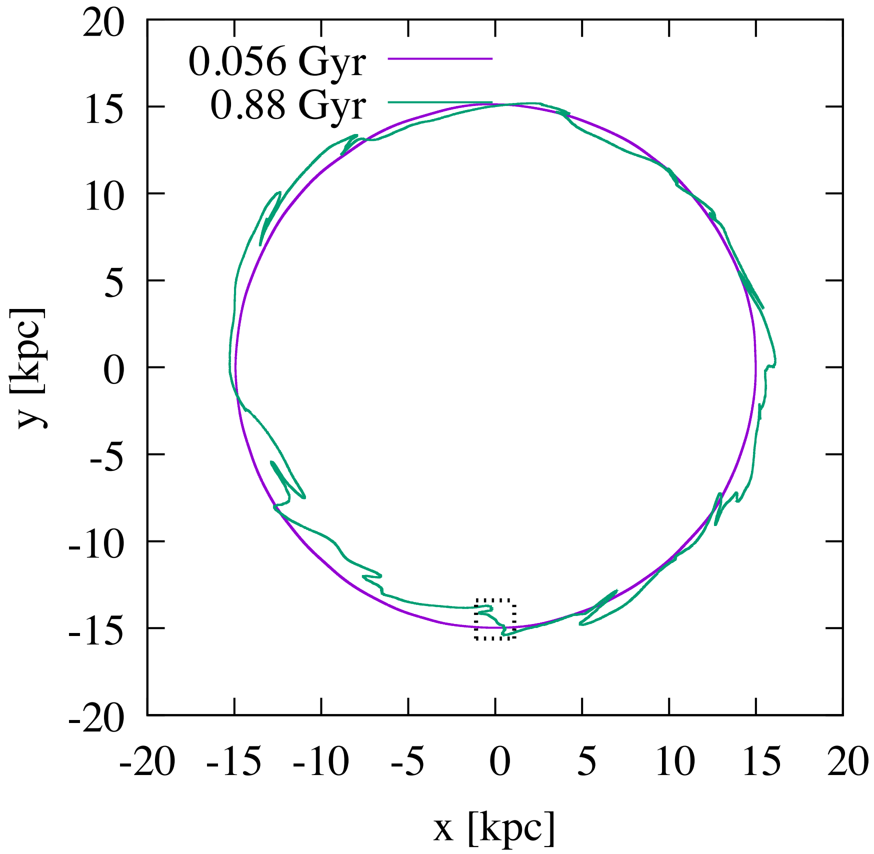
Wave interference can provide such substructure, causing flux anomaly at ten percent level (Chan et al., Broadhurst et al., Alexander et al., LH et al.).

Observational signatures (for ultralight DM)

- Scattering of stars in tidal streams

globular cluster Pal 5 (see Bonaca et al.)





Dalal, Bovy, LH, Li

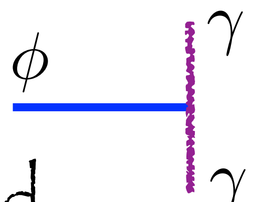
See also Amorisco, Loeb; Bar-Or , Fouvry, Tremaine; Church, Ostriker, Mocz; Schutz

Experimental implications (light DM e.g. QCD axion):

- Experiments mostly make use of

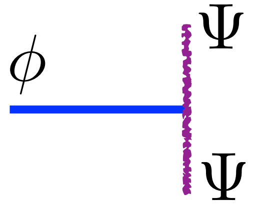
  - coupling of the axion ϕ to photon γ

    e.g. axion produces photon in magnetic field



  - coupling of the axion ϕ to fermion Ψ

    e.g. axion causes fermion spin to precess

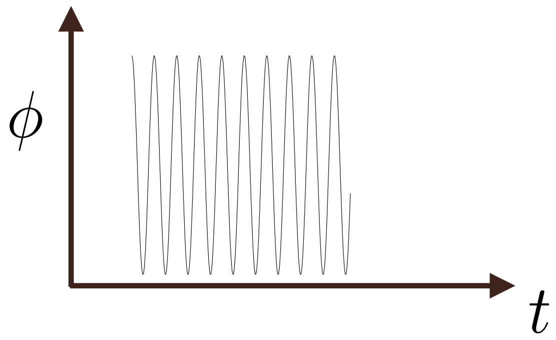


$$\mathcal{L} \sim \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\partial_\mu \phi}{f} \bar{\Psi} \gamma^5 \gamma^\mu \Psi$$

Reviews: Sikivie 2003

Graham et al. 2015, Marsh 2016

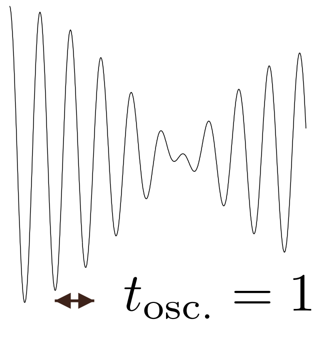
- Instead of



wave interference means:

$$\phi \sim \psi e^{-imt} + \psi^* e^{imt}$$

slow fast



$t_{\rm osc.} = 1/m \sim 10^{-9}\,{\rm s}$

$t_{\rm coherent} = 1/(m\,v^2) \sim 10^{-3}\,{\rm s}$

Derevianko; Foster, Rodd, Safdi;

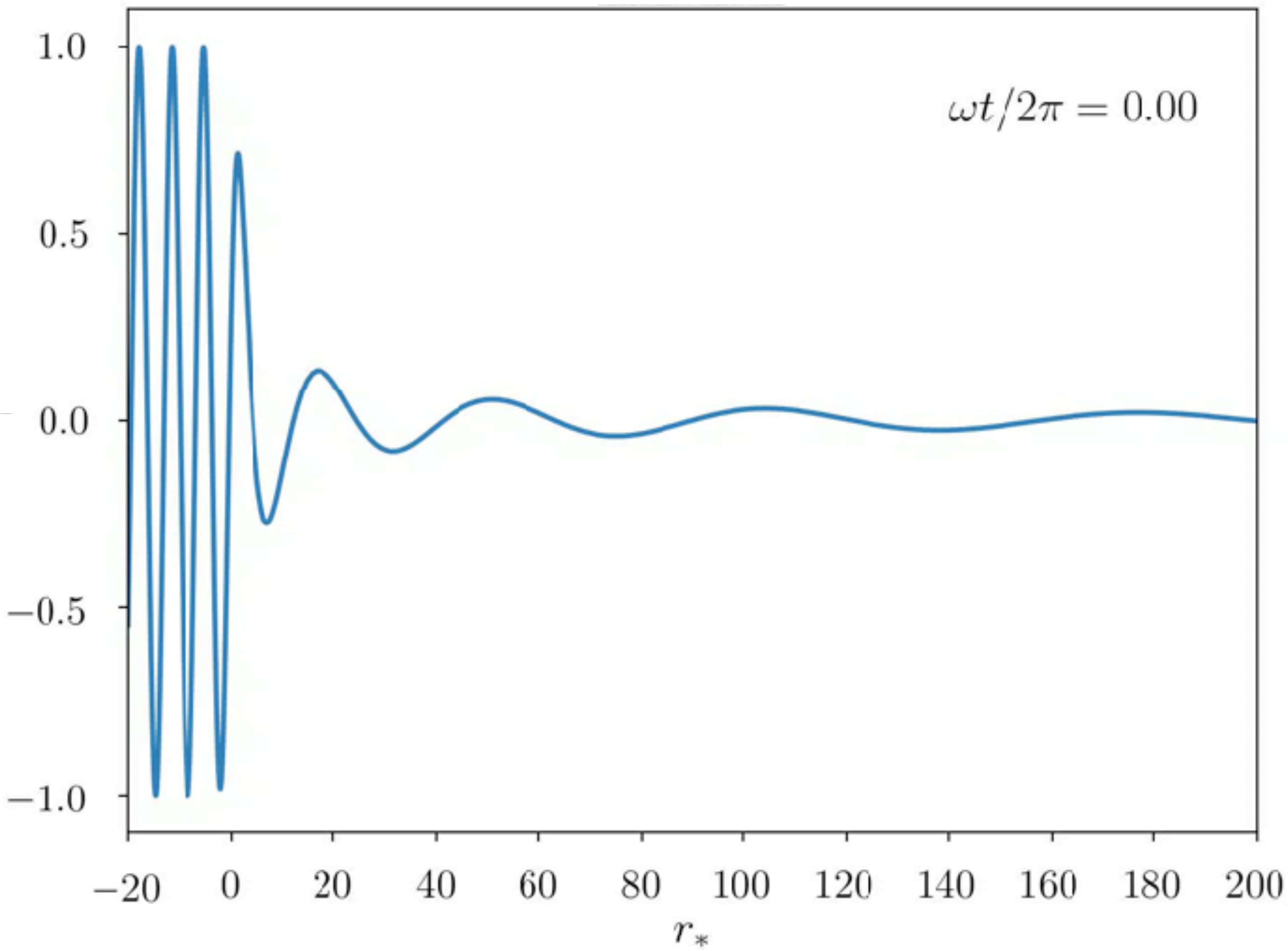
Centers et al.; LH, Joyce, Landry, Li

- Useful to think of axion detection experiment as measuring correlation functions
- At vortices $\phi = 0$ but $\vec{\nabla}\phi \neq 0$.
- Existence of vortices suggests oscillation phase is interesting: $\phi \sim |\psi| \cos(mt - \theta)$

$(-\Box + m^2)\phi = 0$ in Schwarz. bgd.

LH, Kabat, Li, Santoni, Wong

See also Barranco, Bernal et al.



$m = 1/r_{\text{Schwarz.}}$

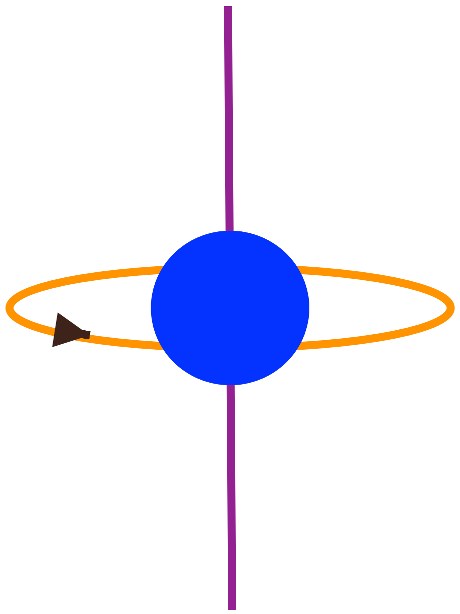
$r_* = r + \ln(r-1) \qquad (r_{\text{Schwarz.}} = 1)$

similar pile − up occurs for $m > 10^{-3}\, r_{\text{Schwarz.}}^{-1}$

Vortex as black hole hair

- Wave dark matter can form a stationary accretion flow around a black hole, donating both mass and angular momentum to the black hole (Clough, Ferreira, Lagos; LH, Kabat, Li, Santoni, Wong; Bamber et al.).

- Under suitable conditions, superradiance can extract mass and angular momentum from the same black hole, growing a superradiance cloud. The combination of accretion and superradiance could lead to a cloud more massive (i.e. comparable to mass of black hole) than that generated by superradiance alone (i.e. < 10 %). This has observational implications e.g. GW.

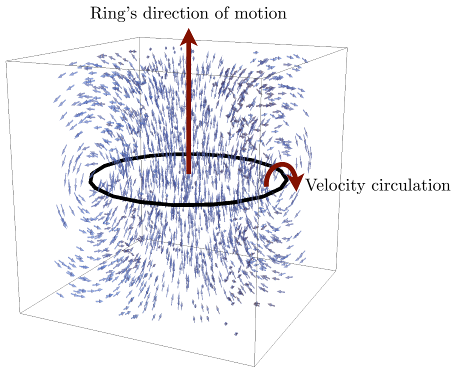


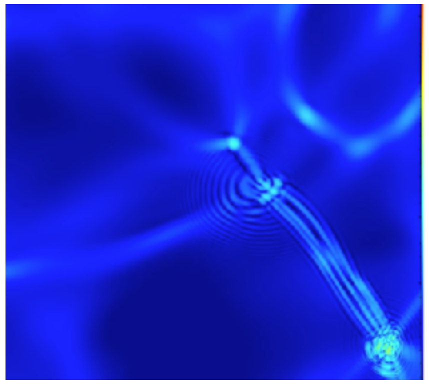
Summary

- For dark matter particle lighter than about 30 eV (e.g. axion/axion-like-particle), wave interference phenomena are unavoidable.

- There are many observational and experimental implications.

  - Lyman-alpha forest
  - solitonic halo core
  - interference substructure
  - vortices
  - dynamical friction
  - evaporation of sub-halos by tunneling
  - direct detection
  - detection by pulsar timing array
  - gravitational lensing
  - scattering of tidal streams
  - soliton oscillations
  - black hole hair





Think about phase!