SOME IDEAS ABOUT COSMIC STRUCTURE ON THE SMALLEST SCALES

largely based on
arXiv:2107.10293
with
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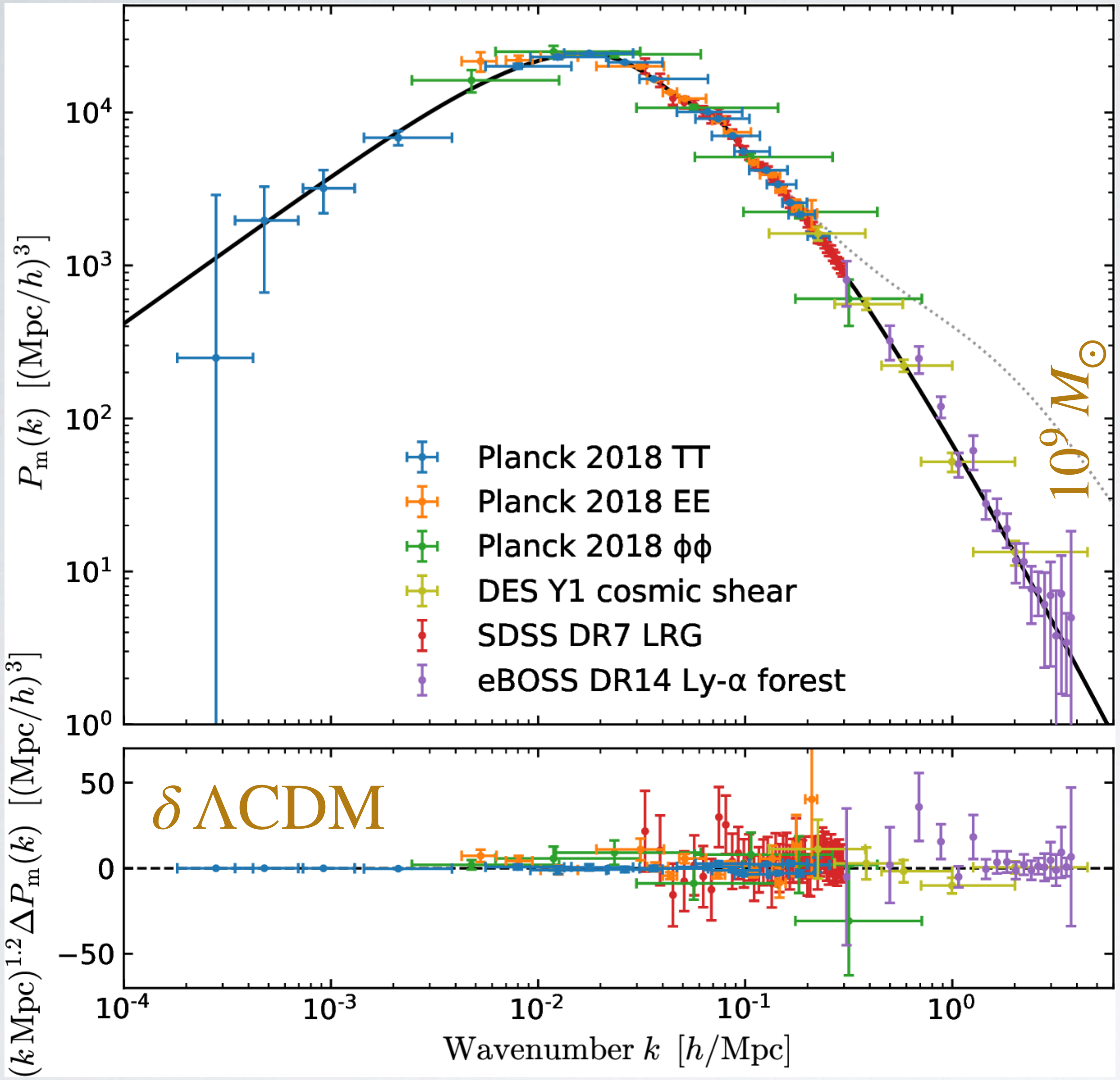
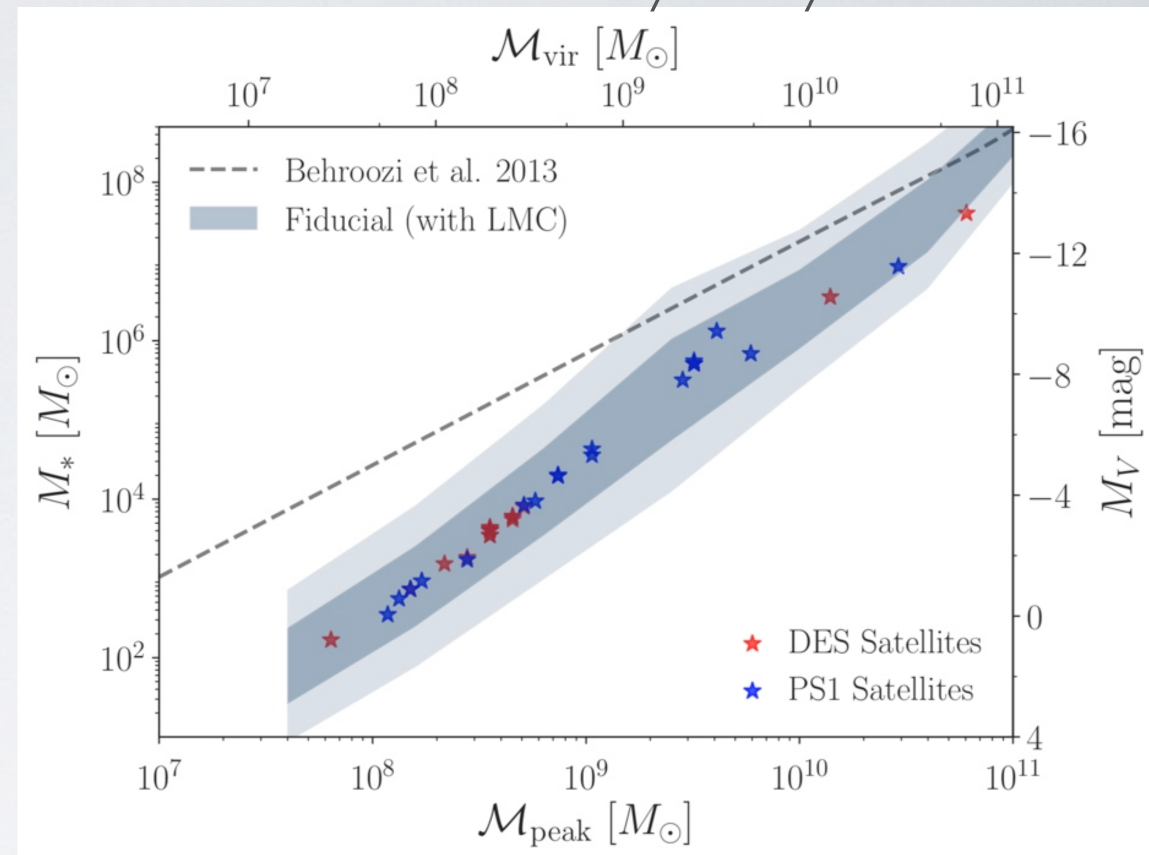
65th Copernicus Webinar
indico.cern.ch/event/936284/
2021-10-17

SMALL-SCALE DARK MATTER FRONTIER



Milky Way Satellites



Nadler et al. 2020

Chabanier Millea Palanque 2019

- ΛCDM works astoundingly well over the observed range of scales.
- Have only probed DM down to ~$10^8 M_\odot$
  - ✳ strong lensing only probes high surface density objects
- DM structure on (much) smaller scales will reveal information on dark matter and the early universe

ΛCDM MICROHALOS

$M \leq 1\, M_{\text{earth}}$

$n \sim 10\, \text{pc}^{-3}$

$f_{\text{CDM}} \sim 5 \times 10^{-4}$

Solar System visitation frequency:

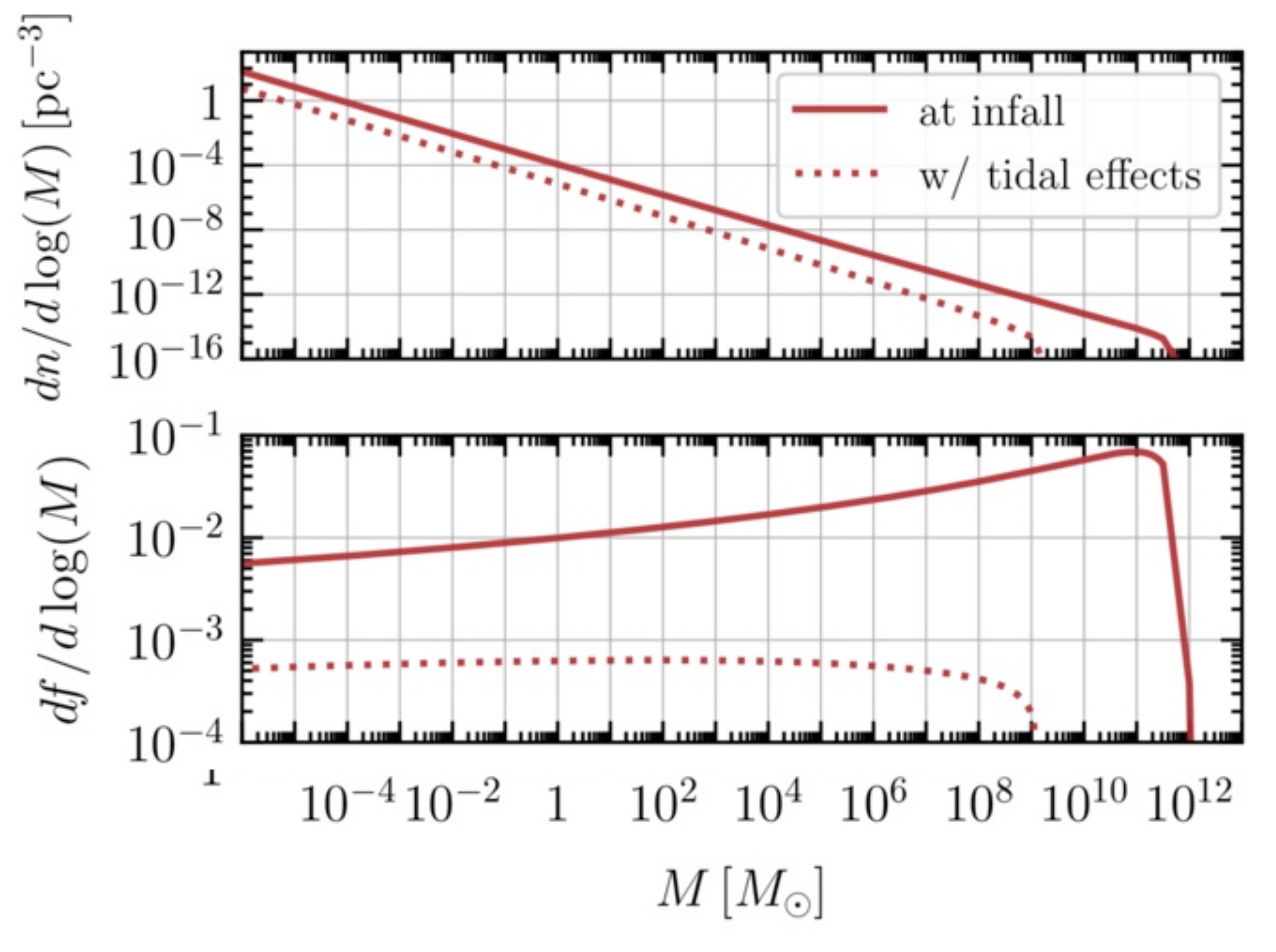
~15/Gyr

$R_{\mu\text{halo}}$=10^3 A.U.

R_{SS}=10^3 A.U.

$v_{\mu\text{halo}}$=200km/sec

t_{visit}~24 years



Lee Mitradate Trickle Zurek 2021

Low mass μhalos tells about particle mass / velocity dispersion of DM

insufficient pulsar timing sensitivity

QCD AXION MINICLUSTERS

$M \leq 2 \times 10^9$ ton

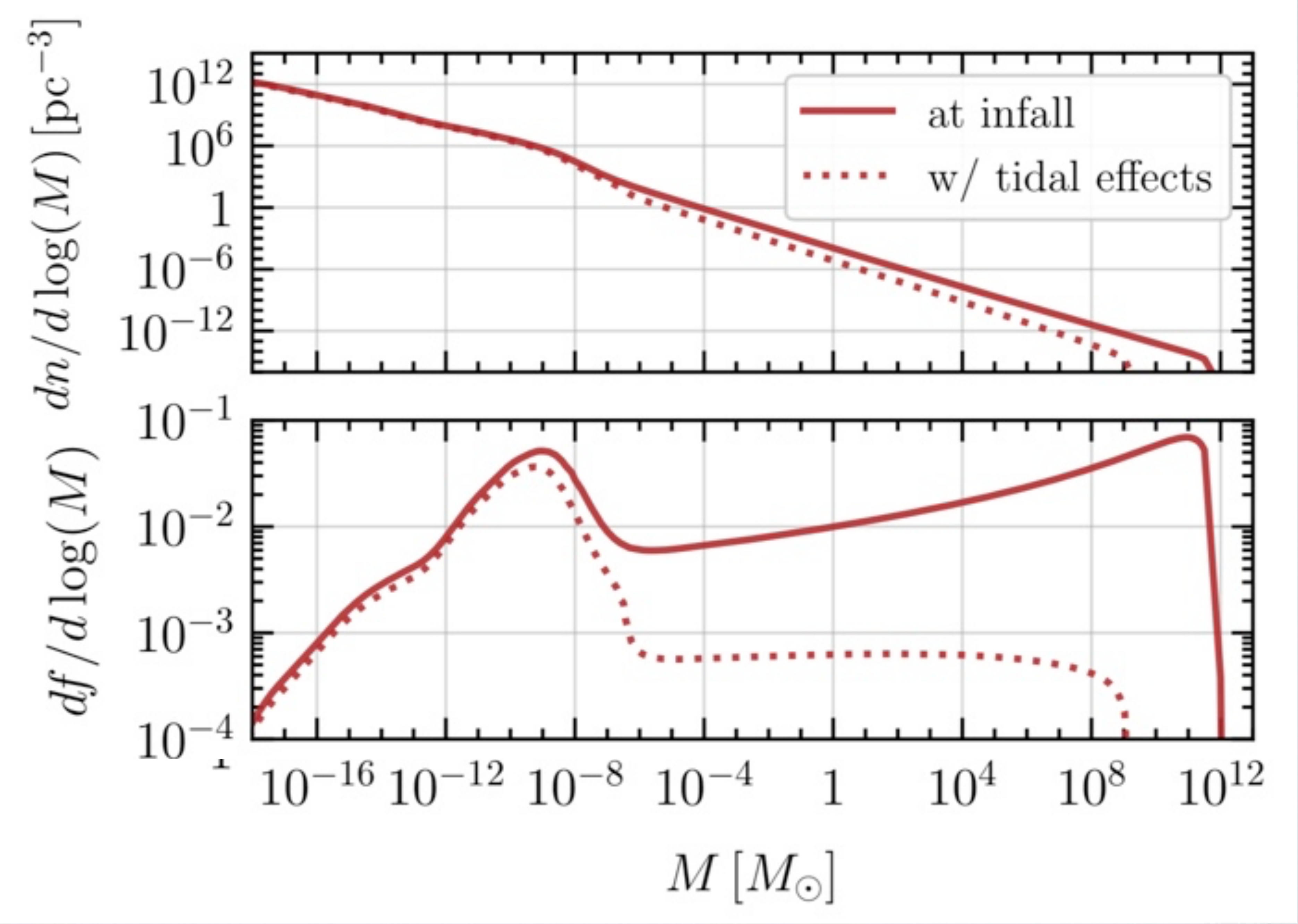
$n \sim 10^{12}\,\mathrm{pc}^{-3}$

$f_{\mathrm{CDM}} \sim 10^{-4}$

Solar System visitation frequency:

~1/year

R_{SS}=10 A.U.
v_{gal}=200km/sec
t_{visit}~100 days



Low mass μhalos tells about clustering of DM during production in early universe

Lee, Mitradate Trickle Zurek 2021

pulsar sensitivity <century

EARLY MATTER DOMINATION MICROHALOS

$T_{\rm RH} = 1\,{\rm GeV}$ long EMD

$M \leq 10^{-15}\,M_\odot \sim 10^{12}\,{\rm ton}$

$n \sim 10^{11}\,{\rm pc}^{-3}$

$f_{\rm CDM} \sim 10^{-2}$

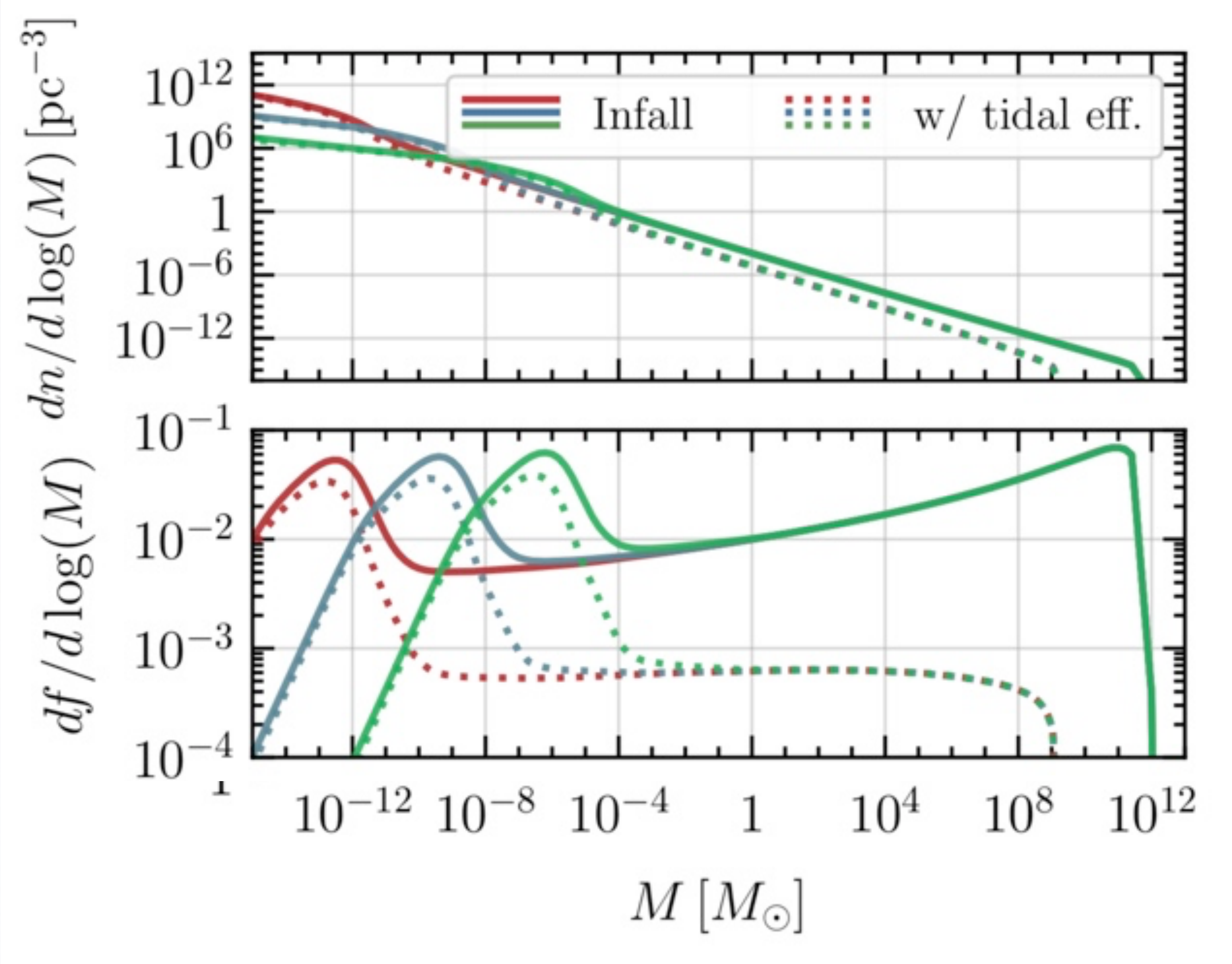
Solar System visitation frequency:

~1/decade

R_{SS}=10 A.U.

$v_{\mu halo}$=200km/sec

t_{visit}~100 days



Lee Mitradate Trickle Zurek 2021

Low mass **μ**halos tells about expansion history of early universe

pulsar timing sensitivity if T_{RH}<0.1GeV

EARLY MATTER DOMINATION MICROHALOS

$T_{\rm RH} = 32\,\mathrm{MeV}$ short EMD

$M \sim 10^{-9}\,M_\odot$ Pluto

$n \sim 10^5\,\mathrm{pc}^{-3}$

$f_{\rm CDM} \sim 10^{-3}$

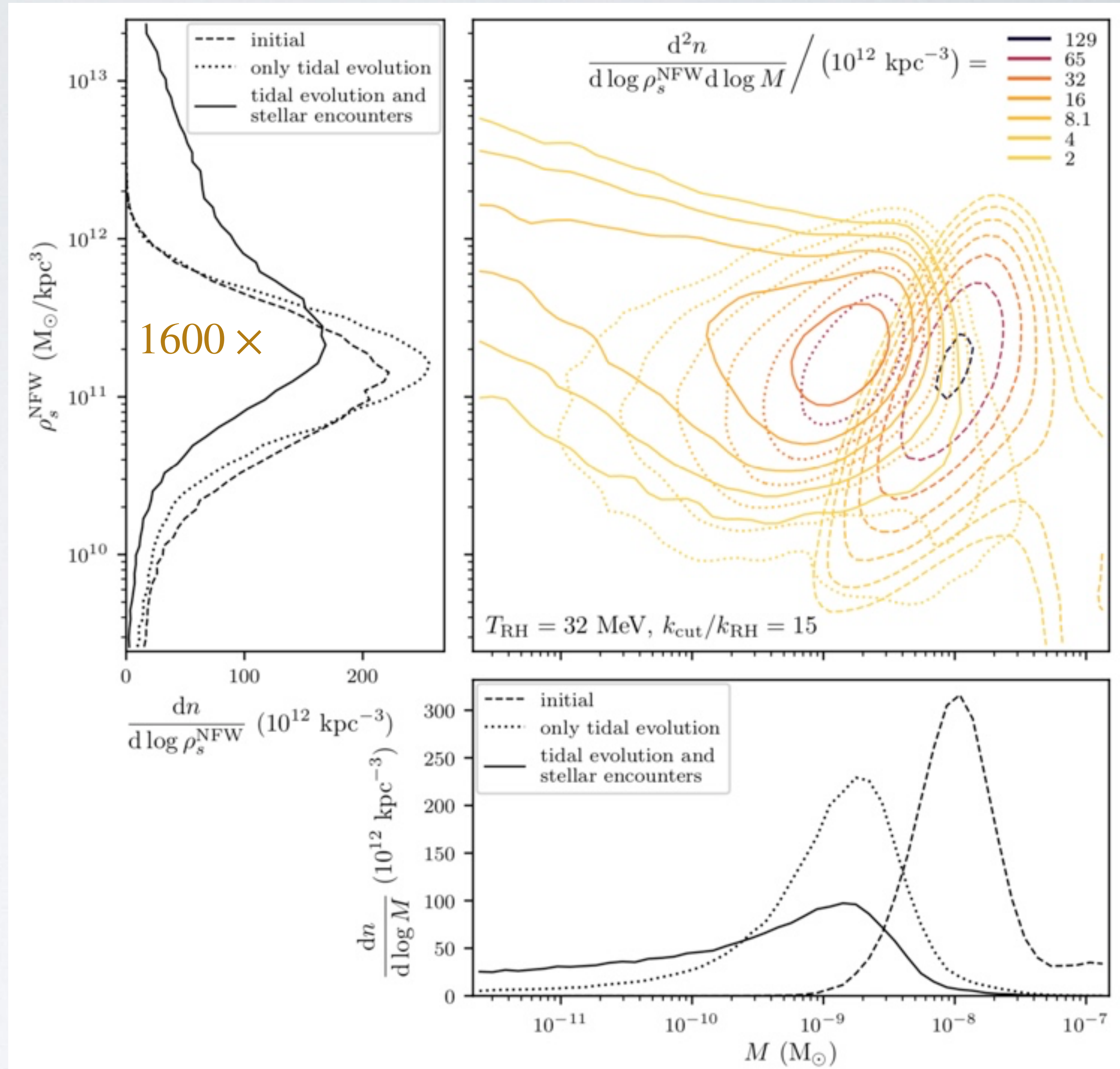
Solar System
visitation frequency:

$\sim 10^2$/Gyr

R_{SS}=10 A.U.

$v_{\mu\mathrm{halo}}$=200km/sec

t_{visit}~100 days



Delos Linden2021

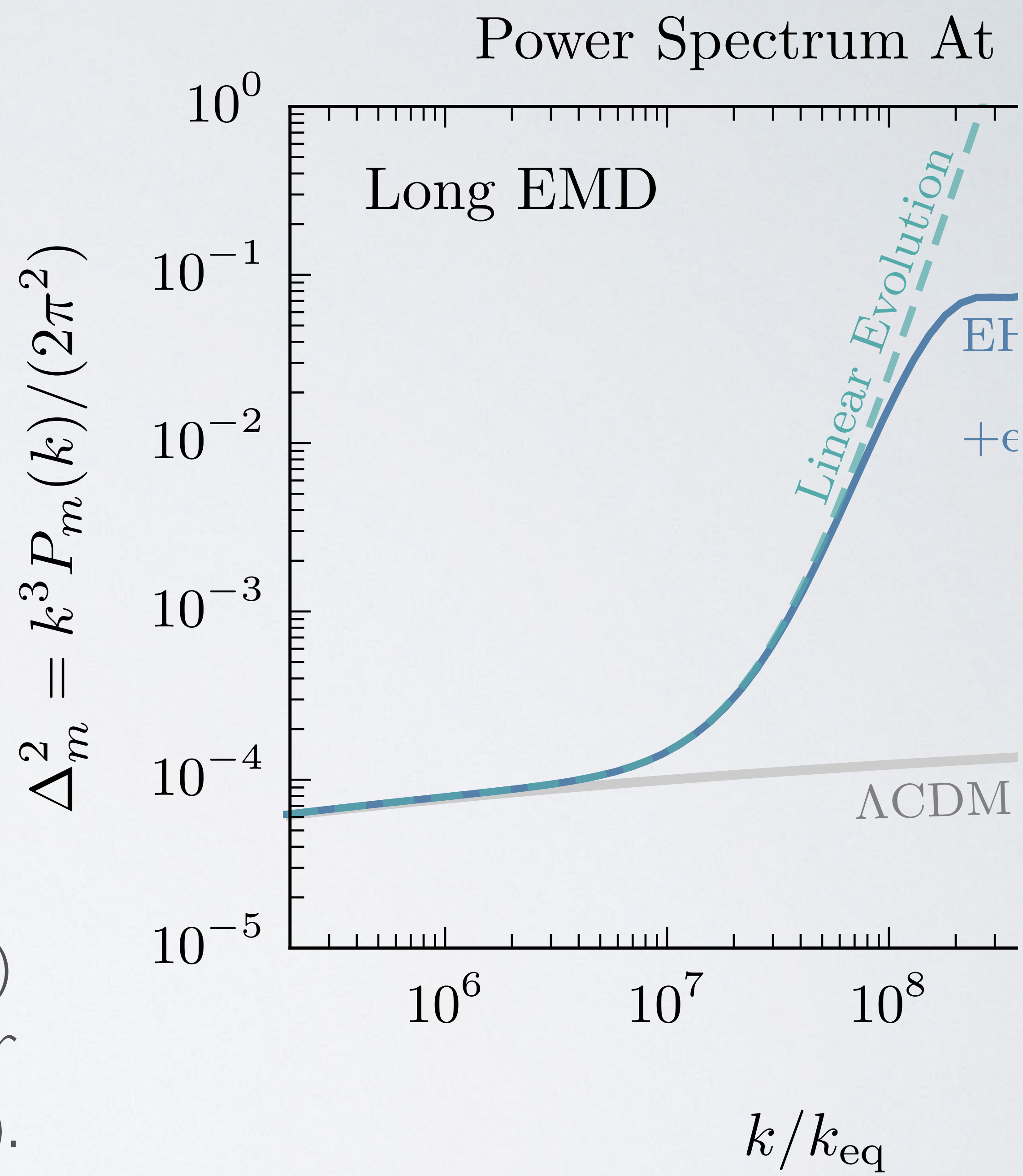
pulsar timing sensitivity <century

WHY EARLY MATTER DOMINATION?

An early period of matter domination leads to additional growth of linear ($\delta\rho/\rho \ll 1$) inhomogeneities for modes within the horizon during early matter domination.

Structure on small scales will collapse earlier than in ΛCDM leading to smaller denser microhalos.

Erickcek Sigurdson 2011
Barenboim Rasero 2014



Motivation for this Work

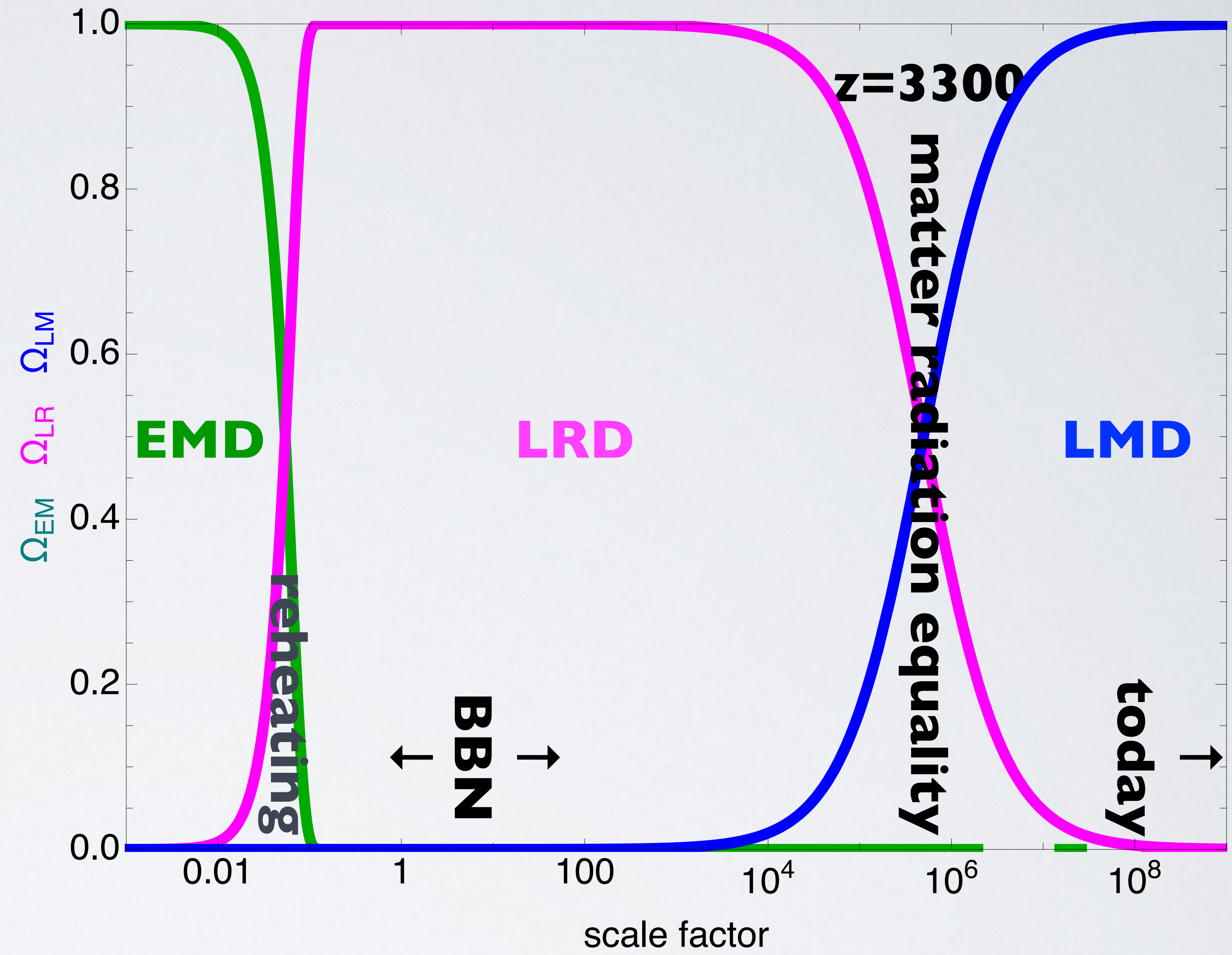
During matter domination nonlinear (bound) inhomogeneities (halos) grow faster ($\delta\rho/\rho \propto a^3$) than linear growth ($\delta\rho/\rho \propto a$). Do remnants of these halos survive leading to even denser microhalos?

Barenboim Blinov AS 2021

EMD SCENARIO

COSMOLOGY

- **ERD** early radiation domination (optional)
  - radiation domination after inflation
- **EMD** early matter domination
  - early period of non-relativistic matter domination
- **LRD** late radiation domination
  - standard "radiation era"
- **LMD** late matter domination
  - standard "matter era"
- **EM** early matter
  - BSM particle, dominates density during EMD
  - decays to LR w/ lifetime τ ("reheating")
- **LR** late radiation
  - standard photons / neutrinos
- **LM** late matter
  - standard dark matter /baryons



CONSTRAINTS

- LM and LR densities match observed ΛCDM values
- T_{RH}>5 MeV to preserve Big Bang Nucleosynthesis
  - LRD must last for expansion factor >10^7

INHOMOGENEITIES

- EM and LM dynamics only gravitational
  - + EMD decay
- EM and LM cluster identically
- m_{EM} and m_{LM} large enough so they are not "fuzzy"
- **EM/LM in gravitational bound halos before reheating**
  - halos are non-relativistic and sub horizon
- EMD LSS **may or may not** be similar to LSS during LMD

✶ LSS = "large scale structure"

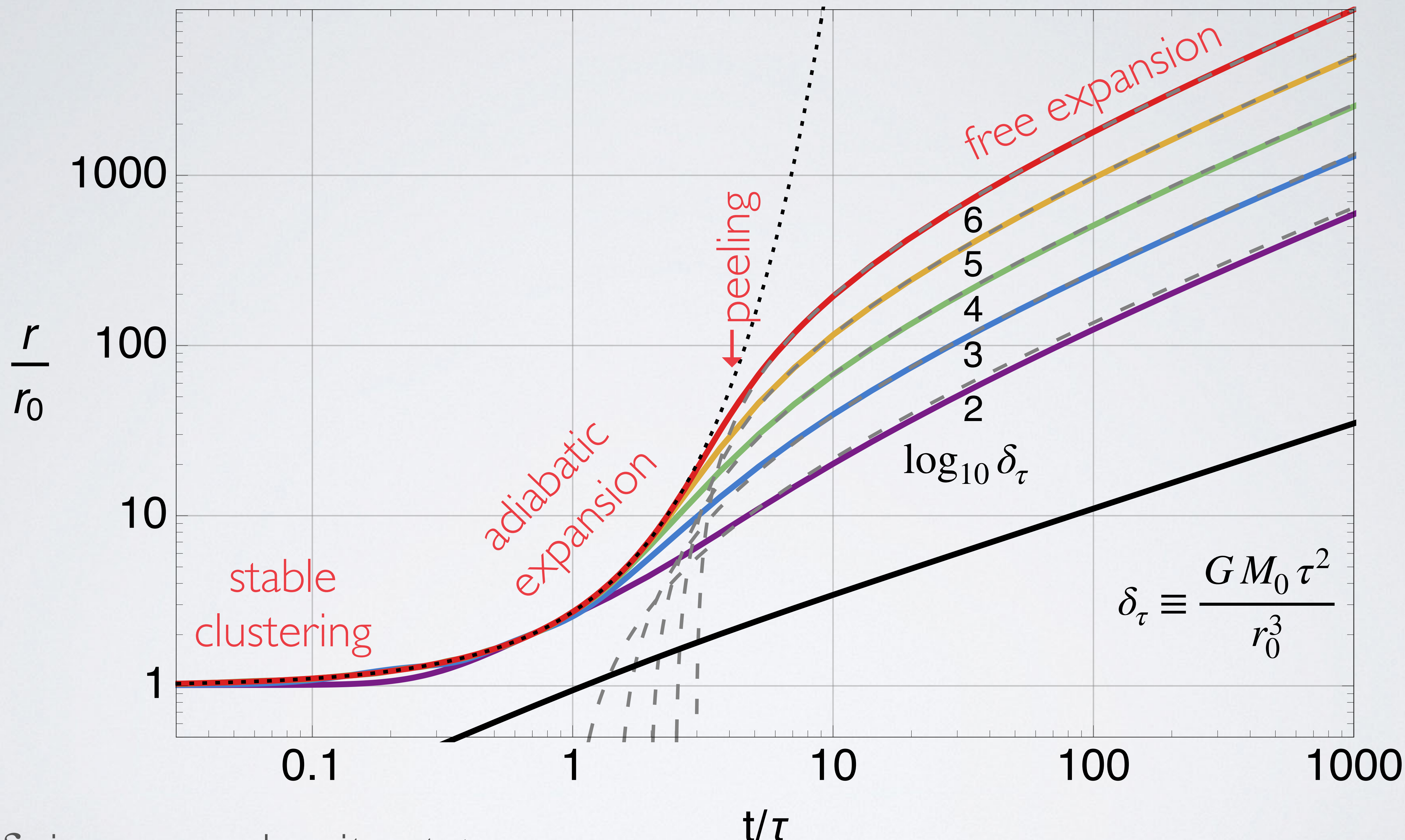
SPHERICAL SHELL MODEL

- Assume spherical isolated halos - evolve *n*-shell as apposed to *n*-body
  - each EM/LM particle will remain in it's own shell
  - isolated halos - good if most matter is in stable clustering regime at time of decay - inner part of halos - outer part too if steep $P(k)$ at non-linear scale so major mergers episodic rather than continual
- Newtonian model exhibiting relevant phenomenology

$$\ddot{r}(t) = -\frac{G\,M_<(r(t),t)}{r(t)^2} + \frac{L^2}{r(t)^3} - \frac{8\pi G}{3}\rho_{\rm rad}(t)\,r(t)$$

- non-relativistic halos means $\rho_{\rm rad}$ is uniform and negligible time dilation for decay
- **further simplification**: initially circular orbits $\dot{r}(t \ll \tau) = 0$
  - shells don't cross so $M_<(r,t) = \left(\frac{1-f}{f}\,e^{-t/\tau} + 1\right) M_{\rm LM}$; $f \equiv \lim_{t \ll \tau} \frac{\Omega_{\rm LM}}{\Omega_{\rm EM} + \Omega_{\rm LM}}$
  - each shell parametrized by LM mass enclosed $M_{\rm LM}$ which is constant

SHELL MODEL SOLUTION



- δ_τ is ~ over density at $t = \tau$
- different curves give evolution of concentric shells of same halo

HOMOLOGOUS ADIABATIC EXPANSION

- Newton-Vlasov equation

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{x}} - \left(\frac{\partial}{\partial \mathbf{x}}\Phi[\mathbf{x},t]\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right) f[\mathbf{x},\mathbf{v},t] = 0$$

$$\Phi[\mathbf{x},t] = -\,G\,d[t]\int d^3\mathbf{x}' \int d^3\mathbf{v}\,\frac{f\left[\mathbf{x}',\mathbf{v},t\right]}{|\,\mathbf{x}-\mathbf{x}'|} \qquad d[t] = \left(1-f_{\mathrm{LM}}\right)e^{-t/\tau} + f_{\mathrm{LM}}$$

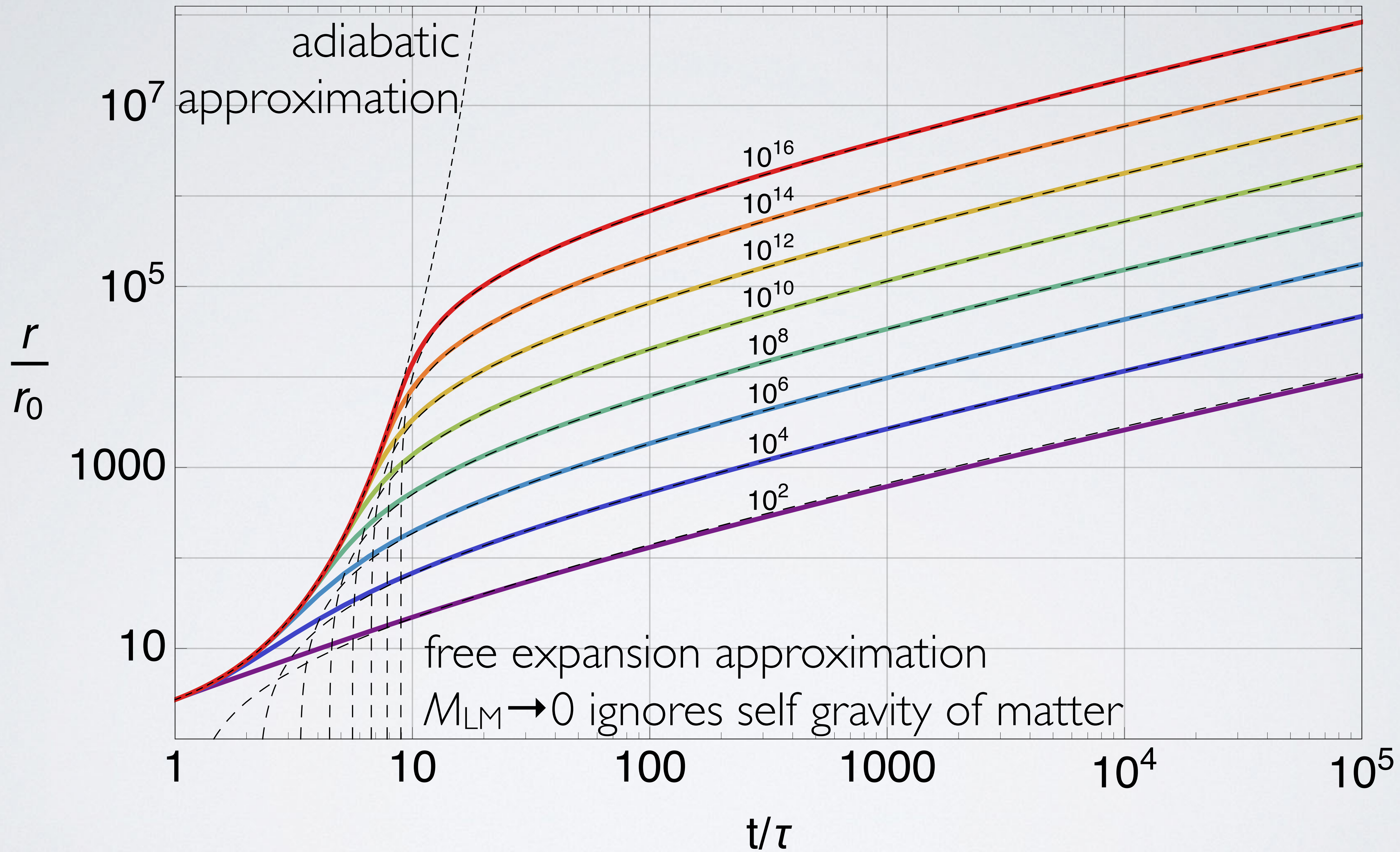
- If $f_{\mathrm{eq}}[\mathbf{x},\mathbf{v}]$ and $\Phi_{\mathrm{eq}}[\mathbf{x}]$ are stationary solutions for $\tau \to \infty$ then an approximate adiabatic solution for $t_{\mathrm{dyn}} \gg \tau$ is

$$f[\mathbf{x},\mathbf{v},t] \cong f_{\mathrm{eq}}\left[d[t]\,\mathbf{x}, \frac{\mathbf{v}}{d[t]}\right] \qquad \Phi[\mathbf{x},t] \cong -\,d[t]\,\Phi_{\mathrm{eq}}\left[d[t]\,\mathbf{x}\right]$$

- since $f \ll 1$ positions grow $\mathbf{x} \propto e^{t/\tau}$ while velocities shrink $\mathbf{v} \propto e^{-t/\tau}$

- N.B adiabatic exponential growth does not depend on spherical symmetry!

PEELING



- peeling happens at logarithmically different time for different shells: when $t_{\rm dyn} \approx \tau$
  - outer shells first then inner shells

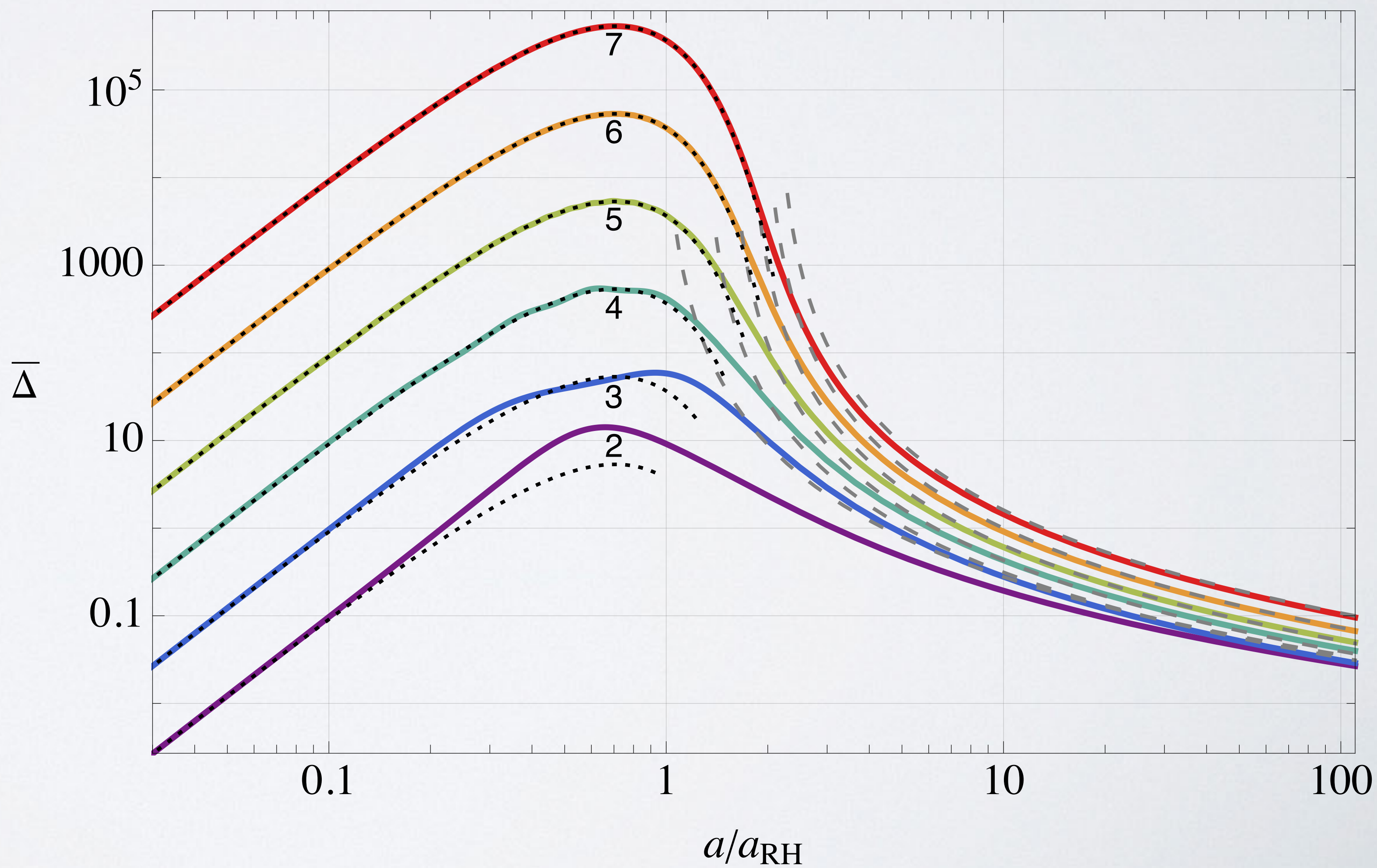
NEWTON IN COSMOLOGICAL CONTEXT

In terms of the interior density ratio $\bar{\Delta}(a(t)) \equiv \dfrac{3M_{\rm LM}}{4\pi\, r^3\, \bar{\rho}_{\rm LM}(t)}$ the shell equation of motion becomes

$$a^2\,\bar{\Delta}'' = \frac{3}{2}\,\Omega_{\rm m}(a)\,\bar{\Delta}\,(\bar{\Delta}-1) - \left(1+\frac{1}{2}\,\Omega_{\rm m}\right)\,a\,\bar{\Delta}' + \frac{4}{3}\,\frac{(a\,\bar{\Delta}')^2}{\bar{\Delta}} - 3\left(\frac{L}{H(a)}\right)^2 \left(\frac{2f}{9\,G\,M_{\rm LM}\,\tau^2}\right)^{4/3} \left(\frac{a_{\rm RH}}{a}\right)^4 \bar{\Delta}^{7/3}$$

where $' \equiv \dfrac{d}{da}$, H is the Hubble parameter and $\Omega_{\rm m}$ is the density parameter for (EM+LM) matter.

- $\bar{\Delta}$ becomes $\ll 1$ even when $\bar{\Delta}_\tau \gg 1$.
- FYI: even in optimal case, $f = 10^{-7}$, one finds that $\bar{\Delta}_{\rm eq} > 1$ only if $\bar{\Delta}_\tau > 10^{20}$.
- Early halos spread out during LRD contributing a small fraction of the mean matter density when they enter LMD.



OVERLAP

Since $\bar{\Delta} \ll 1$ throughout the remnant of a halo the halo remnants must overlap by the identity:

$$\langle \mathcal{N} \rangle \geq \frac{1}{\langle \Delta \rangle} - 1 \gg 1$$

$\langle \mathcal{N} \rangle$ is a mean number of overlapping streams and $\langle \Delta \rangle$ is mass weighted halo average of $\dfrac{\rho_{\rm LM}}{\bar{\rho}_{\rm LM}}$.

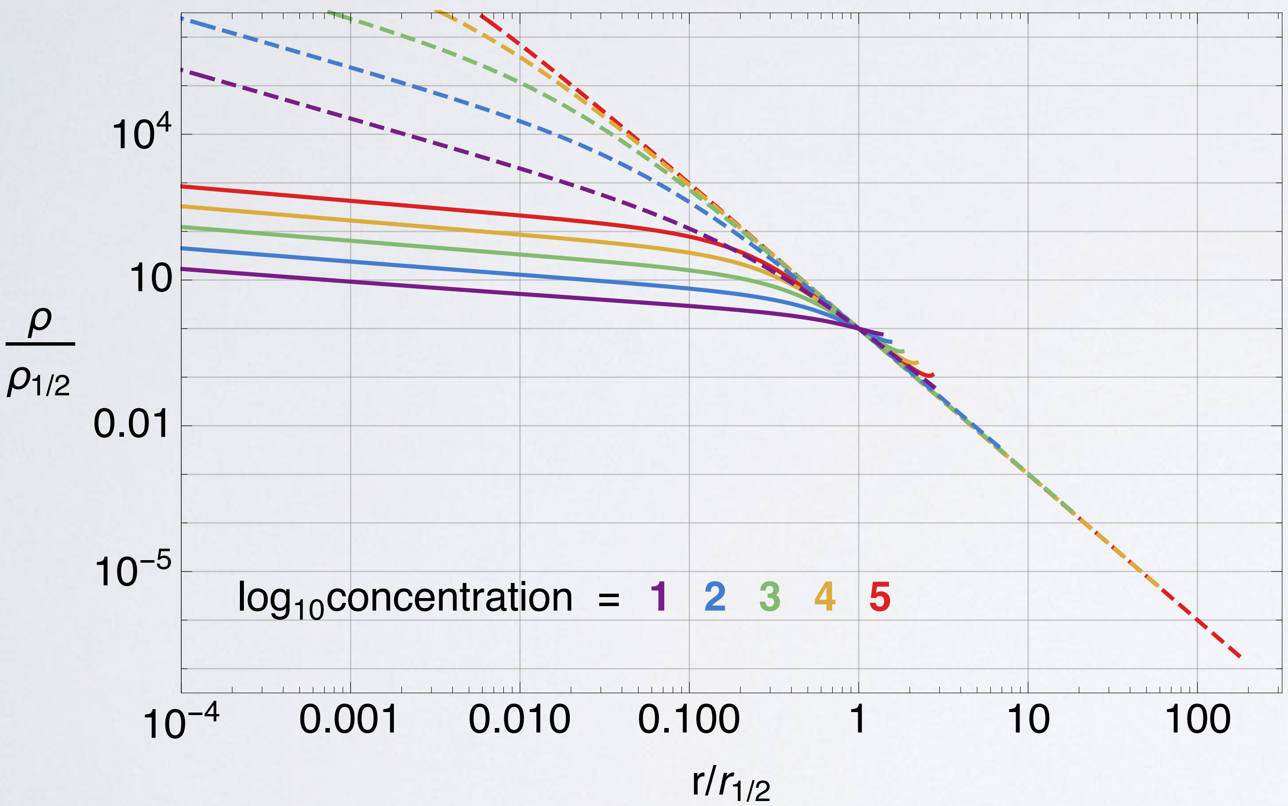
RECOLLAPSE

After entering the LMD the inhomogeneous LM distribution given by the superposition of halos will again grow and eventually recollapse to form new halos not simply related to the halos which existed during EMD.

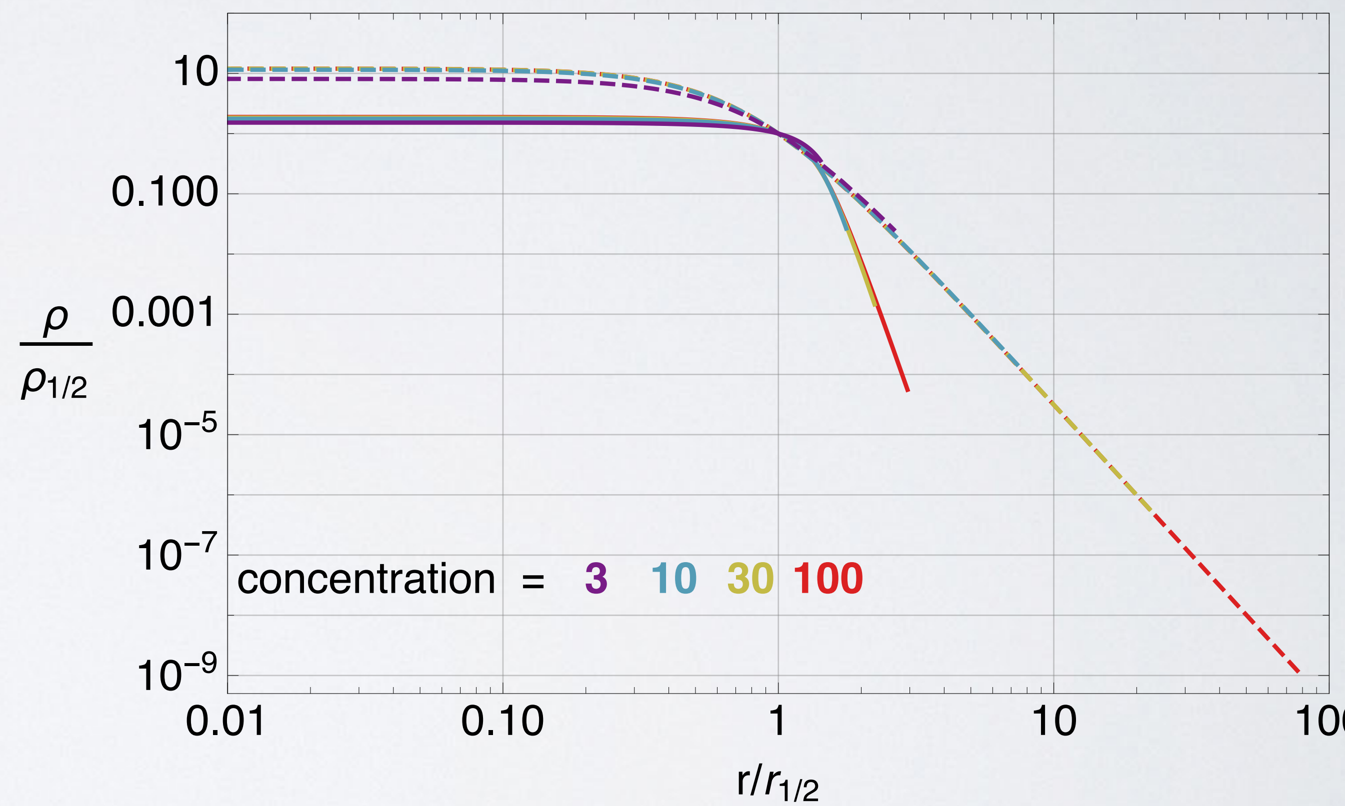
HALO REMNANTS

Semi-Universality: EDM halos evolve to remnants with density profiles close to top hat profiles irrespective of the initial halo density profile.

NFW profile



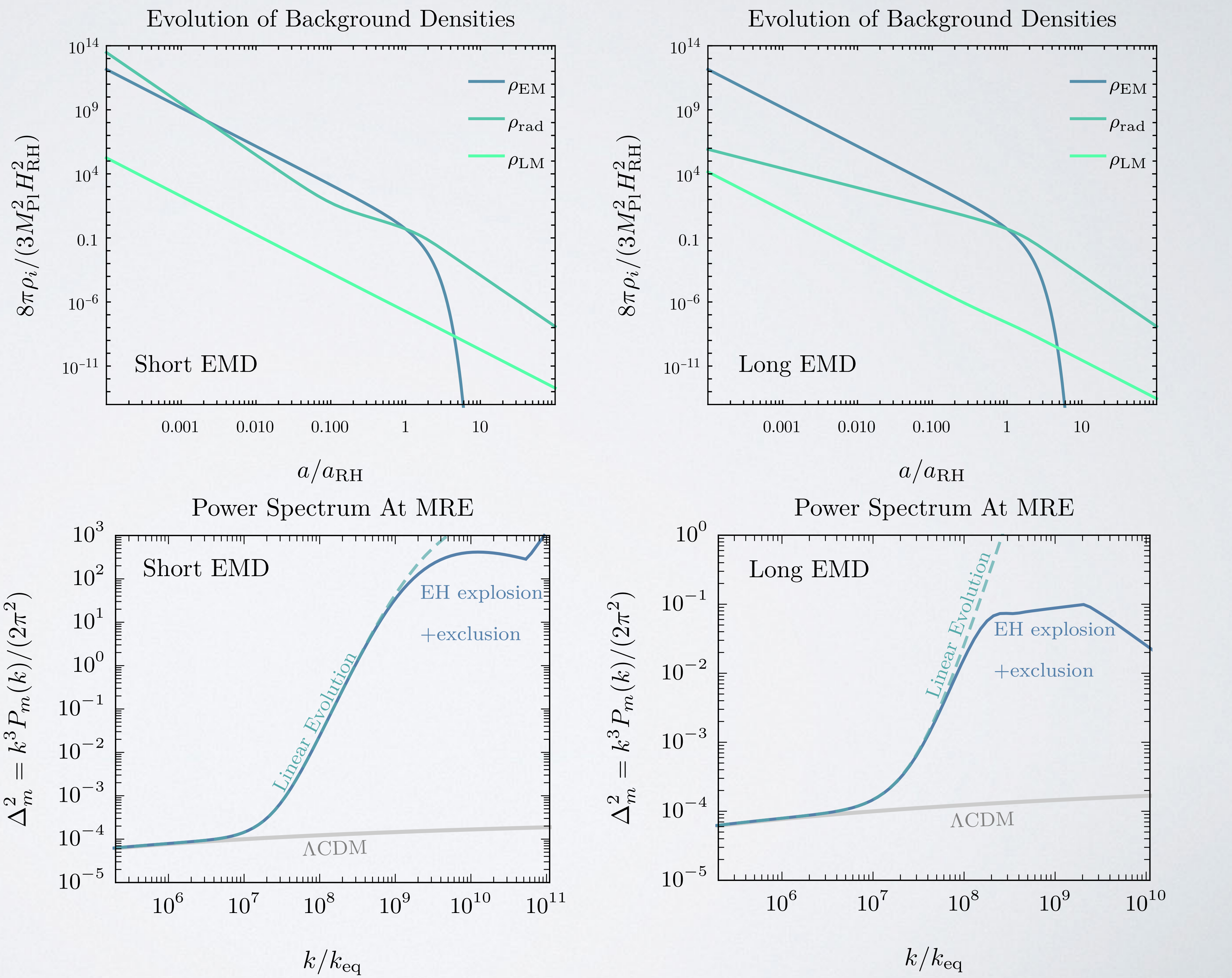
Plummer profile



EXTRAPOLATED ΛCDM

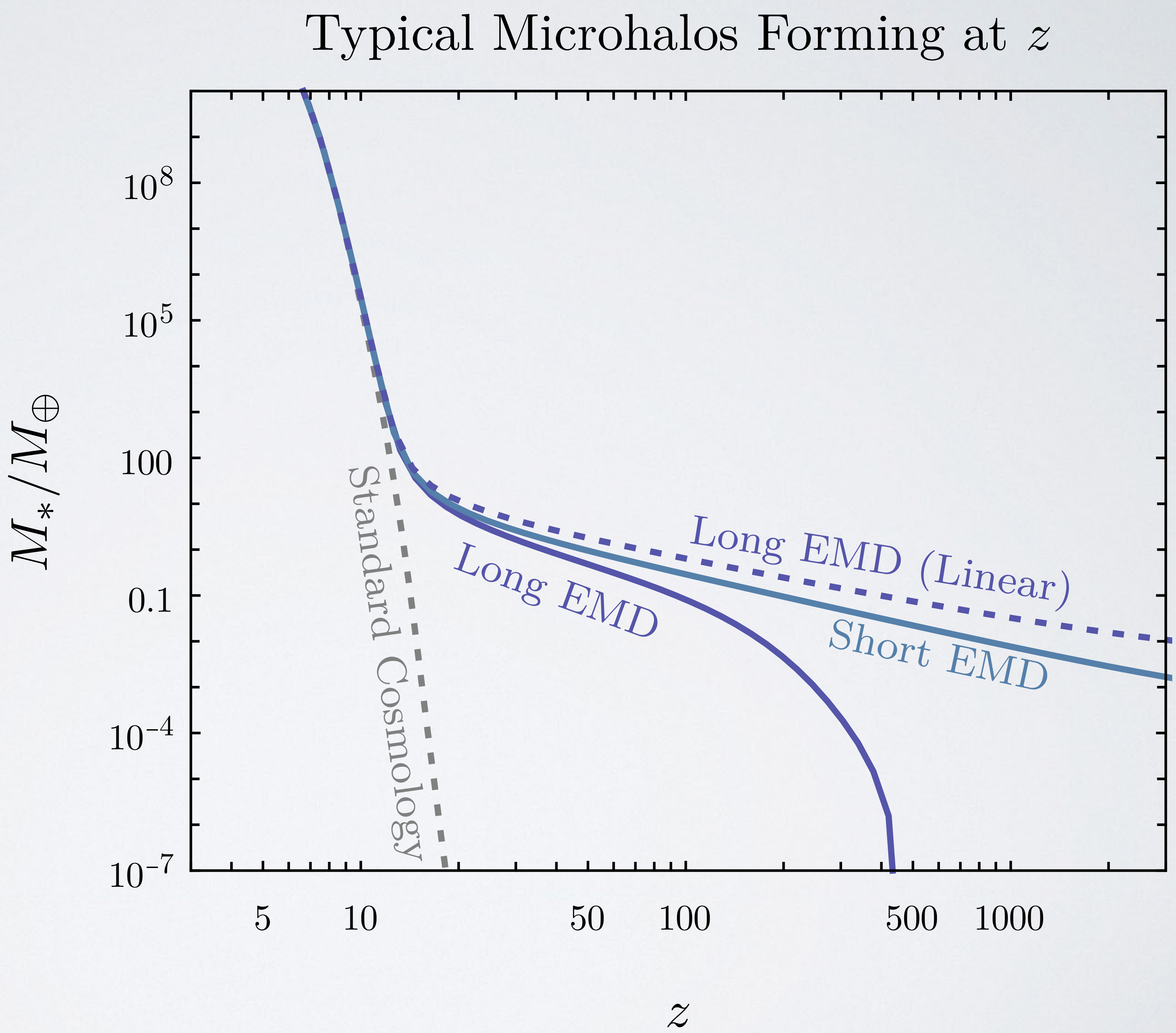
Up to this point we have been agnostic as to how halos formed during EMD. Now assume they formed from adiabatic perturbations of ΛCDM extrapolated to very small scales. In 2 cosmologies:

- Press-Schechter theory used to generate distribution of halos during EMD.
- Final power spectrum always smaller than linear theory.
- The longer the period of EMD the greater the suppression.
- Non-linearities suppress rather than enhance small scale structure in the case that gravitational collapse occurs during EMD.



MICROHALO MASS FUNCTION

- Press-Schechter theory used to estimate distribution of halos formed during LMD from $P(k)$.
  - ✳ due to multi-streaming this isn't really cold dark matter and there are "issues" with Press-Schechter analysis which may over-estimate micro halo densities.
- In these two cosmologies dark matter microhalos with $M \lesssim M_\oplus$ will form at $z > 100$.
  - ✳ subsequent tidal interactions and stellar encounters will strip away much of the mass of microhalos in our Galaxy by today for(Delos).



A BRIEF HISTORY OF NONLINEAR EMD HALOS

- **stable clustering:** individual halo retain constant physical size during EMD, $a \lesssim 0.3\, a_{\rm RH}$. Self gravity dominates and EM has not yet undergone significant decay.
- **adiabatic expansion:** individual halos grow exponentially in physical size retaining their profile during the EMD/LRD transition, $0.3\, a_{\rm RH} \lesssim a \lesssim 10\, a_{\rm RH}$, while a significant fraction of EM has decayed but self gravity still dominates.
- **peeling:** successive outer layers of the halo end adiabatic expansion and begin free expansion soon after LRD begins, $2\, a_{\rm RH} \lesssim a \lesssim 10\, a_{\rm RH}$. The longer orbital timescale of outer layers become comparable to the expansion time invalidating the adiabatic approximation.
- **free expansion:** individual halos grow logarithmically in comoving size during LRD, $2\, a_{\rm RH} \lesssim a \lesssim a_{\rm eq}$. Nearly all EMD has decayed and self-gravity is unimportant. The remnant LM moves ballistically in a radiation dominated universe.
- **halo overlap:** the ballistically expanding halo remnants will overlap with neighboring halos during LRD, $2\, a_{\rm RH} \lesssim a \lesssim a_{\rm eq}$. Rapid expansion evacuates the LM from the initial halo position but this is mostly filled in by LM from neighboring expanding halos.
- **recollapse:** the inhomogeneous LM distribution created by the superposition of overlapping halo remnants will recollapse to form new structures during LMD, $a \gtrsim a_{\rm eq}$, when self gravity of the LM dominates over the LR.

SPACE-TIME JITTER FROM OF ULTRA-LIGHT DARK MATTER

GRAVITATIONAL EFFECTS

Suppose the dark matter is a real (psuedo-)scalar. It's gravitational field is given by it's stress-energy

$$T_{\mu\nu} = \begin{pmatrix} \frac{1}{2}\left(\dot{\phi}^2 + |\nabla\phi|^2 + {m_\phi}^2\,\phi^2\right) & \dot{\phi}\,\nabla\phi \\ \dot{\phi}\,\nabla\phi & \frac{1}{2}\left(\dot{\phi}^2 - \frac{1}{3}\,|\nabla\phi|^2 - {m_\phi}^2\,\phi^2\right)\boldsymbol{I} + \left(\nabla\phi\otimes\nabla\phi - \frac{1}{3}\,|\nabla\phi|^2\,\boldsymbol{I}\right) \end{pmatrix}$$

Unlike for massive dark matter particles for ultra light (fuzzy) dark matter the length scales are macroscopic and the timescales easily resolvable.

STANDING WAVE

Consider the standing wave

$$\phi[t, \boldsymbol{x}] = \varphi \, \mathrm{Cos}[\varpi[\boldsymbol{\kappa}]\, t]\, \mathrm{Cos}[\boldsymbol{\kappa} \cdot \boldsymbol{x}]$$

where $\varpi^2 = m_\phi{}^2 + \kappa^2$, $\kappa \equiv |\,\boldsymbol{\kappa}\,|$ and using $\hat{\boldsymbol{\kappa}} = \boldsymbol{\kappa} / \kappa$

$$T^{\phi}_{\mu\nu} = \begin{pmatrix} \rho & -S\,\hat{\boldsymbol{\kappa}} \\ -S\,\hat{\boldsymbol{\kappa}} & p_{\parallel}\, \hat{\boldsymbol{\kappa}} \otimes \hat{\boldsymbol{\kappa}} + p_{\perp} \left(\boldsymbol{I} - \hat{\boldsymbol{\kappa}} \otimes \hat{\boldsymbol{\kappa}}\right) \end{pmatrix}$$

where

$$\rho = \tfrac{1}{4}\, \varphi^2 \left(\varpi^2 + \left(m_\phi{}^2 - \kappa^2\, \mathrm{Cos}[2\,\varpi\, t]\right) \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\right)$$

$$S = -\tfrac{1}{4}\, \varphi^2\, \kappa\, \varpi\, \mathrm{Sin}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\, \mathrm{Sin}[2\,\varpi\, t]$$

$$p_{\parallel} = \tfrac{1}{4}\, \varphi^2 \left(\kappa^2 - \left(m_\phi{}^2 + \varpi^2\, \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\right) \mathrm{Cos}[2\,\varpi\, t]\right)$$

$$p_{\perp} = \tfrac{1}{4}\, \varphi^2 \left(\kappa^2\, \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}] - \left(\varpi^2 + m_\phi{}^2\, \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\right) \mathrm{Cos}[2\,\varpi\, t]\right)$$

The synchronous gauge linearized metric perturbation is

$$\boldsymbol{h} = -2\,\Phi\,\boldsymbol{I} + \nabla \otimes \nabla \mathcal{P} + \nabla \boldsymbol{\mathcal{V}} + (\nabla \boldsymbol{\mathcal{V}})^T \;+ \boldsymbol{h}^{(\mathrm{T})}$$

$$= \pi\, G\, \varphi^2 \left(\frac{2\, m_\phi{}^2 + \kappa^2}{2\,\kappa^2}\, \boldsymbol{I} - \hat{\boldsymbol{\kappa}} \otimes \hat{\boldsymbol{\kappa}}\, \frac{\kappa^2}{m_\phi{}^2 + \kappa^2}\right) \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]$$

$$+ \pi\, G\, \varphi^2 \left(-\frac{1}{2}\, \boldsymbol{I} + \hat{\boldsymbol{\kappa}} \otimes \hat{\boldsymbol{\kappa}}\, \frac{m_\phi{}^2}{m_\phi{}^2 + \kappa^2}\right) \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\, \mathrm{Cos}[2\,\varpi\, t]$$

where we have used $\varpi[\boldsymbol{\kappa}]^2 = m_\phi{}^2 + |\,\boldsymbol{\kappa}\,|^2$.

In the non-relativistic limit, to 1st order in k

$$\rho \to \tfrac{1}{4}\, m_\phi{}^2\, \varphi^2 \left(1 + \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\right)$$

$$\boldsymbol{S} \to -\tfrac{1}{4}\, \boldsymbol{\kappa}\, \varphi^2\, m_\phi\, \mathrm{Sin}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\, \mathrm{Sin}[2\, m_\phi\, t]$$

$$p_{\parallel}, p_{\perp} \to -\tfrac{1}{4}\, m_\phi{}^2\, \varphi^2 \left(1 + \mathrm{Cos}[2\,\boldsymbol{\kappa} \cdot \boldsymbol{x}]\right) \mathrm{Cos}[2\, m_\phi\, t]$$

The linearized Raychaudhuri equation for expansion is

$$\dot{\theta} \cong -4\,\pi\, G\,(\rho + 3\,p) = 4\,\pi\, G\,\rho\,(3\,\mathrm{Cos}[2\, m_\phi\, t] - 1).$$

There is the usual converging trajectories

$$\overline{\theta}[\boldsymbol{x}, t] \cong \text{constant} - 4\,\pi\, G\,\rho[\boldsymbol{x}, t]\,(t - t_0)$$

(time averaged) plus a high frequency component

$$\tilde{\theta}[\boldsymbol{x}, t] \cong 6\,\pi\, \frac{G\,\rho[\boldsymbol{x}, t]}{m_\phi}\, \mathrm{Sin}[2\, m_\phi\, t]$$

giving an oscillating isotropic strain

$$\tilde{h}[\boldsymbol{x}, t] \cong 3\,\pi\, \frac{G\,\rho[\boldsymbol{x}, t]}{m_\phi{}^2}\, \mathrm{Cos}[2\, m_\phi\, t].$$

The shear is much smaller.

For the local dark matter density

$$\tilde{h} \sim 6 \times 10^{-30} \left(\frac{\mathrm{Hz}}{m_\phi}\right)^2$$

GW-like detectors might be able to see this someday.