A possible connection of blueshifts in the Lemaitre - Tolman and Szekeres models with the gamma-ray bursts

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Contents

1. Motivation and background

In Robertson – Walker spacetimes, a light ray emitted at the Big Bang (BB) reaches all later observers with ***infinite redshift*** ($z_{obs} \to \infty$, $\nu_{obs} \to 0$).

In ***Lemaître – Tolman (L-T)*** and ***Szekeres*** spacetimes, ***some rays*** from the BB reach all observers with ***infinite blueshift*** ($z_{obs} \to -1$, $\nu_{obs} \to \infty$).

Necessary conditions for $\nu_{obs} \to \infty$ are:

(1) At the emission point the BB time $t_B(r)$ has $dt_B/dr \neq 0$ [1].

(2) In L-T, the ray is ***radial*** [2].

→ Rays emitted *close* to the BB can display strong (finite) blueshifts ($\nu_{obs} >> \nu_{em}$).

In the L-T and Szekeres spacetimes, constant BB ($dt_B/dr \equiv 0$) is an exception, and Friedmann models are a subset of this exception.

[1] P. Szekeres, Naked singularities. In: *Gravitational Radiation, Collapsed Objects and Exact Solutions*. Edited by C. Edwards. Springer (Lecture Notes in Physics, vol. 124), New York, pp. 477 -- 487 (1980).
[2] C. Hellaby and K. Lake, The redshift structure of the Big Bang in inhomogeneous cosmological models. I. Spherical dust solutions. *Astrophys. J.* **282**, 1 (1984) + erratum *Astrophys. J.* **294**, 702 (1985).

Impulses of high-frequency electromagnetic radiation (X rays and gamma-ray bursts, GRBs) are observed.

It is known nearly for sure that GRB sources are a few billion light-years away [3].

What if?

the GRBs were emitted simultaneously with the radiation now seen as the CMB, but were blueshifted by the L-T/Szekeres mechanism?

The CMB rays were emitted $\tau \approx 380\,000$ years after the BB [4].

Can any rays emitted then reach us now with a blueshift instead of redshift?

Can the blueshift account for the frequencies of the GRBs?

Yes to both questions! [5 – 7].

[3] D. Perley, http://w.astro.berkeley.edu/~dperley/pub/grbinfo.html
[4] http://astronomy.swin.edu.au/cosmos/e/epoch+of+recombination
[5] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[7] A. Krasiński, Modeling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.

2. Basic properties of gamma-ray bursts (GRBs) [(H) = hypotheses].

(1) The GRB frequencies are contained in the range [8]

$\nu_{\gamma\ min} \approx \mathbf{0.24 \times 10^{19}\ Hz} < \nu < \mathbf{1.25 \times 10^{23}\ Hz} \approx \nu_{\gamma\ max}$. (Converted from keV to Hz by $\nu = E/h$)

(2) GRBs typically last from < 1 second to a few minutes (exceptionally up to 30 hours) [9].

(3) Most GRBs are followed by longer-lived and fainter **afterglows** at longer waves.

(H) It is believed that **all GRBs have afterglows**, but some of them were missed by observers [10].
Nearly all knowledge about GRBs comes from observations of the afterglows [10].

(4) **(H)** GRBs are probably **focussed into narrow jets** [3].

(5) **(H)** Nearly all GRBs come from distances 10^8 ly < d < several billion ly.

Why **(H)**? The distances are calculated from redshifts measured for the afterglows using the Friedmann relations, so they may be grossly underestimated [5] – see Appendix.

(6) About one GRB per day is observed [3], so the sources must be many.

[3] D. Perley, http://w.astro.berkeley.edu/~dperley/pub/grbinfo.html
[5] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[8] A. Goldstein *et al*., The *Fermi* GBM gamma-ray burst spectral catalog: the first two years. *Astrophys. J. Suppl.* **199**, 19 (2012).
[9] S. J. Smartt, A twist in the tale of the γ-ray bursts. *Nature* **523**, 164 (2015).
[10] Gamma Ray Burst Afterglow, http://astronomy.swin.edu.au/cosmos/G/gamma+ray+burst+afterglow

(1) Frequencies in the range $\nu_{\gamma\, min} \approx 0.24 \times 10^{19}$ Hz $< \nu < 1.25 \times 10^{23}$ Hz $\approx \nu_{\gamma\, max}$
(2) Durations from < 1 second to 30 hours.
(3) Afterglows.
(4) Collimation into narrow jets.
(5) Distances > 10^8 ly up to several billion ly.
(6) Many sources.

No explanation of origins of the GRBs is universally accepted.

Explanations exist for different classes of GRBs:

gravitational collapse to a black hole,

supernova explosions,

collisions of ultra-dense neutron stars.

I will show how properties (1) – (6) are accounted for by models using the blueshift mechanism.

3. The Lemaître - Tolman (L-T) models

The metric of the L-T models is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{{R_{,r}}^2}{1+2E(r)}\mathrm{d}r^2 - R^2(t,r)\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2\right) \quad (3.1)$$

where R(t,r) is determined by

$$ {R_{,t}}^2 = 2E(r) + \frac{2M(r)}{R} - \frac{1}{3}\Lambda R^2, \quad (3.2)$$

E(r) and M(r) are arbitrary functions. This is a dust solution of Einstein's equations (p = 0), with the mass density

$$\frac{8\pi G}{c^2}\rho = \frac{2M_{,r}}{R^2 R_{,r}} \ . \quad (3.3)$$

It was found by Lemaître [11] in 1933, then investigated by Tolman [12] in 1934 and Bondi [13] in 1947.

And by > 100 other authors in later years. The number is still growing.

[11] G. Lemaître, L'Univers en expansion [The expanding Universe], *Ann. Soc. Sci. Bruxelles* **A53**, 51 (1933); *Gen. Rel. Grav.* **29**, 641 (1997).
[12] R. C. Tolman, Effect of inhomogeneity on cosmological models, *Proc. Nat. Acad. Sci. USA* **20**, 169 (1934); *Gen. Rel. Grav.* **29**, 935 (1997).
[13] H. Bondi, Spherically symmetrical models in general relativity. *Mon. Not. Roy. Astr. Soc.* **107**, 410 (1947; *Gen. Rel. Grav.* **31**, 1783 (1999).

$$ds^2 = dt^2 - \frac{{R,_r}^2}{1 + 2E(r)} dr^2 - R^2(t,r)\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) \qquad (3.1) \qquad {R,_t}^2 = 2E(r) + \frac{2M(r)}{R} - \frac{1}{3}\Lambda R^2, \qquad (3.2)$$

In integrating (3.2), one more arbitrary function appears:

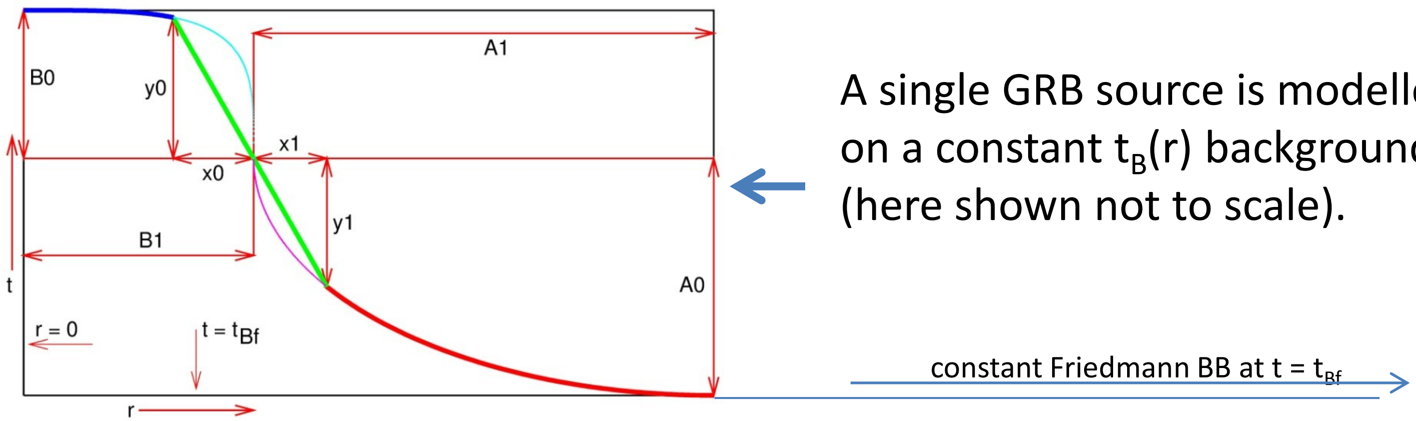
$$\int_0^R \frac{d\widetilde{R}}{\sqrt{2E + 2M/\widetilde{R} + \frac{1}{3}\Lambda \widetilde{R}^2}} = t - t_B(r) \quad ; \qquad (3.4)$$

$t = t_B(r)$ is the (position-dependent) Big Bang.

The Friedmann limit follows when $M/E^{3/2}$ and t_B are constant; then $R(t,r) = M^{1/3}$ Ɽ(t).

I will consider a Friedmann background into which an L-T island is matched.

4. An L-T model of a single GRB source



A single GRB source is modelled by a hump on a constant $t_B(r)$ background (here shown not to scale).

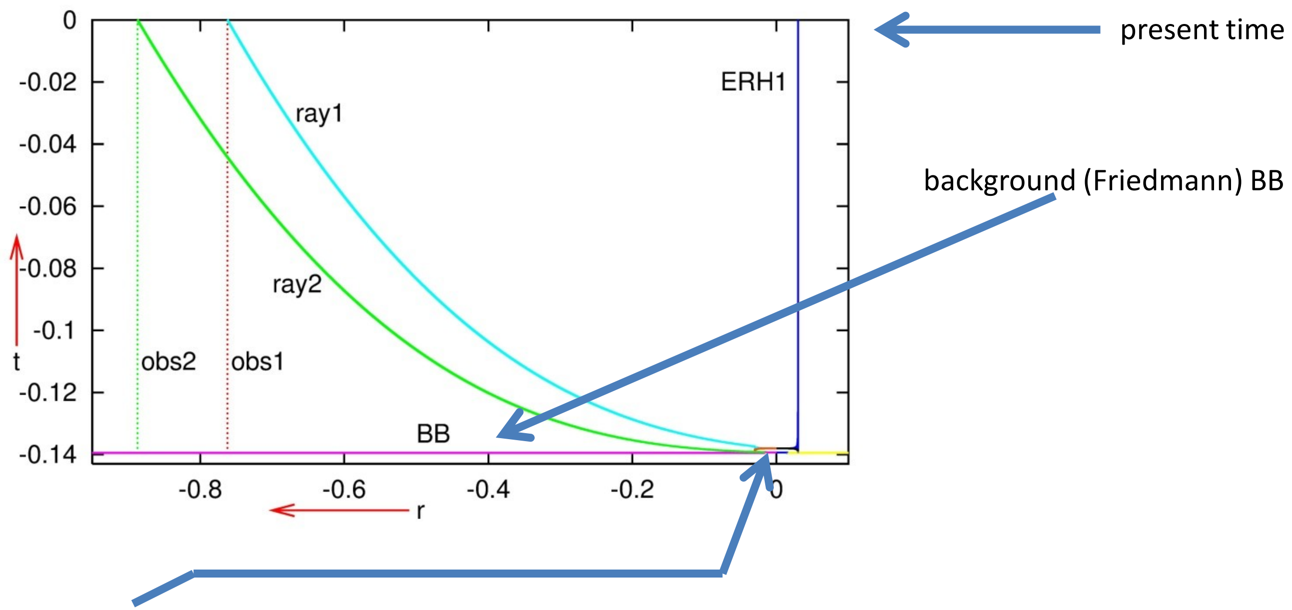
The upper arc is a segment of the curve:

$$\frac{r^4}{{B_1}^4} + \frac{(t - t_{\rm Bf} - A_0)^4}{{B_0}^4} = 1 \quad \text{or} \quad \frac{r^6}{{B_1}^6} + \frac{(t - t_{\rm Bf} - A_0)^6}{{B_0}^6} = 1. \qquad (4.1)$$

The lower arc is a segment of an ellipse.

The straight segment prevents $dt_B/dr \to \infty$ at the junction of full arcs.

The free parameters are A_0, A_1, B_0, B_1 and x_0.



Here two humps are drawn in proportion to the age of the Universe

The lower hump models a GRB source of the lowest observed energy.

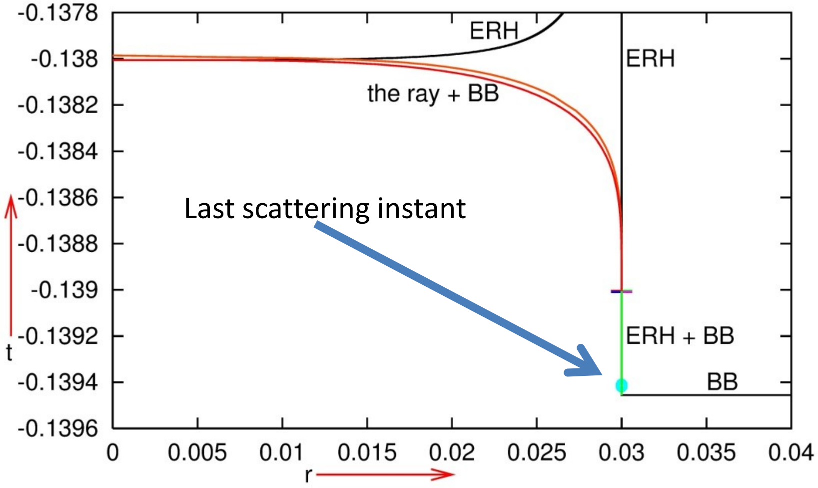
It has the height 8.9×10^{-4} × (the age of the Universe) ≈ 1.23×10^{7} years.

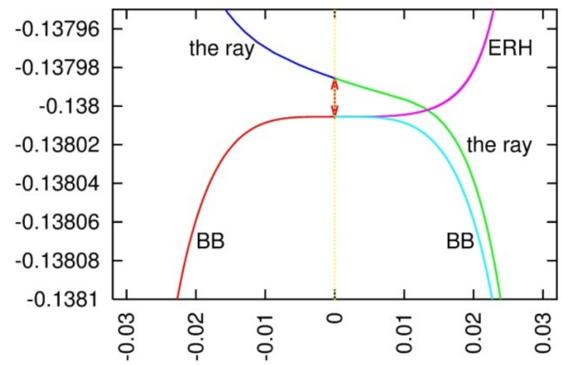
The Universe age taken from the ΛCDM model.

It encompasses the mass ≈ 3.1×10^{6} masses of our Galaxy,

and its upper arc is of 6-th degree.

The higher hump is ≈ 11 times as high and twice as wide; it models a GRB source of the highest energy.





Top of the hump, magnified

The real profile of the lower hump

Backward in time the ray acquires redshift (z increases) up to the first intersection with the ***Extremum Redshift Hypersurface*** (ERH).

Further into the past, the ray acquires blueshift (z decreases) until it intersects the ERH again (or until it hits the BB).

The hump parameters are chosen such that for the „best" ray

$$2.5 \times 10^{-8} < 1 + z_{\text{observed now}} < 1.7 \times 10^{-5}$$

which moves the frequencies from the hydrogen emission range to the GRB range:

$$0.24 \times 10^{19} < \nu_{GRB} < 1.25 \times 10^{23} \text{ Hz.}$$

Improved models of this type account for [5-7,14]:

(1) The observed frequencies of the GRBs [0.24×10^{19}Hz $\leq \nu \leq 1.25 \times 10^{23}$Hz];

(2) Their brief duration [14] (up to 30 hours);

(3) The afterglows (observed durations: up to several hundred days);

(4) The collimation of the GRBs into narrow jets;

(5) The large distances to their sources ($n \times 10^9$ ly);

(6) The multitude of the observed GRBs (observed: about 1/day, the currently best model implies up to ≈11 000 potential sources in the whole sky at present).

Re (3): the afterglows are there, but their durations are too long and the models need still further improvements.

[5] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[7] A. Krasiński, Modeling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.
[14] A. Krasiński, Modeling the durations of the gamma-ray bursts in a quasi-spherical Szekeres metric. In preparation.

5. The quasi-spherical Szekeres (QSS) models

The QSS solutions [15] have the metric

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}t^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} \mathrm{d}r^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(\mathrm{d}x^2 + \mathrm{d}y^2\right), \\ \mathcal{E} &\overset{\text{def}}{=} \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2}, \end{aligned} \tag{5.1}$$

where E(r), M(r), P(r), Q(r) and S(r) are arbitrary functions, and

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{5.2}$$

The mass density and pressure are

$$\kappa\rho = \frac{2\left(M/\mathcal{E}^3\right),_r}{(\Phi/\mathcal{E})^2\,(\Phi/\mathcal{E}),_r}, \qquad \kappa = \frac{8\pi G}{c^2}, \qquad p = 0. \tag{5.3}$$

Eq. (5.2) is the same as in L--T, so

$$\int_0^\Phi \frac{\mathrm{d}\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{5.4}$$

[15] P. Szekeres, A class of inhomogeneous cosmological models. *Commun. Math. Phys.* **41**, 55 (1975).

$$ds^2 = dt^2 - \frac{\mathcal{E}^2 (\Phi/\mathcal{E}),_r{}^2}{1 + 2E(r)} dr^2 - \frac{\Phi^2}{\mathcal{E}^2} \left(dx^2 + dy^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} + \frac{1}{3}\Lambda\Phi^2. \tag{5.2}$$

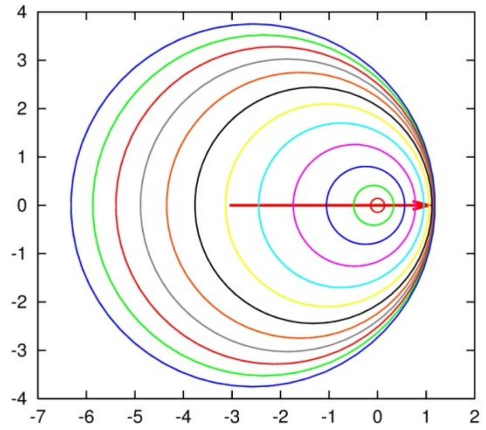
$$\int_0^\Phi \frac{d\widetilde{\Phi}}{\sqrt{2E + 2M/\widetilde{\Phi} + \frac{1}{3}\Lambda\widetilde{\Phi}^2}} = t - t_B(r). \tag{5.4}$$

A general QSS metric has no symmetry.

The surfaces of constant t and r

$ds^2 = \frac{\Phi^2}{\mathcal{E}^2} \left(dx^2 + dy^2\right)$

are ***nonconcentric spheres***.



In the limit of constant (P, Q, S) the spheres become concentric and L-T results.

$$ds^2 = dt^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E}),_r{}^2}{1+2E(r)}dr^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(dx^2+dy^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} \tag{5.2}$$

6. Blueshifts in axially symmetric QSS models

In L-T, z = -1 is possible only on radial rays. But a general Szekeres model has no symmetry, so no radial directions. Can large blueshifts exist in it at all?

In an *axially symmetric* QSS model, a *necessary condition* for infinite blueshift is that the ray is ***axial*** (intersects every space of constant t on the symmetry axis) [6].

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This condition happens to be also ***sufficient***, but this was verified only numerically for an exemplary model:

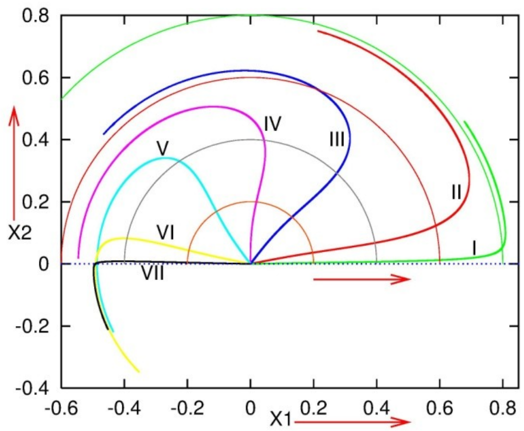
$2E(r) = - k r^2$, with k = -0.4, (6.1)

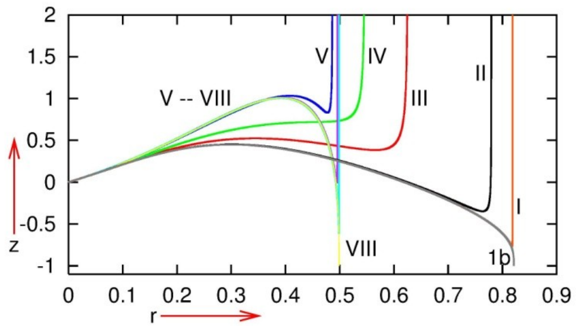
P = Q = 0 (for axial symmetry), $S^2(r) = a^2 + r^2$ (for simplicity), (6.2)

$$t_B(r) = \begin{cases} A\left(e^{-\alpha r^2} - e^{-\alpha {r_b}^2}\right) + t_{BB} & \text{for } r \le r_b, \\ t_{BB} & \text{for } r \ge r_b, \end{cases} \tag{6.3}$$

where A, α, r_b and t_{BB} are constants.

[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).





Rays 1b and VIII are axial.

Rays projected on a surface of constant t and φ (left) and z-profiles along them (right)

$X_2 = 0$ is the projection of the symmetry axis.

$z_{min} \rightarrow -1$ when the ray approaches axial. On rays 1b and VIII, $1 + z_{min} < 10^{-5}$.

Non-axial rays hit the BB hump tangentially to r = constant surfaces, with $z_{obs} \rightarrow \infty$.

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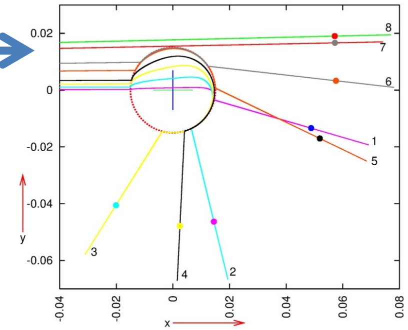
Rays overshooting the hump are strongly deflected and hit the BB in the Friedmann region with $z_{obs} = \infty$.



$$ds^2 = dt^2 - \frac{\mathcal{E}^2(\Phi/\mathcal{E}),_r{}^2}{1+2E(r)}dr^2 - \frac{\Phi^2}{\mathcal{E}^2}\left(dx^2+dy^2\right), \qquad \mathcal{E} = \frac{(x-P)^2}{2S} + \frac{(y-Q)^2}{2S} + \frac{S}{2} \tag{5.1}$$

$$\Phi,_t{}^2 = 2E(r) + \frac{2M(r)}{\Phi} \tag{5.2}$$

7. Blueshifts in nonsymmetric QSS models

In QSS models without symmetry there was no hint whether blueshifted rays exist at all; the search for them had to be done numerically all the way [6].

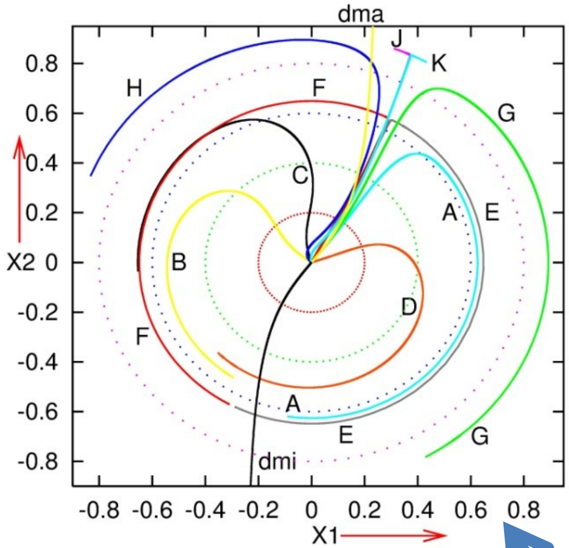
****

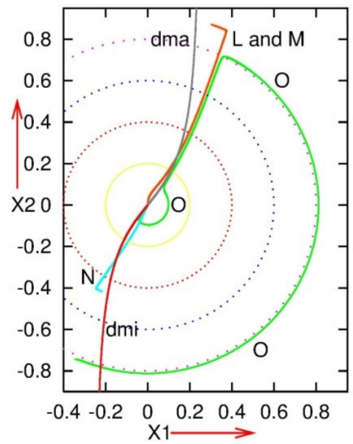
E(r), S(r) and t_B(r) were the same as before, but P and Q had to be nonconstant to destroy the symmetry:

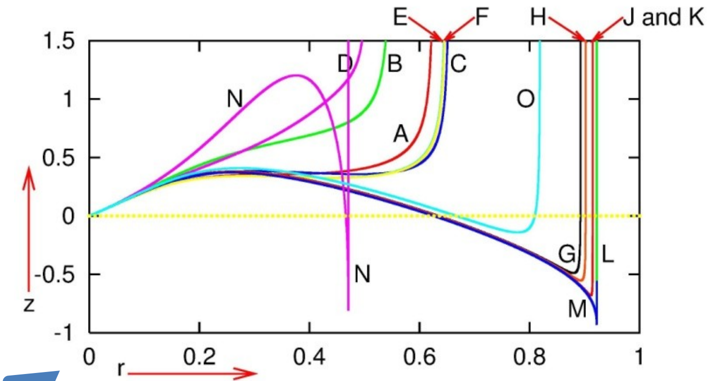
$$P(r) = \frac{pa}{2\left(a^2+r^2\right)}, \qquad Q(r) = \frac{qa}{\sqrt{a^2+r^2}}, \tag{7.1}$$

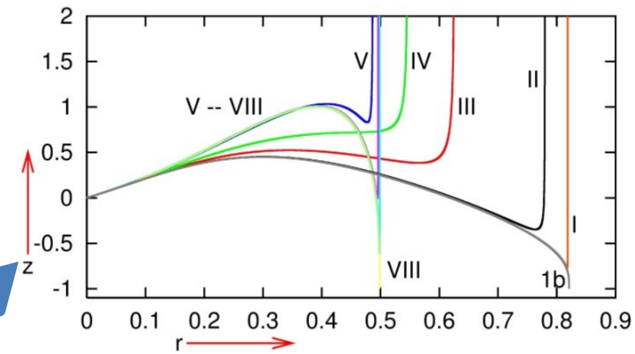
where *p* and *q* are constant parameters.

[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).









Projections of **exemplary rays** and **redshift profiles** along them in a nonsymmetric QSS model.

The z(r) graphs are similar to the axially symmetric case.

➔ In a general Szekeres model blueshifts are strong along two opposite directions.

But these directions ***do not*** coincide with the two principal null directions of the Weyl tensor, except in the axially symmetric case [6].

[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).

8. A realistic QSS model of a GRB

The exemplary QSS models were illustrative, but unrelated to cosmology.

With an axially symmetric QSS superposed on a realistic L–T, $1 + z$ along the axial direction is smaller → the GRBs are accounted for with a lower BB hump [7].

This reduces the angular size of the source and increases its distance from the observer → more sources fit into the observer's field of view [7].

[7] A. Krasiński, Modeling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.

In the current best Szekeres/Friedmann model, the angular radius of a GRB source is

$0.9681^\circ < \vartheta < 0.9783^\circ$,

depending on the direction of observation (the current resolution of GRB detectors is a disk in the sky of radius ≈ 0.5°).

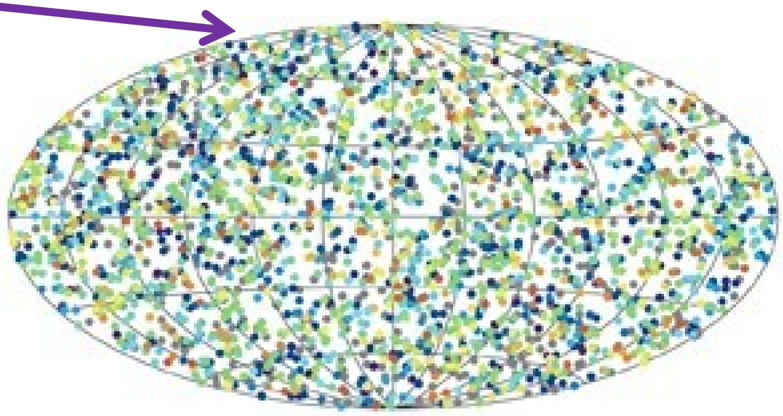
Thus, the whole sky could accommodate

11 005 > *N* > 10 076

such objects (with $\vartheta = 0.5^\circ$ the number ***N* ≈ 44 000**).

The best now-existing detectors (BATSE = Burst And Transient Source Experiment) detected 2704 GRBs during their first 9 years (between 1991 and 1999) [16].

So, during the 27 years up to now the BATSE detectors should have discovered **8112** GRBs.



→ The **numbers** in the model and in the observations are not mutually exclusive.

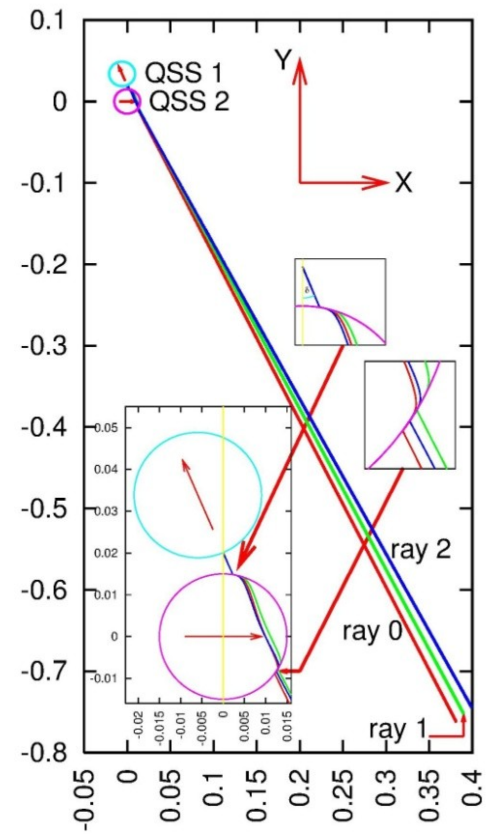
[16] Gamma-Ray Bursts, http://swift.sonoma.edu/about_swift/grbs.html

The Szekeres model can explain short durations of the GRBs via the ***cosmological drift*** [17], [18].

If a non-axial ray propagates above another Szekeres hump between the source and the observer, it is ***deflected***,

and the angle of deflection ***changes with time*** [18].

→*The maximally blueshifted ray* changes direction, so after a while it *will miss the observer* [14].



The rate of change of the ray direction measured by the observer is **4.85×10^{-9} deg/y**.

For comparison, the drift calculated for another cosmological configuration is **2.78×10^{-10} deg/y** [18].

The duration of the afterglows is a problem still to be solved.

[14] A. Krasiński, Modeling the durations of the gamma-ray bursts in a quasi-spherical Szekeres metric. In preparation.

[17] C. Quercellini, L. Amendola, A. Balbi, P. Cabella, M. Quartin, Real-time cosmology. *Phys. Rep.* **521**, 95 -- 134 (2012).

[18] A. Krasiński and K. Bolejko, Redshift propagation equations in the β‘ ≠ 0 Szekeres models. *Phys. Rev*. **D83**, 083503 (2011).

9. Expression of hope

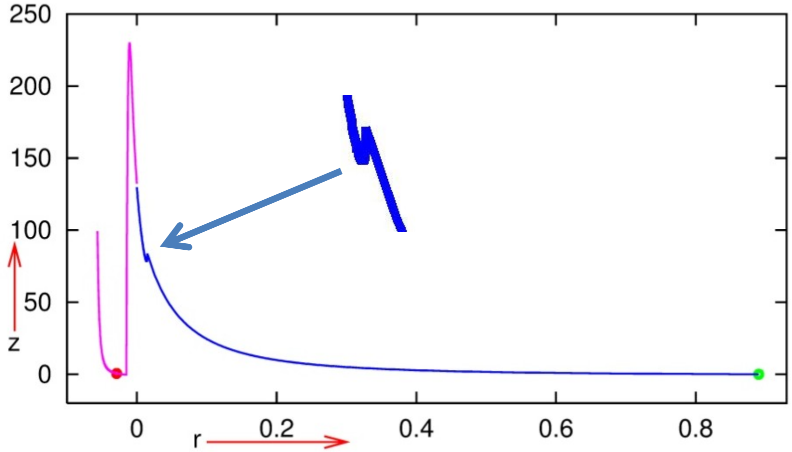
History of science teaches us that if a well-tested theory predicts a phenomenon, then the prediction has to be put to experimental tests.

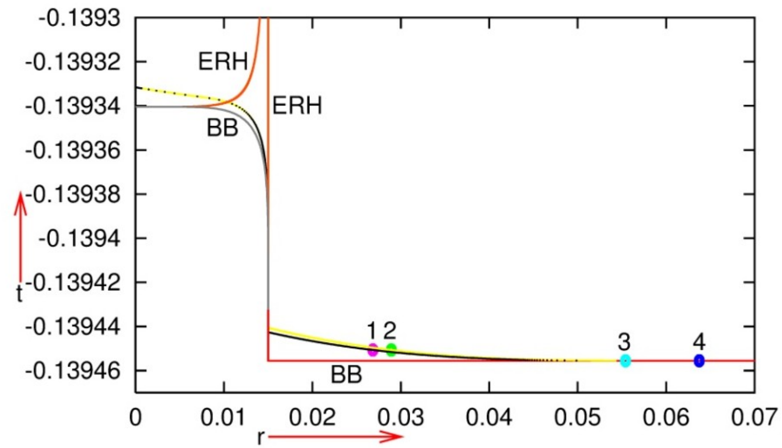
This is what I try to do: verify whether the blueshifts predicted by the L-T and QSS models can be identified among observed effects [5-7].

Perhaps some observers will get convinced to take these attempts seriously (but will it happen during my lifetime?).

[5] A. Krasiński, Cosmological blueshifting may explain the gamma ray bursts. *Phys. Rev.* **D93**, 043525 (2016).
[6] A. Krasiński, Existence of blueshifts in quasi-spherical Szekeres spacetimes. *Phys. Rev.* **D94**, 023515 (2016).
[7] A. Krasiński, Modelling sources of the gamma-ray bursts using quasi-spherical Szekeres metrics. ArXiv 1704.08145, submitted for publication.

10. Appendix: non-monotonicity of redshift along light rays





When local blueshifts are present, ***redshift fails*** to be a distance indicator.

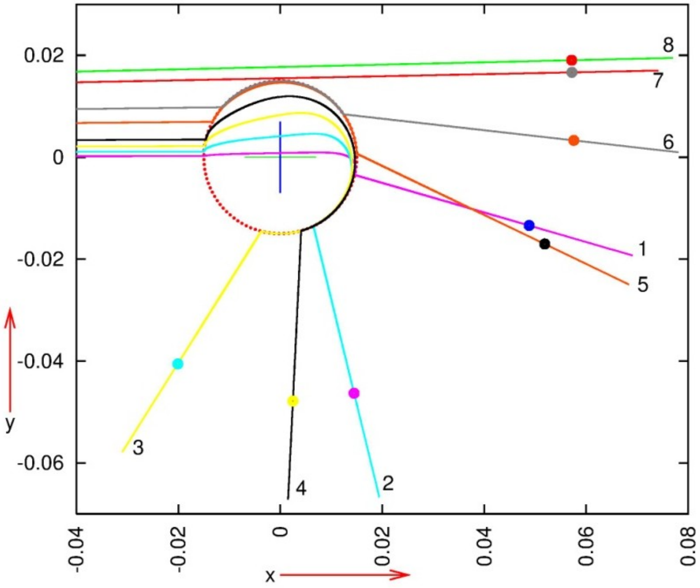
The left graph shows z(r) seen by the observer sitting at r ≈ 0.9 (**green** dot), calculated along the **yellow ray** of the right graph.

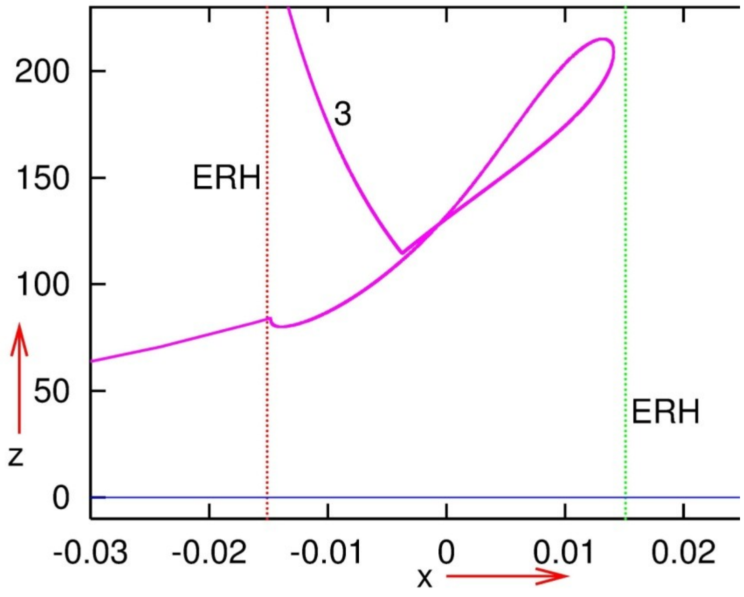
The redshift first increases toward the past, then decreases under the ERH.

At the **red dot** in the left graph z = 0.598. The standard formula [19,20] would imply the source to lie 5.9×10^9 years to the past.

In the L-T model, the source lies 1.37×10^{10} years to the past.

[19] E. L. Wright, A Cosmology Calculator for the World Wide Web. *Publ. Astr. Soc. Pac.* **118**, 1711 (2006).
[20] E. L. Wright, http://www.astro.ucla.edu/~wright/ACC.html





Nonradial rays propagating above the hump toward the same observer (left graph)

Along them, too, z is not monotonic (**right graph is for ray 3**).

The present observer sees all these rays within a 2^o cone around the central ray (the uncertainty in determining the direction to a GRB source is 1^o).

This cone can be made still narrower when the model is improved.

The presence of these rays makes the model falsifiable against observations.