Simple inhomogeneous cosmological model in the context of back-reaction

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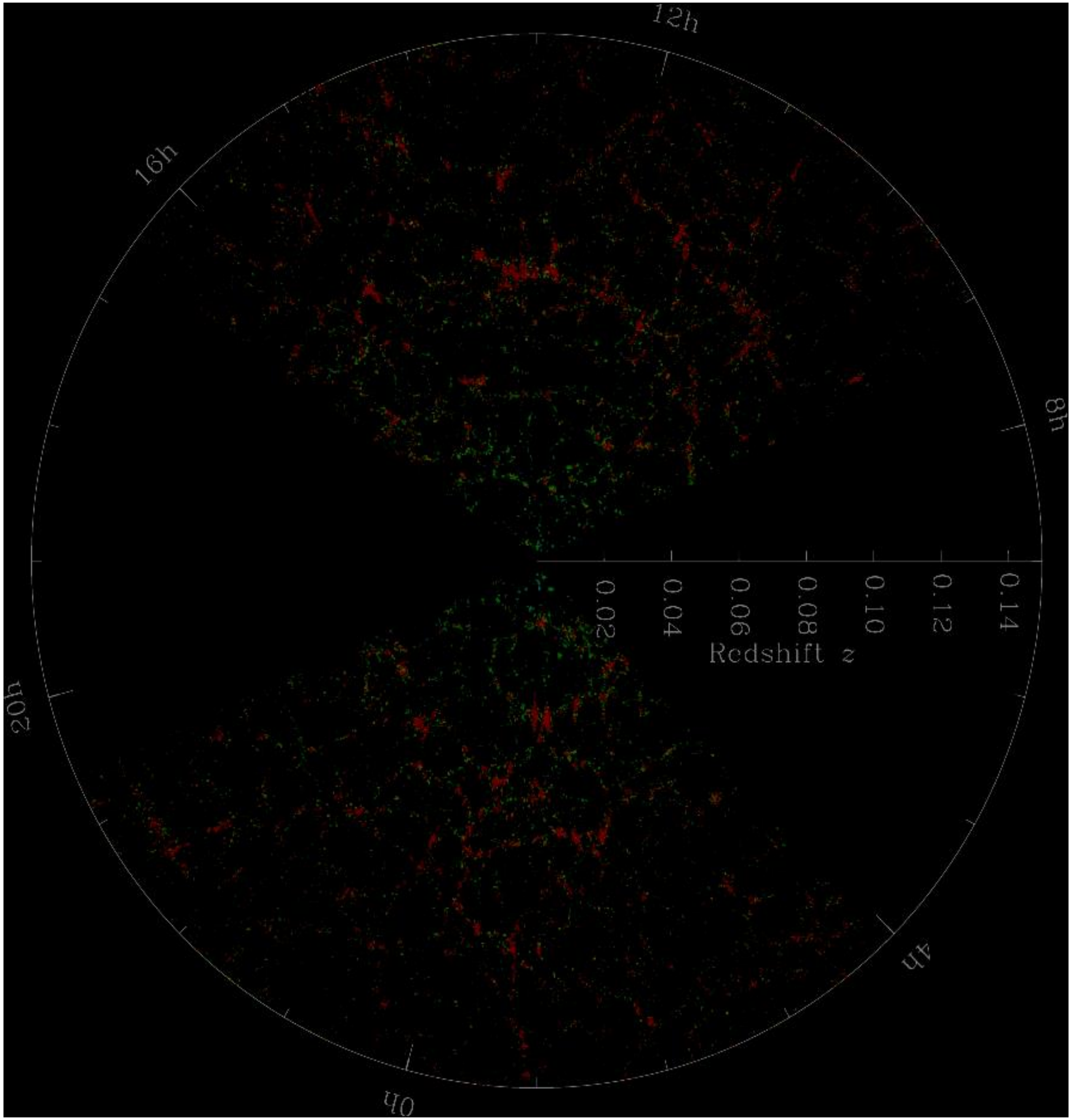
The problem of averaging in cosmology



Buchert averaging

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\downarrow$$

Scalar parts of the Einstein equations

$$\langle \Psi \rangle_{\mathcal{D}} = \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}(t)} \sqrt{\det g^{(3)}}\, \Psi(t, \vec{x})\, d^3x$$

Buchert averaging

$$\begin{cases} 3\dfrac{\ddot{a}_D}{a_D} + 4\pi G \langle \varrho \rangle_D = Q_D \\ 3\left(\dfrac{\dot{a}_D}{a_D}\right)^2 - 8\pi G \langle \varrho \rangle_D + \dfrac{1}{2}\langle R \rangle_D = -\dfrac{1}{2} Q_D \end{cases}$$

$$a_D(t) = \left(\frac{V_D(t)}{V_D(t_0)}\right)^{1/3} \quad \text{effective scale factor}$$

Green-Wald Scheme

Family of metrics $g_{\mu\nu}(\lambda)$ satisfying specified conditions

Background metric $g_{\mu\nu}(\lambda) \xrightarrow[\lambda\to 0]{} g^{(0)}_{\mu\nu}$

Weak limit

$$\underset{\lambda\to 0}{\text{w-lim}}\, A_{\mu\nu}(\lambda) = B_{\mu\nu} \Leftrightarrow \lim_{\lambda\to 0} \int A_{\mu\nu}(\lambda) f^{\mu\nu} = \int B_{\mu\nu} f^{\mu\nu}$$

$$G_{\mu\nu}[g_{\mu\nu}(\lambda)] = 8\pi T_{\mu\nu}$$

$\Downarrow$ weak limit

$$G_{\mu\nu}[g^{(0)}] = 8\pi T^{(0)}_{\mu\nu} + 8\pi t_{\mu\nu}$$

The problem of averaging in cosmology

Motivation of our work

We were looking for a simple inhomogeneous cosmological model,
n explicitly, for which one can apply the Green-Wald scheme and Buchert averaging techni

The metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \lambda h_{\mu\nu}$$

$$g^{(0)}_{\mu\nu} = \mathrm{diag}(-1, a^2(t), a^2(t), a^2(t)), \quad a(t) = \mathcal{C} t^{2/3}$$

$$h_{00} = 0 \quad h_{i0} = 0$$

$$h_{ij} = a^2(t)\left(C_{,ij} - \frac{1}{3}\delta_{ij}(C_{,xx} + C_{,yy} + C_{,zz}) + \delta_{ij} D\right)$$

$$C(t,x,y,z) = -\frac{\mathcal{C}^3}{81t}\left(f(x) + f(y) + f(z)\right)$$

$$D(t,x,y,z) = -\frac{\mathcal{C}^3}{243t}\left(\frac{d^2 f(x)}{dx^2} + \frac{d^2 f(y)}{dy^2} + \frac{d^2 f(z)}{dz^2}\right)$$

$$f(w) = \frac{w^2}{16} + \frac{1}{32B^2}\cos(2Bw)$$

The metric

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \mathcal{C}^2 \sqrt[3]{t} \left(-\frac{\mathcal{C}^3 \lambda}{324} \sin^2(Bx) + t \right) & 0 & 0 \\ 0 & 0 & \mathcal{C}^2 \sqrt[3]{t} \left(-\frac{\mathcal{C}^3 \lambda}{324} \sin^2(By) + t \right) & 0 \\ 0 & 0 & 0 & \mathcal{C}^2 \sqrt[3]{t} \left(-\frac{\mathcal{C}^3 \lambda}{324} \sin^2(Bz) + t \right) \end{bmatrix}$$

The constant $\mathcal{C}$ is determined by the condition $a(t_0) = 1$,

where t_0 is the age of the Einstein-de Sitter universe with a given H_0

The energy-momentum tensor

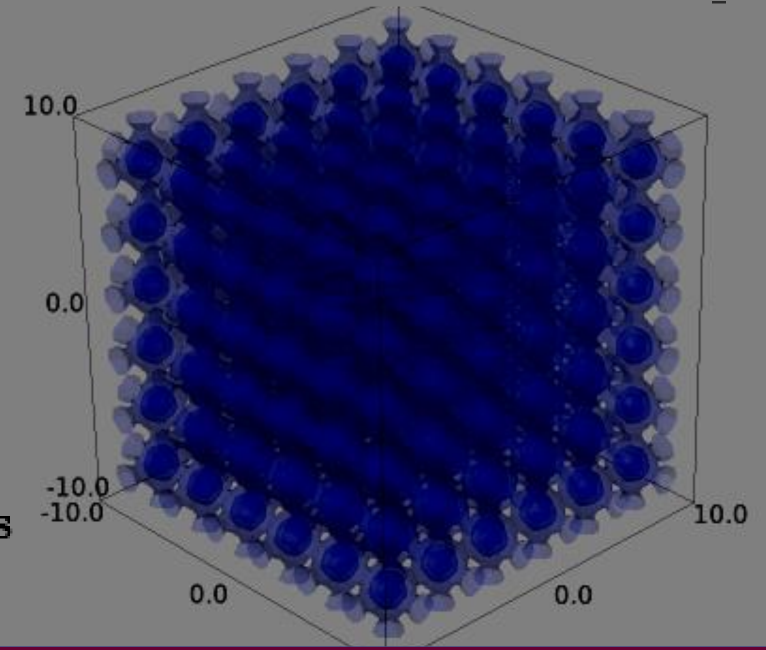
$$G_{\mu\nu} = G^{(0)}_{\mu\nu} + \lambda\, G^{(1)}_{\mu\nu} + \ldots$$

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + \lambda\, T^{(1)}_{\mu\nu} + \ldots$$

$$T^{(k)}_{\mu\nu} = G^{(k)}_{\mu\nu}/8\pi$$

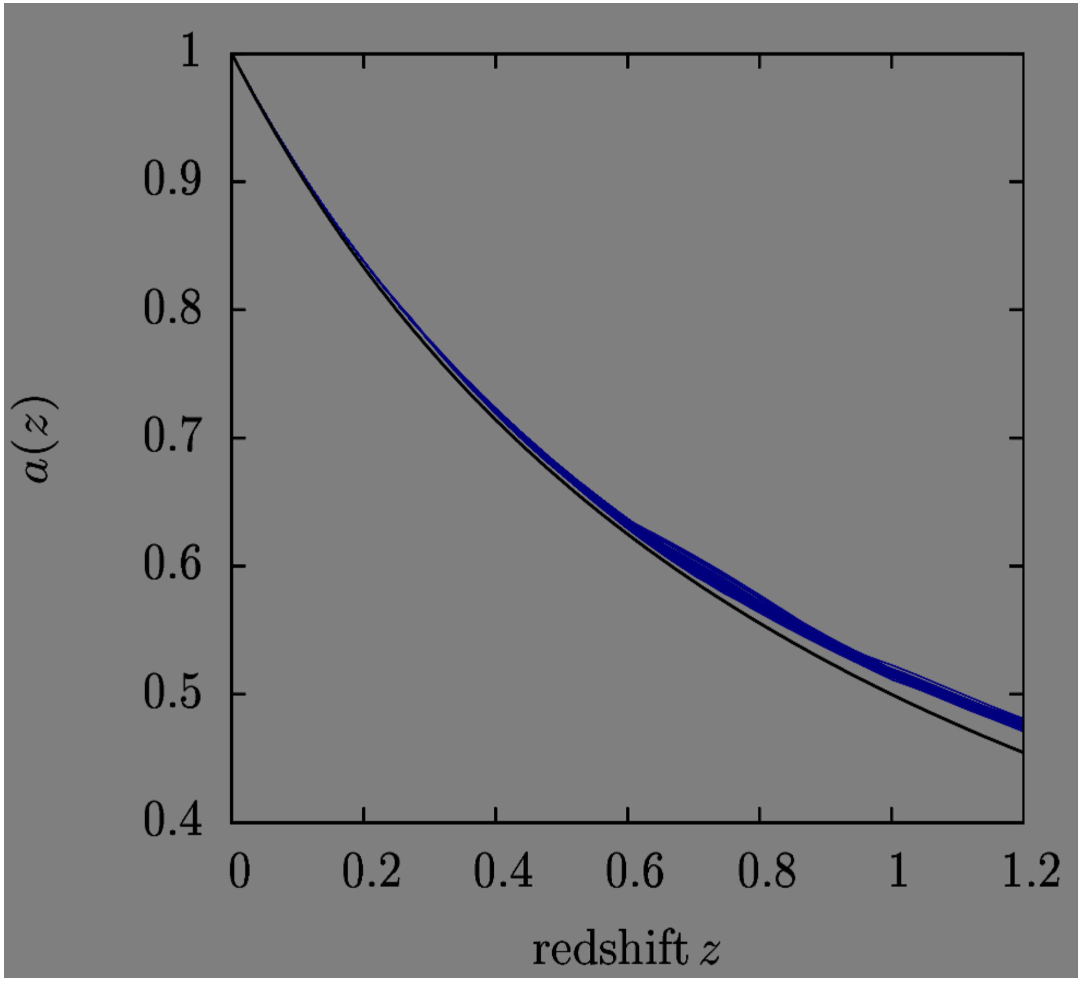
$$T^{(0)}_{\mu\nu} = \rho^{(0)}\, U_\mu\, U_\nu, \quad U^\mu = (1,0,0,0), \quad \rho^{(0)} = \frac{4}{3} t^{-2}$$

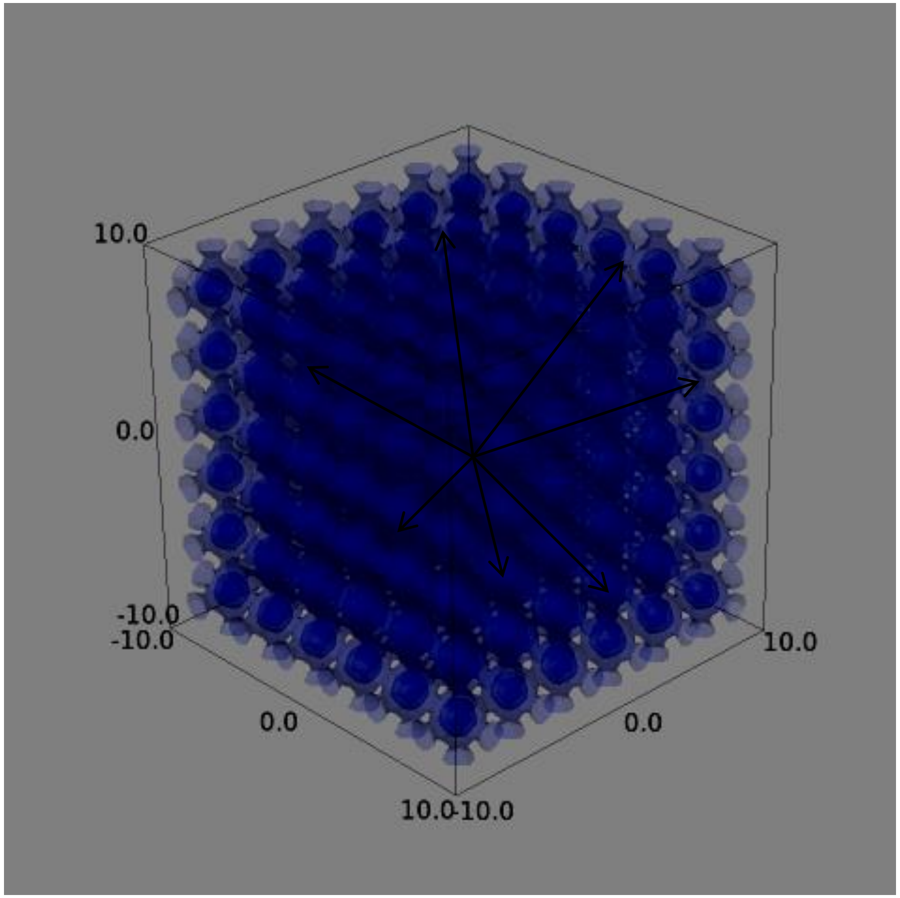
$$T^{(1)}_{\mu\nu} = \rho^{(1)}\, U_\mu\, U_\nu, \quad \rho(1) = \frac{1}{3888\pi t^3}\left(\mathcal{C}^3 \sin^2(Bx) + \mathcal{C}^3 \sin^2(By) + \mathcal{C}^3 \sin^2(Bz)\right)$$



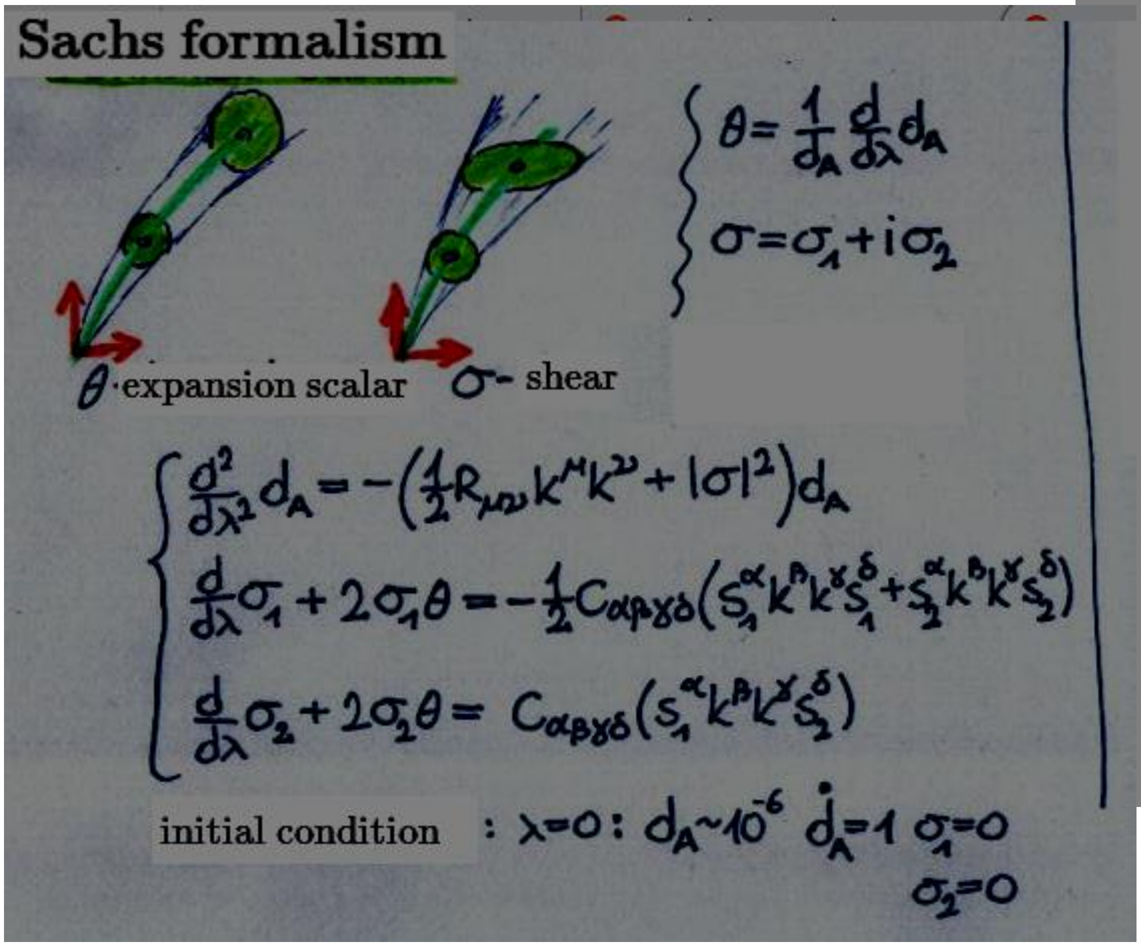
We fixed the scale parameter $B = 1$,
and the amplitude λ so that $\langle \rho^{(0)} \rangle_{\mathcal{D}}$ is 0.04 in critical units

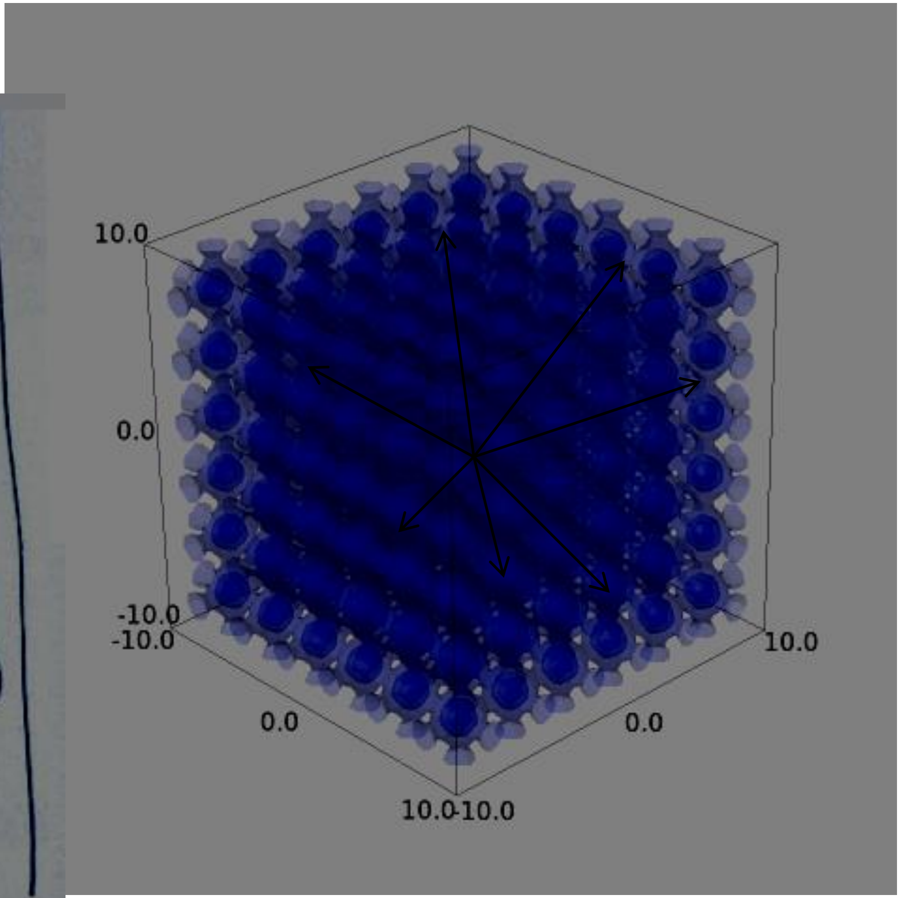
The null geodesics





The angular diameter distance





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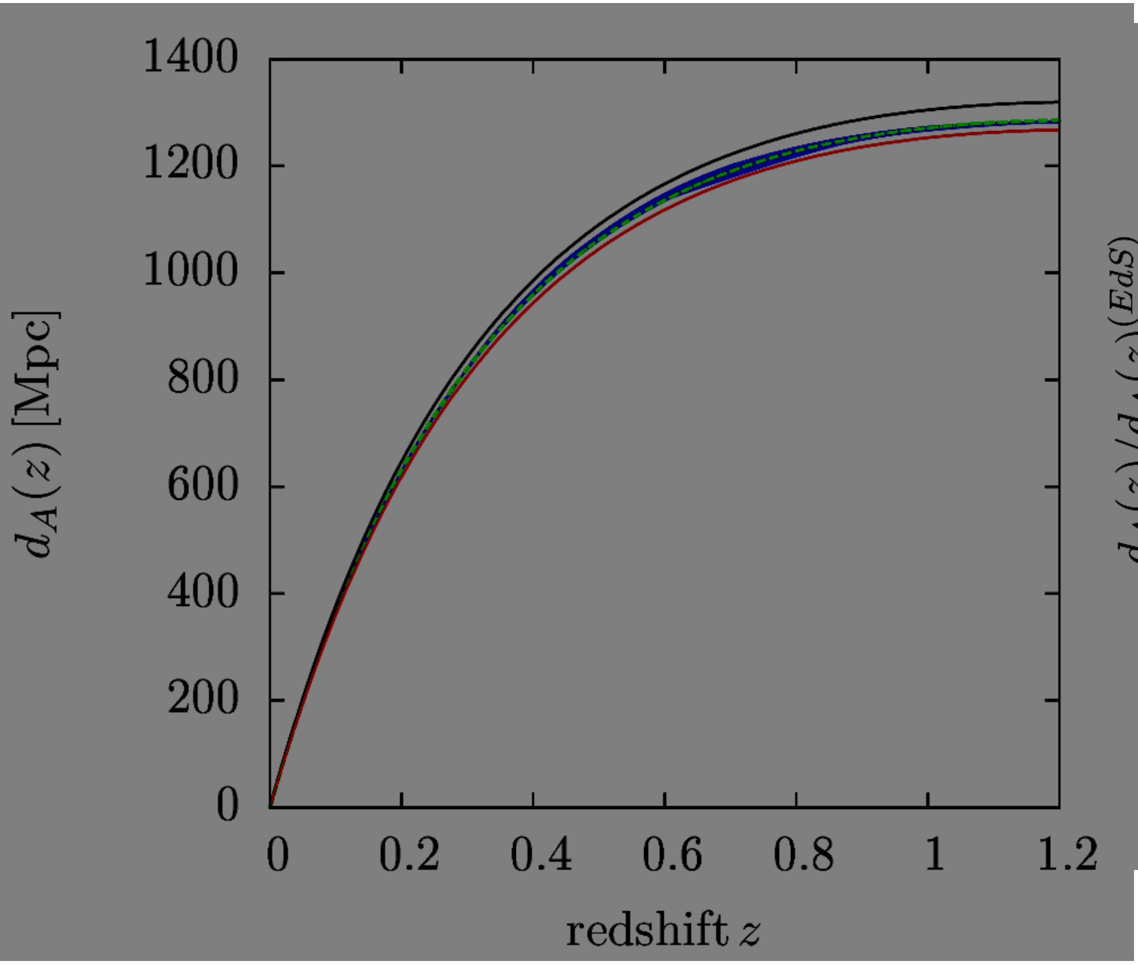

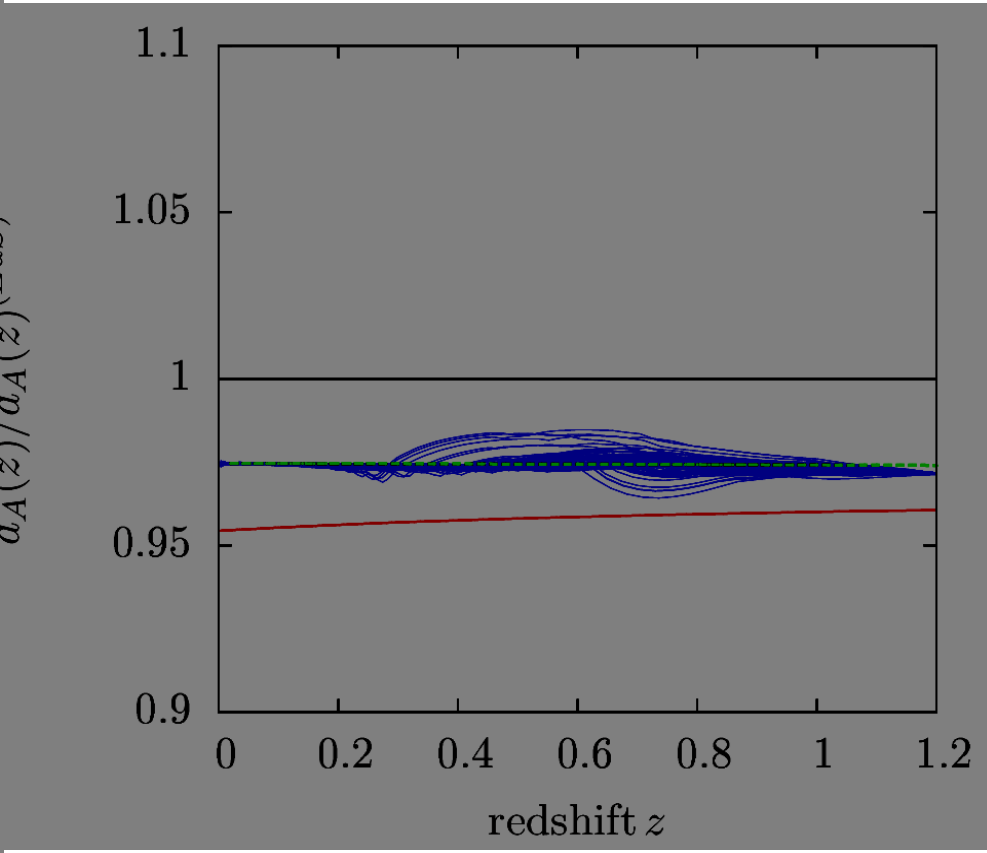

Thank you for your attention.

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