Influence through Mixing

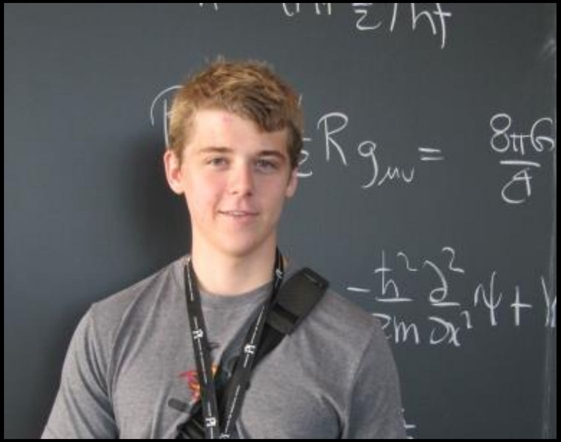
Hotspots as Cartoon Black Holes

CPB @ McMaster University, Perimeter Institute

Copernicus Series, Aug 2021

R. Holman

G. Kaplanek



2106.09854, 2106.10803, 2106.10804

R. Plestid

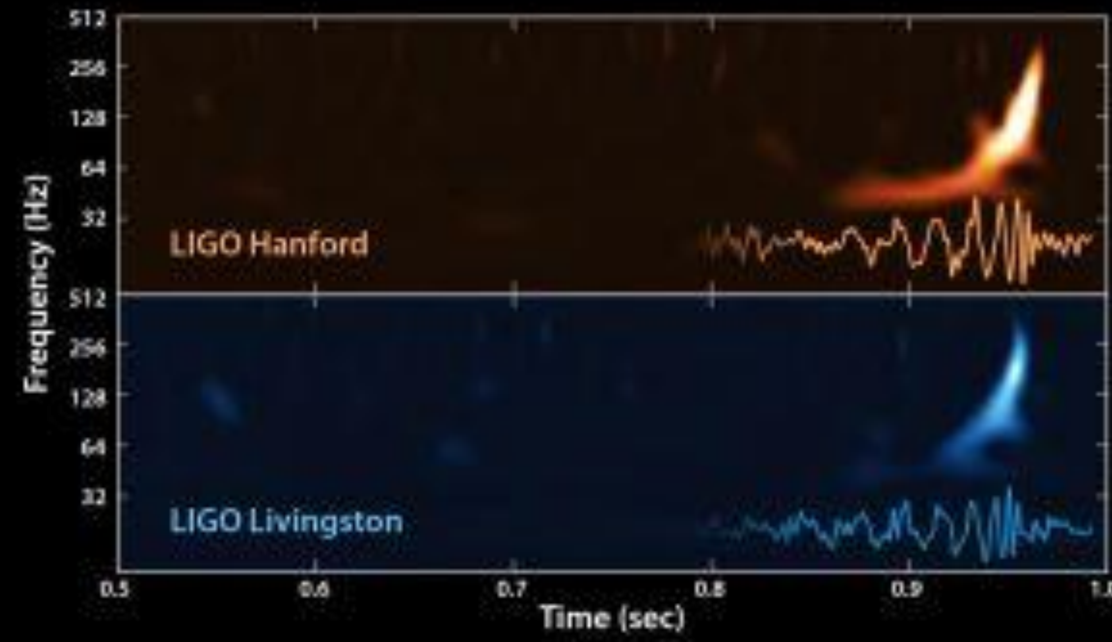
M. Rummel

M. Williams

P. Hayman

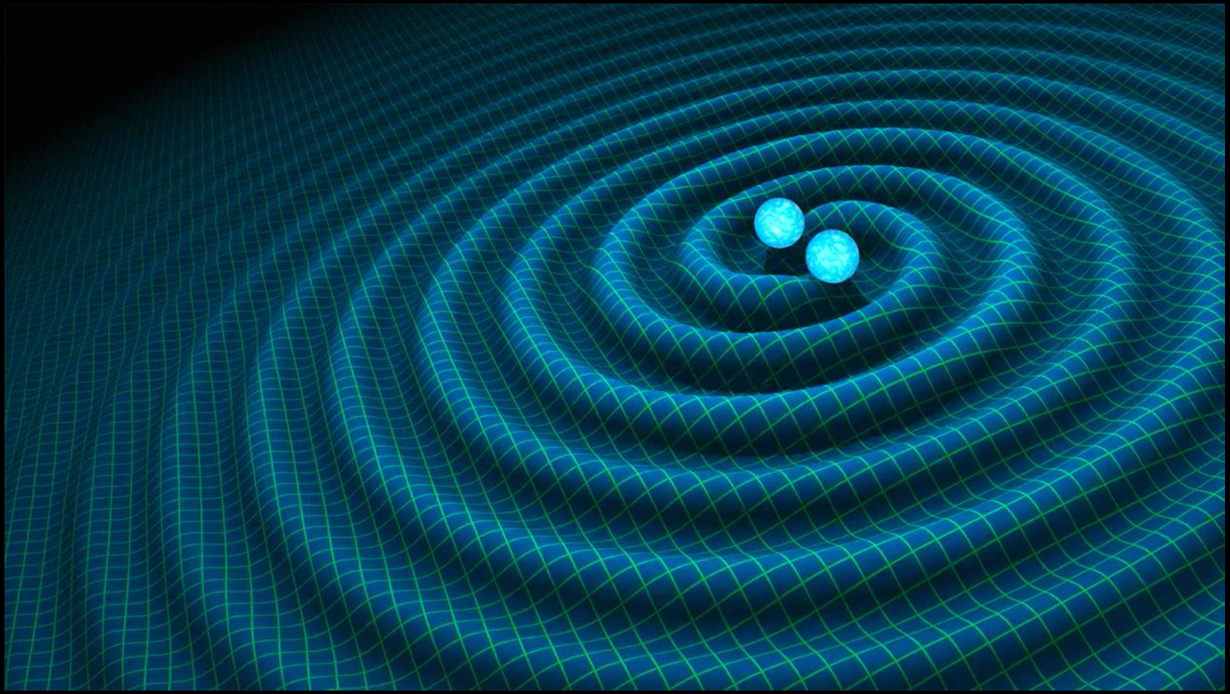
In a nutshell…

Black holes are no longer quite so rare and exotic



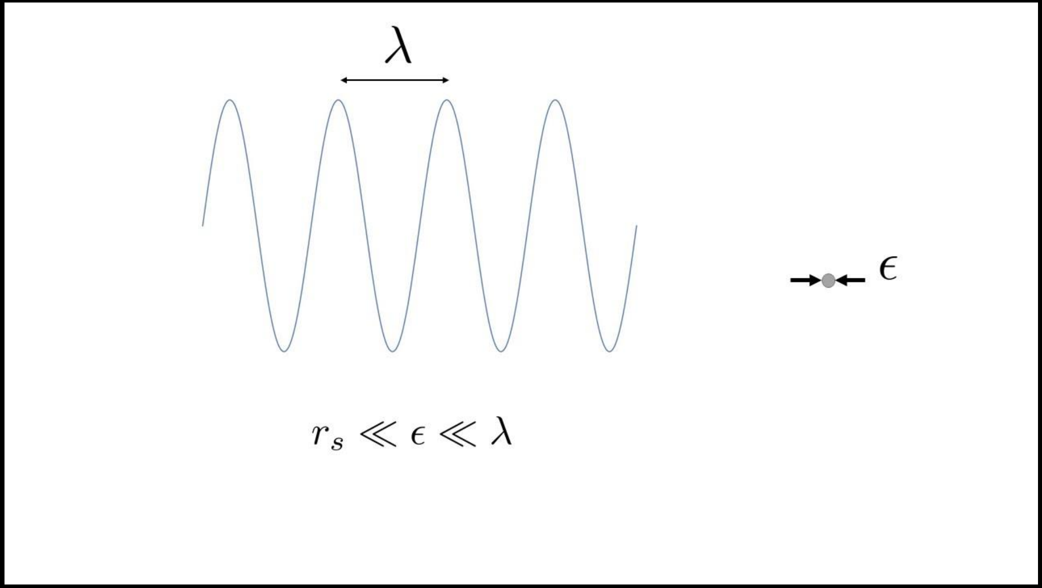
In a nutshell…

Need accurate predictions
to interpret what is seen



In a nutshell…

EFT of 'pointlike' BHs

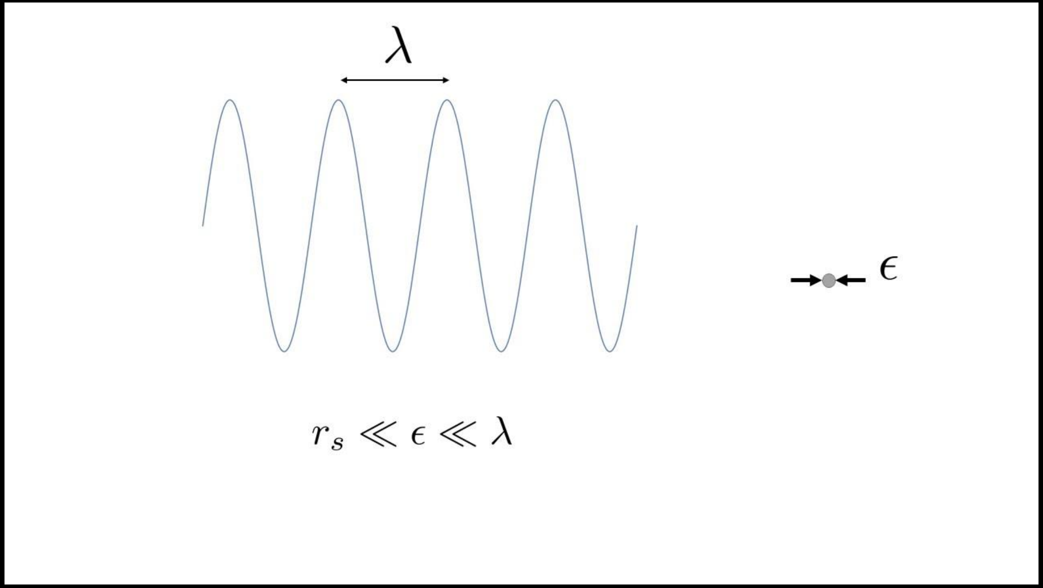


Match eg BH scattering to scattering found from EFT

Goldberger & Rothstein 04
Binnington & Poisson 09
Emparan et al 09
Damour et al 12

In a nutshell…

EFT of ‘pointlike’ BHs



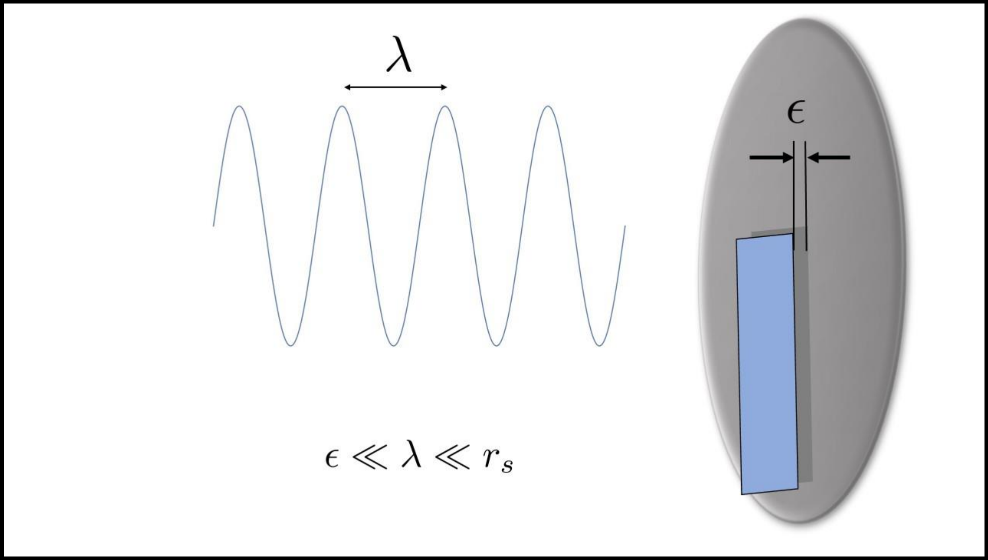
EFT encodes near-BH boundary conditions

CB, Hayman, Rummel & Williams 16

In a nutshell…

Other types of near-horizon EFTs?

CB, Plestid & Rummel 18



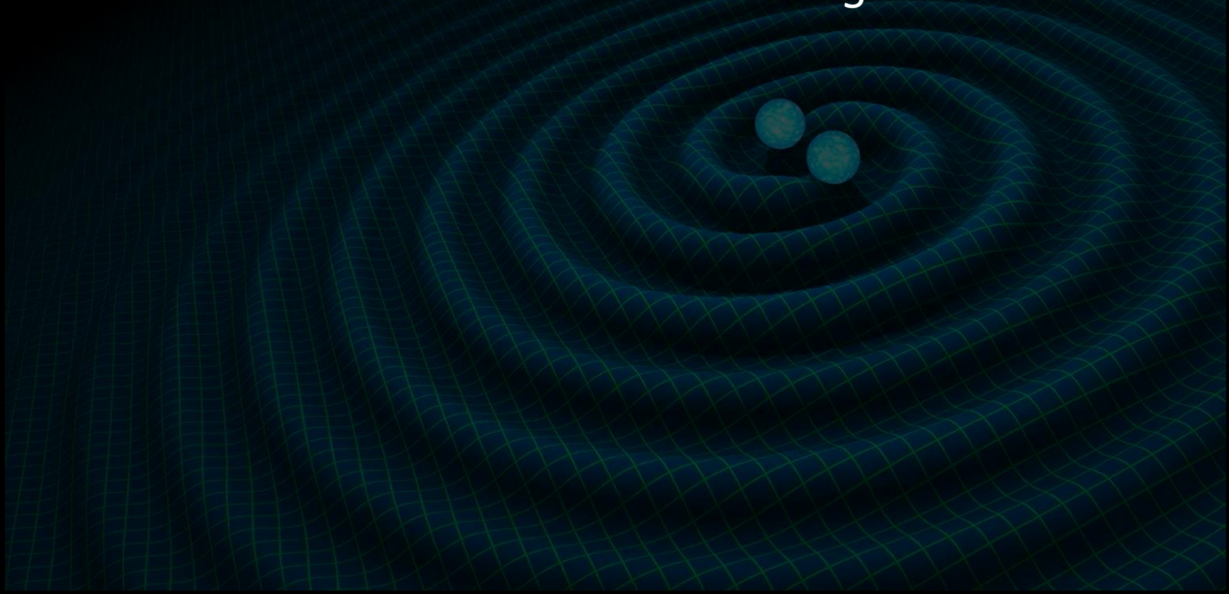
Parameterize new physics near the horizon

Price & Thorne 86 ; Abedi et al 17
Cardoso & Pani 17

In a nutshell…

but BH EFTs have some unusual features…

What about dissipative effects?
Absorption? Locality?
Hawking radiation?



Outline

A Caldeira-Leggett-style Hotspot

The model & its solutions

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Response of an Unruh detector

Late times and thermalization

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A Caldeira-Leggett-style Hotspot

The model & its solutions

Response of an Unruh detector

Late times and thermalization

The state of the field

Locality, decoherence,…

The Hotspot Model

Explicit solutions

Comparisons with perturbative methods

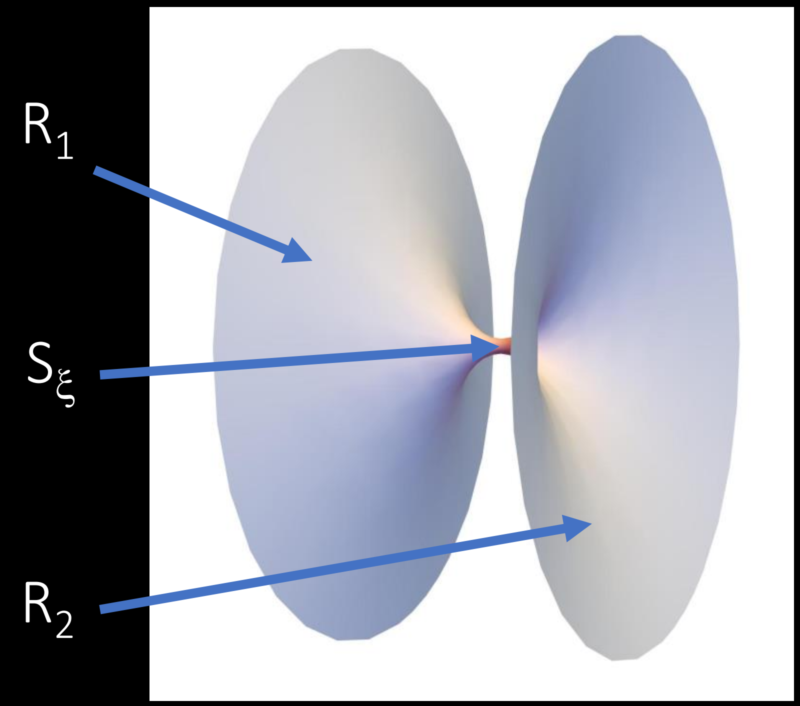
Renormalization issues

The Model

Degrees of freedom

Interior: N thermal fields χ^a

Exterior: single field ϕ



The Model

Action

$$S = S_\chi + S_\phi + S_1$$

$$S_\chi = -\tfrac{1}{2} \sum_a \int_{\mathcal{R}_1} \mathrm{d}^4x \; (\partial \chi_a)^2$$

$$S_\phi = -\tfrac{1}{2} \int_{\mathcal{R}_2} \mathrm{d}^4x \; (\partial \phi)^2$$

$$S_1 = -G_a \int_{\mathcal{S}_\xi} \mathrm{d}^3x \; \chi^a \, \phi$$

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The Model

Point-source limit

$$S_1 = -G_a \int_{\mathcal{S}_\xi} \mathrm{d}^3x \; \chi^a \, \phi$$
$$-\tfrac{1}{2} G_\phi \int_{\mathcal{S}_\xi} \mathrm{d}^3x \; \phi^2$$

$$S_1 \to -g_a \int_{\mathcal{W}} \mathrm{d}t \; \chi^a \, \phi$$
$$-\tfrac{1}{2} \lambda \int_{\mathcal{W}} \mathrm{d}t \; \phi^2$$

$$g_a = 4\pi\xi^2 G_a, \;\; \lambda = 4\pi\xi^2 G_\phi$$

Solution

Heisenberg-picture field equations

$$\Box\chi_a = g_a\phi\,\delta^3(\mathbf{x})$$

$$\Box\phi = \Big(g_a\chi_a + \lambda\phi\Big)\,\delta^3(\mathbf{x})$$

Solution

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$$\Box \phi = \Big(g_a \chi_a + \lambda \phi \Big) \, \delta^3(\mathbf{x})$$

$$\chi_a(x,t) = g_a \int_{\mathcal{W}} \mathrm{d}\tau \; G(x,t;0,\tau) \, \phi(0,\tau)$$

$$G(x,t;x',t') = \frac{\Theta(t-t')}{4\pi|\mathbf{x}-\mathbf{x}'|} \, \delta\Big[(t-t') - |\mathbf{x}-\mathbf{x}'| \Big]$$

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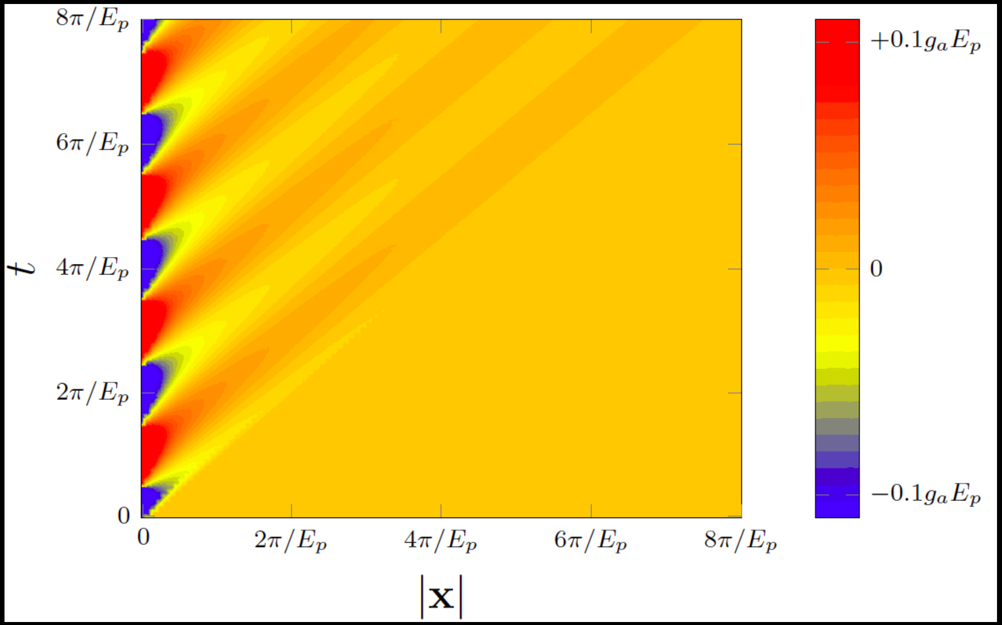
$$\chi_a(x,t) = g_a \int_{\mathcal{W}} \mathrm{d}\tau\; G(x,t;0,\tau)\,\phi(0,\tau)$$

$$= \sum_p \Big[\Big(u_p(x,t)\,a_p + u_p^*(x,t)\,a_p^*\Big) + \Big(v_p(x,t)\,c_p + v_p^*(x,t)\,c_p^*\Big)\Big]$$

Solution

Can solve explicitly for mode functions

$$\phi(x,t) = \sum_p \Big[\Big(u_p(x,t)\, a_p + u_p^*(x,t)\, a_p^* \Big) + \Big(v_p(x,t)\, c_p + v_p^*(x,t)\, c_p^* \Big) \Big]$$



Correlation Functions

Heisenberg-picture state

(or Schrodinger-picture initial state)

$$W(x,t;x',t') = \mathrm{Tr}\left[\phi(x,t)\,\phi(x',t')\,\rho\right]$$

$$\rho = \varrho_\beta \otimes |\mathrm{vac}\rangle\,\langle\mathrm{vac}|$$

ϕ sector

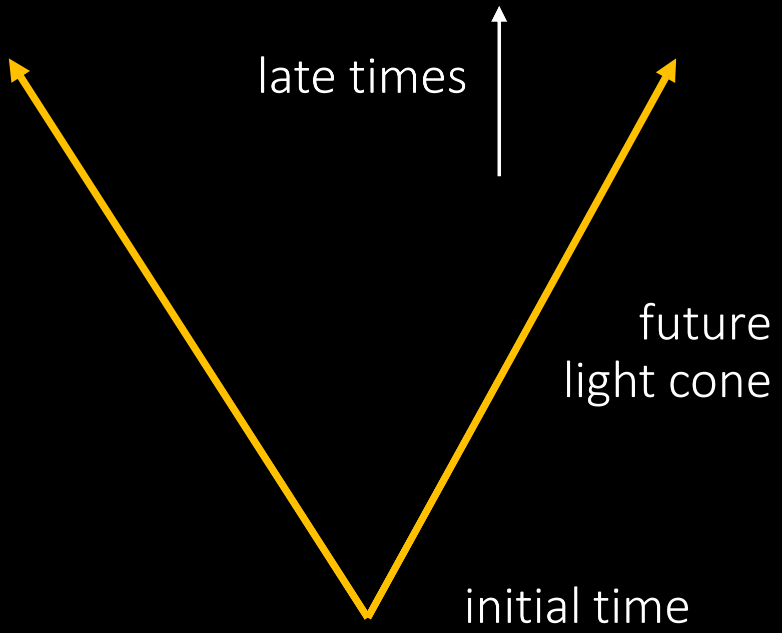
χ Sector (thermal)

$$\varrho_\beta = Z^{-1} e^{-\beta H_\chi}$$

$$Z = \mathrm{Tr}_\chi\, e^{-\beta H_\chi}$$

Correlation Functions

Often useful to avoid transient effects
by quoting late-time limit



Correlation Functions

The free thermal correlation function are also the χ-χ correlation functions in the large-*N* limit

$$\langle \chi_a(x,t)\,\chi_b(x',t')\rangle = \frac{\delta_{ab}}{8\pi\beta|\mathbf{x}-\mathbf{x}'|}\left\{\coth\left[\frac{\pi}{\beta}(t-t'+|\mathbf{x}-\mathbf{x}'|-i\delta)\right] - \coth\left[\frac{\pi}{\beta}(t-t'-|\mathbf{x}-\mathbf{x}'|-i\delta)\right]\right\}$$

so common coupling combination is

$$g_a g_b\langle \chi_a(x,t)\,\chi_b(x',t')\rangle \propto \sum_a g_a^2 = N\tilde{g}^2$$

Correlation Functions

The ϕ correlation function can be computed perturbatively in λ and *g*, with one subtlety

$$\Box\phi = \left(g_a\chi_a + \lambda\phi\right)\delta^3(\mathbf{x})$$

$$\chi_a(0,t) = g_a \int_{\mathcal{W}} \mathrm{d}\tau\; G(0,t;0,\tau)\,\phi(0,\tau)$$

$$G(x,t;x',t') = \frac{\Theta(t-t')}{4\pi|\mathbf{x}-\mathbf{x}'|}\;\delta\Big[(t-t') - |\mathbf{x}-\mathbf{x}'|\Big]$$

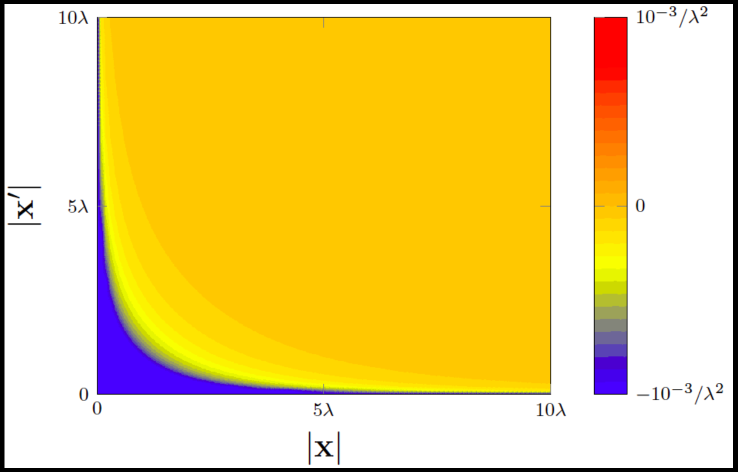
Coulomb-type divergence at r = |x - x’ | = 0 is regularized at r = ε and then renormalized into λ

$$\lambda \to \lambda - \frac{\tilde{g}^2}{4\pi\epsilon}$$

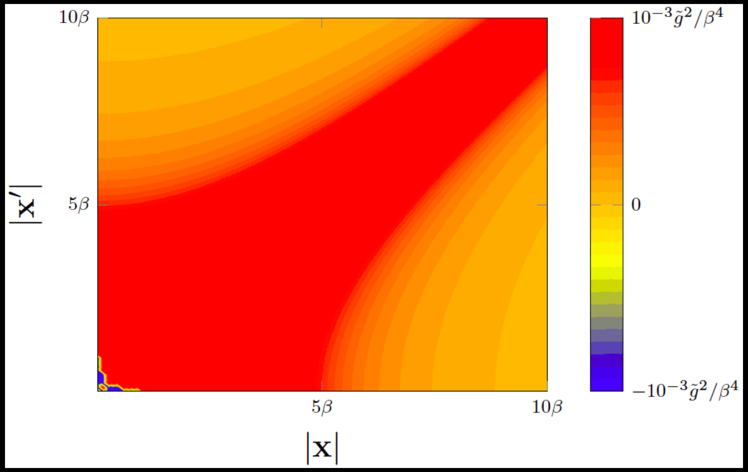
Correlation Functions

eg the perturbative equal-time ϕ correlation function in late-time limit is

$$W(x,t;x',t) = \frac{1}{4\pi^2|\mathbf{x}-\mathbf{x}'|^2} - \frac{\lambda}{16\pi^3|\mathbf{x}||\mathbf{x}'|(|\mathbf{x}|+|\mathbf{x}'|)} + \frac{\tilde{g}^2}{16\pi^4(|\mathbf{x}|^2-|\mathbf{x}'|^2)^2} - \frac{\tilde{g}^2}{64\pi^2\beta^2|\mathbf{x}||\mathbf{x}'|}\,\mathrm{csch}^2\left[\frac{\pi}{\beta}(|\mathbf{x}|-|\mathbf{x}'|)\right]$$



λ dependent term



g^2 dependent term

Regularization Dependence?

Naively ε dependent, but $\lambda(\varepsilon)$ dependence allows use of RG resummation of all orders in $\lambda/(4\pi\varepsilon)$

CB, Hayman, Rummel & Williams 16

$$W(x,t;x',t) = \frac{1}{4\pi^2|\mathbf{x}-\mathbf{x}'|^2} - \frac{\lambda(\epsilon)}{16\pi^3|\mathbf{x}||\mathbf{x}'|(|\mathbf{x}|+|\mathbf{x}'|)} + \frac{\tilde{g}^2}{16\pi^4(|\mathbf{x}|^2-|\mathbf{x}'|^2)^2} - \frac{\tilde{g}^2}{64\pi^2\beta^2|\mathbf{x}||\mathbf{x}'|}\,\mathrm{csch}^2\left[\frac{\pi}{\beta}(|\mathbf{x}|-|\mathbf{x}'|)\right]$$

Amounts to replacing λ with RG invariant combination:

$$\lambda \to \frac{\lambda}{1+\lambda/(4\pi\epsilon)}$$

Correlation Functions

Can verify perturbative results using explicit form for the exact correlation function

$$W(x,t;x',t) = F(x,t;x',t';\ell) + G(x,t;x',t';\beta/\ell)$$

where couplings all appear in the combination

$$\frac{1}{\ell} = \frac{16\pi^2\epsilon}{\tilde{g}^2}\left(1 + \frac{\lambda}{4\pi\epsilon}\right)$$

and resummed perturbative result emerges after expanding in limits

$$t_r - t_r', \beta \gg \ell$$

Detector Response

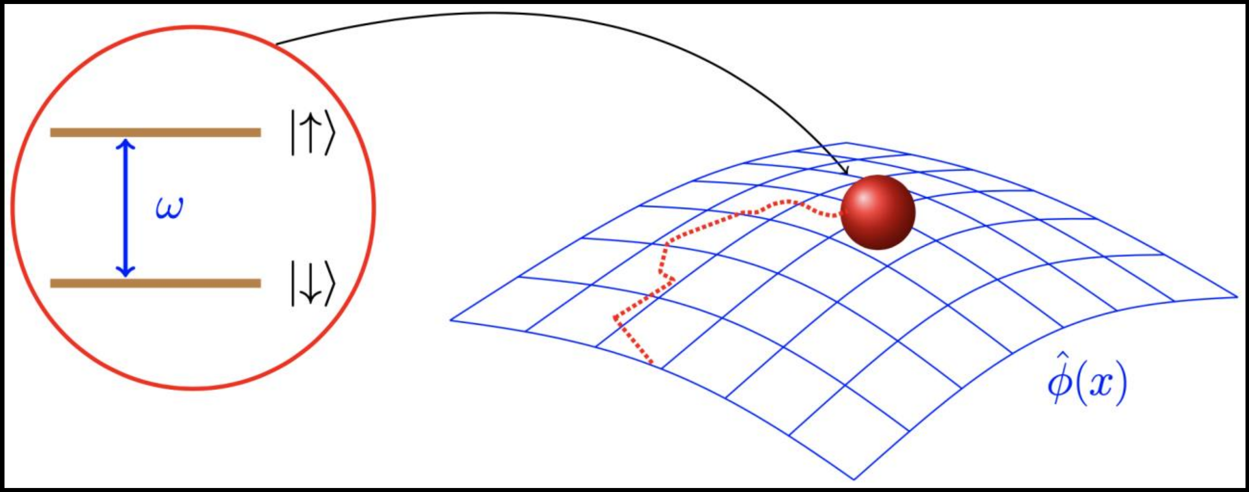
Reliable late-time predictions

Thermalization & Decoherence

Qubit Response

Wightman function appears in the evolution of a qubit coupled to the external field

$$H_{\text{int}} = \lambda_Q\, \phi(x_Q, t) \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Qubit Response

Wightman function controls the evolution of a qubit coupled to the external field once one traces out all field d.o.f.s

$$\varrho(t) = \mathrm{Tr}_{\mathrm{field}}\ \rho$$

$$i\partial_t \varrho(t) = \mathrm{Tr}_{\mathrm{field}}\ \Big[H_{\mathrm{int}}\,,\rho\Big]$$

This form for evolution equation is not that useful because the right-hand side still involves all degrees of freedom

Qubit Response

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Better: eliminate field to obtain evolution equation that refers only to qubit sector

Nakajima & Zwanzig

$$\partial_t \varrho_{\uparrow\uparrow}(t) = 2\lambda_Q^2 \int_0^t \mathrm{d}s \Big(\mathrm{Re}\, W(t,s)\cos[\omega(t-s)] + \mathrm{Im}\, W(t,s)\sin[\omega(t-s)]\Big)$$
$$-4\lambda_Q^2 \int_0^t \mathrm{d}s\ \mathrm{Re}\, W(t,s)\cos[\omega(t-s)]\ \varrho_{\uparrow\uparrow}(s)$$

plus similar equation for $\partial_t \varrho_{\uparrow\downarrow}(t)$

Qubit Response

If the Wightman function is sharply peaked in time relative to the time-scales of interest, the evolution simplifies and becomes approximately Markovian

$$\partial_t \varrho_{\uparrow\uparrow} \simeq \lambda_Q^2 \mathcal{R} - 2\lambda_Q^2 \mathcal{C} \; \varrho_{\uparrow\uparrow}$$

$$\mathcal{R} = \int_{-\infty}^{\infty} \mathrm{d}s \; W(s,0)\, e^{-i\omega s}$$

$$\mathcal{C} = 2 \int_0^{\infty} \mathrm{d}s \; \mathrm{Re}\, W(s,0) \; \cos \omega s$$

Qubit Response

Integrates to give solution

$$\varrho_{\uparrow\uparrow}(t) \simeq \frac{\mathcal{R}}{2\mathcal{C}} + \left[\varrho_{\uparrow\uparrow}(0) - \frac{\mathcal{R}}{2\mathcal{C}}\right] e^{-2\lambda_Q^2 \mathcal{C} t}$$

Qubit Response

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Agrees with perturbative result for small times if prepared in ground state

$$\partial_t \varrho_{\uparrow\uparrow} \simeq \lambda_Q^2 \mathcal{R} - 2\lambda_Q^2 \mathcal{C}\, \varrho_{\uparrow\uparrow}$$

$$\varrho_{\uparrow\uparrow}(t) \simeq \lambda_Q^2 \mathcal{R}\, t$$

Candelas & Sciama

For late times instead get *nonperturbative* relaxation towards diagonal density matrix

Qubit Response

With hotspot Wightman function this simplification occurs if

$$\beta\omega \ll 1 \qquad \frac{\tilde{g}\omega}{4\pi} \ll \frac{4\pi\epsilon}{\tilde{g}} \ll \frac{\epsilon}{|\mathbf{x}_Q|} \ll 1$$

in which case

$$\mathcal{R} = \frac{\tilde{g}^2\omega}{32\pi^2|\mathbf{x}_Q|^2\left[\frac{\tilde{g}^2\omega}{16\pi^2\epsilon}+1\right](e^{\beta\omega}-1)}$$

$$\mathcal{C} \simeq \frac{\omega}{4\pi}\left[1 + \frac{\tilde{g}^2\coth(\frac{\beta\omega}{2})}{16\pi^2|\mathbf{x}_Q|^2[(\omega\tilde{g}^2/16\pi^2\epsilon)^2+1]}\right]$$

Field Response

Reliable late-time predictions

Thermalization & Decoherence

Locality

State of the ϕ field

Working in Schrodinger picture allows determination of the evolution of its state. Use Gaussian ansatz for density matrix:

$$\langle \varphi_1 | \rho | \varphi_2 \rangle = N(t)\, \exp\Big\{ -\tfrac{1}{2} \int \int \mathrm{d}^3 p \, \mathrm{d}^3 q \Big[A_1(p,q)\varphi_1(p)\varphi_1(q) + A_2(p,q)\varphi_2(p)\varphi_2(q) + 2B(p,q)\varphi_1(p)\varphi_2(q) \Big] \Big\}$$

Sign of decoherence

Schrodinger equation implies

$$\partial_t B(p,q) \simeq -\frac{\tilde{g}^2}{2\pi\beta} + B\text{-dependent}$$

so hotspot temperature drives decoherence

Locality

Similarly, mean-field evolution Hamiltonian can be nonlocal

$$H_{\rm mf} = \tfrac{\lambda}{2}\phi^2(0) - i\tilde{g}\phi(0)\int_0^t {\rm d}s\,\mathcal{W}(s)\,\phi(0,t-s)$$

where

$$\mathcal{W}(s) = -\tfrac{1}{4\beta^2}\,{\rm csch}^2\left[\tfrac{\pi}{\beta}(s-i\delta)\right]$$

in general one finds the result is nonlocal in the interaction regime (time and angular directions) but not in radial directions

Conclusions

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EFTs with gravity more Open than Wilsonian

Have phenomena like late-time perturbative failure and decoherence, which Open EFTs can resum reliably

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Simple Caldeira-Leggett-type model instructive

Captures many open black-hole type features, though not all (eg missing redshifts)