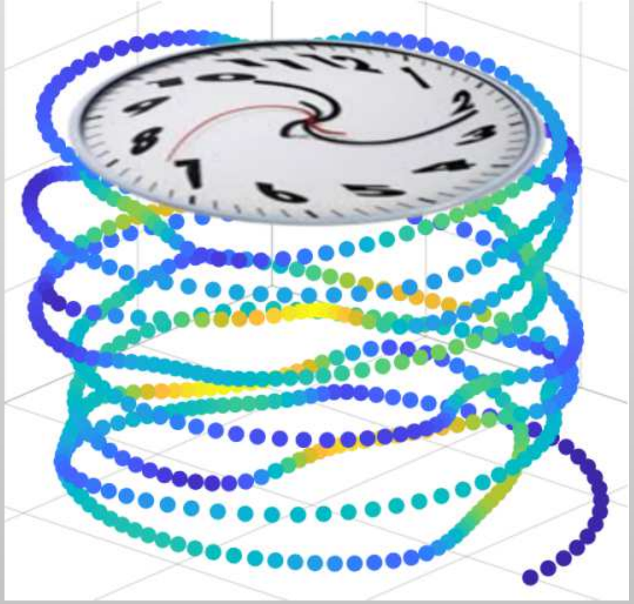
Physical implications of a fundamental period of time

Martin Bojowald

Department of Physics, The Pennsylvania State University



G. Wendel, L. Martínez, MB: PRL 124 (2020) 241301, M. Amaral, MB: Ann. Phys. 388C (2018) 241
MB, A. Tsobanjan, Commun. Math. Phys. to appear

Time

Intuitive experience of time
complicates fundamental understanding.

- → Experience directed monotonic flow. “Time”
- → Measured by periodic processes. “Clock”

Conventional labeling of events by time (and date) reflects this dichotomy.

Is fundamental time monotonic or periodic?

Important in cosmology: Long evolution, high energy initially.

Relativity and quantum mechanics

Relativity:

→ Time coordinate can be transformed.

→ Any specific choice may have to be restricted to a finite range on curved space-time.

Quantum mechanics:

→ Infinite range of time assumed in unitarity requirement, $\exp(-iEt/\hbar)$ valid for all real t.

→ No time operator.

Fundamental time

"The Lagrangian for any dynamical system can be made to satisfy the condition for homogeneous velocities by taking the time t to be an extra coordinate q_0 and using the equation $\dot{q}_0 = 1$ to make the Lagrangian homogeneous of the first degree in all the velocities, including q_0."

P. DIRAC, CAN. J. PHYS. 2 (1950) 129

Relational evolution:
Usual degrees of freedom $q_1, \ldots$ evolve with respect to other (clock) degree of freedom q_0 replacing time coordinate t.

Requiring $\dot{q}_0 = \mathrm{d}q_0/\mathrm{d}t = 1$ assumes monotonic clock.

Some formalism

Position x with momentum p.

Time ϕ (q_0) with momentum $-E$, energy.

Energy equation $E = H(x, p, \phi)$ with Hamiltonian H imposes constraint

$$C = -E + H(x, p, \phi) = 0$$

Only one independent variable.

Hamilton's equation

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\partial C}{\partial(-E)} = 1$$

implies Dirac's identification of time t with clock variable ϕ.

Cosmology

Friedmann equation $(\dot{a}/a)^2 = \frac{8\pi G}{3}\rho$ in canonical form:

$V = a^3$ with momentum $p_V = -\dot{a}/(4\pi G a)$

scalar field ϕ with momentum p_ϕ

$$\begin{aligned} C &= V\rho(\phi, p_\phi) - 6\pi G V p_V^2 \\ &= \frac{1}{2}\frac{p_\phi^2}{V} + \frac{1}{2}V m^2\phi^2 - 6\pi G V p_V^2 \\ &= \left(-p_\phi + H(V, p_V, \phi)\right)\left(p_\phi + H(V, p_V, \phi)\right) \end{aligned}$$

with

$$H(V, p_V, \phi) = \sqrt{12\pi G V^2 (P_V^2 - m^2\phi^2)}$$

Simplifies for $m = 0$ (“deparameterization”), but not realistic as fundamental field: no mass or self-interactions.

Spectator clocks

Massive fields that may be used for imprints of $a(t)$ in cosmological observations. [X. Chen: JCAP 1201 (2012) 038]

[X. Chen, M. H. Namjoo and Y. Wang: JCAP 1602 (2016) 013]

[X. Chen, A. Loeb, Z.-Z. Xianyu: PRL 122 (2019) 121301]

Here:

- → Massive field as fundamental clock, mass may be close to Planckian.
- → Fundamental nature of clock implies constraint. Dynamics different compared with massive field in background time.
- → Different cosmological regimes, perturbative inhomogeneity remain to be studied.
- → New effect on coherence properties of wave function.

Quantization

Introduce operators $\hat{x}$, $\hat{p}$, $\hat{\phi}$ and $\hat{E}$ such that

$$[\hat{x}, \hat{p}] = i\hbar \quad \text{and} \quad [\hat{\phi}, \hat{E}] = -i\hbar$$

and impose quantum constraint

$$\hat{C}\psi(x,\phi) = -\hat{E}\psi(x,\phi) + \hat{H}\psi(x,\phi) = 0$$

on wave functions $\psi(x,\phi)$.

Equivalent to Schrödinger equation if $\hat{E} = i\hbar\partial/\partial\phi$ is used.

Position x and clock ϕ still different conceptually:

- → $\hat{C}$ linear in $\hat{E}$, quadratic in $\hat{p}$ for standard $\hat{H}$
- → ϕ doesn't fluctuate, implication of constraint $\hat{C}$ [MB, Tsobanjan]

Relativity

Relativistic energy equation, modeled on $-E^2 + p^2 = -m^2$:

$$C' = -E^2 + H^2 = 0$$

Hamilton's equations for $(\phi, -E)$:

$$\begin{aligned} \frac{\mathrm{d}\phi}{\mathrm{d}t} &= \frac{\partial C'}{\partial(-E)} = 2E \\ \frac{\mathrm{d}E}{\mathrm{d}t} &= \frac{\partial C'}{\partial \phi} = 2H\frac{\partial H}{\partial \phi} \end{aligned}$$

$\mathrm{d}\phi/\mathrm{d}t$ not constant if H depends on ϕ.

→ $E = \pm H$ may be positive or negative as solution of $C' = 0$.

→ Oscillating $\phi(t)$ possible if Hamiltonian time-dependent.

→ Relativity seems to prefer periodic clocks.

Quantization

1. Time-independent Hamiltonian, $[\hat{E}, \hat{H}] = 0$ in $\hat{C}' = -\hat{E}^2 + \hat{H}^2$:

$$(-\hat{E}^2 + \hat{H}^2)\psi = (-\hat{E} + \hat{H})(\hat{E} + \hat{H})\psi = (\hat{E} + \hat{H})(-\hat{E} + \hat{H})\psi = 0$$

$\hat{E}\psi = i\hbar\partial\psi/\partial\phi = \pm\hat{H}\psi$, two Schrödinger equations.

2. Time-dependent Hamiltonian:

$$(-\hat{E} + \hat{H})(\hat{E} + \hat{H}) = -\hat{E}^2 - [\hat{E}, \hat{H}] + \hat{H}^2 = -\hat{E}^2 + \hat{H}^2 + i\hbar\widehat{\partial H/\partial\phi}$$

$$\neq \quad (\hat{E} + \hat{H})(-\hat{E} + \hat{H}) = -\hat{E}^2 + [\hat{E}, \hat{H}] + \hat{H}^2 = -\hat{E}^2 + \hat{H}^2 - i\hbar\widehat{\partial H/\partial\phi}$$

Cannot impose both $\hat{E}\psi = \hat{H}\psi$ and $\hat{E}\psi = -\hat{H}\psi$ based on a single $\hat{C}'$. Periodic clock impossible?

The problem of time as a Gribov problem

[Amaral, MB]

Choosing ϕ as time, such that $\phi = t$ with a real parameter t, is gauge fixing:

$$\{\phi - t, C'\} = 2E \neq 0$$

as long as $E \neq 0$. ($C' = -E^2 + H^2$)

→ If H is ϕ-independent, $\{E, C'\} = 0$ and E is constant.
Stays non-zero if initially non-zero.

→ If H depends on ϕ, E is not constant, goes through zero as ϕ oscillates.
$\phi - t$ no longer fixes gauge when $E = 0$: Gribov horizon.

Path integral quantization of gauge theories:
Do not cross Gribov horizons in order to avoid overcounting degrees of freedom. $\mathrm{sgn} E$ fixed.

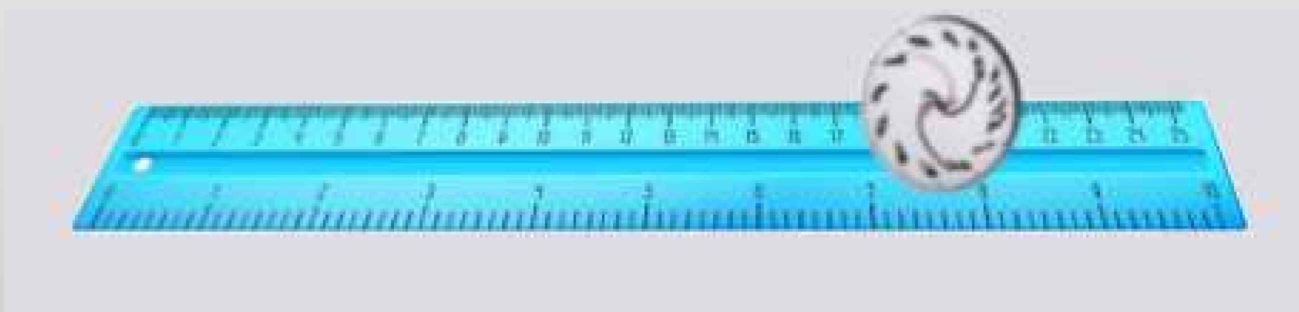
How can ϕ oscillate?

Global evolution

$\mathrm{d}\phi/\mathrm{d}t = 1$ for $C = -E + H = 0$ presupposes direction of ϕ.

More general: $\phi = \tau + A_+$ or $\phi = -\tau + A_-$ at different times.

Periodic clock ϕ, monotonic time-and-date τ.



Schrödinger equation with respect to τ:

$$i\hbar \frac{\partial \psi}{\partial \tau} = i\hbar \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \frac{\partial \psi}{\partial \phi} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \hat{H} \psi$$

Main example

$$C = -E^2 - \lambda^2\phi^2 + H(x,p)^2 = 0$$

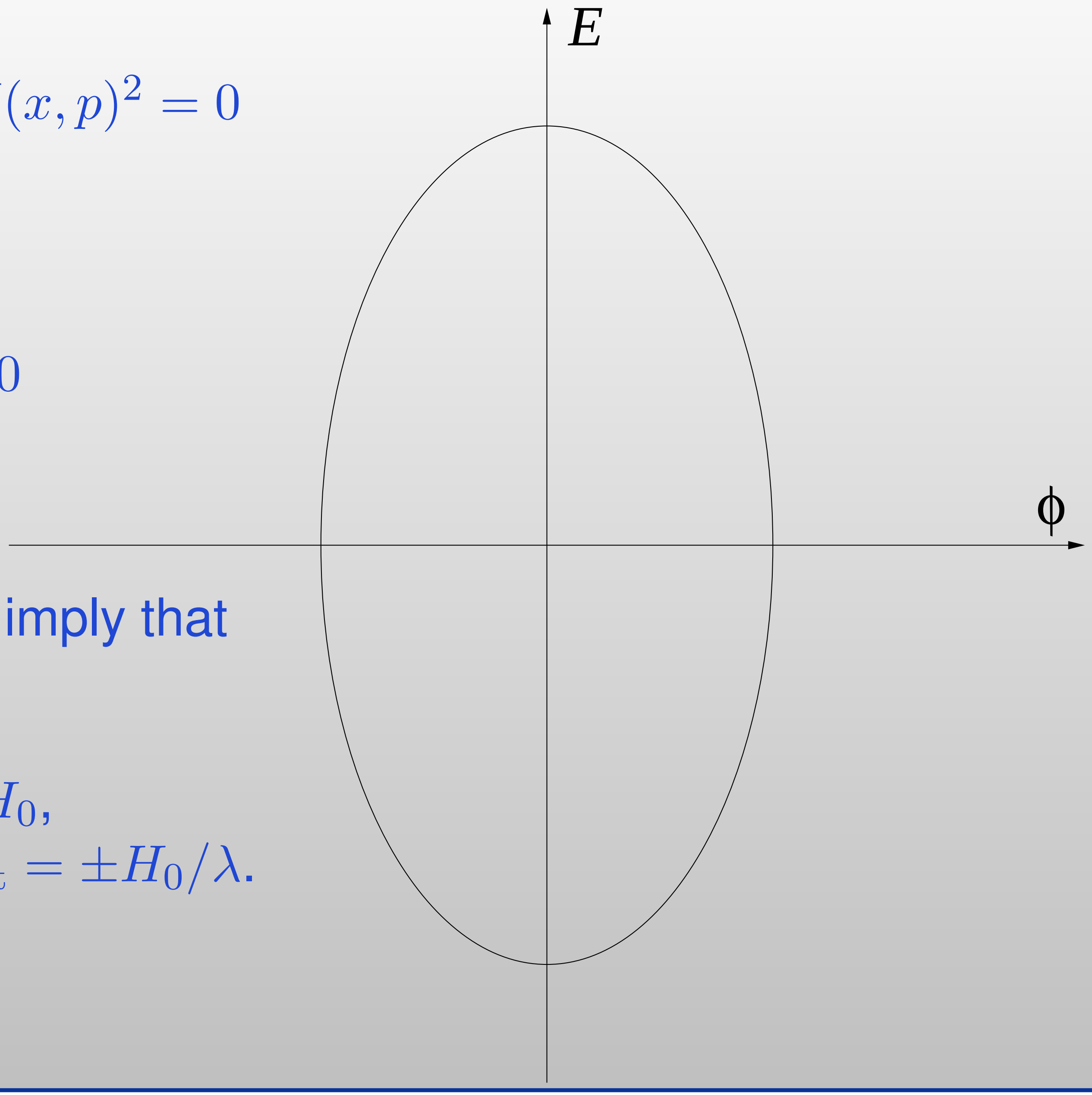
Clock ϕ, system x
coupled minimally:

Energy balance $C = 0$
instead of force.

Hamilton's equations imply that
$H(x,p)$ constant.

For given $H(x,p) = H_0$,
ϕ turns around at $\pm\phi_{\rm t} = \pm H_0/\lambda$.

Clock "frequency" λ
will be large.



Turning point

$C = 0$ implies $E = \sqrt{H^2 - \lambda^2\phi^2}$, choosing $E > 0$.

Stationary states ψ_k of $\hat{H}$:

$$i\hbar\frac{\mathrm{d}\psi_k(\phi)}{\mathrm{d}\phi} = \sqrt{E_k^2 - \lambda^2\phi^2}\,\psi_k(\phi)$$

Solution: $\psi_k(\phi) = \psi_k(0)\exp(i\Theta_k(\phi))$ with

$$\Theta_k(\phi) = -\frac{1}{2\hbar}\left(\phi\sqrt{E_k^2 - \lambda^2\phi^2} + \frac{E_k^2}{\lambda}\arcsin\left(\frac{\lambda\phi}{E_k}\right)\right)$$

Non-linear phase in exponential function replaces standard $\exp(-iE_k t/\hbar)$.

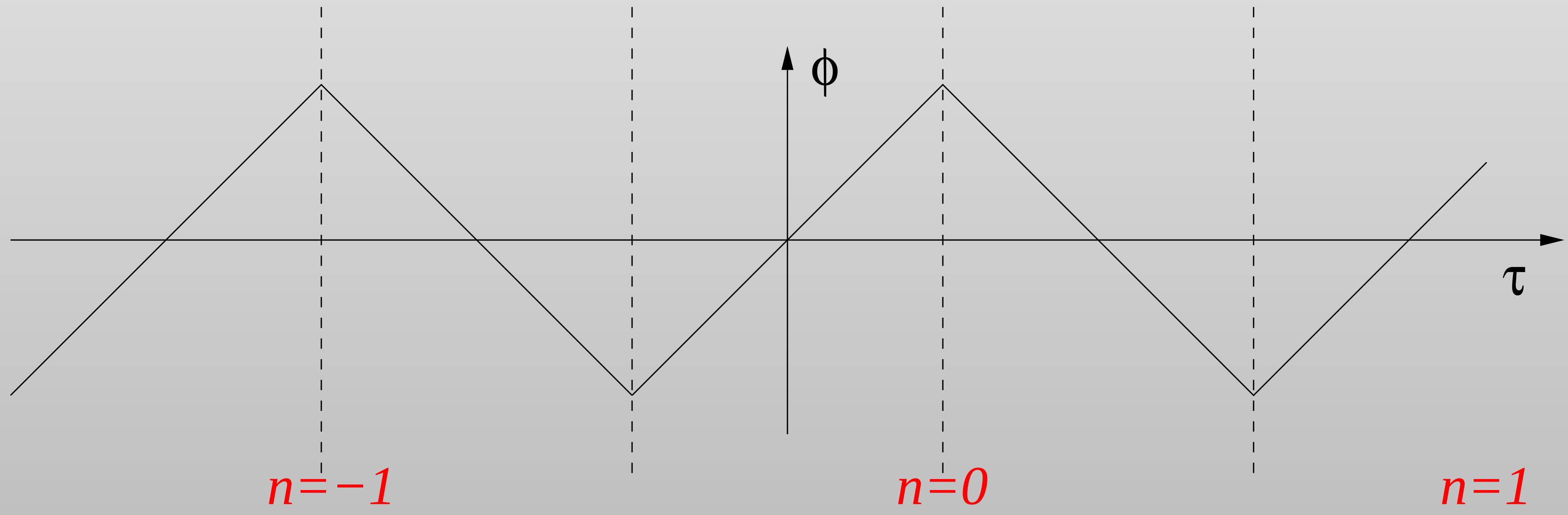
Phase $\Theta_k(\phi)$ real only if $|\phi| \leq \phi_{\mathrm{t}} = E_k/\lambda$: ϕ-evolution not unitary.

Unwinding time

For fixed k, define

$$\phi(\tau) = \begin{cases} \tau - 4n\phi_{\rm t} & \text{if } 4n-1 \le \tau/\phi_{\rm t} \le 4n+1 \\ (4n+2)\phi_{\rm t} - \tau & \text{if } 4n+1 \le \tau/\phi_{\rm t} \le 4n+3 \end{cases}$$

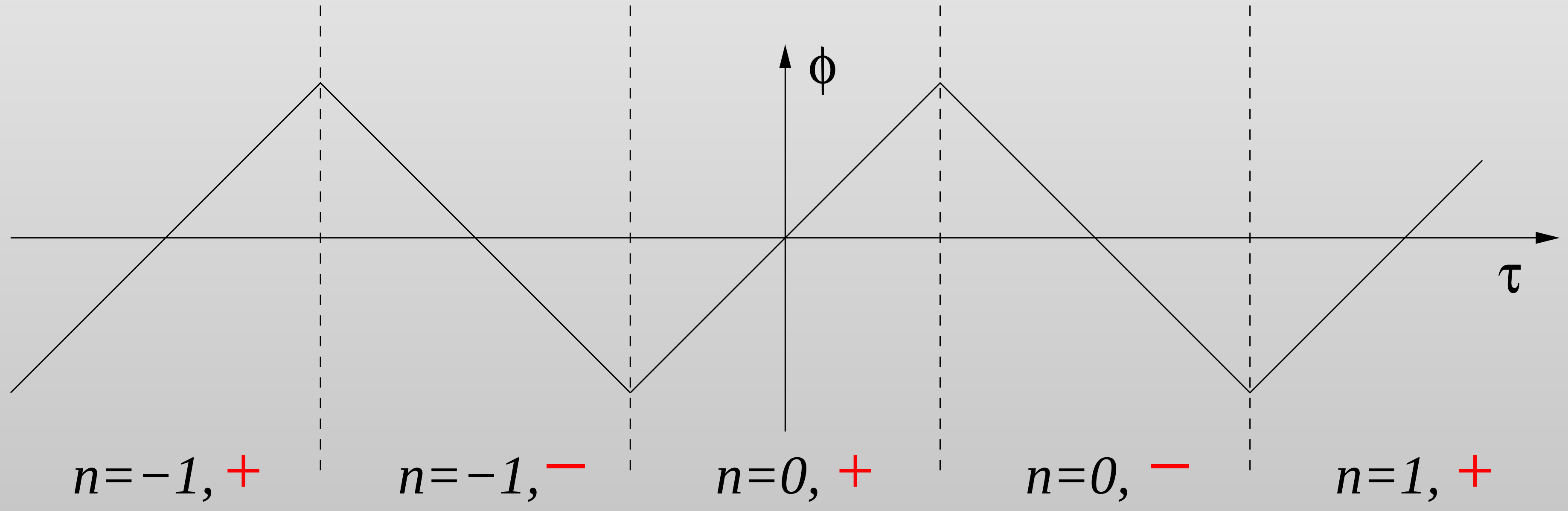
such that $|\phi(\tau)| \le \phi_{\rm t} = E_k/\lambda$ for all τ, $n = \left\lfloor \frac{1}{4}(1+\tau/\phi_{\rm t}) \right\rfloor$ cycles.



Global evolution

$$i\hbar \frac{\mathrm{d}\psi_k(\tau)}{\mathrm{d}\tau} = \frac{\mathrm{d}\phi}{\mathrm{d}\tau} \sqrt{E_k^2 - \lambda^2 \phi(\tau)^2}\, \psi_k(\tau)$$

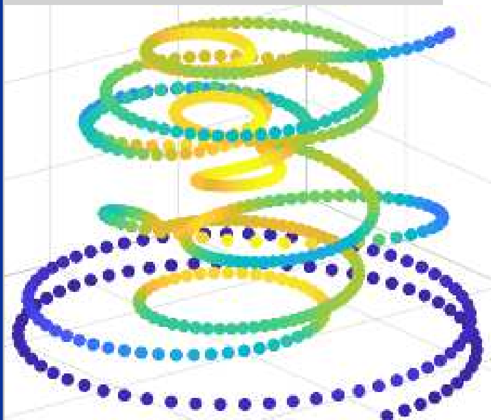
Replace $\Theta_k(\phi)$ with $\mathrm{sgn}(\mathrm{d}\phi/\mathrm{d}\tau)\Theta_k(\phi(\tau))$ in stationary solutions.

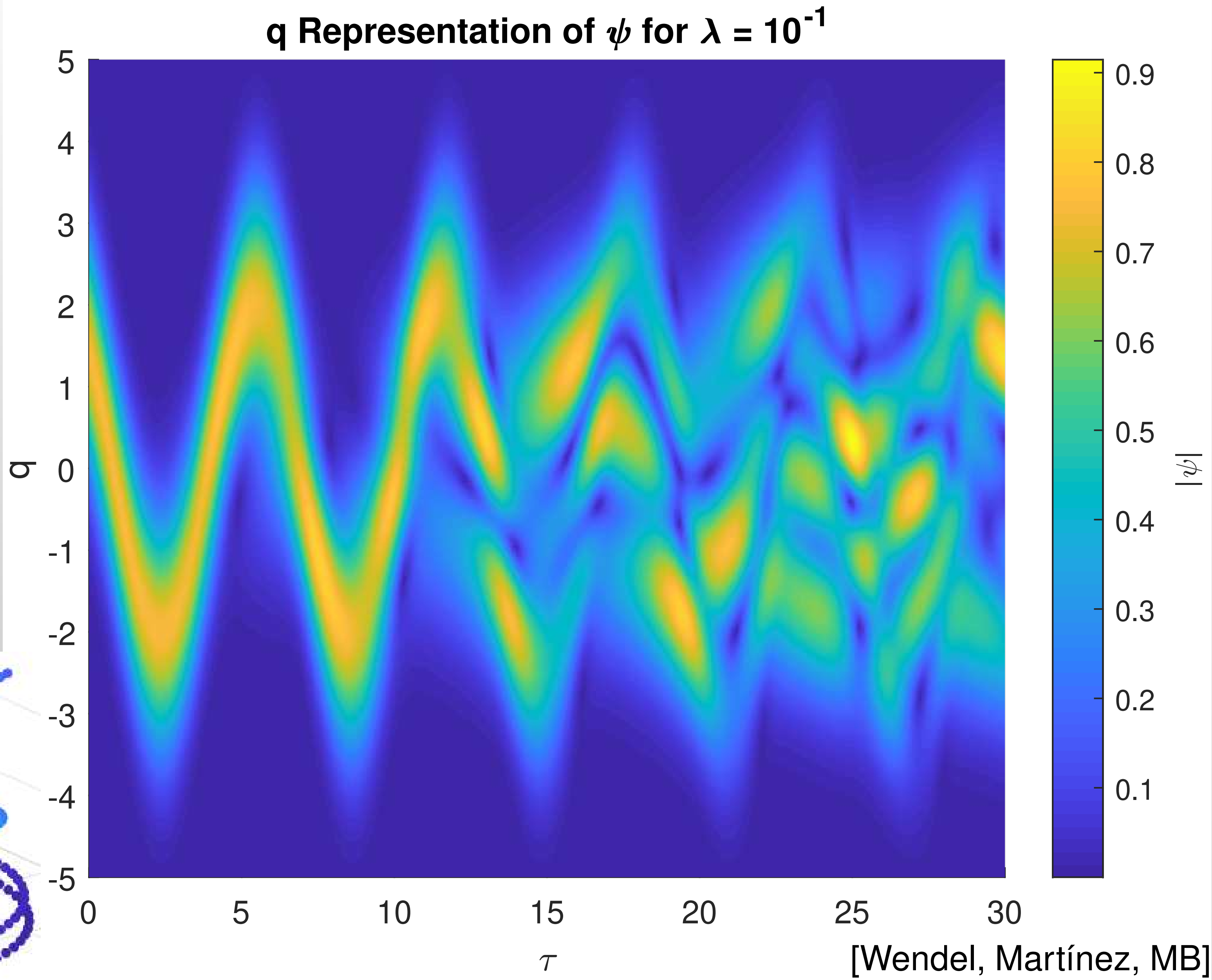


Superposition of stationary states $\psi_k(\tau)$ according to initial state.

Lost coherence: Harmonic oscillator

$$\hat{H} = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$$

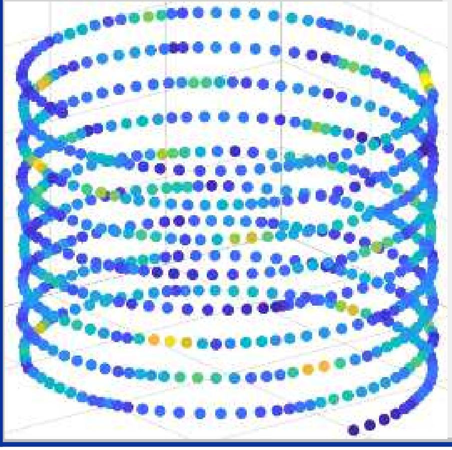


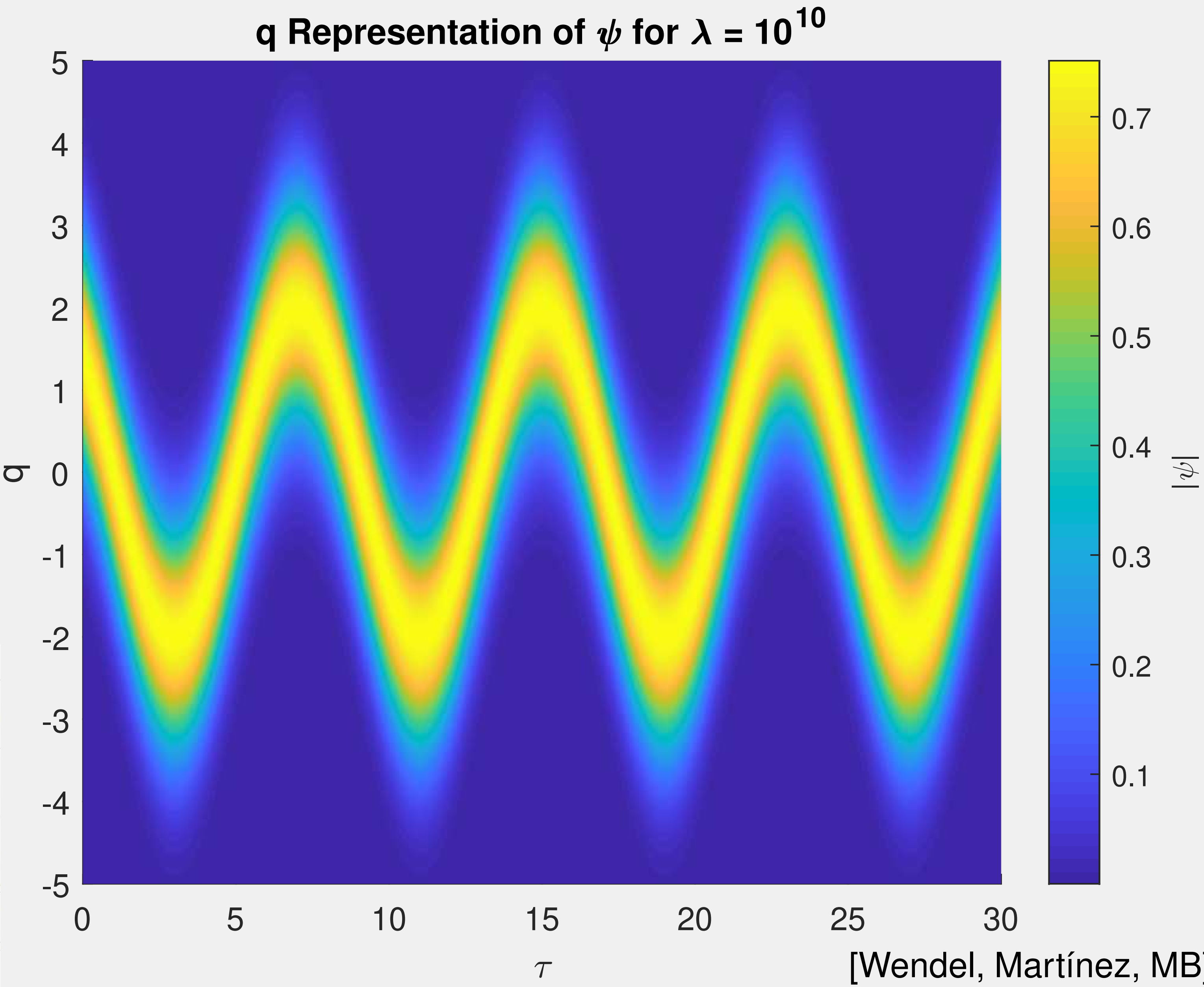


[Wendel, Martínez, MB]

Regained coherence: short clock period

$\hat{H} = \frac{1}{2}(\hat{p}^2 + \hat{q}^2)$





[Wendel, Martínez, MB]

Rescaled system period

Large λ, short clock period.

Clock cycles:

$$\frac{n}{\lambda} = \frac{\lfloor 1/4(1+\lambda\tau/E_k)\rfloor}{\lambda} \sim \frac{\tau}{4E_k}$$

Accumulated phase at time τ:

$$\Delta\Theta_k = 2n(\Theta_k(\phi_{\mathrm{t}}) - \Theta_k(-\phi_{\mathrm{t}})) = -\frac{\pi E_k^2}{\hbar}\frac{n}{\lambda} \sim -\frac{\pi}{4}\frac{E_k\tau}{\hbar}$$

System period $2\pi/\omega_k = 2\pi\hbar/E_k$ rescaled to $8/\omega_k$.

Not observable: Can be absorbed in “bare” system frequency.

PENNSTATE

Phase variations

Non-linear $\Theta_k(\phi(\tau))$ at finite $\lambda \neq 0$:

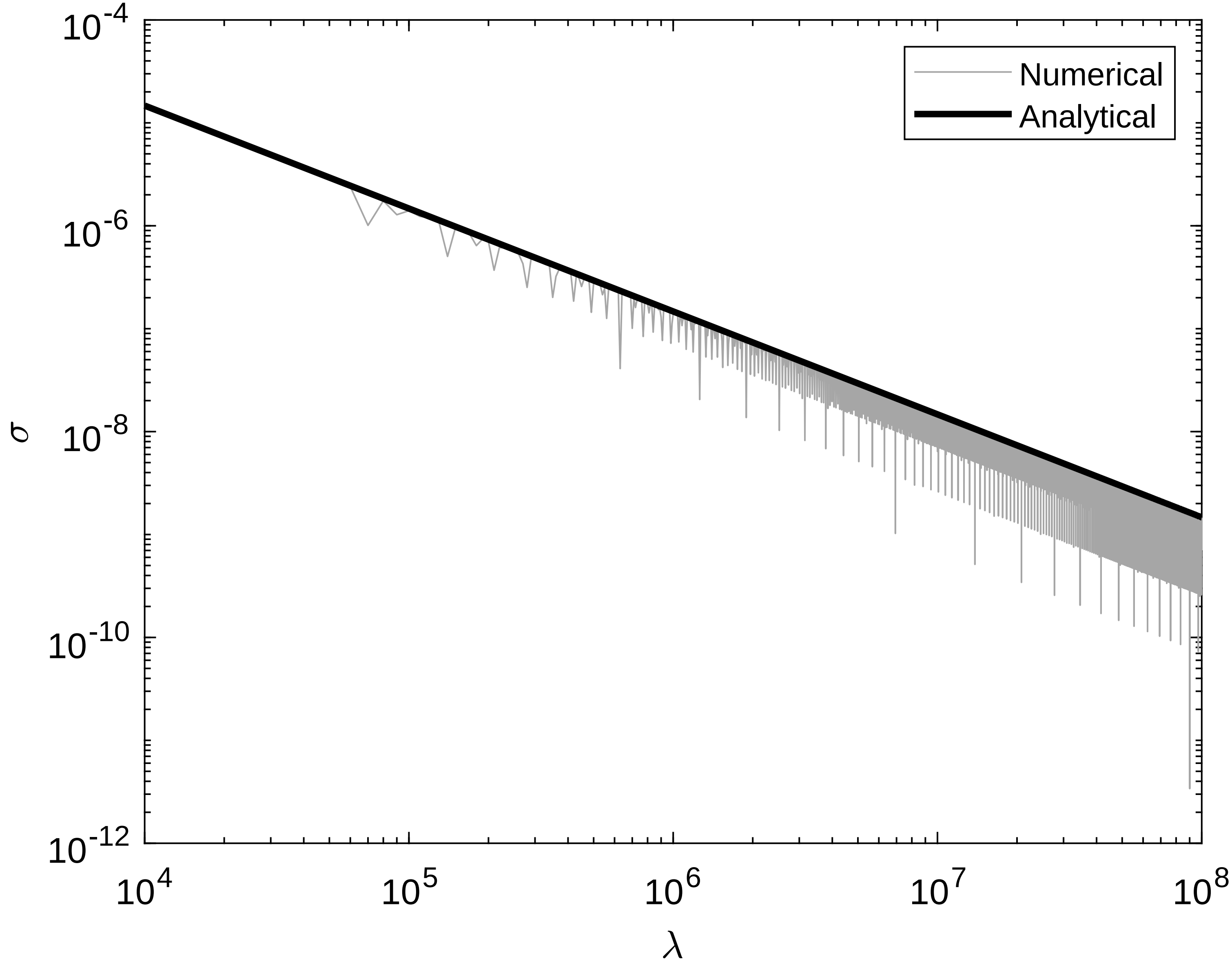
$$\sigma^2 = \frac{1}{\phi_{\rm t}} \int_0^{\phi_{\rm t}} \left(\Theta_k(\phi(\tau)) - (\text{accumulated phase})\right)^2 d\tau$$
$$= \frac{E_k^4(21\pi^2 - 1024/5)}{24^2 \lambda^2 \hbar^2}$$

Clock period $T_{\rm C} = 4\phi_{\rm t} = 4E_k/\lambda$ and system period $T_{\rm S} = 2\pi\hbar/E_k$ related by

$$T_{\rm C} = \frac{48\sigma T_{\rm S}}{\pi\sqrt{21\pi^2 - 1024/5}} \approx 9.7\sigma T_{\rm S} \qquad \text{for all } k$$

New dephasing effect largely model-independent. Requires non-linear $\Theta_k(\tau)$, always realized for ϕ-dependent Hamiltonian.

Standard deviation



Upper bound

$T_\mathrm{C} \approx 9.7\sigma T_\mathrm{S}$ based on theory of fundamental clock.

If T_S can be measured with accuracy σ, then T_C cannot be greater than $9.7\sigma T_\mathrm{S}$.

Latest atomic clocks: $\sigma \approx 10^{-19}$ at system period of $T_\mathrm{S} \approx 2\,\mathrm{fs}$.

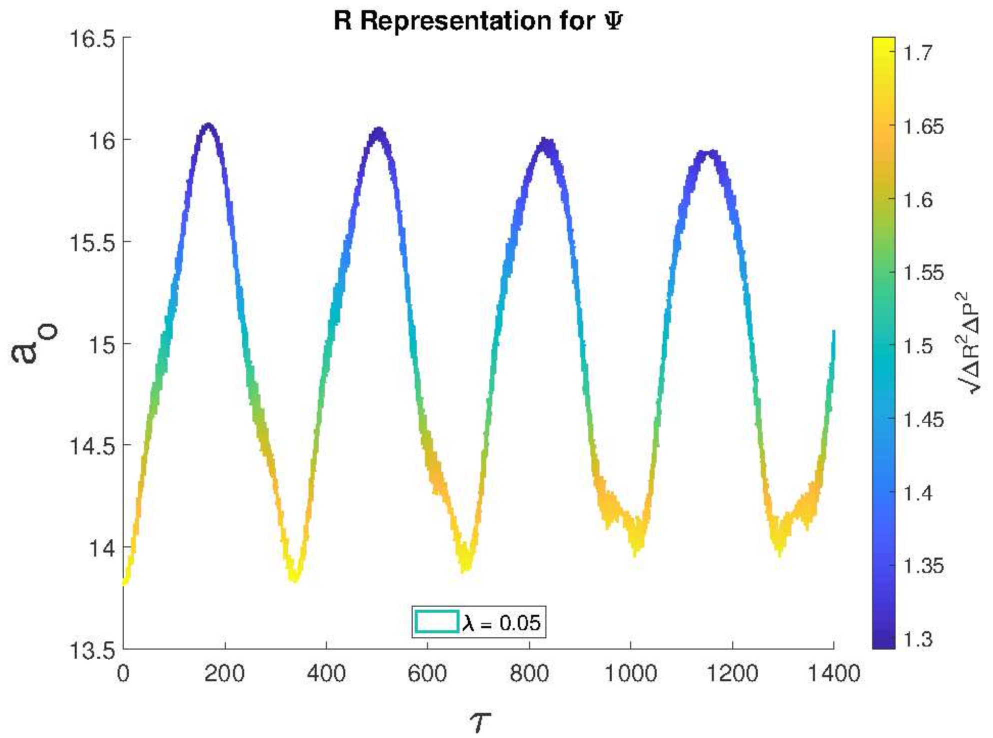
[Campbell et al. *Science* 358 (2017) 98]

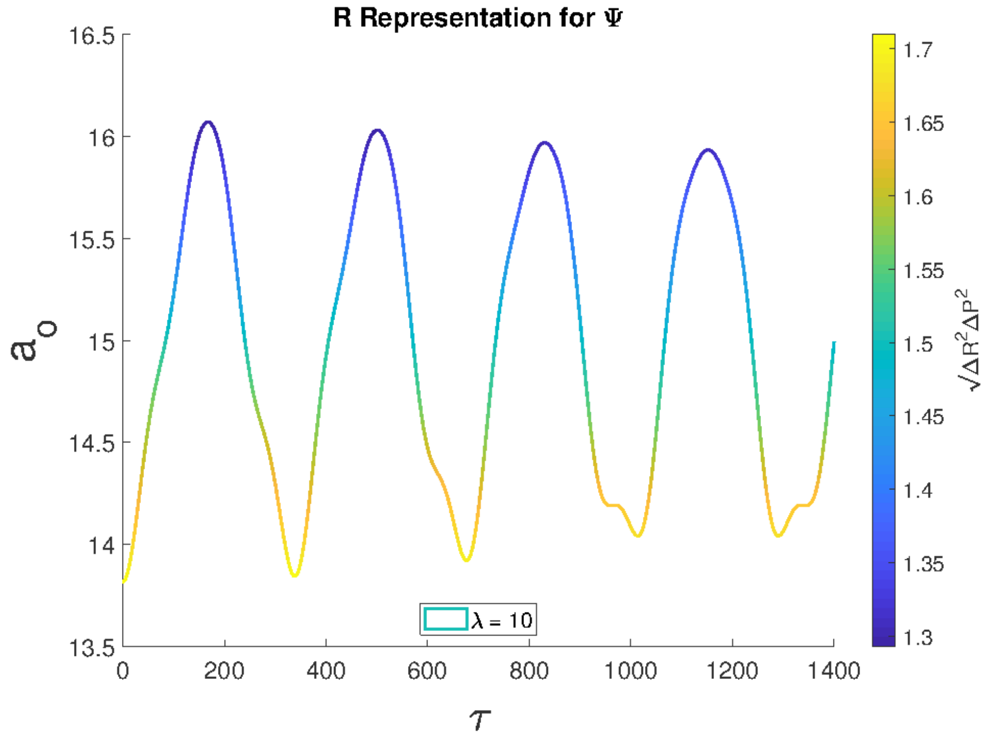
Therefore,

$$T_\mathrm{C} < 2 \cdot 10^{-33}\,\mathrm{s} \approx 0.5 \cdot 10^{11} t_\mathrm{P}$$

Particle accelerators probe spatial distances of $10^{-20}\mathrm{m} \approx 10^{16}\ell_\mathrm{P}$. Corresponds to $\approx 10^{-28}\,\mathrm{s}$.

Anharmonic system or clock





Cosmology

Planck-mass clock field ϕ (not inflaton): [with Ding, Martínez]

$$C = -\frac{1}{2}\frac{p_\phi^2}{V} - \frac{1}{2}Vm^2\phi^2 + 6\pi GVp_V^2 + \cdots$$

→ Hubble friction: ϕ does not oscillate if $\dot{V}/V$ large ("small" V).

- Monotonic clock at Planck density: deparameterized.
- As V increases, first turning points of ϕ appear. Enhanced decoherence of V-state or matter state.

  Can this wash out trans-Planckian fatures?
- Stabilizes when mass term dominates.

  Something like T-duality as used in string gas cosmology?

→ Possible effective description at large λ: varying period.

New stochastic term of the form $(1+\xi)\mathrm{d}/\mathrm{d}t$, noise ξ.

Quantum cosmology with periodic time

$$C = -\frac{1}{2}\frac{p_\phi^2}{V} - \frac{1}{2}V m^2\phi^2 + 6\pi G V p_V^2 + \cdots$$

Quantization requires two non-commuting operators

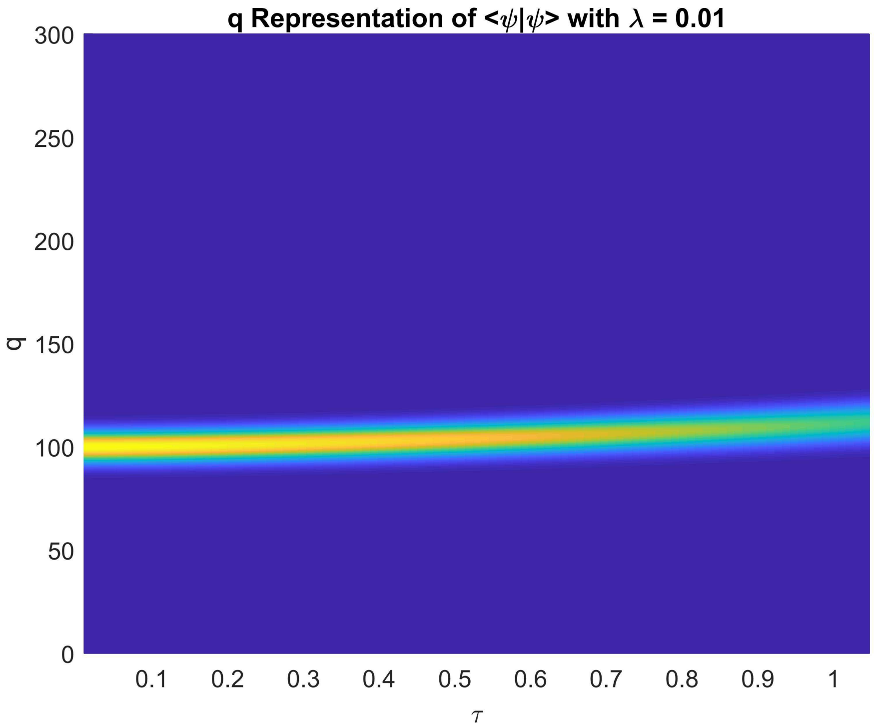
$$\hat{H} = \sqrt{12\pi G}\widehat{|Vp_V|} \quad \text{and} \quad \hat{V}^2$$

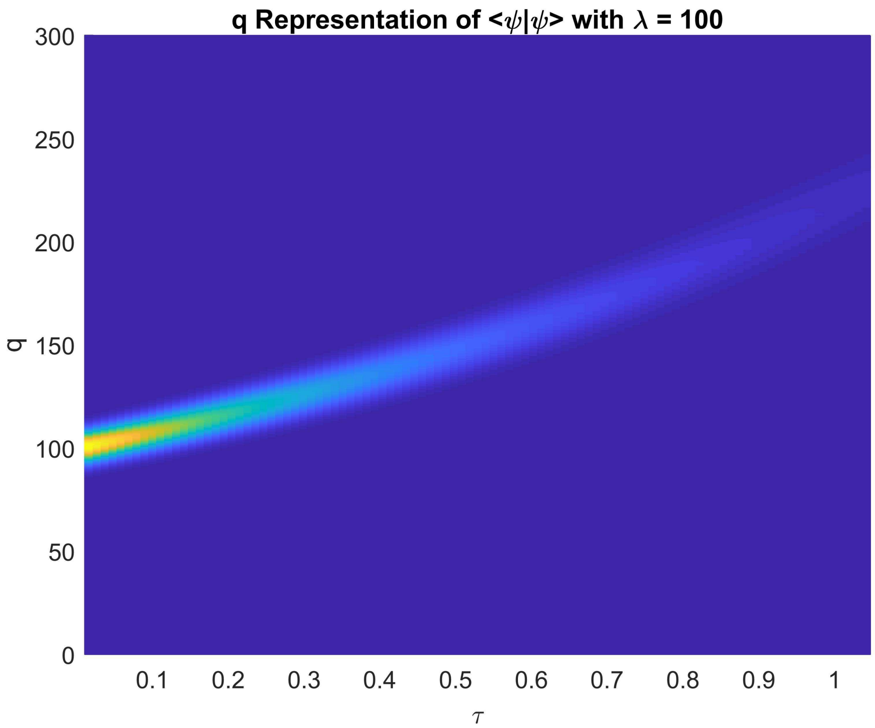
(or $\hat{V}$-dependent λ).

Stationary states of $\hat{H}$ do not decouple modes, ϕ-dependent evolution operator:

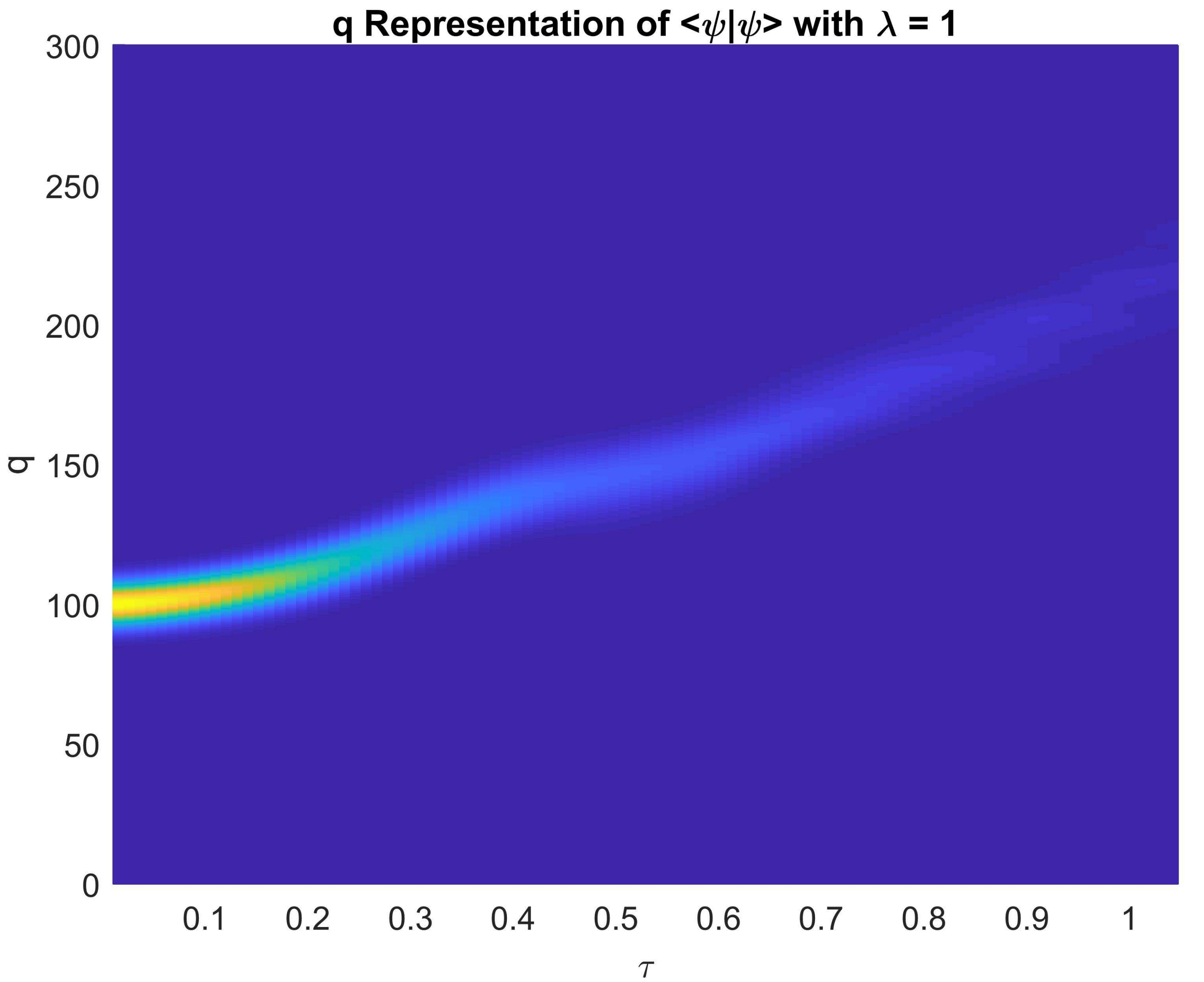
Numerics requires diagonalization at every time step, combined with implementation of turning-point condition for phase.

Small and large mass





Intermediate mass



PENNSTATE

Some open questions

→ Detailed cosmological analysis, inclusion of perturbative inhomogeneity.

→ Good effective theory to express discrete process of phase reflections in terms of observables for large λ.