VACUUM GENERAL RELATIVITY WITH A POSITIVE COSMOLOGICAL CONSTANT Λ AS A GAUGE THEORY

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Summary

We show that the vacuum general relativity action (and Lagrangian) in recent Einstein-Palatini formulation is equivalent to the action (and Lagrangian) of a gauge field.
Firstly, we present the Einstein-Palatini action (with $\Lambda \neq 0$) and derive Einstein field equations from it. Then we consider the Einstein-Palatini action integral in vacuum with a positive cosmological constant Λ in terms of the corrected curvature Ω_{cor}. We will see that in terms of Ω_{cor} this action takes the form typical for a gauge field.
Finally, we give a geometrical interpretation of the curvature Ω_{cor}

1. Einstein – Palatini action for general relativity

The geometric part of the Einstein – Palatini action with cosmological constant Λ in new formulation reads

$$S_{E\text{-}P} = \frac{1}{4\varkappa}\int_{D}\left(\vartheta^i \wedge \vartheta^j \wedge \Omega^{kl} - \frac{\Lambda}{6}\vartheta^i \wedge \vartheta^j \wedge \vartheta^k \wedge \vartheta^l\right)\eta_{ijkl} \qquad (1)$$

where Ω is the curvature of the spin connection ω and $\varkappa = \frac{8\pi G}{c^4}$.

All indices take values $(0,1,2,3)$ and D means an established 4-dimensional compact domain in spacetime. ϑ^a denote 1-forms of the Lorentzian coreper in terms of which the spacetime looks locally Minkowskian, i.e., $g = \eta_{ik}\,\vartheta^i \otimes \vartheta^k$; $\eta_{ik} = \mathrm{diag}(1,-1,-1,-1)$.

η_{ijkl} is completely antysymmetric Levi-Civita pseudotensor: $\eta_{0123} = \sqrt{|g|}$, where $g := \det(g_{ik})$.

In a Lorentzian coreper $|g| = 1$.

Spin connection ω is a general metric connection (or Levi-Civita connection) in Lorentzian coreper ϑ^a.

For the geometrical units $G = c = 1$ the formula (1) takes the form

$$S_{EP} = \frac{1}{32\pi}\int_{D}\left(\eta_{ijkl}\,\vartheta^i \wedge \vartheta^j \wedge \Omega^{kl} - \frac{\Lambda}{6}\eta_{ijkl}\,\vartheta^i \wedge \vartheta^j \wedge \vartheta^k \wedge \vartheta^l\right). \qquad (2)$$

Adding to the geometric part S_{EP} the matter action $S_m = \int_D L_{mat}(\phi^A, D\phi^A, \vartheta^i)$, (3)

where L_{mat} is 4-form of the matter Lagrangian ϕ^A means tensor-valued matter form and $D\phi^A$ its absolute exterior derivative,

we obtain full action

$$S = S_{EP} + S_m = \frac{1}{32\pi}\int_D \eta_{ijkl}\left(\vartheta^i\wedge\vartheta^j\wedge\Omega^{kl} - \frac{\Lambda}{6}\vartheta^i\wedge\vartheta^j\wedge\vartheta^k\wedge\vartheta^l\right) + \int_D L_{mat}(\phi^A, D\phi^A, \vartheta^i). \quad (4)$$

After some calculations one gets that the variation $\delta S = \delta S_{EP} + \delta S_m$ with respect ϑ^i, $\omega^i{}_j$, and ϕ^A reads

$$\delta S = \int_D \Big[\frac{1}{8\pi}\delta\vartheta^i\wedge\left(\frac{1}{2}\Omega^{kl}\wedge\eta_{kli} - \Lambda\eta_i + 8\pi t_i\right) + \frac{1}{2}\delta\omega^i{}_j\wedge\left(\frac{1}{8\pi}D\eta_i{}^j + s_i{}^j\right) + \delta\phi^A\wedge L_A + \underbrace{\text{an exact form}}_{\text{total exact form}}\Big]. \quad (5)$$

The tree-forms: energy-momentum, t_i; classical spin, $s_i{}^j$, and L_A are defined by the following form of the variation δL_m

(6). $\delta L_m = \delta\vartheta^i\wedge t_i + \frac{1}{2}\delta\omega^i{}_j\wedge s_i{}^j + \delta\phi^A\wedge L_A +$ an exact form,

η_{kli}, $\eta_i{}^j$, η_i mean the forms introduced in past by A. Trautman.

The variations δV^i, $\delta\omega^i{}_j$ and $\delta\phi^A$ are vanishing on the boundary ∂D of the compact domain D.

Hamiltonian Principle

$$\delta S = 0 \tag{7}$$

leads us to the following sets of the field equations

$$\frac{1}{2}\Omega^{kl}\wedge\eta_{kli} - \Lambda\eta_i = (-)\,8\pi t_i \tag{8}$$

$$\mathcal{D}\eta_i{}^j = (-)\,8\pi S_i{}^j \tag{9}$$

$$L^A = 0. \tag{10}$$

$L_A = 0$ represent equations of motion for matter field. These equations are not intrinsic in further considerations, so we will omit them. We are interested only in the gravitational field equations which are given by the equations (8)–(9).

In vacuum where $t_i = S_i{}^j = 0$ also $\mathcal{D}\eta_i{}^j = 0$ and we get standard vacuum Einstein's equations with cosmological constant Λ

$$\frac{1}{2}\Omega^{kl}\wedge\eta_{kli}-\Lambda\eta_i=0 \qquad (11)$$

and pseudoriemannian geometry.

In general, we have the Einstein-Cartan equations and Riemann-Cartan geometry (= a metric geometry with torsion).

The standard **GR** we obtain also if

$$\frac{\delta L_m}{\delta\omega^i{}_k}=0 \Longrightarrow S_i{}^k=0 \quad \text{and} \quad \underline{D\eta_i{}^k=0},$$

i.e., <u>if we confine to vanishing dynamical spin</u> $S_i{}^k$.

Namely, one has in the case the following gravitational equations

$$\frac{1}{2}\Omega^{kl}\wedge\eta_{kli}-\Lambda\eta_i=(-)\,8\pi\, t_i\,. \qquad (12)$$

One can show that $\frac{1}{2}\Omega^{kl}\wedge\eta_{kli}=(-)G_i{}^s\eta_s$ where the Einstein tensor $G_i{}^s$ is defined as follows

$$G_i{}^s:=R_i{}^s-\frac{1}{2}\delta_i^s R\,. \qquad (13)$$

Putting $t_i=T_i{}^s\eta_s$, we get from (12)

$$-G_i{}^s\eta_s-\Lambda\delta_i^s\eta_s=-8\pi\, T_i{}^s\eta_s\,, \qquad (14)$$

or

$$\underline{G_i{}^s+\Lambda\delta_i^s=8\pi T_i{}^s}\,. \qquad (15)$$

(15) are standard Einstein equations with cosmological constant Λ in tensorial notation with symmetric matter tensor: $T^{ik} = T^{ki}$. This tensor is obtained from a canonical one by the Belintante-Rosenfeld procedure.

2. Einstein-Palatini action for GR in vacuum and with positive cosmological constant Λ as action for a gauge field.

Now, getting back to Einstein-Palatini action in vacuum

$$S_{EP} = \frac{1}{4\varkappa} \int_{D} \left(\vartheta^i \wedge \vartheta^j \wedge \Omega^{kl} - \frac{\Lambda}{6} \vartheta^i \wedge \vartheta^j \wedge \vartheta^k \wedge \vartheta^l \right) \eta_{ijkl}$$

$$= \frac{1}{4\varkappa} \int_{D} \left(\vartheta^i \wedge \vartheta^j \wedge \Omega^{kl} \eta_{ijkl} - \frac{\Lambda}{6} \vartheta^i \wedge \vartheta^j \wedge \vartheta^k \wedge \vartheta^l \eta_{ijkl} \right) \quad (16)$$

and defining the duality operator $*$

$$* := (-) \frac{\eta_{ijkl}}{2} \Longrightarrow \eta_{ijkl} = (-) 2 *, \quad (17)$$

one gets

$$\eta_{ijkl} \Omega^{kl} = -2 * \Omega_{ij} \quad (18)$$

$$\eta_{ijkl} \vartheta^k \wedge \vartheta^l = -2 * (\vartheta_i \wedge \vartheta_j). \quad (19)$$

Thus, the Einstein-Palatini action has the following form

$$S_{EP} = (-)\frac{1}{2\varkappa}\int_{\mathcal{D}}\left(\vartheta^i\wedge\vartheta^j\wedge *\Omega_{ij} - \frac{\Lambda}{6}\vartheta^i\wedge\vartheta^j\wedge *(\vartheta_i\wedge\vartheta_j)\right)$$

$$= (-)\frac{1}{2\varkappa}\int_{\mathcal{D}} tr\left(\vartheta\wedge\vartheta\wedge *\Omega - \frac{\Lambda}{6}\vartheta\wedge\vartheta\wedge *(\vartheta\wedge\vartheta)\right). \quad (20)$$

Let us introduce the corrected curvature

$$(21). \quad \Omega_{cor} := \Omega - \frac{\Lambda}{3}\vartheta\wedge\vartheta \rightarrow \vartheta\wedge\vartheta = \frac{3}{\Lambda}(\Omega - \Omega_{cor})$$

Substituting the last formula into Einstein-Palatini action we get

$$S_{EP} = (-)\frac{1}{2\varkappa}\int_{\mathcal{D}} tr\left(\vartheta\wedge\vartheta\wedge *\Omega - \frac{\Lambda}{6}\vartheta\wedge\vartheta\wedge *(\vartheta\wedge\vartheta)\right)$$

$$= (-)\frac{1}{2\varkappa}\int_{\mathcal{D}} tr\left[\frac{3}{\Lambda}(\Omega - \Omega_{cor})\wedge *\Omega - \frac{\Lambda}{6}\frac{9}{\Lambda^2}(\Omega - \Omega_{cor})\wedge *(\Omega - \Omega_{cor})\right]$$

$$= (-)\frac{3}{4\Lambda\varkappa}\int_{\mathcal{D}} tr\left[\Omega\wedge *\Omega - \Omega_{cor}\wedge *\Omega + \Omega\wedge *\Omega_{cor} - \Omega_{cor}\wedge *\Omega_{cor}\right]. \quad (22)$$

Because $(-)\Omega_{cor}\wedge{*}\Omega+\Omega\wedge{*}\Omega_{cor}$ reduces, then we finally have

$$S_{EP}=(-)\frac{3}{4\Lambda\varkappa}\int_D tr\left[\Omega\wedge{*}\Omega-\Omega_{cor}\wedge{*}\Omega_{cor}\right]. \quad (23)$$

The expression $tr\left(\Omega\wedge{*}\Omega\right)=\eta_{ijkl}\Omega^{ij}\wedge\Omega^{kl}$ is in four dimension a topological invariant called Euler's form, which does not influence the equations of motion.

Hence, the $\overset{\text{vacuum}}{\vee}$ Einstein-Palatini action is equivalent to

$$S_{EP}=\frac{3}{4\Lambda\varkappa}\int_D tr\left(\Omega_{cor}\wedge{*}\Omega_{cor}\right). \quad (24)$$

We see that the Einstein-Palatini action is efectively the functional which is quadratic function of the corrected Riemannian curvature Ω_{cor}, i.e., it has a form of the action for a gauge field.

Only one difference is that in (24) we have the star operator, $*$, which is different from Hodge star operator. Namely, our star operator acts onto interior (= tetrads) indices, not onto forms as Hodge duality operator does.

3. Geometrical interpretation of the corrected curvature Ω_{cor}

Let $P(M_4; GdS)$ denotes the principial bundle of the de Sitter basis over a manifold M_4 (= spacetime) with de Sitter group, GdS, as a structure group.

The GdS is isomorphic to the group $SO(4;1)$.

Let $\tilde{\omega}$ be 1-form of connection in the principle fibre bundle $P(M_4; GdS)$.

The form $\tilde{\omega}$ has values in the algebra $\mathfrak{g}$ of the group GdS. This algebra splits (as a vector space) into direct sum

$$\mathfrak{g} = so(3;1) \oplus R^{(3,1)}. \tag{25}$$

$so(3;1) =: \mathbf{h}$ denotes here algebra of the group $SO(3;1)$, which is isomorphic to Lorentz group $\mathcal{L}$, and $R^{(3;1)} =: \mathbf{p}$ is a 4-dimensional vector space of generalized translations in the <u>curved de Sitter spacetime</u>.

One can identify this spacetime with the quotient

$$\underbrace{\frac{SO(4;1)}{SO(3;1)} = \text{de Sitter with } \Lambda > 0}. \tag{26}$$

or with the group space of the group of generalized translations which acts transitively on the de Sitter space.

One has

$$\mathfrak{g} = h \oplus p \,, \qquad (25')$$

and

$$[h, h] \subset h \,, \quad [h, p] \subset p \,, \quad [p, p] \subset h \,. \qquad (27)$$

This means that the Lie algebra $\mathfrak{g}$ is a symmetric Lie algebra.

On the other hand, the spaces which satisfy (27) are called globally symmetric Riemannian spaces.

Let $\mathbf{P}(M_4; L)$ denotes the principial bundle of Lorentz basis over the manifold M_4 (= spacetime).

There exists a morphism of principal bundles

$$f : P(M_4; L) \longrightarrow P(M_4; GdS) \,. \qquad (28)$$

This morphism creates pull-back $f_* \tilde{\omega}$ of the form $\tilde{\omega}$ onto the bundle $\mathbf{P}(M_4; L)$.

Let us denote this pull-back by $\mathbf{A}$.

$\mathbf{A}$ is a 1-form on $\mathbf{P}(M_4; L)$ with values in the direct sum

$$\underbrace{so(3;1) \oplus R^{(3,1)}}\; \equiv h \oplus p \; (= \mathfrak{g}) \,. \qquad (29)$$

Hence, we have a natural decomposition

$$A = f_* \tilde{\omega} = \omega + \Theta \,, \qquad (30)$$

where ω is a 1-form on $P(M_4;L)$ with values in the algebra $so(3;1) \equiv \mathbf{h}$ and Θ is a 1-form on $P(M_4;L)$ with values on $R^{(3;1)} \equiv \mathbf{p}$.

ω is a connection (with values in $\mathbf{h}$) on the bundle $P(M_4;L)$ and Θ can be identified with Lorentzian coreper ϑ om the spacetime $(M_4;g)$.

Let us compute a 2-form curvature $\tilde{\Omega}$ of the pulled back A.

From definition one has (we have already put $\Theta = \vartheta$)

$$\tilde{\Omega} = dA + \frac{1}{2}[A,A] = d(\omega+\vartheta) + \frac{1}{2}[\omega+\vartheta, \omega+\vartheta]$$

$$= d\omega + \frac{1}{2}[\omega,\omega] + d\vartheta + \frac{1}{2}[\omega,\vartheta] + \frac{1}{2}[\vartheta,\omega] + \frac{1}{2}[\vartheta,\vartheta].$$

Now we introduce to our equations bases **(31)** $\tilde{M}_{ik} = (-1)\tilde{M}_{ki}$ of the algebra $so(3;1) \equiv \mathbf{h}$ and $\{e_i\}$ of the vector space $R^{(3;1)} \equiv \mathbf{p}$. In these bases, we have

$$\omega = \omega^i{}_{\cdot k}\tilde{M}^k{}_{\cdot i} = \omega^{ik}\tilde{M}_{ki}, \quad \vartheta = \vartheta^i e_i \qquad (32)$$

$(\tilde{M}_{ik}, e_i)$ form together the basis of the algebra $\mathfrak{g} = \mathbf{h} \oplus \mathbf{p} = so(4;1)$.

In terms of the elements $(\tilde{M}_{ik}, e_i)$ the comutational relations for the algebra $so(4;1) = \mathfrak{g}$ read

$$[\tilde{M}_{ij}, \tilde{M}_{kl}] = \frac{1}{2}\left(\eta_{ik}\tilde{M}_{jl} + \eta_{jl}\tilde{M}_{ik} - \eta_{il}\tilde{M}_{jk} - \eta_{jk}\tilde{M}_{il}\right)$$

$$[e_i, \tilde{M}_{jk}] = \frac{1}{2}\left(\eta_{ik}e_j - \eta_{ij}e_k\right)$$

$$[e_i, e_j] = \frac{2\tilde{M}_{ij}}{R^2}. \tag{33}$$

R is the radius of the de Sitter spacetime. Its connection with $\Lambda > 0$ reads: $\Lambda = \frac{3}{R^2}$.

The following commutation relations are important in the further our considerations

$$[\tilde{M}_{ki}, e_l] = \frac{1}{2}\left(\eta_{kl}e_i - \eta_{il}e_k\right) \tag{34}$$

$$[e_i, e_k] = \frac{2\tilde{M}_{ik}}{R^2}. \tag{35}$$

Namely, by using (34)-(35) one gets from (31)

$$\tilde{\Omega} = \Omega_\omega + \frac{1}{2}\omega^i{}_{\cdot k}\wedge\vartheta^l[\tilde{M}^k{}_{\cdot i}, e_l] + \frac{1}{2}\vartheta^l\wedge\omega^i{}_{\cdot k}[e_l, \tilde{M}^k{}_{\cdot i}]$$

$$+ d\vartheta^i e_i + \frac{1}{2}\vartheta^i\wedge\vartheta^k[e_i, e_k] = \underline{\Omega_\omega} + \omega^i{}_{\cdot k}\wedge\vartheta^l[\tilde{M}^k{}_{\cdot i}, e_l]$$

$$+ \underbrace{\vartheta^i\wedge\vartheta^k\frac{\tilde{M}_{ik}}{R^2}}_{} + d\vartheta^i e_i = \underbrace{\Omega_\omega^{ik}\tilde{M}_{ki} - \frac{\vartheta^i\wedge\vartheta^k\tilde{M}_{ki}}{R^2}}_{}$$

$$+ \left(d\vartheta^i + \omega^i{}_{\cdot k}\wedge\vartheta^k\right)e_i = \Omega_{cor}^{ik}\tilde{M}_{ki} + (D_\omega\vartheta^i)e_i =$$

$$= \Omega_{cor}^{ik}\tilde{M}_{ki} + \Theta^i e_i, \tag{36}$$

or after leaving basises

$$\tilde{\Omega} = \Omega_{cor} + \Theta \, . \qquad (37)$$

$$\Omega_{cor} := \Omega_{\omega} - \frac{\vartheta \wedge \vartheta}{R^2} = \Omega_{\omega} - \frac{\Lambda}{3}\vartheta \wedge \vartheta \qquad (38)$$

and it denotes the corrected curvature of the connection ω on the bundle $P(M_4; L)$ and $\Theta = D_{\omega}\vartheta$ is a torsion of the connection ω. $\Lambda = \frac{3}{R^2}$.

Here $\Omega_{\omega} = d\omega + \frac{1}{2}[\omega, \omega]$ is the curvature 2-form of the connection ω, and $\Lambda > 0$ means the cosmological constant.

Now we adjust the connection $\tilde{\omega}$ in such a way that the connection ω is torsionless ($\Theta = 0$).

Then, the connection ω is Levi-Civita connection, and we get (after leaving the basis of $so(3;1) = \mathfrak{h}$)

$$\tilde{\Omega} = \Omega_{cor} = \Omega_{\omega} - \frac{\Lambda}{3}\vartheta \wedge \vartheta \, . \qquad (39)$$

In Section 2 we gave the definition of the corrected curvature Ω_{cor} as follows

$$\Omega_{cor} = \Omega_{\omega} - \frac{\Lambda}{3}\vartheta \wedge \vartheta \, . \qquad (40)$$

As one can see this curvature is a curvature of the pulled back connection

$$A = f_{*}\tilde{\omega} = \omega + \theta = \omega + \vartheta .$$

if $\Theta_{\omega} = 0$.

4. Conclusion

In our lecture we have shown that in four dimensions the vacuum action integral for GR with a positive cosmological constant Λ can be written in an analogical form to the form of the action integral for a typical gauge field. However, there is one difference – the star. Instead of the Hodge star, we have slightly different star called "the duality operator".

Our result shows that there is no need to generalize GR and construct very complicated gravitational theories to obtain gravitational theory as a gauge theory.

The ordinary vacuum GR with cosmological constant $\Lambda > 0$ formulated in terms of Lorentzian coreper, ϑ, and spin connection, ω, already is a gauge theory.

We would like to emphasize that the formulation of the EP action in the form (24) can be important for quantizing of general relativity because gauge fields are quantized.

Remarks: ① We have used only the standard theory of connection which was created by Ehresmann. His approach is commonly used in differential geometry and in relativity. Some authors came to the similar conclusions as ours by using Cartan's approach to connection. But this approach is unknown for majority of geometrists (except Sharp) and relativists. 15

(2.) <u>Introducing 5-dimensional notation connected with algebra $so(4;1)=\mathcal{G}$.</u>

$$A^{IJ} = (-) A^{JI} = \begin{cases} A^{ab} = \omega^{ab} \\ A^{a4} = \vartheta^{a} \\ A^{4a} = (-)\vartheta^{a} \\ A^{44} = 0 \end{cases} \qquad \begin{matrix} (I,J = 0,1,2,3,4) \\ (a,b = 0,1,2,3) \end{matrix}$$

$$\tilde{\Omega}^{AB} = \begin{cases} \Omega_{cor}^{ab} \\ \tilde{\Omega}^{A4} = (-)\tilde{\Omega}^{4A} = 0 \end{cases} \qquad (A,B,C,D = 0,1,2,3,4)$$

one can $\underbrace{}_{(formally)}$ write the geometric part (owing the fact: $\tilde{\Omega}^{A4} = (-)\tilde{\Omega}^{4A} = 0$)

$$S_{EP} = \frac{3}{4\Lambda\varkappa}\int_{\mathcal{D}} \eta_{ABCD}\,\tilde{\Omega}^{AB}\wedge\tilde{\Omega}^{CD} = \int_{\mathcal{D}} \tilde{\Omega}^{AB}\wedge *\tilde{\Omega}_{AB}$$

$$\left(\overset{\text{effectively}}{=} \frac{3}{4\Lambda\varkappa}\int_{\mathcal{D}} \Omega_{cor}^{ab}\wedge *\Omega_{cor\,ab}\right)$$

This is $SO(3;1)$ invariant procedure only

From that one has

$$\delta S_{EP} = \boxed{\frac{3}{4\Lambda\varkappa}\int_{\mathcal{D}} \frac{\delta(\eta_{ABCD}\,\tilde{\Omega}^{AB}\wedge\tilde{\Omega}^{CD})}{\delta A^{IJ}}\,\delta A^{IJ}} =$$

$$= \frac{3}{4\Lambda\varkappa}\int_{\mathcal{D}} \left[\frac{\delta(\eta_{ABCD}\,\tilde{\Omega}^{AB}\wedge\tilde{\Omega}^{CD})}{\delta A^{ab}}\,\delta A^{ab} + \right.$$

$$\left. + \frac{\delta(\eta_{ABCD}\,\tilde{\Omega}^{AB}\wedge\tilde{\Omega}^{CD})}{\delta A^{a4}}\,\delta A^{a4} \right] =$$

$$= \frac{3}{4\Lambda\varkappa} \int_{\mathcal{D}} \left[\frac{\delta(\eta_{abcd}\, \Omega_{cor}^{\ ab} \wedge \Omega_{cor}^{\ cd})}{\delta\omega^{ef}} \delta\omega^{ef} \right.$$

$$\left. + \frac{\delta(\eta_{abcd}\, \Omega_{cor}^{\ ab} \wedge \Omega_{cor}^{\ cd})}{\delta\vartheta^{e}} \delta\vartheta^{e} \right].$$

Finally,

Hamiltonian Principle

$\delta S_{EP} = 0 \rightarrow$ Equations of the gravitational field in vacuum

results in the vacuum Einstein equations with cosmological constant

$$\frac{1}{2}\Omega^{kl} \wedge \eta_{kli} - \Lambda \eta_i = 0.$$

So, following this way, one can obtain formally vacuum Einstein equations as a gauge field equations for a gauge field having potentials $A^{IJ} = -A^{JI}$ and strengths $\tilde{\Omega}^{AB} = -\tilde{\Omega}^{BA}$.

$(A^{44} = 0)$ $\qquad (\tilde{\Omega}^{A4} = -\tilde{\Omega}^{4A} = 0)$.

3. The case $\Lambda < 0$ will be considered in near future. (This case has probably small physical meaning.)

4. SO(4;1) invariant action

$$S_{EP} = \frac{3}{4\Lambda \varkappa} \int_{\mathcal{D}} \Big[V^I \eta_{IABCD} \tilde{\Omega}^{AB} \wedge \tilde{\Omega}^{CD}$$

$$+ \sigma (V_I V^I + 1) \Big], \qquad (*)$$

where V^I is an auxilliary vector field: $V^I V_I = -1$.
[We use signature $(+----)$ in 5-dimensional pseudoeuclidean spacetime $E^{4;1}$ in which the SO(4;1) group acts].
σ is an arbitrary 4-form serving as a Lagrange multiplier.

In the gauge $V^a = 0$, $V^4 = (+)1$ one has for $V^I V_I = -1$:

$$S_{EP} \overset{(*)}{=} \frac{3}{4\Lambda \varkappa} \int_{\mathcal{D}} \eta_{abcd} \, \Omega_{cor}^{ab} \wedge \Omega_{cor}^{cd}$$

$$= \frac{3}{4\Lambda \varkappa} \int_{\mathcal{D}} \mathrm{tr} \left(\Omega_{cor} \wedge {}^* \Omega_{cor} \right),$$

i.e., one has Einstein-Palatini action (24).

(*) leads to de Sitter gauge theory of gravity.

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