Memory effect in FLRW cosmology

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Axial versus polar gravitational waves

- It is well known that (**Chandrasekhar and Ferrari 1991**)

  Chandrasekhar, Subrahmanyan, and Valeria Ferrari. "On the Non-Radial Oscillations of Slowly Rotating Stars Induced by the Lense-Thirring Effect." Proceedings: Mathematical and Physical Sciences 433, no. 1888 (1991): 423-40.
- *Axial gravitational waves incident onto a uniform (fluid star) do not excite fluid motion*
- *Polar gravitational waves incident onto a uniform (fluid star) do excite fluid motion*.

Axial versus polar gravitational waves

- These conclusions of (**Chandrasekhar and Ferrari 1991**) are true also in FLRW spacetimes filled with fluids Gundlach and Martin Garcia PRD 2000; EM and Wylężek CQG 2004; Clarkson et al. JCAP 2009
- — under a differentiability condition — (Wojciech Kulczycki, EM, CQG 2017)
- *Smooth axial gravitational waves do not excite fluid motion*
- *Polar gravitational waves do excite fluid motion.*

Main goal

- Find a class of axial waves that interact with cosmological fluids.
- **More specifically, show that:**
- initial axial data (twice differentiable but less than C^2) in the radiation epoch of FLRW cosmology generate wave pulses that force radiation matter to rotate.
- **this rotation is permanent — it persists after the passage of the gravitational pulse.**

Axial gravitational waves in FLRW cosmology

- The FLRW geometry (spherical coordinates (η, r, θ, ϕ), the conformal time coordinate, $k = 0$ universe)
-
$$ds^2 = a^2(\eta)\left(-d\eta^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right).$$
- The perturbed components of the metric
$$g_{\mu\nu} = a^2(\eta)\eta^{(0)}_{\mu\nu} + h_{\mu\nu},$$
where $\eta^{(0)}_{\mu\nu}$ is the Minkowski metric. Perturbing terms can be expanded in terms of tensorial spherical harmonics H_{lm} Regge and Wheeler 1950's.

Axial gravitational waves in FLRW cosmology

- Let P be the parity operator acting on a rank 2 tensor H as follows
$$P(H(\theta,\phi)_{lm} \rightarrow \tilde{H}(\pi-\theta,\pi+\phi)_{lm}$$
For the axial perturbations $\tilde{H}(\theta,\phi)_{lm} = -(-1)^l H(\pi-\theta,\pi+\phi)_{lm}$.
- The only nonzero components of $h_{\mu\nu}$ in the Regge-Wheeler gauge:
$$h_{0\phi} = h_0(\eta,r)\sin\theta\ \partial_\theta Y_{lm} \qquad h_{r\phi} = h_1(\eta,r)\sin\theta\ \partial_\theta Y_{lm}.$$
(No loss of generality if $m=0$, in later considerations.)

Axial gravitational waves in FLRW cosmology

- **Perfect fluid** (ρ_0 and p_0 are the background mass density and pressure)

$$T_{\mu\nu} = (\rho_0 + p_0) u_\mu u_\nu + p_0 g_{\mu\nu} - \Lambda g_{\mu\nu};$$

- **only 4-velocity can be perturbed by the incident axial wave in the linear approximation** (Kulczycki, EM CQG 2017)

$$u_0 = -a(\eta), u_\phi = \sin\theta \cdot u(\eta, r) Y',$$

$Y' \equiv \partial_\theta Y_{l0}.$

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$$\partial_r \partial_\eta h_1 - \partial_r^2 h_0 - 2H \partial_r h_1 + \frac{2}{r} \partial_\eta h_1 - \frac{4H}{r} h_1 + \\ + \frac{l(l+1)}{r^2} h_0 = -2a^3 (\rho_0 + p_0) u \quad (1)$$

Axial gravitational waves in FLRW cosmology

- Define

$$h_1(\eta, r) = ra(\eta)Q(\eta, r);$$

- Q solves the master equation

$$\partial_\eta^2 Q - \partial_r^2 Q + \frac{l(l+1)}{r^2} Q - \frac{1}{2} a^2 \left(\frac{1}{3}\rho_0 - p_0 + \frac{4}{3}\Lambda \right) Q = 0,$$

- while

$$h_0(\eta, r) = \int_{\eta_0}^{\eta} h_1'(\tau, r) d\tau,$$

- where the conformal time η_0 characterizes the initial hypersurface.
- Solving the master equation completely specifies the full axial-wave solution.

Axial GW in the radiation epoch

- For the radiation fluid $p_0 = \frac{\rho_0}{3}$ and $\Lambda = 0$; the conformal factor $a(\eta) \propto \eta$ and $\rho_0 \propto \eta^{-4}$.
- The quadrupole GW master equation reads

$$\partial_\eta^2 Q - \partial_r^2 Q + \frac{6}{r^2} Q = 0.$$

- Its general solution has the form (see, for instance, Wylężek,EM CQG 2004)

$$Q(r, \eta) = r^2 \, \partial_r \, \frac{1}{r} \, \partial_r \left(\frac{g + h}{r} \right)$$

- where functions g and h depend on the combinations $r - \eta$ or $r + \eta$, respectively.

Exemplary solution

- Take the outgoing gravitational wave

$$Q(r,\eta) = r^2 \partial_r \frac{1}{r} \partial_r \left(\frac{g(r-\eta)}{r} \right)$$

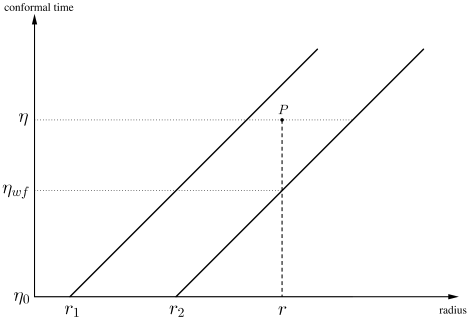

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Figure: Space-time diagram showing the propagation of the considered gravitational pulse ($\eta_{wf} = r - r_2 + \eta_0$). The dashed line shows the integration contour for h_0.

Memory effect

- Assume compact initial data g as in the figure, smooth everywhere except at the wave front, where the fourth derivative is allowed to have a jump. The wave front of this initial pulse is represented by the sphere $r = r_2 + \eta - \eta_0$. Let $a(\eta) = \eta$.
- If the wave front passes through a point P with coordinates (r, η), (that is $r > r_2$ — see fig. 1), then P starts to rotate
- $$u(r, \eta)\rho_0 a^3 = \frac{3r}{8}(r - r_2 + \eta_0)\partial_\eta^4 g(r_2 - \eta)|_{\eta_0}.$$
- The function u and the angular velocity $\Omega = \frac{u^\phi}{u^0} = C\frac{u\cos\theta}{ar^2}$ become nonzero.
- the passage of a wave front through a particle of fluid forces it to rotate around the symmetry axis, with the angular velocity Ω. This effect is permanent — the rotation persists after the wave leaves the region.

Main lesson

- **Gravitational axial waves, that are twice differentiable but which are not C^2 interact with matter in the radiation cosmological era**
- **and enforce a large-scale permanent cosmological rotation.**

Memory effect: sources of GW's

- What kind of physical processes would be responsible for discontinuities such as discussed above?
- They can result due to non-gravitational forces, associated with violent processes. In the modern cosmological era, the non-symmetric explosion of a supernova would constitute an example.

Manifestation of the axial GW memory effect in CMBR

- Zones influnced by GW's created during inflation era would be of the order (at least) of the particle horizon at the recombination era, — their angular sizes would be larger than (roughly) 1 degree. The rotation induced by this pulse would manifest itself through the Doppler-related perturbations of the spectra of the CMB radiation.
- Assume:
  i) a simplified evolution picture, where the Universe experiences a sudden transition from the radiation to the dust dominated epoch at the recombination time.
  ii) that the gravitational radiation originates during inflation.
- Then:
  the amplitude of the rotation velocity $|v|$ can be estimated,

$$|v| < \frac{|\partial_\eta^4 g(r_2 - \eta)|_{\eta_0}|}{H_0^2(1 + z_R)^3}.$$

H_0 is the present value of the Hubble constant and z_R is the redshift at the recombination epoch.

Manifestation of the axial GW memory effect in CMBR

- Waves of the type considered here might be present in the primordial (inflationary) gravitational radiation discussed in 1980's (Abbott and Wise, Nucl. Phys. B; Starobinsky SAL; Rubakov aet al., Phys. Lett. B) and recently reinvestigated by several researchers (Takahashi and Soda, PRL 2009; Book and Kamionkowski, PRL 2011; Masui et al., PRL 2017
- Its amplitude would be strongest close to the boundary of an elliptic region and weakest at its center, but the net effect would obviously depend on the orientation of the symmetry axis of the gravitational pulse and on a specific behaviour of the angular velocity Ω.

Conclusions

- **Smooth axial gravitational waves do not interact with cosmological fluids.**
- **Gravitational axial waves, that are twice differentiable but which are not C^2 interact with matter in the radiation cosmological era.**
- **They enforce a large-scale cosmological rotation**
- **that is permanent — it persists after the passage of the gravitational pulse.**

Appendix

- After (Wojciech Kulczycki EM (CQG 2017)):

$$\partial_r(f^2 \cdot (21)) - \partial_\eta(f^2 \cdot (20)) - (l(l+1)-2) \cdot (19) =$$

$$f^2 \partial_\eta \left(2a^3 \left(\rho_0 + p_0\right) u\right);$$

- the lhs vanishes for smooth slns, which implies $u = 0$ for compact pulses.