Testing the equivalence principle on cosmological scales

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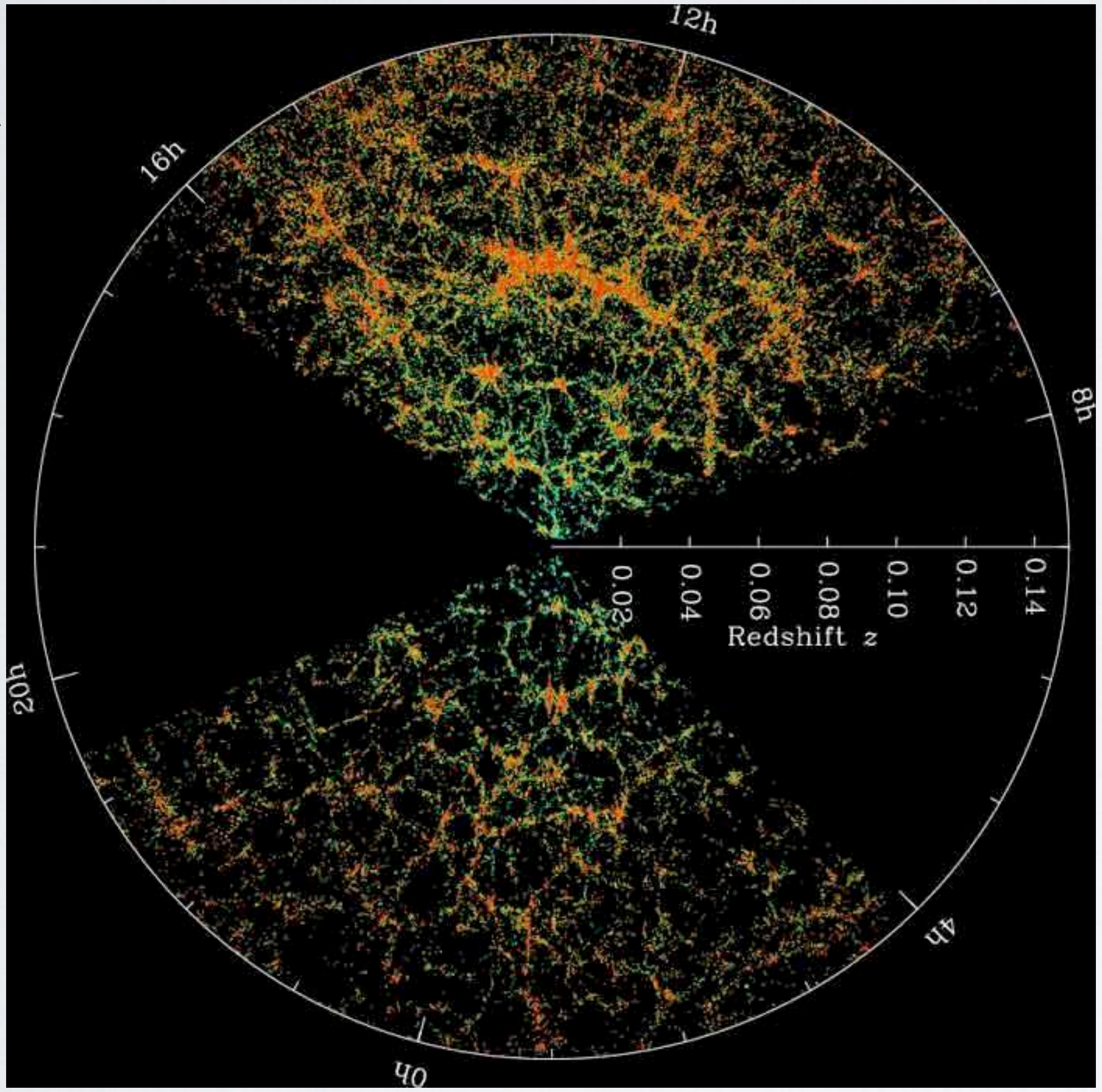
Galaxy survey

The **distribution** of galaxies is sensitive to:



Credit: M. Blanton, SDSS

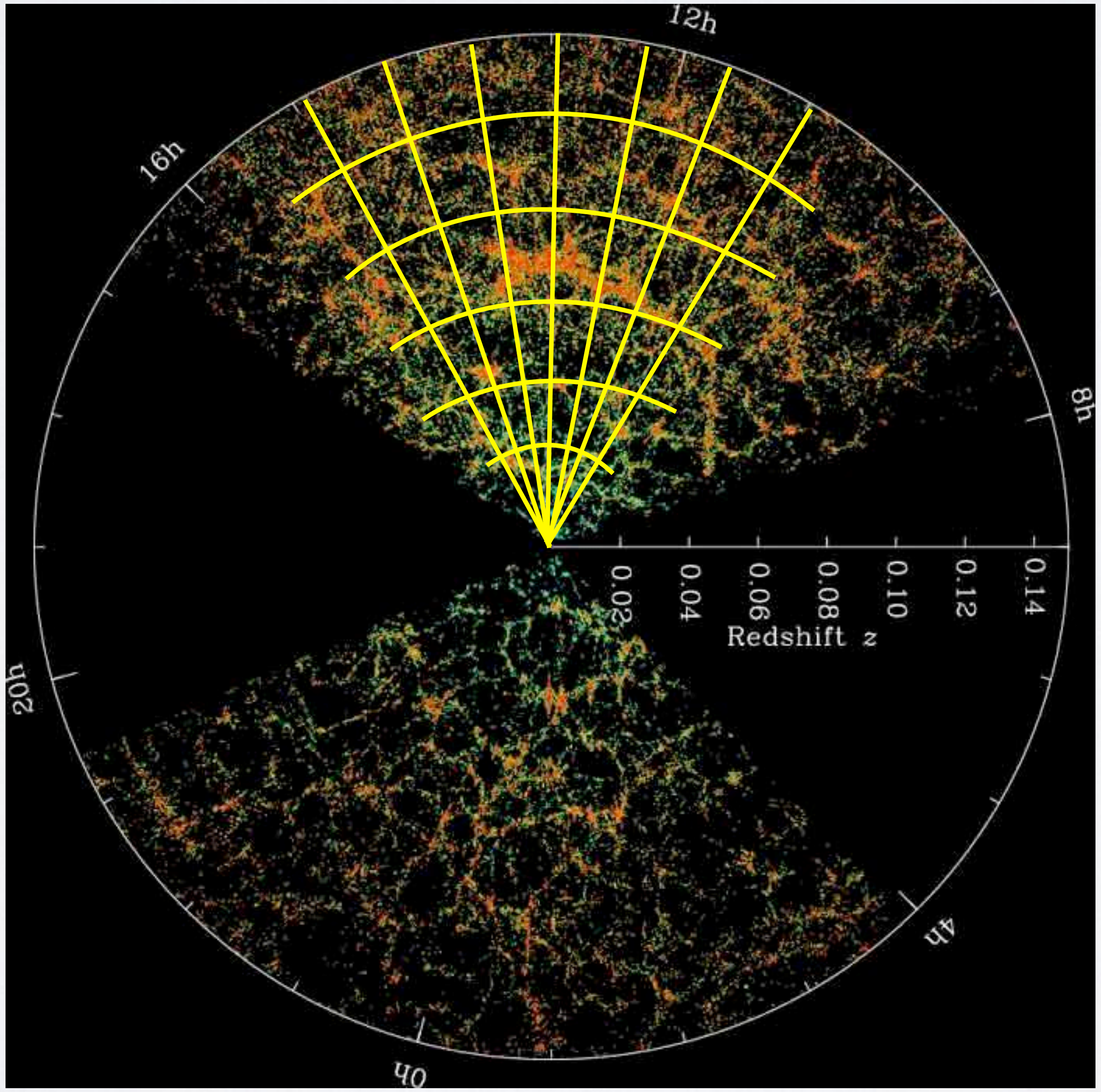
- the initial conditions
- the theory of gravity
- the content of the universe

→ The large-scale structure contains valuable **information**

To interpret properly this information, we need to understand **what** we are **measuring**.

Galaxy survey

- We count the number of **galaxies** per **pixel**: $\Delta = \dfrac{N - \bar{N}}{\bar{N}}$
- How is Δ related to: the initial conditions, the theory of gravity and dark energy?

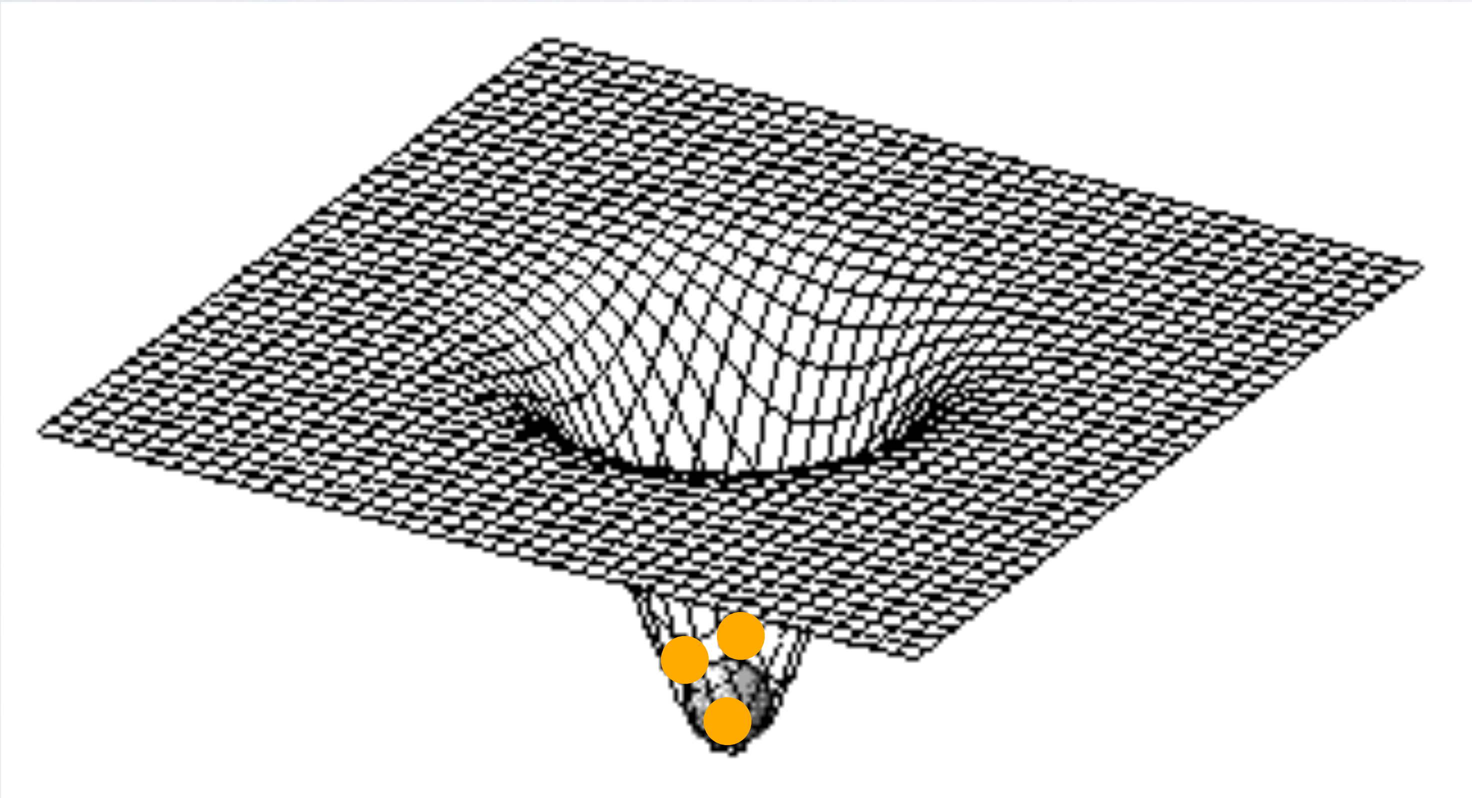


Credit: M. Blanton, SDSS

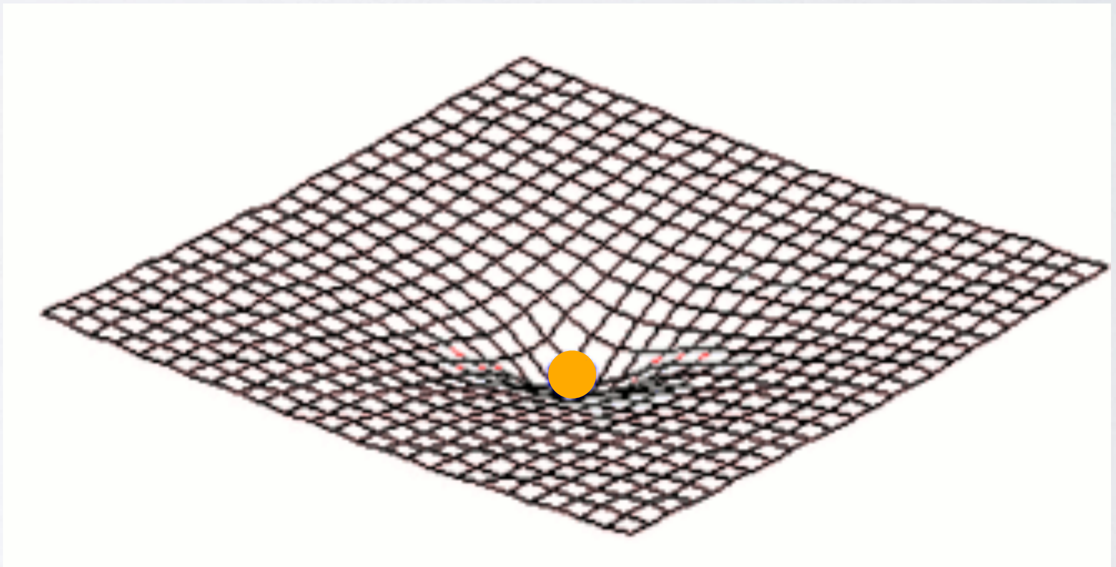
Galaxy distribution

Simple picture:
- ♦ **dark matter** is not homogeneously distributed
- ♦ it creates **gravitational potential** wells
- ♦ **baryons** fall into them and form galaxies

More dark matter



Less dark matter



$$\Delta = \frac{\delta\rho}{\bar{\rho}} \equiv \delta$$

ρ dark matter energy density

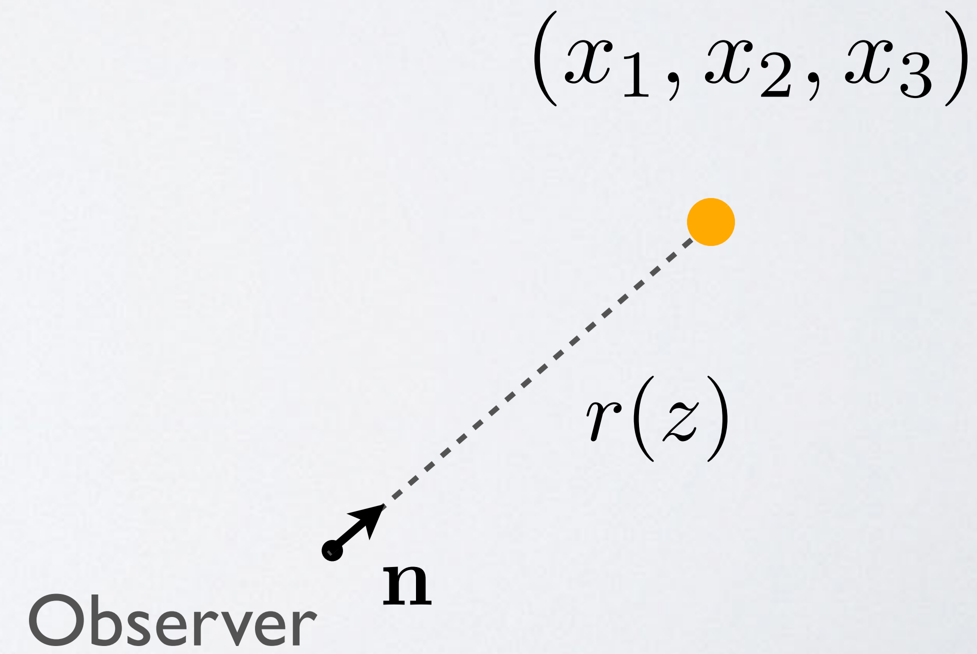
Complications

- **Bias**: the distribution of galaxies does not trace directly the distribution of dark matter $\Delta = b \cdot \delta$

- We never observe directly the **position** of galaxies, we observe the **redshift** z and the **direction** of incoming photons $\mathbf{n}$.

In a **homogeneous** universe:

- we calculate the distance $r(z)$
- light propagates on straight lines

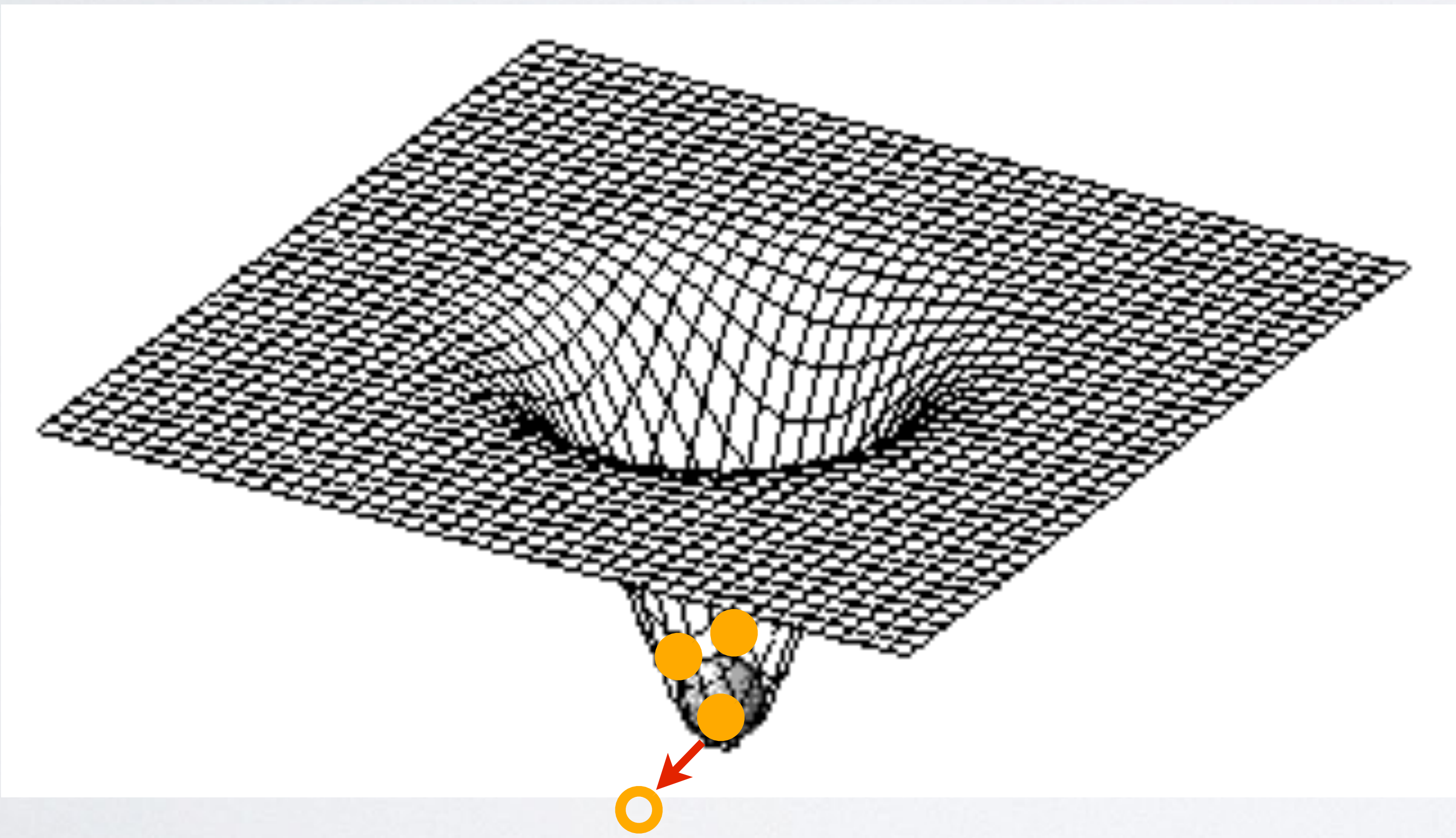


Distortions: radial

The trajectory of the **photons** emitted by the galaxies is **distorted** by the structures along the way.

→ Distortions in our coordinates: example Doppler effect.

More dark matter



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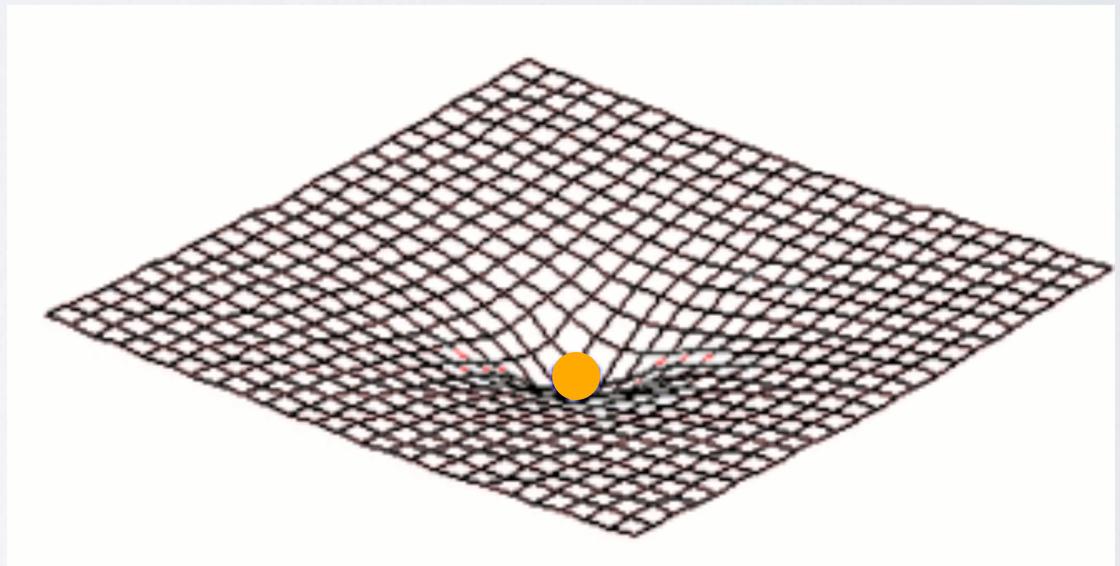


Redshift distortions

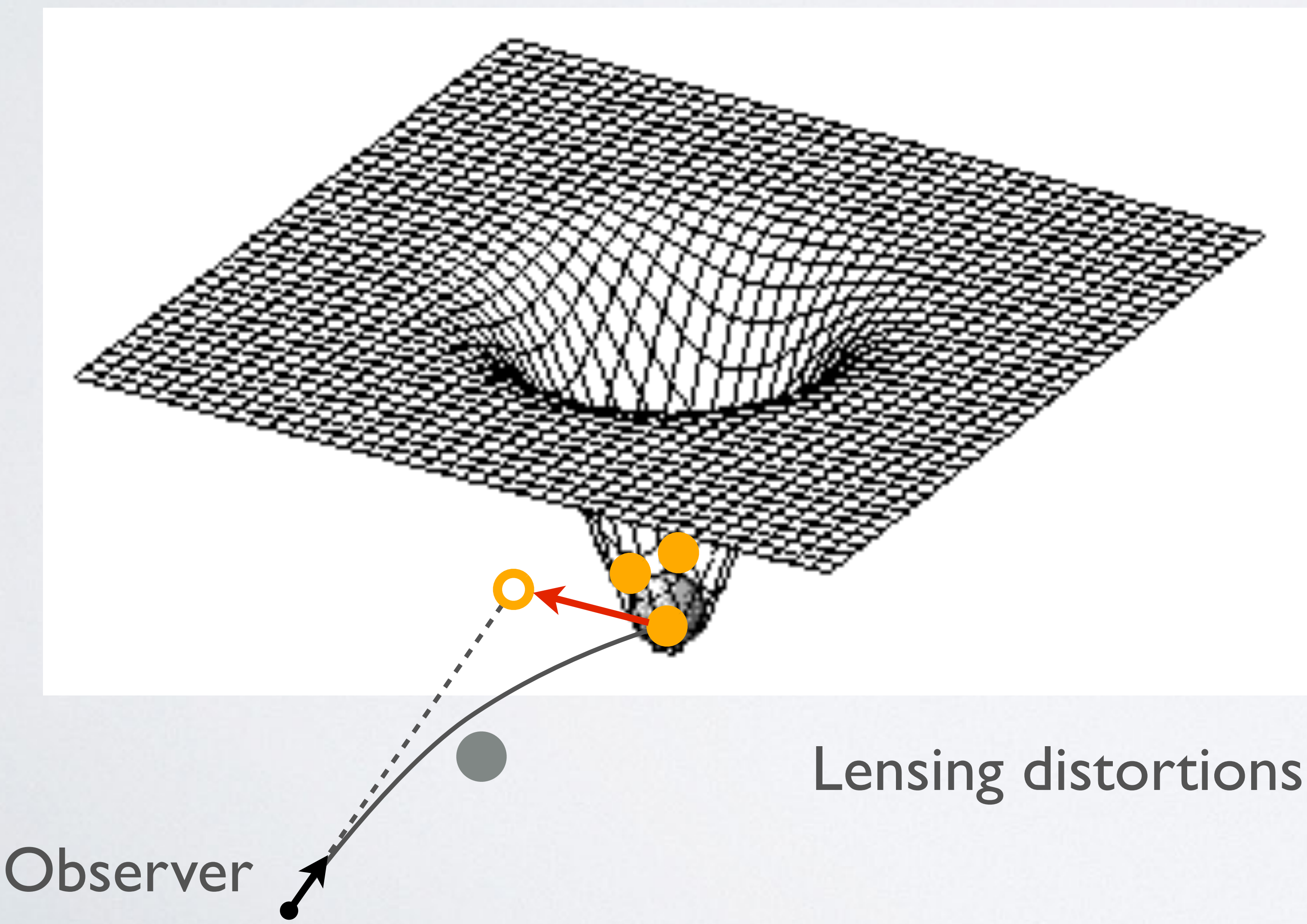
Observer

Distortions: transverse

The trajectory of the **photons** emitted by the galaxies is **distorted** by the structures along the way.

→ Distortions in our coordinates: example lensing effect.

More dark matter



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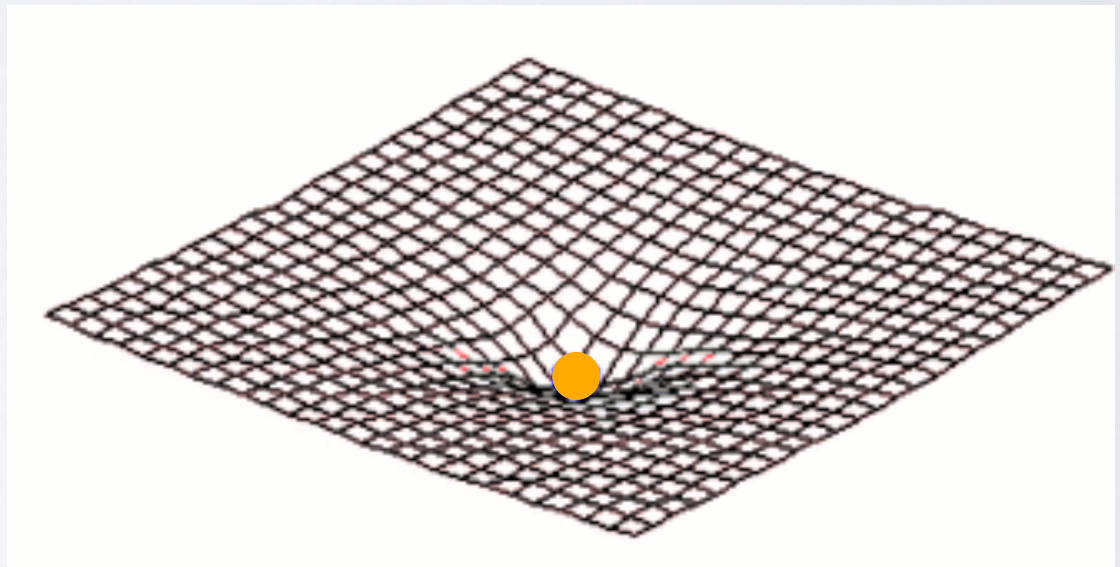
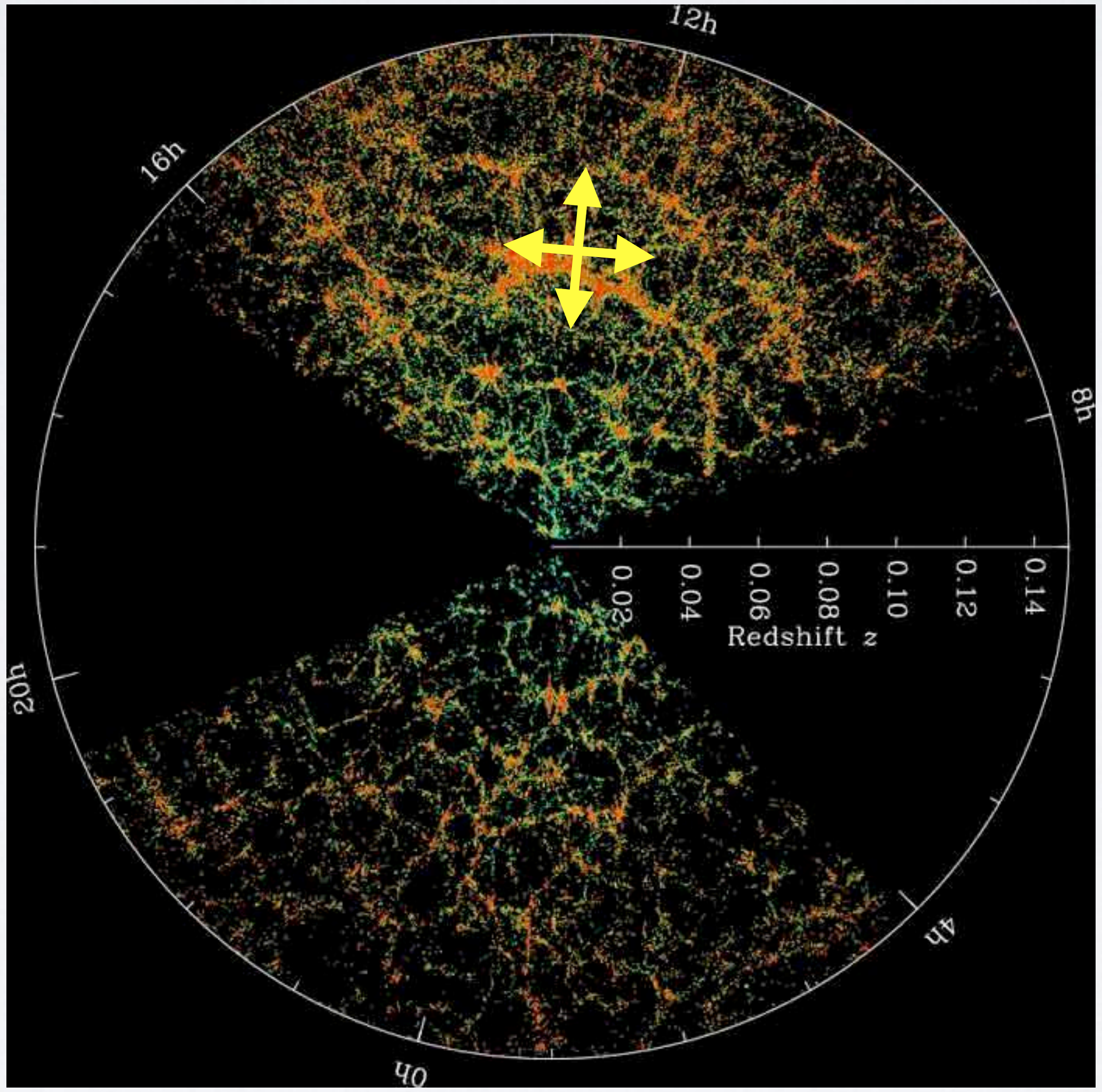


Galaxy distribution

The **structures** seen on a galaxy map do **not reflect** directly the underlying dark matter structures. The observed **position** of galaxies are **shifted** radially and transversally.

Credit: M. Blanton, SDSS



To extract **information** from a galaxy map, we need to understand exactly which **distortions** there are.

Outline

- Expression for Δ encoding all **distortions** at linear order

$$\Delta = \text{density} + \text{redshift distortions} + \text{lensing} + \text{relativistic effects}$$

- The **relativistic** effects contains additional information

- We can use them to test for **deviations** from **General Relativity** in a model-independent way

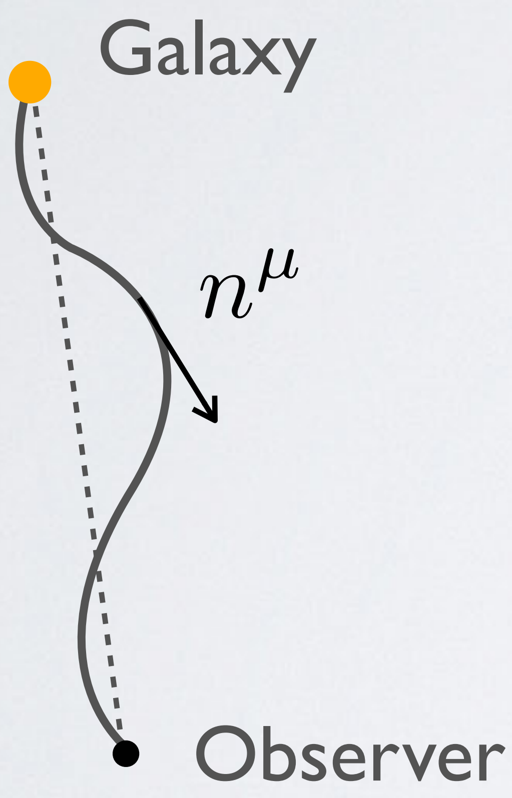
  - **Euler** equation

  - **Anisotropic stress**

Calculation of the distortions

Perturbed Friedmann universe:

$$ds^2 = -a^2\Big[(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\Big]$$



We calculate the **propagation** of **photons**, i.e. the null geodesics and infer:

- the change in **energy**
- the change in **direction**

→ distortions in $(z, \mathbf{n})$

What we really observe

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$
\begin{aligned}
\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
+ (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi) \\
+ \left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
+ \frac{2-5s}{r}\int_0^r dr'(\Phi+\Psi) + 3\mathcal{H}\nabla^{-2}(\nabla\mathbf{V}) + \Psi + (5s-2)\Phi \\
+ \frac{1}{\mathcal{H}}\dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s\right)\left[\Psi + \int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}
$$

What we really observe

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

$$\Delta(z,\mathbf{n}) = b\cdot\delta \; - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Velocities

$$+ (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$

$$+\left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

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Lensing

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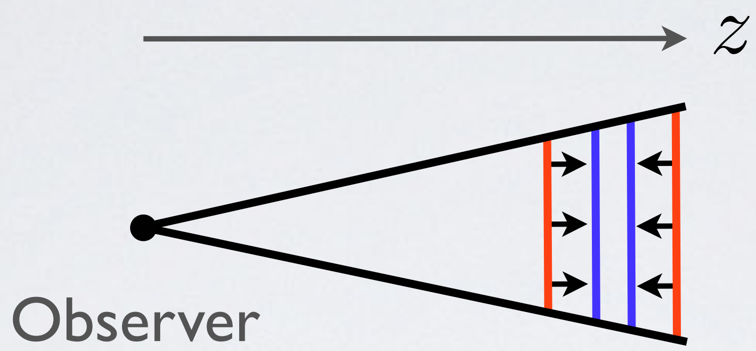
Potentials

Distortions

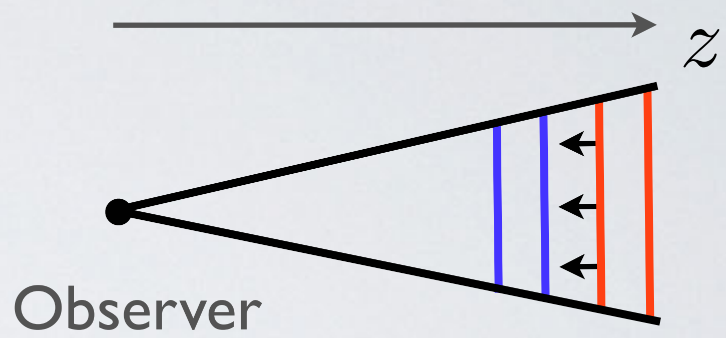
♦ Velocities

Kaiser (1987), Lilje & Efstathiou (1989), Hamilton (1992)

Change in the bin size:
Redshift distortions



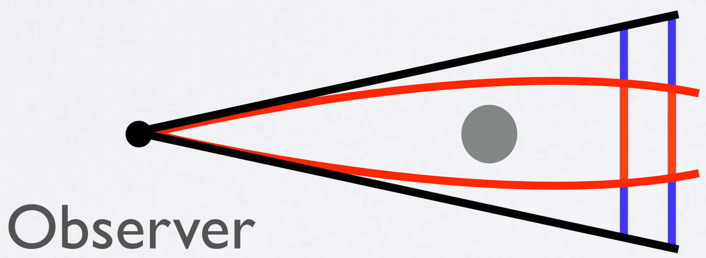
Change in the bin position:
Doppler effect



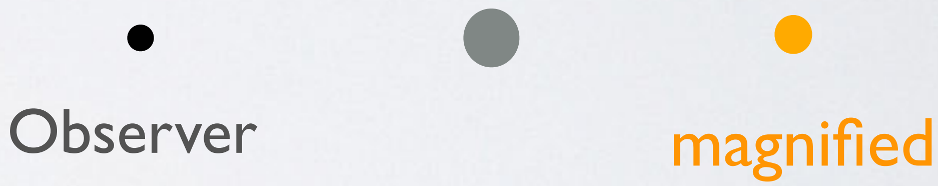
♦ Lensing

Gunn (1967), Schneider (1989), Broadhurst, Taylor & Peacock (1995)

Change in the solid angle



Change in the flux

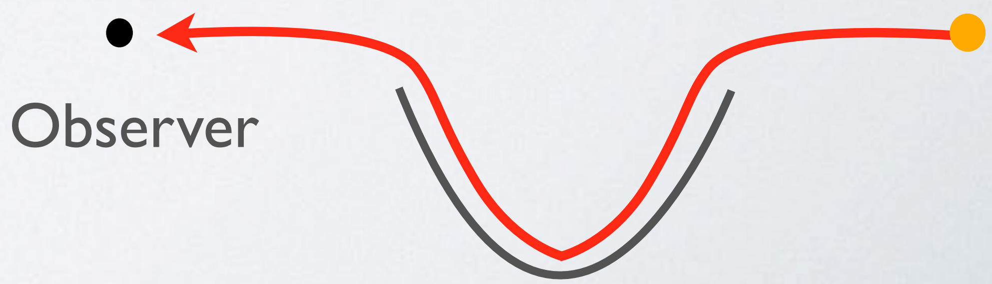


♦ Potentials

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

Local terms:
e.g. gravitational redshift

Integrated terms: e.g. Shapiro time-delay and Integrated Sachs-Wolfe



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Challinor and Lewis (2011)

$$\Delta(z,\mathbf{n}) = b\cdot\delta \; -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Velocities

$$+(5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$

Lensing

$$+\left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

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Potentials

What we really observe

redshift-space distortions

$$\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

current standard analyses

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What we really observe

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lensing: measured with quasars and relevant for future surveys

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relativistic effects

What we really observe

Relativistic effects:

$\Delta(z, \mathbf{n})$ Can we **measure** them with upcoming surveys? Do they bring new **information** compared to redshift-space distortions and lensing?

$$+\left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\partial_r\Psi$$

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relativistic effects

Measured quantities

- δ and V measured through **redshift-space** distortions
- $\Phi + \Psi$ measured with **quasars** or with **cosmic shear**
- We have 4 variables but only 3 measurements

In **General Relativity** we know the relations between these quantities. In general we want to **test** them.

δ continuity V

Poisson Euler

Φ anisotropic stress Ψ

Measured quantities

- δ and V measured through **redshift-space** distortions
- $\Phi + \Psi$ measured with **quasars** or with **cosmic shear**

We **cannot** test four relations with only three independent measurements

Current method: **assume** the **validity** of **Euler** and **continuity** equation and test other relations

	δ	continuity	V	
Poisson				Euler
	Φ	anisotropic stress	Ψ	

Measured quantities

- δ and V measured through **redshift-space** distortions
- $\Phi + \Psi$ measured with **quasars** or with **cosmic shear**

> **Relativistic effects** provide a measurement of the potential Ψ
>
> We can test **all relations** without assumptions

	δ	continuity	V
Poisson			Euler
	Φ	anisotropic stress	Ψ

What we really observe

$$\Delta(z,\mathbf{n}) = b\cdot\delta \;-\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

redshift-space distortions

$$+(5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi)$$

lensing

gravitational redshift

$$+\left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n}+\frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n}+\boxed{\frac{1}{\mathcal{H}}\partial_r\Psi}$$

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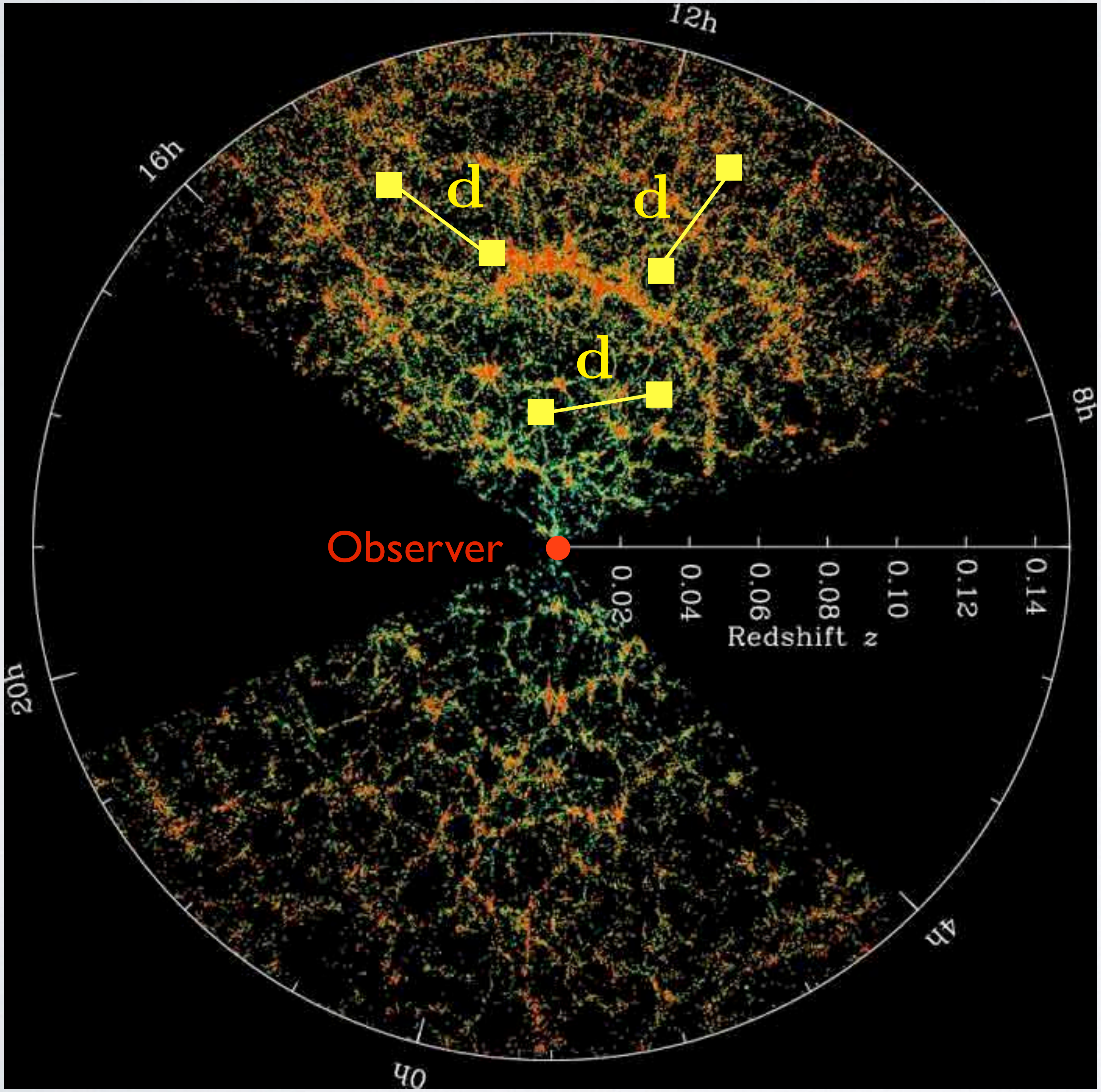
relativistic effects

Correlation function

Credit: M. Blanton, SDSS



$$\xi = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}')\rangle$$

The dark matter fluctuations generate isotropic correlations

$$\Delta = b \cdot \delta$$

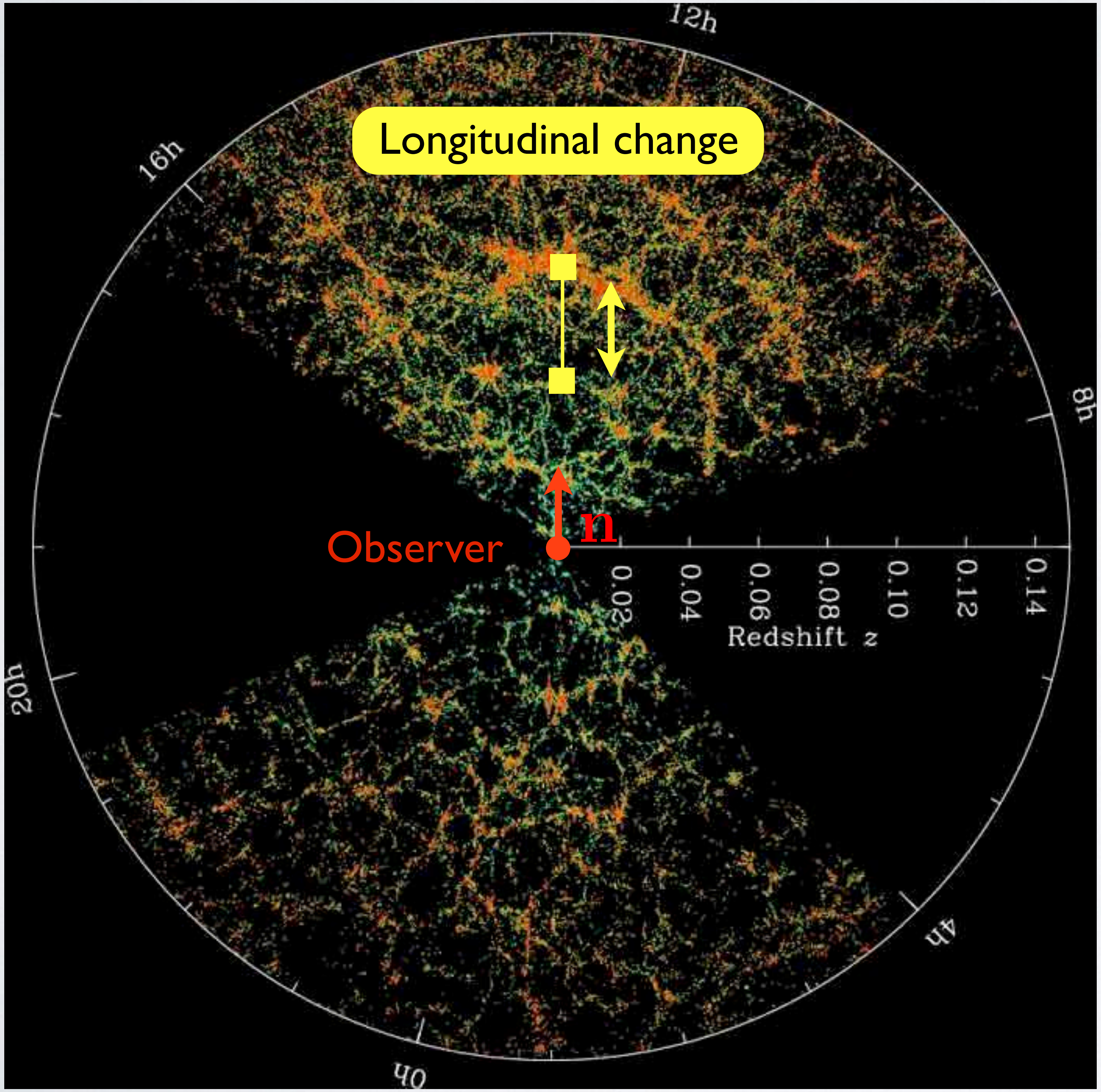
$$\xi(d) = C_0(d)$$

Redshift-space distortions

Kaiser (1987)
Hamilton (1992)

Redshift-space distortions **break** the **isotropy** of ξ.

Credit: M. Blanton, SDSS



$$\Delta = -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Changes the **redshift** separation but not the **angular** separation.

$$\xi = C_0(d) + C_2(d)P_2(\cos\beta) + C_4(d)P_4(\cos\beta)$$

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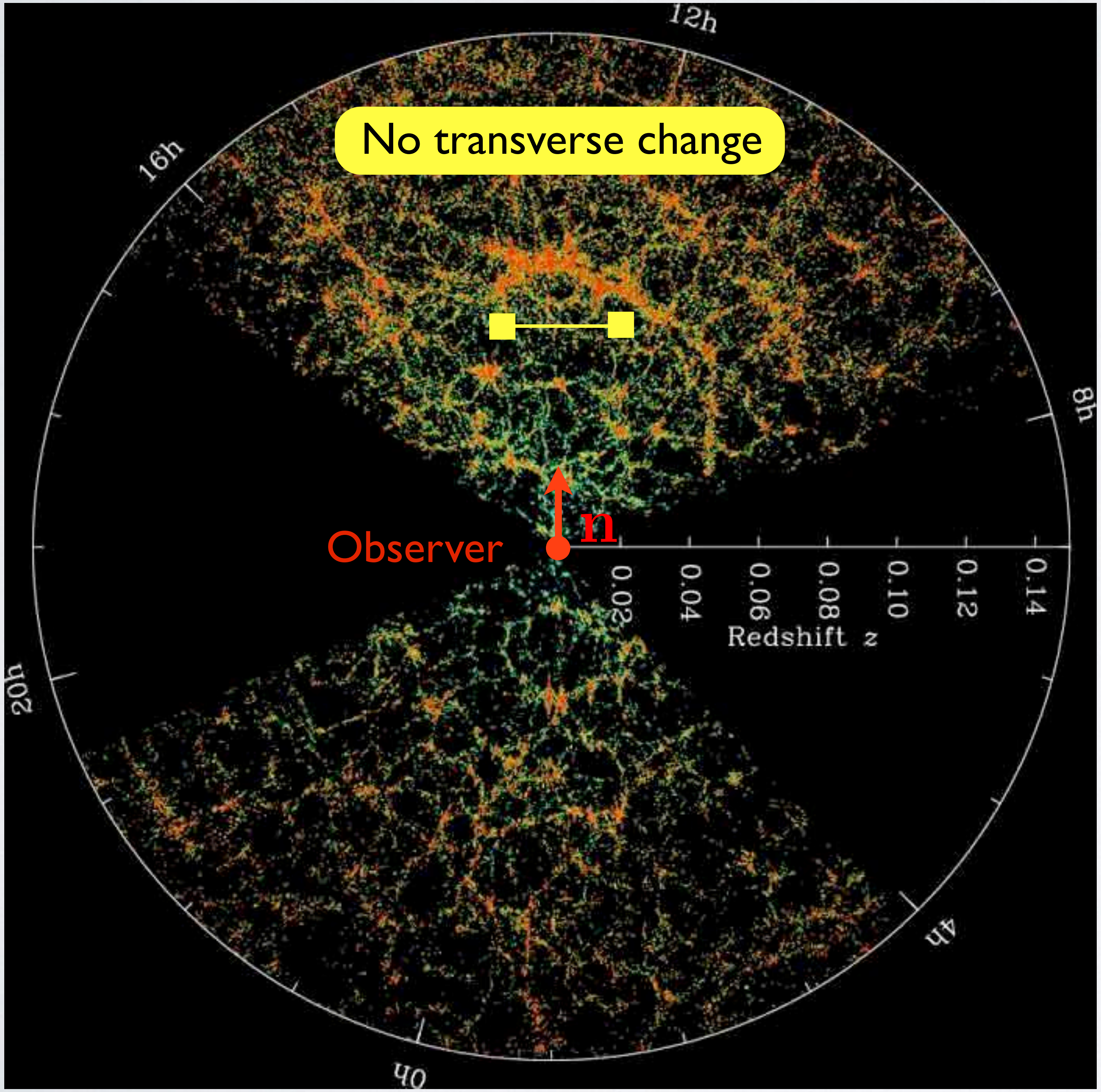

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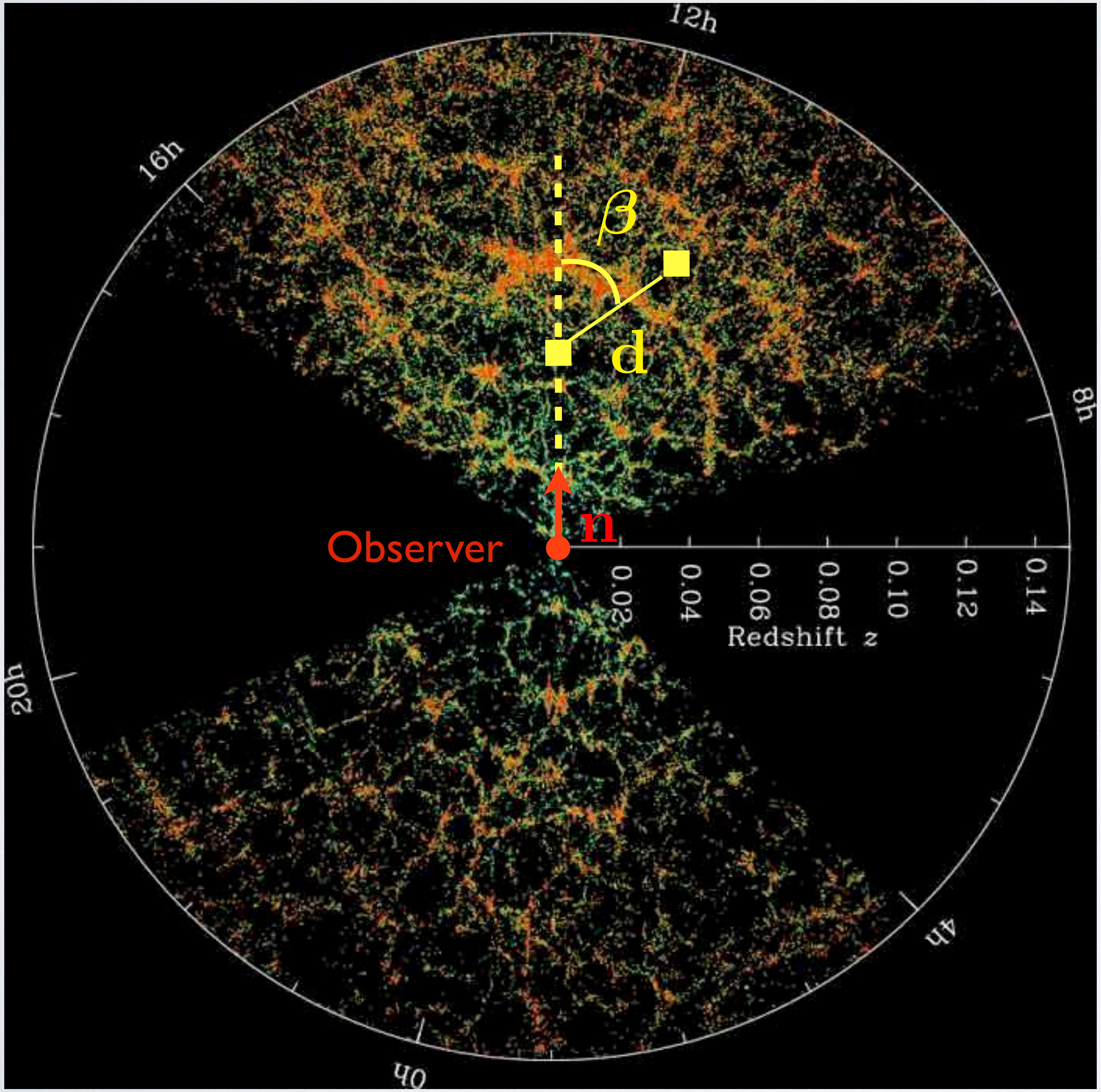

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Legendre polynomials

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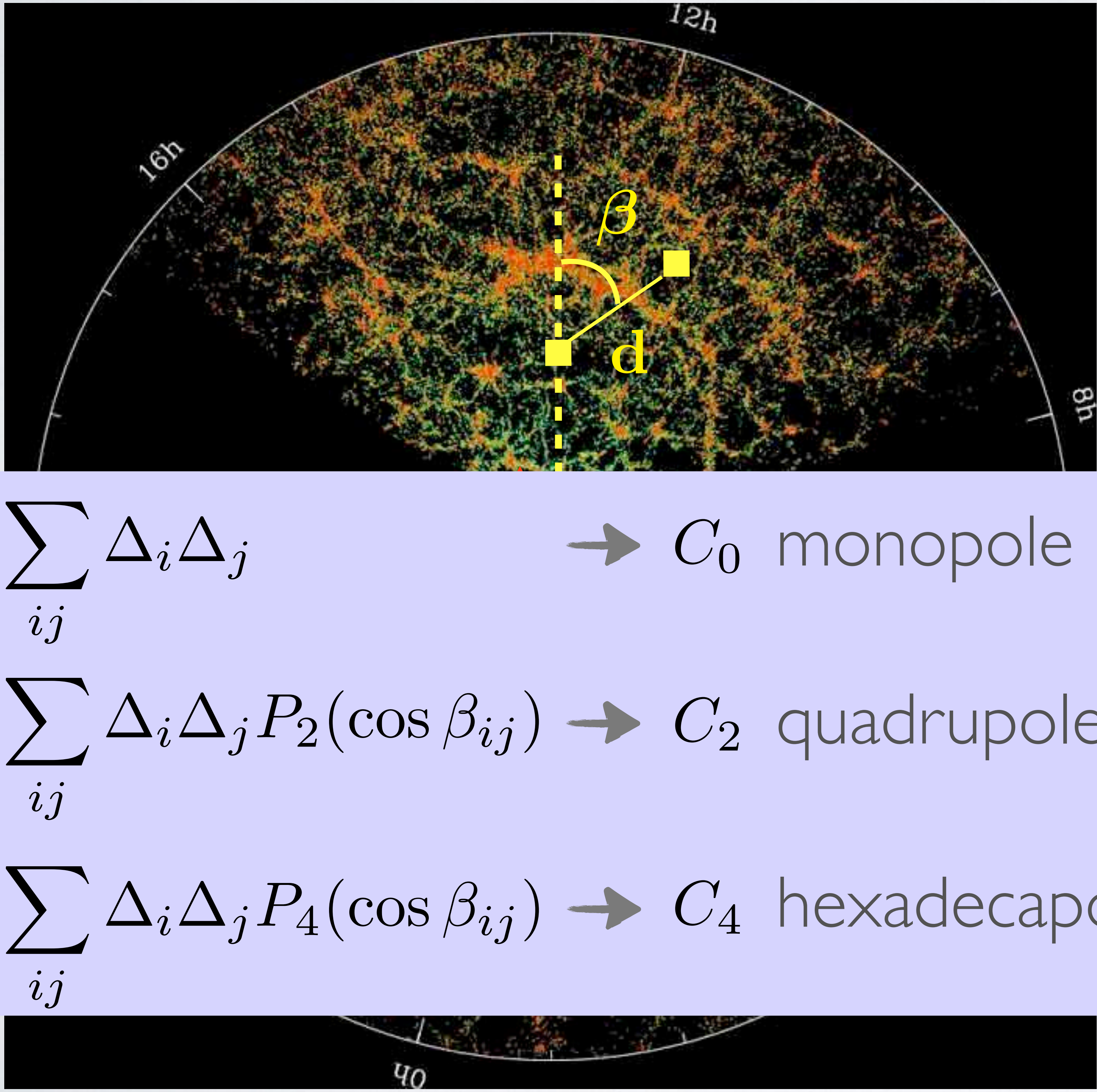

$$\sum_{ij}\Delta_i\Delta_j \rightarrow C_0 \text{ monopole}$$

$$\sum_{ij}\Delta_i\Delta_j P_2(\cos\beta_{ij}) \rightarrow C_2 \text{ quadrupole}$$

$$\sum_{ij}\Delta_i\Delta_j P_4(\cos\beta_{ij}) \rightarrow C_4 \text{ hexadecapole}$$

$$\Delta = -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Changes the **redshift** separation but not the **angular** separation.

$$= C_0(d) + C_2(d)P_2(\cos\beta) + C_4(d)P_4(\cos\beta)$$

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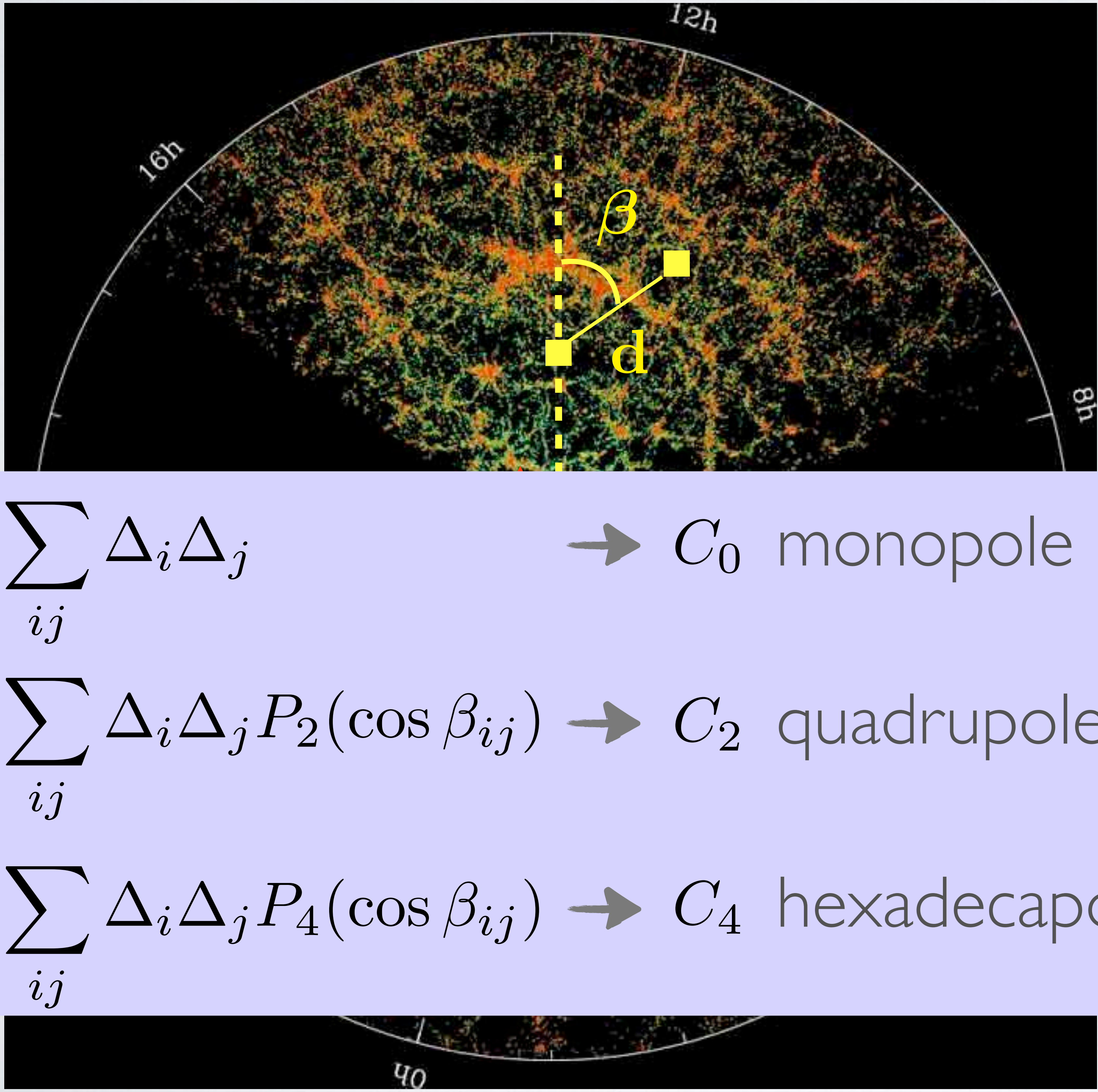


$$\Delta = -\frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n})$$

Changes the **redshift** ... the

$\sum_{ij}\Delta_i\Delta_j$ → C_0 monopole

$\sum_{ij}\Delta_i\Delta_j P_2(\cos\beta_{ij})$ → C_2 quadrupole

$\sum_{ij}\Delta_i\Delta_j P_4(\cos\beta_{ij})$ → C_4 hexadecapole

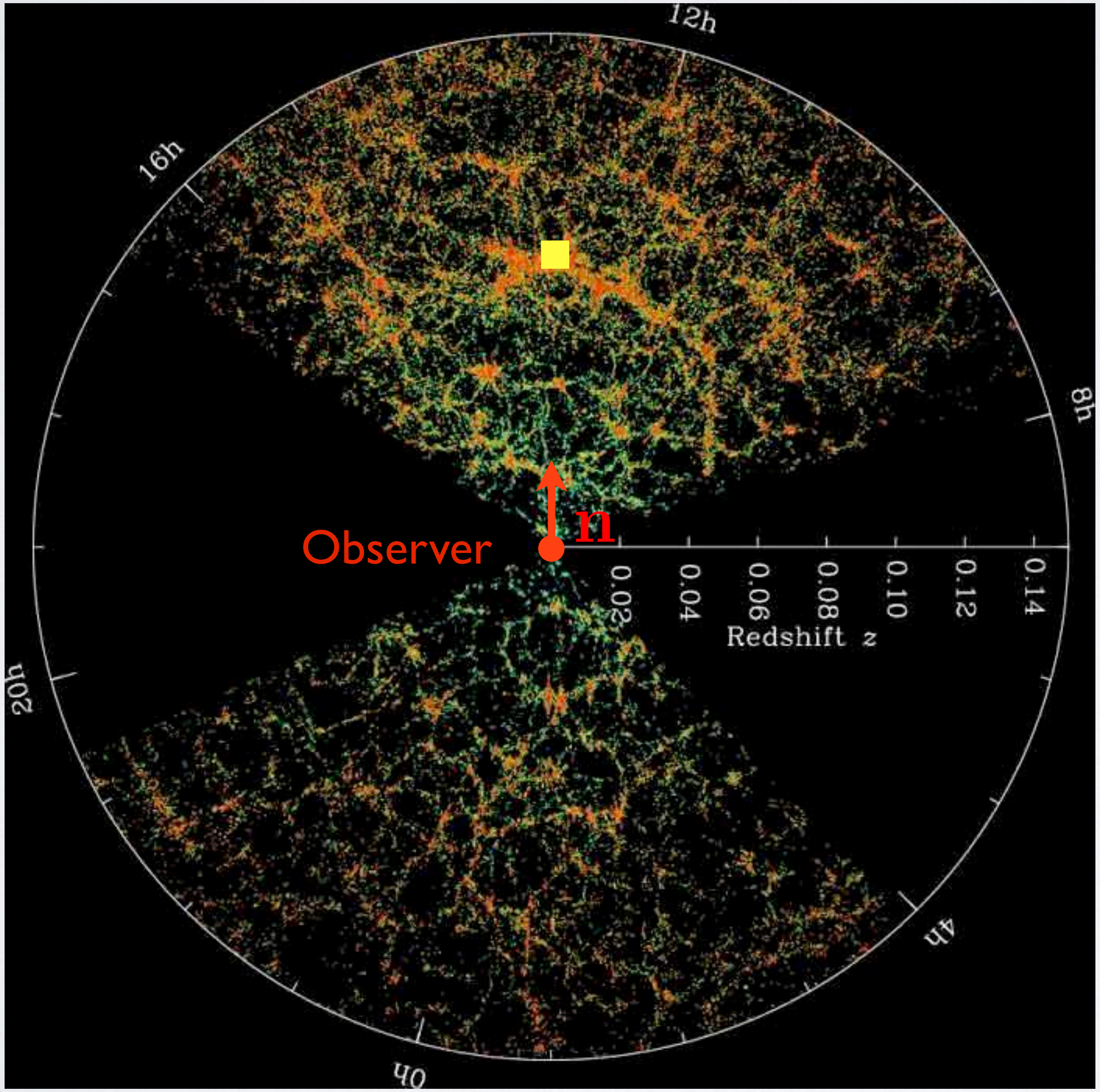
} measure δ and V

BOSS, Wigglez

Relativistic effects

Relativistic effects **break** the **symmetry** of ξ

Credit: M. Blanton, SDSS



Gravitational redshift

$$\Delta = \frac{1}{\mathcal{H}}\partial_r \Psi$$

CB, Hui & Gaztanaga (2014)

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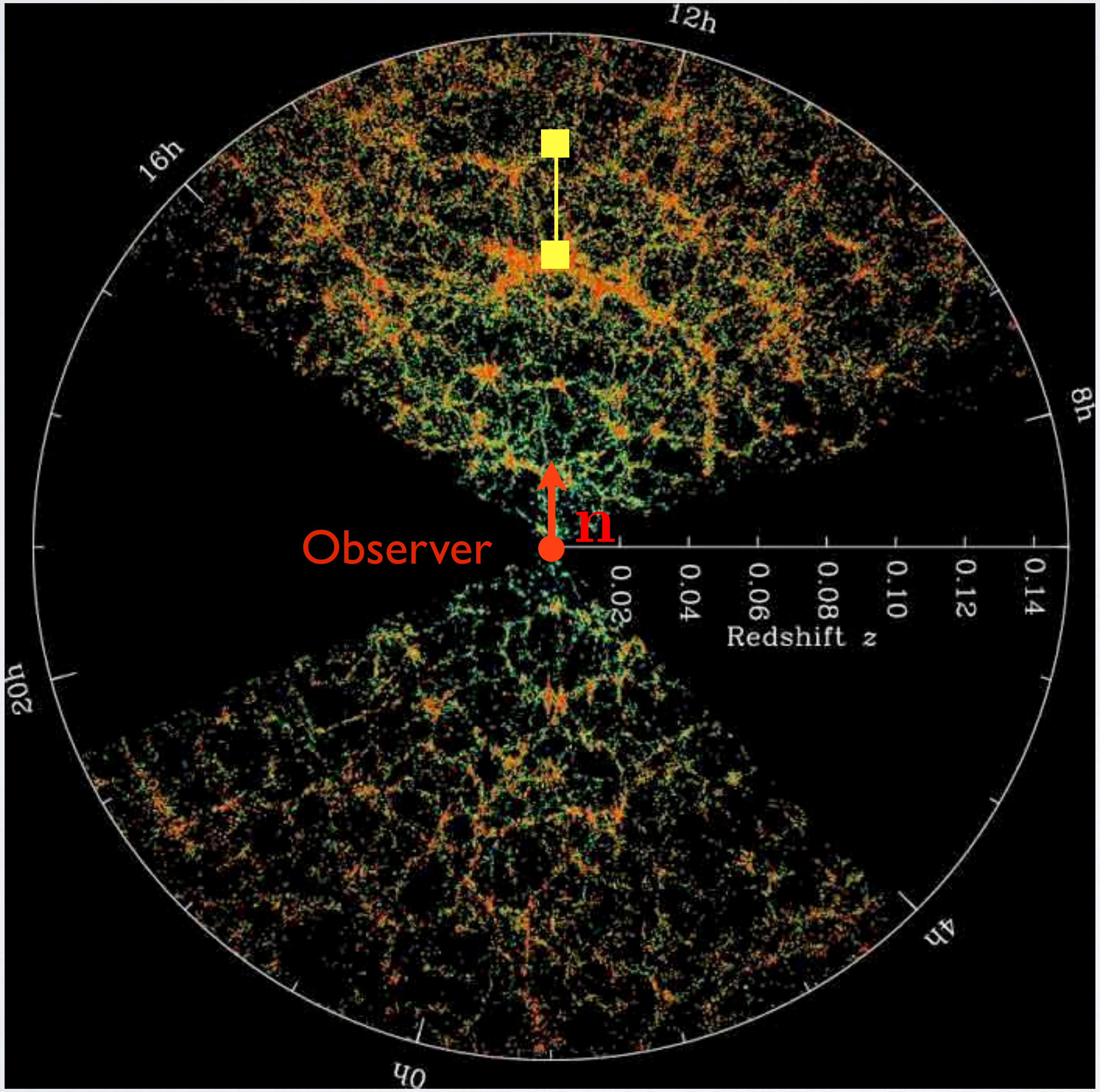
Credit: M. Blanton, SDSS



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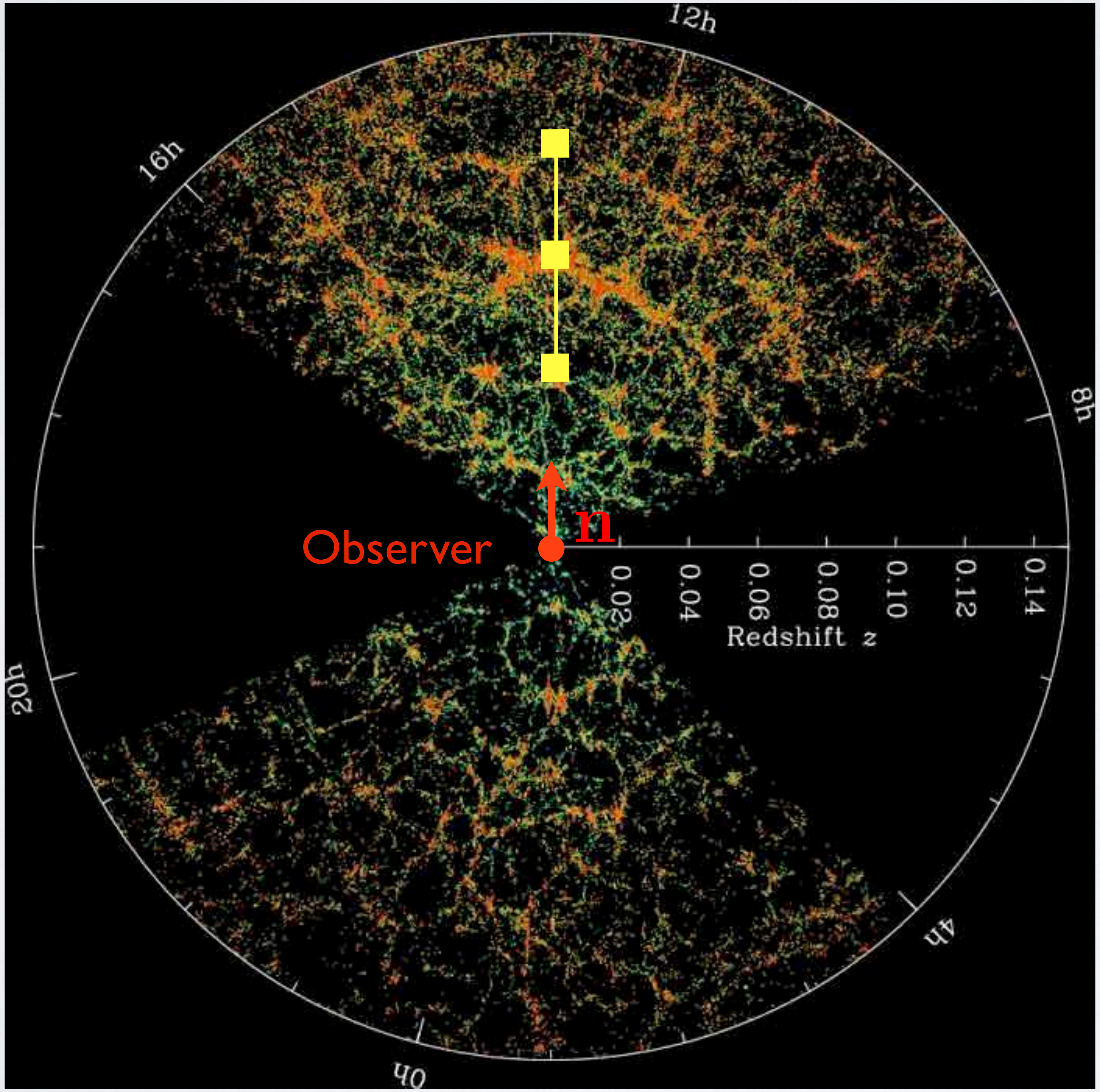
Relativistic effects **break** the **symmetry** of ξ

Credit: M. Blanton, SDSS



Gravitational redshift

$$\Delta = \frac{1}{\mathcal{H}}\partial_r \Psi$$

Relativistic effects

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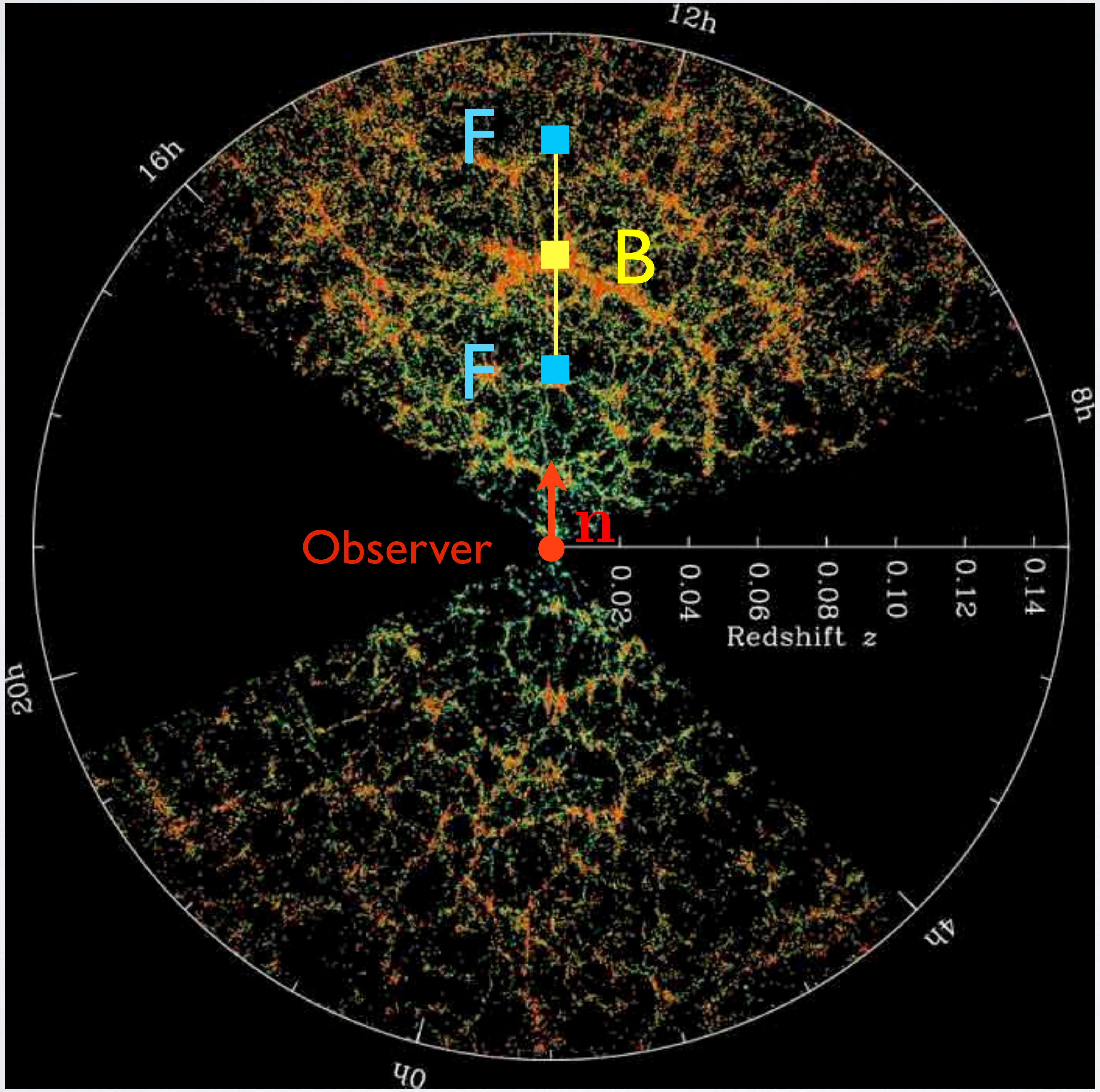
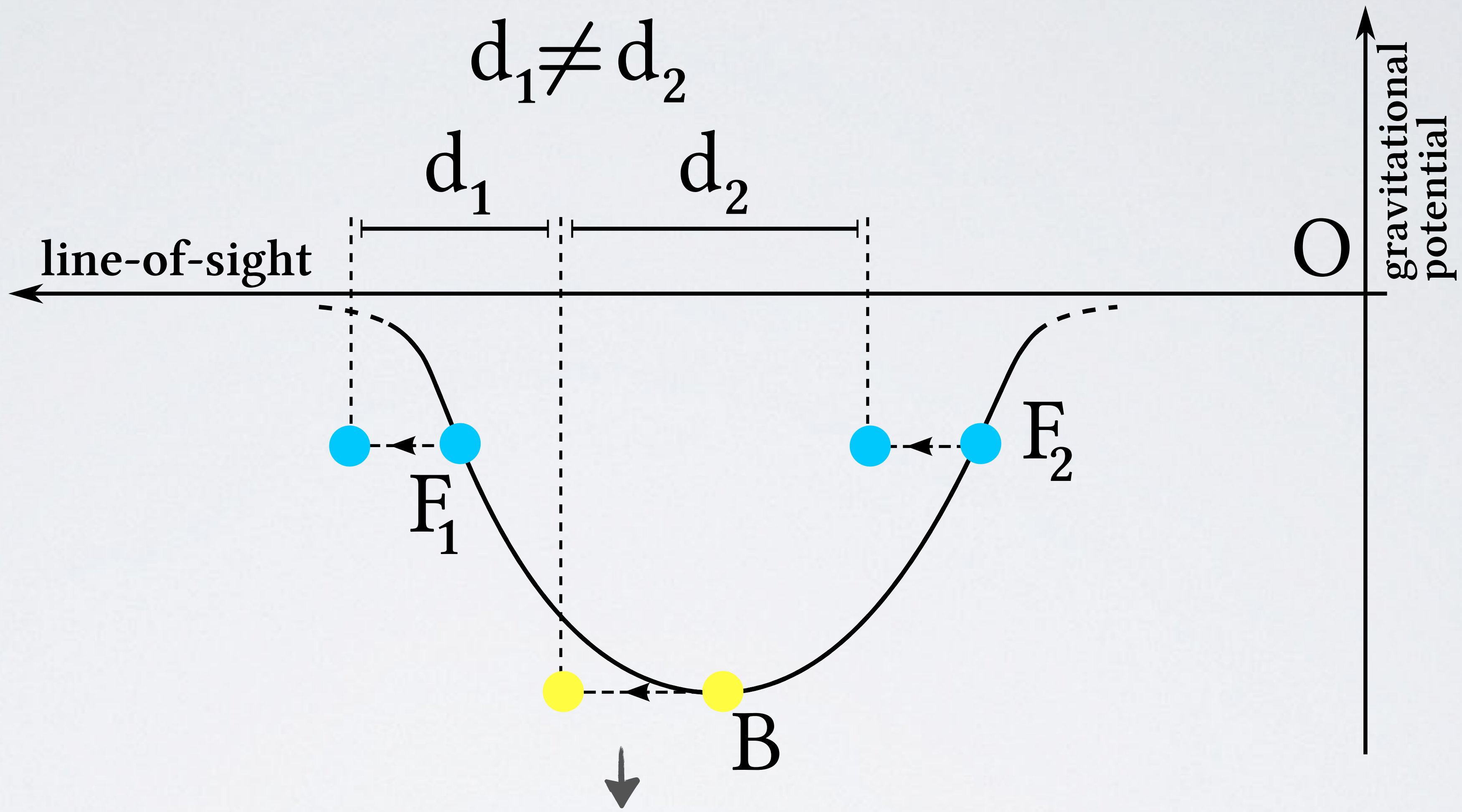


Gravitational redshift

$$\Delta = \frac{1}{\mathcal{H}}\partial_r \Psi$$

Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)



shift in position due to gravitational redshift

$$\xi = \frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{D_1}{D_{10}}\right)^2 \Big[(b_\mathrm{B}-b_\mathrm{F})\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)+3(s_\mathrm{F}-s_\mathrm{B})f^2\left(1-\frac{1}{r\mathcal{H}}\right)$$

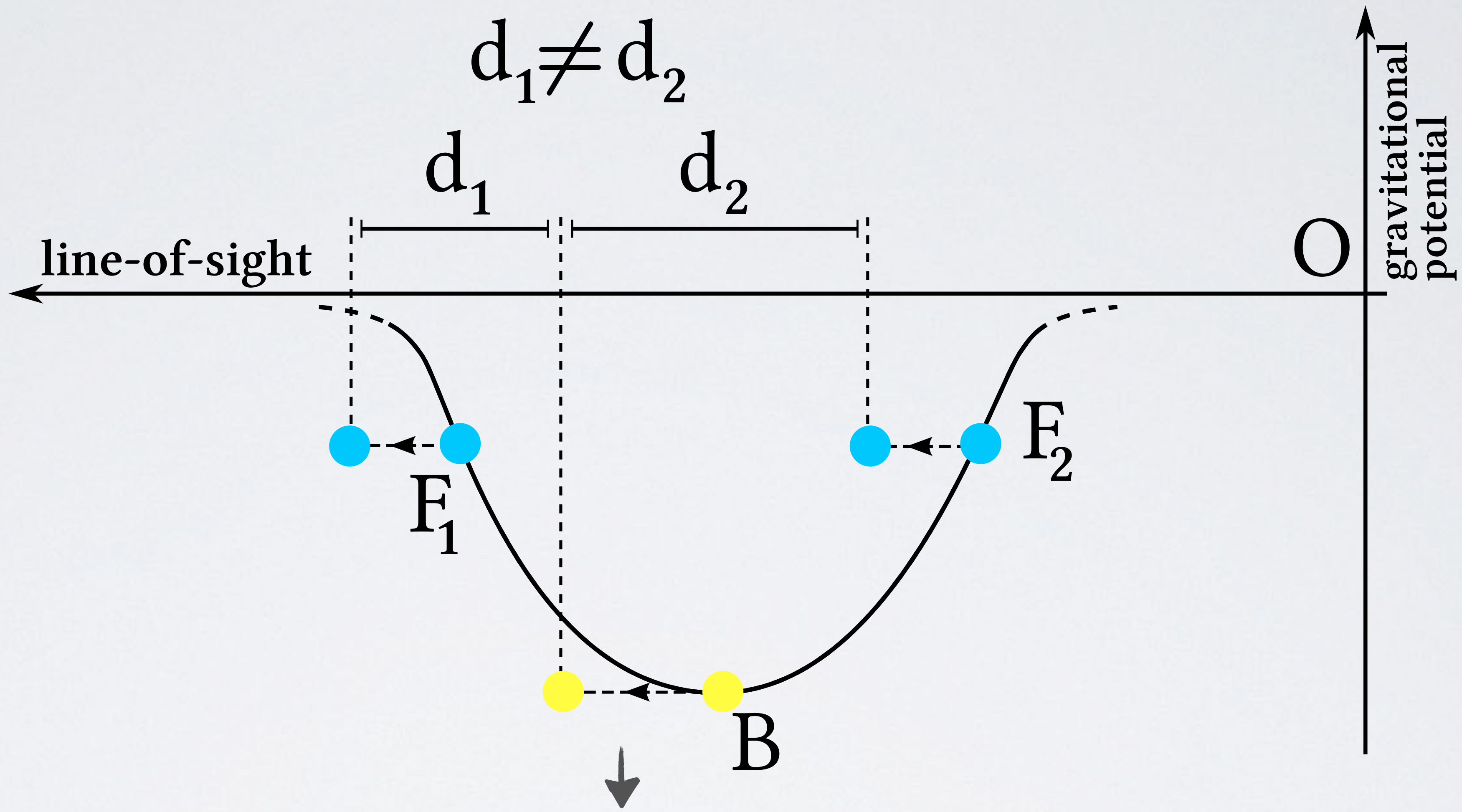
$$+5(b_\mathrm{B}s_\mathrm{F}-b_\mathrm{F}s_\mathrm{B})f\left(1-\frac{1}{r\mathcal{H}}\right)\Big]\nu_1(d)\cos(\beta)$$

The number count at linear order

$$
\begin{aligned}
\Delta(z,\mathbf{n}) = b\cdot\delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{V}\cdot\mathbf{n}) \\
+ (5s-2)\int_0^r dr'\frac{r-r'}{2rr'}\Delta_\Omega(\Phi+\Psi) \\
+ \left(1-5s-\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{5s-2}{r\mathcal{H}}\right)\mathbf{V}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\dot{\mathbf{V}}\cdot\mathbf{n} + \frac{1}{\mathcal{H}}\partial_r\Psi \\
+ \frac{2-5s}{r}\int_0^r dr'(\Phi+\Psi) + 3\mathcal{H}\nabla^{-2}(\nabla\mathbf{V}) + \Psi + (5s-2)\Phi \\
+ \frac{1}{\mathcal{H}}\dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}+\frac{2-5s}{r\mathcal{H}}+5s\right)\left[\Psi+\int_0^r dr'(\dot{\Phi}+\dot{\Psi})\right]
\end{aligned}
$$

Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)



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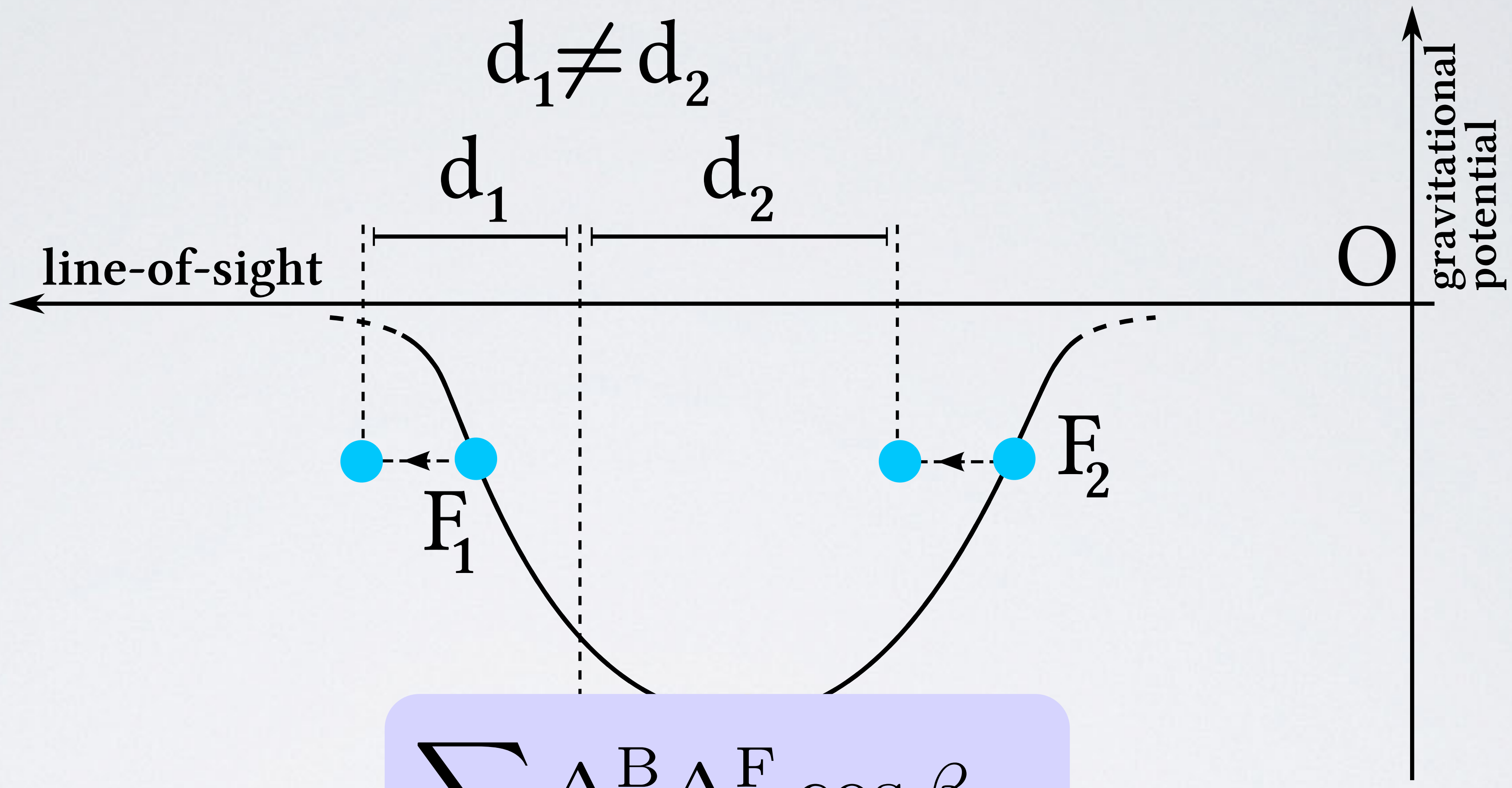
Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)



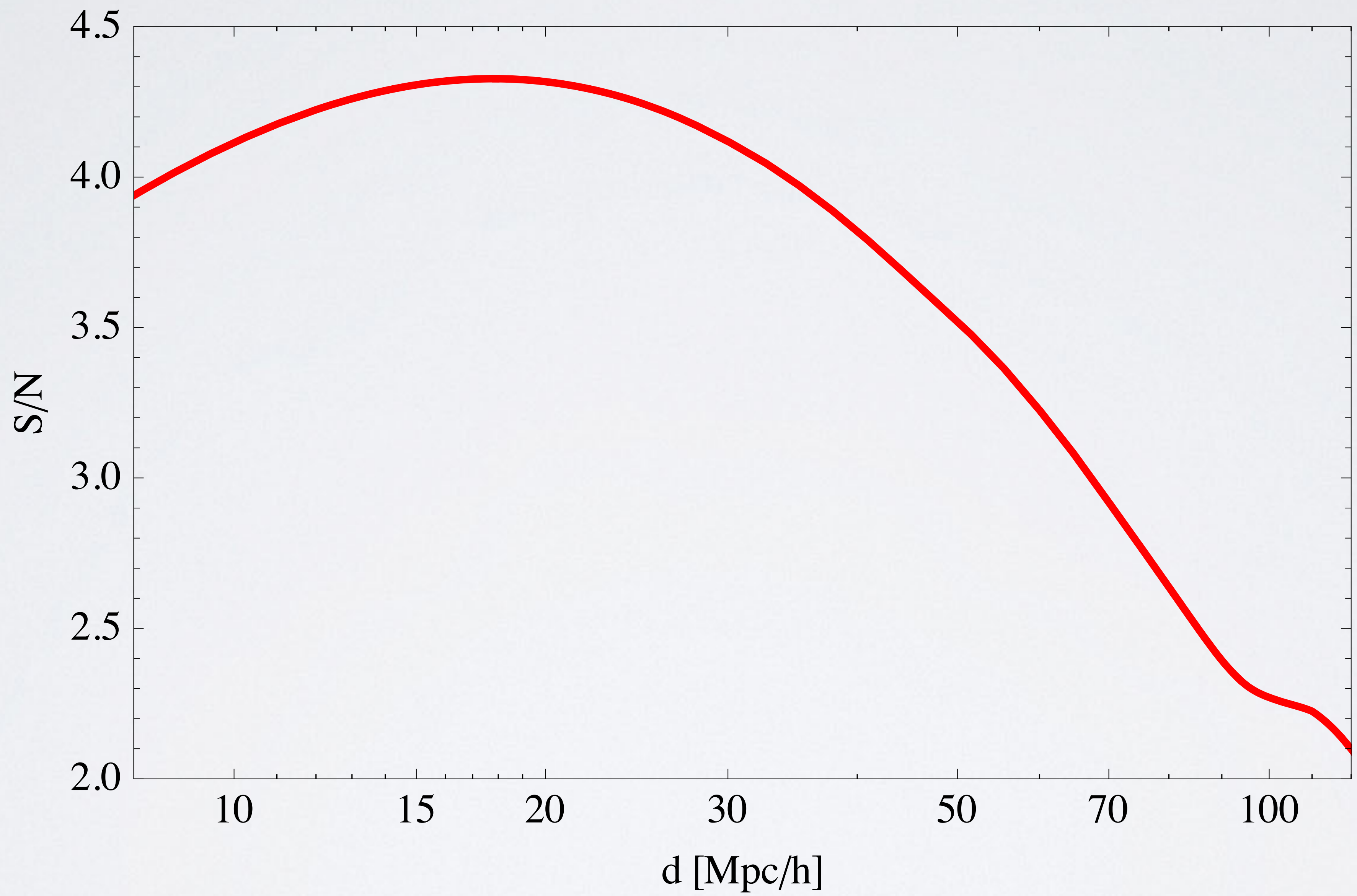
$$\sum_{ij}\Delta_i^{\rm B}\Delta_j^{\rm F}\cos\beta_{ij}$$

$$\xi=\frac{\mathcal{H}}{\mathcal{H}_0}\left(\frac{D_1}{D_{10}}\right)^2\Big[(b_{\rm B}-b_{\rm F})\left(\frac{2}{r\mathcal{H}}+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)+3(s_{\rm F}-s_{\rm B})f^2\left(1-\frac{1}{r\mathcal{H}}\right)$$

$$+5(b_{\rm B}s_{\rm F}-b_{\rm F}s_{\rm B})f\left(1-\frac{1}{r\mathcal{H}}\right)\Big]\nu_1(d)\cos(\beta)$$

Forecasts

DESI Bright Sample (2021): 10 million galaxies at $z \leq 0.3$



Cumulative signal-to-noise of 7.4

Forecasts

CB, Hui & Gaztanaga, (2015)

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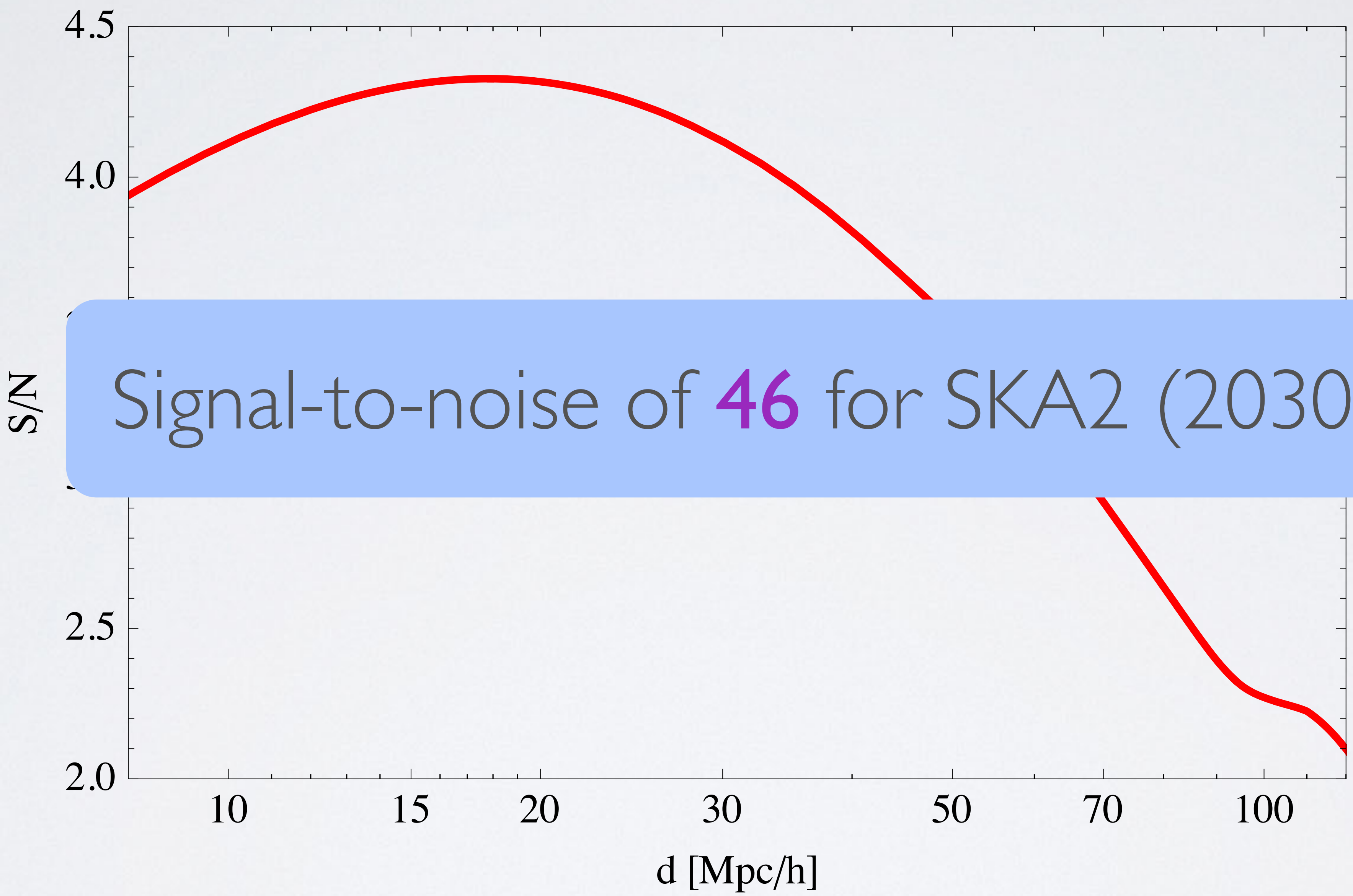


Signal-to-noise of **46** for SKA2 (2030)

Cumulative signal-to-noise of 7.4

Euler equation

- Which **deviations** from General Relativity can we **test**?

- By comparing Ψ with $V \equiv \mathbf{V}\cdot\mathbf{n}$, we can test **Euler equation**

$$\dot{V} + \mathcal{H}V + \partial_r\Psi = 0$$

- Theories that break the **equivalence principle** violate Euler equation: the way objects fall into a gravitational potential is modified

$$\dot{V} + \mathcal{H}V + \partial_r\Psi = E^{\mathrm{break}}$$

- We have strong **constraints** on **baryons** and **photons** but not on **dark matter**

CB & Fleury (2018)

Example

- New degrees of freedom propagating gravity: **scalar** field or **vector** field

- We couple this degree of freedom non-minimally to **dark matter**

$$\dot{V} + \mathcal{H}V + \partial_r \Psi = E^{\rm break}$$

$$E^{\rm break} = -\Theta(z)\mathcal{H}V - \Gamma(z)\partial_r\Psi$$

friction ($\Theta(z)$) — gravitational-like force ($\Gamma(z)$)

- How can we test for a non-zero $E^{\rm break}$

Null test of the equivalence principle

- We **combine** the **multipoles** in such a way that the combination **vanishes** if Euler equation is valid.

$$\dot{V} + \mathcal{H}V + \partial_r\Psi = E^{\rm break}$$

- Dipole: $\langle\delta\dot{V}\rangle$ $\langle\delta\,\partial_r\Psi\rangle$ $\langle(\delta+V)V\rangle$
- Quadrupole: $\langle\delta V\rangle$ $\langle VV\rangle$
- Hexadecapole: $\langle VV\rangle$

$$\mathcal{N}_{\rm E} = A\,\xi_1^{\rm BF} + C\,\xi_2^{\rm BB} + D\,\xi_2^{\rm FF} + G\,\xi_4 \;=\; \begin{cases} \nearrow 0 \\ \searrow (b_{\rm B}-b_{\rm F})\Upsilon(E^{\rm break}) \end{cases}$$

CB, Franco & Fleury (2020)

Null test of the equivalence principle

- We **combine** the **multipoles** in such a way that the combination **vanishes** if Euler equation is valid.

$$\langle \delta \dot{V} \rangle + \mathcal{H}\langle \delta V \rangle + \langle \delta \partial_r \Psi \rangle = \langle \delta E^{\rm break} \rangle$$

- Dipole: $\langle \delta \dot{V} \rangle \quad \langle \delta\, \partial_r \Psi \rangle \quad \langle (\delta + V) V \rangle$
- Quadrupole: $\langle \delta V \rangle \quad \langle V V \rangle$
- Hexadecapole: $\langle V V \rangle$

$$\mathcal{N}_{\rm E} = A\, \xi_1^{\rm BF} + C\, \xi_2^{\rm BB} + D\, \xi_2^{\rm FF} + G\, \xi_4 \; = \begin{cases} \nearrow 0 \\ \searrow (b_{\rm B} - b_{\rm F})\Upsilon(E^{\rm break}) \end{cases}$$

CB, Franco & Fleury (2020)

Forecasts

- Calculation of the **variance** of the null test
    - Cosmic variance
    - Shot noise

- **Ideal survey** with 30'000 square degrees: we can detect at 3 sigma $\Upsilon = 0.005$

- **SKA2**: we can detect at 3 sigma $\Upsilon = 0.83$

First direct test of the equivalence principle for dark matter

Anisotropic stress

- We can use the dipole to test the **relation** between the two gravitational potentials Φ and Ψ

- In ΛCDM and in many dark energy models: $\Phi = \Psi$

- In modified theories of gravity: $\Phi \neq \Psi$

Saltas, Sawicki, Amendola & Kunz (2014)

→ non-zero **anisotropic stress**

Sobral Blanco & CB (2021)

Anisotropic stress

Combine the dipole with other multipoles to **isolate** Ψ

- Dipole: $\langle \delta \dot{V} \rangle$ $\langle \delta \Psi \rangle$ $\langle (\delta + V) V \rangle$
- Monopole: $\langle \delta \delta \rangle$ $\langle \delta V \rangle$ $\langle V V \rangle$
- Quadrupole: $\langle \delta V \rangle$ $\langle V V \rangle$
- Hexadecapole: $\langle V V \rangle$

$\langle \delta \dot{V} \rangle$ obtained from **redshift variation** of the multipoles

Anisotropic stress

Sobral Blanco & CB (2021)

- Combination of multipoles gives measurement of $(b_{\rm B}-b_{\rm F})\langle\delta\Psi\rangle$
- **Lensing**: correlate **shapes** at high redshift with **galaxies** at low redshift

Galaxy-galaxy lensing: $\langle\Delta_{\rm B}\kappa\rangle-\langle\Delta_{\rm F}\kappa\rangle\propto(b_{\rm B}-b_{\rm F})\langle\delta(\Phi+\Psi)\rangle$

Ratio: $\dfrac{(b_{\rm B}-b_{\rm F})\langle\delta(\Phi+\Psi)\rangle}{(b_{\rm B}-b_{\rm F})\langle\delta\Psi\rangle}$ ↗ 2

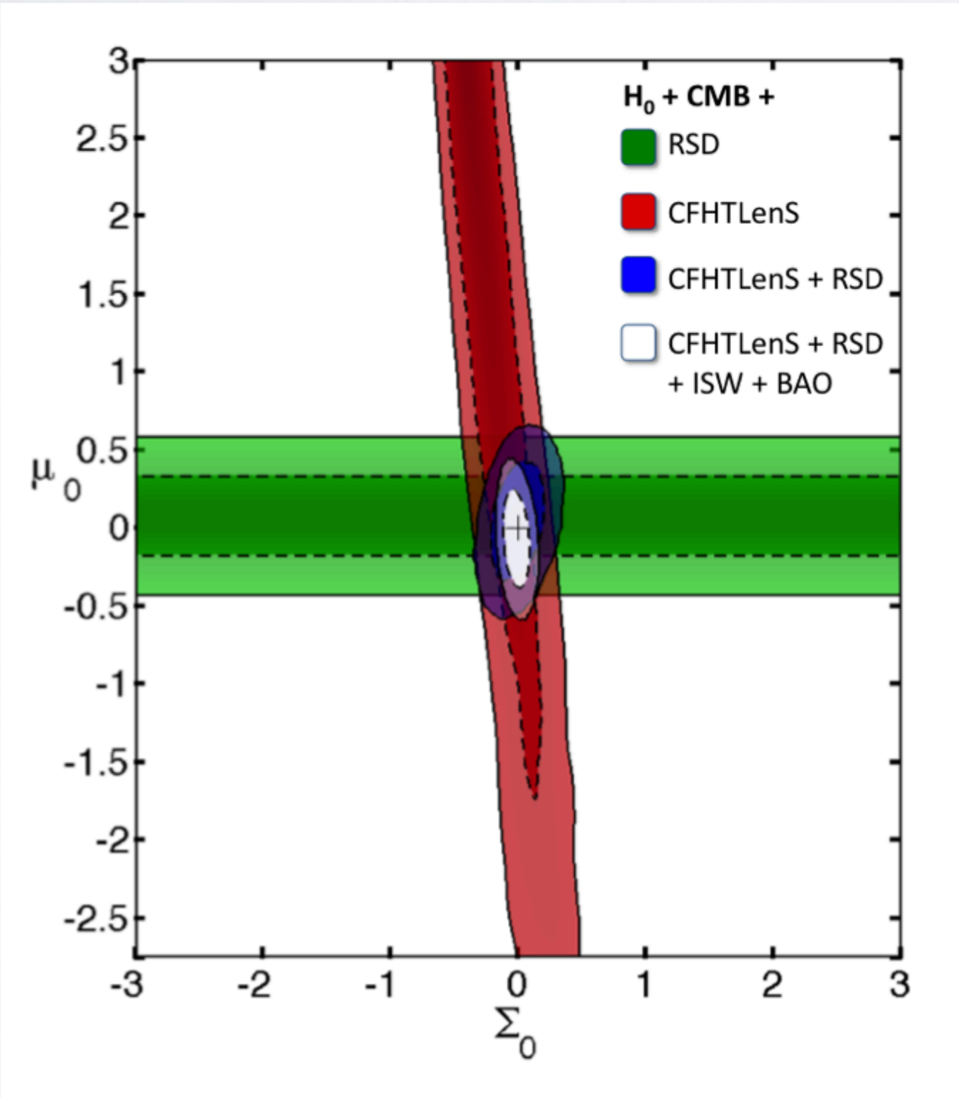
↘ $1+\eta$ $\quad\Phi=\eta\Psi$

Current measurements

Deviations from GR: $\Phi = \eta\Psi$ and $-k^2\Psi = \frac{3}{2}\mathcal{H}^2\Omega_m\mu\,\delta$

- **Lensing** measures $\Sigma = (1+\eta)\mu$
- **Redshift-space distortions** measure μ

Figure from Simpson et al (2012)



δ continuity V

Poisson Euler

Φ anisotropic stress Ψ

Assumptions

- If the **continuity** and **Euler equations** are valid then redshift-space distortions directly measure μ

- Evolution equation for δ

  - $$\frac{\partial^2\delta}{\partial(\ln a)^2} + \left(2 + \frac{\partial \ln H}{\partial \ln a}\right)\frac{\partial\delta}{\partial \ln a} = \frac{3}{2}\left(\frac{H_0}{H}\right)\frac{\Omega_m}{a^3}\mu\delta$$

  - $$V = -\frac{aH}{k}\frac{\partial\delta}{\partial \ln a}$$

- **RSD** measure δ and V → measure μ

Assumptions

- If the **continuity** and **Euler equations** are modified then redshift-space distortions does not directly measure μ

- Evolution equation for δ

  Euler equation modified

  - $$\frac{\partial^2\delta}{\partial(\ln a)^2}+\left(2+\frac{\partial\ln H}{\partial\ln a}\right)\frac{\partial\delta}{\partial\ln a}=\frac{3}{2}\left(\frac{H_0}{H}\right)\frac{\Omega_m}{a^3}\mu\delta$$ + extra terms $\propto E^{\rm break}$

  - $$V=-\frac{aH}{k}\frac{\partial\delta}{\partial\ln a}$$

- **RSD** measure δ and V → measure μ + modifications of Euler eq.

  → We **cannot** reconstruct η from lensing and RSD

Relativistic effects

- Our method **does not rely** on the validity of Euler equation or continuity equation

  → **model-independent**

- **Drawback**: the dipole is more difficult to measure than the even multipoles

  → Constraints on η will be **weaker**

Sobral Blanco & CB in progress

Conclusion

- The distribution of galaxies is affected by **relativistic distortions**
- These effects have the particularity to **break** the **symmetry** of the correlation function → we can **isolate** them
- This provides a **measurement** of Ψ
- Combining this with redshift-space distortion and lensing we can build new **tests** of **gravity**
    - **Euler** equation
    - **Anisotropic stress**