Relativistic low angular momentum accretion

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Contents

Motivation

- There are a lot of galactic centers hosting supermassive black holes
- There is a lot of matter in the vicinity of the supermassive black holes
- The vast majority of galaxy is inactive
- Possible explanation: low angular momentum accretion

Previous works

- Newtonian: **Begelman, Proga**, Janiuk, Kurosawa, Sznajder, Mościbrodzka.
- post-Newtonian: Paczyński, Wiita.
- GR: Font, Daigne, Gammie, McKinney, Bambi, Yoshida, Harada, Takahashi.

Setup

- Central black hole in the middle and perfect fluid around
- Background geometry: Schwarzschild or Kerr solution
- Horizon-penetrating coordinates: Eddington-Finkelstein or Kerr-Schild coordinates
- Bondi-type (radial) accretion flow as an initial data (in GR it is known as a *Michel solution*)
- Perturbation by adding a small amount of angular momentum

Basic equation

Hydrodynamic:

- Continuity equation:

$$\nabla_\mu(\rho u^\mu) = 0$$

- Euler equation:

$$\nabla_\mu T^{\mu\nu} = \nabla_\mu \left(\rho h u^\mu u^\nu + p g^{\mu\nu}\right) = 0$$

- Equation of state:

$$p = (\Gamma - 1)\rho\epsilon$$

  where:

  - ρ is the rest-mass density
  - $h = 1 + \epsilon + p/\rho$ is the specific enthalpy
  - ϵ is the specific internal energy
  - p is the pressure
  - u^μ is the four-velocity of the fluid
  - Γ is a constant

Equations of hydrodynamic – Valencia formulation

3+1 separation:

$$\alpha = \sqrt{-\tfrac{1}{g^{00}}},\ \beta_i = g_{0i},\ \beta^i = -\tfrac{g^{0i}}{g^{00}},\ \gamma_{ij} = g_{ij},\ \gamma^{ij} = g^{ij} + \tfrac{\beta^i \beta^j}{\alpha^2}$$

where:

- three-velocity

$$v^i = \frac{u^i}{\alpha u^0} + \frac{\beta^i}{\alpha}$$

- Lorentz factor

$$W = \alpha u^0 = \frac{1}{\sqrt{1 - v_i v^i}}$$

Equations of hydrodynamic – Valencia formulation

- The vector of conserved quantities

$$\mathbf{q} = (D, S_j, \tau) = \left(\rho W, \rho h W^2 v_j, \rho h W^2 - p - \rho W\right)$$

- The flux vectors

$$\mathbf{F}^i = \left(D\left(v^i - \frac{\beta^i}{\alpha}\right), S_j\left(v^i - \frac{\beta^i}{\alpha}\right) + p\delta^i_j, \tau\left(v^i - \frac{\beta^i}{\alpha}\right) + pv^i\right)$$

- The vector of source terms

$$\boldsymbol{\Sigma} = \left(0, T^{\mu\nu}\left(\partial_\mu g_{\nu j} - \Gamma^\delta_{\mu\nu} g_{\delta j}\right), \alpha\left(T^{\mu 0}\partial_\mu \ln\alpha - T^{\mu\nu}\Gamma^0_{\mu\nu}\right)\right)$$

Equations of hydrodynamic – Valencia formulation

$$\frac{1}{\sqrt{-g}}\partial_t\left(\sqrt{\gamma}\mathbf{q}\right)+\frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}\mathbf{F}^i\right)=\mathbf{\Sigma}$$

where:

- $g = \det g_{\mu\nu}$
- $\gamma = \det \gamma_{ij}$
- $\sqrt{-g} = \alpha\sqrt{\gamma}$

Initial data

- Stationary, spherically symmetric Bondi-Michel-type accretion solution. [H. Bondi, **1952**], [F. C. Michel, **1972**]
- $l = -u_\phi/u_t$ [Kozłowski, Jaroszyński & Abramowicz **1978**]
- Some amount of angular momentum $l = f(\theta) = l_0(1 - |\cos\theta|)$ [D. Proga & M. Begelman, **2003**]

Boundary conditions

- We use spherical grid (with the radial variable ranging from $r_{\rm in}$ to $r_{\rm out}$) and horizon-penetrating coordinates
- On the inner boundary of the grid (beneath the horizon of the black hole) we use outflowing boundary conditions
- On the outer boundary of the grid:
  - every quantities is fixed as the values characteristic for the initial conditions, except following two
  - v_r - radial component of the velocity evolves freely
  - v_θ - polar component of the velocity is fixed or evolves freely

Numerical code – technical issues.

- Solver for Riemann problem – HLLE (Harten, Lax, van Leer and Einfeldt)
- Time evolution – Runge-Kutta method
- Output – vtk-file.

Coordinates

- Eddington-Finkelstein coordinates ($c = G = 1$):

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{4M}{r} dtdr + \left(1 + \frac{2M}{r}\right) dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

- 3+1 separation:

$$ds^2 = -(\alpha^2 - \beta_i \beta^i) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j,$$

where

  - $\alpha = \sqrt{r/(r + 2M)}$,
  - $\beta_r = 2M/r$,
  - $\gamma_{rr} = 1 + 2M/r$,
  - $\gamma_{\theta\theta} = r^2$,
  - $\gamma_{\phi\phi} = r^2 \sin^2\theta$.

Hydrodynamic equations in Eddington-Finkelstein coordinates

We have one equation

$$\partial_t \mathbf{q} + \partial_r (\alpha \mathbf{F}^r) + \partial_\theta \left(\alpha \mathbf{F}^\theta\right) + \partial_\phi \left(\alpha \mathbf{F}^\phi\right) = \alpha \mathbf{\Sigma} - \left(2 - \frac{M\alpha^2}{r}\right) \frac{\alpha}{r} \mathbf{F}^r - \cot\theta \alpha \mathbf{F}^\theta,$$

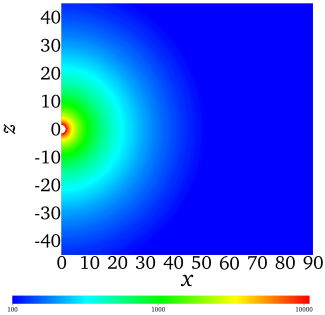
where the sources $\mathbf{\Sigma}$ read:

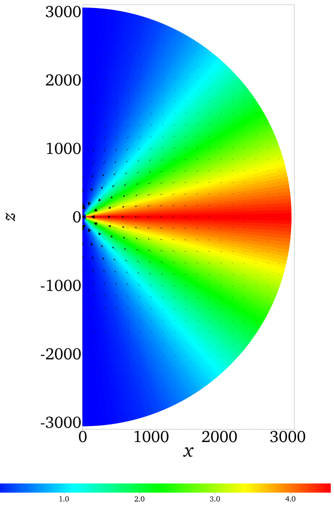
$$\begin{aligned}
\Sigma_1 &= 0, \\
\Sigma_2 &= \frac{2p}{r} + \frac{\rho h W^2}{r}\left(v_\theta v^\theta + v_\phi v^\phi - \frac{M}{r}(v^r + \alpha)^2\right), \\
\Sigma_3 &= \cot\theta \left(p + \rho h W^2 v_\phi v^\phi\right), \\
\Sigma_4 &= 0, \\
\Sigma_5 &= \frac{M\alpha}{r^2}\left(2\left(1 + \frac{3M}{r}\right)\alpha^2 p + \rho h W^2 \left(2\left(v_\theta v^\theta + v_\phi v^\phi\right)\right.\right. \\
&\qquad \left.\left. -2\alpha^2\left(1 + \frac{2M^2}{r^2} + \frac{3M}{r}\right)(v^r)^2 - \alpha v^r\right)\right).
\end{aligned}$$

Some particullar configuration which we investigated

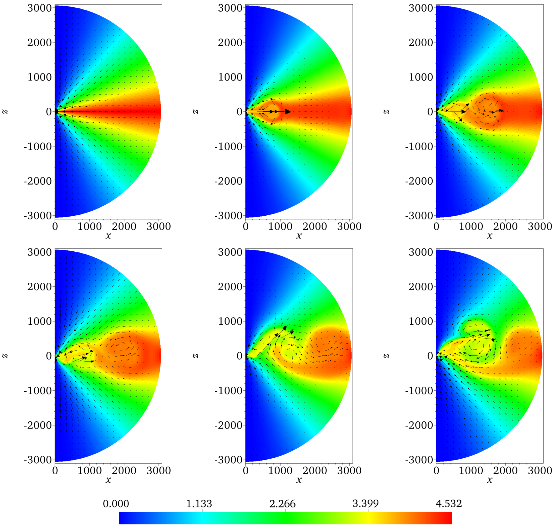
No.	r'_S	$c_{s,\infty}$	r_B	r_{out}	l_0	v^θ
1a	10^{-2}	0.07071	200	300	6.657	free
2a	10^{-3}	0.02236	2000	3000	4.5714	free
3a	10^{-3}	0.02236	2000	3000	10.204	free
4a	$10^{-3.5}$	0.01257	6324	9487	14.285	free
1b	10^{-2}	0.07071	200	300	6.657	fixed
2b	10^{-3}	0.02236	2000	3000	4.5714	fixed
3b	10^{-3}	0.02236	2000	3000	10.204	fixed
4b	$10^{-3.5}$	0.01257	6324	9487	14.285	fixed

Initial state of the system: Three-velocity $\mathbf{v} = (v^r, 0, 0)$ (Michel solution), angular momentum $l = l_0(1 - |\cos\theta|)$.

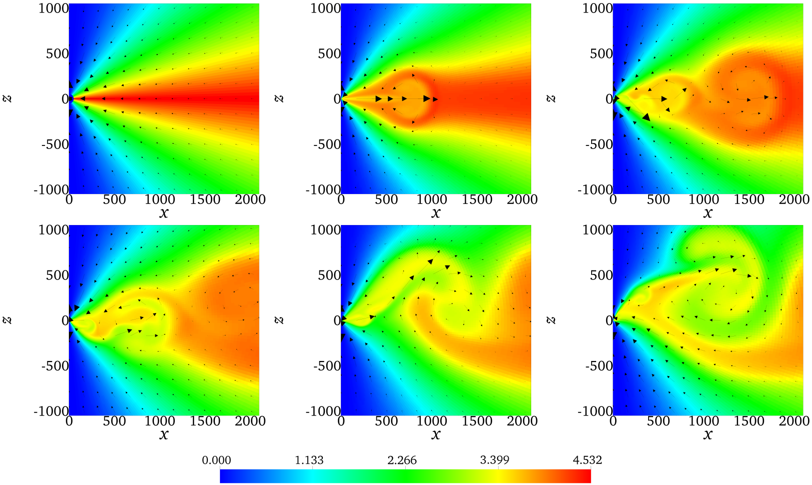




Angular momentum during the evolution (free boundary conditions).

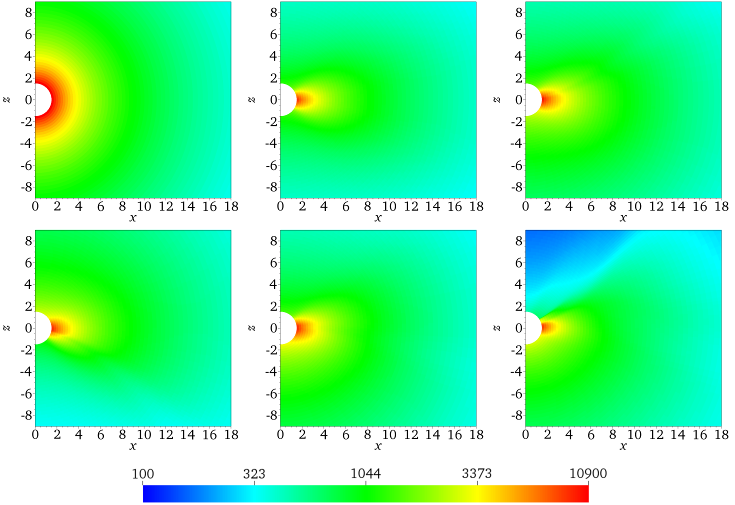


Angular momentum during the evolution (free boundary conditions).

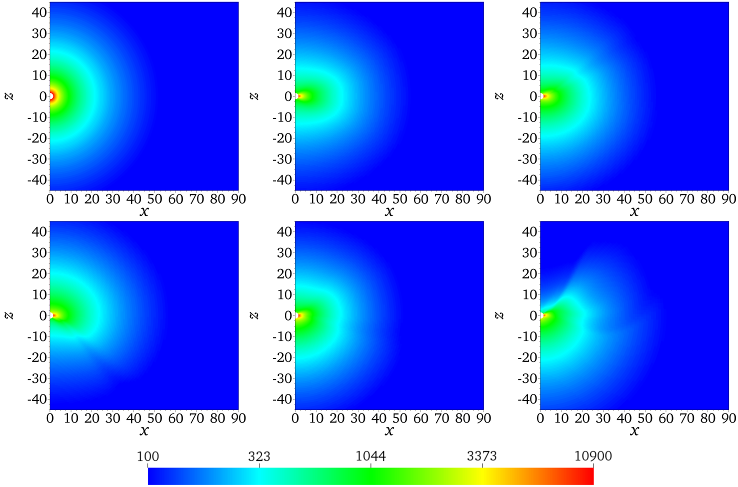


Start movie

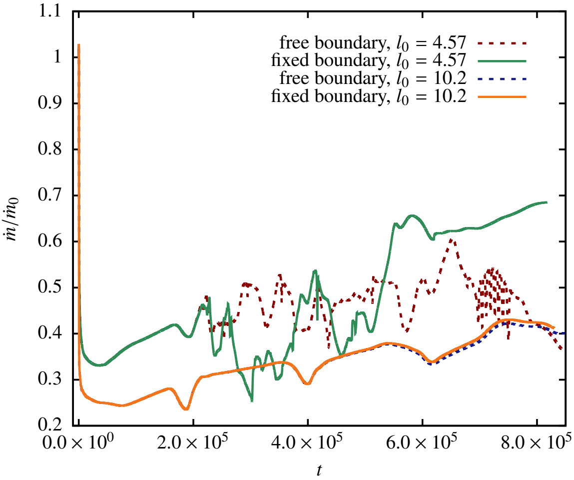
Density during the evolution (free boundary conditions).



Density during the evolution (free boundary conditions).



mass accretion rate during the evolution



Progai i Begelmana's original result of mass accretion rate during the evolution

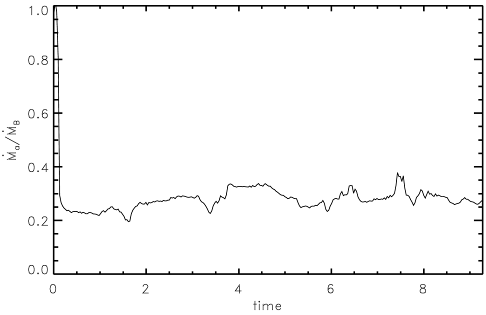


FIG. 5.—Time evolution of the mass accretion rate in units of the Bondi rate, for model B04f1a.

Summary

- Our results confirm the original Newtonian results.
- The mass accretion rate is smaller than the mass accretion rate for Bondi-type (Michel) solution in all cases which we considered.
- Outer boundary conditions do not influence the behavior of the flow in the center of the system.

A Thorne–Żytkow object.