Universal electromagnetic fields[1]

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[1] Joint work with V. Pravda, arXiv:1708.08017

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Modified electrodynamics

Maxwell's theory (in vacuum)

- $L = I_1 \qquad (I_1 \equiv F_{ab}F^{ab},\ I_2 \equiv F_{ab}{}^{*}F^{ab})$
- $\mathrm{d}\boldsymbol{F} = 0,\ {*\mathrm{d}} * \boldsymbol{F} = 0$

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"Corrections"

1. Classical theories (to cure infinite self-energies)
   - Mie 1912
   - Born-Infeld 1933, 1934:
     $L = \sqrt{b^4 + 2b^2 I_1 + I_2^2} - b^2 \qquad (\to I_1 \text{ for } b \to \infty)$
   - general NLE: $L = L(I_1, I_2)$ (e.g. [Plebański'70])
   - Bopp 1940, Podolsky 1942: $L = I_1 + a^2(\nabla \boldsymbol{F})^2$

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2. Vacuum polarization in QED: Heisenberg-Euler (NLE)
   [Heisenberg-Euler'36, Weisskopf'36, Schwinger'51]
3. String theory: Born-Infeld as a low-energy Lagrangian
   [Fradkin-Tseytlin'85, Bergshoeff-Sezgin-Pope-Townsend'87, . . .]

Schrödinger's "universal" solutions

Field equations of NLE:

$$\mathrm{d}\boldsymbol{F} = 0, \qquad *\mathrm{d} * \tilde{\boldsymbol{F}} = 0,$$

where $\tilde{\boldsymbol{F}} \equiv \alpha \boldsymbol{F} + \beta * \boldsymbol{F}, \quad \alpha = \alpha(I_1, I_2), \ \beta = \beta(I_1, I_2).$

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all null Maxwell fields satisfy any NLE! [Schrödinger'35,'43]

E.g. Born-Infeld:

$$F_{[ab,c]} = 0, \quad \left(\frac{F^{ab} - \cancel{\mathcal{G}\,{}^*F^{ab}}}{\sqrt{1 + \cancel{\mathcal{F}} - \cancel{\mathcal{G}^2}}} \right)_{;b} = 0 \qquad (\mathcal{F} \equiv \tfrac{1}{2} I_1, \ \mathcal{G} \equiv \tfrac{1}{4} I_2).$$

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- $\boldsymbol{F} = \boldsymbol{\ell} \wedge \boldsymbol{\omega}$, with $\ell_a \ell^a = 0 = \ell^a \omega_a$
- plane waves [Schwinger'51, Synge'55]
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The previous argument breaks down when $\tilde{\boldsymbol{F}}$ contains $\nabla^{(k)} \boldsymbol{F}$ . . .

Universal solutions

Definition: $\boldsymbol{F}$ universal

$$\mathrm{d}\boldsymbol{F} = 0, \qquad *\mathrm{d} * \tilde{\boldsymbol{F}} = 0,$$

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Theorem

$\boldsymbol{F}$ is universal provided

1. *$\boldsymbol{F}$ is null: $\boldsymbol{F} = \boldsymbol{\ell} \wedge \boldsymbol{\omega}$*
2. (a) *$\boldsymbol{\ell}$ is a Kundt vector field ($\kappa = \sigma = \omega = \theta = 0$)*
   (b) *the spacetime is of Petrov type III (w.r.t. $\boldsymbol{\ell}$)*
   (c) *$S_{ab}\ell^b = 0 = S_{[ab}\ell_{c]}$, where $S_{ab} \equiv R_{ab} - \frac{R}{4} g_{ab}$ (traceless-Ricci).*

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2(b,c) simply mean: $R_{abcd} = \frac{R}{6} g_{a[c} g_{d]b} + $ (terms with "many" $\boldsymbol{\ell}$)

Explicitly:

$$\mathrm{d}s^2 = 2P^{-2}\mathrm{d}\zeta\mathrm{d}\bar{\zeta} - 2\mathrm{d}u\left(\mathrm{d}r + W\mathrm{d}\zeta + \bar{W}\mathrm{d}\bar{\zeta} + H\mathrm{d}u\right),$$
$$\boldsymbol{F} = \mathrm{d}u \wedge \left[f(u,\zeta)\mathrm{d}\zeta + \bar{f}(u,\bar{\zeta})\mathrm{d}\bar{\zeta}\right] \qquad (\ell_a \mathrm{d}x^a = \mathrm{d}u),$$

where (some constraints omitted . . .)

$$P = P(u,\zeta,\bar{\zeta}), \qquad W = rg^{(1)}(u,\zeta,\bar{\zeta}) + g^{(0)}(u,\zeta,\bar{\zeta}),$$
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- also p-forms in n-dimensions

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- all the invariants of $\boldsymbol{F}$ vanish (including derivatives)
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- also p-forms in n-dimensions
- holds in Minkowski, (A)dS (plus possible waves)
- test fields (no backreaction)
- coupling to gravity also possible: further restrictions
  cf. also [Güven'87, Horowitz-Steif'90, Coley'02]
  NLE in [Kichenassamy'59, Kremer-Kichenassamy'60, Peres'60]

A simple example:

$$\mathrm{d}s^2 = 2\mathrm{d}u\Big[\mathrm{d}r + \tfrac{1}{2}x\,(r - e^x)\,\mathrm{d}u\Big] + e^x(\mathrm{d}x^2 + e^{2u}\mathrm{d}y^2)$$

$$\boldsymbol{F} = e^{x/2}c(u)\mathrm{d}u \wedge \left(-\cos\frac{ye^u}{2}\mathrm{d}x + e^u \sin\frac{ye^u}{2}\mathrm{d}y\right)$$

- $\boldsymbol{\ell} = \partial_r$ is Kundt and *recurrent* ($\ell_{a;b} \sim \ell_a\ell_b$)
- Petrov type III, Ricci-flat [Petrov'62]
- $\boldsymbol{F}$ is universal

Sketchy proof

Recall: $\tilde{\boldsymbol{F}} =$ "polynomial" in $\boldsymbol{F}, \boldsymbol{F}^2, \ldots, \nabla^{(k)}\boldsymbol{F}, (\nabla^{(k)}\boldsymbol{F})^2, \ldots$

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  e.g. $\tilde{F}_{ab} = c_1 F_{ab} + c_2 F_{ab;c}{}^{c} + c_3 \underbrace{F_{[a|c;}{}^{c}{}_{|b]}}_{=0!} + c_4 F_{ab;cd}{}^{cd} + \ldots$

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- $\Rightarrow$ $\tilde{F}_{ac;}{}^{c} = 0$, Q.E.D.

Gravity analog: universal spacetimes

- Certain type N vacuum *pp*-waves solve *any* $L(\mathrm{Riem}, \nabla\mathrm{Riem}, \ldots)$
  e.g. quadratic gravity, Lovelock gravity, string corrections, etc.
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- results for type III/N and II in 4D and HD
  for example: *all Kundt Einstein spacetimes of Weyl type N are universal*
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- slightly different approach [Gürses-Şişman-Tekin'14,'17]