Minimalism in modified gravity

1. Introduction
2. Minimally modified gravity (MMG)
3. Examples of type-I & type-II MMG theories
4. D→4 EGB gravity with 2 dof
5. Massive gravity
6. Summary

Shinji Mukohyama (YITP, Kyoto U)

Based on collaborations with
Katsuki Aoki, Nadia Bolis, Sante Carloni, Antonio De Felice, Francesco Di Filippo, Andreas Doll, Justin Feng, Tomohiro Fujita, Xian Gao, Mohammad Ali Gorji, Sachiko Kuroyanagi, Francois Larrouturou, Chunshan Lin, Shuntaro Mizuno, Karim Noui, Michele Oliosi, Masroor C. Pookkillath, Zhi-Bang Yao

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INTRODUCTION

Why modified gravity?

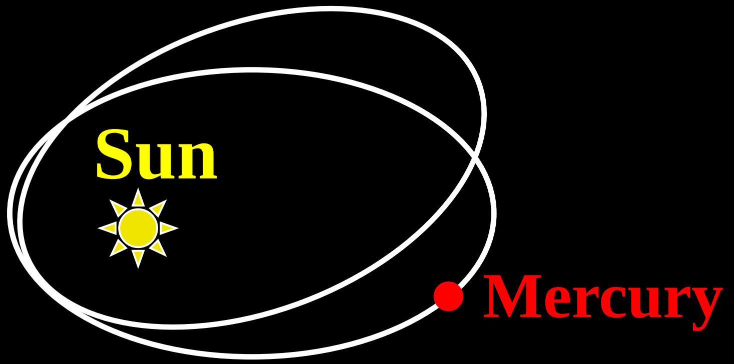
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

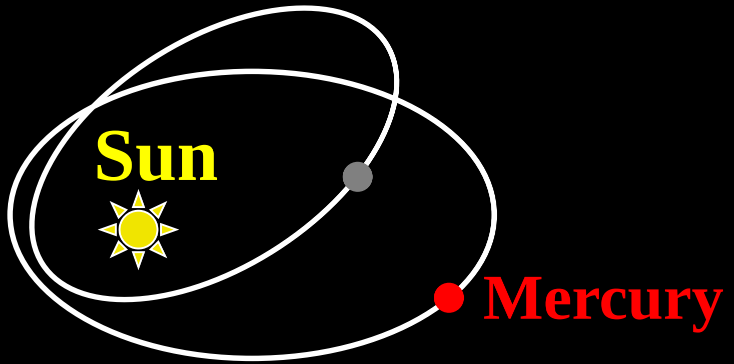
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion
observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



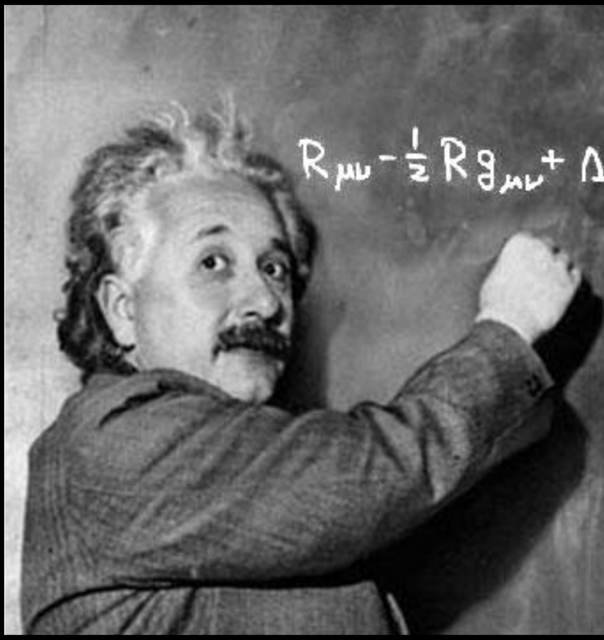
But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Why modified gravity?

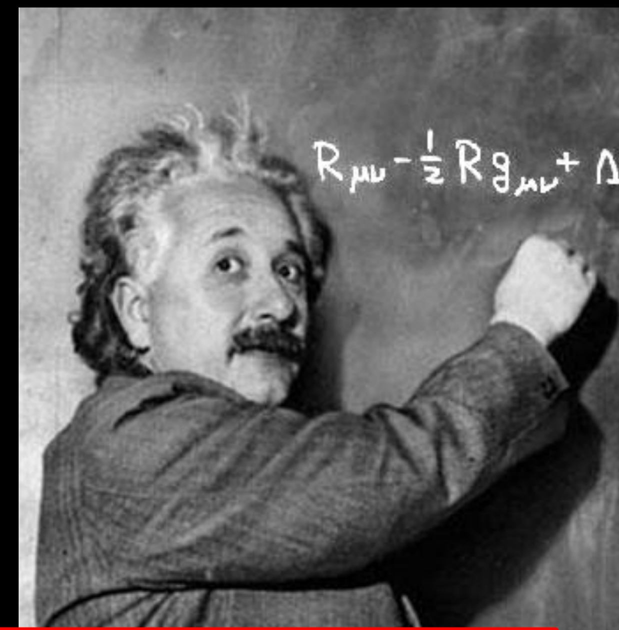
- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.

How to unify Quantum Theory with General Relativity?



How to unify Quantum Theory with General Relativity?



Probably we need to modify GR at short distances

Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.

Why modified gravity?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

of d.o.f. in general relativity

- 10 metric components → 20-dim phase space @ each point

ADM decomposition

- Lapse N, shift N^i, 3d metric h_{ij}

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- Einstein-Hilbert action

$$I = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\,{}^{(4)}R$$
$$= \frac{M_{\rm Pl}^2}{2}\int dt d^3\vec{x} N\sqrt{h}\left[K^{ij}K_{ij} - K^2 + {}^{(3)}R\right]$$

- Extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\partial_t h_{ij} - D_i N_j - D_j N_i)$$

of d.o.f. in general relativity

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of d.o.f. in general relativity

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  All constraints are independent of N & N^i → π_N & π_i "commute with" all constraints → 1^{st}-class

1^{st}-class vs 2^{nd}-class

- **2^{nd}-class constraint S**
  $\{ S , C_i \} \not\approx 0$ for $^{\exists} i$
  Reduces 1 phase space dimension
- **1^{st}-class constraint F**
  $\{ F , C_i \} \approx 0$ for $^{\forall} i$
  Reduces 2 phase space dimensions
  Generates a symmetry
  Equivalent to a pair of 2^{nd}-class constraints

$\{ C_i \mid i = 1,2,\ldots \}$: complete set of independent constraints

$A \approx B$ ⟺ $A = B$ when all constraints are imposed

(weak equality)

of d.o.f. in general relativity

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- 4 generators of 4d-diffeo: 1^{st}-class constraints
- 20 – (4+4) x 2 = 4 → 4-dim physical phase space @ each point → 2 local physical d.o.f.

of d.o.f. in general relativity

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Minimal # of d.o.f. in modified gravity = 2

Can this be saturated?

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MINIMALLY MODIFIED GRAVITY (MMG)

Is general relativity unique?

- Lovelock theorem says **"yes"** if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only; (iv) up to 2nd-order eom's of the form $E_{ab}=0$.
- Effective field theory (derivative expansion) says **"yes"** at low energy if we assume:
  (i) 4-dimensions; (ii) diffeo invariance; (iii) metric only.
- However, cosmological backgrounds break 4d-diffeo while keeping 3d-diffeo.
- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)

Example: simple scalar-tensor theory

- Covariant action

$$I = \frac{1}{2}\int d^4x\sqrt{-g}\left[\Omega^2(\phi)^{(4)}R + P(X,\phi)\right] \qquad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- ADM decomposition

$$ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$g^{\mu\nu} = \begin{pmatrix} -\frac{1}{N^2} & \frac{N^i}{N^2} \\ \frac{N^j}{N^2} & h^{ij} - \frac{N^iN^j}{N^2} \end{pmatrix}$$

- Unitary gauge

$$\phi = t \quad \Rightarrow \quad X = \frac{1}{2}\frac{1}{N^2}$$

This is a good gauge iff derivative of ϕ is timelike.

- Action in unitary gauge

$$I = \int dt d^3\vec{x} N\sqrt{h}\left\{f_1(t)\left[K^{ij}K_{ij} - K^2 + {}^{(3)}R\right] + \frac{2}{N}\dot{f}_1(t)K + f_2(N,t)\right\}$$

$$\Omega^2(\phi) = f_1(t) \qquad P(X,\phi) = f_2(N,t)$$

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- A metric theory with 3d-diffeo but with broken 4d-diffeo typically has 3 local physical d.o.f. (e.g. scalar-tensor theory, EFT of inflation/dark energy, Horava-Lifshitz gravity)
- Is GR unique when we assume: (i) 4-dimensions; (ii) 3d-diffeo invariance; (iii) metric only; (iv) 2 local physical d.o.f. (= 2 polarizations of TT gravitational waves)?
- **Answer is "no" → Minimally modified gravity (MMG)**

EXAMPLES OF TYPE-I & TYPE-II MMG THEORIES

Type-I & type-II modified gravity

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, JCAP 01 (2019) 017

- Jordan (or matter) frame

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\rm J}}\,\left[\Omega^2(\phi)\,R[g^{\rm J}] + \cdots\right] + I_{\rm matter}[g^{\rm J}_{\mu\nu};{\rm matter}]$$

- Einstein-frame $\quad g^{\rm E}_{\mu\nu} = \Omega^2(\phi)g^{\rm J}_{\mu\nu}$ $\quad$ K.Maeda (1989)

$$I = \frac{1}{2}\int d^4x\sqrt{-g^{\rm E}}\,\left[R[g^{\rm E}] + \cdots\right] + I_{\rm matter}[\Omega^{-2}(\phi)g^{\rm E}_{\mu\nu};{\rm matter}]$$

- Do we call this GR? No. This is a modified gravity because of **non-trivial matter coupling → type-I**
- There are more general scalar tensor theories where there is **no Einstein frame → type-II**

Type-I & type-II modified gravity

Katsuki Aoki, Antonio De Felice, Chunshan Lin, SM and Michele Oliosi, JCAP 01 (2019) 017

- **Type-I:**
  There exists an Einstein frame
  Can be recast as GR + extra d.o.f. **+ matter, which couple(s) non-trivially**, by change of variables
- **Type-II:**
  No Einstein frame
  Cannot be recast as GR + extra d.o.f. + matter by change of variables

Type-I minimally modified gravity (MMG)

Katsuki Aoki, Chunshan Lin and SM, PRD98 (2018) 044022

- **# of local physical d.o.f. = 2**
- There exists an Einstein frame
- Can be recast as GR **+ matter, which couple(s) non-trivially**, by change of variables
- **The most general change of variables = canonical tr.**
- Matter coupling just after canonical tr. → breaks diffeo → 1st-class constraint downgraded to 2nd-class → leads to extra d.o.f. in phase space → inconsistent
- Gauge-fixing after canonical tr. → splits 1st-class constraint into pair of 2nd-class constraints
- **Matter coupling after canonical tr. + gauge-fixing → a pair of 2nd-class constraints remain → consistent**

A type-I MMG fitting Planck data better than ΛCDM

Katsuki Aoki, Antonio De Felice, SM, Karim Noui, and Michele Oliosi, Masroor C. Pookkillath
arXiv:2005.13972

- f(ℋ) theory with f'(C) = $f_{,C}$

Carballo-Rubio, Di Filippo & Liberati 2018
Mukohyama & Noui 2019

(ℋ < 0)

$$f_{,C} = 1 + \frac{1}{2}a_1 - \frac{1}{2}a_1 \tanh\left[\frac{1}{a_3}\left(\frac{C}{H_0^2} + a_2\right)\right]$$

$$a_3 = \beta\, a_2$$

- 3 additional parameters determined by data (Planck, JLA, BAO, HST)
$a_1 \neq 0$ → deviation from ΛCDM
$\log_{10} a_2 \simeq 8.94$ → transition @ $z \simeq 743$

Parameters	95% limits
a_1	$0.0028^{+0.0006}_{-0.0023}$
$\log_{10} a_2$	$8.95^{+0.20}_{-1.33}$
$\log_{10} \beta$	< -3.5
$10^2\omega_b$	$2.284^{+0.019}_{-0.036}$
$\tau_{\rm reio}$	$0.052^{+0.013}_{-0.015}$
n_s	$0.9778^{+0.0058}_{-0.0092}$
H_0	$69.19^{+0.67}_{-0.90}$
Ω_m	$0.2952^{+0.0104}_{-0.0090}$

Type-II minimally modified gravity (MMG)

- **# of local physical d.o.f. = 2**
- **No Einstein frame**
- Cannot be recast as GR + matter by change of variables
- **Is there such a theory? Yes!**
- Example: Minimal theory of massive gravity
  [Antonio De Felice and SM, PLB752 (2016) 302; JCAP1604 (2016) 028; PRL118 (2017) 091104]
- Another example:
  arXiv 2004.12549 w/ Antonio De Felice and Andreas Doll

VCDM: a theory of type-II MMG

Antonio De Felice, Andreas Doll and Shinji Mukohyama [arXiv 2004.12549]

- Simple construction with **a free function V(ϕ)**
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (including CDM)

$$\mathcal{L} = N\sqrt{\gamma}\left[\frac{M_{\rm P}^2}{2}\left(R + K_{ij}K^{ij} - K^2 - 2V(\phi)\right) - \frac{\lambda^i_{\rm gf}}{N}M_{\rm P}^2\,\partial_i\phi - \frac{3M_{\rm P}^2\lambda^2}{4} - M_{\rm P}^2\lambda\,(K+\phi)\right]$$

- No Einstein frame, equivalent to cuscuton

  [arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo] Afshordi-Chung-Geshnizjani 2007
- V(ϕ) reconstructed from FLRW background
- $c_{GW} = 1$, no extra dof
- Can reduce H_0 tension

  [arXiv: 2009.08718 w/ Antonio De Felice & Masroor C. Pookkillath]
- Extension to address both H_0&S_8 tensions? [arXiv:2011.04188 w/ Antonio De Felice]

Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

	Having Einstein frame	No Einstein frame
$c_T^2(k^2) = 1$	**Type-Ia** $g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$	**Type-IIa**
$c_T^2(k^2) \neq 1$	**Type-Ib** $g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$	**Type-IIb** MTMG: $\omega^2 = k^2 + m^2$ 4DEGB: $\omega^2 = k^2 + k^4/\Lambda^2$

Proof of the absence of Einstein frame in cuscuton/VCDM

1. GWs → cuscuton/VCDM is of type-Ia or type-IIa
2. GR + conformal-type canonical tr. → most general type-Ia MMG
3. Vacuum Bianchi-I universes → cuscuton/VCDM is not of type-Ia
4. 1 & 3 → cuscuton/VCDM is of type-IIa, thus no Einstein frame

Refined classification

[arXiv: 2103.15044 w/ Katsuki Aoki & Francesco Di Filippo]

	Having Einstein frame	No Einstein frame
$c_T^2(k^2) = 1$	**Type-Ia** $g_{\mu\nu} \propto \tilde{g}_{\mu\nu}$	**Type-IIa** Cuscuton/VCDM: $\omega^2 = k^2$
$c_T^2(k^2) \neq 1$	**Type-Ib** $g_{\mu\nu} \not\propto \tilde{g}_{\mu\nu}$	**Type-IIb** MTMG: $\omega^2 = k^2 + m^2$ 4DEGB: $\omega^2 = k^2 + k^4/\Lambda^2$

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Weaker gravity for DM: VCCDM

Antonio De Felice and Shinji Mukohyama [arXiv 2011.04188]

- Simple construction with **free functions $f_0(\phi)$ & $f_1(\phi)$**
  1. Hamiltonian of GR with 3+1 decomposition
  2. Canonical tr to a new frame
  3. Add a cosmological const **& dark matter** in the new frame
  4. Gauge fix
  5. Inverse canonical tr back to the original frame
  6. Legendre tr to Lagrangian
  7. Add minimally-coupled matter fields (no dark matter here)

$$\mathcal{L} = N\sqrt{\gamma}\left[\frac{M_{\rm P}^2}{2}\left(R + K_{ij}K^{ij} - K^2 - 2V(\phi)\right) - \frac{\lambda^i_{\rm gf}}{N} M_{\rm P}^2\,\partial_i\phi - \frac{3M_{\rm P}^2\lambda^2}{4} - M_{\rm P}^2\lambda\,(K+\phi)\right]$$

SM metric: $$g_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

DM metric: $$g^{\rm eff}_{\mu\nu}dx^\mu dx^\nu = -\frac{N^2}{f_1^2}dt^2 + \frac{\gamma_{ij}}{f_0}(dx^i + N^i dt)(dx^j + N^j dt)$$

- $f_0(\phi)$ & $f_1(\phi)$ reconstructed from $H(z)$ & $G_{DM}(z)/G_N$
- $c_{GW} = 1$, $G_{SM} = G_N$, no extra dof
- May reduce H_0 & S_8 tensions

$$V(\phi) \equiv \frac{\bar{\Lambda}}{f_1 f_0^{3/2}}$$

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D→4 EGB GRAVITY WITH 2 DOF

Refs. arXiv:2005.03859 & 2005.08428 w/ Katsuki Aoki & Mohammad Ali Gorji
arXiv:2010.03973 w/ Katsuki Aoki, Mohammad Ali Gorji & Shuntaro Mizuno

EGB theory and D→4

$$S_{\mathrm{EGB}} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}\left[\mathcal{R} - 2\Lambda + \alpha\mathcal{R}^2_{\mathrm{GB}}\right]$$

$$\mathcal{R}^2_{\mathrm{GB}} = \mathcal{R}^2 - 4\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}$$

- For D=4, the GB term is total derivative and thus does not contribute to eom's.
- D → 4 with $\tilde{\alpha} = (D-4)\,\alpha$ kept fixed?
  0/0 = finite?
  [Glavan&Lin, PRL124, 081301 (2020)]
- Maybe yes, but requires either extra dof. or Lorentz violation due to Lovelock theorem
- **The best we can do without extra d.o.f. is to keep 3d diffeo** → MMG framework

Hamiltonian of 4D theory with 2 dof

$$H^{\rm 4D}_{\rm EGB} = \int d^3x \left(N\,{}^3\mathcal{H}_0 + N^i\mathcal{H}_i + \lambda^0\pi_0 + \lambda^i\pi_i + \lambda_{\rm GF}\,{}^3\mathcal{G}\right)$$

$${}^3\mathcal{H}_0 = \frac{\sqrt{\gamma}}{2\kappa^2}\left[2\Lambda - \mathcal{M} + \tilde{\alpha}\left(4\mathcal{M}_{ij}\mathcal{M}^{ij} - \frac{3}{2}\mathcal{M}^2\right)\right] \qquad \mathcal{H}_i = -2\sqrt{\gamma}\gamma_{ik}D_j\left(\frac{\pi^{jk}}{\sqrt{\gamma}}\right)$$

$$\mathcal{M}_{ij} := R_{ij} + \mathcal{K}^k{}_k\mathcal{K}_{ij} - \mathcal{K}_{ik}\mathcal{K}^k{}_j$$

$$\pi^i{}_j = \frac{\sqrt{\gamma}}{2\kappa^2}\left[\mathcal{K}^i_j - \mathcal{K}\delta^i_j - \frac{8}{3}\tilde{\alpha}\delta^{ikl}_{jrs}\mathcal{K}^r_k\left(R^s_l - \frac{1}{4}\delta^s_l R + \frac{1}{2}\left(\mathcal{M}^s_l - \frac{1}{4}\delta^s_l\mathcal{M}\right)\right)\right]$$

- $1^{\rm st}$ class x 6

$$\pi_i \approx 0\,, \quad \mathcal{H}_i \approx 0$$

- $2^{\rm nd}$ class x 4

$$\pi_0 \approx 0\,, \quad {}^3\mathcal{H}_0 \approx 0\,, \quad {}^3\mathcal{G} \approx 0\,, \quad {}^3\dot{\mathcal{G}} \approx 0$$

- **10x2 – 6x2 – 4 = 4 → 2 dof**

5 properties of 4D theory

4D theory is unique up to a choice of $^3\mathcal{G}$.

i. 3D spatial diffeo invariance is respected
ii. # of dof = 2
iii. Reduces to GR when $\tilde{\alpha} = 0$
iv. Correction terms are 4th-order in derivatives
v. If the Weyl tensor of the spatial metric and the Weyl part of $K_{ik}K_{jl}–K_{il}K_{jk}$ vanish for a solution of (d+1)-dim EGB, then the d→3 limit of the solution satisfies eoms of 4D theory.

➡ **A consistent theory of D→4 EGB gravity**

Lagrangian of 4D theory with 2 dof

$$\mathcal{L}^{4\mathrm{D}}_{\mathrm{EGB}} = \frac{1}{2\kappa^2}\left(-2\Lambda + \mathcal{K}_{ij}\mathcal{K}^{ij} - \mathcal{K}^i_i\mathcal{K}^j_j + R + \tilde{\alpha}R^2_{4\mathrm{DGB}}\right)$$

$$R^2_{4\mathrm{DGB}} = -\frac{4}{3}\left(8R_{ij}R^{ij} - 4R_{ij}\mathcal{M}^{ij} - \mathcal{M}_{ij}\mathcal{M}^{ij}\right) + \frac{1}{2}\left(8R^2 - 4R\mathcal{M} - \mathcal{M}^2\right)$$

$$\mathcal{K}_{ij} = K_{ij} - \frac{1}{2N}\gamma_{ij}D^2\lambda_{\mathrm{GF}} \qquad \mathcal{M}_{ij} := R_{ij} + \mathcal{K}^k_k\mathcal{K}_{ij} - \mathcal{K}_{ik}\mathcal{K}^k_j$$

- Valid for specific choice: ${}^3\mathcal{G} = \sqrt{\gamma}D_kD^k(\pi^{ij}\gamma_{ij}/\sqrt{\gamma})$ compatible with cosmology & static sol
- d→3 limit of any solutions of (d+1)-dim EGB with conformally flat spatial metric and vanishing Weyl part of $K_{ik}K_{ji}-K_{il}K_{jk}$ are solutions (e.g. FLRW & spherical sol of Glavan&Lin)

Constraints

- Stability of scalar perturbation

$$\dot{H} < 0$$

- Stability of tensor perturbation

$$\tilde{\alpha} > 0$$

- Propagation of gravitational waves

$$\tilde{\alpha} \lesssim (10\,\mathrm{meV})^{-2}$$

- Properties of neutron stars

$$\tilde{\alpha} \lesssim 10^{22}\,\mathrm{eV}^{-2}$$

MASSIVE GRAVITY

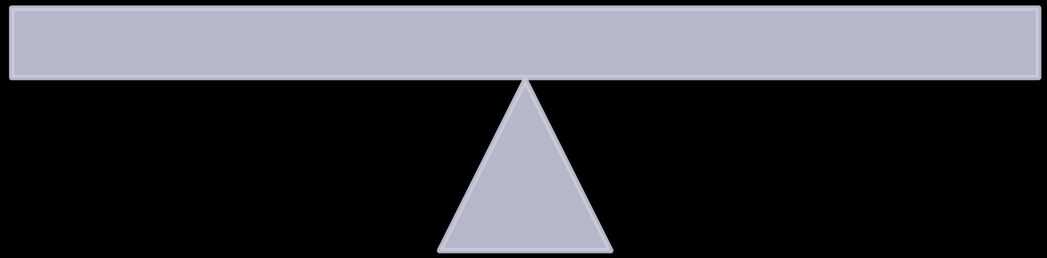
Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?



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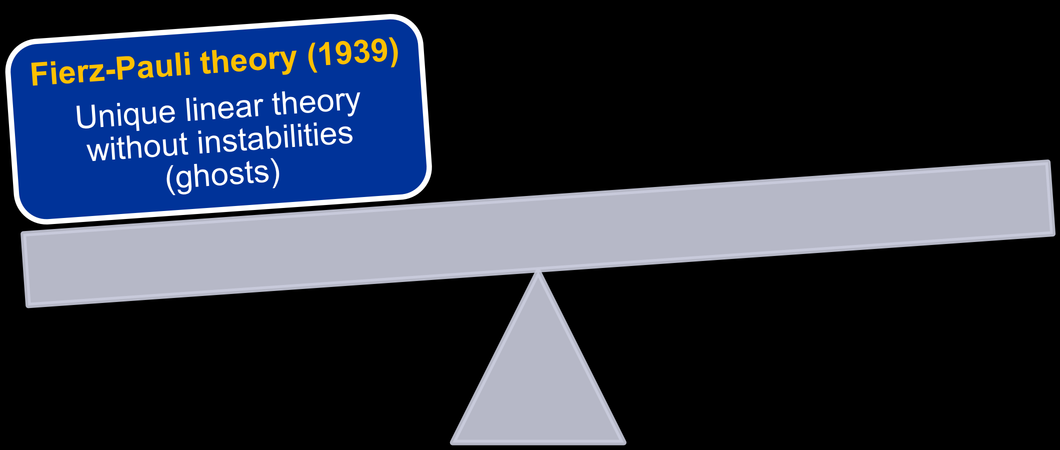

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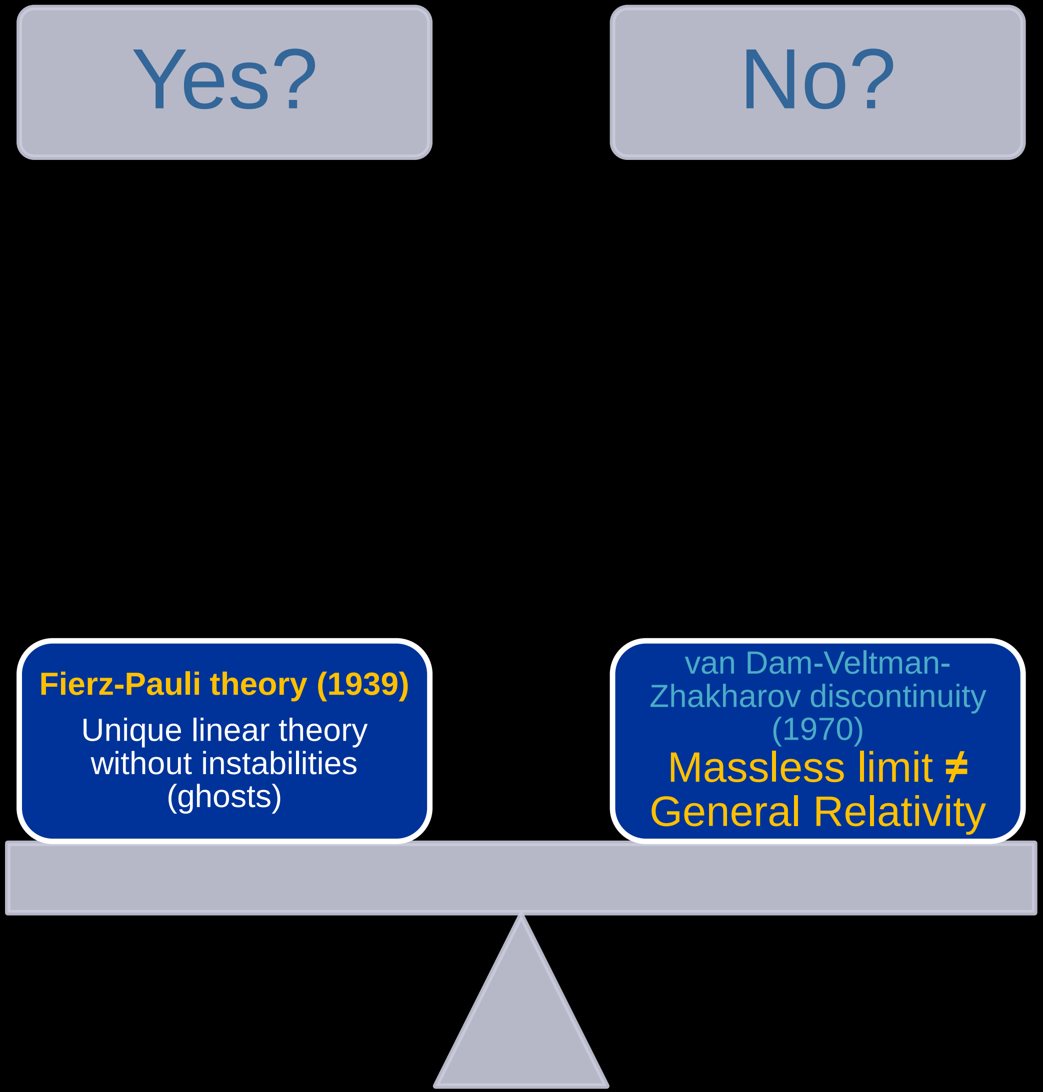

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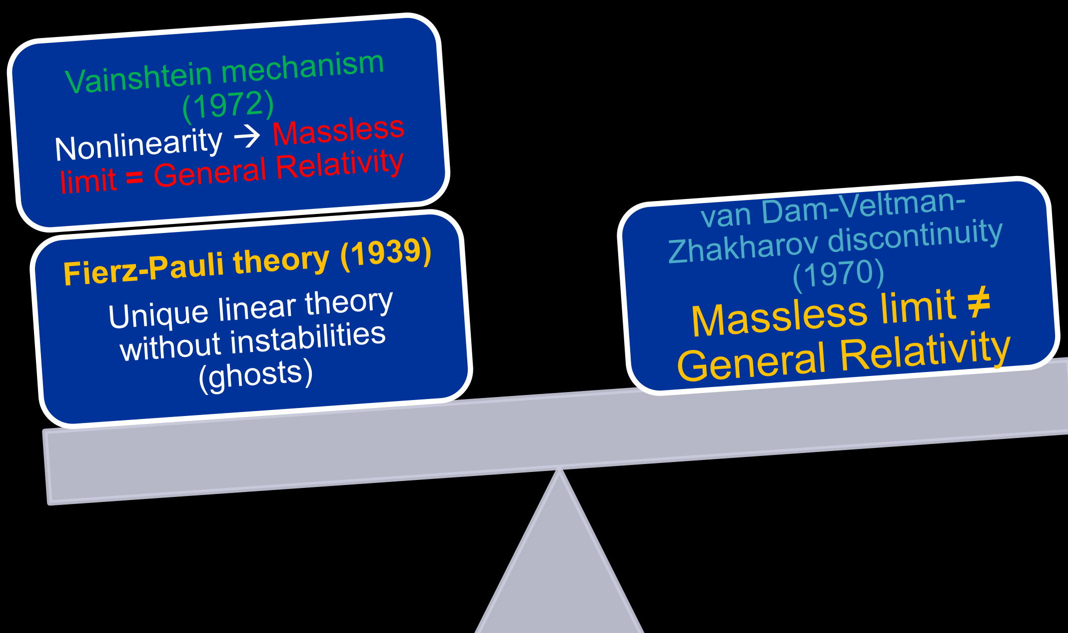

Massive gravity in a nutshell

Simple question: Can graviton have mass?

May lead to acceleration without dark energy

Yes?

No?

Vainshtein mechanism (1972)
Nonlinearity → Massless limit = General Relativity

Fierz-Pauli theory (1939)
Unique linear theory without instabilities (ghosts)

Boulware-Deser ghost (1972)
6^{th} d.o.f.@Nonlinear level → Instability (ghost)

van Dam-Veltman-Zhakharov discontinuity (1970)
Massless limit ≠ General Relativity

Massive gravity in a nutshell

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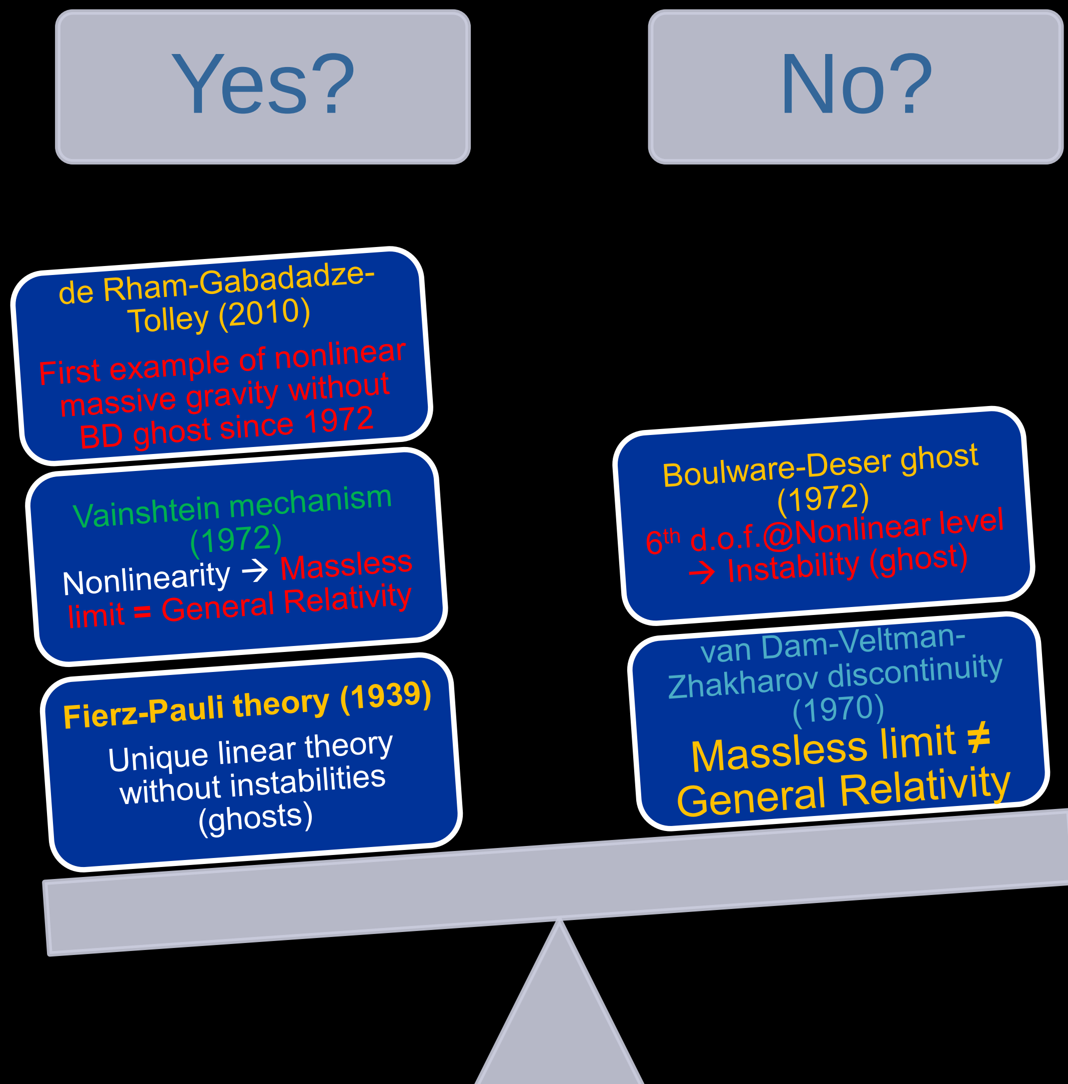
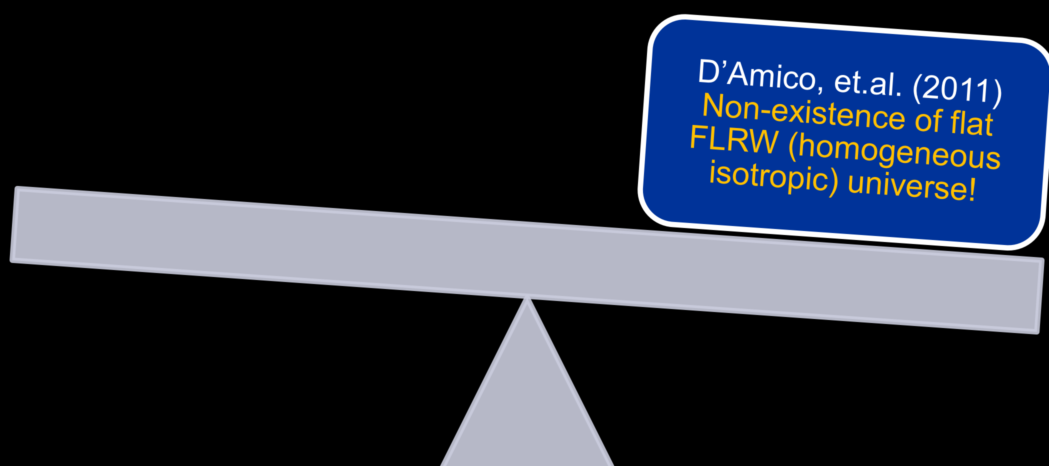


Cosmological solutions in nonlinear massive gravity

Good?

Bad?

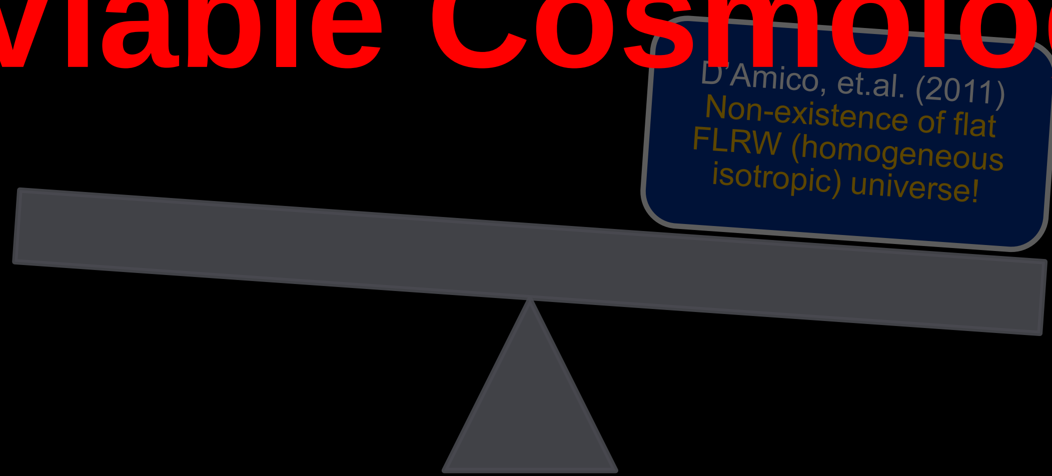


Cosmological solutions in nonlinear massive gravity

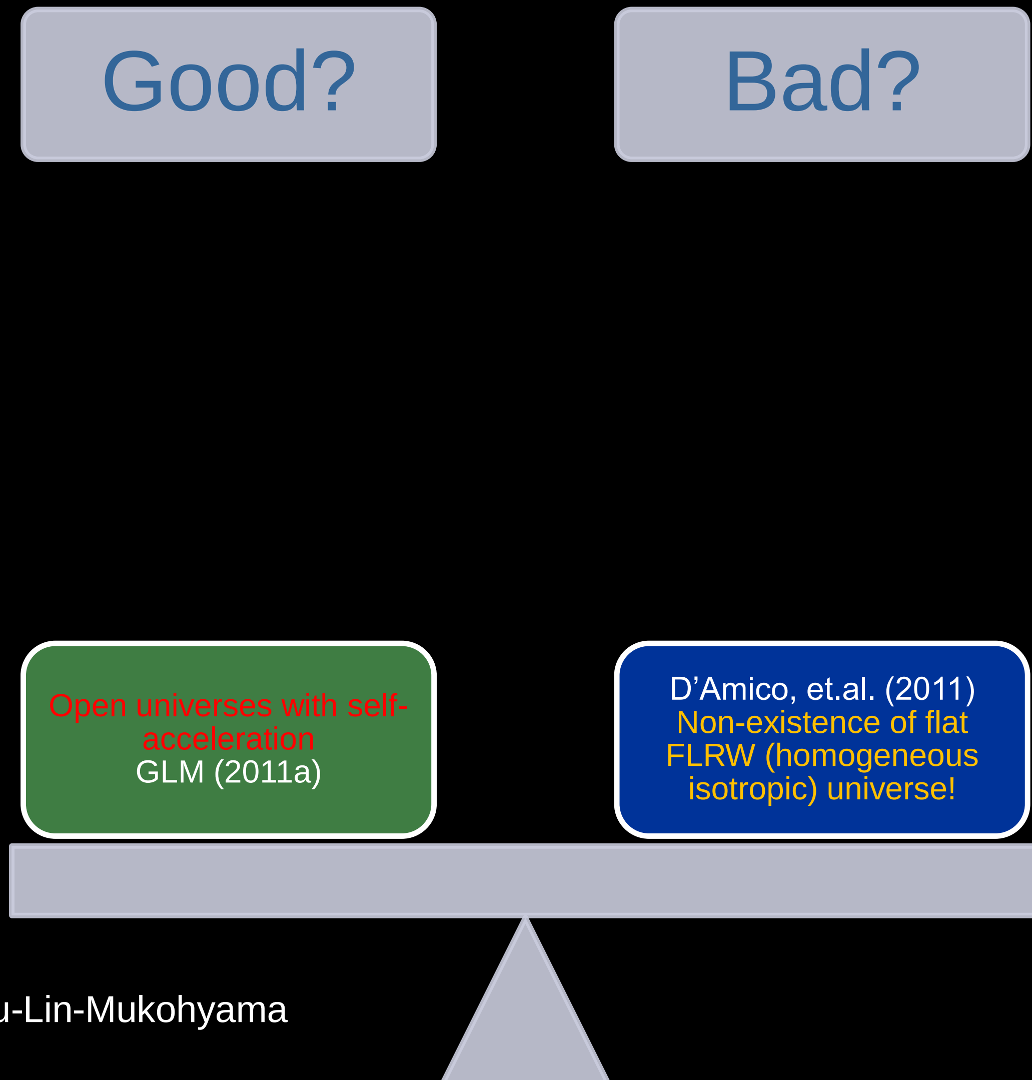
Good? Bad?

Consistent Theory found in 2010 but

No Viable Cosmology?



Cosmological solutions in nonlinear massive gravity

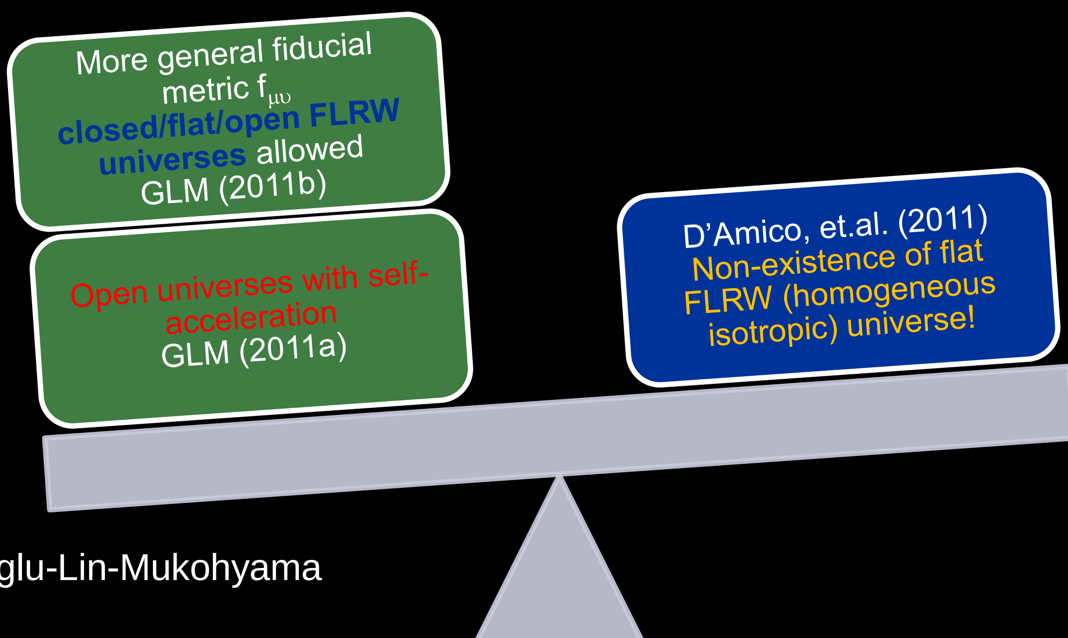


GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

Bad?



GLM = Gumrukcuoglu-Lin-Mukohyama

Cosmological solutions in nonlinear massive gravity

Good?

More general fiducial metric $f_{\mu\nu}$
closed/flat/open FLRW universes allowed
GLM (2011b)

Open universes with self-acceleration
GLM (2011a)

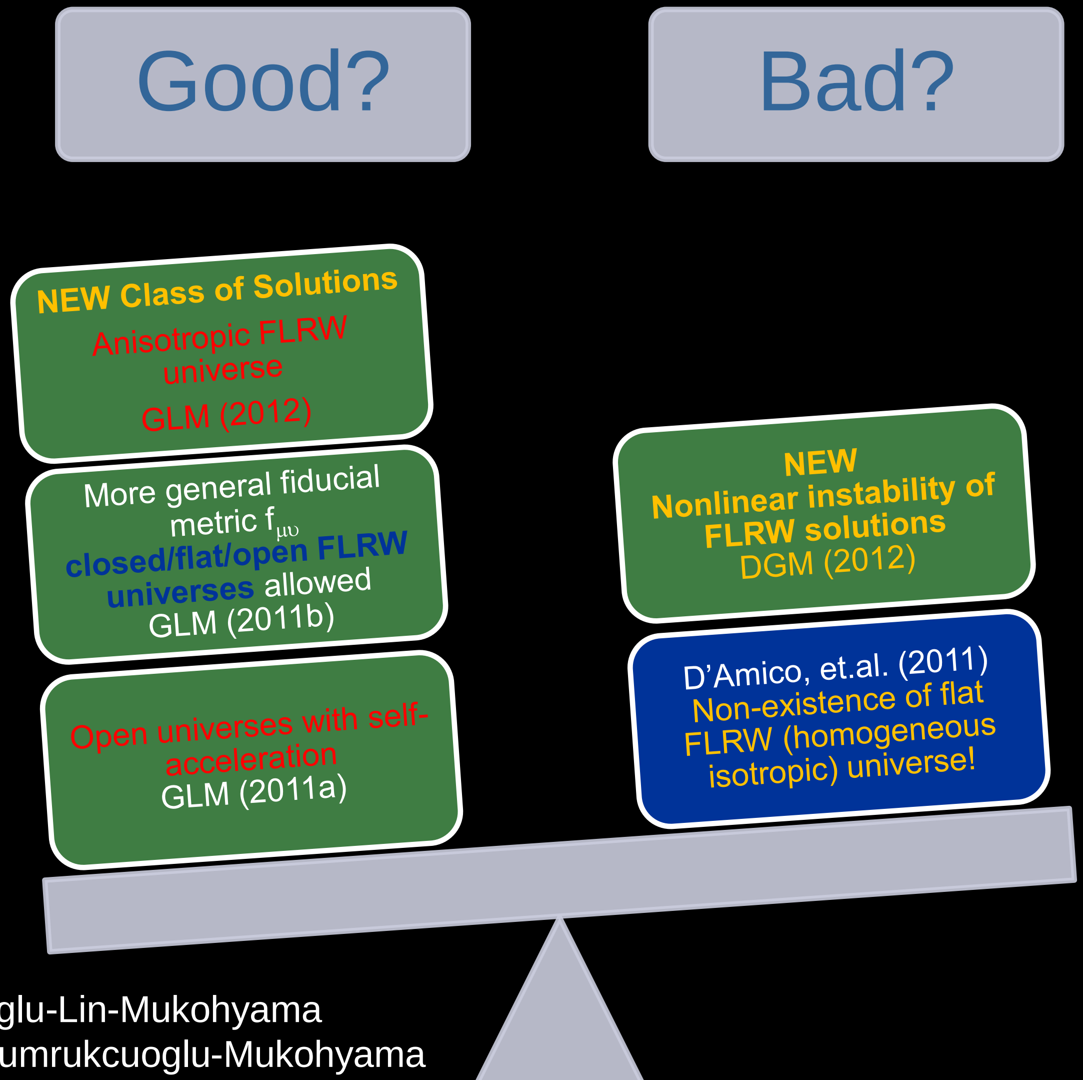
Bad?

NEW
Nonlinear instability of FLRW solutions
DGM (2012)

D'Amico, et.al. (2011)
Non-existence of flat FLRW (homogeneous isotropic) universe!

GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Minimal theory of massive gravity (MTMG)

De Felice & Mukohyama, PLB752 (2016) 302; JCAP1604 (2016) 028

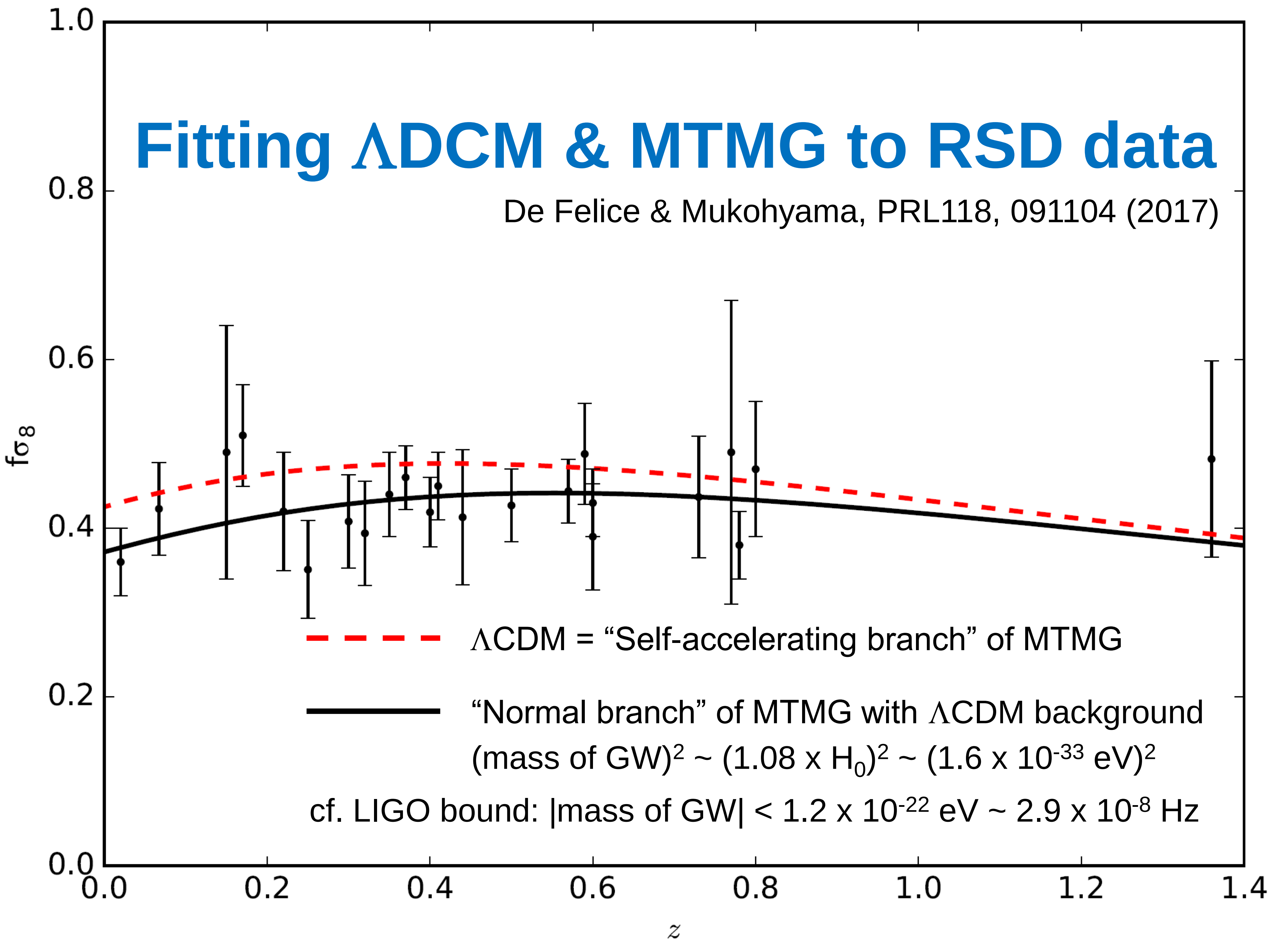
- 2 physical dof only = massive gravitational waves
- exactly same FLRW background as in dRGT
- no BD ghost, no Higuchi ghost, no nonlinear ghost
- positivity bound does not apply

Three steps to the Minimal Theory

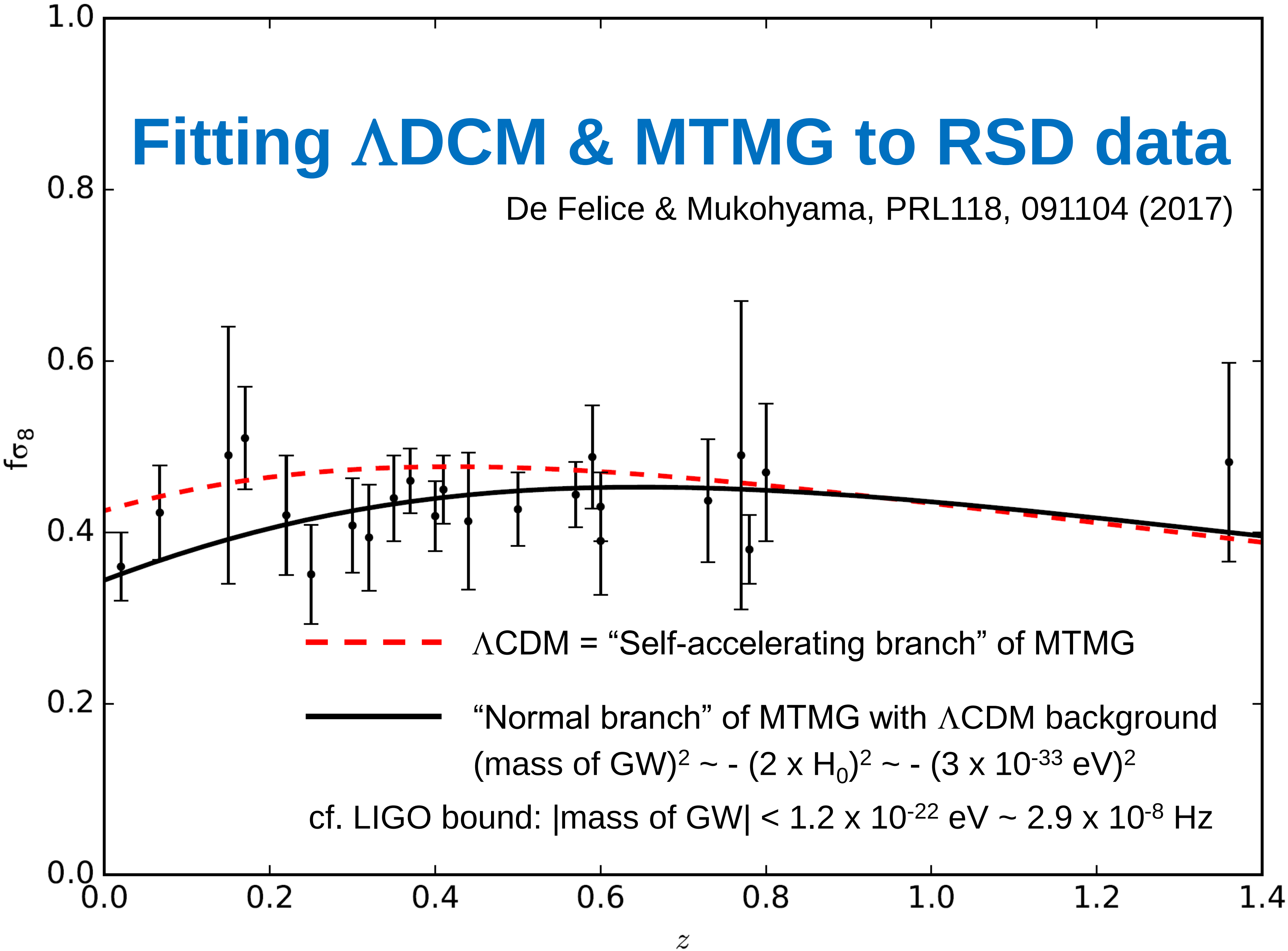
1. Fix local Lorentz to realize ADM vielbein in dRGT
2. Switch to Hamiltonian
3. Add 2 additional constraints

(It is easy to go back to Lagrangian after 3.)

Lorentz-violation due to graviton loops is suppressed by m^2/M_{Pl}^2 and thus consistent with all constraints for $m = O(H_0)$



Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, PRL118, 091104 (2017)
ΛCDM = “Self-accelerating branch” of MTMG
“Normal branch” of MTMG with ΛCDM background
(mass of GW)^2 ~ (1.08 x H_0)^2 ~ (1.6 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

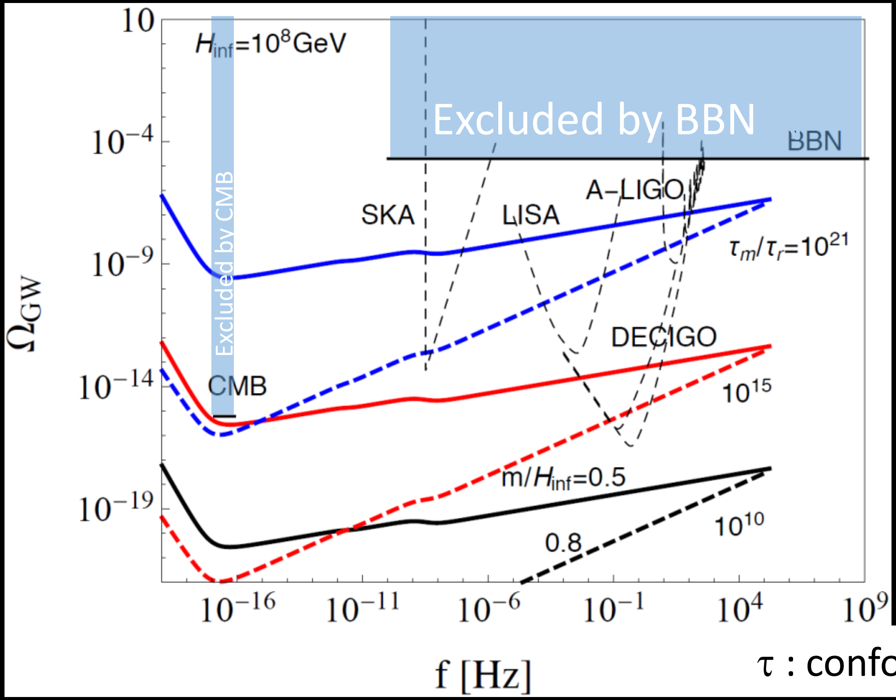

Fitting ΛDCM & MTMG to RSD data
De Felice & Mukohyama, PRL118, 091104 (2017)
ΛCDM = “Self-accelerating branch” of MTMG
“Normal branch” of MTMG with ΛCDM background
(mass of GW)^2 ~ - (2 x H_0)^2 ~ - (3 x 10^{-33} eV)^2
cf. LIGO bound: |mass of GW| < 1.2 x 10^{-22} eV ~ 2.9 x 10^{-8} Hz
$f\sigma_8$
z
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

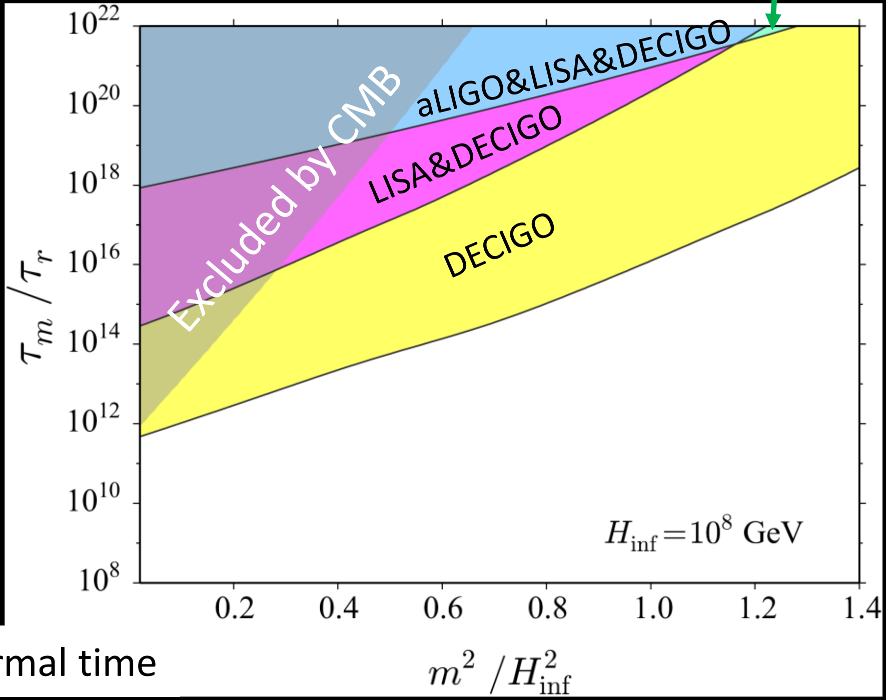
Blue-tilted & amplified primordial GW from MTMG

Fujita, Kuroyanagi, Mizuno, Mukohyama, PLB789 (2019) 215

Fujita, Mizuno, Mukohyama, JCAP 01 (2020) 023

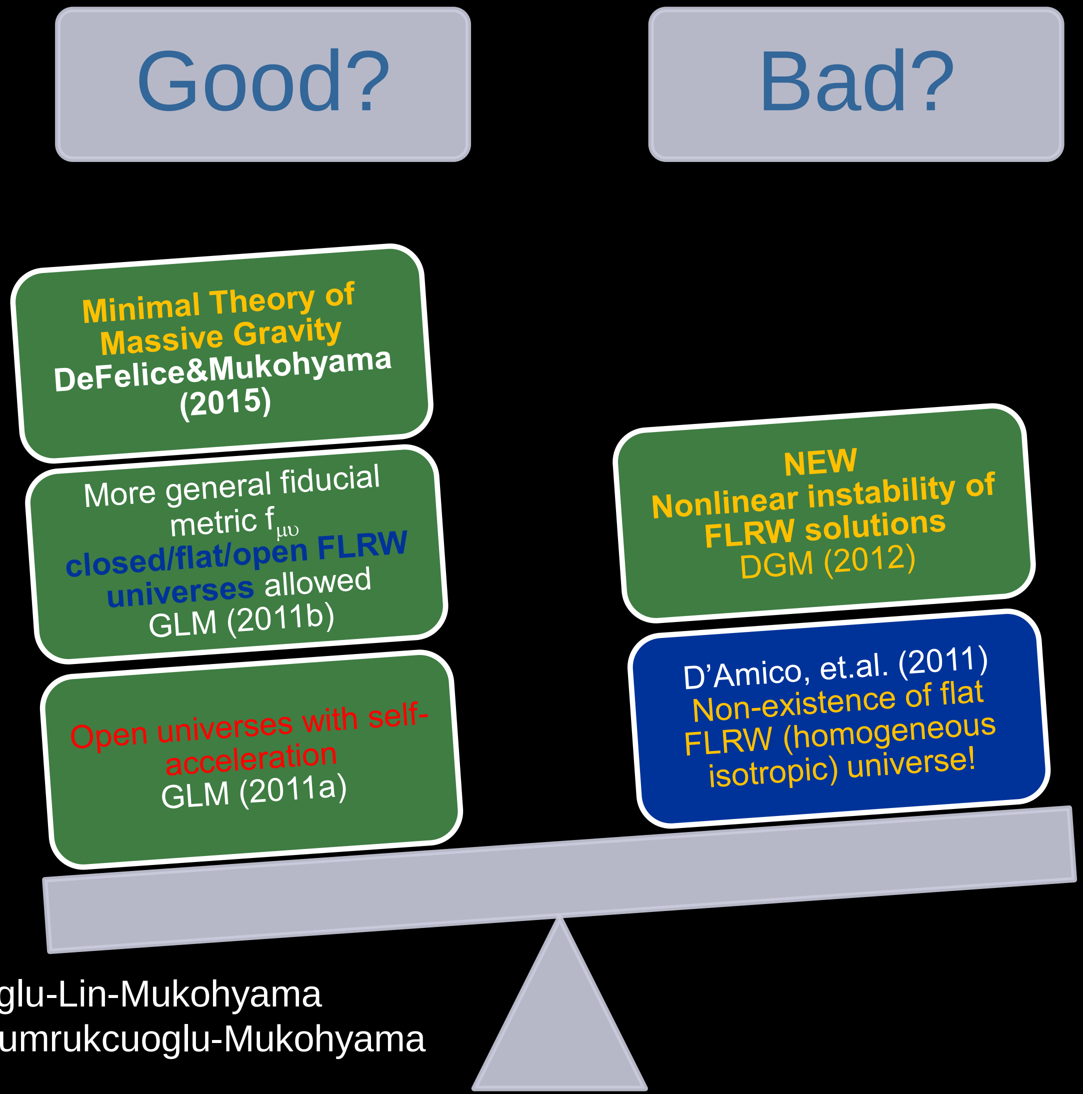
- Simple extension: $c_i \rightarrow c_i(\phi)$ with $\phi = \phi(t)$
- m large until t_m ($t_{reh} < t_m < t_{BBN}$) but small after t_m
  cf. no Higuchi bound in MTMG
- Suppression of GW in IR due to large m → **blue spectrum**
- $\rho_{GW} \propto a^{-3}$ for $t_{reh} < t < t_m$ → **amplification relative to GR**





τ : conformal time

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama

Minimal theory of bigravity (MTBG)

De Felice, Larrouturou, Mukohyama, Oliosi, arXiv:2012.01073.

- 4 physical dof only = massless & massive GWs
- exactly same FLRW backgrounds as in HRBG
- no BD ghost, no Higuchi ghost, no strong coupling

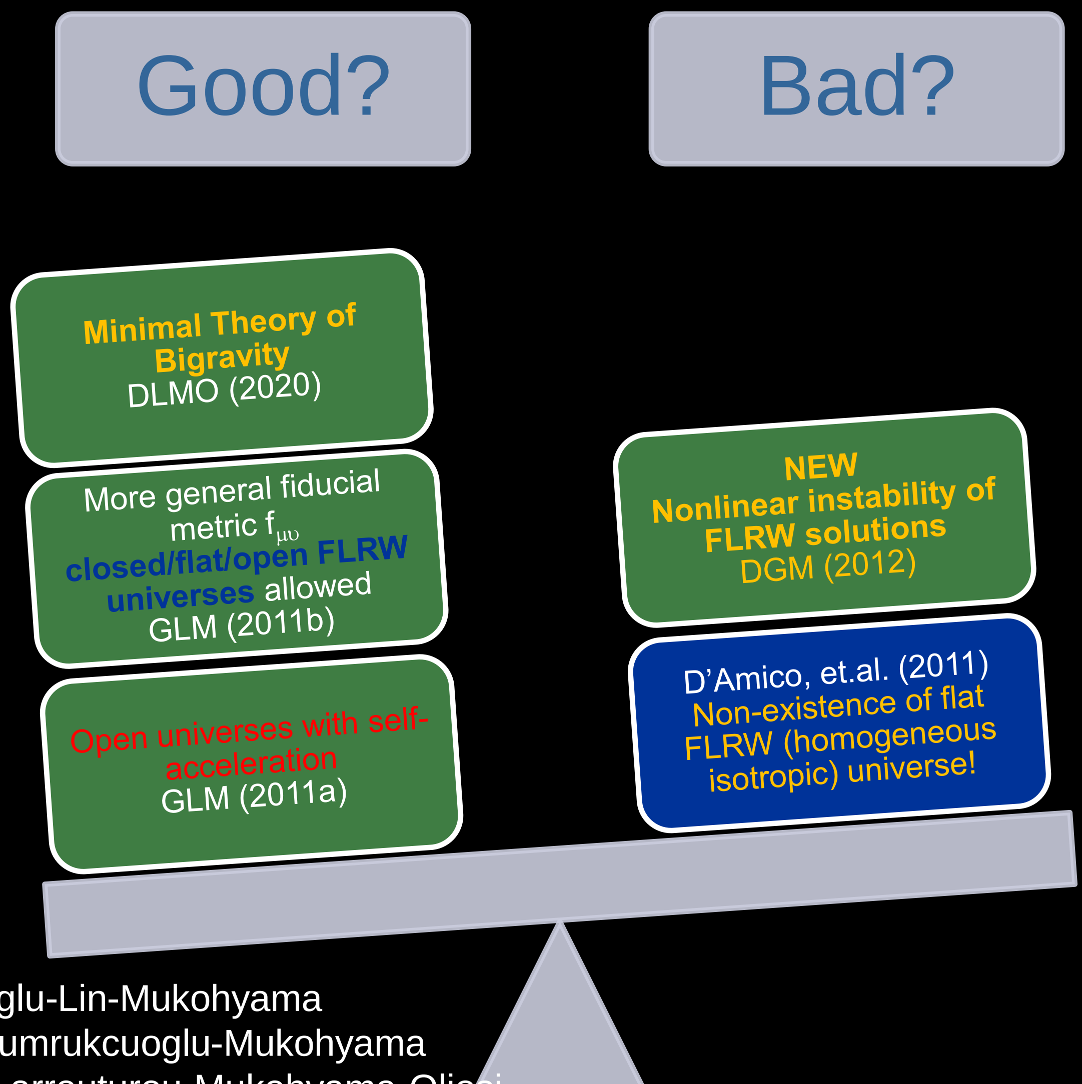
Three steps to the Minimal Theory

1. Fix local Lorentz to realize ADM vielbeins in HRBG
2. Switch to Hamiltonian
3. Add 4 (= 5-1) additional constraints carefully

(It is easy to go back to Lagrangian after 3.)

The very first example of completely stable & cosmologically viable theory of nonlinear bigravity. A testing ground for gravitational phenomena, e.g. graviton oscillation, that can be probed by GWs.

Cosmological solutions in nonlinear massive gravity



GLM = Gumrukcuoglu-Lin-Mukohyama
DGM = DeFelice-Gumrukcuoglu-Mukohyama
DLMO = DeFelice-Larrouturou-Mukohyama-Oliosi

SUMMARY

1. Introduction
2. Minimally modified gravity (MMG)
3. Examples of type-I & type-II MMG theories
4. D→4 EGB gravity with 2 dof
5. Massive gravity
6. Summary

Minimalism in modified gravity

- Minimal # of d.o.f. in modified gravity = 2
  can be saturated → **minimally modified gravity (MMG)**
- Type-I MMG: ∃ Einstein frame
  Type-II MMG: no Einstein frame
- Examples of type-I MMG
  GR + canonical tr. + gauge-fixing + adding matter
  Rich phenomenology: w_{DE}, G_{eff}, etc.
  f(H) theory can fit Planck data batter than ΛCDM
- An example of type-II MMG
  Minimal theory of massive gravity (MTMG)
- Another example of type-II MMG: cuscuton/VCDM
  GR + canonical tr. + cc + gauge-fixing + inverse canonical tr.
  $V(\phi)$ reconstructed from FLRW background
  May reduce H_0 tension
  Extension (VCCDM) may address both H_0&S_8 tensions

Summary of massive gravity part

- Nonlinear massive gravity (dRGT 2010)
  free from BD ghost
- FLRW background
  No closed/flat universe
  Open universes with self-acceleration!
- More general fiducial metric $f_{\mu\upsilon}$
  closed/flat/open FLRW universes allowed
- All FLRW solutions in the original dRGT theory have strong coupling and ghost instability
- Stable cosmology requires either (i) new class of cosmological solutions or (ii) extended theories
- MTMG and MTBG provide nonlinear completion of dRGT self-accelerating cosmology

Thank you!