Chemical-Potential-Assisted Particle Production in FRW Spacetimes

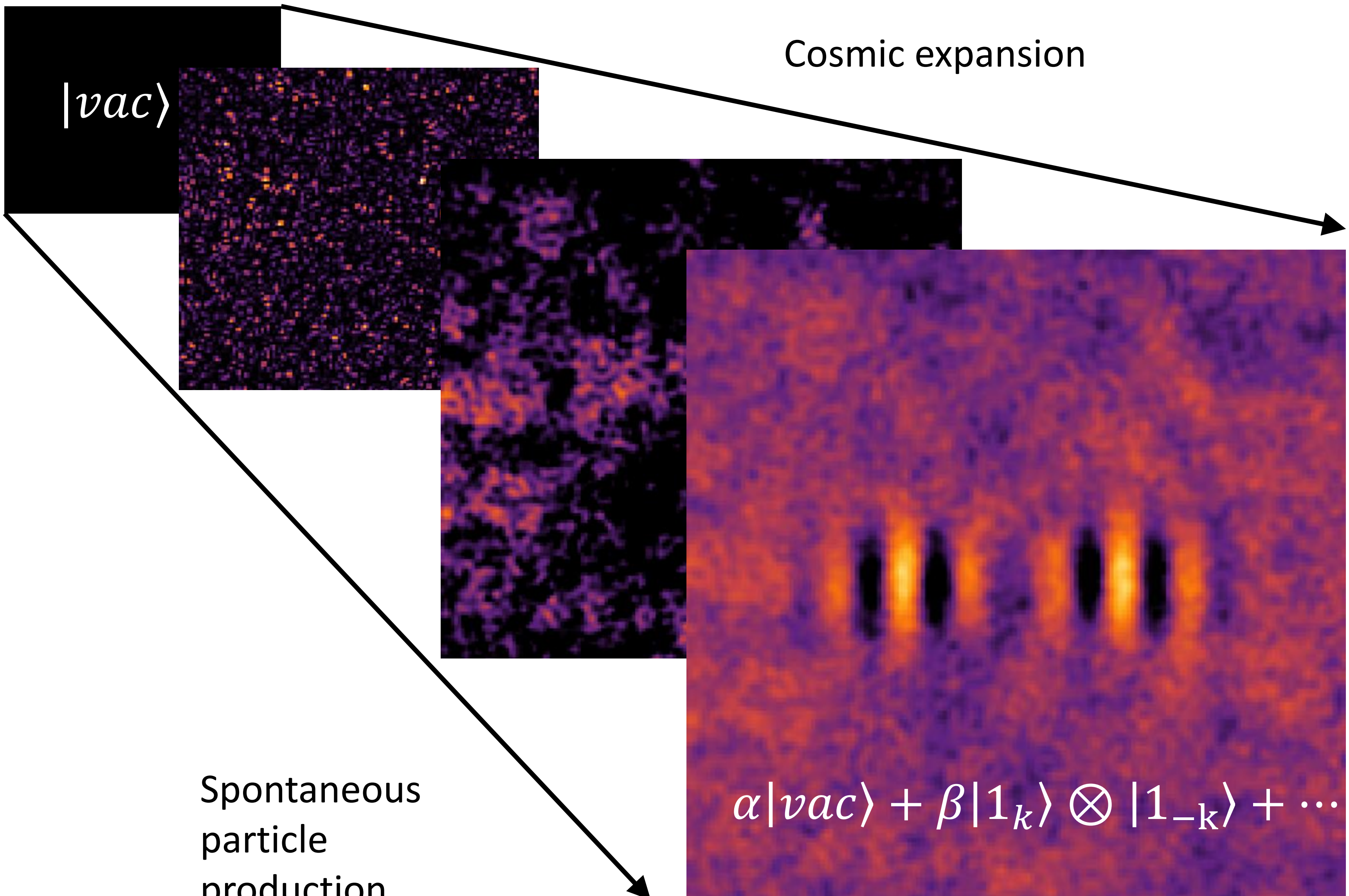
Xi Tong (童曦)
Department of Physics, HKUST

With Chon Man Sou and Yi Wang
Based on arXiv: 2104.08772

Outline

- Motivation
- Chemical potential: Definition and examples
- Stokes phenomenon and particle production
- Analysis in FRW spacetimes
- Conclusion & outlooks

Gravitational particle production



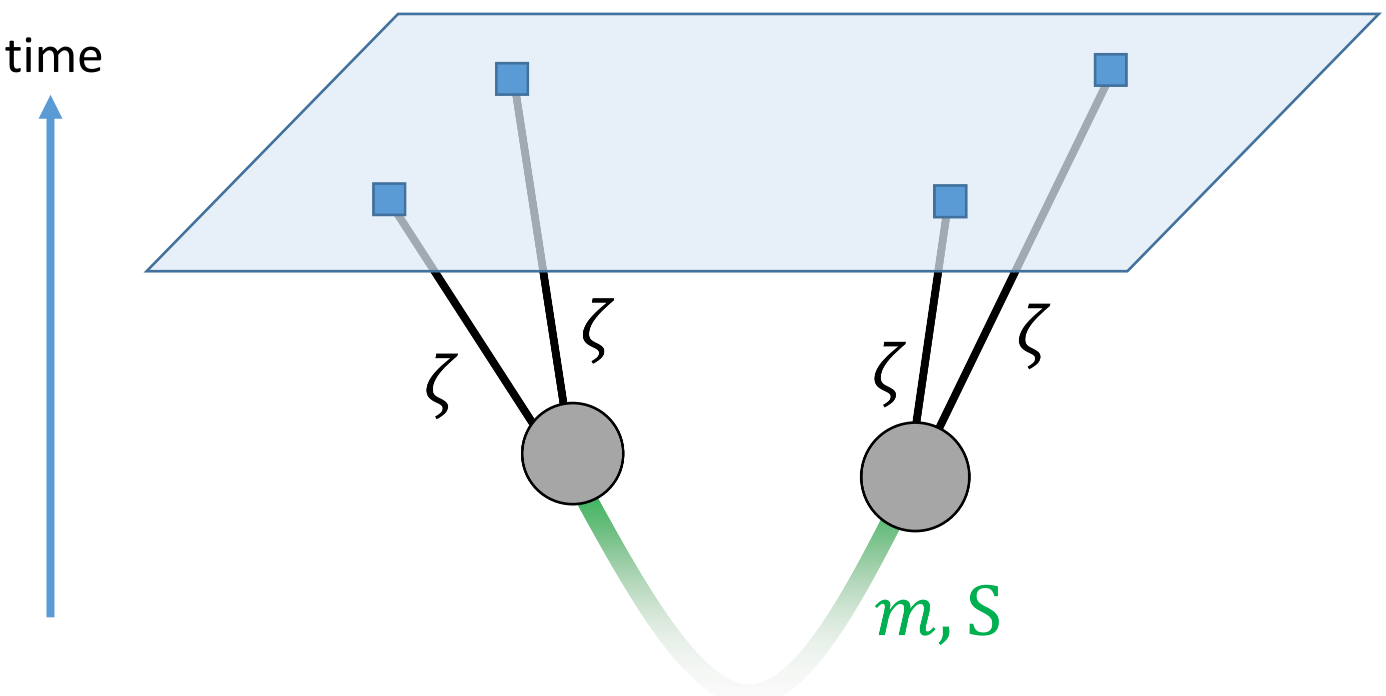
Inflation and the cosmological collider

[Arkani-Hamed&Maldacena, 2015] [Chen&Wang, 2009] [Baumann&Green, 2011]

$$\langle\zeta^4\rangle' \sim \lambda^2 \left(\frac{k_1+k_2}{k_I}\right)^{i\sigma} P_S(\cos\theta) f(\phi) \times e^{-\pi\, m/H}$$

$\sigma = \mu + i\delta$

- Fix k_3, k_4 → $\sim$
- coupling → λ^2
- mass → μ
- decay width → δ
- spin → $P_S(\cos\theta)$
- P&CP → $f(\phi)$



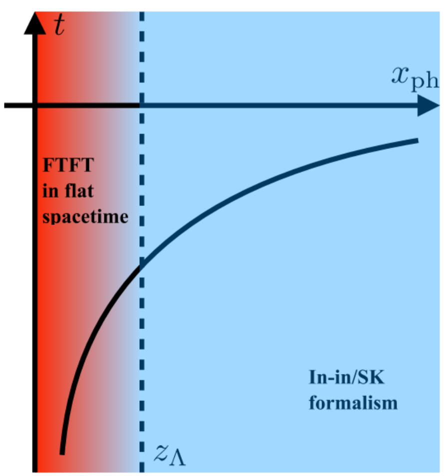
No.1 challenge of CC: Hopeless if $m > O(10)H$

Signal enhancement: One way or another

$$SNR \sim |\beta| \sim e^{-\pi \frac{m}{H}}$$

[Barnaby & Peloso, 2011]
[Liu et al., 2019]
[Adshead & Sfakianakis, 2015]
[Chen, Wang & Xianyu, 2018]

Raise the temperature
e.g. Warm inflation

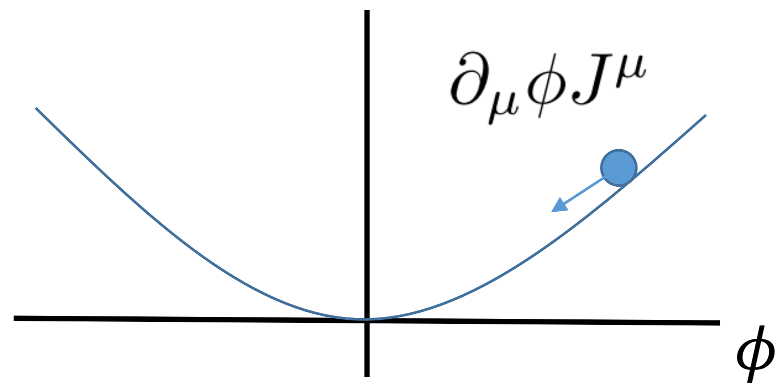


$$e^{-\pi \frac{m}{H}} \xrightarrow[\text{prod}]{\text{Thermal}} \left(\frac{T}{z_\Lambda H}\right)^{\#}$$

[Tong, Wang & Zhou, 2018]

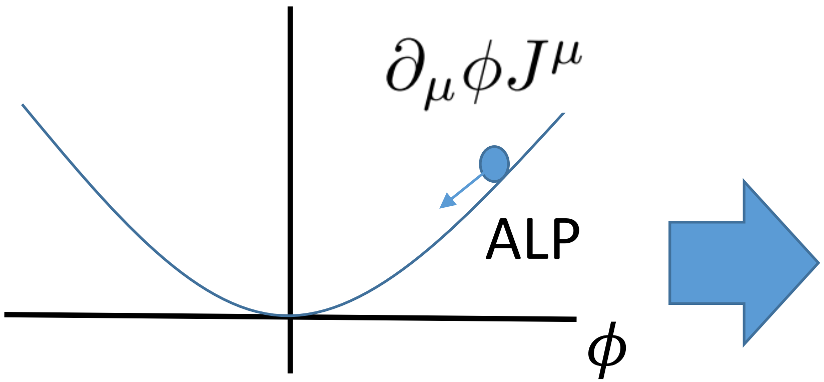
Lower the mass:
e.g. Chemical potential

$$m \to m - \kappa$$



$$e^{-\pi \frac{m}{H}} \xrightarrow[\text{potential}]{\text{Chemical}} e^{-\pi \frac{m-\kappa}{H}}$$

Chemical potential in the late universe



- Gauge fields sector: Tachyonic instability
  - ➢ Efficient ALP to DPDM conversion

    [Co, Pierce, Zhang, & Zhao, 2018]
  - ➢ Gravitational wave production

    [Machado, Ratzinger, Schwaller, & Stefanek, 2018, 2019]
- Fermion sector
  - ➢ Helicity asymmetry & baryogenesis

    [Adshead & Sfakianakis, 2015, 2016]

Therefore:

- Chemical-potential-assisted particle production is a generic phenomenon.
- A systematic investigation is necessary

Chemical potential: A formal discussion

- What do we mean by chemical potential?

Chemical potential

From Wikipedia, the free encyclopedia

In thermodynamics, **chemical potential** of a species is energy that can be absorbed or released due to a change of the particle number of the given species, e.g. in a

$$\kappa = \left(\frac{\partial U}{\partial N}\right)_{S,V}$$

J. W. Gibbs, 1876

- Grand canonical ensemble

$$Z = e^{-(H-\kappa N)/T}$$

- + Locality & Lorentz covariance

$$\Delta\mathcal{L}_{\rm chem} \equiv \kappa_\mu(x) J^\mu(x)$$

External chemical potential provided by a local field

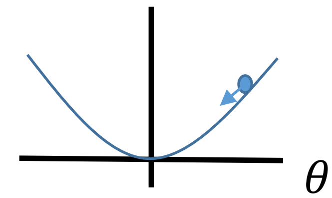
E.g., $\kappa_\mu \propto \partial_\mu \phi$

$$Z = \int \mathcal{D}\phi\, e^{i\int dt d^3x \sqrt{|g|}(\mathcal{L}(\phi,\partial\phi)+\Delta\mathcal{L}_{\rm chem})}$$

Chemical potential: Examples in dS

- Spin-1 (Non-conserved current, closed κ)

Chern-Simons current: $J^{\mu}_{CS} = \mathcal{E}^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}$



Choose $\kappa_\mu = a(\tau)\kappa\delta^0_\mu$

$$S_1 = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu - \frac{1}{4}\theta(\tau)\mathcal{E}^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right]$$

⇩ Massive axion electrodynamics

[Barnaby & Peloso, 2011]
[Liu et al., 2019]
[Wang & Xianyu, 2020]

In dS, EoM is *exactly solvable*:

Number density: $|\beta_\pm(k)|^2 \xrightarrow{\mu \gg 1} e^{-2\pi(\mu \pm \tilde{\kappa})}$

linear and *biased*

Chemical potential: A short summary

- General form: $\Delta\mathcal{L}_{\text{chem}} \equiv \kappa_\mu(x) J^\mu(x)$
- Powered by: E.g., a bg rolling scalar.
- Physical effects:
  - A *linear* and *biased* shift of mass
  - Non-linear features for fermions (Pauli blocking)

- dS mode functions are solvable, but other spacetimes?
- Even if $|\beta|^2$ is solvable in dS, but production time, production duration?
- A systematic way to analyze CPAP?

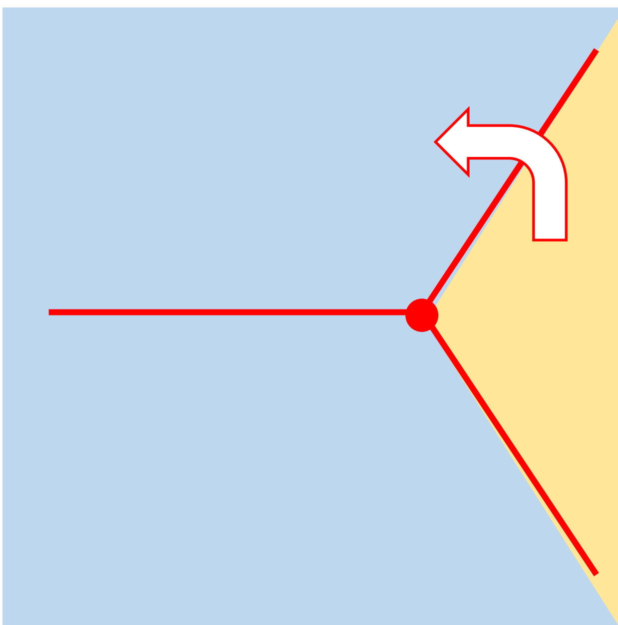
Stokes phenomenon

$$\frac{d^2 f}{dz^2} - z\, f = 0$$

Solution

Airy functions

$$f = c_1\, Ai(z) + c_2 Bi(z)$$



$$Ai(z) \sim \frac{e^{-\frac{2}{3}z^{3/2}}}{z^{-1/4}} \sum_n (-1)^n c_n z^{-3n/2}$$

Crossing the Stokes line: $\mathrm{Im}\, z^{3/2} = 0$

Emergence of a negative frequency mode

[Stokes, 1864, 1871, 1889, 1902]
[Dingle, 1973]

$$Ai(z) \sim \frac{e^{-\frac{2}{3}z^{3/2}}}{z^{-1/4}} \sum_n (-1)^n c_n z^{-\frac{3n}{2}} - i \frac{e^{\frac{2}{3}z^{3/2}}}{z^{-1/4}} \sum_n (-1)^n c_n z^{-3n/2}$$

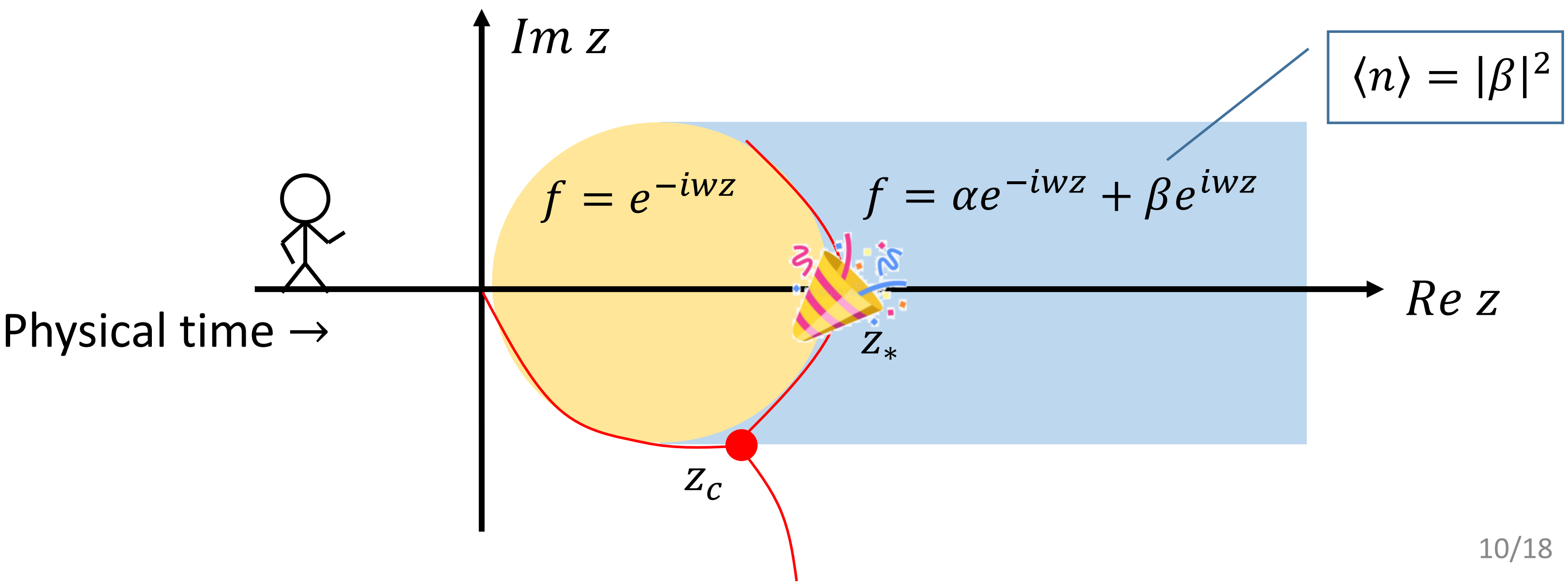
Stokes phenomenon and particle production

EoM: $$\frac{d^2 f}{dz^2} + w^2(z)\, f = 0$$

Near a simple root: $w^2(z) \sim A\,(z - z_c) + \cdots$

$$\frac{d^2 f}{dz^2} + A(z - z_c)\, f = 0$$

⇒ Same Stokes phenomenon

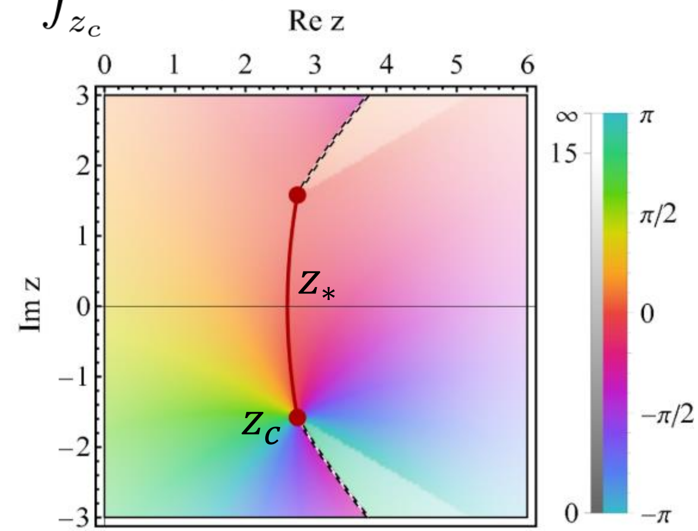


Stokes phenomenon and particle production

$$\frac{d^2 f}{dz^2} + w^2(z)\, f = 0 \quad \Rightarrow \quad \text{Singulant } F(z) = -2i\lambda \int_{z_c}^{z} w(z_1)\, dz_1$$

$$\beta(z) \approx -\frac{i e^{-F(z_i)}}{2}\left[1 + \mathrm{Erf}\left(-\frac{\mathrm{Im}F(z)}{\sqrt{2\mathrm{Re}F}}\right)\right]$$

[Berry, 1988, 1989]



- Production amount

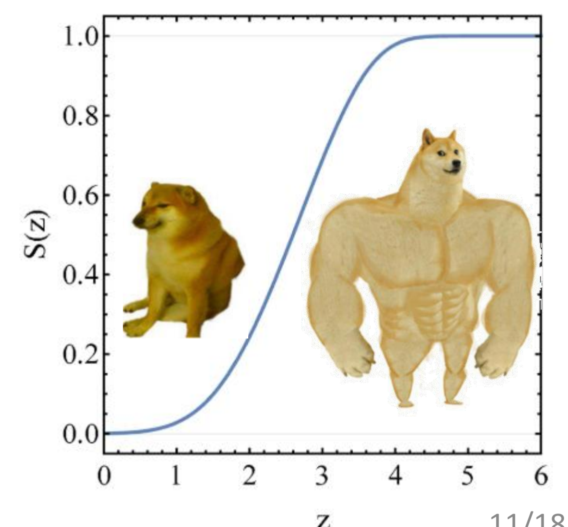
$$|\beta|^2 = e^{-2\mathrm{Re}\, F(z_i)}$$

- Production time

$$\mathrm{Im}\, F(z_*) = 0$$

- Production width

$$\Delta z_* = \frac{2\sqrt{2|\mathrm{Re}\, F(z_*)|}}{|\mathrm{Im}\, F'(z_*)|}$$



Stokes-line method for fermions

EoM: $$i\frac{\partial}{\partial\tau}\begin{pmatrix} u_s \\ v_s \end{pmatrix} = \begin{pmatrix} -sk - a\kappa & am \\ am & sk + a\kappa \end{pmatrix}\begin{pmatrix} u_s \\ v_s \end{pmatrix}$$

- A 1st order ODE
- A two-level system (Landau-Zener)

$$\beta(z) \approx \frac{e^{-F(z_i)}}{2}\left[1 + \mathrm{Erf}\left(-\frac{\mathrm{Im}F(z)}{\sqrt{2\mathrm{Re}F}}\right)\right]$$

[Berry, 1990]

with $$F(z) = -2i\int_{z_c}^{z} E(z_1)dz_1$$

and $$E(z) = \sqrt{1 + \frac{2s\kappa a}{k} + \frac{(m^2+\kappa^2)a^2}{k^2}}$$

Compare to bosons:

$$F(z) = -2i\int_{z_c}^{z} w(z_1)dz_1$$

$$w(z) = \sqrt{1 + \frac{2s\kappa a}{k} + \frac{m^2 a^2}{k^2}}$$

A recipe for cooking particles

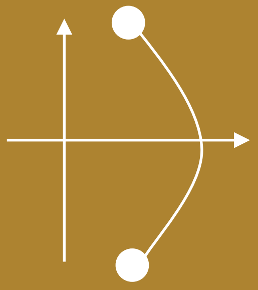
Step.1

- Find the simple roots

$$w^2(z_c) = 0$$

Step.2

- Draw the Stokes line



Step.3

- Calculate the singulant

$$F = ?$$

Step.4

- Obtain the production history

$$|\beta|^2 = ?$$
$$z_* = ?$$
$$\Delta z_* = ?$$

Step.5

- Fermion replacement rule

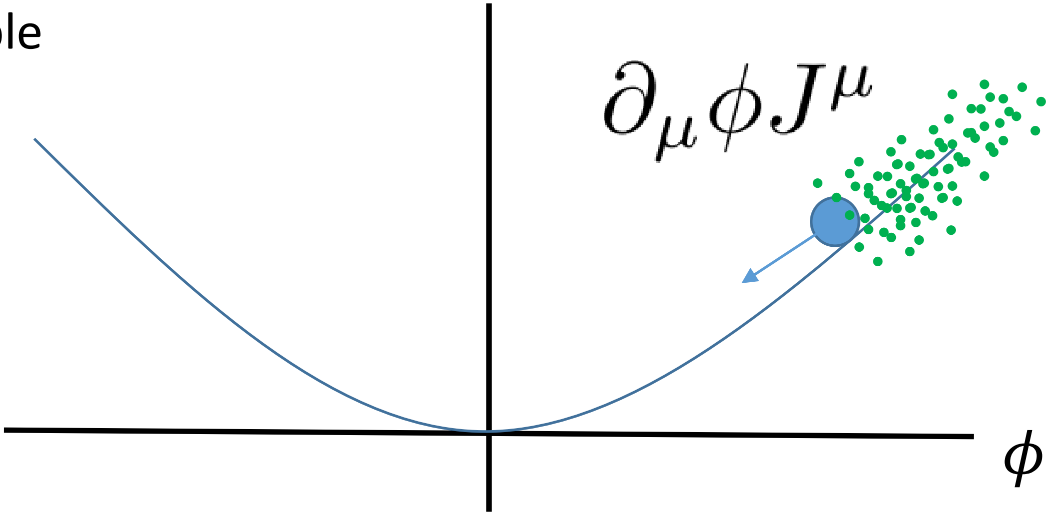
$$m^2 \leftrightarrow m^2 + \kappa^2$$

Application to CPAP in FRW

- Goal: An analytical/semi-analytical computation of the fine-grained production histories in common FRW.

- Assumption: A *constant* external chemical potential

  Justification:
  1. In some cases, it is so.
  2. LO approximation
  3. Mathematically simple



Application to CPAP in FRW

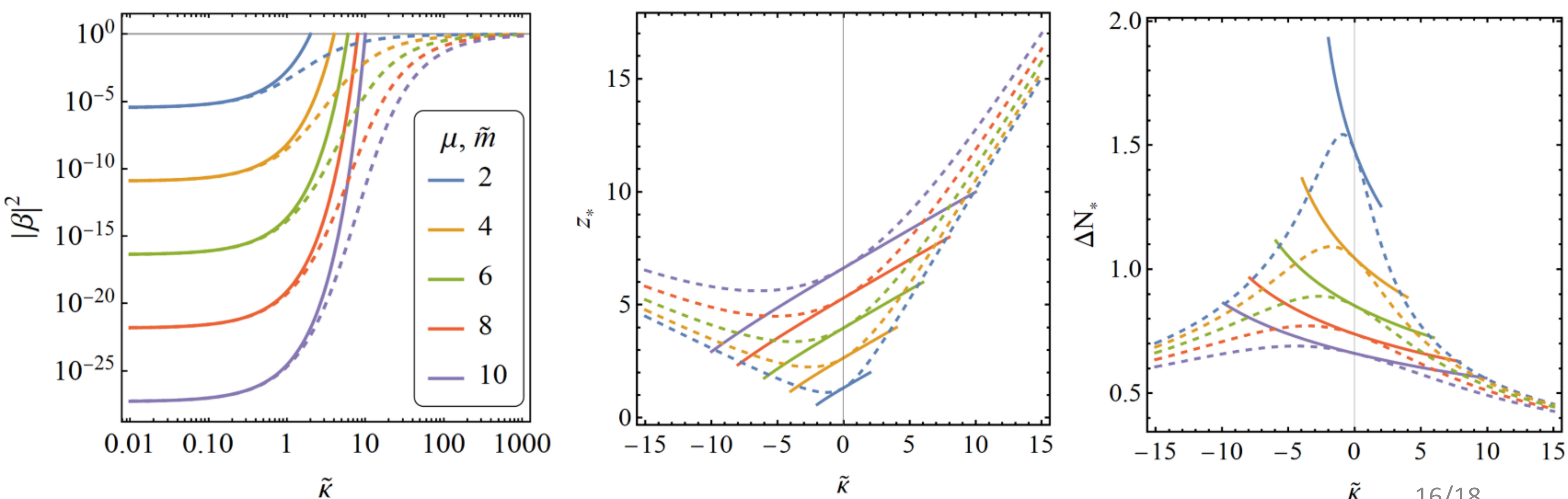
A systematic and classification

	Amount	Time	Width	Plot	Valid region
dS	=(145) $e^{-2\pi(m-\kappa)}$	≃(146) $0.6m + 0.3\kappa + \cdots$	≃(147) $\frac{2.1}{\sqrt{m}} - \frac{0.4\kappa}{m^{3/2}} + \cdots$	FIG. 9	
ϵ-dS	=(157) $e^{-2\pi(m-\kappa)[1+\epsilon(\ln k+\cdots)]}$	≈(159) $0.6m + 0.3\kappa + \cdots + \epsilon(0.8m + \cdots)$	≈(161) $\frac{2.1}{\sqrt{m}} - \frac{0.4\kappa}{m^{3/2}} + \cdots + \frac{\epsilon}{\sqrt{m}}(-\ln k + \cdots)$	FIG. 12	$\epsilon \ll 1$
η-dS	=(176) $e^{-2\pi(m-\kappa)[1+c(k)(\cdots)]}$	≈(178) $(1 + c(k))(0.6m + 0.3\kappa + \cdots) + c(k)\eta_i(0.8m + \cdots)$	≈(180) $\left(1 - \frac{c(k)}{2}\right)\left(\frac{2.1}{\sqrt{m}} - \frac{0.4\kappa}{m^{3/2}} + \cdots\right) + \frac{c(k)\eta_i}{\sqrt{m}}\left(\ln \frac{m}{H_i} + \cdots\right)$	FIG. 16	$c(k), \eta_i \ll 1$
RD	=(189) $e^{-\frac{\pi k^2}{c_r m^3}(m^2-\kappa^2)}$	=(190) $\frac{k^2\kappa}{c_r m^2}$	=(191) $\sqrt{\frac{\pi k^2}{c_r m}}$	FIG. 22	Ginzburg criterion: (192)
MD	=(199) $e^{-1.0\sqrt{\frac{2\pi k^3}{c_m m}}(1-\frac{0.7\kappa}{m}+\cdots)}$	≃(200) $\sqrt{\frac{k^3}{c_m m}}\left(0.6 + \frac{0.5\kappa}{m} + \cdots\right)$	≃(201) $\left(\frac{k^3}{c_m m}\right)^{1/4}\left(1.5 - \frac{0.3\kappa}{m} + \cdots\right)$	FIG. 26	Ginzburg criterion: (202)

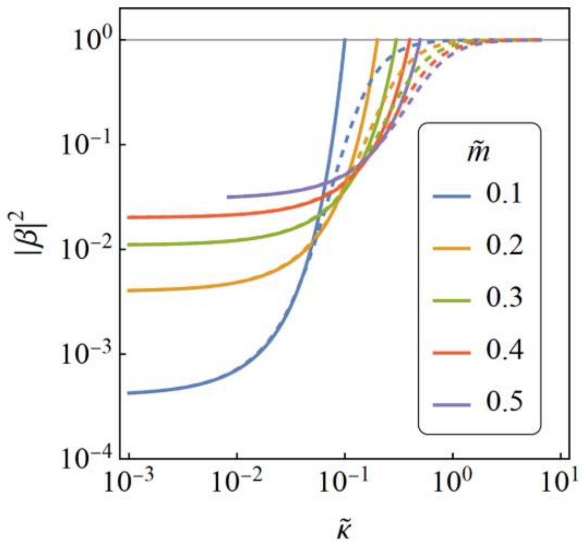
Application to CPAPP in FRW: dS

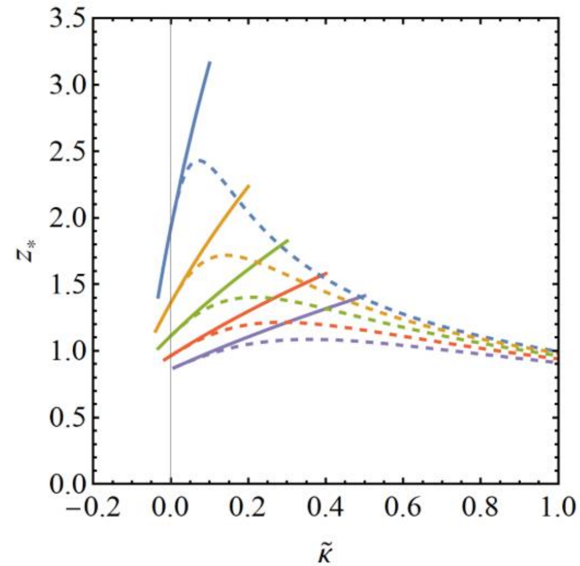
Exactly solvable singulant: $F(z) = 2i\left[\sqrt{\tilde{m}^2 - 2z\tilde{\kappa} + z^2} + \tilde{m}\tanh^{-1}\left(\frac{z\tilde{\kappa} - \tilde{m}^2}{\tilde{m}\sqrt{\tilde{m}^2 - 2z\tilde{\kappa} + z^2}}\right) + \tilde{\kappa}\tanh^{-1}\left(\frac{\tilde{\kappa} - z}{\sqrt{\tilde{m}^2 + z(z - 2\tilde{\kappa})}}\right)\right] + \pi(\tilde{m} - \tilde{\kappa})\,,$

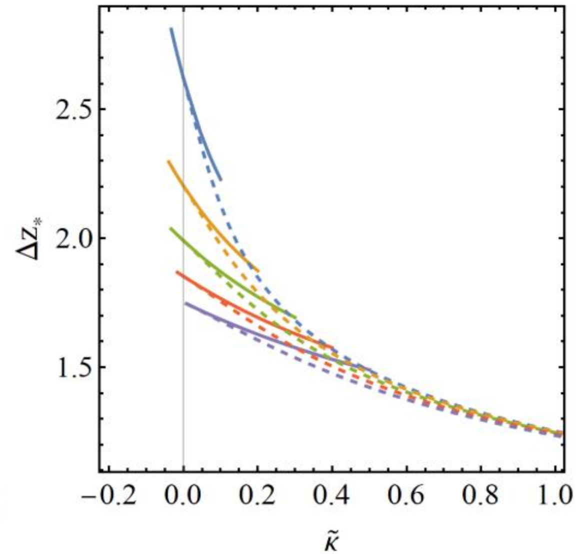
- Amount: $|\beta|^2 = e^{-2\pi(\mu - \tilde{\kappa})}$
- Time: $z_*(\mu, \tilde{\kappa}) \simeq 0.6627\mu + 0.3435\tilde{\kappa} - 0.0102\frac{\tilde{\kappa}^2}{\mu} + 0.0064\frac{\tilde{\kappa}^3}{\mu^2} + \cdots$
- Width: $\Delta N_* \simeq \frac{2.0895}{\sqrt{\mu}} - \frac{0.4131\tilde{\kappa}}{\mu^{3/2}} + \frac{0.1323\tilde{\kappa}^2}{\mu^{5/2}} - \frac{0.0523\tilde{\kappa}^3}{\mu^{7/2}} + O\left(\frac{\tilde{\kappa}^4}{\mu^{9/2}}\right)$



Application to CPAPP in FRW: MD

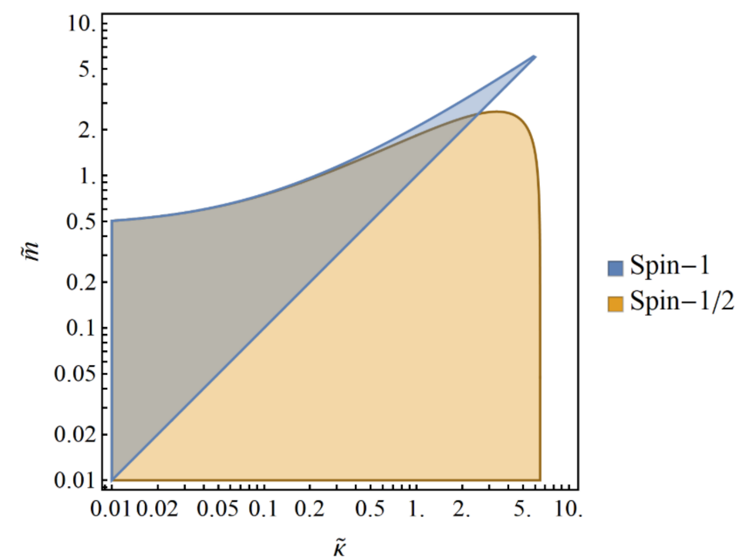






Vacuum initial condition requires a Ginzburg criterion:

$$0 < \frac{\Delta z_*}{2} \lesssim z_*$$



Notice:
Modes in MD need not to start from vacuum.
Initial condition is *model-dependent*.

Conclusion & outlooks

- ✓ CPAPP is common in both the early & late universe
- ✓ Examples in dS
- ✓ Stokes phenomenon and a recipe for cooking particles
- ✓ Application: A systematic and analytical classification of CPAPP

- ☐ A better treatment of $|\beta|^2 \sim 1$?
- ☐ Dynamical chemical potential?
- ☐ Non-trivial initial conditions?
- ☐ Understanding cosmological collider physics (loop estimations, etc.)?
- ☐ Stokes-line method for parametric resonance? [Yamada,2021]
- ☐ Higher spins?

(In progress, with Zhong-Zhi Xianyu)

Thank you for listening!