An Order-Unity Correction to Hawking Radiation

Éanna Flanagan, Cornell

Copernicus Online Seminar
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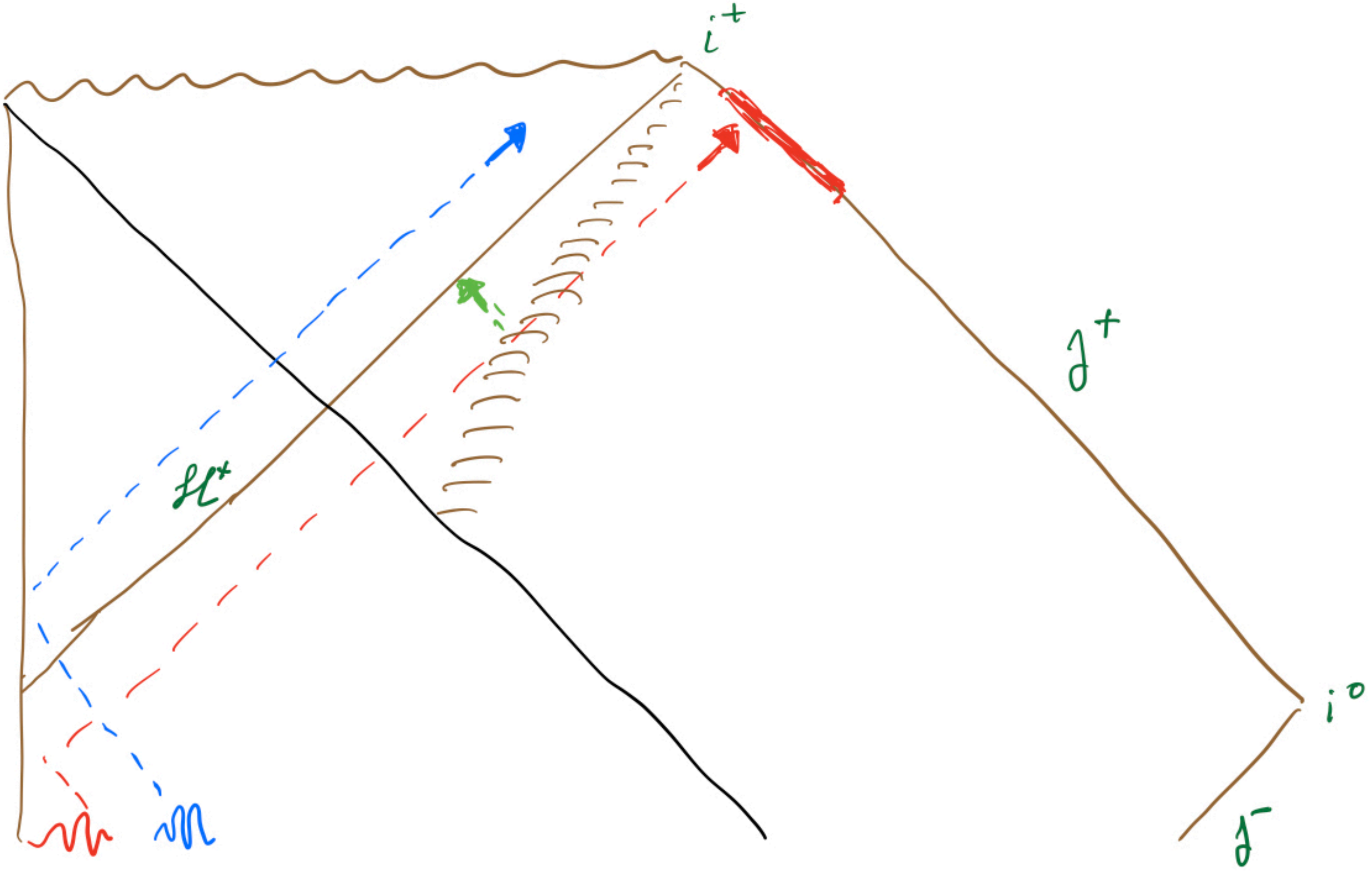
Hawking Radiation



Progress on the Black Hole Information Puzzle

★ Celebrated advances, many indirect

- AdS | CFT → Unitarity
- State counting → Bekenstein Hawking entropy
- Euclidean Path integrals → Page curve
- Soft hair

★ This talk: a new modification to Hawking radiation: large, elementary, innocuous?

Outline

1. Description of new effect
2. Evolution of fluctuations in black hole charges
3. Effect on Hawking Radiation
4. Implications

Context

- General relativity in 3+1 dimensions, $\Lambda = 0$
- Schwarzschild black hole, massless scalar field
- Energy scales:

Classifying Corrections to Hawking Radiation

- Instantaneous or secularly growing
- Does it modify entanglement entropy?
- Size of correction!

Correction $\Delta\rho$ to density matrix, algebra $\mathcal{A}$ of operators

$$\varepsilon \equiv \max_{A \in \mathcal{A}} \frac{\mathrm{tr}[\Delta\rho\, A]}{\mathrm{tr}[\rho A]}$$

Two cases:

$\varepsilon_{n\sim M^2}$, $\varepsilon_{n \ll M^2}$

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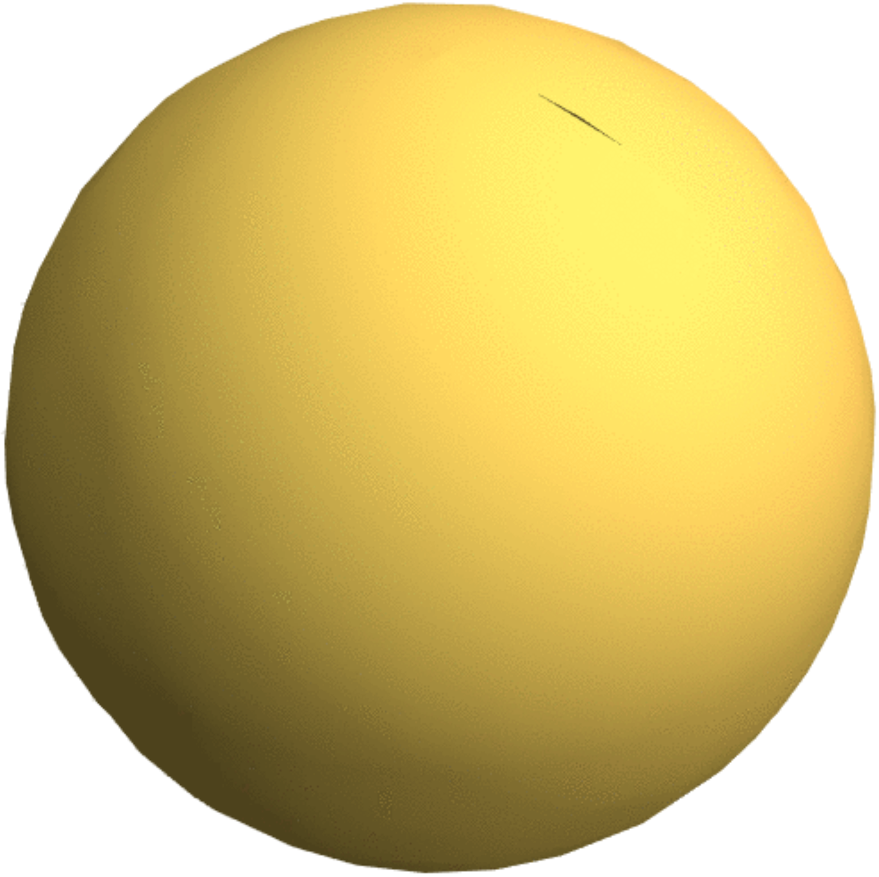
$\epsilon_{n\sim M^2}$

$\epsilon_{n\ll M^2}$

	$O(e^{-1/M^2})$	$O(M^{-p})$	$O(1)$
$O(e^{-\frac{1}{M^2}})$	✓	✓	generally expected
$O(M^{-p})$	▨	✓	
$O(1)$	▨	▨	new

The Standard Prediction at $\mathscr{I}^+$

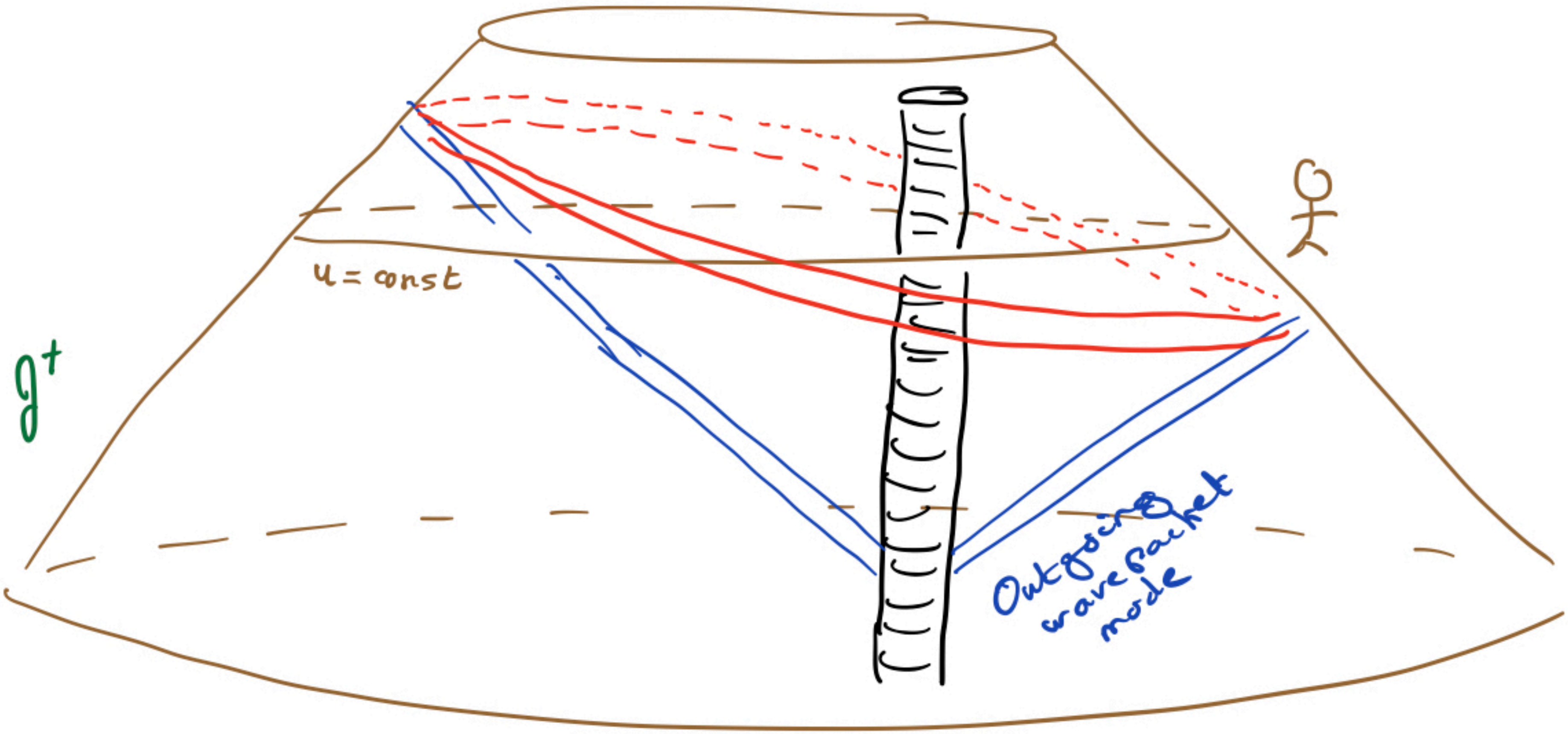


The Standard Prediction at $\mathscr{I}^+$

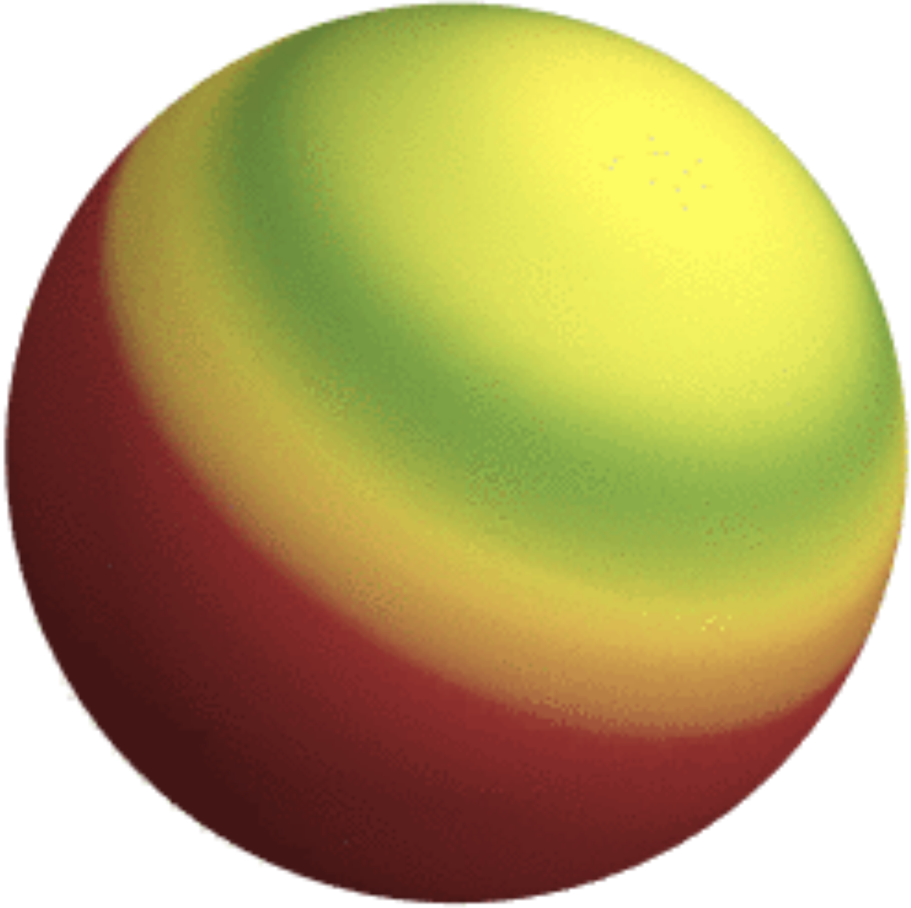
- Radiation characterized by a single timescale $R_s \sim GM$ and angular scale $\Delta\theta \sim 1$

- Follows from dimensional analysis, computed in the limit $G \to 0$ with $R_s = GM$ fixed

A Transversely Displaced Black Hole



A Transversely Displaced Black Hole



Corrected Unruh State at Future Null Infinity

- Combining this effect with spreading of wavefunction of black hole center of mass (Page, 1980) gives a state which is stationary, spherically symmetric, non Gaussian

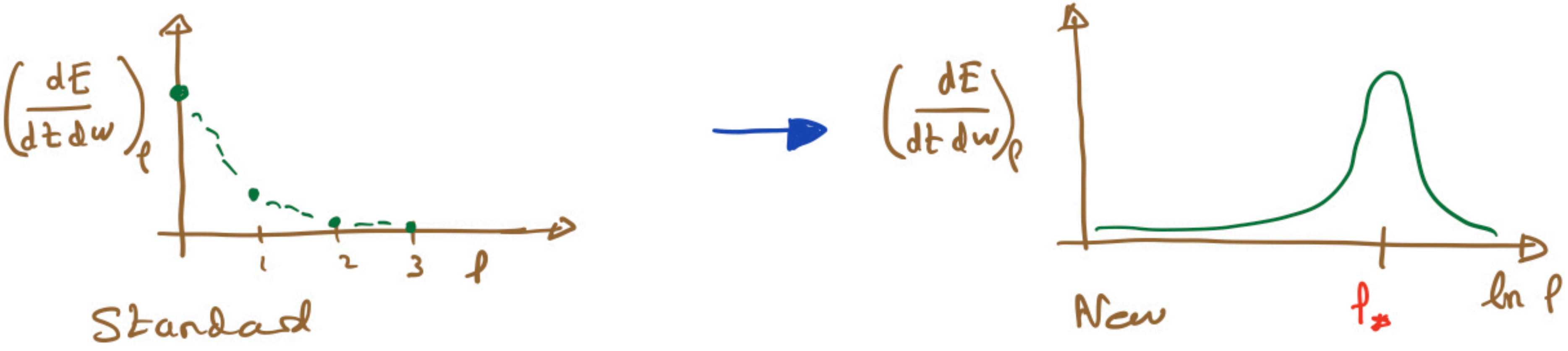
- Definitions: $\Phi(u, r, \underset{\sim}{\theta}) = \frac{1}{r}\varphi(u, \underset{\sim}{\theta}) + O\left(\frac{1}{r^2}\right)$

$$G(\Delta u, \gamma) = \left\langle \varphi(u, \underset{\sim}{\theta})\, \varphi(u+\Delta u, \underset{\sim}{\theta}') \right\rangle - {}_{out}\left\langle 0 \right| \varphi(u, \underset{\sim}{\theta})\, \varphi(u+\Delta u, \underset{\sim}{\theta}') \left| 0 \right\rangle_{out}$$

$$= \int d\omega\, e^{i\omega \Delta u} \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2\pi} P_\ell(\cos\gamma)\, \tilde{G}(\omega, \ell)$$

$$\tilde{G}(\omega, \ell) = \frac{2\pi}{\omega^2(2\ell+1)} \left(\frac{dE}{dt\, d\omega}\right)_\ell$$

Corrected Unruh State

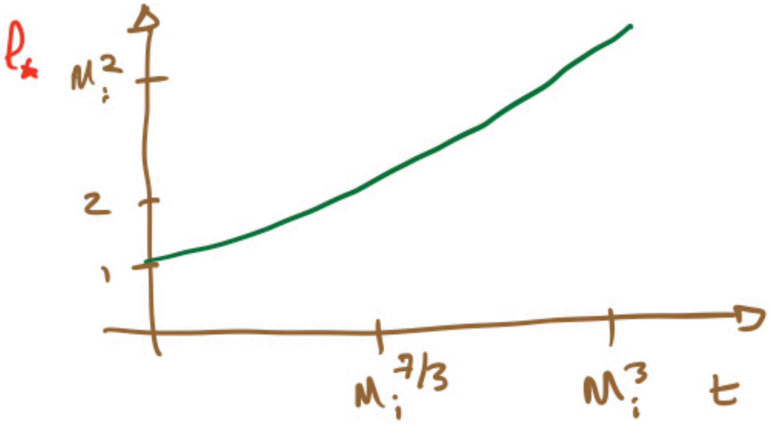


★ Characteristic angular scale

$$\ell_*^2 \sim \overbrace{1}^{\text{Standard}} + \overbrace{\frac{\hbar^2 t^3}{G^5 M_i^7}}^{\text{new}}$$

$$\sim 1 + \frac{G M_i^2}{\hbar}$$

after evaporation time



Outline

1. Description of new effect ✓
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4. Implications

Basis of Effect: Gravitational Backreaction

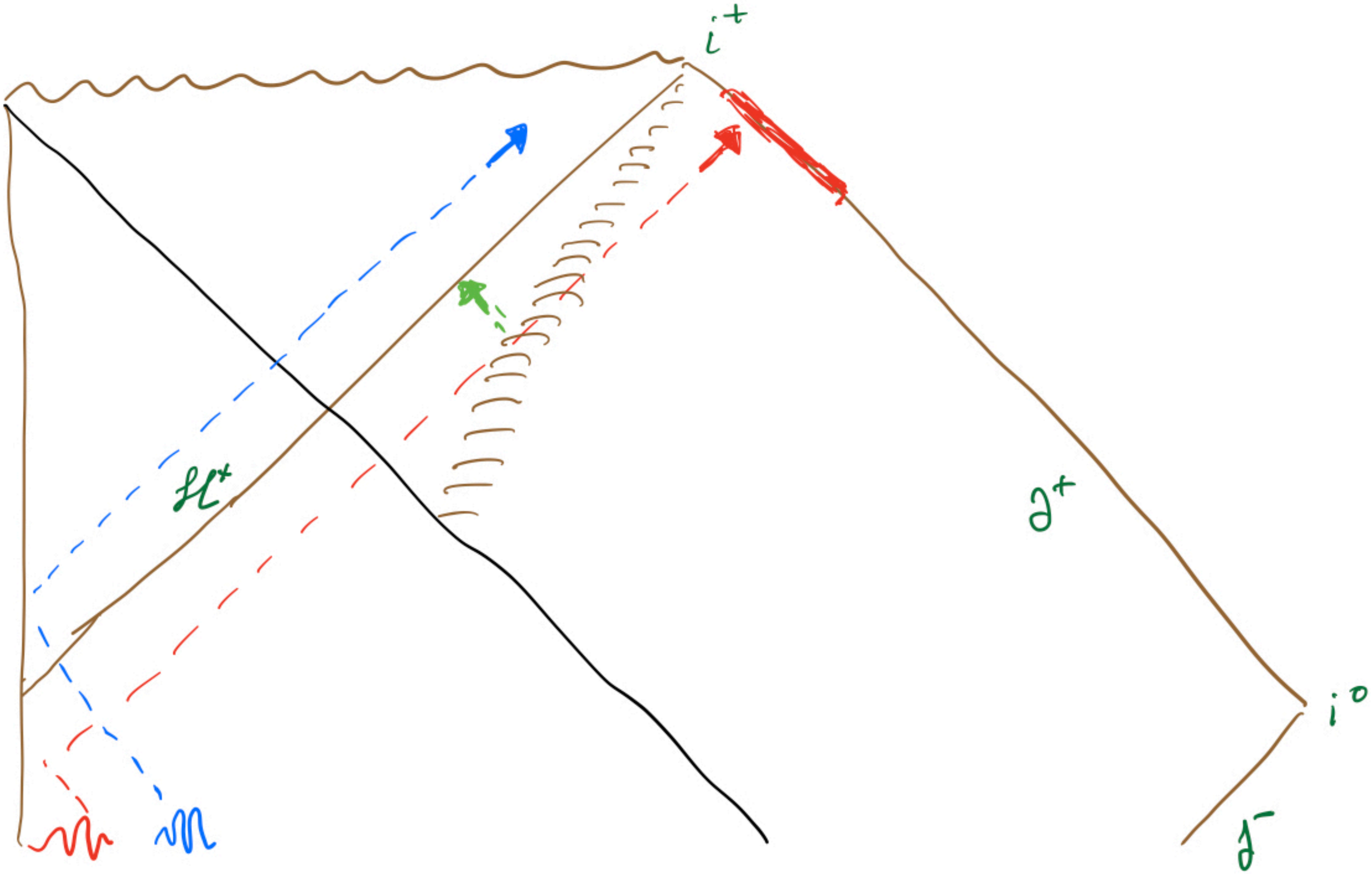
(Page, 1980)

- Effects small but can accumulate secularly for example $M(t)^3 = M_i^3 - t$, and spin evolution

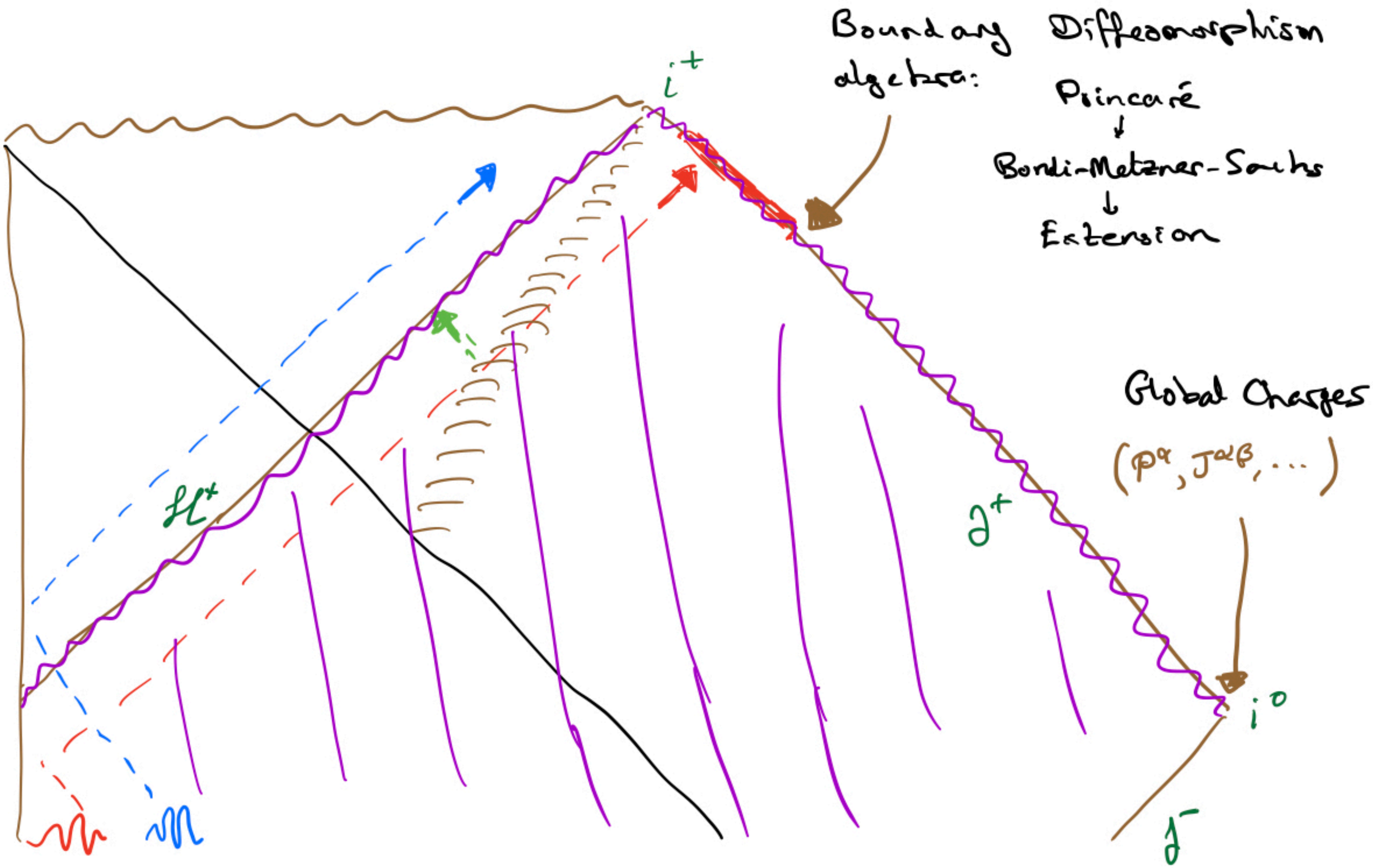
- Are there other similar effects? Not within semiclassical theory

- Idea: augment semiclassical theory by quantizing a small number of infrared degrees of freedom, whose variances evolve secularly

Boundary Symmetries and Charges



Boundary Symmetries and Charges



Boundary Diffeomorphism algebra:

Poincaré

↓

Bondi-Metzner-Sachs

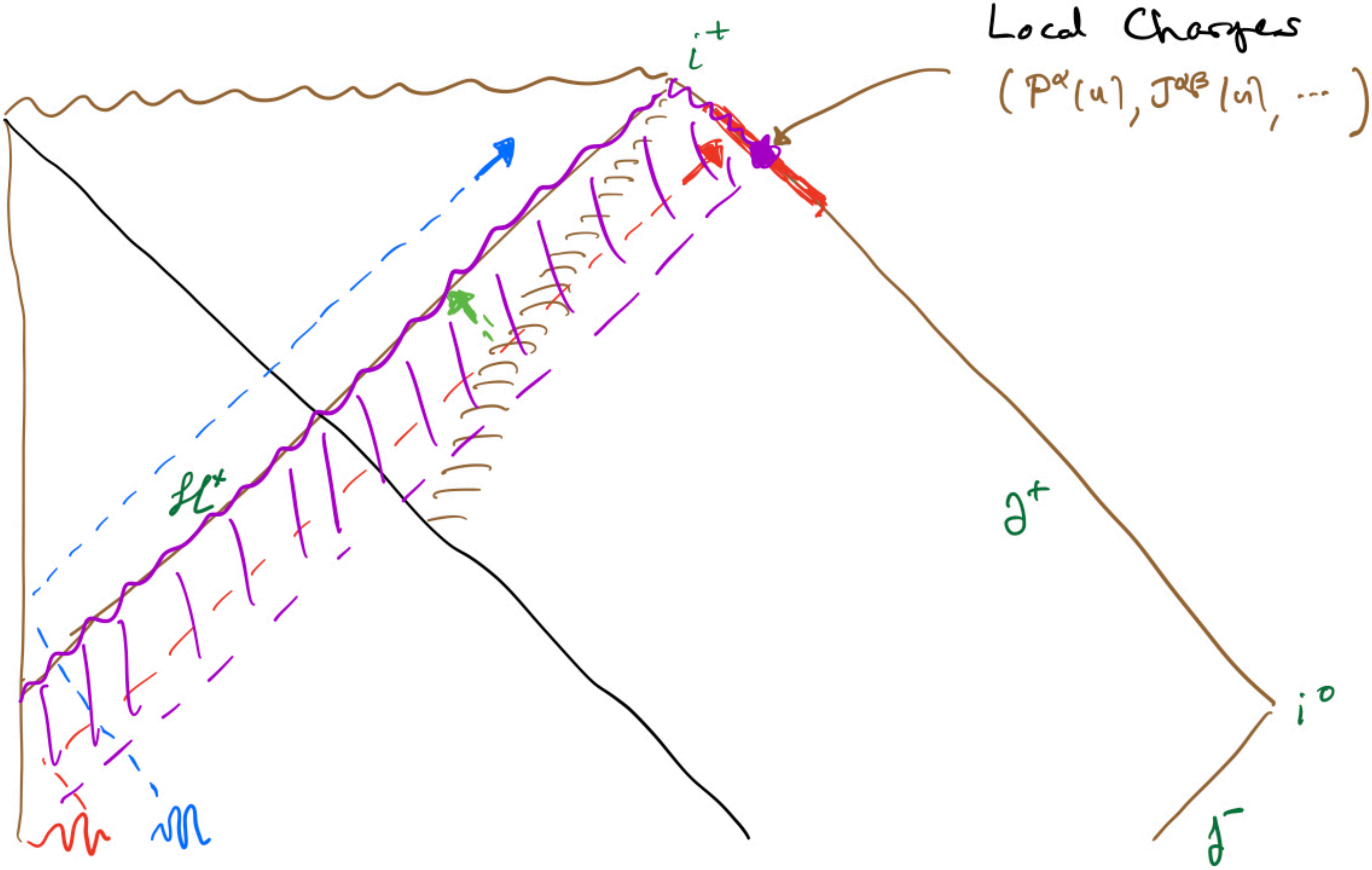
↓

Extension

Global Charges

$(P^\alpha, J^{\alpha\beta}, \dots)$

Boundary Symmetries and Charges



Local Charges

$(P^\alpha(u), J^{\alpha\beta}(u), \cdots)$

Simple Evolution Model

- Idealize as a sequence of transitions, one per quantum
- Einsteins equations impose

$$P^{\alpha}_{n+1} = P^{\alpha}_{n} - \Delta P^{\alpha}_{n}$$

$$J^{\alpha\beta}_{n+1} = J^{\alpha\beta}_{n} - \underbrace{\Delta J^{\alpha\beta}_{n}}_{\text{make order of magnitude estimates}}$$

- Evolution:

$$M_{n+1} = M_n - \delta_n M_n^{-1}$$

$$p_{n+1} = p_n + \epsilon_n \delta_n M_n^{-1}$$

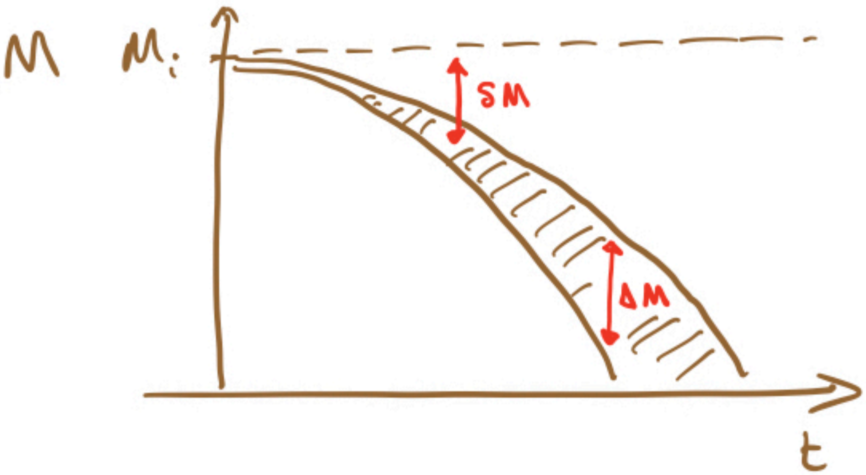
$$t_{n+1} = t_n + M_n$$

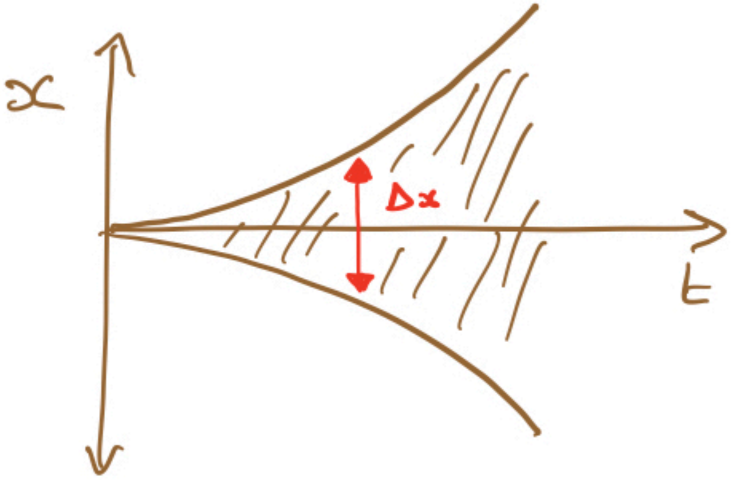
$$x_{n+1} = x_n + \frac{p_n}{M_n}\left(t_{n+1} - t_n\right)$$

$$\delta_n = \begin{cases} 0 & \text{no emission} \\ 1 & \text{emission} \end{cases}$$

$$\epsilon_n = \begin{cases} -1 & \text{left} \\ +1 & \text{right} \end{cases}$$

Results of Evolution Model





- Two regimes:

  $\delta M \ll M_i$ (early)

  $M \ll M_i$ (late)

- Results:

$$\Delta x \sim \begin{cases} \delta M^{3/2} M_i^{1/2} \\ M_i^2 \end{cases}$$

- $\Delta M \sim M$ when $M \lesssim M_i^{2/3}$

- Two approximation methods:

  $\delta M \ll M_i$ (Page)

  $\Delta M \ll M$

$$\Delta M \sim \begin{cases} \sqrt{\delta M / M_i} \\ M_i^2 / M^2 \end{cases}$$

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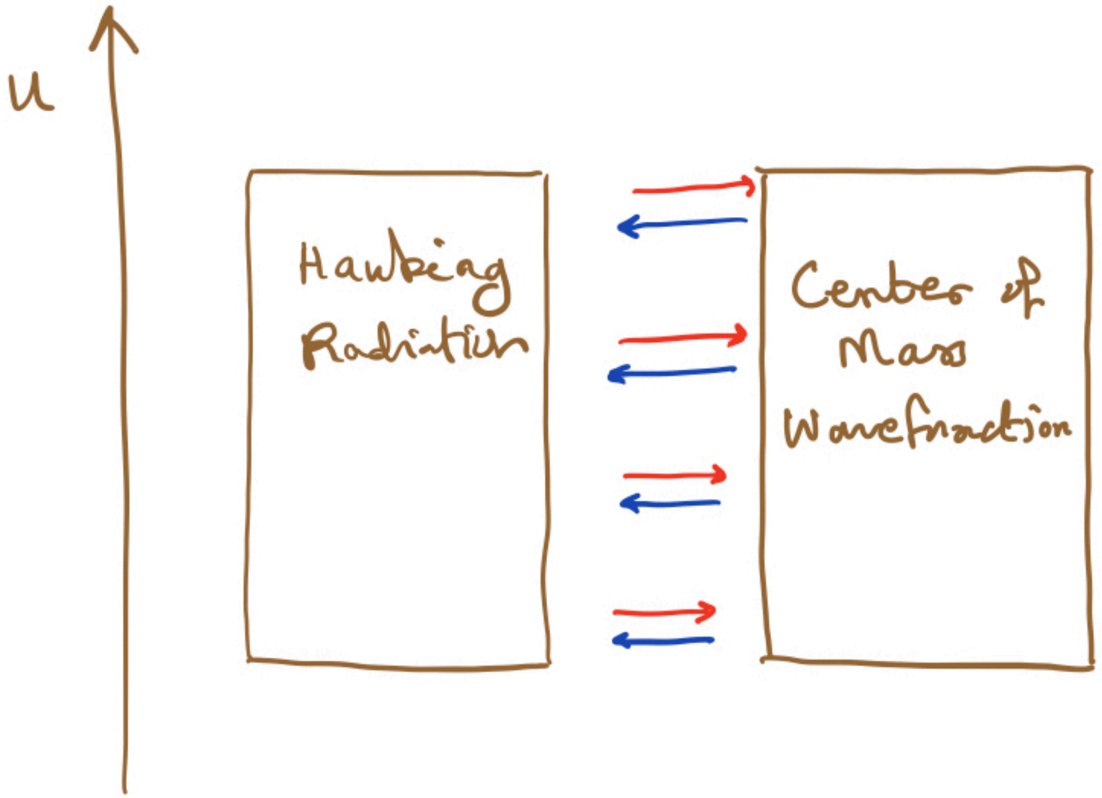
Effect of Center of Mass Motion on Hawking Radiation

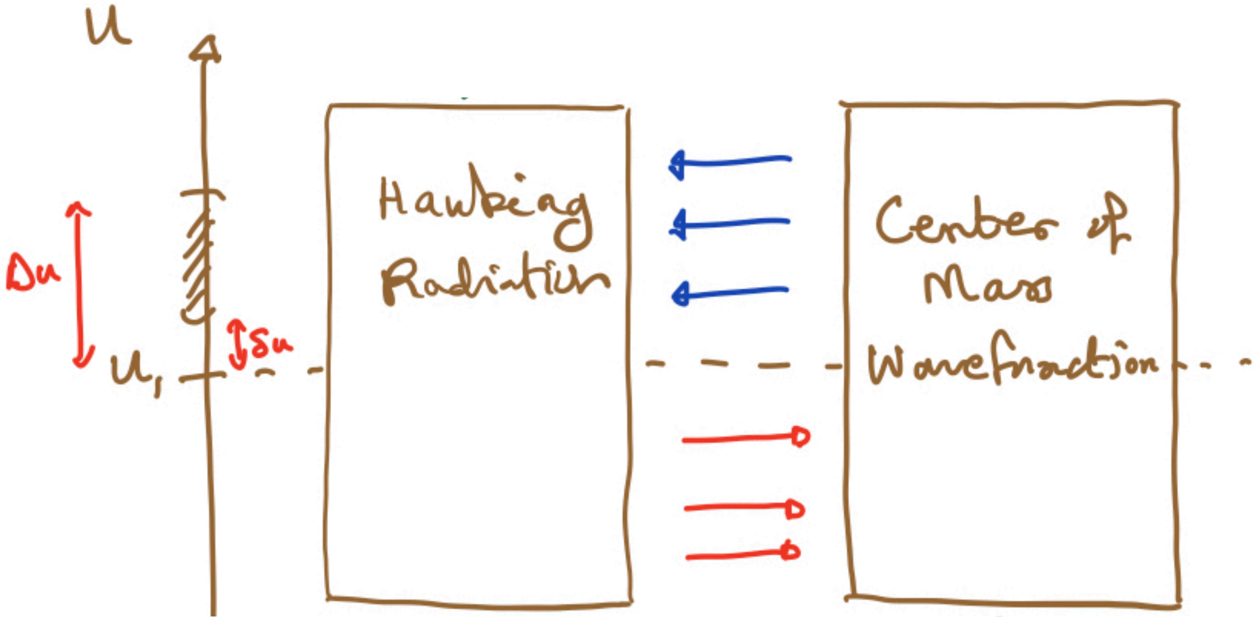


Effect of Center of Mass Motion on Hawking Radiation

Require

$\delta u \gg M_i^2$

$\Delta u \ll M^3$



Effect of Center of Mass Motion on Hawking Radiation

State is $\rho_{corr} = \int d^3\Delta \, \tilde{W}(\Delta,0) \, U_\Delta \, \rho \, U_\Delta^\dagger$

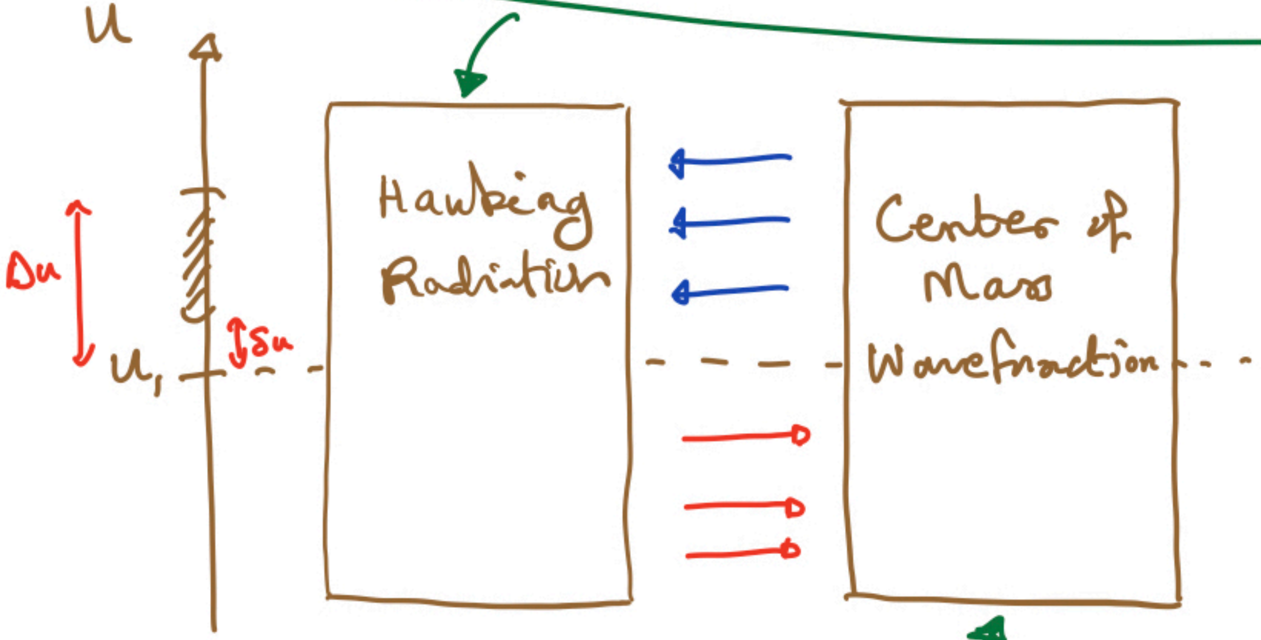
Unruh state

Displacement operator

Require

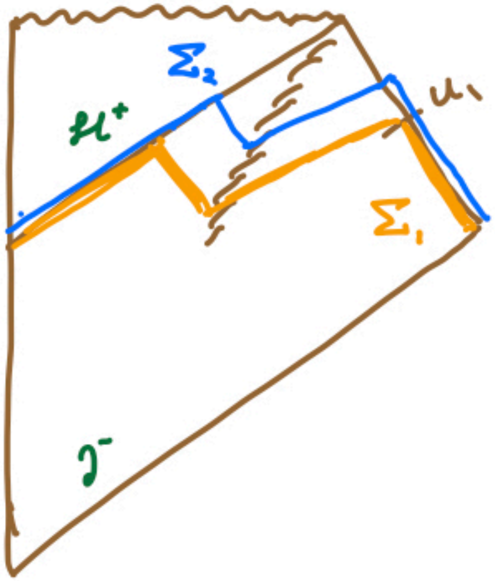
$\delta u \gg M_i^2$

$\Delta u \ll M^3$



State at $u = u_1$ is $\int d^3\Delta \int d^3\xi \, \underbrace{\tilde{W}(\Delta,\xi)}_{\text{Wigner function, Gaussian}} \, |\Delta - \xi/2\rangle \underbrace{\langle \Delta + \xi/2|}_{\text{Position eigenstate}}$

Derivation



With no interaction:

$$|\psi\rangle_{\mathcal{J}^-} = |\Delta\rangle\,|0\rangle_{\mathcal{J}^-}$$

$$\rightarrow \quad |\psi\rangle_{\Sigma_1} = |\Delta\rangle \sum_j |j\rangle_{\mathcal{H}^+}\, U_\Delta\, |j\rangle_{\mathcal{J}^+}$$

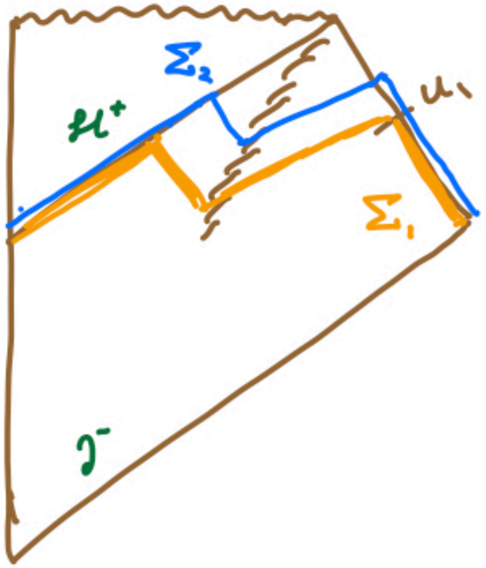
Trace over horizon:

$$\rightarrow \quad |\Delta\rangle\langle\Delta| \;\; U_\Delta\, \underbrace{\rho}_{\text{Unruh state}}\, U_\Delta^\dagger$$

$$\rho = \sum_j c_j^2\, |j\rangle_{\mathcal{J}^+}\; {}_{\mathcal{J}^+}\langle j|,$$

$$c_j^2 = {}_{\mathcal{H}^+}\langle j|j\rangle_{\mathcal{H}^+}$$

Derivation with interaction



$$|\psi\rangle_{\mathcal{I}^-} = |0\rangle|0\rangle$$

$$\rightarrow |\psi\rangle_{\Sigma_1} = \int d^3\Delta \, |\Delta\rangle \sum_{\substack{j \\ \text{early}}} |j\rangle_{\mathcal{H}^+} |j\rangle_{\mathcal{I}^+} f_j(\Delta)$$

$$\rightarrow |\psi\rangle_{\Sigma_2} = \int d^3\Delta \, |\Delta\rangle \sum_{\substack{j \\ \text{early}}} |j\rangle_{\mathcal{H}^+} |j\rangle_{\mathcal{I}^+} f_j(\Delta)$$

$$\otimes \sum_{\substack{\alpha \\ \text{late}}} |\alpha\rangle_{\mathcal{H}^+} U_\Delta |\alpha\rangle_{\mathcal{I}^+}$$

Trace over early modes and horizon $\rightarrow$

$$\int d^3\Delta \int d^3\xi \, \tilde{W}(\Delta,\xi) \, |\Delta-\xi/2\rangle\langle\Delta+\xi/2| \; U_{\Delta-\xi/2} \, \rho \, U^\dagger_{\Delta+\xi/2}$$

Trace over center of mass $\rightarrow$

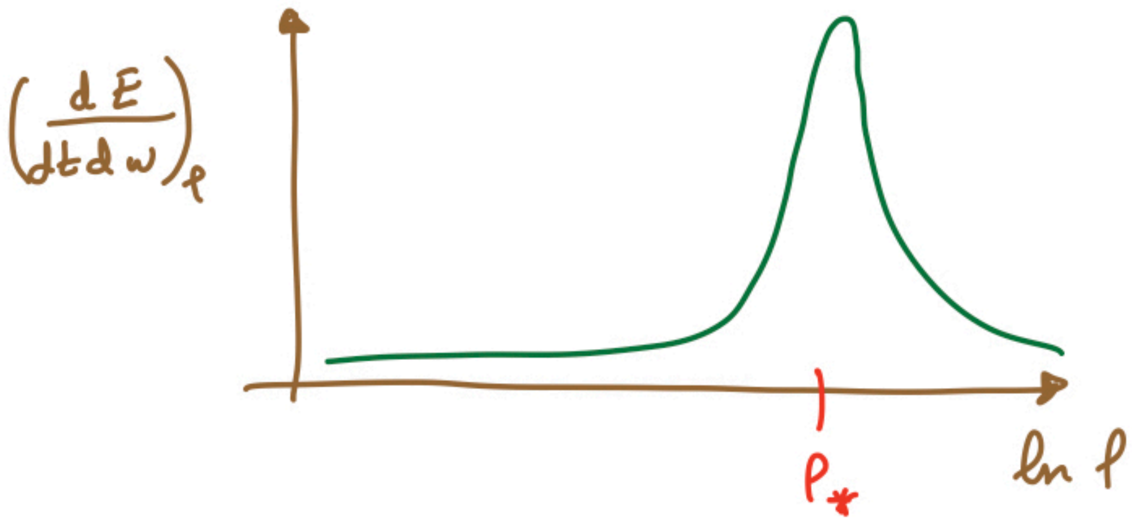
$$\int d^3\Delta \, \tilde{W}(\Delta,0) \, |\Delta\rangle\langle\Delta| \; U_\Delta \, \rho \, U^\dagger_\Delta$$

Angular Spectrum of Corrected Unruh State

$$G_{corr}(\Delta u, \gamma) = \int d^3\Delta \, \underbrace{\tilde{\mathcal{D}}(\Delta, 0)}_{\text{Gaussian with width } \sigma_\Delta} \, G(\Delta u + (\underline{n}-\underline{n}')\cdot\underline{\Delta}, \gamma)$$

$$\rightarrow \quad \tilde{G}_{corr}(\omega, \gamma) = \exp\left[-2\omega^2 \sigma_\Delta^2 \sin^2(\gamma/2)\right] \underbrace{G(\omega, \gamma)}_{\simeq \, G(\omega, 0)}$$

→



$$\ell_* \sim \omega \, \sigma_\Delta$$

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Discussion and Implications

- Entanglement entropy unchanged, mutual information generated
- A parallel effect involving supertranslations is negligible
- Pitfalls of thinking in terms of a single semiclassical spacetime: corrected state extrapolated to horizon is Planckian

- Black holes posess higher-ℓ analogs of center-of-mass, soft hair.
  Strominger suggested that Hawking radiation can be purified by entanglement with soft hair. Are there sufficient accessible degrees of freedom?

$$\Phi(u_0,\theta) \sim \int^{u_0} du \; \varphi_{,u}(u,\underset{\sim}{\theta})^2$$

Hawking Radiation