Surprising properties of geometric Dirac observables in General Relativity

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PoToR 4
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Plan

- The geometric observables programm
- The symmetry group of the Gauss coordinates: deformed Poincare
- The symmetry group of the Trautman coordinates: smaller than expected, new relation between metric tensors

Problem of spacetime points in GR

Given spacetime M, g

and fields $\varphi_1, \dots, \varphi_I, \dots$

A point $p \in M$

does not have any physical meaning: evaluation observable

$$(g, \varphi_I) \quad \mapsto \quad \varphi_5(p)$$

is not physical. The reason is that

$$\mathrm{Diff}(M)$$

is contained in the group of the gauge transformations of General Relativity.

Physical points defined by deparametrization

Promote 4 fields to spacetime coordinates

$$x^0 := \varphi_1,\ x^1 := \varphi_2,\ x^2 := \varphi_3,\ x^3 := \varphi_4,$$

or apply that method partially:

$$x^0 := \varphi_1$$

and deal with the remaining diffeomorphisms.

Kijowski, Kuchar,Torre, Brown, Husain, Rovelli,Smolin,…,

Alternatively, follow ***Cartan*** and instead of the mater fields use 4 invariants constructed from

$$R_{\alpha\beta\gamma\delta}$$

and/or it's derivatives

$$\nabla_\mu R_{\alpha\beta\gamma\delta}$$

Relational observables

Rovelli, Thiemann, Dittrich, …

important errors were found and clarified in:
Dapor, Kamiński, Lewandowski, Świeżewski 2013

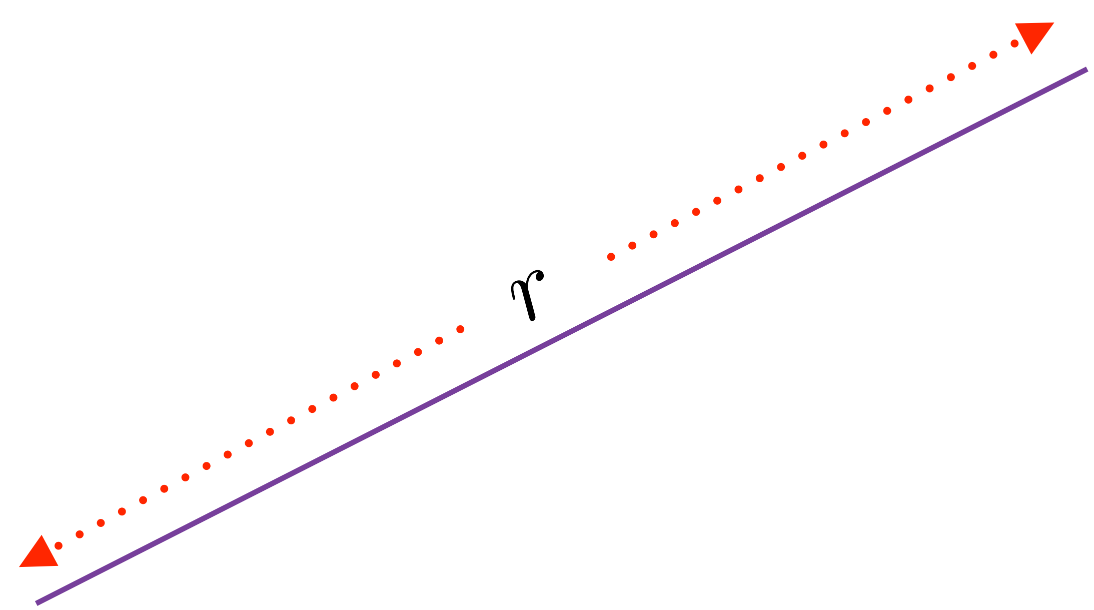
application to perturbations in cosmology:
Dapor, Lewandowski, Puchta 2013

Quantum emergence of observables:
Dziendzikowski, Domagała, Lewandowski 2012

Observables for General Relativity related to geometry

Duch, Kamiński, Lewandowski, Świeżewski, Bodendorfer, Kolanowski

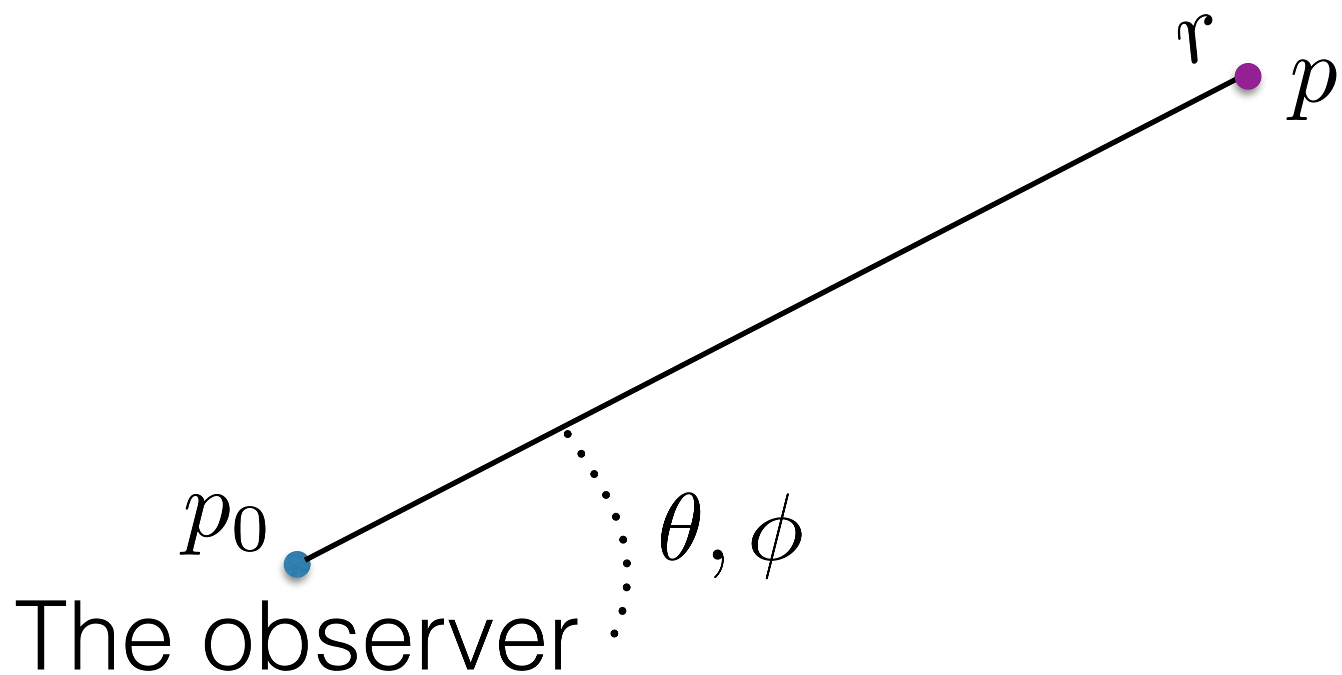
Define spacetime points by their position with respect to an observer. Use geodesics, distances, times, affine parameters.



Space like case: radial coordinates

Σ - 3d manifold (space)

q - a metric tensor



Given a scalar field φ metric tensor q and $(r_1, \theta_1, \phi_1) \in \mathbb{R}^3$ an observable is

$$(q, \varphi) \quad \mapsto \quad \varphi(p(r_1, \theta_1, \phi_1; q))$$

The Resulting radial gauge:

$$q_{ra} = \delta_{ra},$$

Space-like case: radial gauge

Applied to theories with time deparametrized by a scalar field.

Results:

GR in radial gauge, canonical structure, reduced phase space, dynamics.

Duch, Kamiński, Lewandowski, Świeżewski 2015
Bodendorfer, Lewandowski, Świeżewski 2015

A quantum reduction to spherical symmetry in loop quantum gravity,

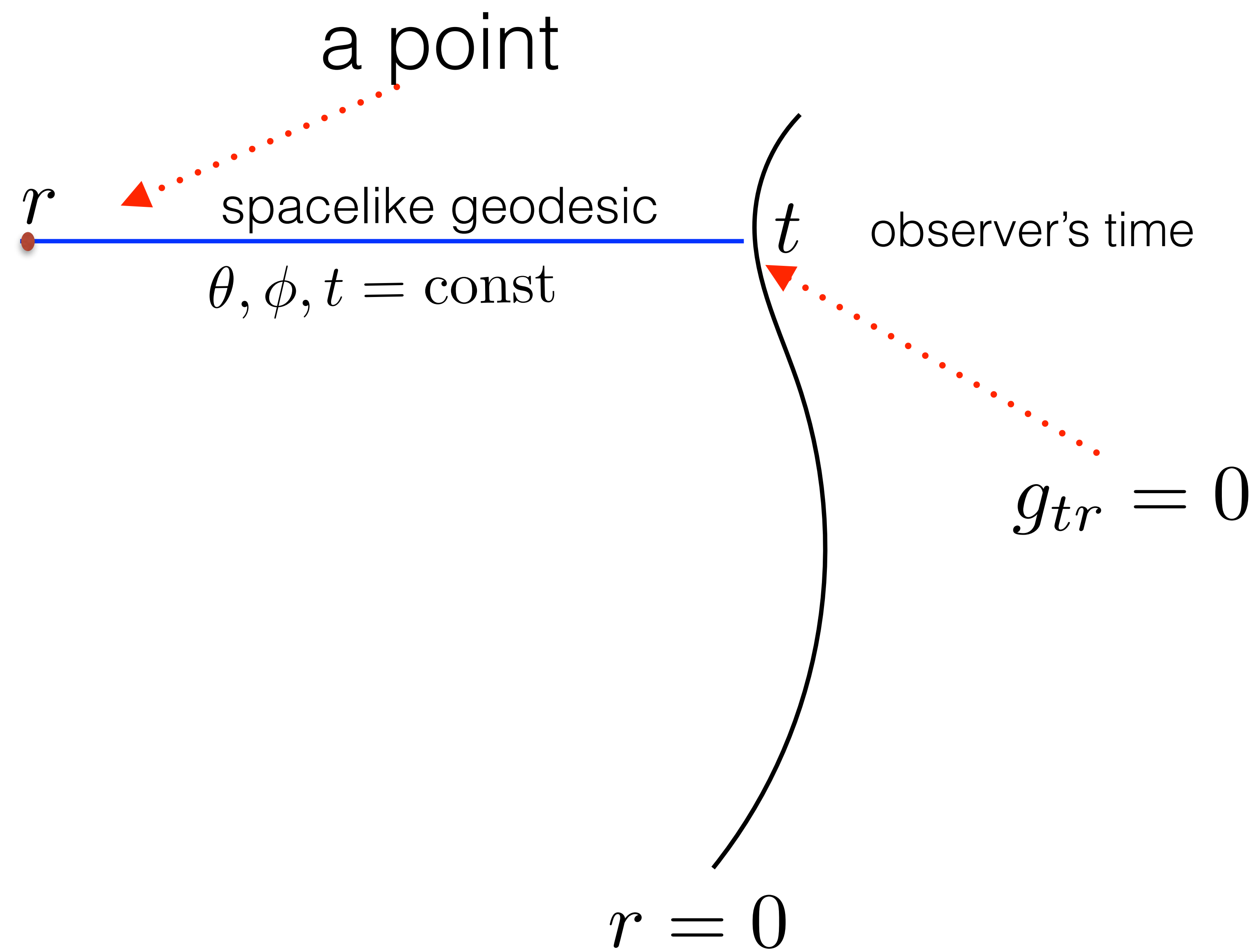
Bodendorfer, Lewandowski, Świeżewski 2015

Space-Time case: Gauss coordinates

M - spacetime manifold

g - metric tensor

(t, r, θ, ϕ) - the coordinates



Observer's world line
timelike geodesic

θ, ϕ - coordinates on the observer's sphere of null directions parallelly transported along the world line

Space-Time case: Gauss coordinates

Results:

Combining with the canonical framework. Equivalence with a gauge choice on ADM variables:

$$q_{ra} = \delta_{ra}, \qquad K_{rr} = 0$$

Calculation of the Poisson bracket between the resulting observables

Bodendorfer, Duch, Lewandowski, Świeżewski 2016

Symmetries of those gauge conditions

Duch, Lewandowski, Świeżewski 2016

The Gauss gauge: symmetries

Given $M, g,$ and GB coordinates (t, r, θ, ϕ)

a symmetry of the GB gauge is $f \in \mathrm{Diff}(M)$

such that (t, r, θ, ϕ) are still the GB coordinates for

$$f^* g$$

That is, in a neighborhood of the observer:

$$(f^* g)_{rt} = 1, \quad (f^* g)_{rr},\, (f^* g)_{r\theta},\, (f^* g)_{r\phi} = 0$$

And at the observer:

$$(f^* g)_{tt}|_{r=0} = -1,$$

$$(f^* \nabla)_{\partial_t} \partial_t|_{r=0} = (f^* \nabla)_{\partial_t} \partial_r|_{r=0} = 0$$

The Gauss observer point of view

The observer does not see the diffeomorphisms

$$f \in \mathrm{Diff}(M)$$

Such that

$$t[f^* g] \;=\; f^* t[g]$$

$$r[f^* g] \;=\; f^* r[g]$$

$$\theta[f^* g] \;=\; f^* \theta[g]$$

$$\phi[f^* g] \;=\; f^* \phi[g]$$

and (s)he views residual diffeomorphisms as the symmetries of the Gauss gauge we have just defined.

The Gauss gauge: infinitesimal symmetries

A GB gauge symmetry generator $X = X^\mu \partial_\mu$
is a solution to the equation

$$X_{\mu;r} + X_{r;\mu} = 0$$

with suitable initial conditions at $r = 0$

It is characterized by the initial data at the point $p_o \in M$
such that $(t, r) = (0, 0)$:

$$X^\mu(p_0), X^\mu{}_{,\nu}(p_0)$$

subject to the condition:

$$g_{\mu\alpha}(p_0) X^\alpha{}_{,\nu}(p_o) \;=\; -g_{\nu\alpha}(p_0) X^\alpha{}_{,\mu}(p_o)$$

The Gauss gauge: infinitesimal symmetries, algebra?

Every pair

$$t \in T_{p_0}M, \qquad l \in so(g(p_0))$$

Defines an infinitesimal symmetry:

$$X^{(t,l)}$$

The symmetries do not close to an algebra though:

$$[X^{(t,l)}, X^{(t',l')}] \neq \sum_I a_I X^{(t_I,l_I)}$$

Deformed Poincare algebra

What does close are infinitesimal generators induced on the space of metric tensors:

$$\tilde{X}^{(t,l)} = \int_M d^3x \, \mathcal{L}_{X^{(t,l)}} g_{\mu\nu}(x) \frac{\delta}{\delta g_{\mu\nu}(x)}$$

Indeed:

$$[\tilde{X}^{(0,l)}, \tilde{X}^{(0,l')}] = \tilde{X}^{(0,[l',l])}$$

However

$$[\tilde{X}^{(t,0)}, \tilde{X}^{(t',0)}] = \tilde{X}^{(0,l")}$$

$$l"^{\mu}_{\nu} = t^{\alpha} t'^{\beta} R_{\alpha\beta\nu}{}^{\mu}(p_0)$$

The same algebra in the Gauss gauge case, and similar in the radial gauge case

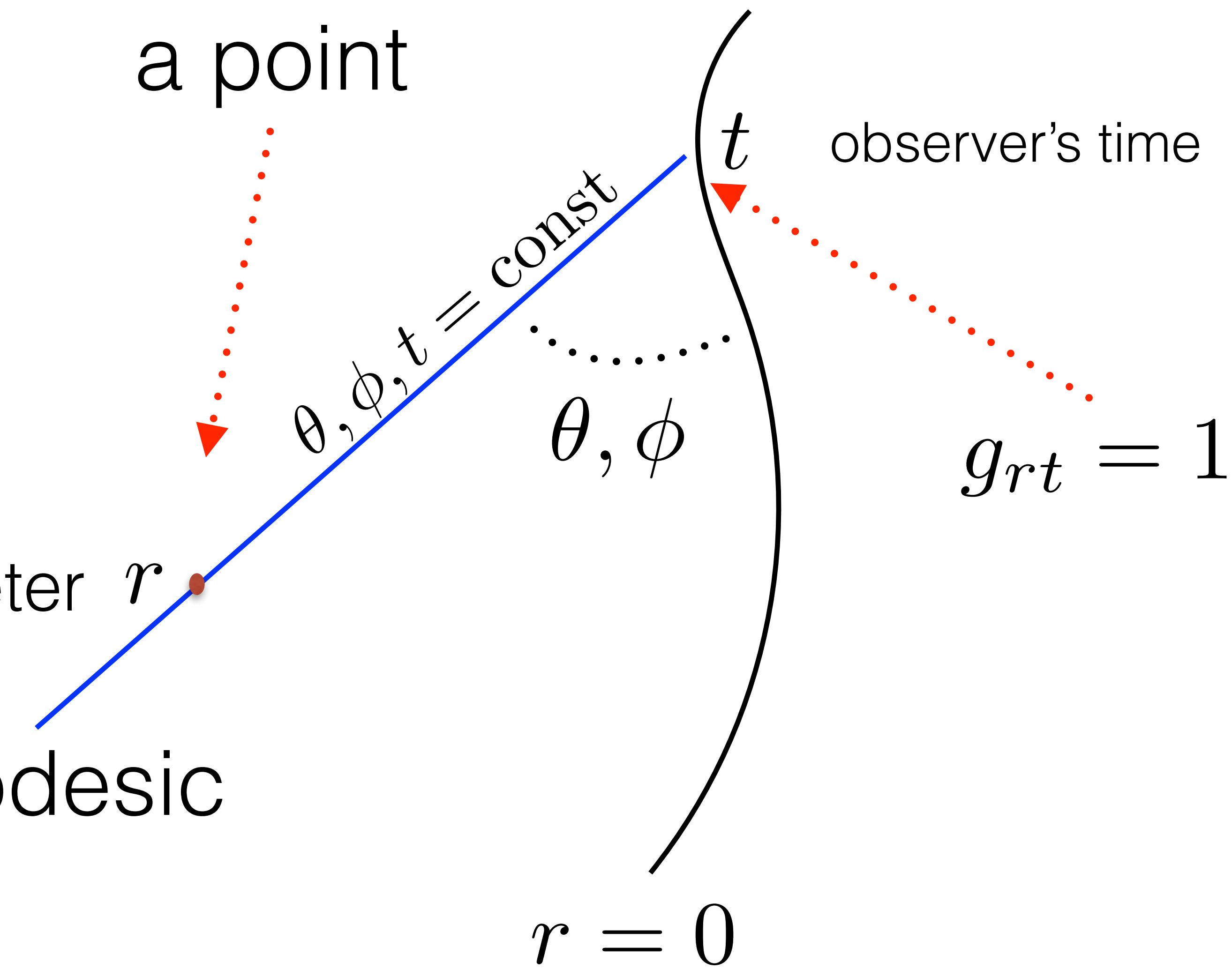
Space-Time case: Trautman coordinates

Kolanowski, Lewandowski 2017

M - spacetime manifold

g - metric tensor

(t, r, θ, ϕ) - the coordinates



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The Trautman coordinates

Minkowski spacetime:

$$g = -dt^2 + 2dtdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

General spacetime:

1) In a neighborhood of the observer:

$$g_{rt} = 1, \qquad g_{rr} = g_{r\theta} = g_{r\phi} = 0$$

2) At the observer:

$$g_{tt}|_{r=0} = -1$$

$$\nabla_{\partial_t}\partial_t|_{r=0} = \nabla_{\partial_t}\partial_r|_{r=0} = 0$$

The Trautman coordinates: symmetries

The observer does not see the diffeomorphisms

$$f \in \mathrm{Diff}(M)$$

Such that

$$t[f^*g] \;=\; f^*t[g]$$

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$$\phi[f^*g] \;=\; f^*\phi[g]$$

and (s)he views residual diffeomorphisms as the symmetries of the Trautman coordinates.

The Trautman coordinates: symmetries

1. The symmetries are differentiable

2. There are examples of symmetries that fail to be twice differentiable

3. New equivalence relation:

$$(g, t, r, \theta, \phi) \sim (g', t', r', \theta', \phi')$$

iff the symmetry they define is smooth

4. Example: if g' is flat, then a necessary condition on (g, t, r, θ, ϕ) reads:

$$R_{t\alpha\beta\gamma|_{r=0}} = 0$$

Comparison of Gauss with BMS

Bondi-Mertzner-Sachs:

Observer is a null conformal boundary in infinity

Uniquely defined subalgebra of translations

Many subalgebras SO(1,3)

Well defined energy-momentum
Problem with angular momentum

Gauss:

Observer is a timelike geodesic curve

Uniquely defined subalgebra SO(1,3)

Non-commuting translations

Well defined angular momentum?
Energy momentum?

Discuussion of the Trautman coordinates case

Technically:
a new relation between spacetimes and observers

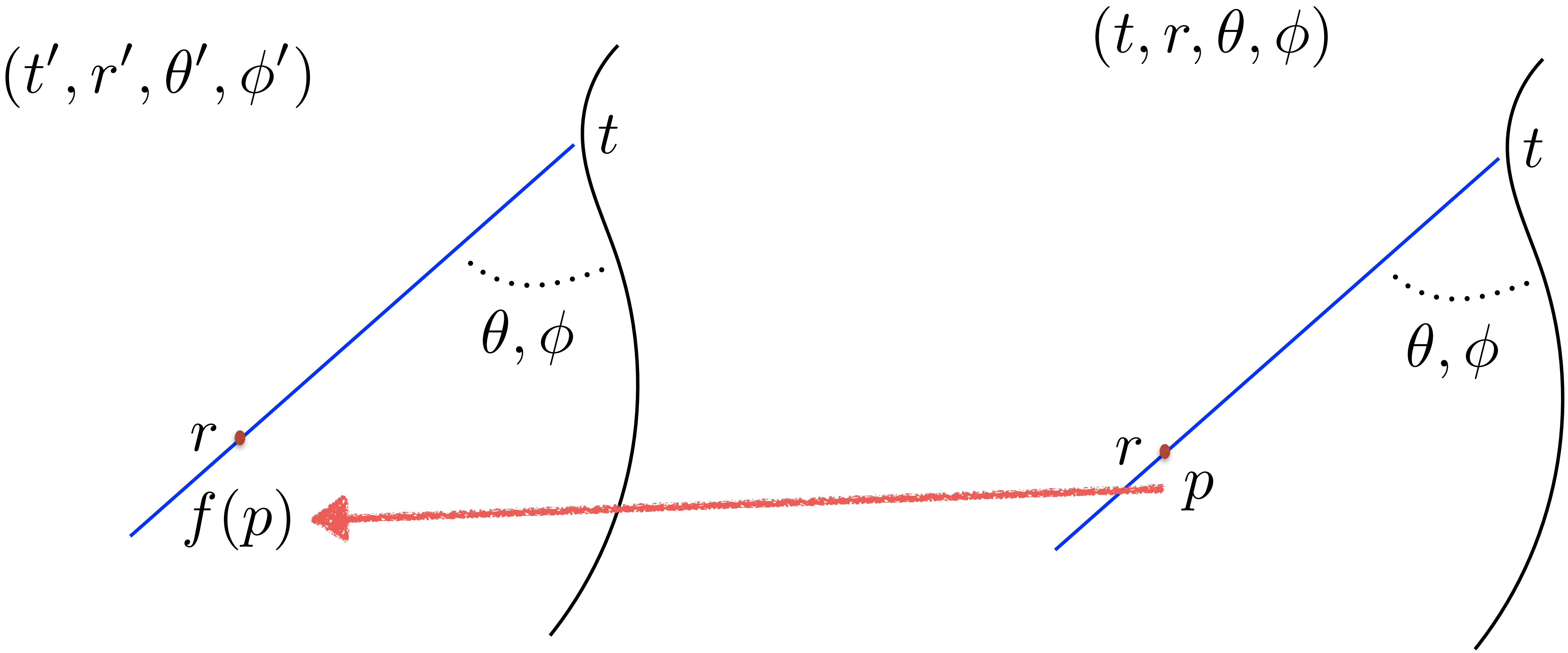
The gauge fixing point of view:
better than expected, less symmetries

The observer point of view:
the smooth structure of spacetime at r=0 expressed
in a complicated way

Thank You!

The Gauss-Bondi gauge: a general solution to the symmetry problem

Other GB coordinates: Our GB coordinates:



$$t'(f(p)) = t(p), \quad r'(f(p)) = r(p), \quad \theta'(f(p)) = \theta(p), \quad \phi'(f(p)) = \phi(p),$$

The result is a 10-dim family of f's modulo the differentiability

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