KAVLI IPMU
INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

PROBING DARK ENERGY AND INFLATION WITH GRAVITATIONAL WAVES

VALERI VARDANYAN

wpi
World Premier International Research Center Initiative

科研費
KAKENHI

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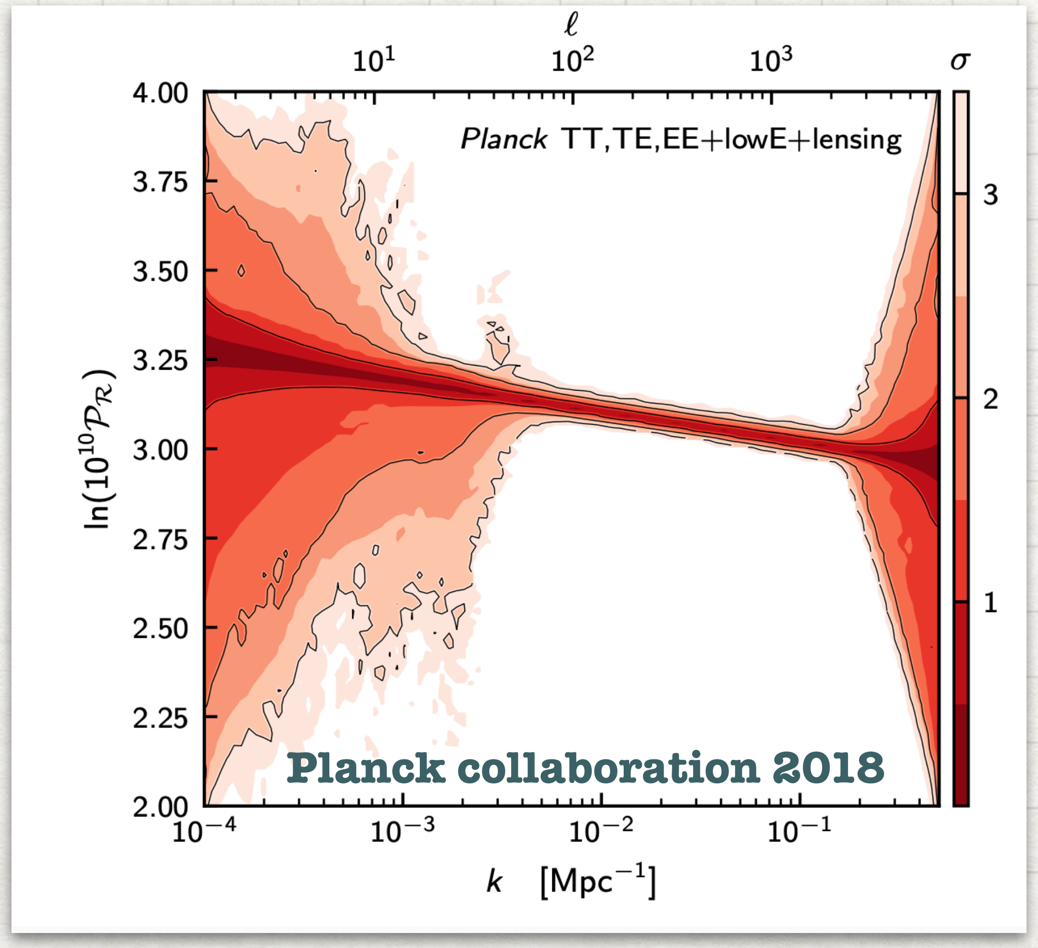
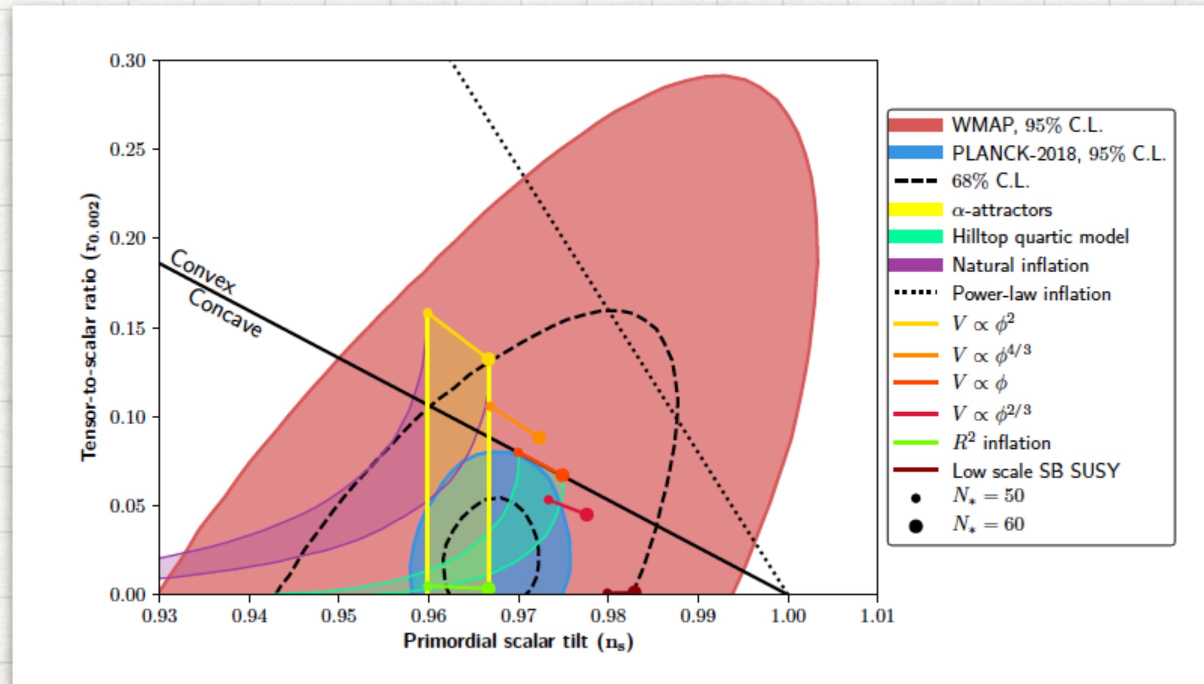
STANDARD MODEL OF THE UNIVERSE

- **Inflation in the early universe**
- **Solves several important issues of non-inflationary scenario (Horizon problem, flatness problem)**
- **Quantum fluctuations explain the observed CMB anisotropies and source the LSS of the late universe**
- **Simplest working scenario: single field ϕ with a flat potential**
- **Produces almost scale-invariant curvature power spectrum**



$$P_{\mathscr{R}}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2 = A_s k^{n_s - 1}$$

PRIMORDIAL GRAVITATIONAL WAVES

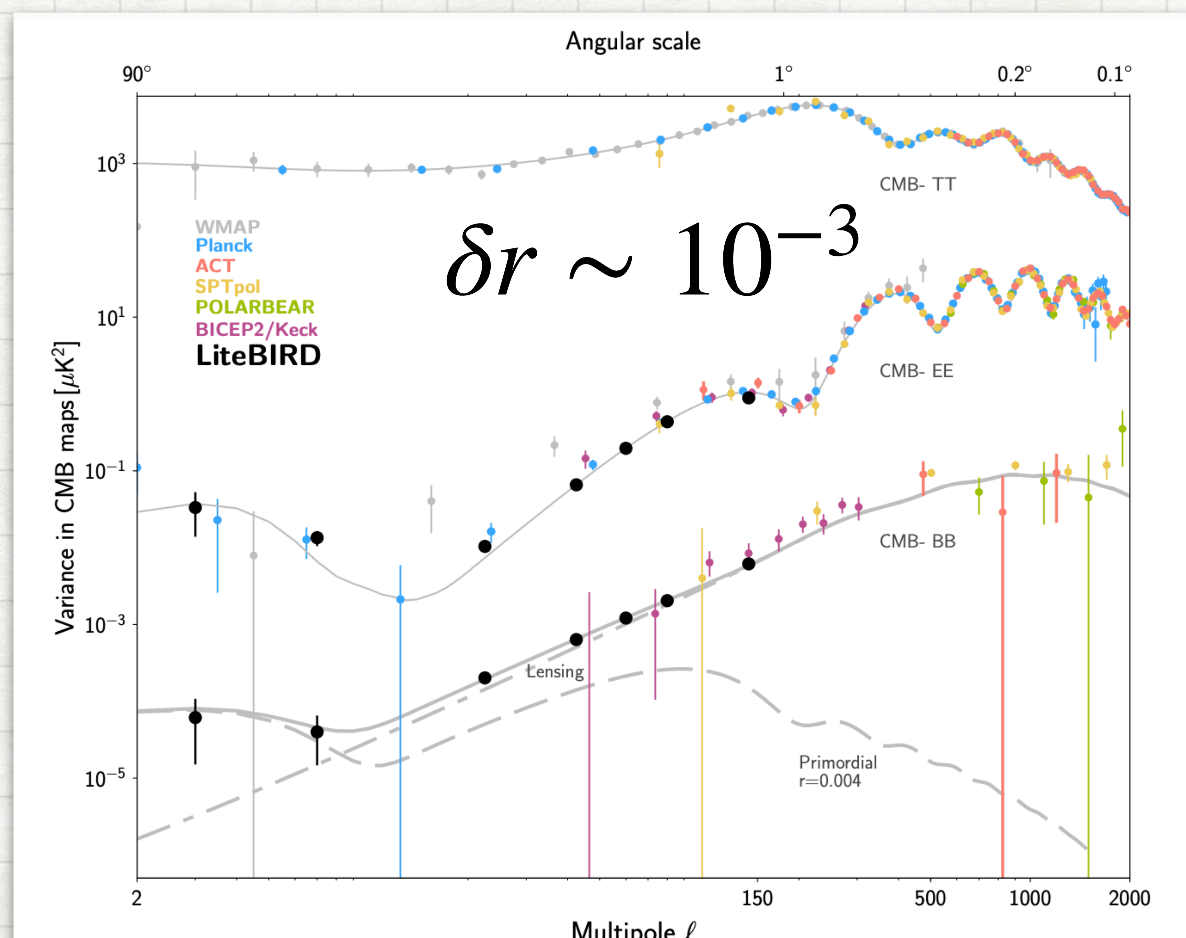


$$P_h(k) = \frac{8}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

Vacuum GWs. Lyth bound:

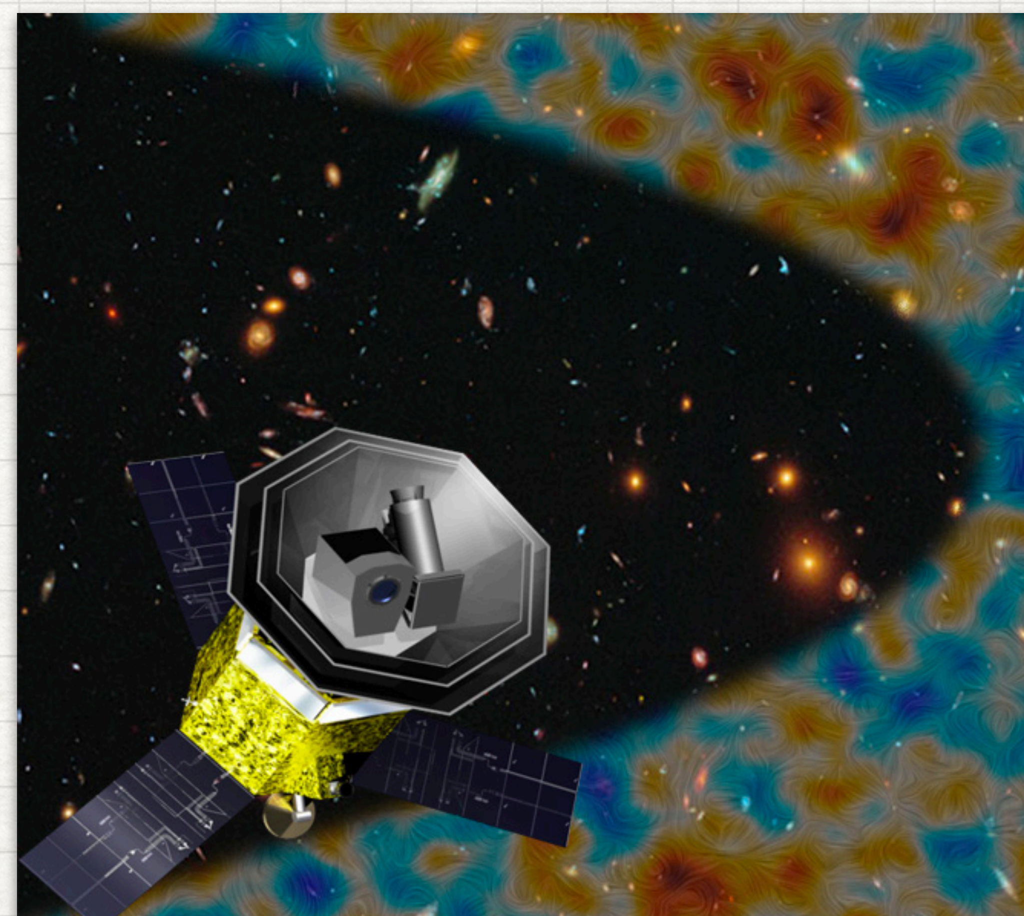
$$\frac{\Delta\phi}{M_{\rm Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2}, \; V^{1/4} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/4} 10^{16}\text{GeV}$$

LiteBIRD mission

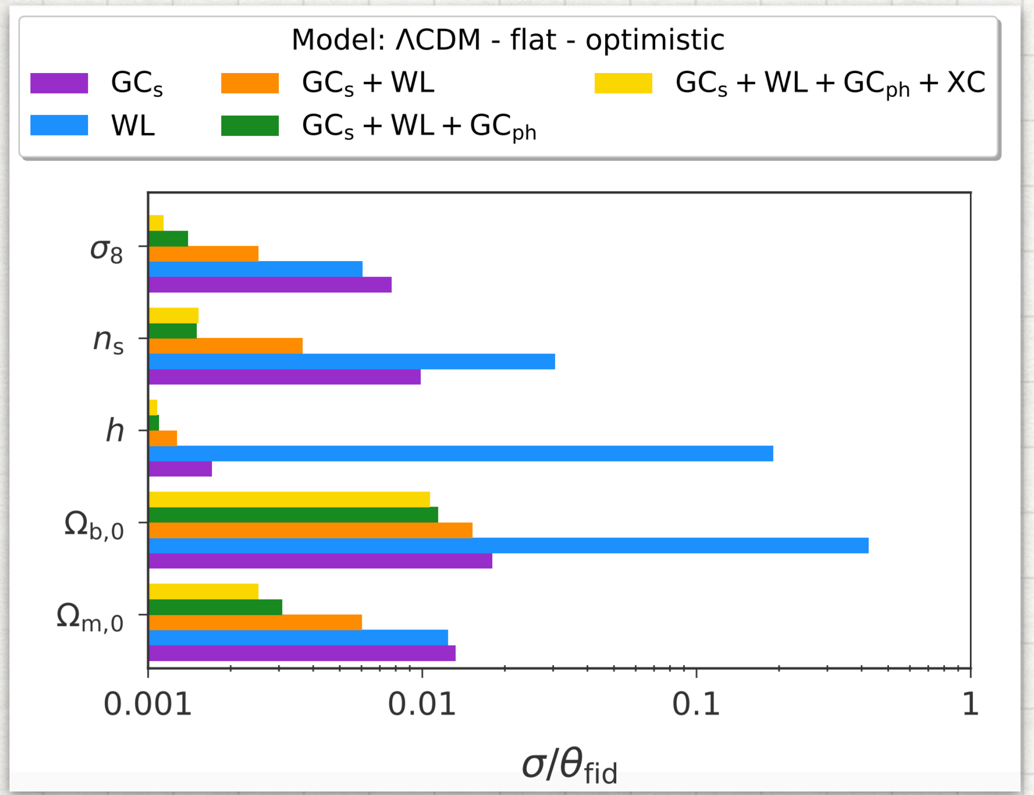


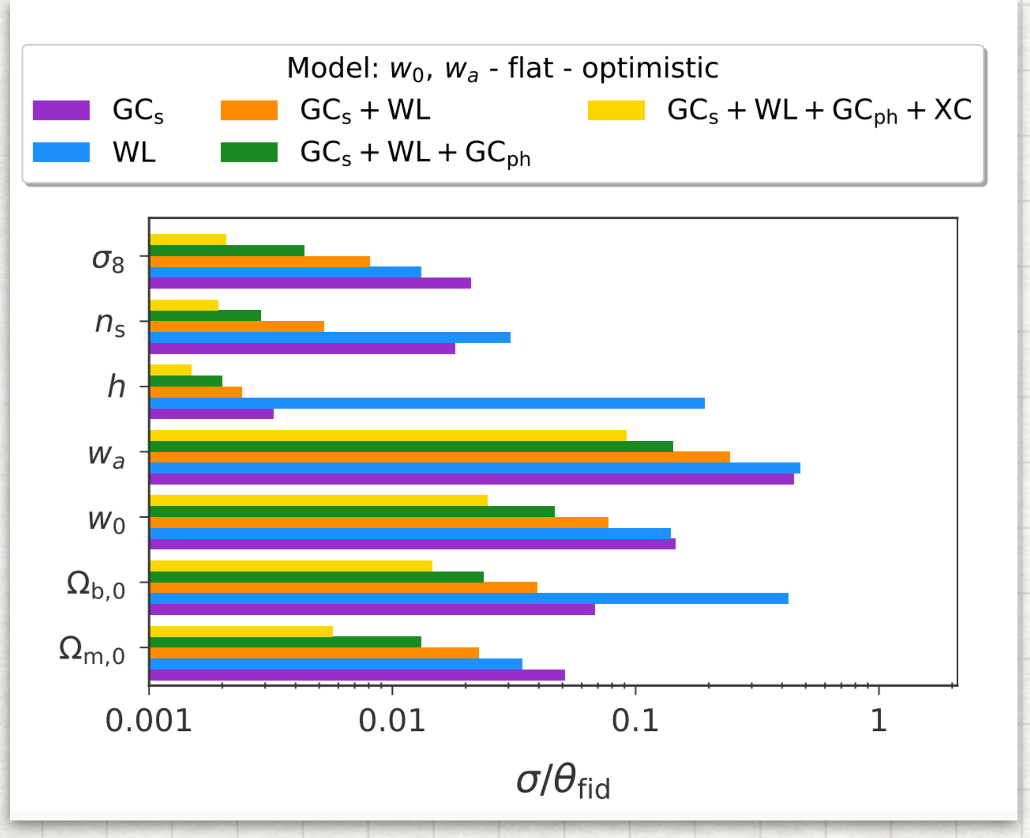
$$\delta r \sim 10^{-3}$$

CMB B-mode polarization



STANDARD MODEL OF THE UNIVERSE

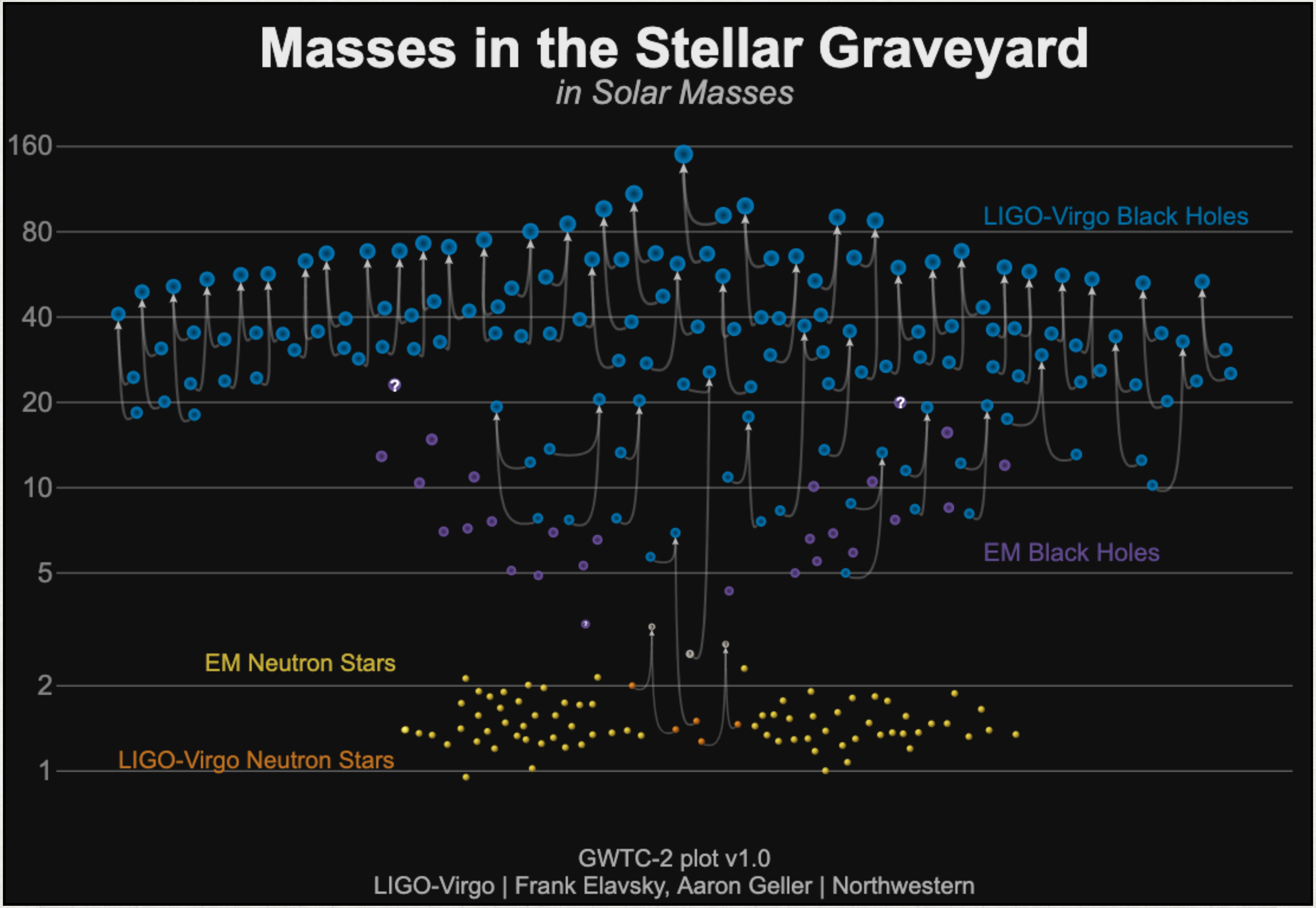




EUCLID collaboration, forecast

- **LCDM in the late universe**
- **Assumes GR + cosmological constant**
- **Still large wiggle-room for interesting alternatives**

- **Modified background history (e.g. $w_0 - w_a$ extension)**
- **Extra degrees of freedom at cosmological scales (e.g. scalar-tensor theories)**


Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

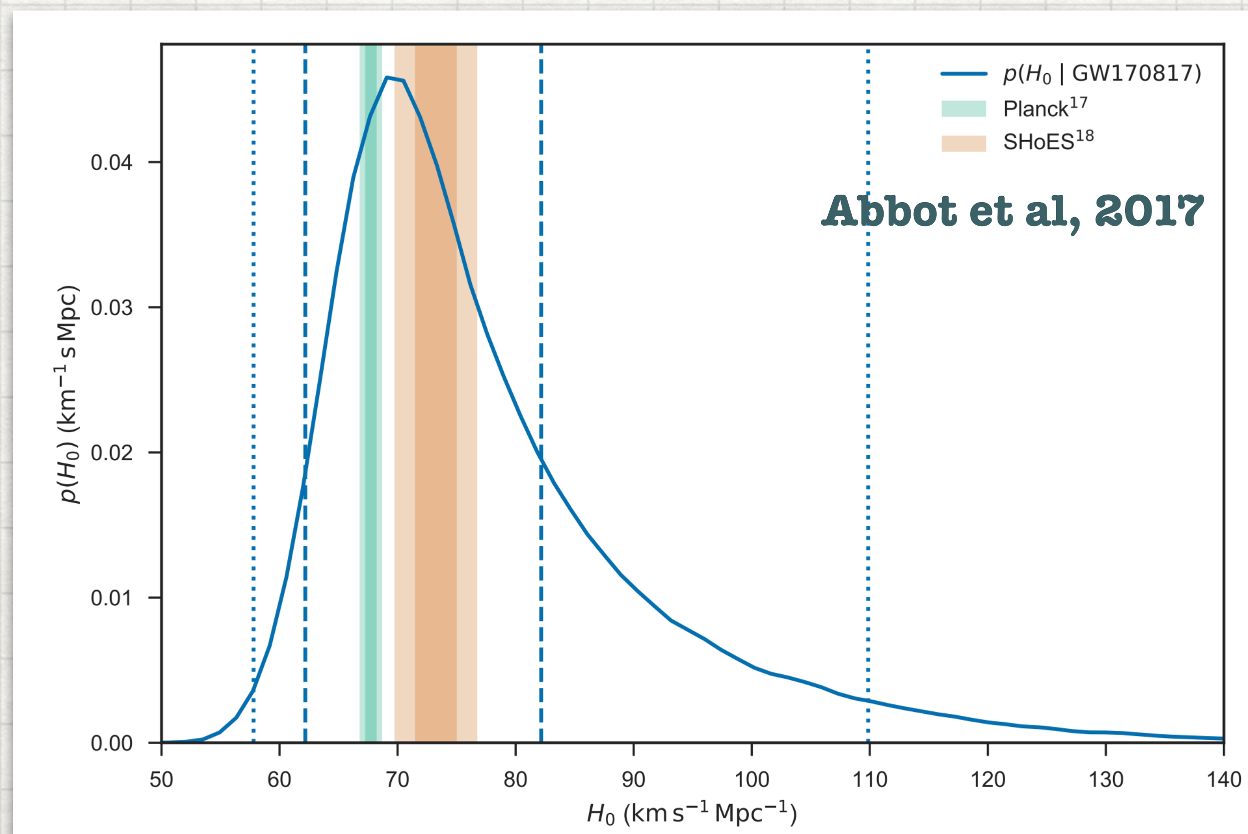
STANDARD SIRENS (SCHUTZ, 1986)

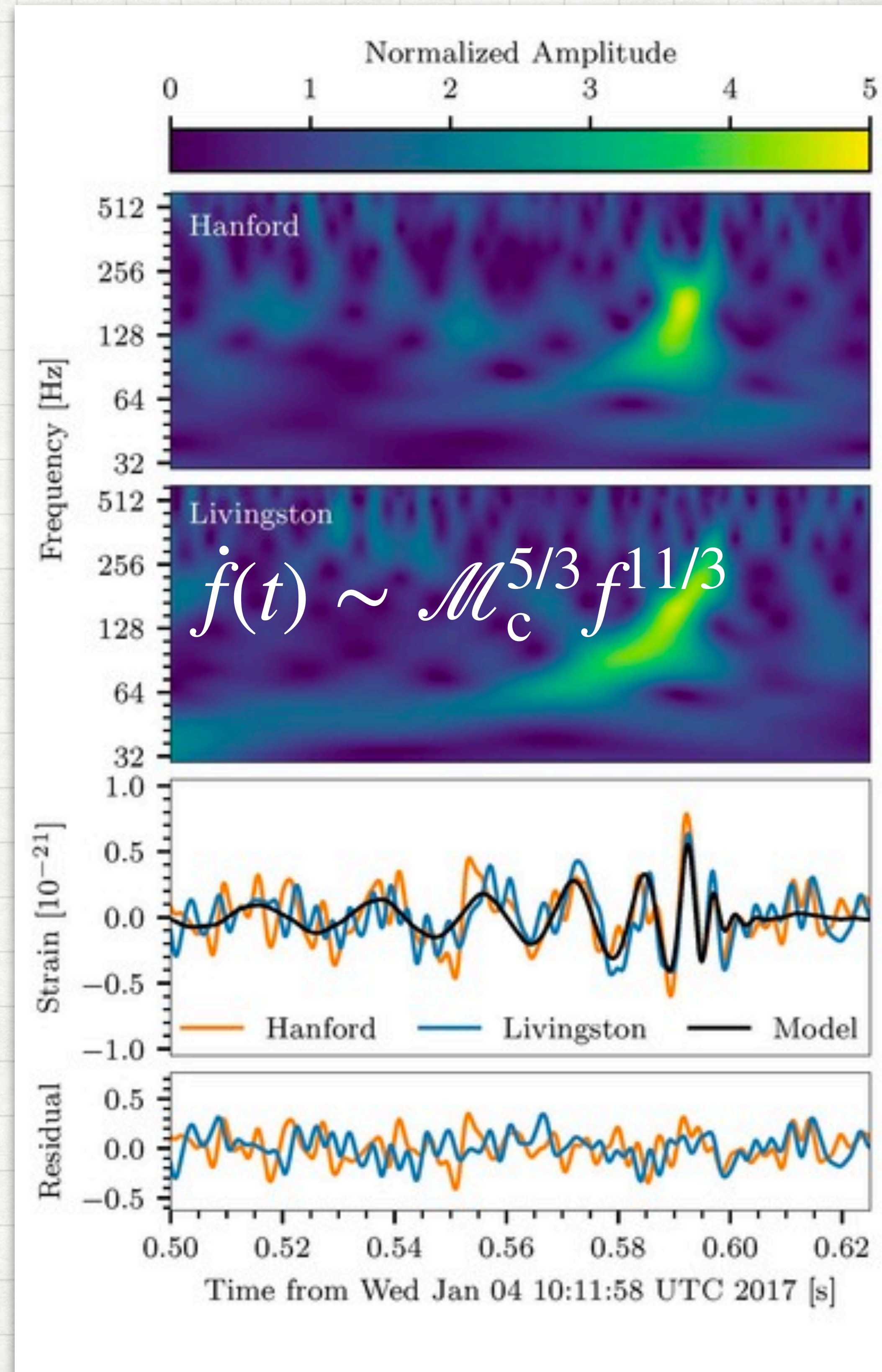
- **GWs measure the luminosity distance**

$$h_\times(t) \sim \frac{\cos\iota}{D_\mathrm{L}(z)} \left(\mathcal{M}_\mathrm{c}\right)^{5/3} f(t)^{2/3} \sin\Phi(t)$$

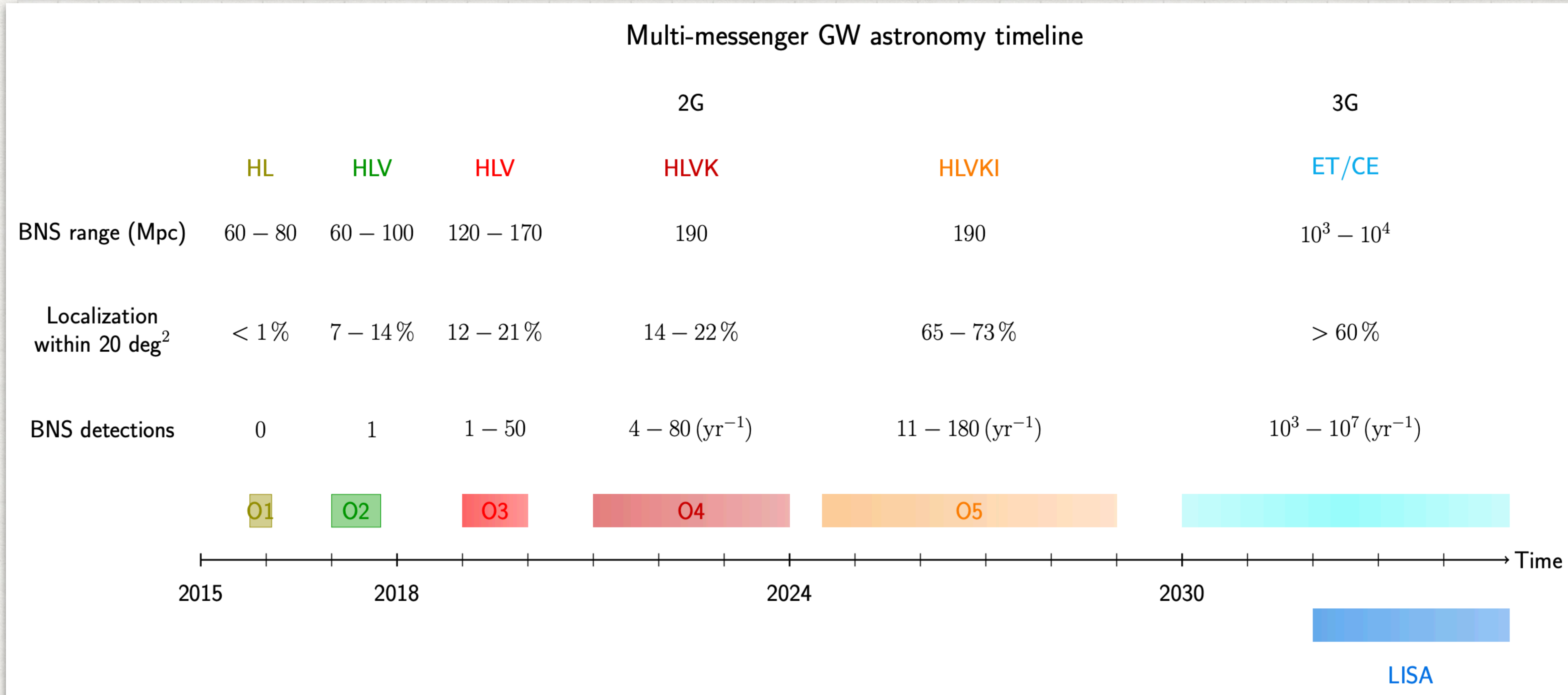
$$h_+(t) \sim \frac{1+\cos^2\iota}{D_\mathrm{L}(z)} \left(\mathcal{M}_\mathrm{c}\right)^{5/3} f(t)^{2/3} \cos\Phi(t)$$

- **Need electromagnetic counterpart?**





ANTICIPATED TIMELINE



Summary from Ezquiaga, Zumalacarregui, 2018

IN THIS TALK

- **Part 1: Clustering of black hole binaries — cosmology without electromagnetic counterparts. Testing the GW propagation in the late universe.**

$$h''_{ij} + \left[2 + \alpha_M(\eta)\right]\mathcal{H}h'_{ij} + k^2 h_{ij} = 0$$

Canas-Herrera, Contagian, Vardanyan, PRD 2020
Canas-Herrera, Contagian, Vardanyan, APJ 2021

- **Part 2: What can GWs teach us about the early universe? Tensor modes from matter sources — beyond the vacuum**

$$P_h(k) = \frac{8}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2 \longrightarrow P_h^{\rm induced}(k)$$

Cai, Jiang, Sasaki, Vardanyan, Zhou, arXiv:2105.12554

- **Part 3: Primordial black hole production and the associated GWs.**

Sasaki, Takhistov, Vardanyan, Zhang, In prep

Quick intro to general GW propagation

$$h''_{ij} + \left[2 + \alpha_M(\eta)\right] \mathscr{H} h'_{ij} + \left[\left(1 + \alpha_{\mathrm{T}}(\eta)\right) k^2 + m_{\mathrm{T}}^2(\eta)\right] h_{ij} = \Pi_{ij}$$

Dumping of GWs

GW speed

Massive gravitons

Anisotropic stress from matter

Connection to large-scale structure

$$h_\alpha'' + \left[2 + \alpha_M(z)\right] \mathcal{H} h_\alpha' - \vec{\nabla}^2 h_\alpha = 0$$

$$\alpha_M(z) \equiv \mathrm{d}M_{\mathrm{eff}}^2/\mathrm{d}\ln a$$

$$S_\phi = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}M_{\mathrm{eff}}^2\left(\phi\right)R + K\left(\phi, X\right) - G_3\left(\phi, X\right)\Box\phi\right)$$

$$\nabla^2\Psi \sim \mu(\eta, k)\rho\delta$$

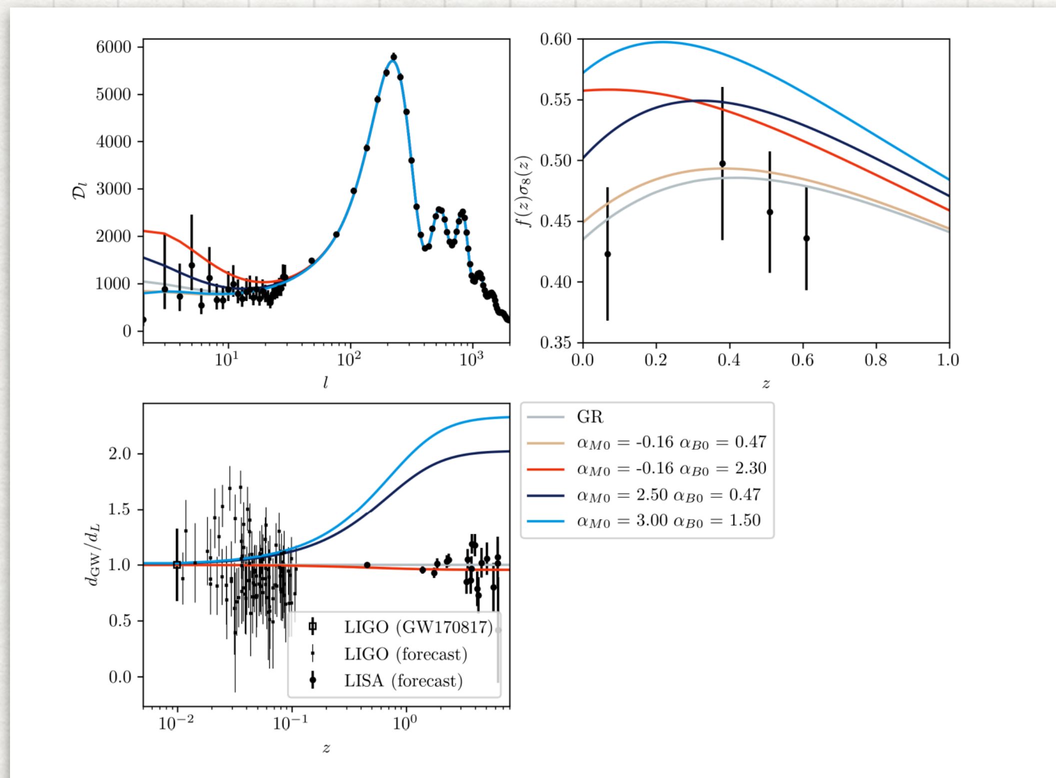
$$\nabla^2\left(\Psi + \Phi\right) \sim \Sigma(\eta, k)\rho\delta$$

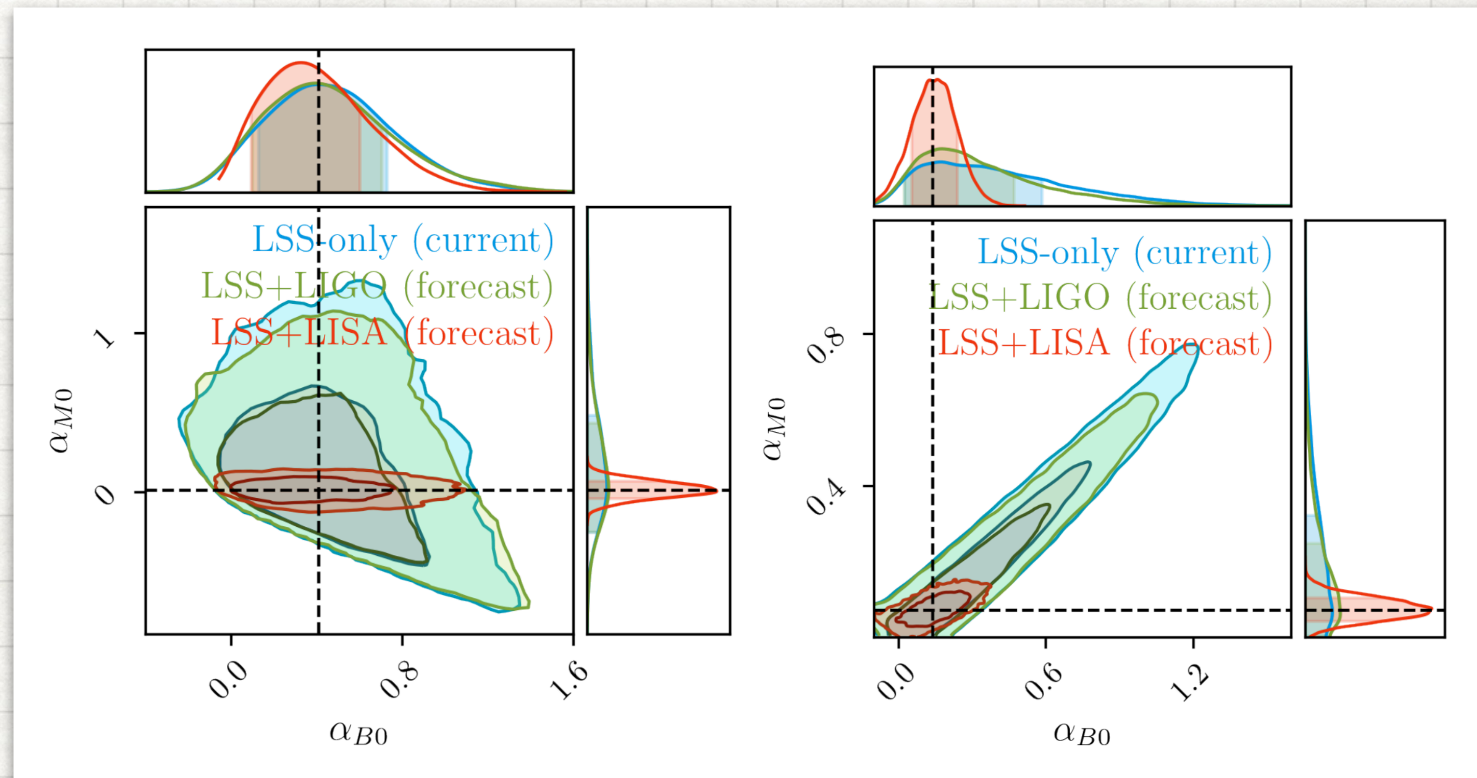
$$\eta \equiv \frac{\Phi}{\Psi} = 2\frac{\Sigma(\eta, k)}{\mu(\eta, k)} - 1$$

$\eta \neq 1 \rightarrow \alpha_{\mathrm{M}} \neq 0$ **or** $\alpha_{\mathrm{T}} \neq 0$

Saltas, Sawicki, Amendola, Kunz, 2014

Connection to large-scale structure





Baker, Harrison, 2020

Tomographic cross-corrs: what are they good for?

$$h''_\alpha + \left[2 + \alpha_M(z)\right] \mathcal{H} h'_\alpha - \vec{\nabla}^2 h_\alpha = 0$$

$$\alpha_M(z) \equiv \mathrm{d}M^2_{\mathrm{eff}}/\mathrm{d}\ln a$$

$$S_\phi = \int \mathrm{d}^4x \sqrt{-g} \left(\frac{1}{2} M^2_{\mathrm{eff}}(\phi) R + K(\phi, X) - G_3(\phi, X) \Box \phi \right)$$

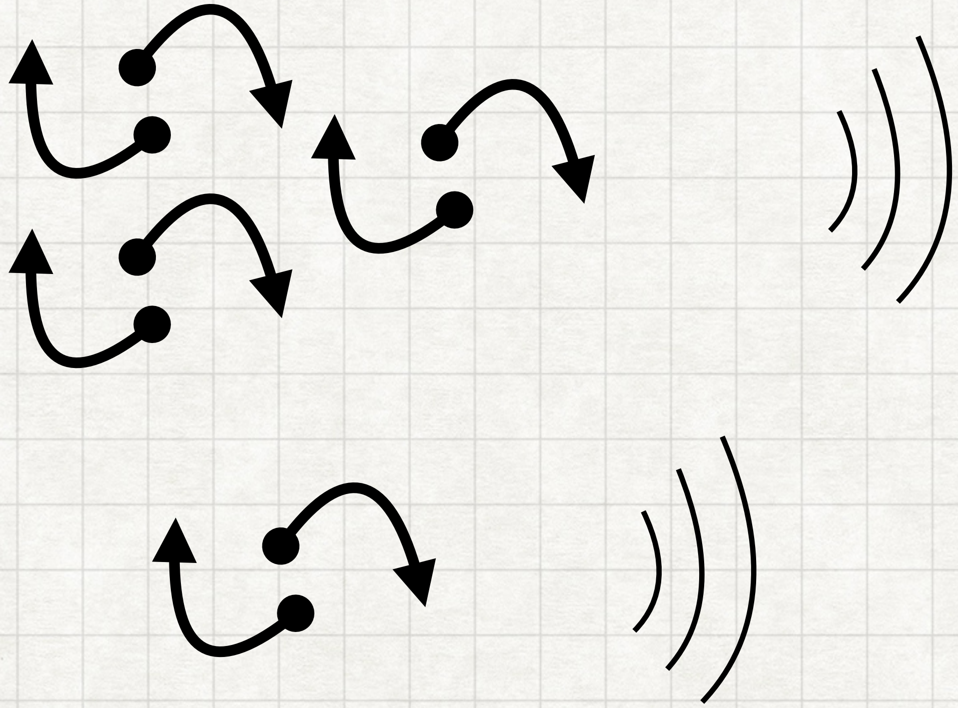
$$\frac{D_{\mathrm{L,GW}}(z)}{D_{\mathrm{L,EM}}(z)} = \exp\left\{ -\frac{1}{2} \int_0^z \mathrm{d}\tilde{z} \, \frac{\alpha_M(\tilde{z})}{(1+\tilde{z})} \right\} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$

Common parametrization

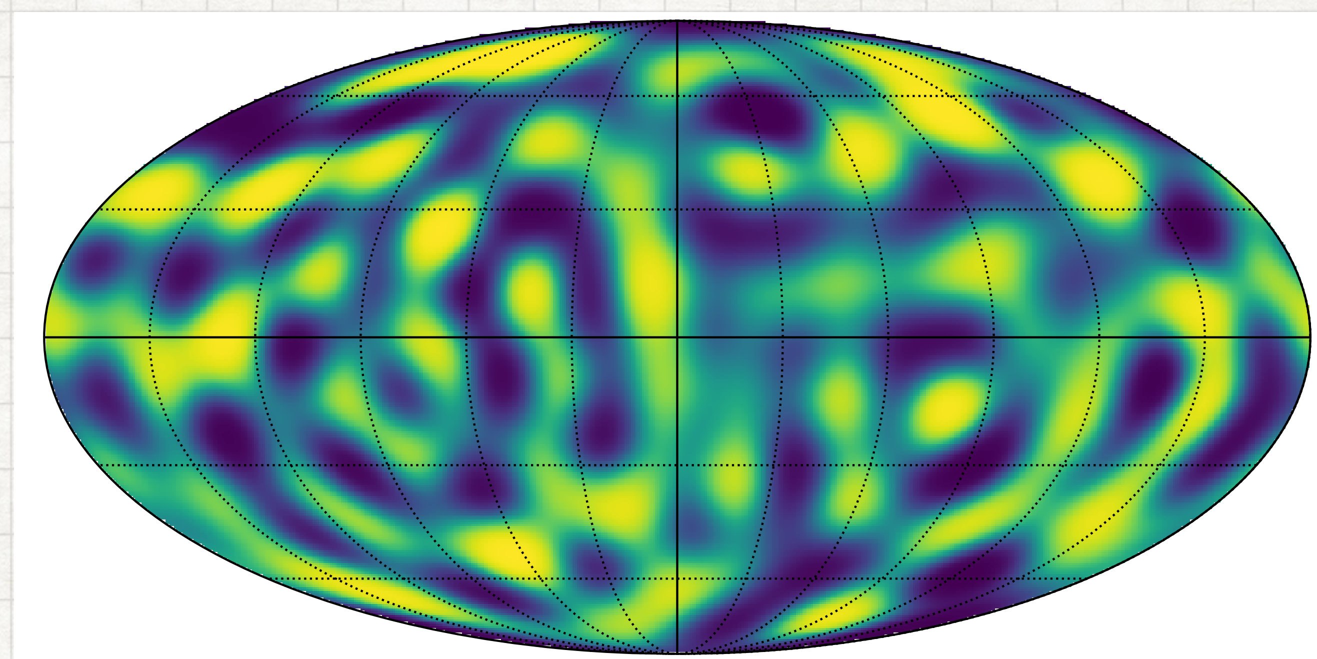
Belgacem, Dirian, Foffa, Maggiore, 2018

Part 1: Resolved GW sources

Based on: **Learning how to surf: reconstructing the origin and propagation of GWs with Gaussian Processes, arXiv: 2105.04262**

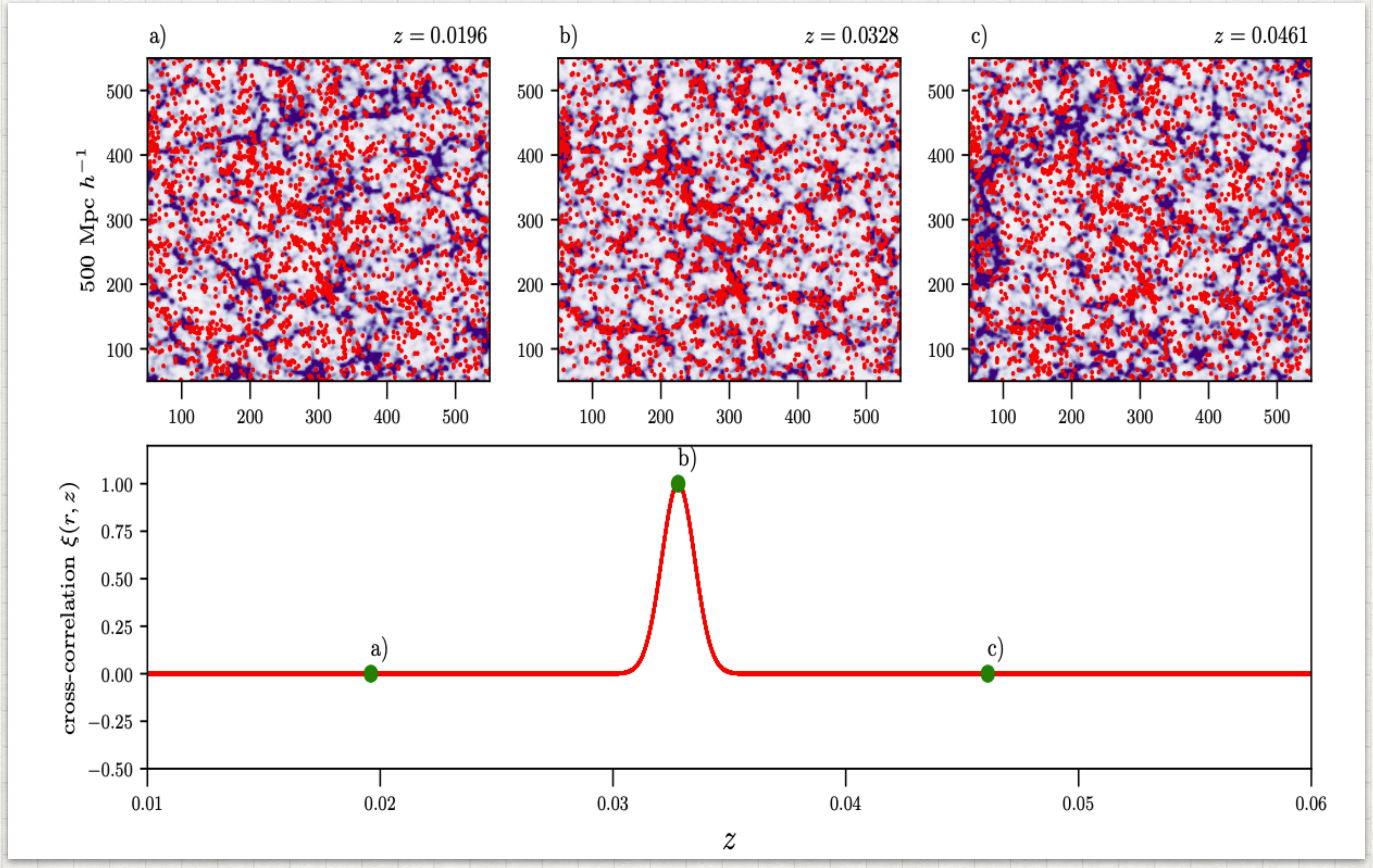


Clustering of resolved compact object binaries



The resulting observable is the projected number count.

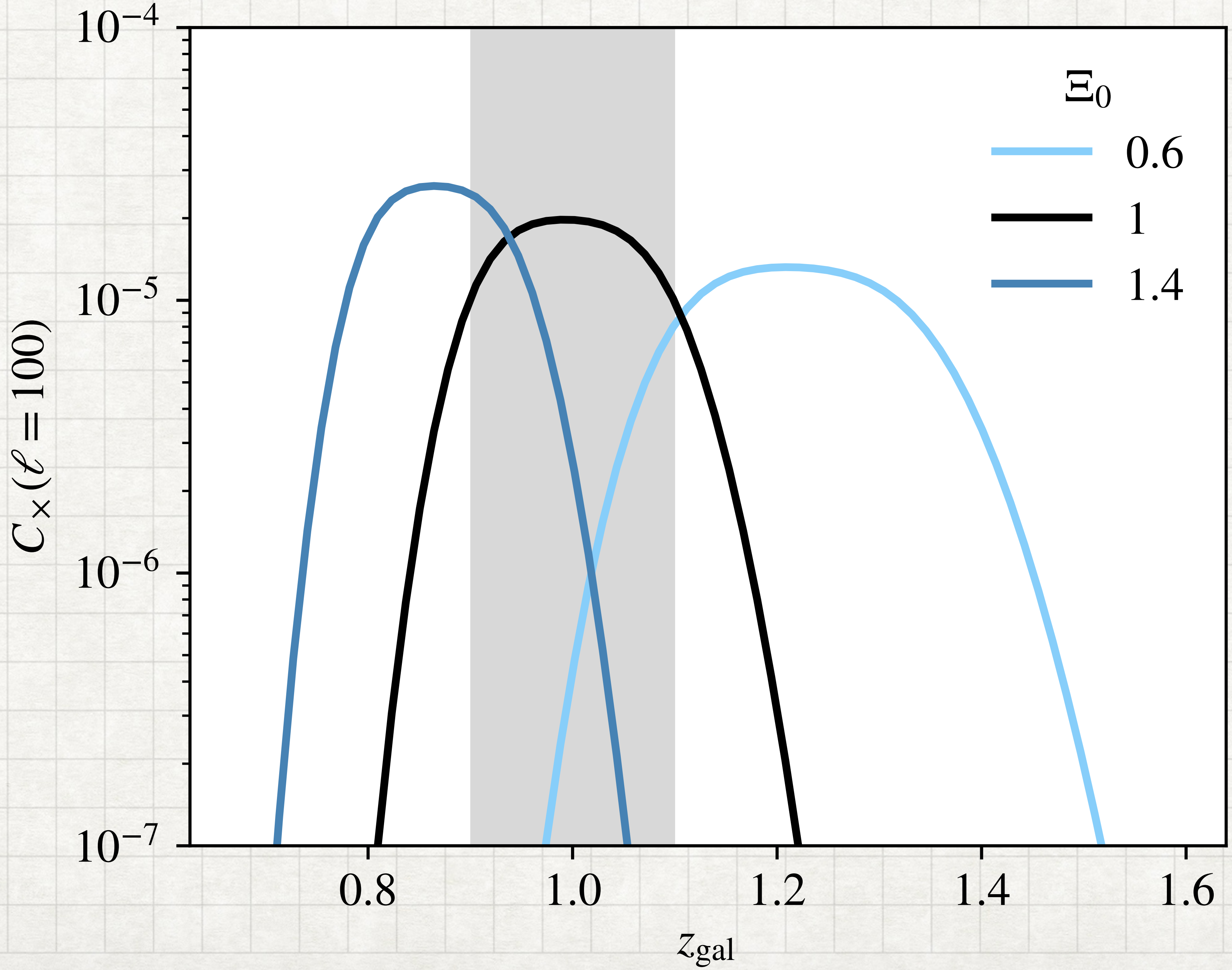
Redshift estimation



From Bera et al, 2020

Tomographic cross-corrs: what are they good for?

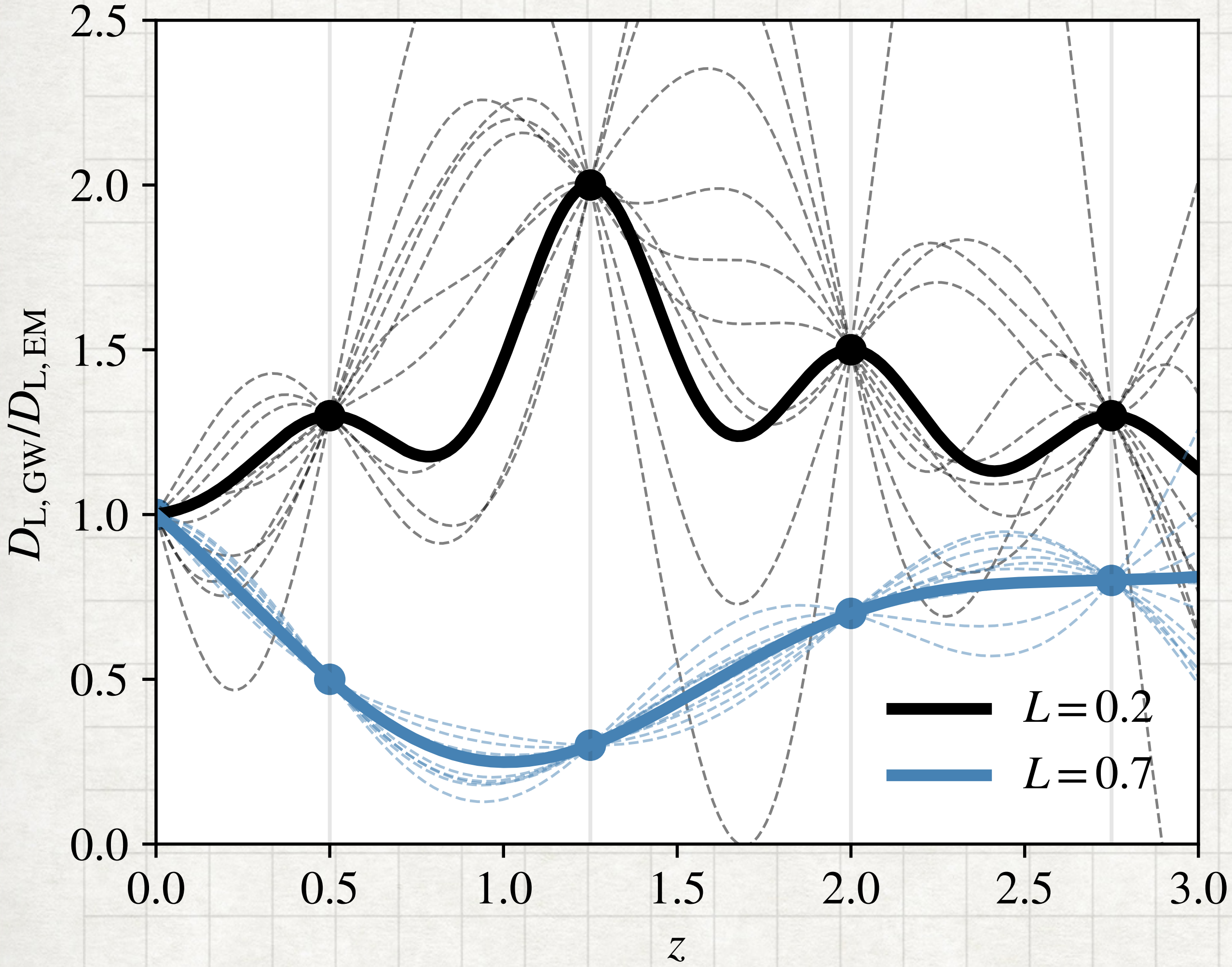
$$\frac{D_{\mathrm{L,GW}}(z)}{D_{\mathrm{L,EM}}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$



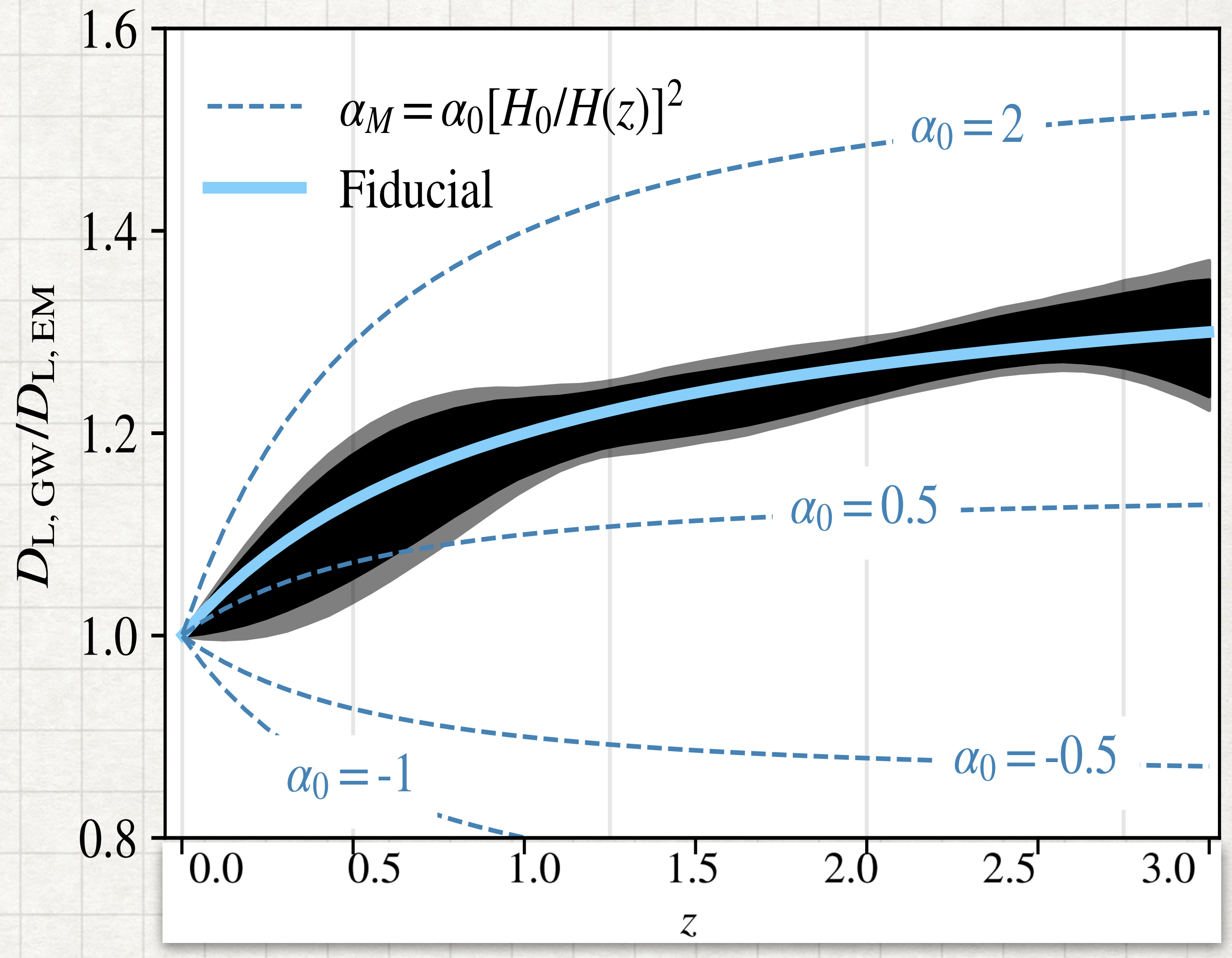
Gaussian process reconstruction

Not very general

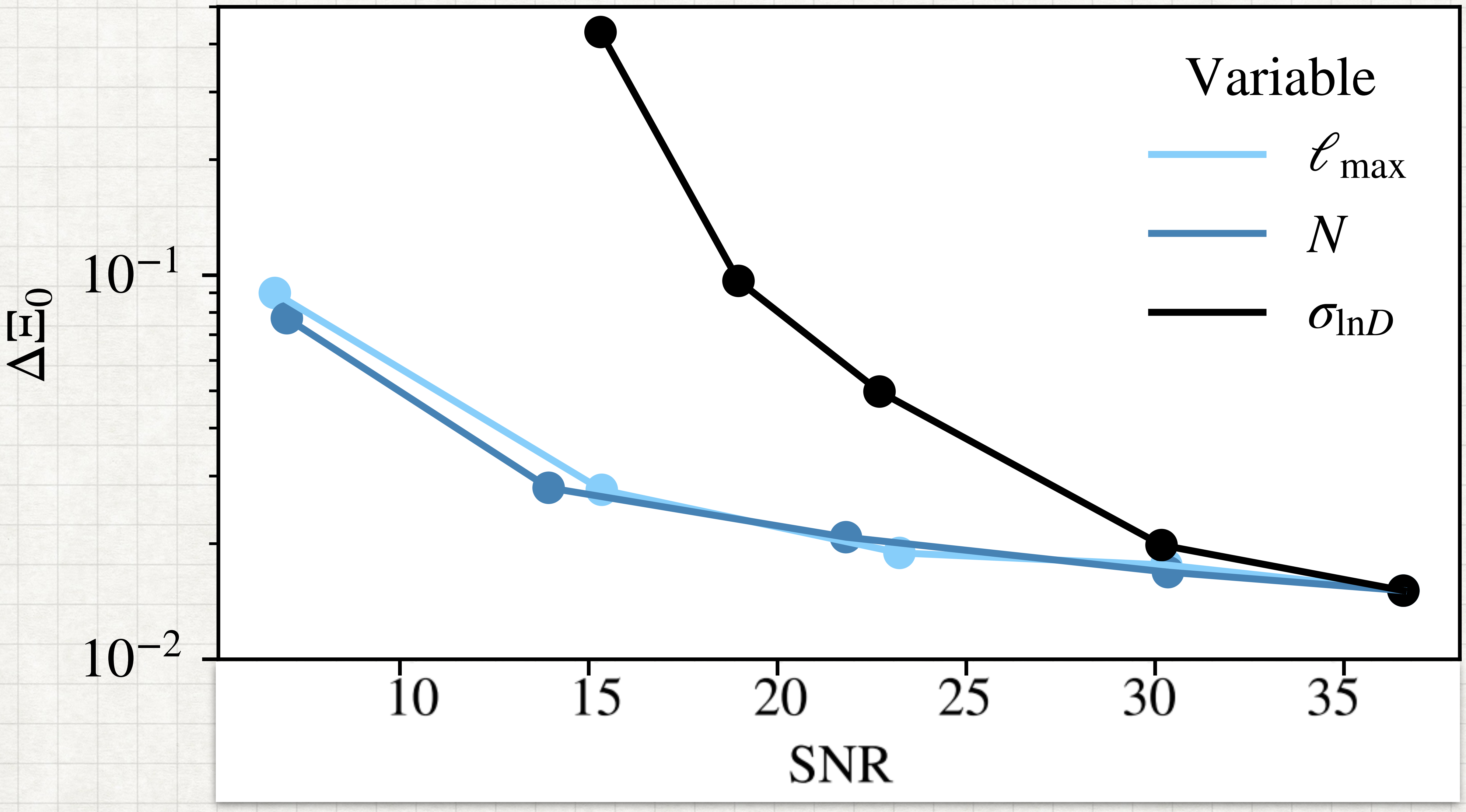
$$\frac{D_{L,GW}(z)}{D_{L,EM}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n}$$



Reconstruct the shape model independently



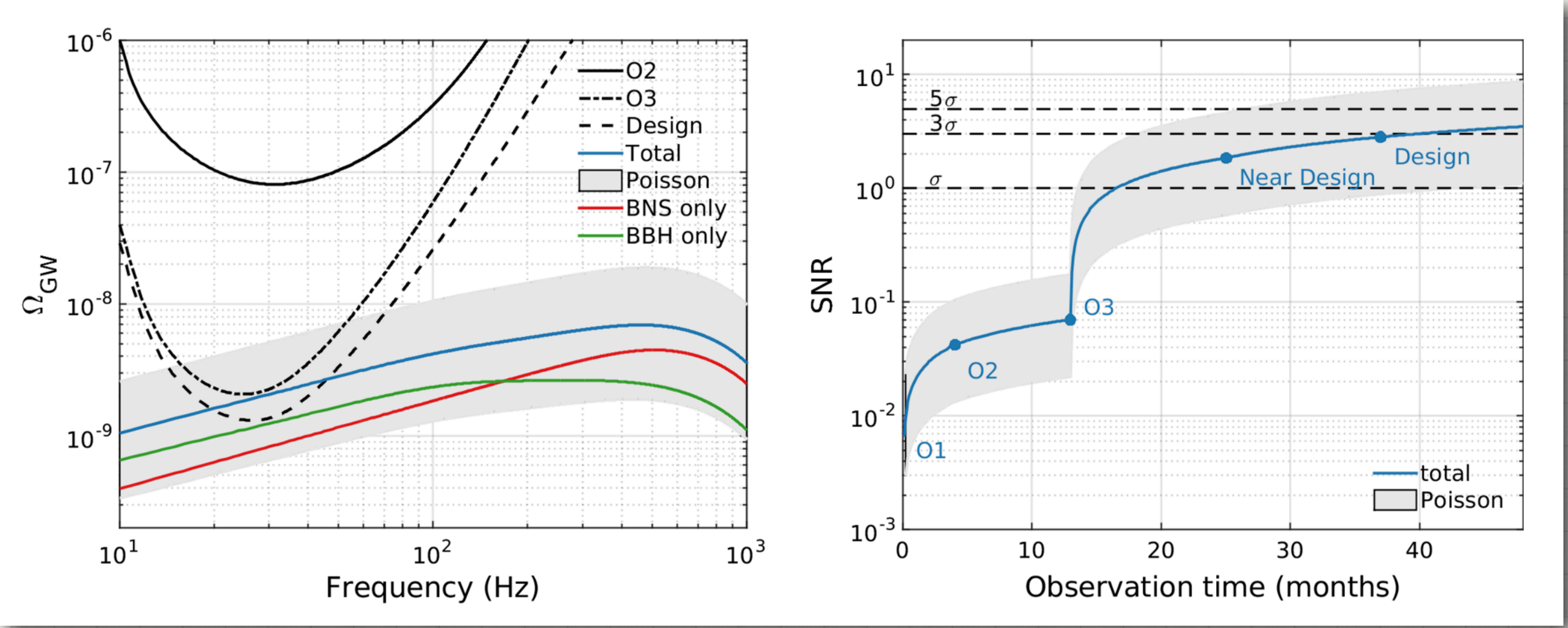
Observational strategy



Astrophysical GW background

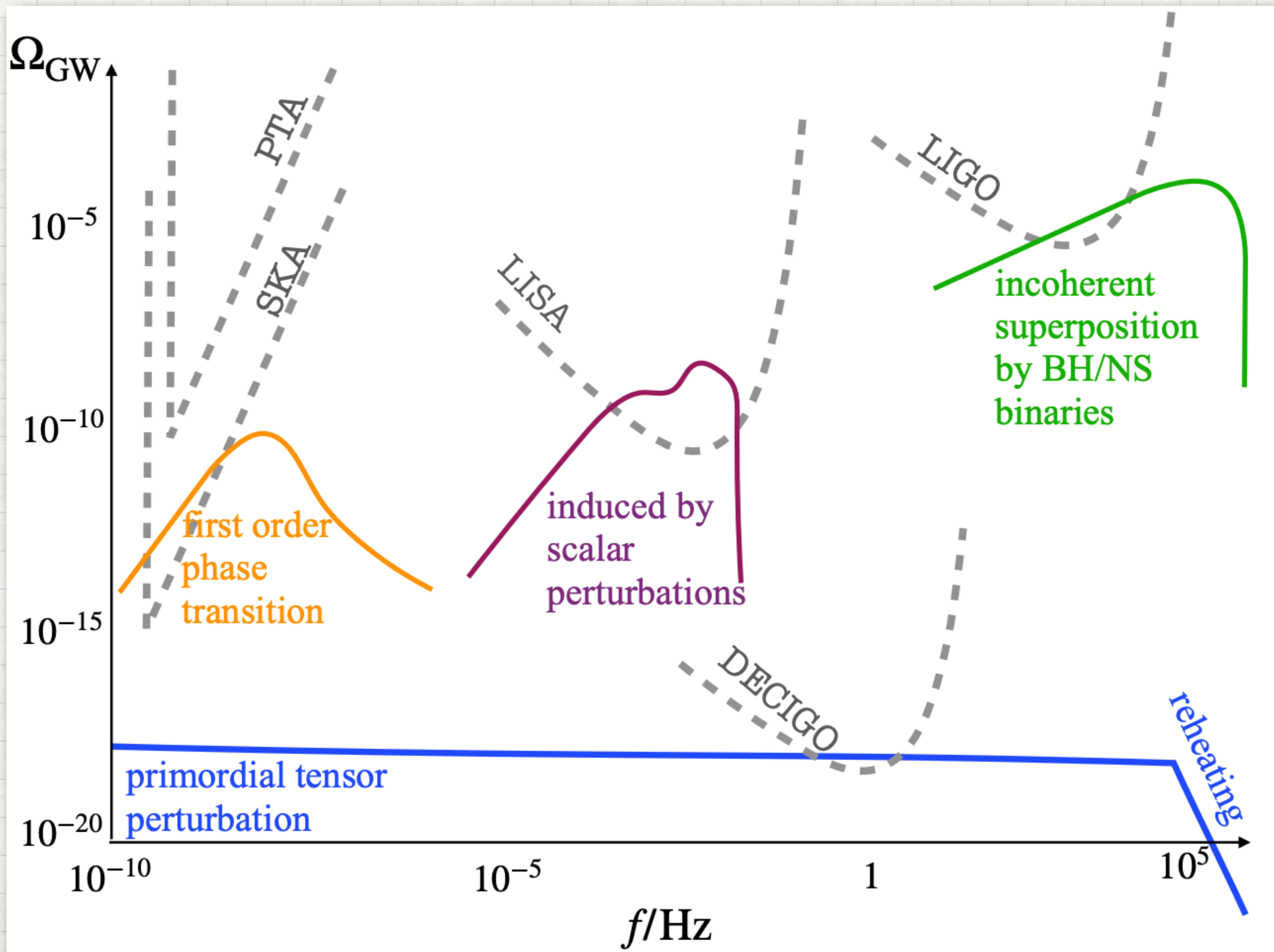
Based on: Cross-correlation of the AGWB with galaxy clustering, arXiv:1910.08353

$$\Omega_{\rm GW}(f,\theta) = \frac{f}{\rho_c H_0}\int_0^{z_{\rm max}} dz \frac{R_m(z;\theta)dE_{\rm GW}(f_s;\theta)/df_s}{(1+z)E(\Omega_M,\Omega_\Lambda,z)}.$$



LIGO/VIRGO Collaboration: "GW170817: Implications for the Stochastic Gravitational-Wave Background from Compact Binary Coalescences"

Multitude of sources



Modelling the signal

The map on the sky → $$\Omega_{\rm GW}(\nu_0,\hat{r}) = \frac{\nu_0}{\rho_c}\frac{d\rho_{\rm GW}(\nu_0,\hat{r})}{d\nu_0 d^2\hat{r}}$$

Let's model this as → $$\Omega_{\rm GW}(\hat{r}) \equiv \int dr\, r^2 \mathcal{K}(r) n(\vec{r})$$

$$\Omega_{\rm GW}(\hat{r}) = \int dr\, r^2 \mathcal{K}(r)\bar{n}(r)\left(\delta_{\rm GW}(\vec{r})+1\right)$$

Astrophysical kernel

Modelling the signal

$$\Omega_{\rm GW}(\hat{r}) = \int {\rm d}r\; r^2 \mathscr{K}(r) \bar{n}(r) \left(\delta_{\rm GW}(\vec{r}) + 1\right)$$

Astrophysical kernel

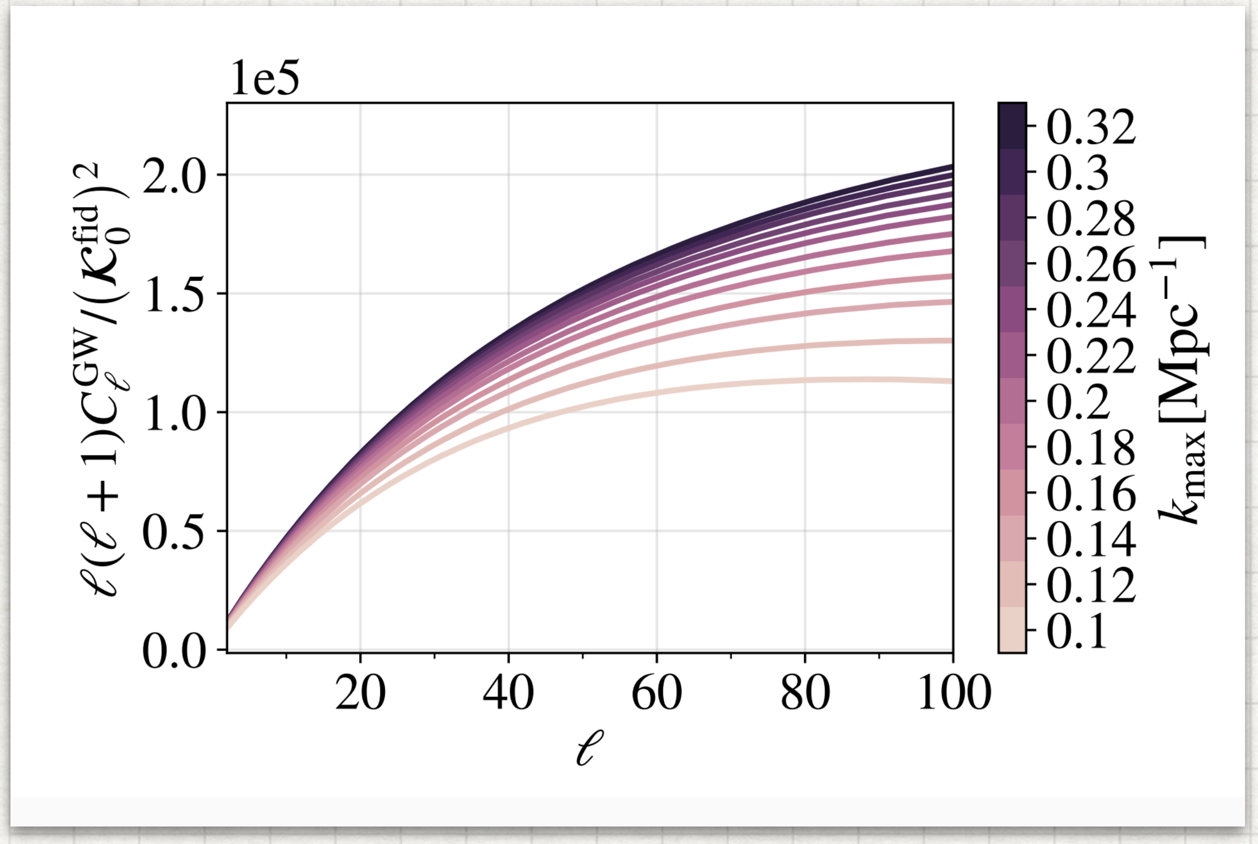
From real space to angular correlations

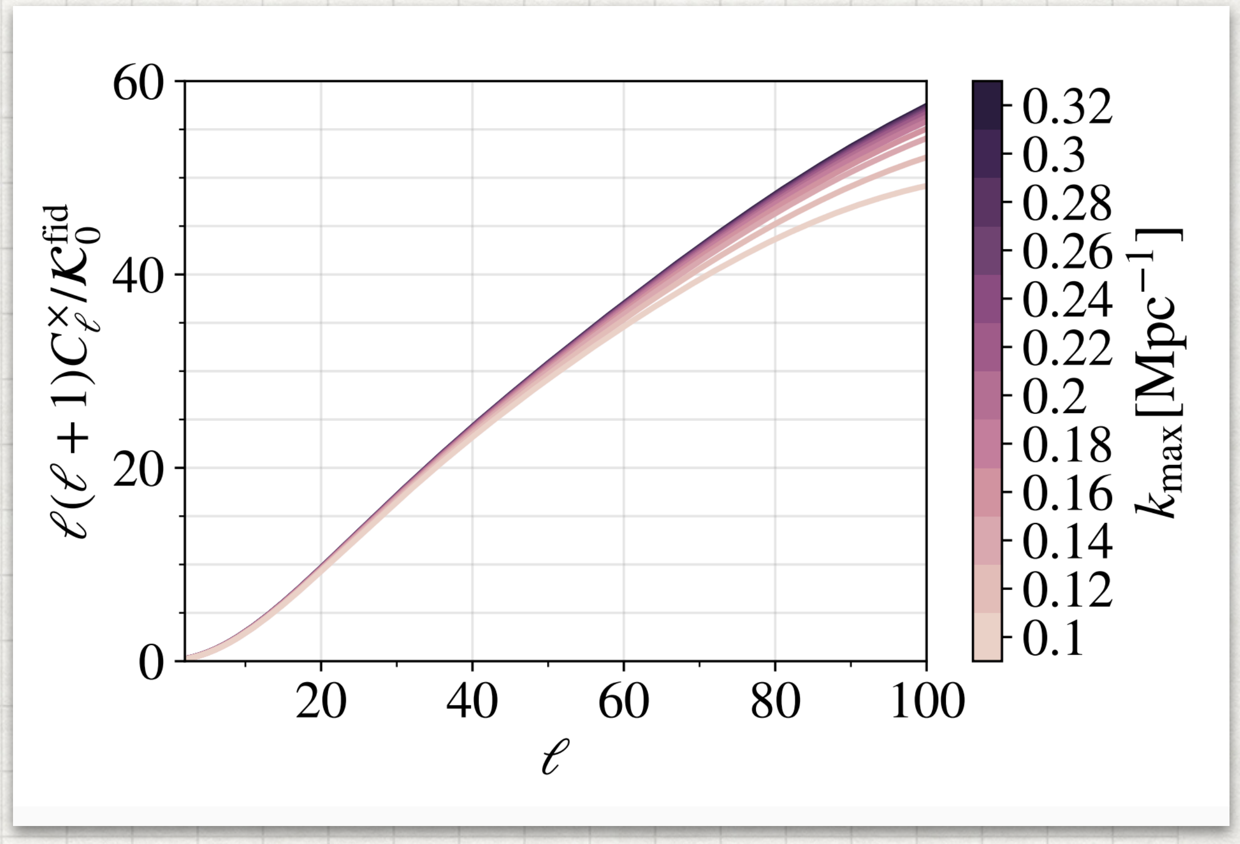
$$C_\ell^{\rm GW} = 4\pi \int_{k_{\rm min}}^{k_{\rm max}} \frac{dk}{k} \,|\, \delta\Omega_\ell \,|^2 \mathscr{P}(k) + B_\ell^{\rm GW}$$

- **Integrands** $\delta\Omega_\ell(k) = \int {\rm d}r\; r^2 \mathscr{K}(r) \bar{n}(r) T_{\rm GW}(k, r) j_\ell \left(kr\right)$

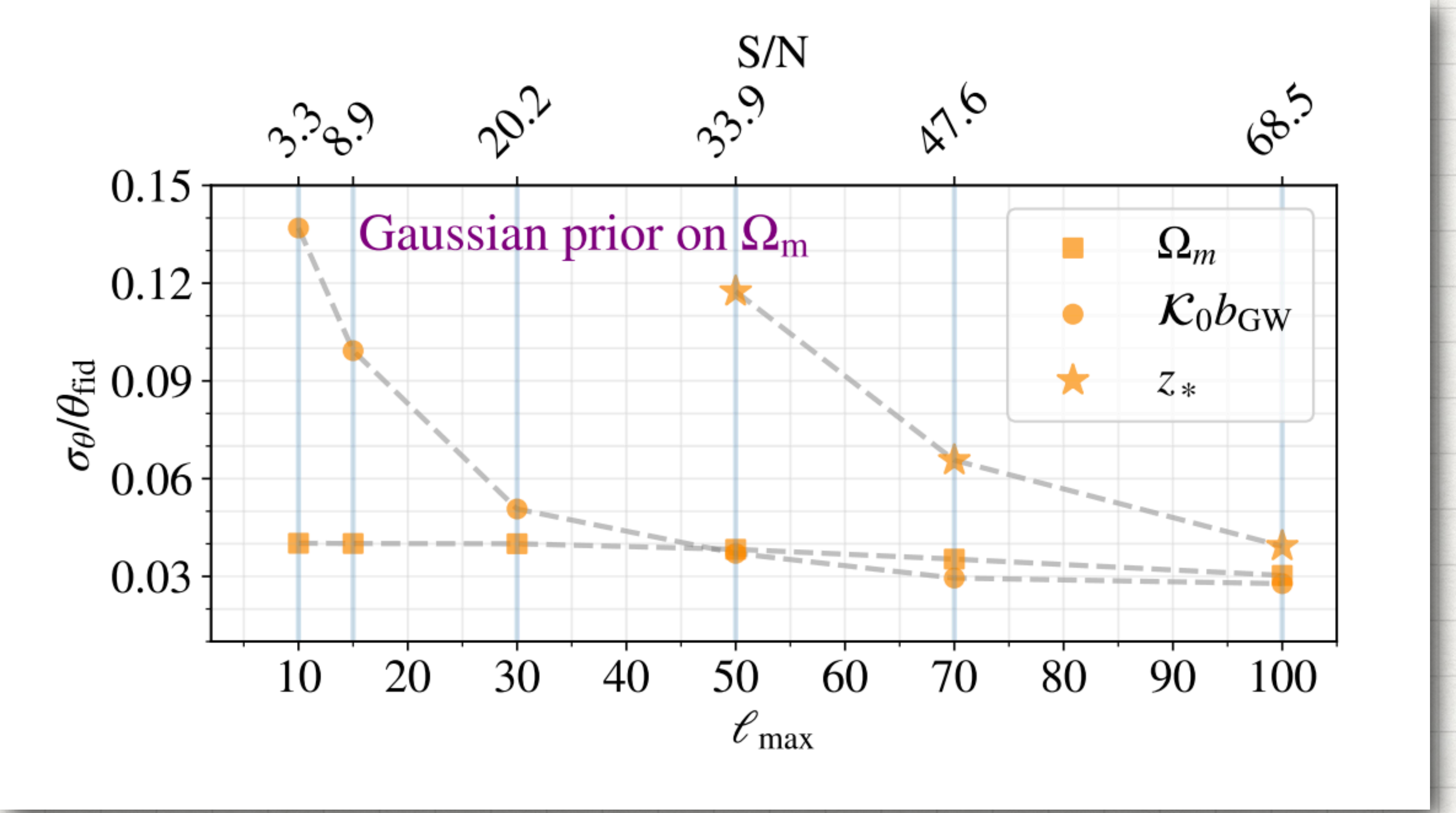
No relativistic corrections for now!

Sensitivity on non-linearities



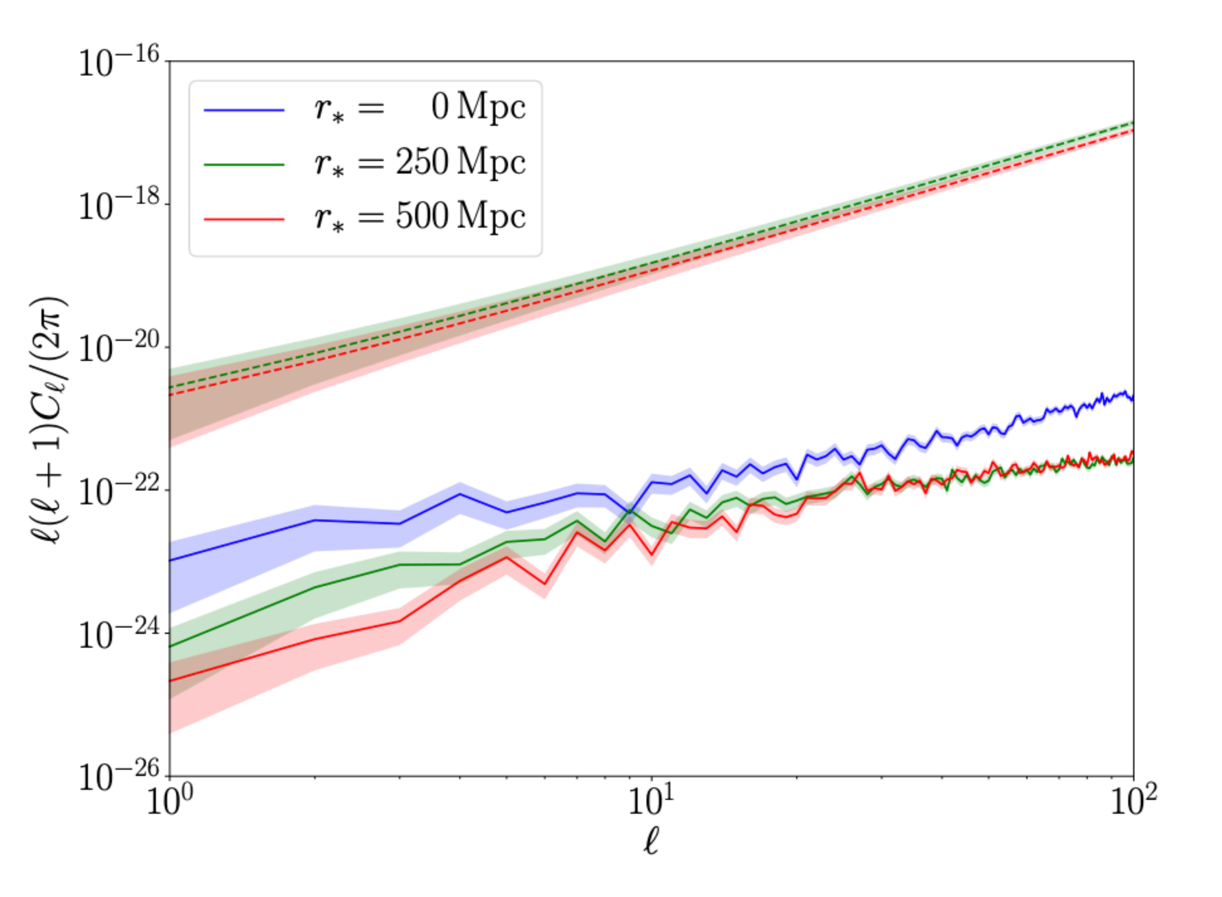


What to expect?



- **The anisotropies will first be detected via the cross-correlation**
- **Sensitive to the features in the astrophysical kernel** $\mathscr{K}(z)$
- **Useful for cosmology if** $\ell_{\rm max} \sim 100$ **is detected**

The shot-noise issue



$$B_\ell^{\rm GW} = \int dr \frac{\tilde{\mathcal{K}}^2(r)}{\bar{n}(r) r^2}\left[1 + \frac{1+z(r)}{R(r) T_O}\right]$$

Jenkins, Sakellariadou, 2019
Canas-Herrera, Contigiani, Vardanyan, 2019
Alonso, Cusin, Ferreira, Pitrou, 2020

$$\mathrm{Var} C_\ell^\times = \frac{\left(C_\ell^{\rm GW} + B_\ell^{\rm GW}\right)\left(C_\ell^{\rm GC} + B_\ell^{\rm GC}\right) + \left(C_\ell^\times + B_\ell^\times\right)^2}{2\ell + 1}$$

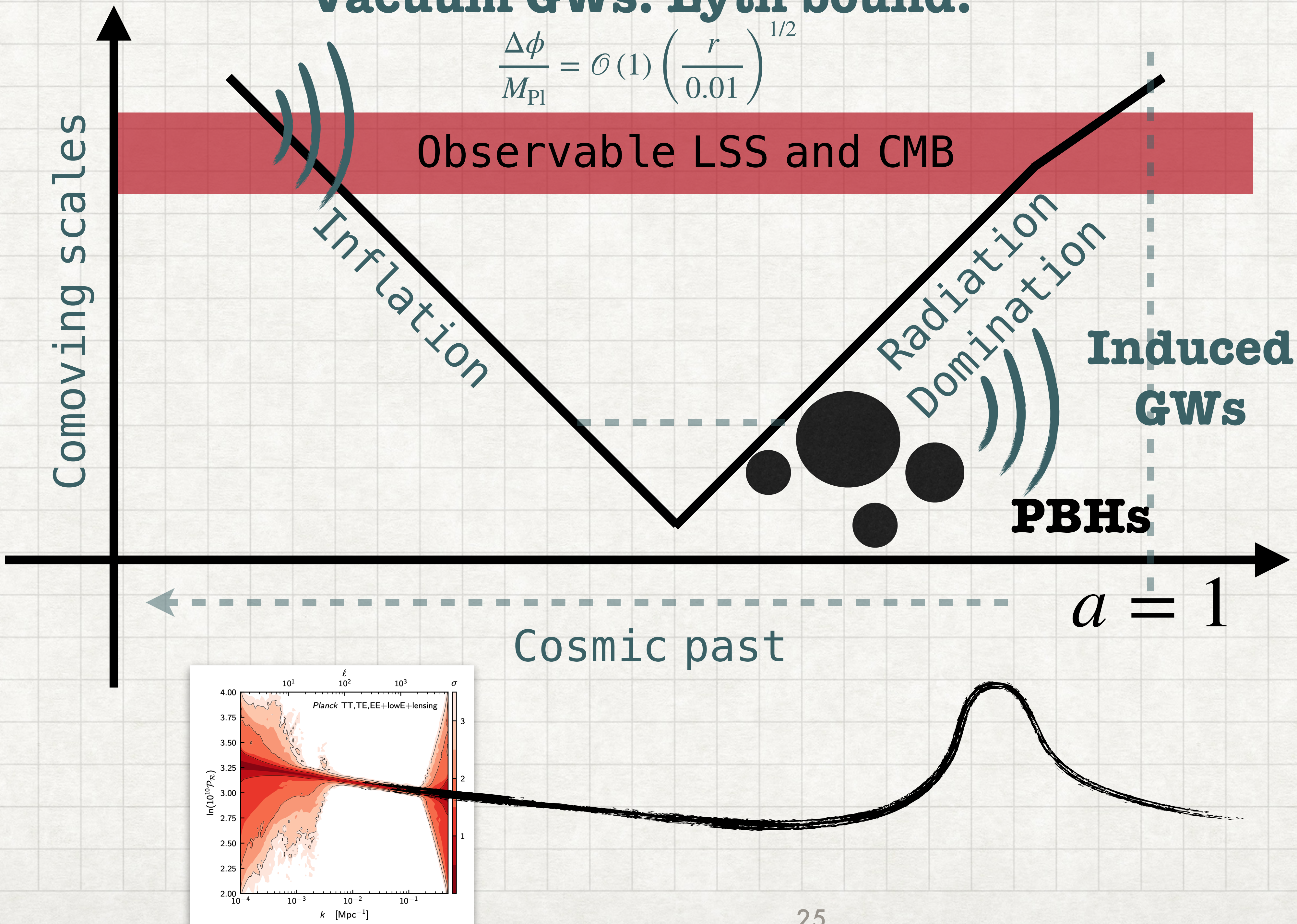
Cross-corr variance can be huge due to contamination from the auto-corr

Part 2: Primordial GWs

Based on: **Beating the Lyth bound by parametric resonance during inflation, arXiv: 2105.12554**

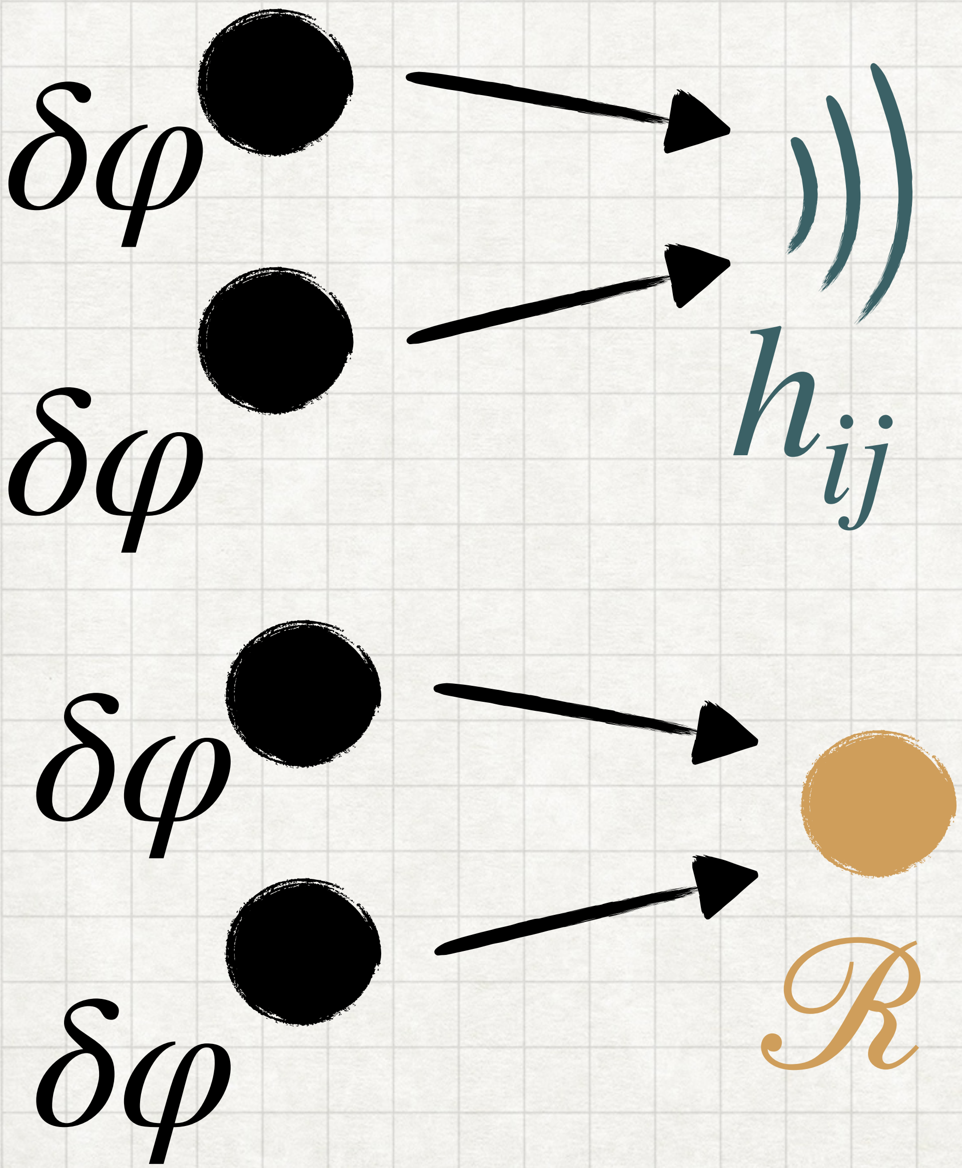
Vacuum GWs. Lyth bound:

$$\frac{\Delta\phi}{M_{\rm Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2}$$



Inducing GWs during inflation

$$h_{ij}'' + 2\mathscr{H} h_{ij}' - \vec{\nabla}^2 h_{ij} \sim \Pi_{ij}^{ab} \partial_a \delta\varphi \partial_b \delta\varphi$$



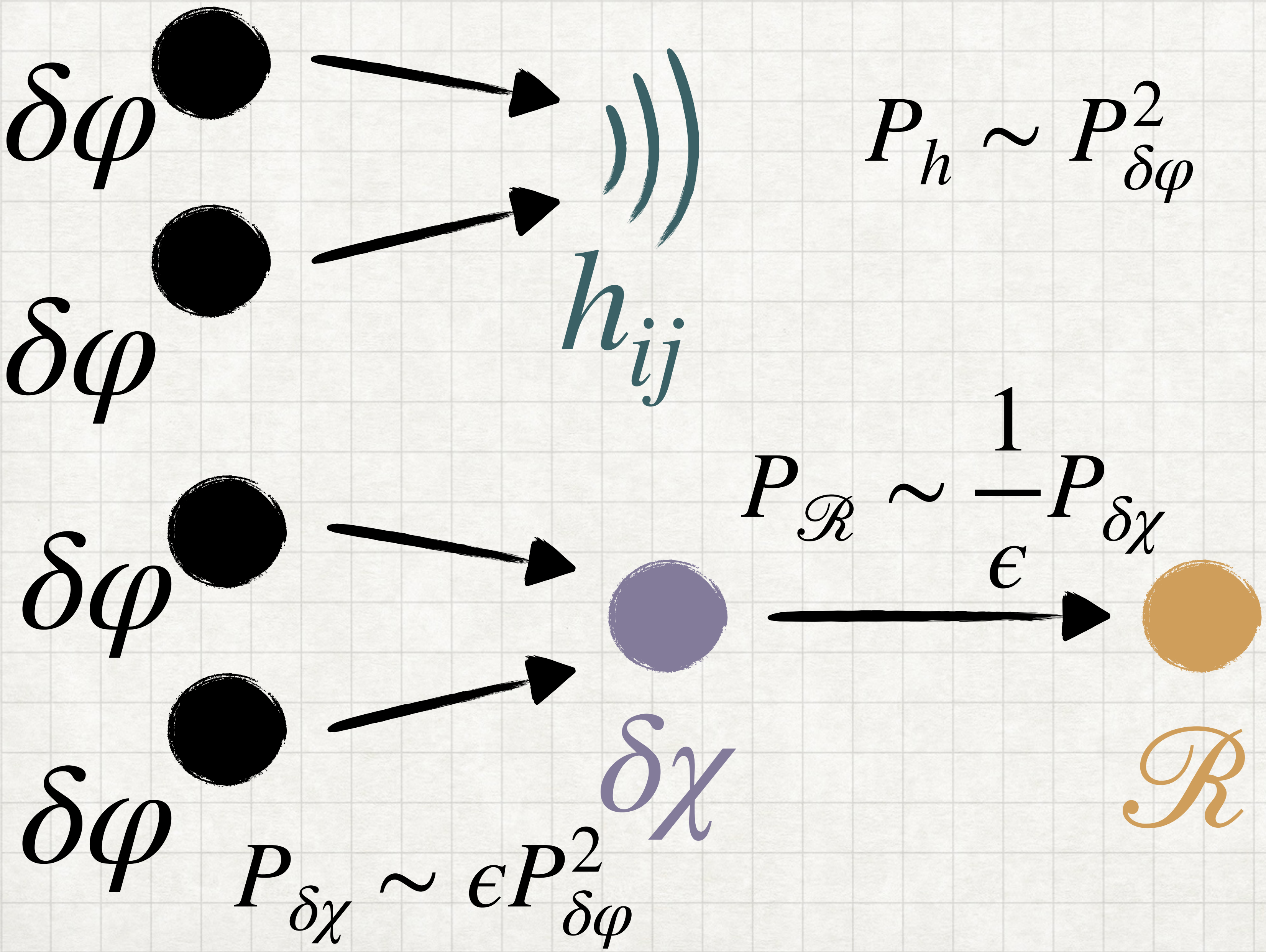
$$P_h \sim P_{\delta\varphi}^2$$

$$P_{\mathscr{R}} \sim \frac{1}{\epsilon^2} P_{\delta\varphi}^2$$

$$r^{\mathrm{ind}} \sim \epsilon^2$$

Inducing GWs during inflation

$$h_{ij}'' + 2\mathcal{H} h_{ij}' - \vec{\nabla}^2 h_{ij} \sim \Pi_{ij}^{ab} \partial_a \delta\varphi \partial_b \delta\varphi$$



$$r^{\text{ind}} \sim \epsilon^0$$

Inducing GWs during inflation

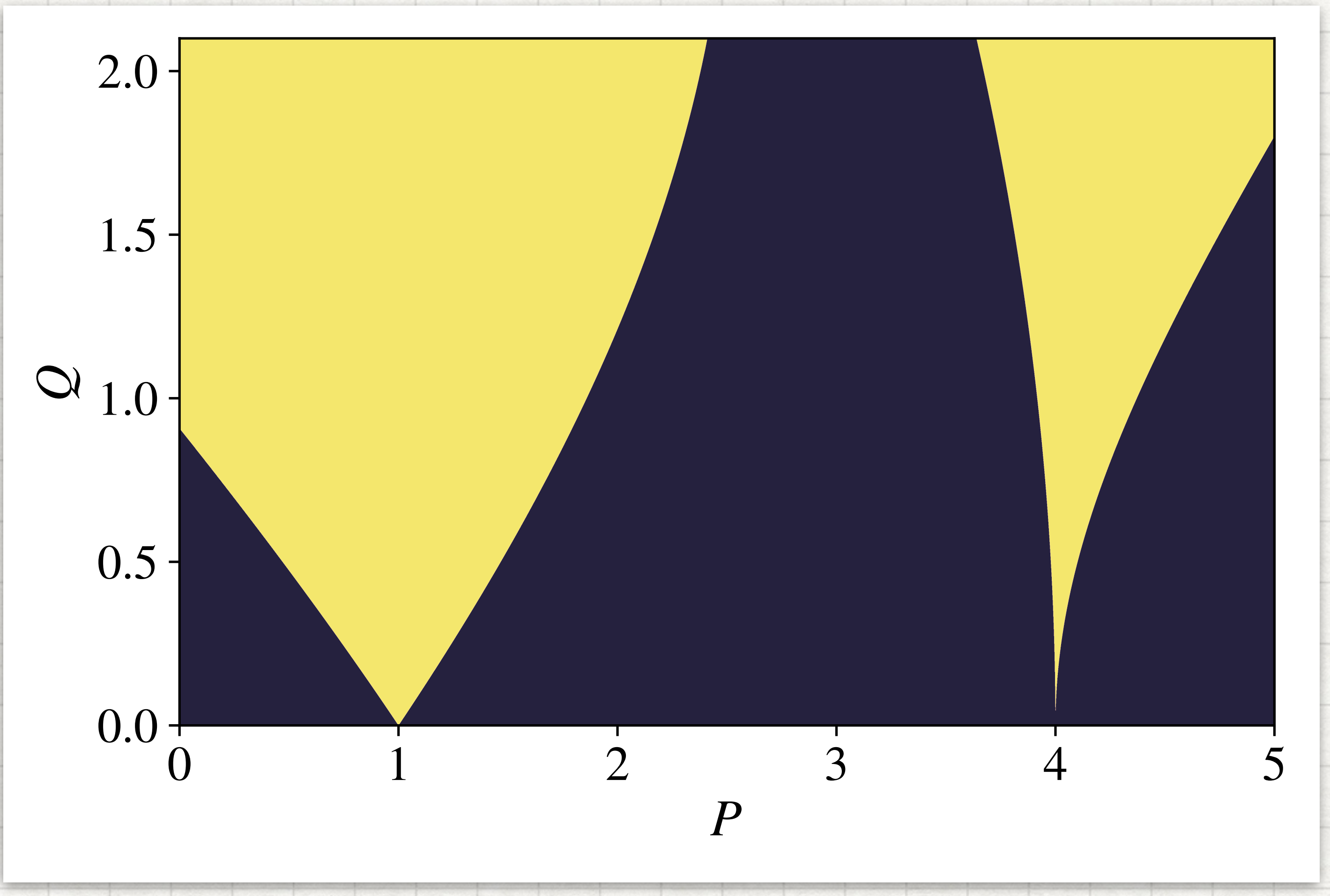
- $\delta\chi$ — massless field, controls the curvature perturbations
- $\delta\phi$ — massive field, resonantly amplified, gravitationally coupled to $\delta\chi$

- Three main conditions on the effective mass of $\delta\phi$

1. Resonance should occur inside horizon
2. The resonant field should decay later
3. Shortly after horizon crossing the resonant field mass should be small

Inducing GWs during inflation

- **Parametric resonance** $u(\tau)'' + \left[P - 2Q\cos(2\tau)\right] u(\tau) = 0$
- **Exponential enhancement if in the resonance band**



- **Consider** $\mathscr{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}\left(\partial\chi\right)^2 - V(\phi,\chi)$

$V(\chi)$ → **Sources inflation**

$V(\phi) = -V_0\sqrt{2\epsilon_\phi}\frac{\phi}{M_{\rm Pl}} + \Lambda(\phi)\cos\left(\frac{\phi}{f_{\rm a}}\right) + V_{\rm m}(\phi)$ → **Oscillating field**

Inducing GWs during inflation

- **Parametric resonance** $u(\tau)'' + \left[P - 2Q\cos(2\tau)\right]u(\tau) = 0$
- **Exponential enhancement if in the resonance band**
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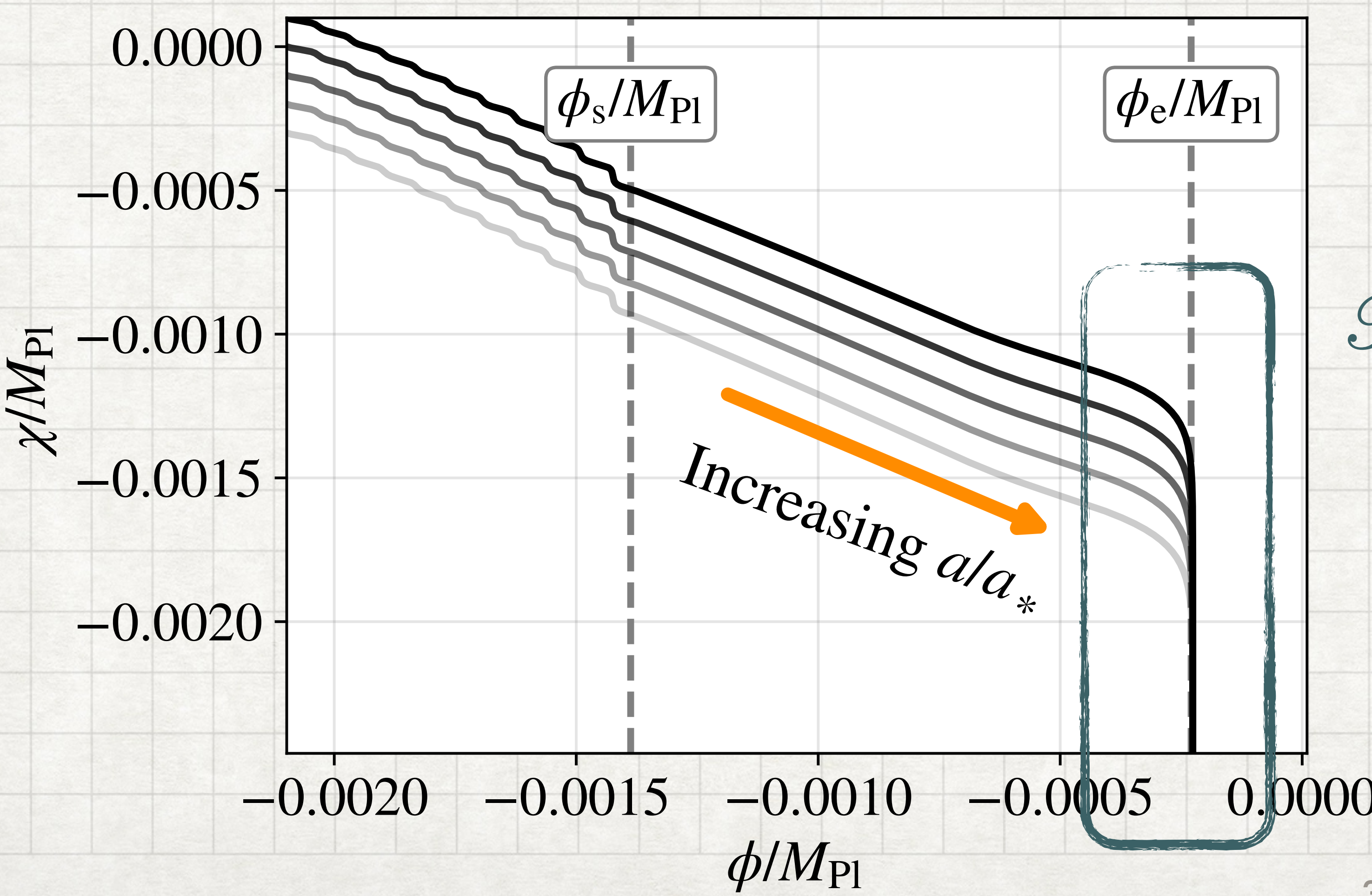


$$\mathscr{R}_{\rm c} \sim \frac{\partial N}{\partial\phi^a}\delta\phi^a \sim \frac{\partial N}{\partial\chi}\delta\chi$$

Mode functions

Non-linear source

$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \frac{k^2}{a^2}\delta\chi_k = \frac{\sqrt{2\epsilon_\chi}}{M_{\rm Pl}}\left[\ddot{\phi}\delta\phi_k + \mathcal{S}_k\right],$$

Harmonic oscillations

$$\ddot{\delta\phi}_k + 3H\dot{\delta\phi}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2_{\rm eff}\right)\delta\phi_k = 0$$

$$\delta\varphi_k \equiv a^{3/2}\delta\phi_k \longrightarrow \ddot{\delta\varphi}_k + \omega_k^2(t)\,\delta\varphi_k = 0$$

$$\omega_k^2(t) \approx \frac{k^2}{a^2} - \frac{9}{4}H^2 - \frac{\Lambda^4}{f_{\rm a}^2}\cos\left(\frac{\phi}{f_{\rm a}}\right)$$

$$|\,\delta\phi_{k_*}| \propto e^{\lambda_{k_*}Ht},\;\; \lambda_{k_*} = \mu_{k_*}M_{\rm Pl}\sqrt{2\epsilon_\phi}/2f_a - \frac{3}{2}$$

$$\mathscr{A} \equiv |\,\delta\phi_{k_*}|/|\,\delta\phi_{k_*,vac}| \gg 1 \longrightarrow$$ **Amplification**

Casting into Matheu equation

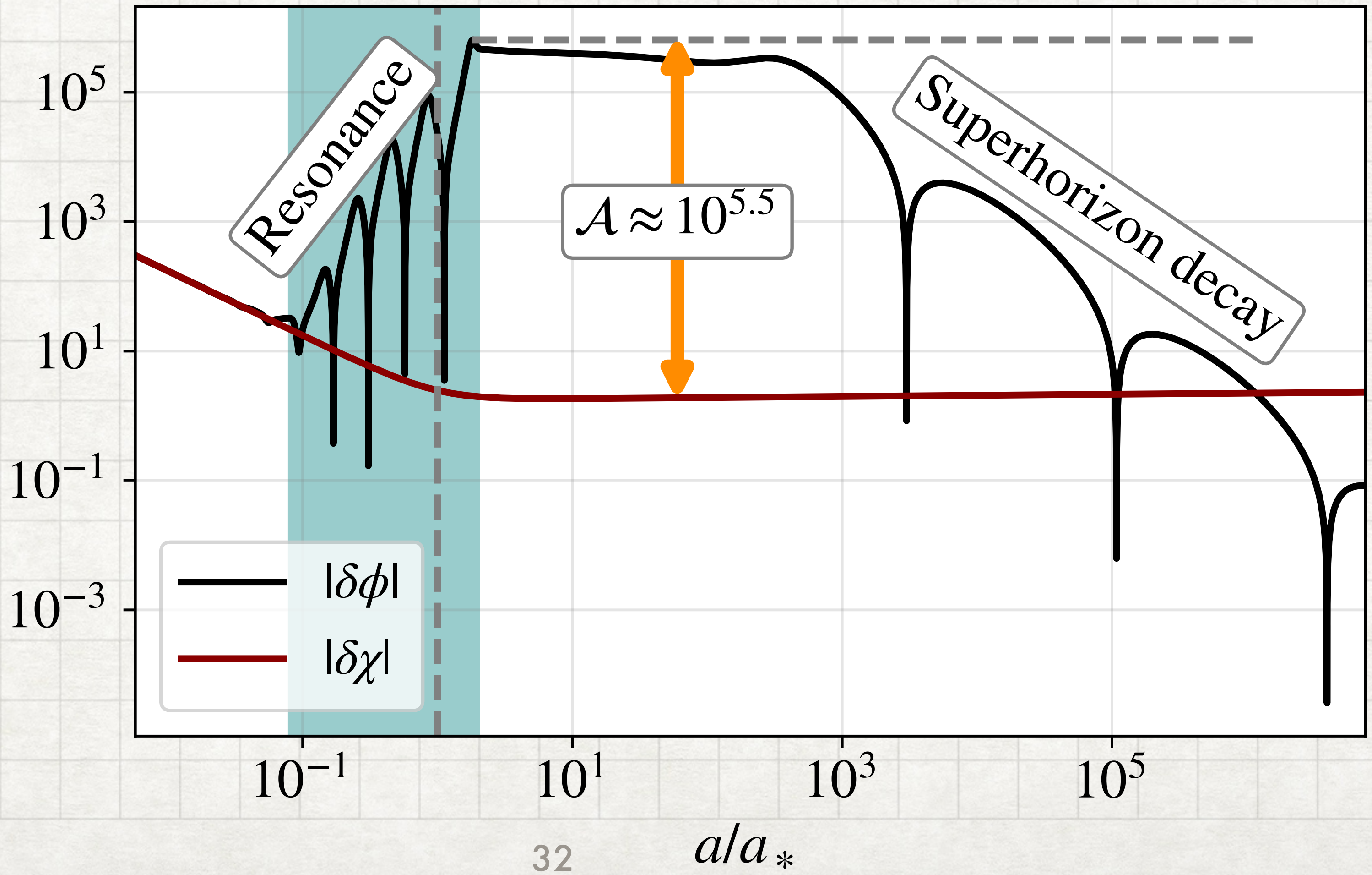
$$\ddot{\delta\chi}_k + 3H\dot{\delta\chi}_k + \frac{k^2}{a^2}\delta\chi_k = \frac{\sqrt{2\epsilon_\chi}}{M_{\rm Pl}}\left[\ddot{\phi}\delta\phi_k + \mathcal{S}_k\right]$$

$$\mathcal{F}_{\rm L} \equiv \mathcal{A}\,|\ddot{\phi}/M_{\rm Pl}|\sqrt{2\epsilon_\chi}/H^2 \approx 6\mathcal{A}\sqrt{\epsilon_\phi\epsilon_\chi} \ll 1$$

$$\epsilon_\chi \to R^{2p}\epsilon_\chi,\ \epsilon_\phi \to R^{2q}\epsilon_\phi,\ H^2 \to R^{2p}H^2,\ f_{\rm a} \to R^q f_{\rm a}$$

$$\mathcal{F}_{\rm L} \to R^{p+q}\mathcal{F}_{\rm L}$$

$\mathcal{A}, A_s$: unchanged

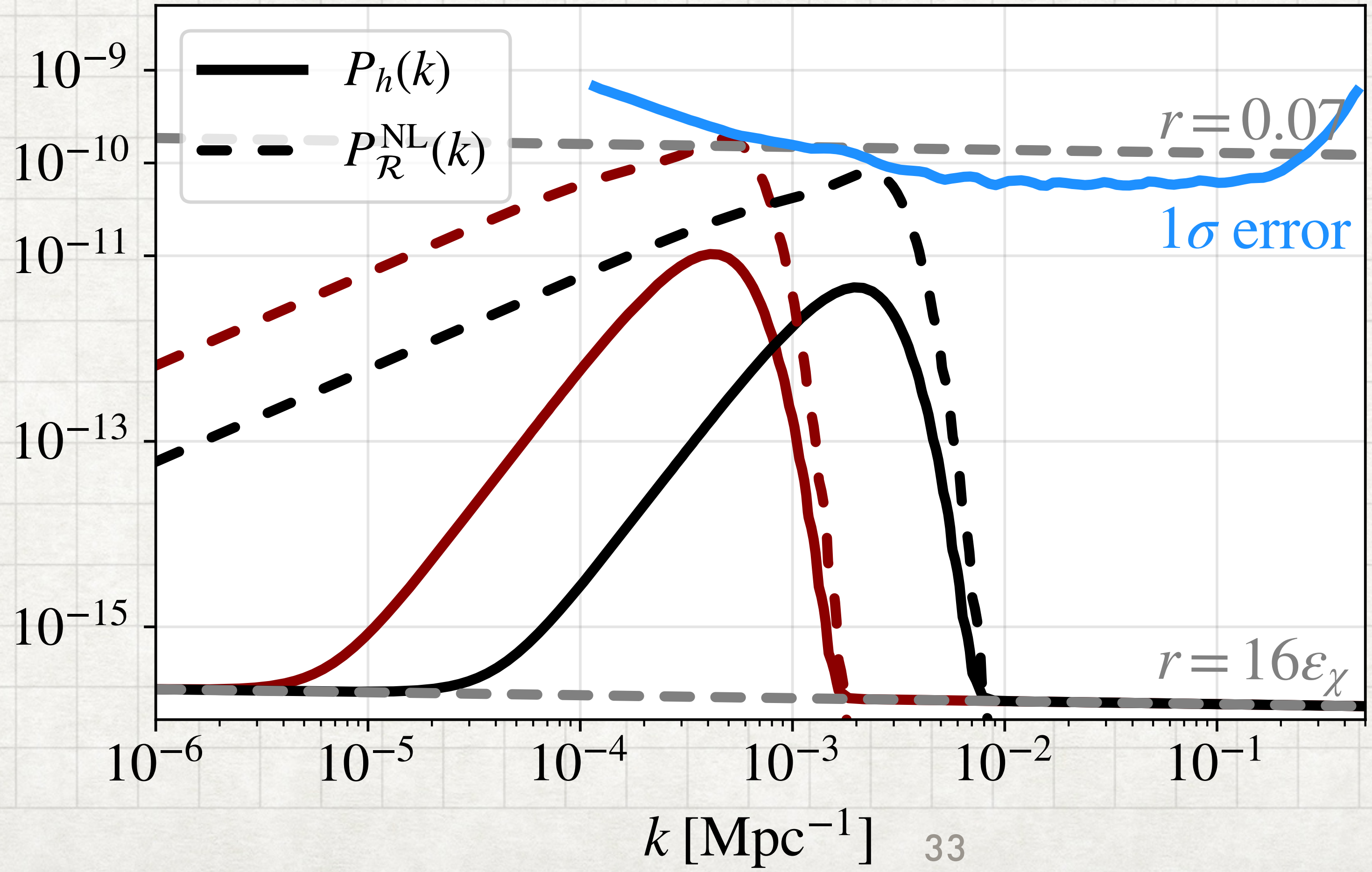


Enhanced tensor modes

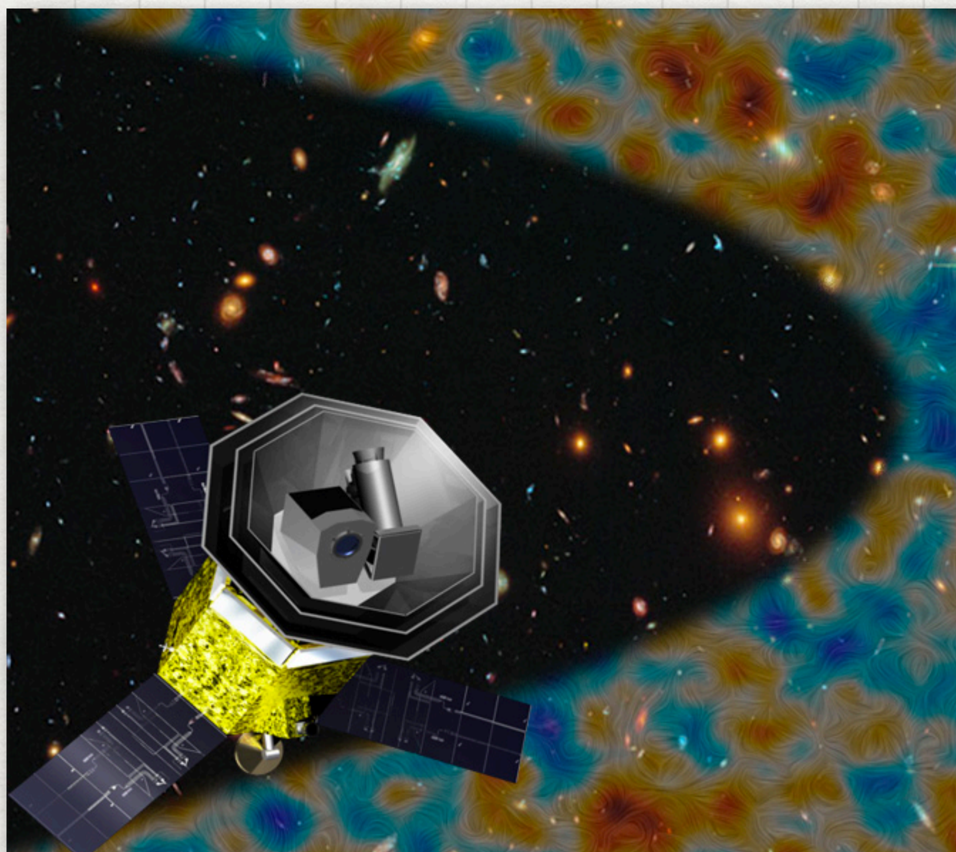
$$\ddot{h}_k^\lambda + 3H\dot{h}_k^\lambda + \frac{k^2}{a^2}h_k^\lambda = \mathcal{T}_k^\lambda(t)$$

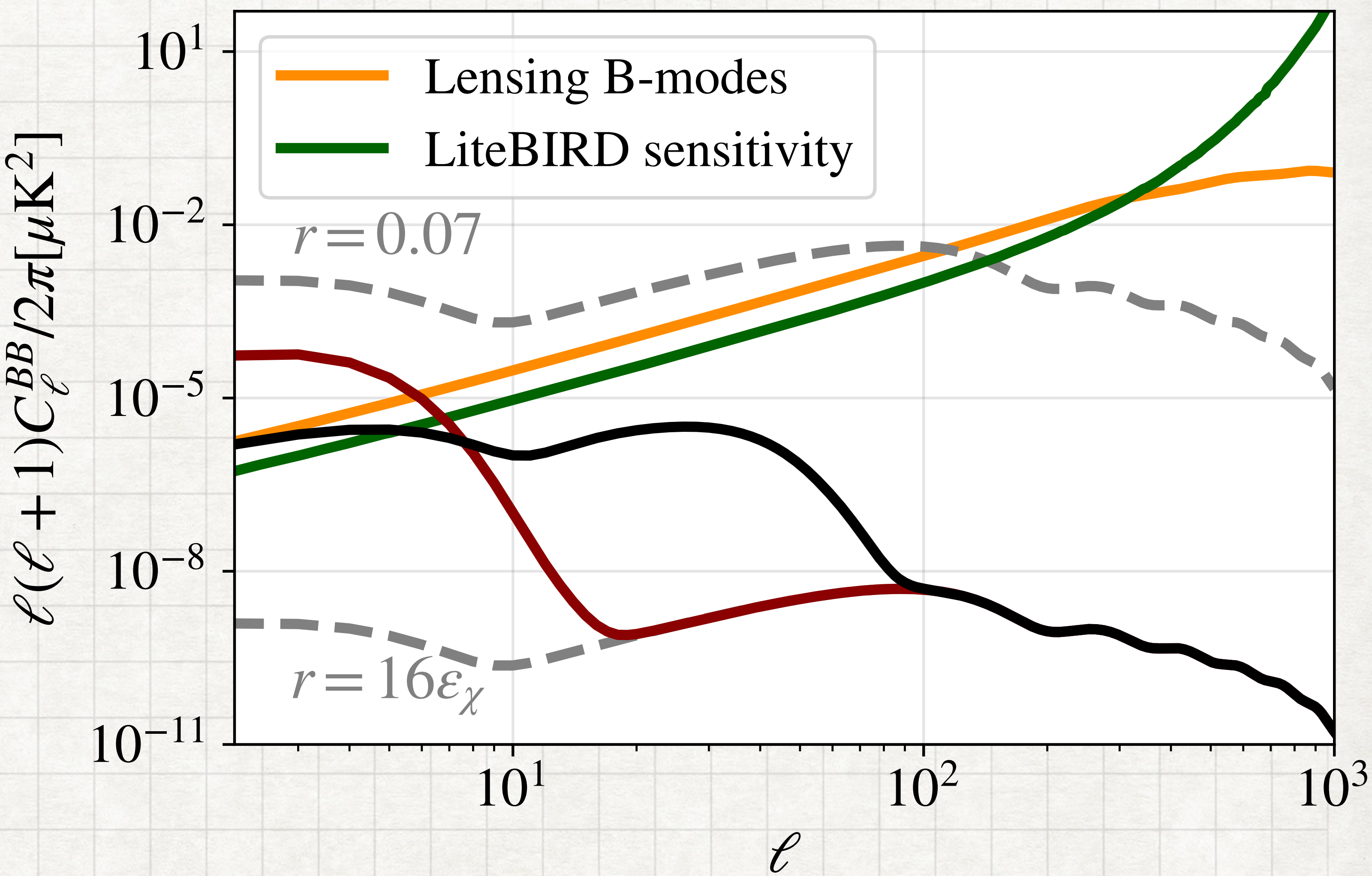
Non-linear source

$$\mathcal{T}_k^\lambda(t) = \frac{2}{M_p^2}\int \frac{d^3\mathbf{p}}{(2\pi)^3} e_{ij}^\lambda(\mathbf{k}) \frac{p_i p_j}{a^2} \delta\phi_{|\mathbf{p}|}\delta\phi_{|\mathbf{k}-\mathbf{p}|}$$



Detectability





Possible to source linearly?

In our case:
non-linear scalar

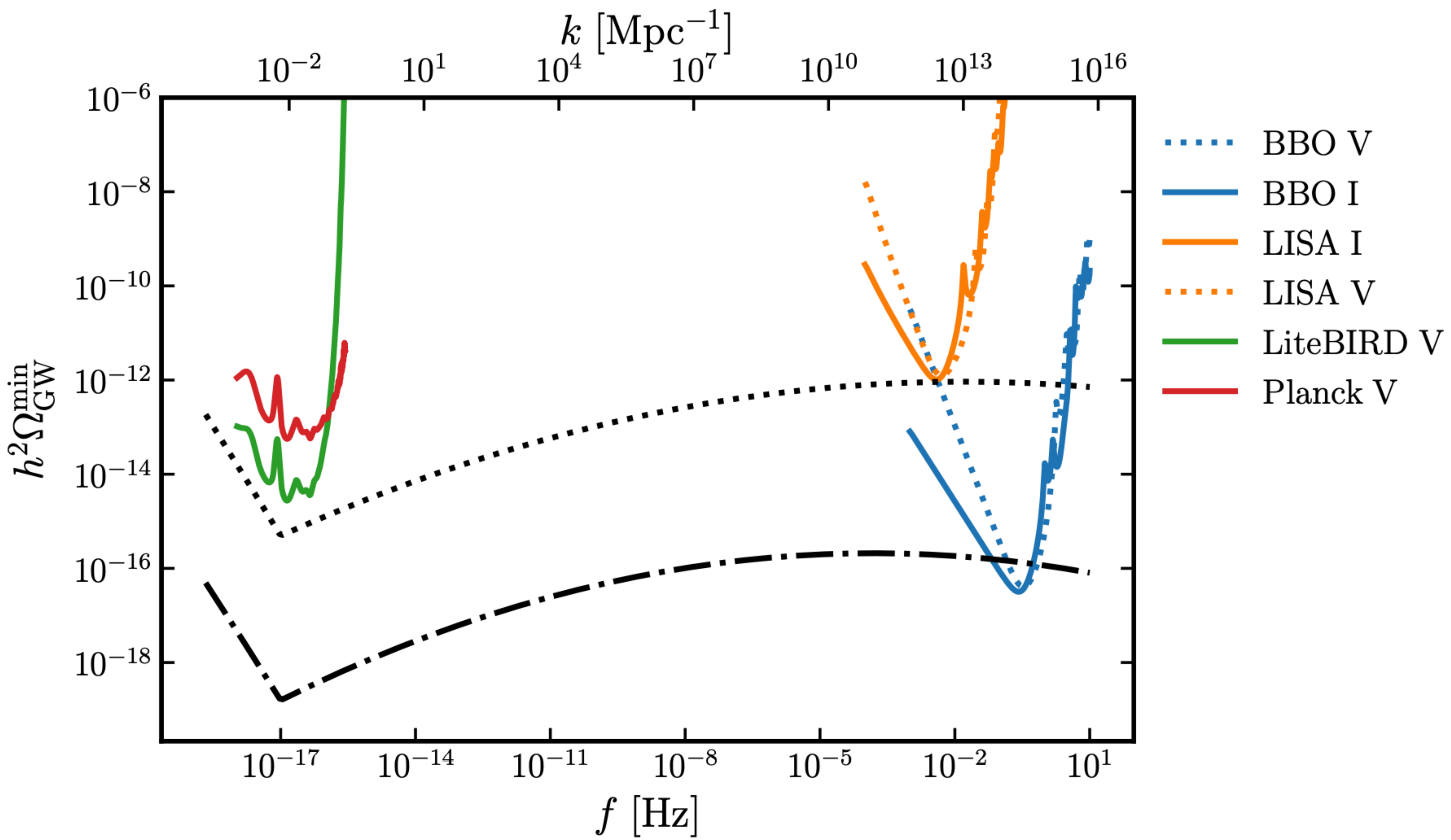
$$h''_{ij} + 2\mathscr{H} h'_{ij} - \vec{\nabla}^2 h_{ij} \sim \Pi_{ij}$$

$$\mathscr{L}_{\text{matter}} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \mu^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{\lambda}{4f} \phi F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$$

$SU(2)$ **Gauge sector**

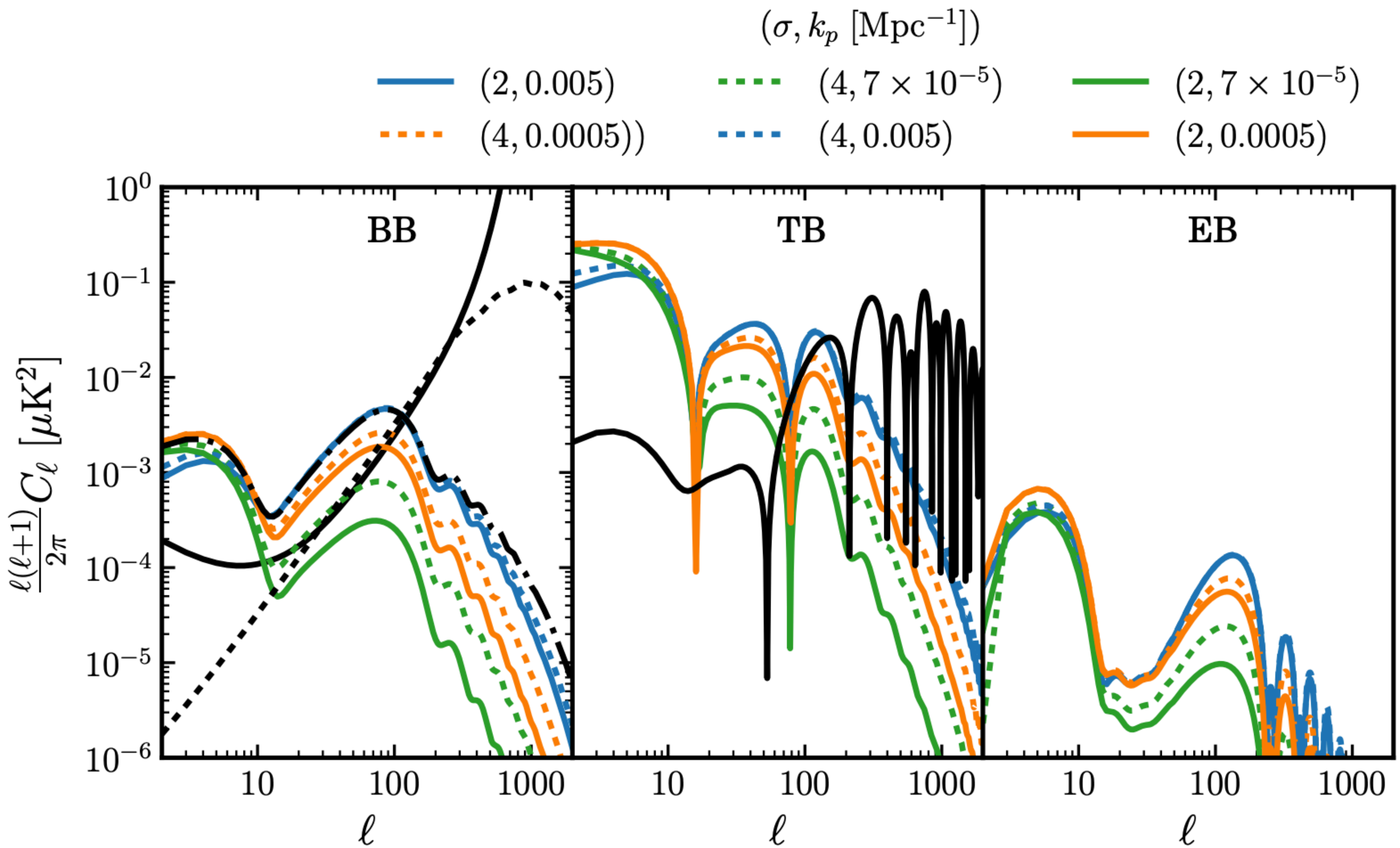
e.g. Maleknejad, Sheikh-Jabbari, 2017
Dimastrogiovanni, Fasiello, Fujita, 2017
Agrawal, Fujita, Komatsu, 2018

Gauge fields during inflation



Thorne, Fujita, Hazumi, Katayama, Komatsu, Shiraishi, 2017

Gauge fields during inflation



Part 3: PBH formation

Radiation dominated epoch

$$h_{\boldsymbol{k}}^{\lambda''}(\tau) + 2\mathcal{H}h_{\boldsymbol{k}}^{\lambda'}(\tau) + k^2 h_{\boldsymbol{k}}^{\lambda}(\tau) = S_{\boldsymbol{k}}^{\lambda}(\tau),$$

where the source term $S_{\boldsymbol{k}}^{\lambda}(\tau)$ is given by

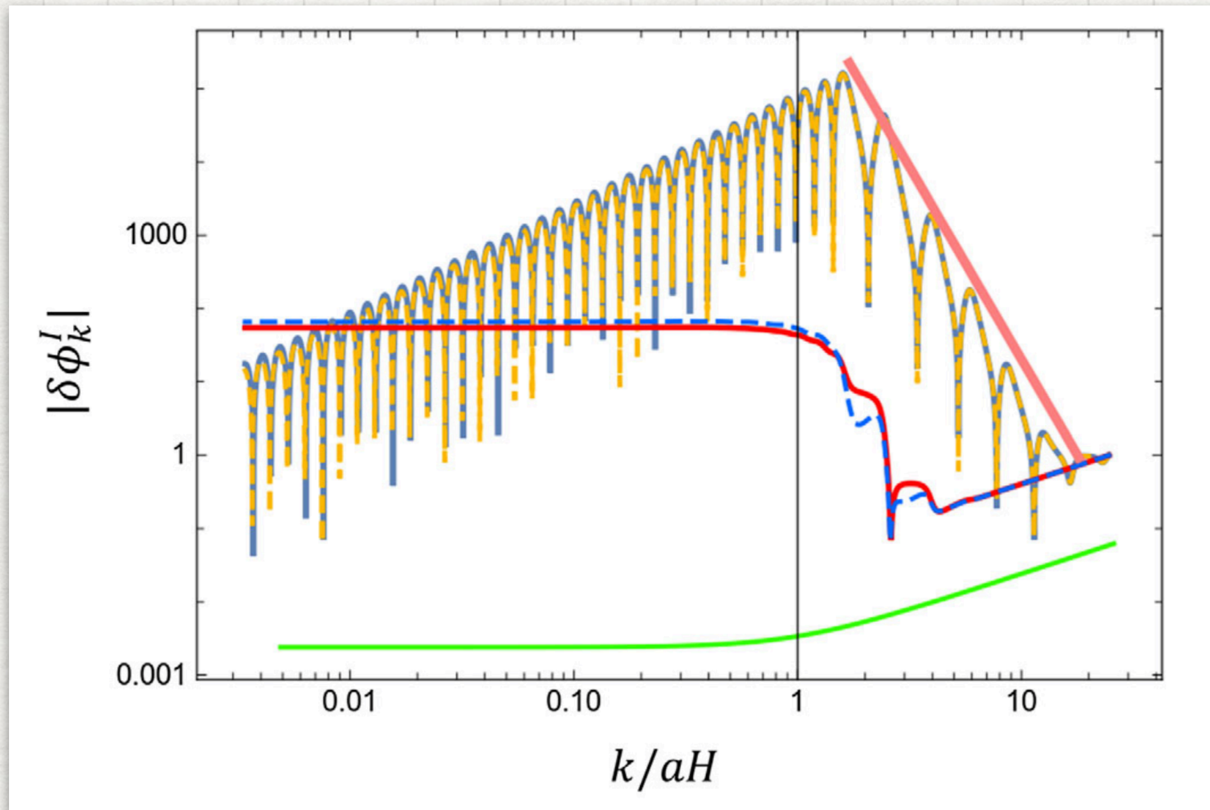
$$S_{\boldsymbol{k}}^{\lambda}(\tau) = 2\int \frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}} \mathbf{e}^{\lambda}(\boldsymbol{k},\boldsymbol{p}) \Big[2\Phi_{\boldsymbol{p}}(\tau)\Phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + \left(\Phi_{\boldsymbol{p}}(\tau) + \frac{\Phi'_{\boldsymbol{p}}(\tau)}{\mathcal{H}}\right)\left(\Phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + \frac{\Phi'_{\boldsymbol{k}-\boldsymbol{p}}(\tau)}{\mathcal{H}}\right)\Big]$$

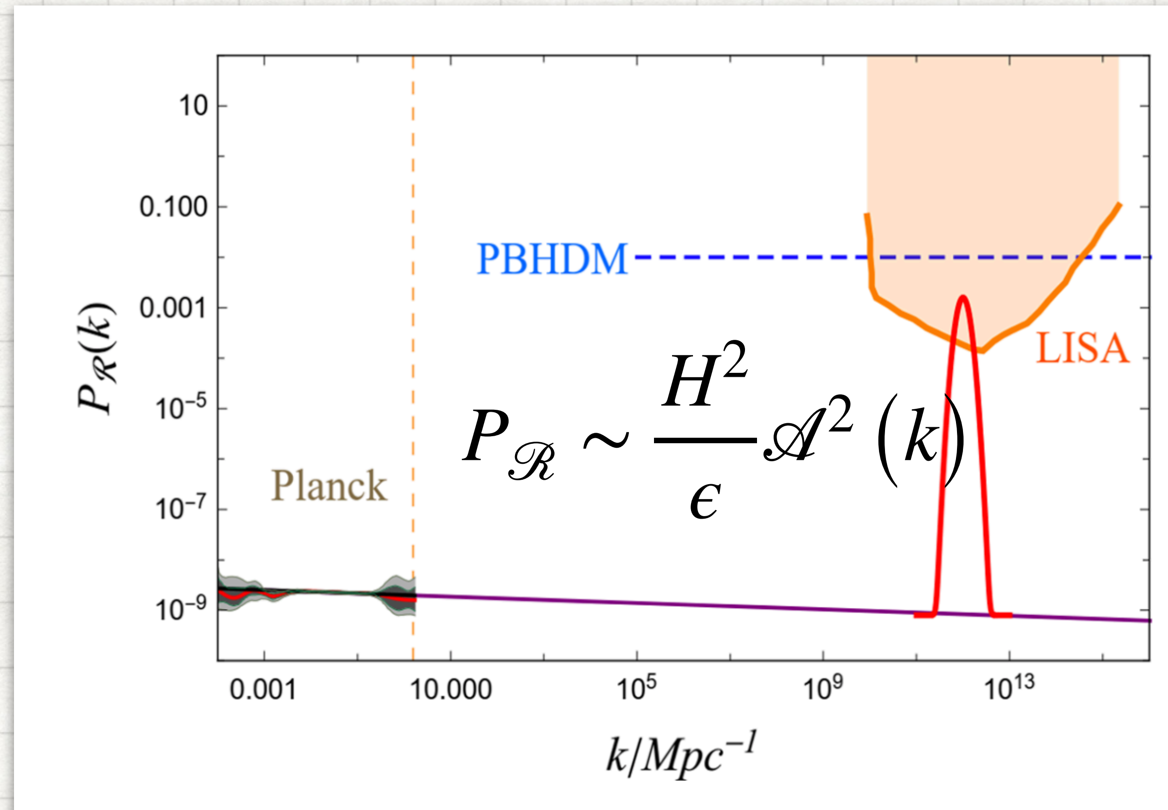
Inflationary epoch

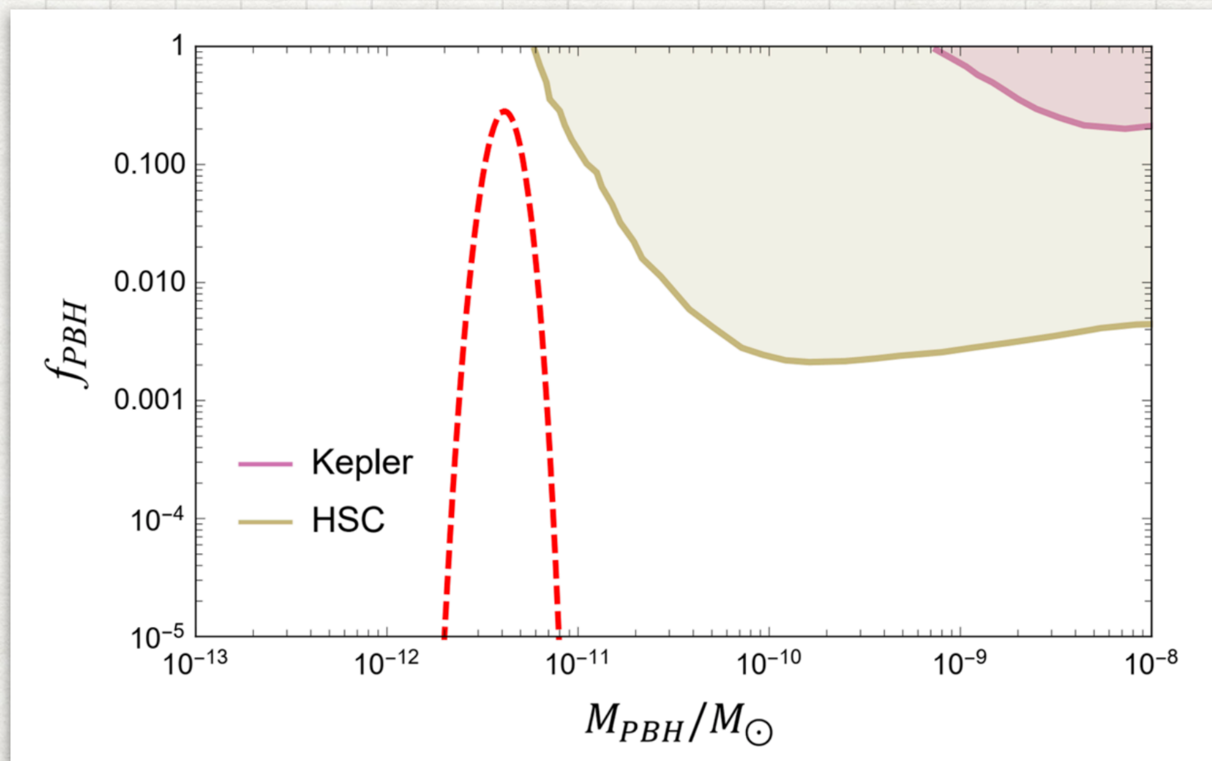
$$S_{\boldsymbol{k}}^{\lambda}(\tau) = \frac{2}{M_p^2}\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \mathbf{e}^{\lambda}(\boldsymbol{k},\boldsymbol{p})\delta\phi_{\boldsymbol{p}}(\tau)\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}(\tau) + (\phi \leftrightarrow \chi).$$

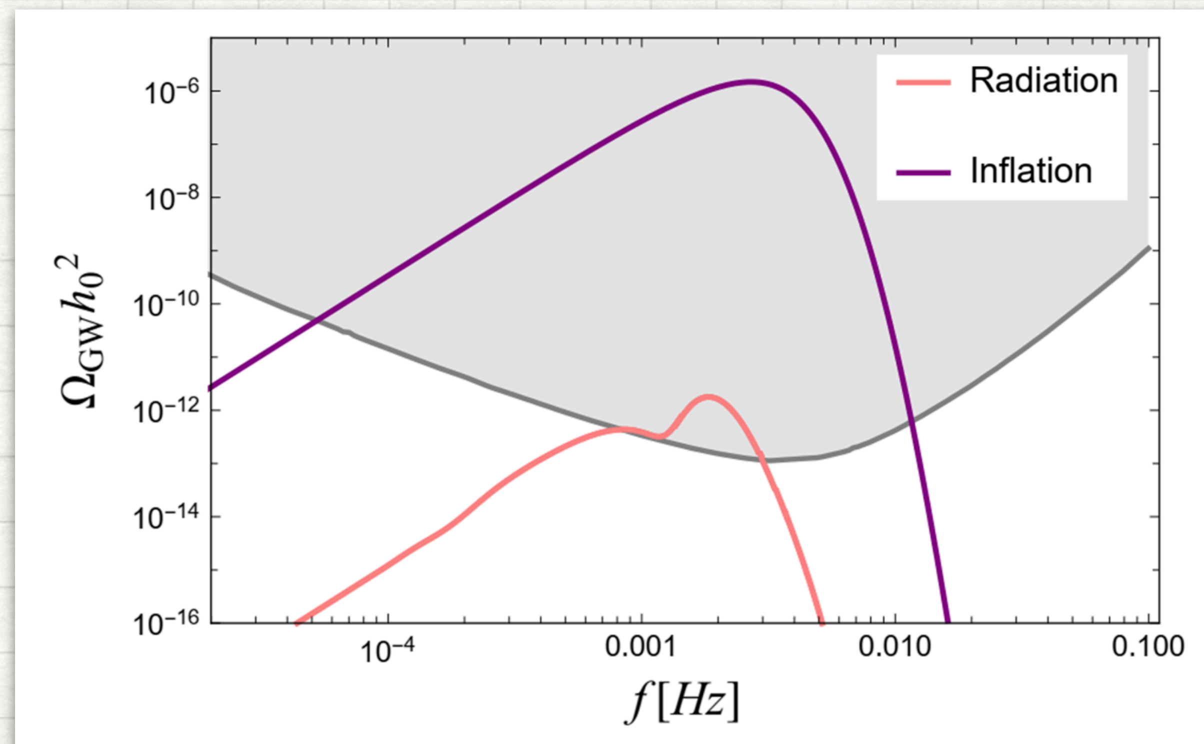
Zhou, Jiang, Cai, Sasaki, Pi, PRD 2020

PBH formation









Summary

- 1. Astrophysical black hole clustering and correlations with galaxy distribution: how to use redshift-unknown sources for cosmology? AGWB: a complementary probe. Important for interpreting possible future detection.

- 2. How to enhance primordial GWs: parametric resonance during inflation and induced gravitational waves. Scale-dependent counter example of the Lyth bound with scalar fields.

- 3. PBH production and the associated GWs: both induced and through mergers.