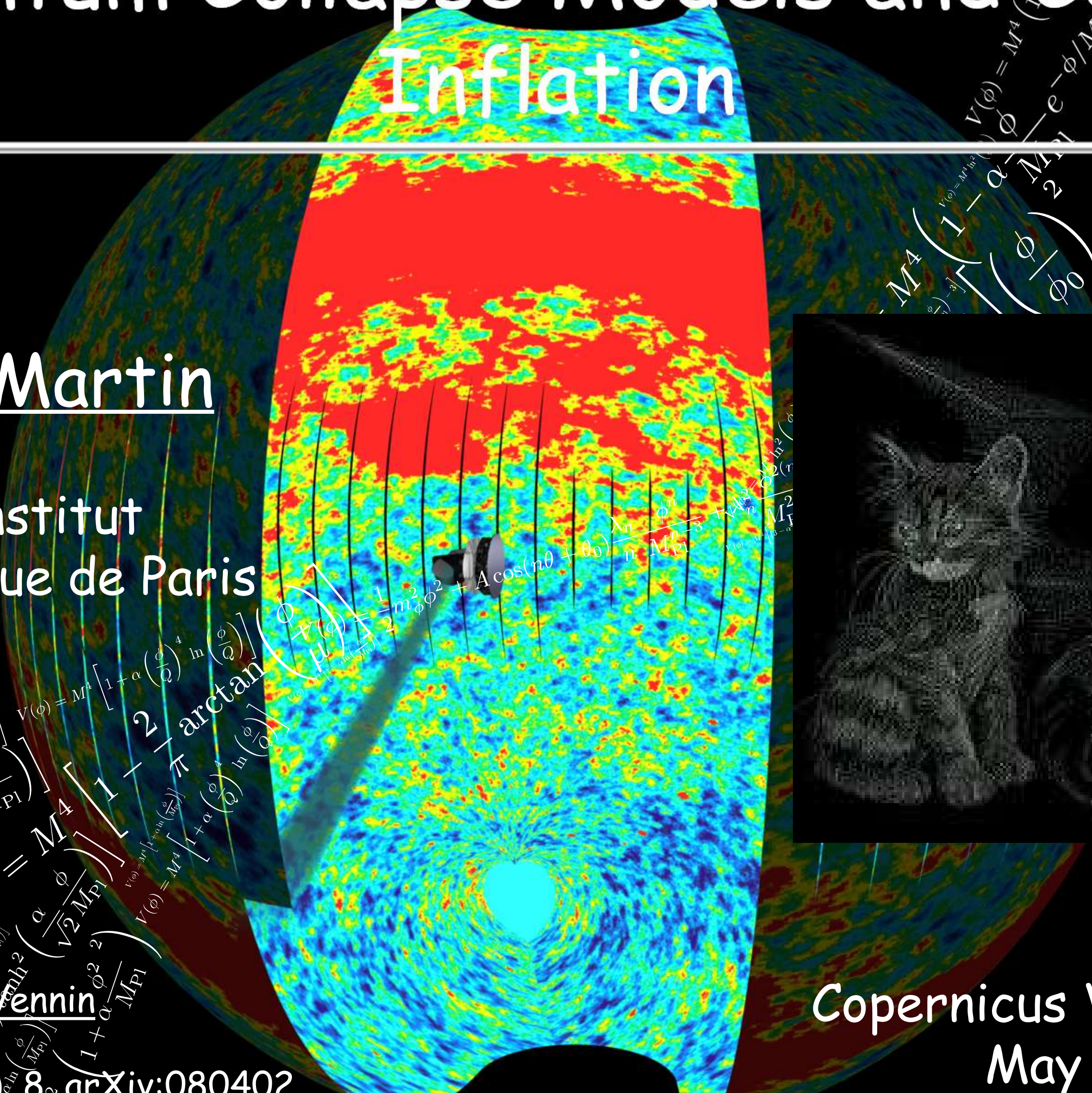
Quantum Collapse Models and Cosmic Inflation



Jerome Martin

CNRS/Institut d'Astrophysique de Paris

J. Martin & V. Vennin

- PRL 124 (2020), 8, arXiv:080402
- Fundam. Theor. Phys. 198 (2020), 269-290, arXiv:1912.07429
- Eur. Phys. J. C. 81 (2021), 64, arXiv:2010.04067
- arXiv:2103.01697

Copernicus Webinar Series
May 18, 2021

Outline

- Introduction & Motivations
- Quantum collapse models in brief
- Application to cosmology and perturbation theory
- Conclusions

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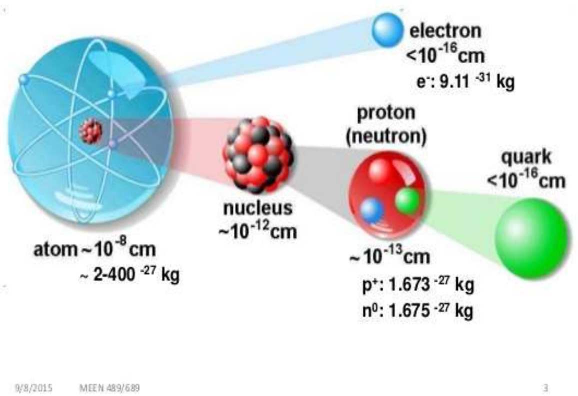

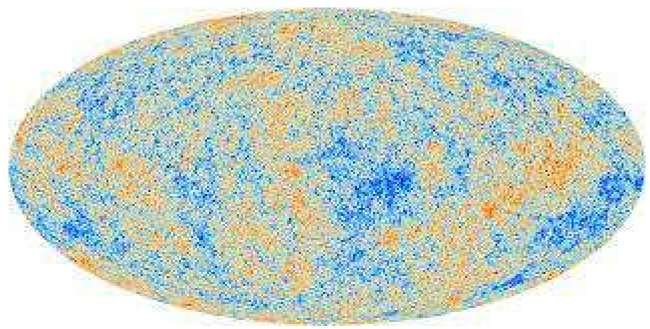

Motivations

- There is no doubt that Quantum Mechanics is a very successful theory
- There is still, however, a debate about how the theory should be understood

"I don't believe I've ever seen a scientific paper defended quite as vigorously as this one!"

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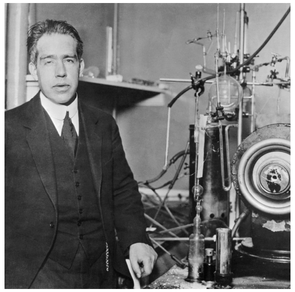
- There is no doubt that Quantum Mechanics is a very successful theory
- There is still, however, a debate about how the theory should be understood
- This has given rise to a large variety of interpretations …

  - Copenhagen



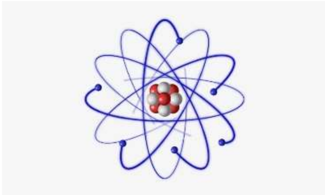
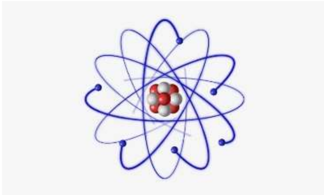
  - MWI

  - Bohm

  - etc …

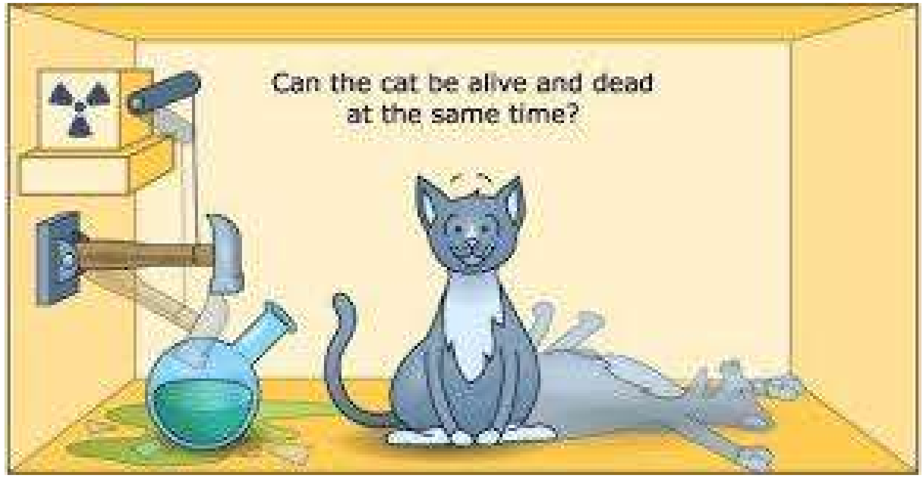
Motivations

The problem originates from quantum superpositions

 **+** 

✓ Experimentally seen

Quantum mechanics is linear



✓ Macroscopic superpositions
but … never seen

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- Collapse models are falsifiable

- First motivation: since collapse models are falsifiable, can Cosmology help contraining them?

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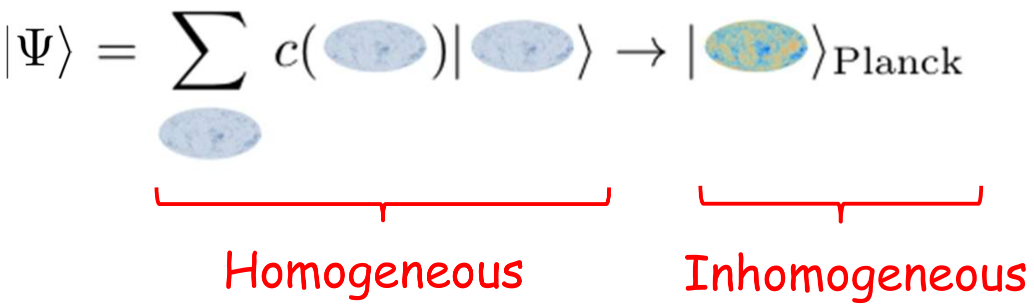
- For instance



$$\left[\hat{H}, \hat{P}_\mu\right] = 0$$

$$\hat{P}_\mu = -\int \mathrm{d}^3\boldsymbol{x}\sqrt{{}^{(3)}g}\,\hat{T}^0{}_\mu$$

Why and how in the early Universe??

- First motivation: since collapse models are falsifiable, can Cosmology help contraining them?

- Second motivation: can collapse models help improving our understanding of the behavior of quantum perturbations in the early Universe?

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A first approach: the GRW model

Main idea: The wave-function undergoes random "flashes" in time and space, with frequency λ, that localizes the state vector on a spatial scale r_c

$$\Psi(t,x) \to \frac{\hat{L}_q \Psi(t,x)}{||\hat{L}_q \Psi(t,x)||} \qquad \text{with probability} \quad ||\hat{L}_q \Psi(t,x)||^2$$

$$\hat{L}_q = \frac{1}{\pi^{3/4} r_c^{3/2}} e^{-(q-\hat{X})^2/(2r_c^2)}$$

Ghirardi, Rimini, Weber, Phys. Rev. D 34 (1986), 470

Bassi, Ghirardi, Phys. Rept. 379 (2003), 257, quant-ph/0302164

Bassi, Lochan, Satin, Singh, Ulbricht, Rev. Mod. Phys. 85 (2013), 471, arXiv: 1204.4325

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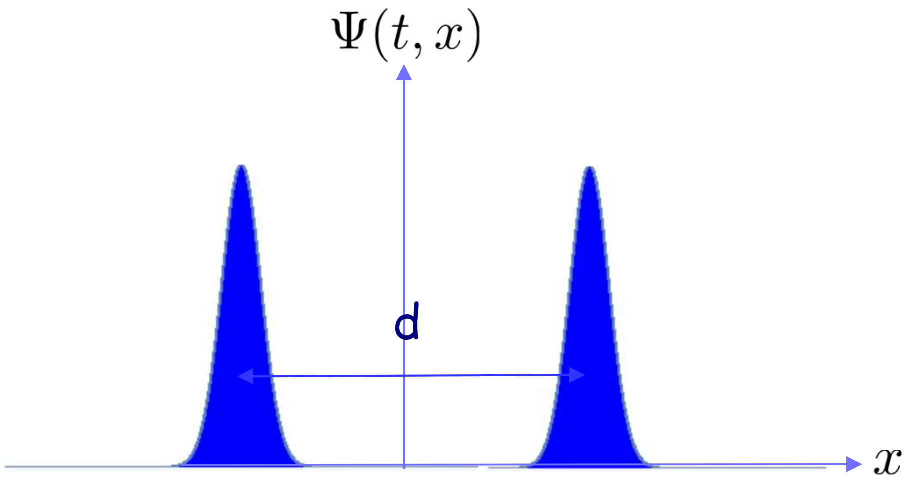

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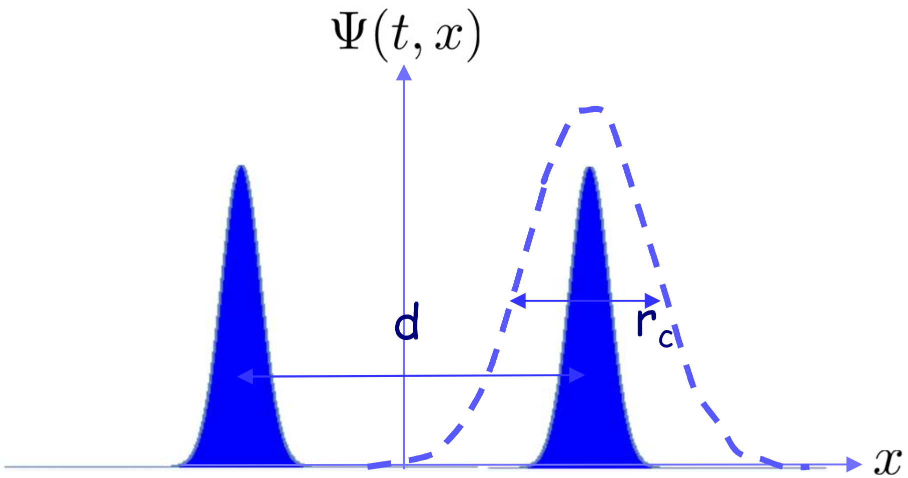

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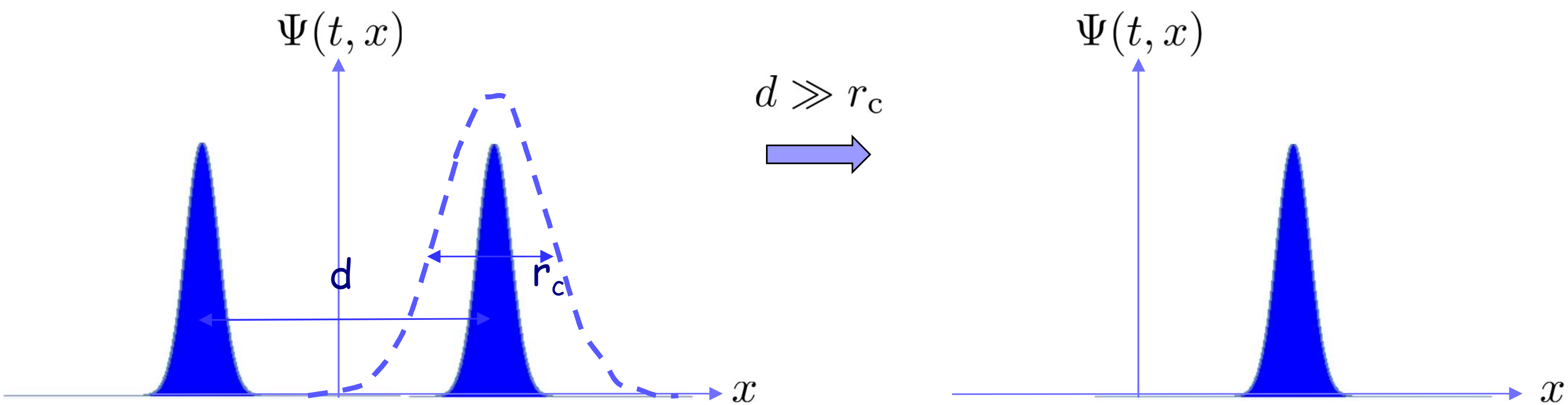

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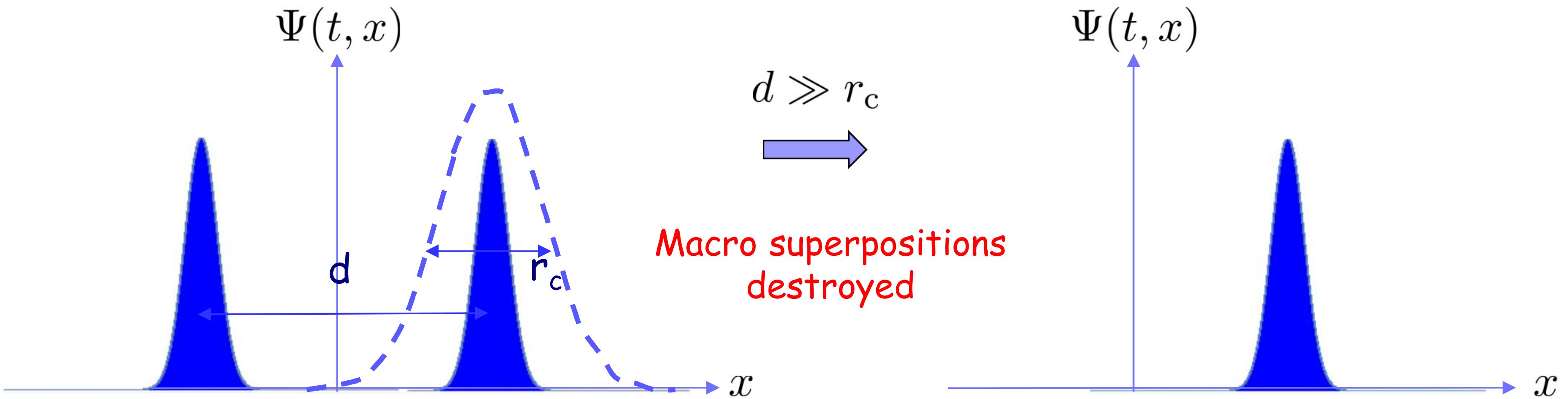

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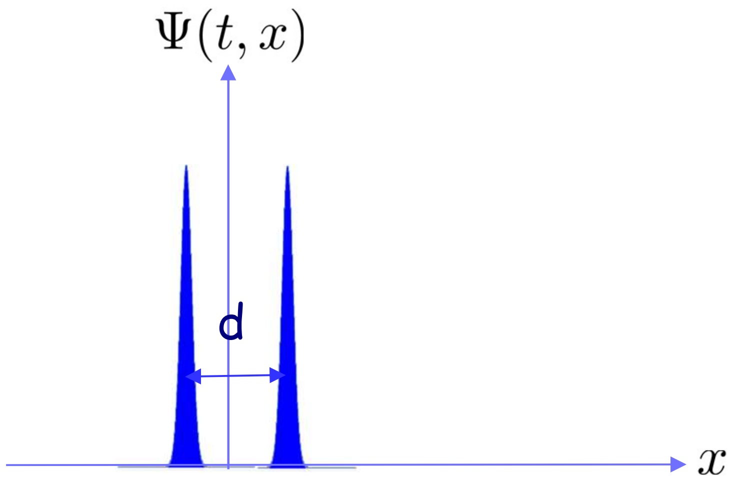

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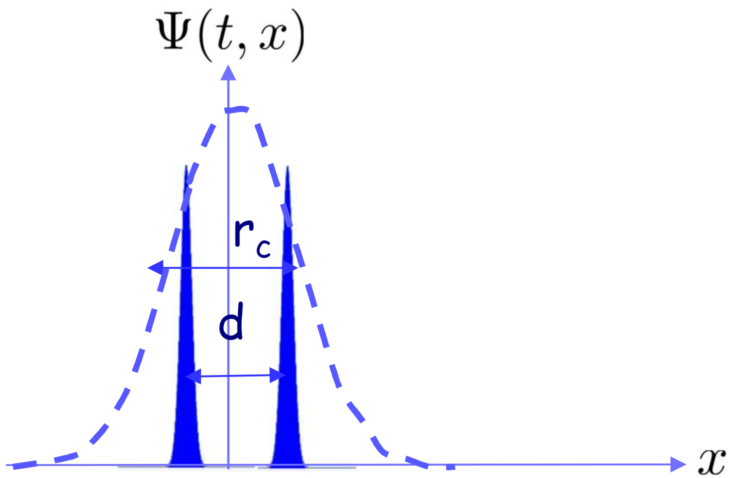

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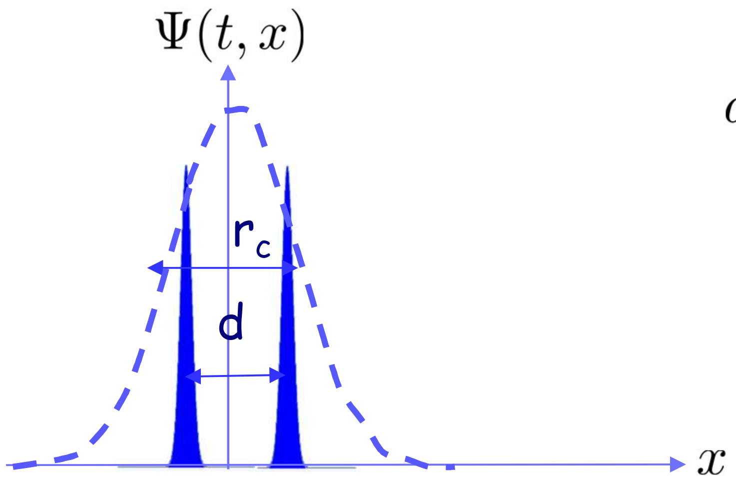
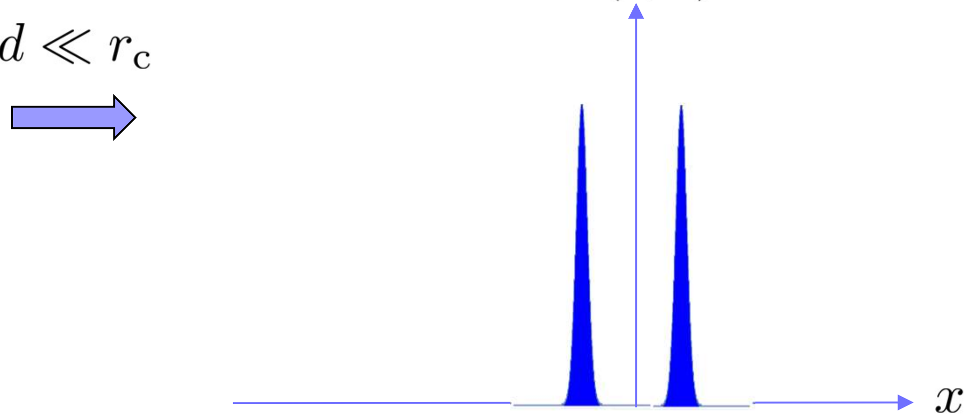


$d \ll r_c$



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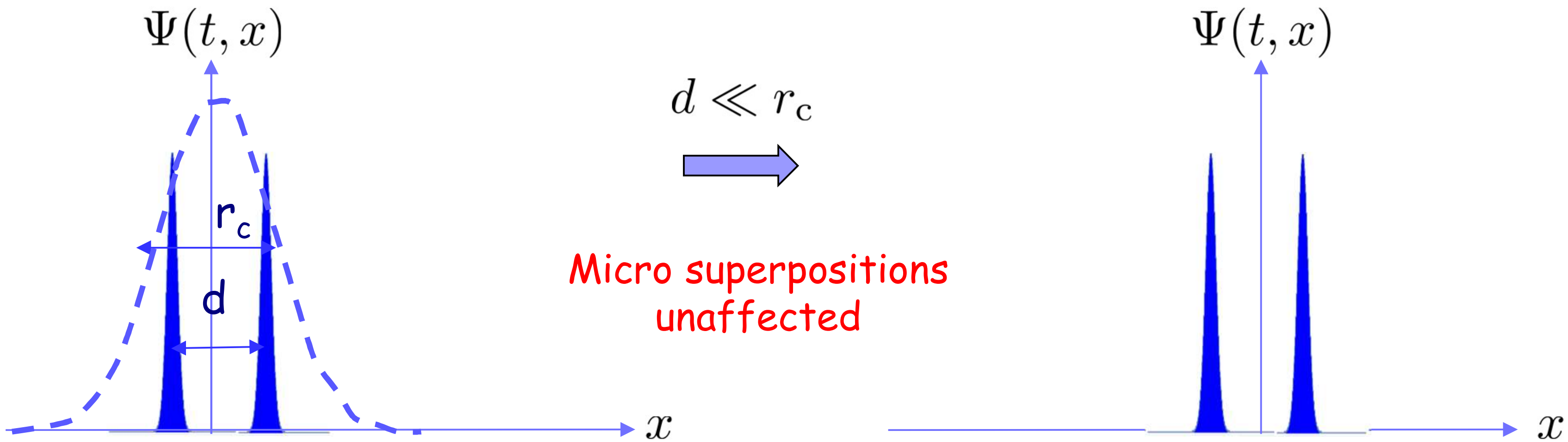

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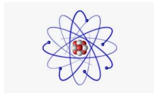
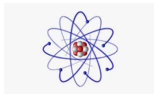
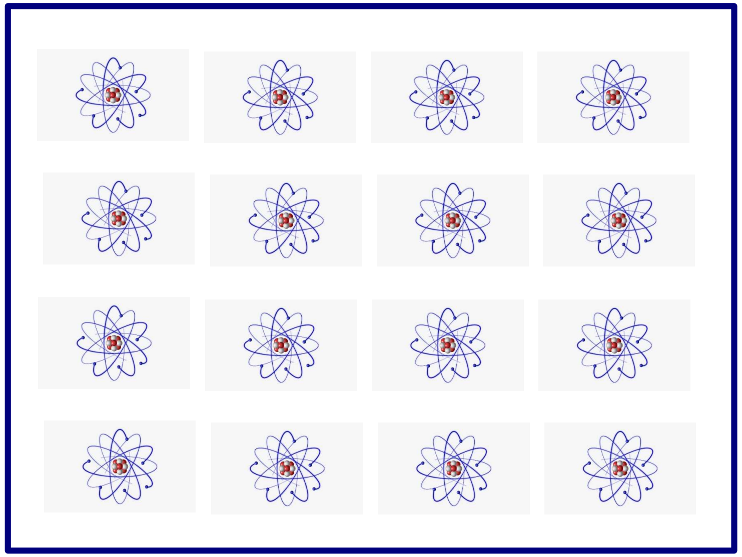
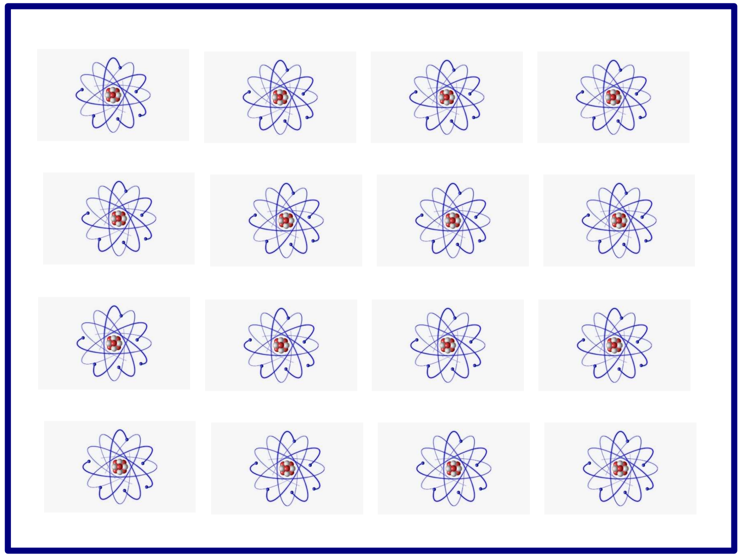
There is an amplification mechanism: the destruction of big superposition proceeds with a much larger rate

 + 

rate: λ

 + 

rate: $\lambda_{\text{eff}} = N\lambda \gg \lambda$

Stochastic evolution in Hilbert space

$$\mathrm{d}|\Psi(t,\boldsymbol{x})\rangle = \left[\underbrace{-i\hat{H}\mathrm{d}t}_{\text{Schrödinger term}} \underbrace{- \frac{\gamma}{2}\sum_i \left(\hat{C}_i - \left\langle \hat{C}_i \right\rangle\right)^2 \mathrm{d}t + \sqrt{\gamma}\sum_i \left(\hat{C}_i - \left\langle \hat{C}_i \right\rangle\right) \mathrm{d}W_i(t)}_{\text{New terms}}\right] |\Psi(t,\boldsymbol{x})\rangle$$

- $\{C_i\}$: set of collapse operators
- γ: new parameter, sets the strength of the collapse
- Non-linear evolution:

$$\left\langle \hat{C}_i \right\rangle = \left\langle \Psi | \hat{C}_i | \Psi \right\rangle$$

- Stochastic evolution:

$$\mathbb{E}\left[\mathrm{d}W_i(t)\right] = 0$$

$$\mathbb{E}\left[\mathrm{d}W_i(t)\mathrm{d}W_j(t')\right] = \delta_{ij}\delta(t-t')$$

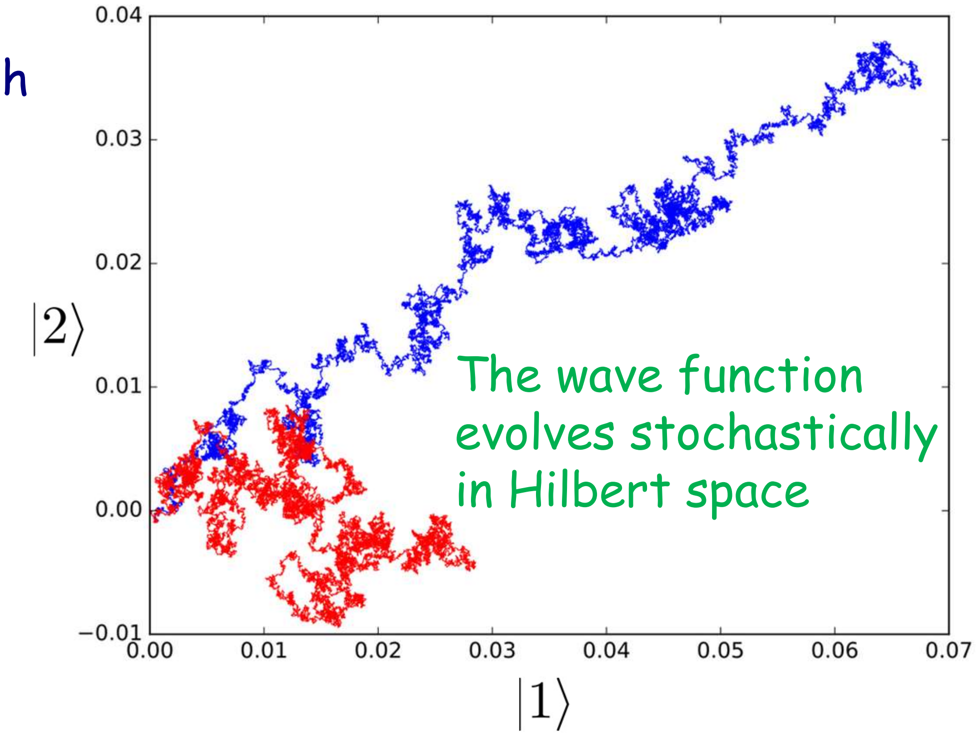


Illustration: collapse of the state vector

$$\mathrm{d}|\Psi(t,\boldsymbol{x})\rangle = \left[-i\hat{H}\mathrm{d}t - \frac{\gamma}{2}\sum_i\left(\hat{C}_i - \left\langle\hat{C}_i\right\rangle\right)^2\mathrm{d}t + \sqrt{\gamma}\sum_i\left(\hat{C}_i - \left\langle\hat{C}_i\right\rangle\right)\mathrm{d}W_i(t)\right]|\Psi(t,\boldsymbol{x})\rangle$$

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$$\hat{C} = \sum_\sigma a_\sigma |\sigma\rangle\langle\sigma|$$

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$$\mathrm{d}z_\sigma = 2z_\sigma \sum_{\sigma'} z_{\sigma'}\left(a_\sigma - a_{\sigma'}\right)\mathrm{d}W_t$$

$$\sum_\sigma z_\sigma = 1$$

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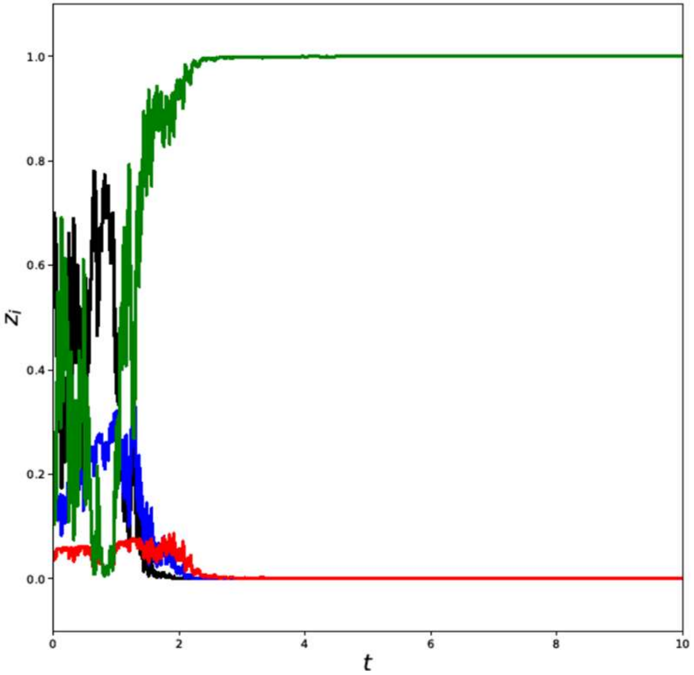
- Solution



Continuous Spontaneous Localization (CSL) models

$$\mathrm{d}|\Psi\rangle = \Big\{ -\frac{i}{\hbar}\hat{H}\mathrm{d}t - \frac{\gamma}{2m_0^2}\int \mathrm{d}\boldsymbol{x}_\mathrm{p} \left[\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) - \left\langle \hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) \right\rangle\right]^2 \mathrm{d}t + \frac{\sqrt{\gamma}}{m_0}\int \mathrm{d}\boldsymbol{x}_\mathrm{p} \left[\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) - \left\langle \hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) \right\rangle\right] \mathrm{d}W_t \Big\} |\Psi\rangle$$

- Collapse operator is the “smeared” mass density operator

$$\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) = \frac{1}{(2\pi)^{3/2} r_\mathrm{c}^3}\int \mathrm{d}\boldsymbol{y}_\mathrm{p}\, \hat{\rho}(\boldsymbol{x}_\mathrm{p} + \boldsymbol{y}_\mathrm{p})\, e^{-|\boldsymbol{y}_\mathrm{p}|^2/(2r_\mathrm{c}^2)}$$

- m_0 is a reference mass (usually taken as the nucleon mass)

- Two parameters model: $\lambda = \frac{\gamma}{8\pi^{3/2} r_\mathrm{c}^3}$ and r_c

- $[\lambda]$ = time^{-1}, gives the frequency/collapse and $[r_c]$ = length, gives localization scale

Balance sheet

Pros

Cons

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Pros	Cons

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- Born rule derived
- It is falsifiable
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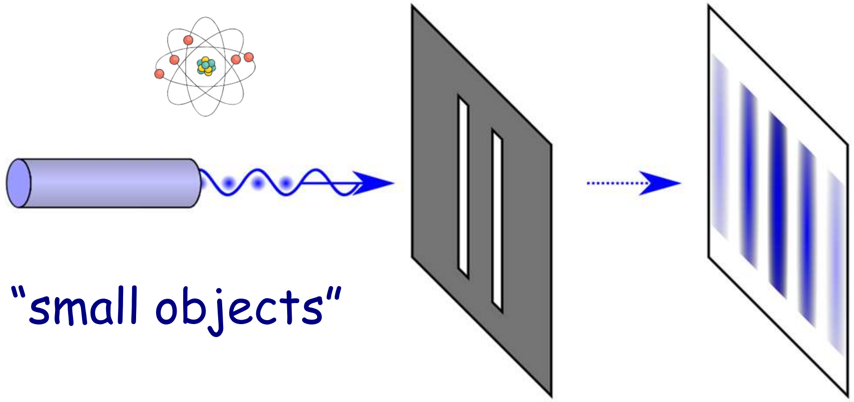
Cons

- Interpretation of the noise?
- Energy not conserved
- No easy relativistic generalization

CSL is falsifiable



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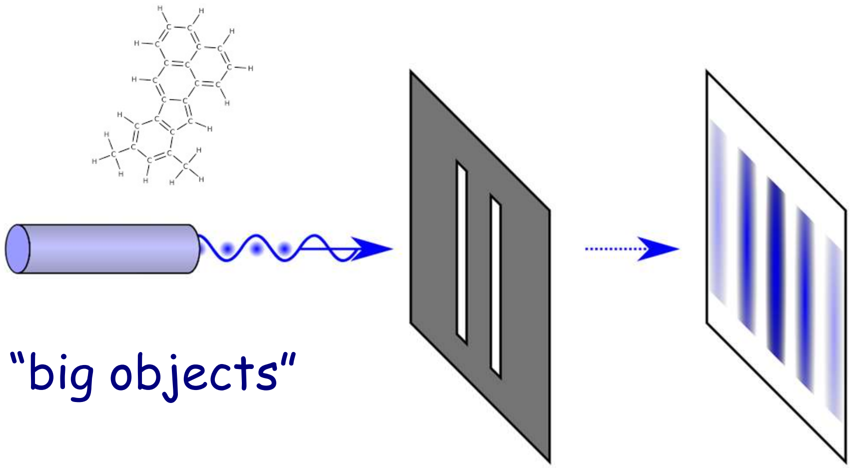
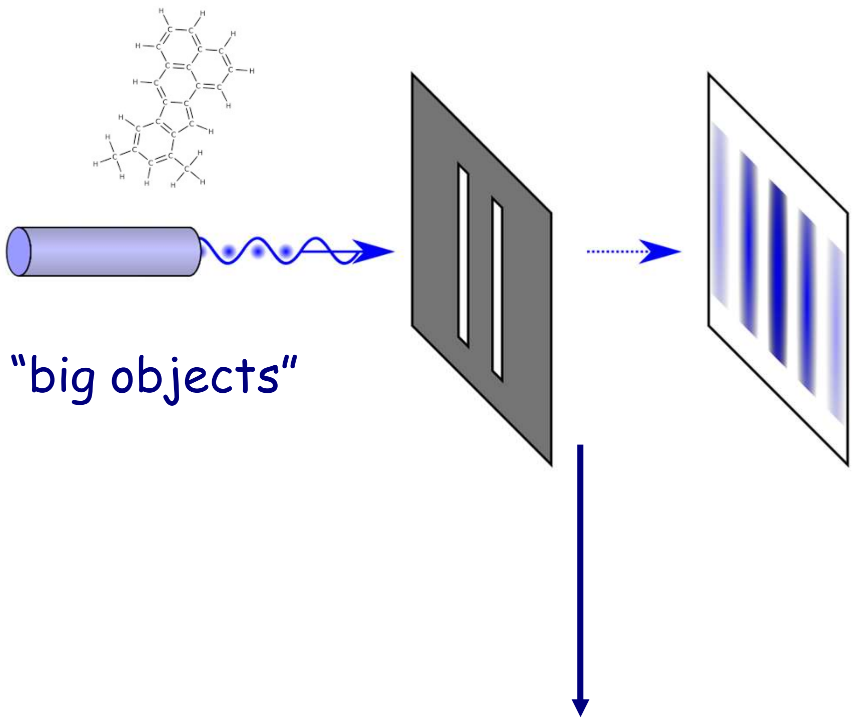


"big objects"

CSL is falsifiable



If one increases the mass, the superposition produced after the two slits will be destroyed by spontaneous collapse

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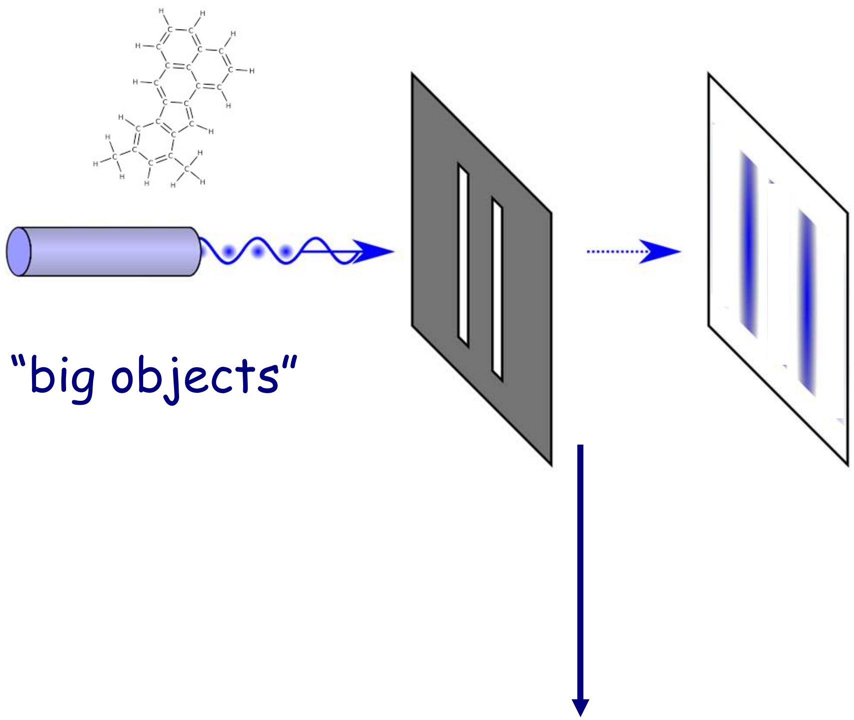

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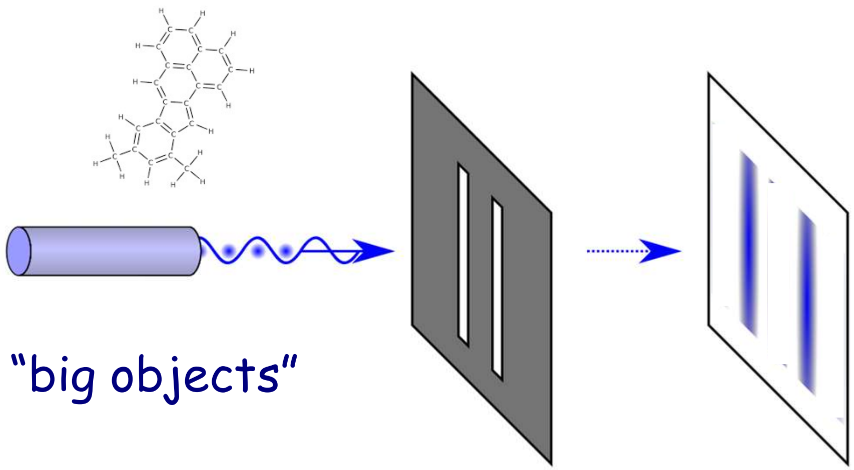
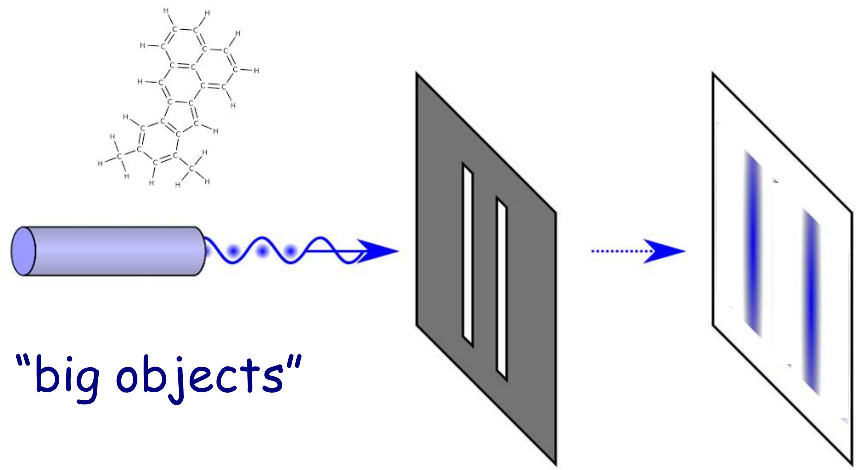


- 1999: Fullerene, 60 atoms, 720 amu

CSL is falsifiable



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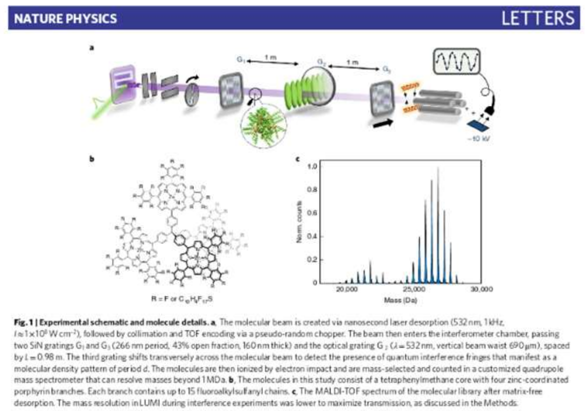
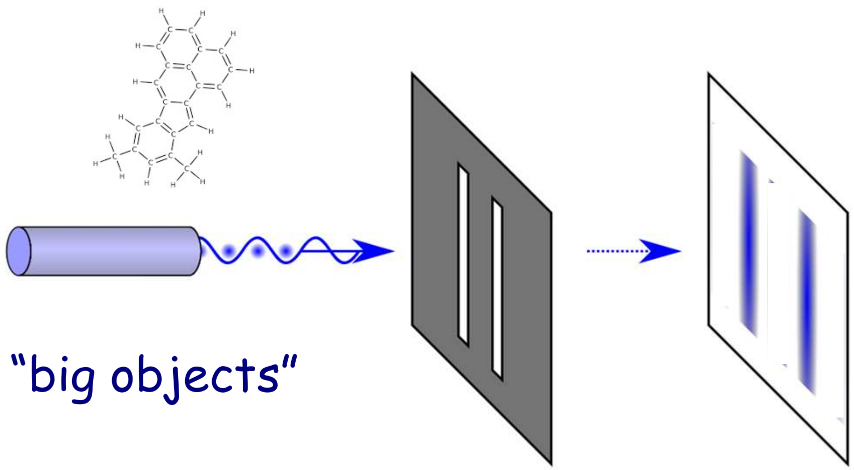
- 1999: Fullerene, 60 atoms, 720 amu

- 2019: large molecules, 2000 atoms, 26 777 amu



CSL is falsifiable



"big objects"

- 1999: Fullerene, 60 atoms, 720 amu

- 2019: large molecules, 2000 atoms, 26 777 amu

- Transition expected around 10^6-10^9 amu

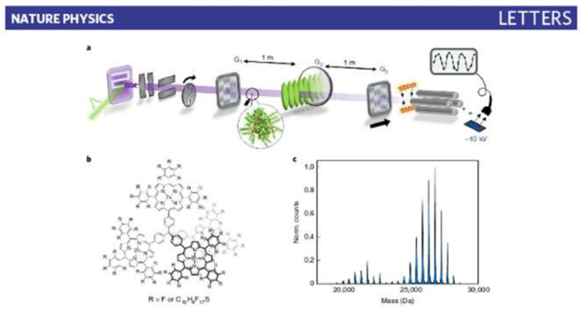
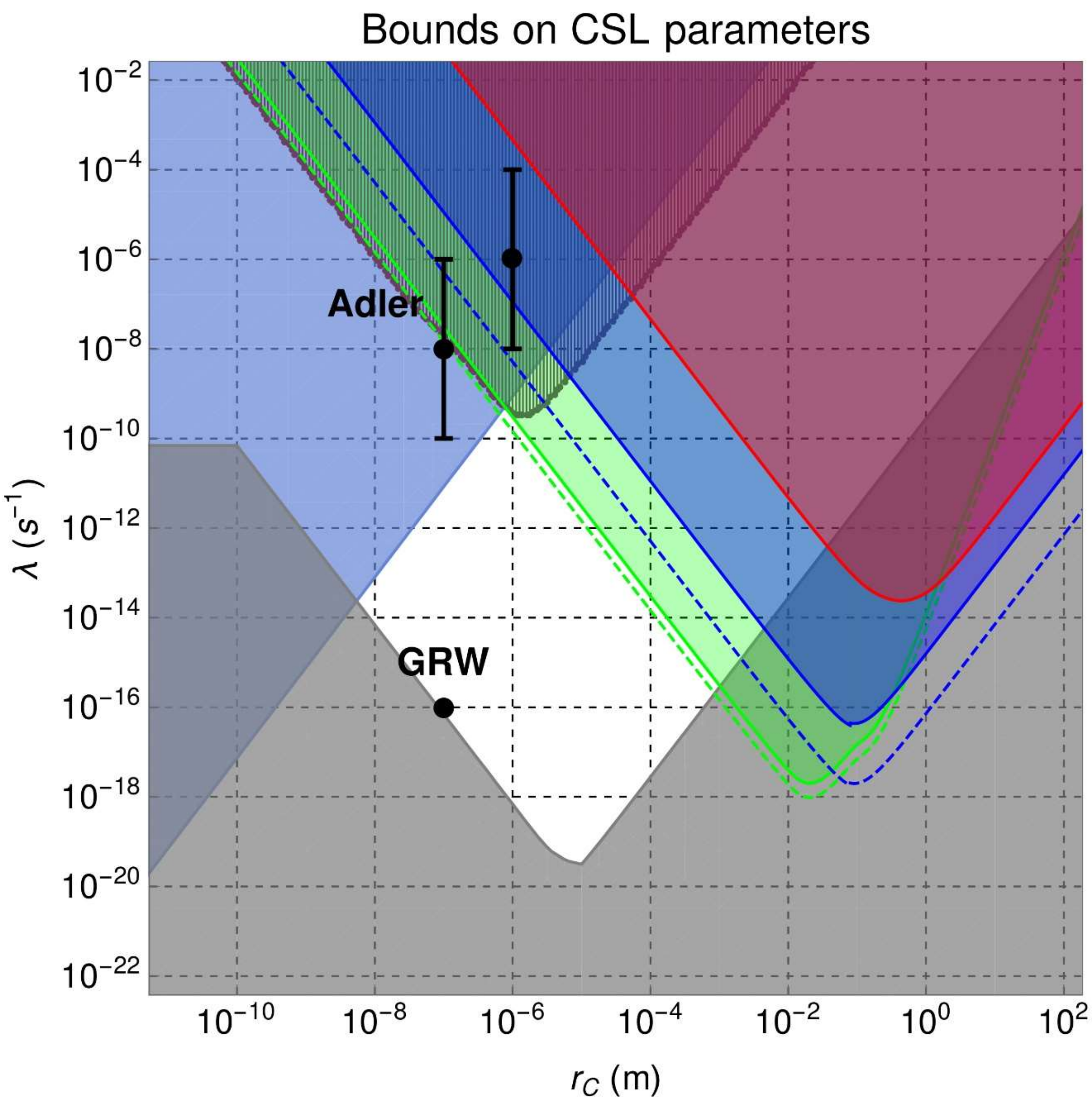


Fig. 1 | Experimental schematic and molecule details. a, The molecular beam is created via nanosecond laser desorption (532 nm, 1 kHz, $I \approx 1\times10^8$ W cm^{-2}), followed by collimation and TOF encoding via a pseudo-random chopper. The beam then enters the interferometer chamber, passing two SiN gratings G_1 and G_3 (266 nm period, 43% open fraction, 160 nm thick) and the optical grating G_2 ($\lambda = 532$ nm, vertical beam waist 690 μm), spaced by $L = 0.98$ m. The third grating shifts transversely across the molecular beam to detect the presence of quantum interference fringes that manifest as a molecular density pattern of period d. The molecules are then ionized by electron impact and are mass-selected and counted in a customized quadrupole mass spectrometer that can resolve masses beyond 1 MDa. **b**, The molecules in this study consist of a tetraphenylmethane core with four zinc-coordinated porphyrin branches. Each branch contains up to 15 fluoroalkylsulfanyl chains. **c**, The MALDI-TOF spectrum of the molecular library after matrix-free desorption. The mass resolution in LUMI during interference experiments was lower to maximize transmission, as discussed in the Methods.



Bounds on CSL parameters
Adler
GRW
$\lambda\ (s^{-1})$
r_C (m)
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}
10^{2}

Outline

- Introduction & Motivations
- Quantum collapse models in brief
- Application to cosmology and perturbation theory
- Conclusions

- According to inflation, the perturbations originate from quantum fluctuations of the gravitational and scalar fields, then amplified by gravitational instability and stretched by cosmic expansion

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$$\hat{H} = \int_{\mathbb{R}^{3+}} \mathrm{d}^3\boldsymbol{k}\,\left[\hat{p}_{\boldsymbol{k}}^2 + \omega^2(k,\eta)\hat{v}_{\boldsymbol{k}}^2\right]$$

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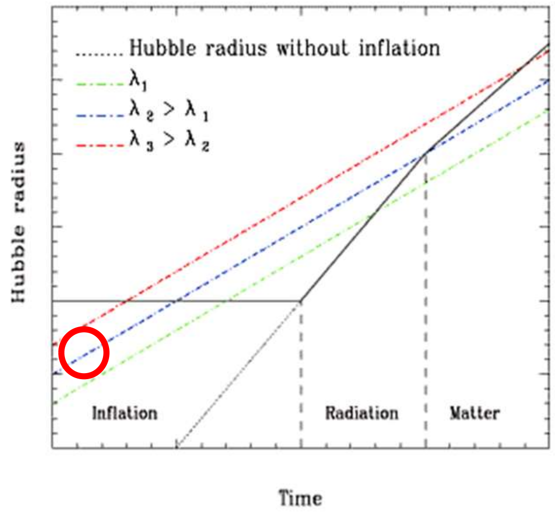
✓ Parametric oscillator

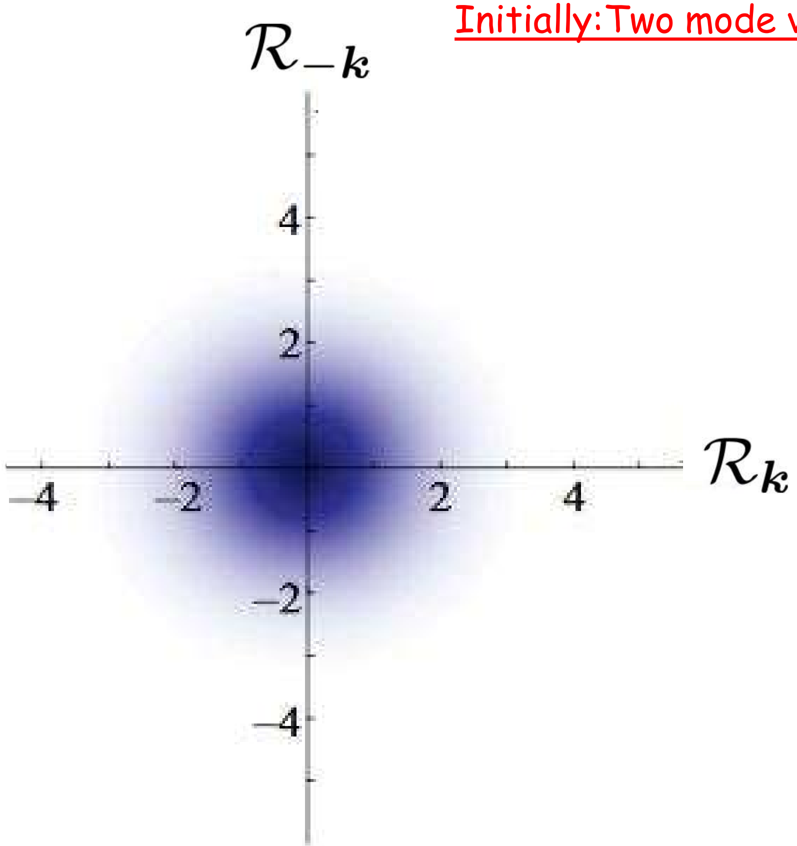
$$v_{\boldsymbol{k}}'' + \omega^2(k,\eta)v_{\boldsymbol{k}} = 0$$

✓ Gaussian squeezed state

$$\Psi[\mathcal{R}] = \prod_{\boldsymbol{k}\in \mathrm{R}^{3+}} \Psi(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}})$$

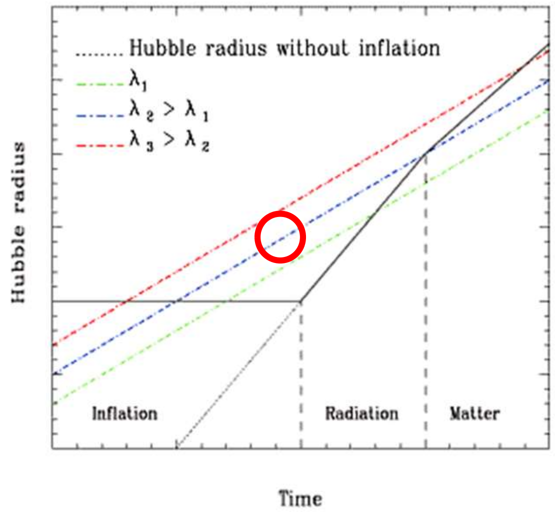


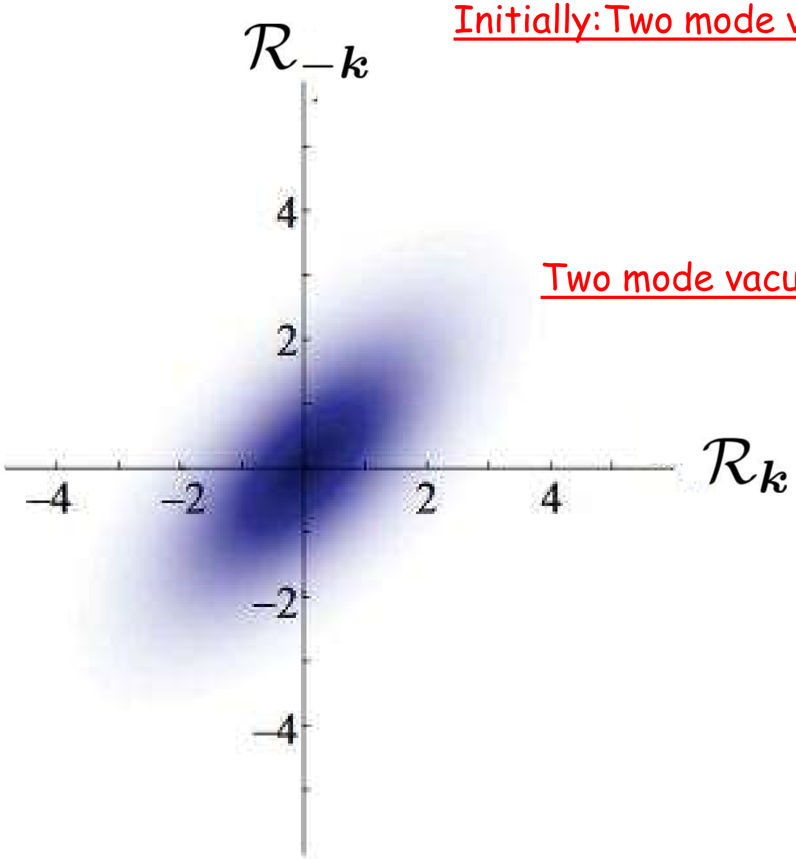
Initially: Two mode vacuum coherent state



$$\Psi_0(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}}) = \frac{1}{\pi^{1/4}} e^{-\mathcal{R}_{\boldsymbol{k}}^2/2 - \mathcal{R}_{-\boldsymbol{k}}^2/2} = \frac{1}{\sqrt{\pi}} e^{-(\mathcal{R}_{\boldsymbol{k}} - \mathcal{R}_{-\boldsymbol{k}})^2/4} e^{-(\mathcal{R}_{\boldsymbol{k}} + \mathcal{R}_{-\boldsymbol{k}})^2/4}$$

$$\Psi[\mathcal{R}] = \prod_{\boldsymbol{k}\in \mathrm{R}^{3+}} \Psi(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}})$$





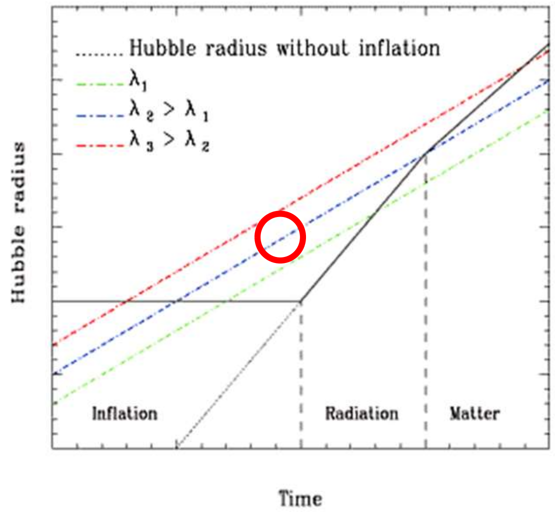
Initially: Two mode vacuum coherent state

↓

Two mode vacuum squeezed state

$$\Psi_R\left(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}}\right) = \frac{1}{\sqrt{\pi}} e^{-(\mathcal{R}_{\boldsymbol{k}}-\mathcal{R}_{-\boldsymbol{k}})^2/(4R^2)} e^{-R^2(\mathcal{R}_{\boldsymbol{k}}+\mathcal{R}_{-\boldsymbol{k}})^2/4}$$

$$\Psi[\mathcal{R}] = \prod_{\boldsymbol{k}\in \mathrm{R}^{3+}} \Psi(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}})$$



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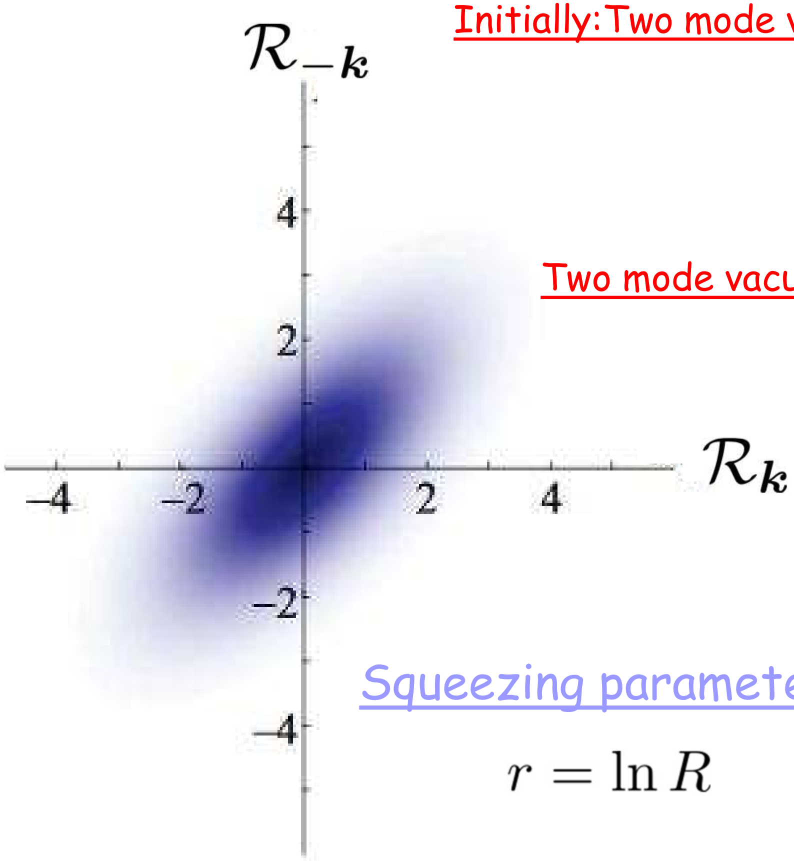
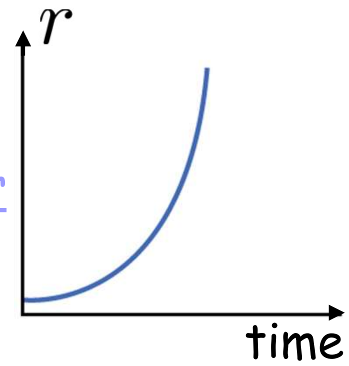


Squeezing parameter

$$r = \ln R$$

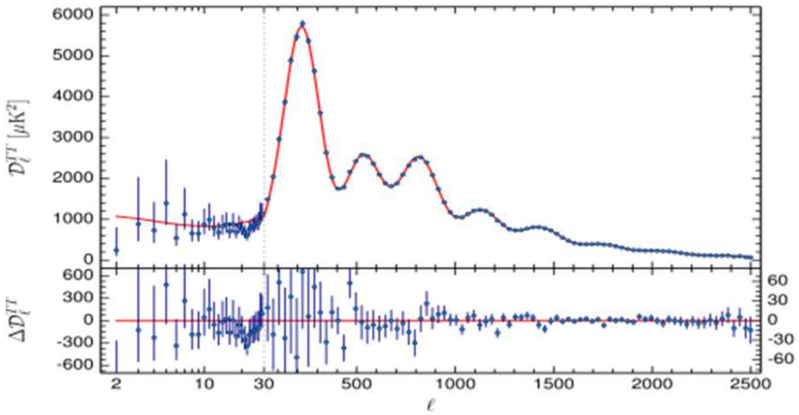


$$\Psi_R\left(\mathcal{R}_{\boldsymbol{k}}, \mathcal{R}_{-\boldsymbol{k}}\right) = \frac{1}{\sqrt{\pi}} e^{-(\mathcal{R}_{\boldsymbol{k}} - \mathcal{R}_{-\boldsymbol{k}})^2/(4R^2)} e^{-R^2(\mathcal{R}_{\boldsymbol{k}} + \mathcal{R}_{-\boldsymbol{k}})^2/4}$$

Planck data

- Universe spatially flat $\Omega_{\mathcal{K}} = -0.040^{+0.038}_{-0.041}$

- Phase coherence



- Adiabatic perturbations $\alpha^{(2,2500)}_{\mathcal{RR}} \in [0.985, 0.999]$

- Gaussian perturbations $f^{\rm loc}_{\rm NL} = 0.8 \pm 5$

- Almost scale invariant power spectrum $n_{\rm S} = 0.9645 \pm 0.0049$

- Background of quantum gravitational waves $r < 0.08$

Single field slow-roll models, with minimal kinetic terms, are preferred

CSL: application to Cosmology & inflation

- No (fully satisfactory) relativistic version of CSL: application to cosmology is necessarily phenomenological …

$$\mathrm{d}|\Psi[v(\boldsymbol{x}_\mathrm{p}),t]\rangle = \left\{-i\hat{H}\mathrm{d}t + \frac{\sqrt{\gamma}}{m_0}\int \mathrm{d}\boldsymbol{x}_\mathrm{p}\left[\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) - \langle\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p})\rangle\right]\mathrm{d}W_t(\boldsymbol{x}_\mathrm{p}) - \frac{\gamma}{2m_0^2}\int \mathrm{d}\boldsymbol{x}_\mathrm{p}\left[\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) - \langle\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p})\rangle\right]^2\mathrm{d}t\right\}|\Psi[v(\boldsymbol{x}_\mathrm{p}),t]\rangle$$

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- Which collapse operator? Energy density: $\hat{\rho} = \bar{\rho} + \widehat{\delta\rho}$

  - $\hat{C} - \langle\hat{C}\rangle \to \widehat{\delta\rho} - \langle\widehat{\delta\rho}\rangle$

  - $\hat{C}_\mathrm{sm}(\boldsymbol{x}_\mathrm{p}) = \frac{1}{(2\pi)^{3/2}r_\mathrm{c}^3}\int \mathrm{d}\boldsymbol{y}_\mathrm{p}\,\hat{\rho}(\boldsymbol{x}_\mathrm{p}+\boldsymbol{y}_\mathrm{p})\,e^{-|\boldsymbol{y}_\mathrm{p}|^2/(2r_\mathrm{c}^2)}$

CSL: application to Cosmology & inflation

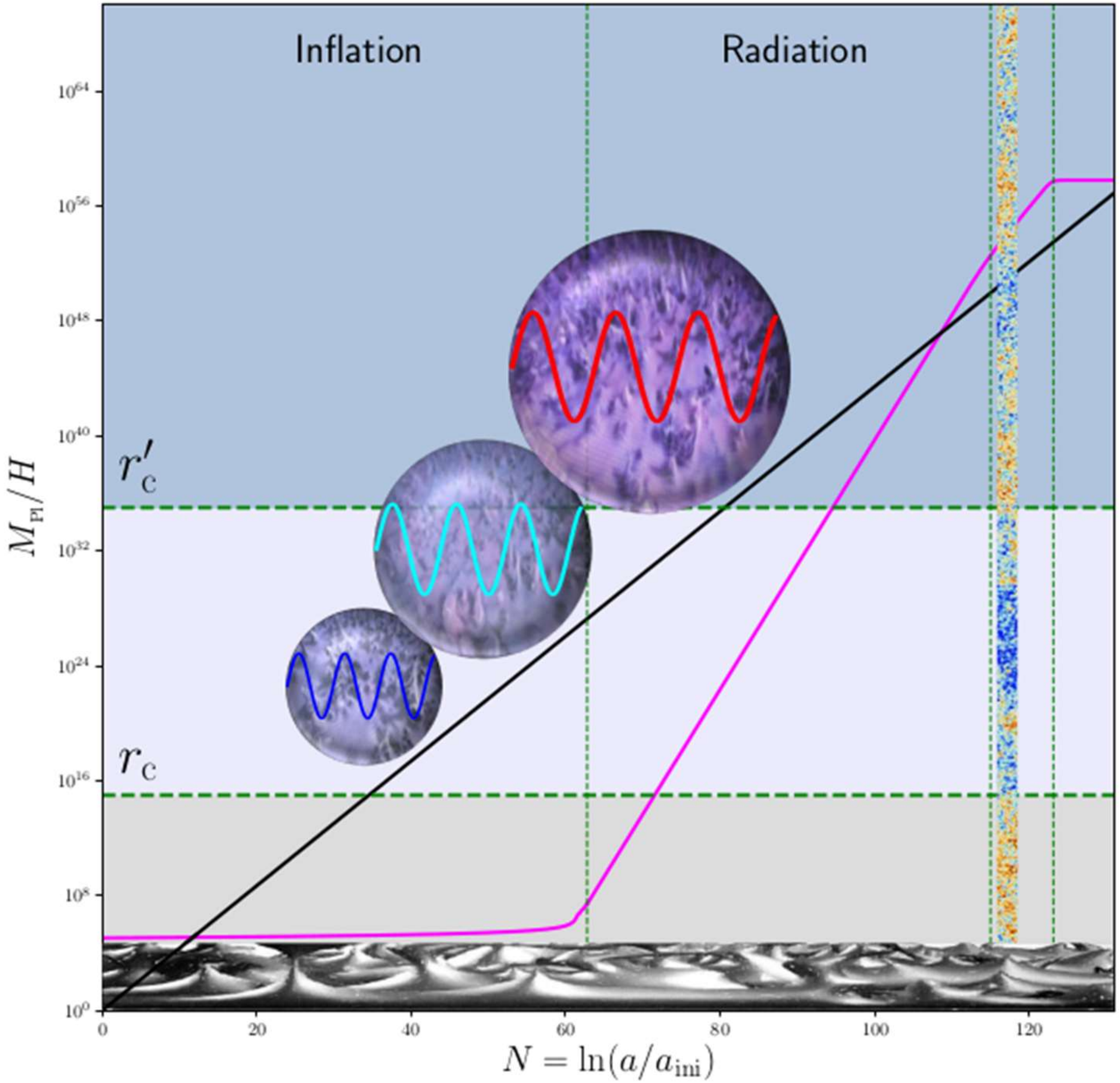
- In Fourier space, this leads to

$$\mathrm{d}|\Psi[v_{\boldsymbol{k}},t]\rangle = \left\{-iH_{\boldsymbol{k}}\mathrm{d}t + \frac{\sqrt{\gamma a^3}}{m_0}\left[\hat{C}_{\mathrm{sm}}(\boldsymbol{k}) - \langle\hat{C}_{\mathrm{sm}}(\boldsymbol{k})\rangle\right]\mathrm{d}W_t(\boldsymbol{k}) - \frac{\gamma a^3}{2m_0^2}\left[\hat{C}_{\mathrm{sm}}(\boldsymbol{k}) - \langle\hat{C}_{\mathrm{sm}}(\boldsymbol{k})\rangle\right]^2\mathrm{d}t\right\}|\Psi[v_{\boldsymbol{k}},t]\rangle$$

$$\hat{C}_{\mathrm{sm}}(\boldsymbol{k}) = 3M_{\mathrm{Pl}}^2\frac{\mathcal{H}^2}{a^2}\underbrace{e^{-k^2r_{\mathrm{c}}^2/(2a^2)}}\frac{\widehat{\delta\rho}}{\bar{\rho}}(\boldsymbol{k})$$

- Cosmological amplification mechanism: the collapse operator is effective only if $\lambda \gg r_{\mathrm{c}}$

- Btw: no problem with the initial conditions!


Inflation
Radiation
10^{64}
10^{56}
10^{48}
10^{40}
10^{32}
10^{24}
10^{16}
10^{8}
10^{0}
r_c'
r_c
$M_{\rm Pl}/H$
0
20
40
60
80
100
120
$N = \ln(a/a_{\rm ini})$

CSL: application to Cosmology & inflation

- Which energy density contrast??

$$\delta_{\mathrm{p}} \propto \delta_{\mathrm{g}} \left(\frac{k}{aH}\right)^{p}$$

with 0<p<2

- p=0, Newtonian density contrast $$\delta_{\mathrm{g}} = \delta + \frac{\rho'}{\rho}(B - E')$$

- p=2, Flat threading, density contrast $$\delta_{\mathrm{m}} = \delta + \frac{\rho'}{\rho}(v + B)$$

Stochastic evolution of the cosmological wavefunction:

- Gaussian state:

$$\Psi(v_{\boldsymbol{k}},\eta) = |N_{\boldsymbol{k}}(\eta)| \exp\left\{-\Re\Omega_{\boldsymbol{k}}\left[v_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}}(\eta)\right]^2 + i\sigma_{\boldsymbol{k}}(\eta) + i\chi_{\boldsymbol{k}}(\eta)v_{\boldsymbol{k}} - i\Im\Omega_{\boldsymbol{k}}v_{\boldsymbol{k}}^2\right\}$$

- $\Omega_{\boldsymbol{k}}(\eta)$, $\bar{v}_{\boldsymbol{k}}(\eta)$, $\sigma_{\boldsymbol{k}}(\eta)$, $\chi_{\boldsymbol{k}}(\eta)$ — Random variables

- Mean value $\langle \hat{v}_{\boldsymbol{k}} \rangle = \bar{v}_{\boldsymbol{k}}(\eta)$

- Variance $\left\langle (\hat{v}_{\boldsymbol{k}} - \bar{v}_{\boldsymbol{k}})^2 \right\rangle = (4\Re\Omega_{\boldsymbol{k}})^{-1}$

Stochastic evolution of the cosmological wavefunction:

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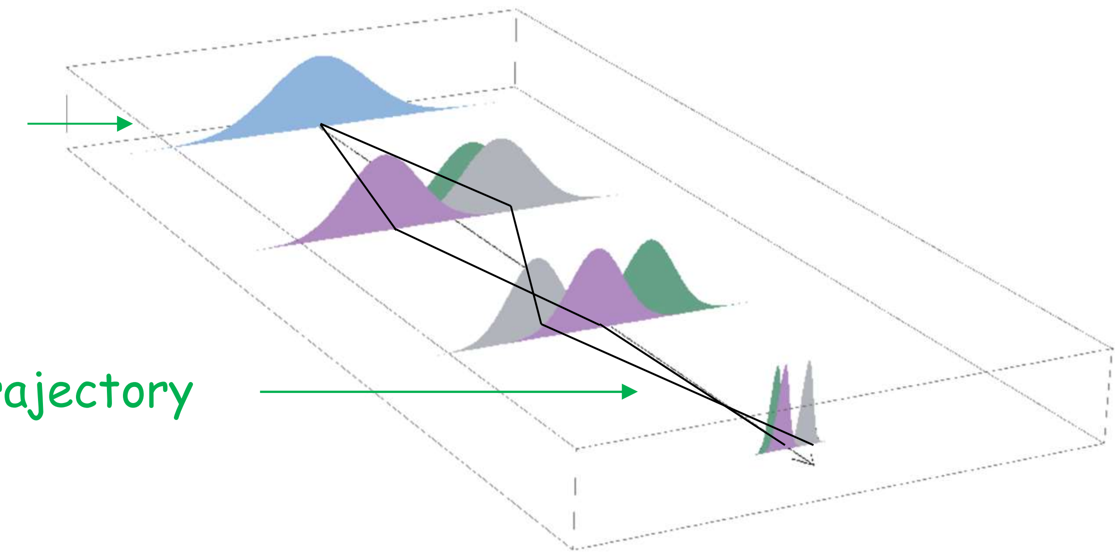
Initial state

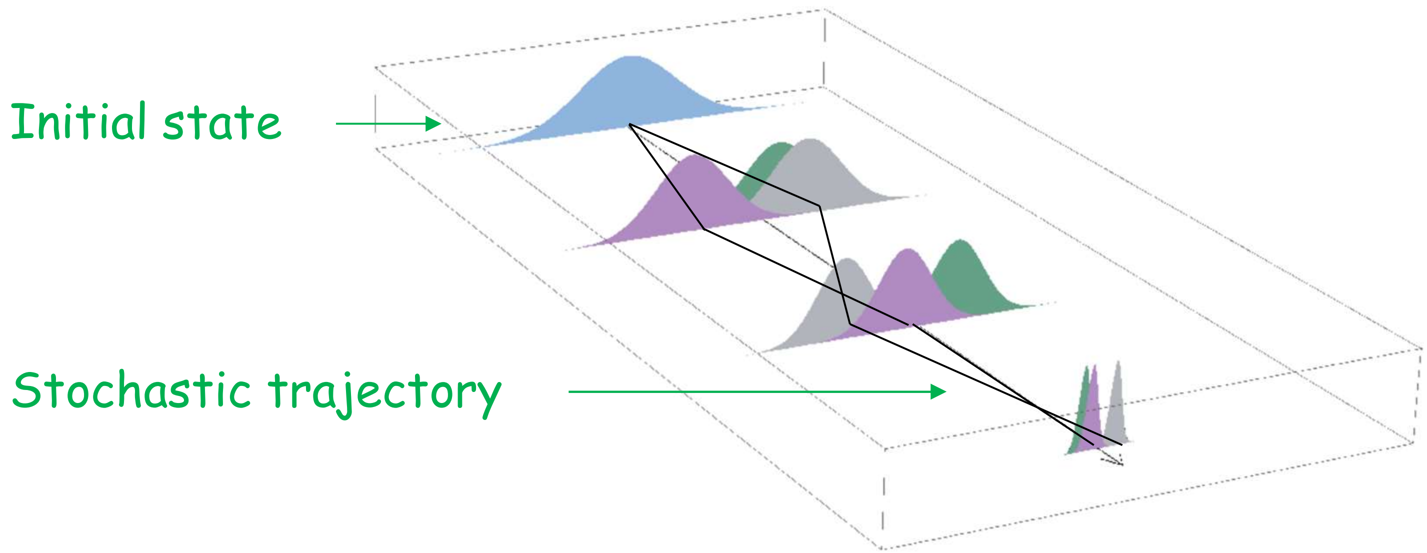
Stochastic trajectory



Stochastic evolution of the cosmological wavefunction:

Two requirements

- ✓ The CSL power spectrum is scale-invariant
- ✓ The wavefunction collapses

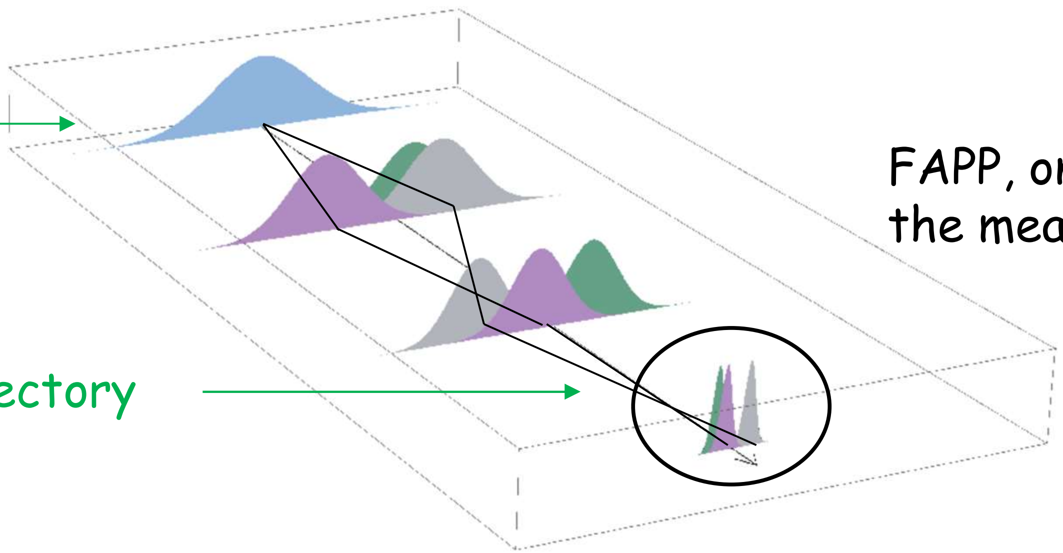


Stochastic evolution of the cosmological wavefunction:

✓ The CSL power spectrum is scale-invariant

$$\mathcal{P}_v \propto \mathbb{E}\left(\langle \hat{v}_{\boldsymbol{k}} \rangle^2\right) - \mathbb{E}^2\left(\langle \hat{v}_{\boldsymbol{k}} \rangle\right)$$

Initial state



Stochastic trajectory

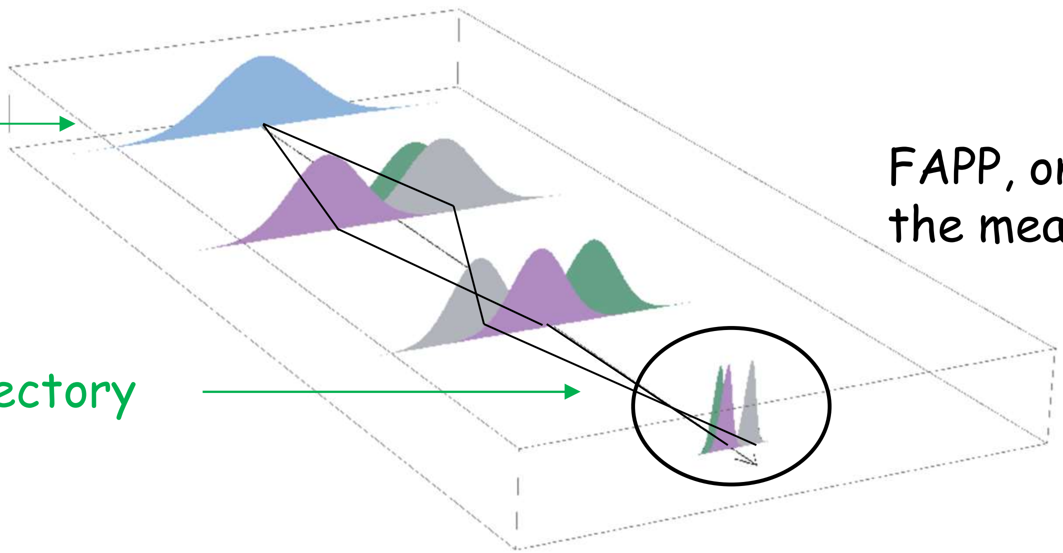
FAPP, one measures the mean values

Stochastic evolution of the cosmological wavefunction:

✓ The CSL power spectrum is scale-invariant

$$\mathcal{P}_v(k) \simeq \mathcal{P}_v(k)|_{\rm std} \begin{cases} 1 + 448\dfrac{\gamma}{m_0^2} M_{\rm Pl}^2 H^2 \epsilon_1 \left(\dfrac{k}{aH}\right)_{\rm end}^{-1}, & r_c \text{ crossed during inflation} \\ 1 + \dfrac{35408}{143}\dfrac{\gamma}{m_0^2} M_{\rm Pl}^2 H^2 \epsilon_1 \left(\dfrac{r_{\rm c}}{\ell_{\rm H}}\right)_{\rm end}^{-9} \left(\dfrac{k}{aH}\right)_{\rm end}^{-10}, & r_c \text{ crossed during radiation epoch} \end{cases}$$

Initial state



Stochastic trajectory

FAPP, one measures the mean values

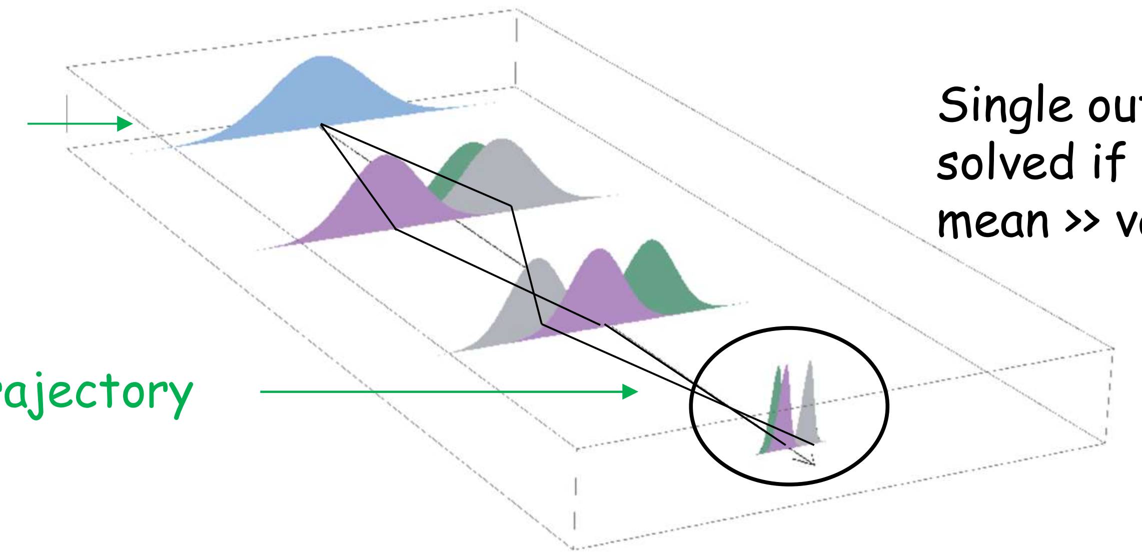
Stochastic evolution of the cosmological wavefunction:

✓ The wavefunction collapses

$$R = \frac{\mathbb{E}\left[\left\langle\left(\hat{v}_{\boldsymbol{k}} - \langle\hat{v}_{\boldsymbol{k}}\rangle\right)^2\right\rangle\right]}{\mathbb{E}\left(\langle\hat{v}_{\boldsymbol{k}}\rangle^2\right)} \ll 1$$

Initial state

Stochastic trajectory



Single outcome problem solved if dispersion of the mean >> variance

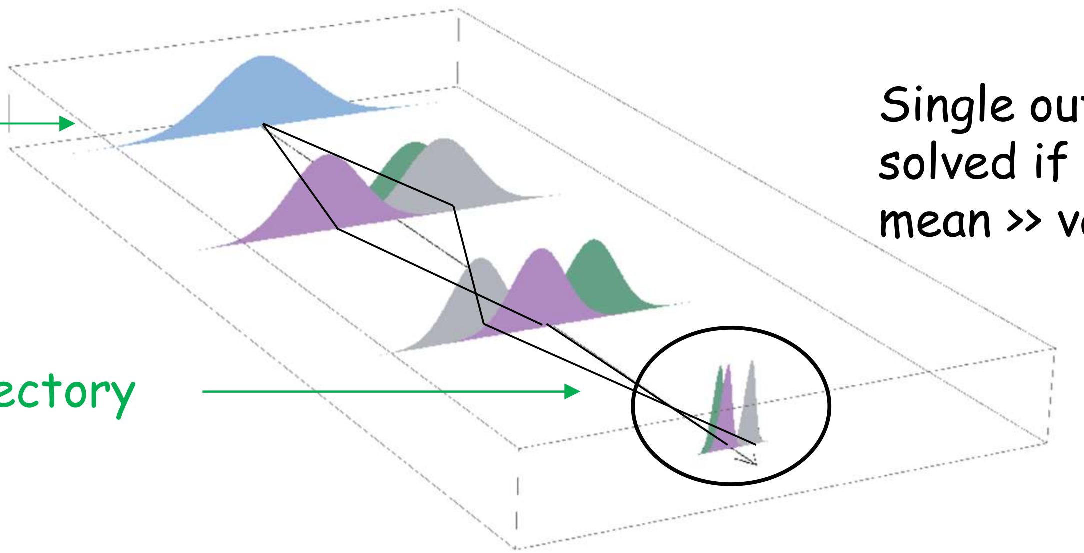
Stochastic evolution of the cosmological wavefunction:

✓ The wavefunction collapses

$$R(k) = \begin{cases} \dfrac{1}{1+3456\dfrac{\gamma}{m_0^2}M_{\rm Pl}^2H^2\left(\dfrac{k}{aH}\right)_{\rm end}^{-7}} + \mathcal{O}(\gamma) & r_c \text{ crossed during inflation} \\ \dfrac{1}{1+\dfrac{21792}{11}\dfrac{\gamma}{m_0^2}M_{\rm Pl}^2H^2\left(\dfrac{r_{\rm c}}{\ell_{\rm H}}\right)^{-7}\left(\dfrac{k}{aH}\right)^{-14}} + \mathcal{O}(\gamma) & r_c \text{ crossed during radiation epoch} \end{cases}$$

Initial state

Stochastic trajectory



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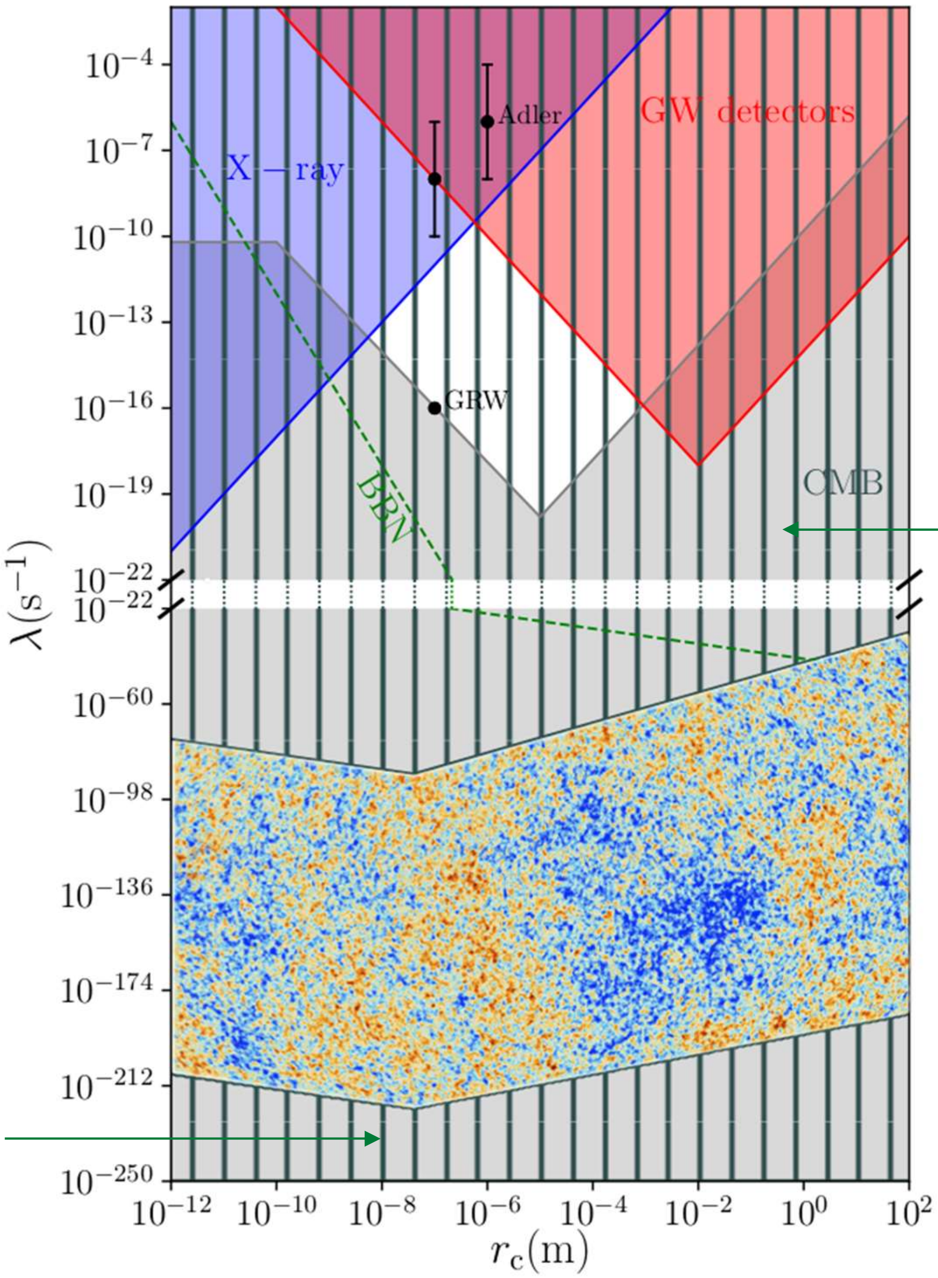

Power spectrum is not scale-invariant

The wavefunction does not collapse

Additional results

- Particular case: if the flat threading density contrast is used then the contraints become compatible with those of lab experiments

- One can show that $\bar{v}_{\boldsymbol{k}}$ exactly follows a Gaussian statistics

- Other collapse operators related to stress energy tensor lead to (almost) the same result

- Non linear collapse operators do not collapse the wave function and/or do not lead to Gaussian statistics: flat threading density contrast seems to be the "good" collapse operator for cosmology

Recap

- Quantum collapse models (e.g. CSL) are falsfiable alternatives to QM
- Cosmology can help QM because these models can be constrained by inflation
- Collapse models can help Cosmology because they can be used in order to understand puzzling issues of inflation
- CSL with a flat threading density contrast as collapse operator seems to be a good model. Any other choice faces severe difficulties.
- Take away message: inflation is not only a successful scenario of the early Universe, it is also a very interesting playground for foundational issues of quantum mechanics